- In the last lecture we introduced pre-conditions and postconditions

- We also identified rules involving pre-consitions and postconditions

- Rules:
  - Precondition Strengthening
  - Postcondition Weakening

- In this lecture we will:

  1. Formalise these ideas on precondition strengthening and postcondition weakening

  2. Consider correctness proofs in loopless code

- When is one assertion stronger or weaker than another

- Definition: If R and S are two assertions, then R is said to be stronger than S if $R \Longrightarrow S$. If R is stronger than S, then S is weaker than R

- Stronger means more selective or more specific

- Weaker means more frequent or more general

- Give some examples

- Precondition Strengthening

  - Suppose that $\{P\}C\{Q\}$ is correct and $P_1 \implies P$ has been proven. We can conclude that $\{P_1\}C\{Q\}$ is correct. This results in the following rule:

$$\frac{\begin{array}{c} P_1 \implies P \\ \{P\}C\{Q\} \end{array}}{\{P_1\}C\{Q\}}$$

- Postcondition Weakening

  - Suppose that $\{P\}C\{Q\}$ is correct and $Q \implies Q_1$ has been proven. We can conclude that $\{P\}C\{Q_1\}$ is correct. This results in the following rule:

$$\frac{\{P\}C\{Q\} \quad Q \implies Q_1}{\{P_1\}C\{Q\}}$$

- Conjunction Rule
  - If C is a piece of code and $\{P_1\}C\{Q_1\}$ and $\{P_2\}C\{Q_2\}$ then one can conclude $\{P_1 \wedge P_2\}C\{Q_1 \wedge Q_2\}$
  - $$\frac{\{P_1\}C\{Q_1\} \quad \{P_2\}C\{Q_2\}}{\{P_1 \wedge P_2\}C\{Q_1 \wedge Q_2\}}$$
- Examples applying the conjunction rule given in lecture

- Disjunction Rule

  - If C is a piece of code and $\{P_1\}C\{Q_1\}$ and $\{P_2\}C\{Q_2\}$ then one can conclude $\{P_1 \vee P_2\}C\{Q_1 \vee Q_2\}$

  - $$\frac{\{P_1\}C\{Q_1\} \quad \{P_2\}C\{Q_2\}}{\{P_1 \vee P_2\}C\{Q_1 \vee Q_2\}}$$

- Examples applying the disjunction rule given in lecture

- Correctness of Assignment statements
  - Assignment statements are of the form $V = E$
  - Must consider the values of variables at different stages
  - E_0 might be the value of the expression E using the initial value of the variables involved.
  - Examples given in lecture

- Assignment Rule: Let E be an expression and C be a program variable. If C is a statement of the form $V = E$ with postcondition $\{Q\}$, then the precondition of C can be found by replacing all instances of V in Q by E. This expression can be written as Q_E^V.
  $\{Q_E^V\}V = E\{Q\}$

- Examples given in lecture

- Concatenation Rule:

$$\frac{\{P\}C_1\{R\} \quad \{R\}C_2\{Q\}}{\{P\}C_1; C_2\{Q\}}$$

- Examples given in lecture

- Modified Concatenation Rule:

$$\frac{\begin{array}{c}\{P\}C_1\{R\} \\ \{S\}C_2\{Q\} \\ R \implies S\end{array}}{\{P\}C_1; C_2\{Q\}}$$

- Examples given in lecture