UNIVERSITY of LIMERICK

OLLSCOIL LUIMNIGH

FACULTY of SCIENCE and ENGINEERING

Department of Computer Science
and Information Systems

Final Assessment Paper

Academic Year:	2008/2009	Semester:	Spring
Module Title:	Data Structures and Algorithms	Module Code:	CS4115
Duration of Exam:	$2\frac{1}{2}$ hours	Percent of Semester Marks:	60
Lecturer:	P. Healy	Paper marked out of:	100

Instructions to Candidates:

- There are three sections to the paper: Multiple Choice Questions, Short Questions and Long Questions
- The mark distribution is 40 marks for Multiple Choice Questions, 20 marks for Short Questions and 40 marks for the Long Questions
- Answer all questions in all sections

Section 1. Multiple Choice Answers (40 marks).

Use the machine-readable multiple-choice question grid that has been provided to answer these questions. Please completely mark in black exactly one circle on the grid for each answer. A penalty will be charged for wrong answers. Mark the **X** bubble for those questions you wish to skip.

1. In an AVL tree of height h, the number of probes made in an unsuccessful search will be
   (a) $O(1)$
   (b) $O(\log n)$
   (c) $O(n)$
   (d) Depends on the way the AVL was first created

2. Given $f(n) = n$ and $g(n) = 2n$, which one of the following statements is *false*.
   (a) $f(n) = O(g(n))$
   (b) $f(n) = \Omega(f(n))$
   (c) $f(n) = \Theta(f(n))$
   (d) $f(n) = o(g(n))$

3. How many multiplications are used in the calculation of X^{2^n}, using the "fast" exponentiation algorithm?
   (a) $O(\log n)$
   (b) $O(n)$
   (c) $O(2^n)$
   (d) $O(n!)$

4. What is the vertex-connectivity of the graph drawn in Figure 1(a)?
   (a) 2
   (b) 3
   (c) 4
   (d) 5

5. How many cut vertices does the graph drawn in Figure 1(b) have?
   (a) 1
   (b) 3
   (c) 4
   (d) 5

6. On input of size N, the running time of programs A is always less than $212 \times N$. On input of size N, the running time of program B is always less than $212 \times log(2^N)$. Is the average running time of program A greater than the average running time of program B, for $N = 100,000$.
   (a) Yes, the average running time of program A is greater than the average running time of program B
   (b) No, the average running time of program A equals the average running time of B, for $N = 100,000$
   (c) No, the average running time of program B is greater than the average running time of program A, for $N = 100,000$
   (d) There is not enough information to tell

7. Consider what happens after 6 is inserted into the AVL tree drawn in Figure 2. The tree becomes unbalanced and needs to be re-balanced using a single-rotation. Following this single-rotation what node(s), if any, are the children of 6?
   (a) node 7 and node 8
   (b) just node 7
   (c) just node 8
   (d) node 6 will have no children

8. Consider an arbitrary binary tree B where every node is either full or is a leaf. Let F represent the number of full nodes in B and let L represent the number of leaves in B. (Recall a full node is a node with two children, and a leaf is a node with no children). Which of the following statements is true, no matter what the choice of B?
   (a) $F = L + 1$
   (b) $L = F + 1$
   (c) $L = 2F$
   (d) $F = 2L$

9. For which of the following sorting algorithms, is it important that the input is random (rather than presorted or in reverse order for example)?
   (a) *quicksort*, where the first element is chosen as the pivot
   (b) *quicksort*, where the pivot is chosen using median-of-three partitioning
   (c) *mergesort*
   (d) *insertion sort*

10. In an AVL tree of height h, the number of probes made in a *successful* search will be (most accurately)
    (a) $O(1)$
    (b) $O(\log n)$
    (c) $O(n)$
    (d) Depends on the way the AVL was created

Section 2. Short Questions (5×4 marks).

- Please put your answers to these questions in the answer book provided to you, labelling your answers 2.1, 2.2, etc.

1. When solving the *weighted shortest path* a priority queue is used but uses a non-standard operation. What is this non-standard operation? ____________.
2. In O-notation, what is the analogue of a linear-time algorithm if the input is a graph $G = (V, E)$? __________.
3. The height of an AVL tree is no worse than _______ times the optimal height.
4. Given a graph, $G = (V, E)$, what is the largest number of edges exactly a graph can have in terms of $|V|$, the number of nodes? ________.
5. Ordinarily the most appropriate way to represent a graph internally is with __________; however, if many queries are of the form "Is node u adjacant to node v?" then the most appropriate representation may be ________________

Section 3. Long Questions (40 marks).

- Please put your answers to these questions in the answer book provided to you
- Label your answers 3.1, 3.2, and 3.3 in your answer books

1. Use induction to show that $n! > 2^n$, for $n > 3$. **(10 marks.)**

2. What does the function `what()` below return? Give a clear explanation referring to a worked example. Hint: recall the tutorial problem of decomposing an integer into its binary representation. **(10 marks.)**

```
int what(int a, int b)
{
  int p = 0;
  while (a != 0) {
    if (a%2 == 1) p += b;
    a = a / 2;
    b = b * 2;
  }
  return p;
}
```

3. **(20 marks.)**

   Given a graph $G = (V, E)$, where $n = |V|$ and $m = |E|$

   (a) Show that every spanning tree of G has $n - 1$ edges. (8 marks.)

   (b) Use the previous result to show that there are $m - (n - 1)$ different cycles in G. (6 marks.)

   (c) A graph $G = (V, E)$ is called d-regular if every vertex has degree d. Figure 1 below illustrates a 4-regular graph. Does there exist a 3-regular graph G on 5 vertices? Either give an example or prove that one cannot exist. (6 marks.)

(a) A 4-regular graph. (b) A graph with cutvertices.

Figure 1: Some example graphs.

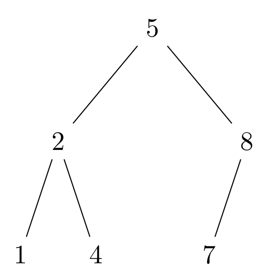

Figure 2: An AVL Tree.