UNIVERSITY OF LIMERICK

COLLEGE OF INFORMATICS AND ELECTRONICS

Department of Computer Science
and Information Systems

Assessment Paper

Semester 1
Module Code: CS4111
Duration of Exam: 2 $\frac{1}{2}$ Hours
Lecturer: C. Ryan
Calculators **are** allowed.

Academic Year: 2007/08
Module Title: Computer Science
% of Total Marks: 50%

Instructions to Candidates:
- Answer any **four** questions
- All questions carry equal marks

1. (a) Explain, with examples, each of the following notations : **prefix, postfix, superfix, infix.** (4 marks)

   (b) What are the standard rules of precedence for arithmetic operators? Describe the relative precedences of the standard arithmetic operators, i.e. $+ - / * ()$. Evaluate the following expression using these rules. *Show* **all** *your work.*

   $$3 + (2 + (3 + 1 * 4 * 2) * (2 * 3 + 1))$$

   (5 marks)

   (c) Convert the following expression into prefix, and then draw an Abstract Syntax Tree for the resulting expression. Evaluate the AST, showing **all** your work.

   $$(3 * 1 + 2) + (4 + 2) * 2$$

   (6 marks)

(d) Convert the following expression into prefix, and then draw an Abstract Syntax Tree for the resulting expression. Explain how your prefix notation handles sub-expressions such as b^2.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(5 marks)

(e) In how many ways can (+ (+ 5 6) (+ 3 1)) be drawn so that it evaluates to the same result? What about (+ (/ 2 2) (/ 4 1))? Do not change the arity of the operators in either case. Which of the following evaluates to 32?

a (+ (- (* 2 (* 2 2)) (* 2 (* 2 2))) (+ (* 3 5) 1))
b (* (- (* 2 (* 2 2)) (* 2 (* 2 2))) (+ (* 3 5) 1))
c (* (- (* 2 (* 2 2)) (* 2 (* 2 2))) (+ (* 3 5) 1))
d (+ (+ (* 2 (* 2 2)) (* 2 (* 2 2))) (+ (* 3 5) 1))

(5 marks)

2. (a) What is the difference between a **parameter** and an **argument**?

(4 marks)

(b) Draw an Abstract Syntax Tree for each of the following expressions:

i. (λ xy. (+ (* x x) (+ y y)))
ii. (λ xyz. + (* x z) (+ y z)) 2 3 4
iii. (λ x. + 2 (λ y. + 5 y) x) 4

(5 marks)

(c) Evaluate, showing all your work, each of the λ **expressions** above. If any of them can't be evaluated, explain why. (5 marks)

(d) Evaluate, showing all your work, the AST you produced for expression iii in part (b). (5 marks)

(e) What is the difference between *lazy* and *eager* evaluation? Show the relative advantages of each. Evaluate (λ y. * y y) (+ 2 3) using each of lazy and eager evaluation. (6 marks)

3. (a) What are **bound** and **free** variables? (2 marks)

(b) Given the following definitions in scheme, identify the free and bound variables in each of the functions.

```
(define x 1)
(define y 2)
(define z 3)
(define add3 (lambda (y) (+ y 3)))
(define addy (lambda (z) (+ z y)))
(define addz (lambda (x) (+ (addy z) x )))
(define messy (lambda (x z) (+ (addy z) (addz x))))
```

(5 marks)

(c) Given the definitions in part (b), what do each of the following calls return?

i. `(add3 3)`
ii. `(add3 x)`
iii. `(addy 4)`
iv. `(addz 5)`
v. `(define x (add3 y))`
vi. `(messy 1 2)`
vii. `(define x (messy z y))`
viii. `(messy (addz y) (addy y))`

(6 marks)

(d) Using the definitions from part (a), write out the values of x, y and z after each of the following is executed.
(*Hint:* Each of x, y and z are global, so if their value changes after one expression, it will keep that value for the start of the next one).

i. `(addz x)`
ii. `(define x (add3 x))`
iii. `(define x (addz x))`
iv. `(define z y)`
v. `(define x (messy x z))`
vi. `(messy x z)`
vii. `(messy (add3 x) (messy (addz z) (addy z)))`

(6 marks)

(e) Identify the free and bound variables in each of the following λ calculus functions. Make sure to show which λ you are talking about:

i. $(\lambda x. + 2 (\lambda y. + 5\ x)\ y)$
ii. $(\lambda xy. + (\lambda y. + x\ 3)\ x\ (\lambda x. + y\ 4))$
iii. $(\lambda xy. + (\lambda z. + x\ z)\ x\ y)$

(6 marks)

4. (a) Explain each of the following terms : **β-conversion, α-conversion, δ conversion, redex.** (3 marks)

   (b) Reduce each of the following expressions as much as possible, showing **all** your work. Where appropriate, use α-conversions before evaluating the expressions.

   i. $(\lambda xy.(\lambda\ y.x\ y\ (\lambda x.\ y\ x\ y))x\ y)A\ B$
   ii. $(\lambda x.(\lambda\ xy.(\lambda\ y.\ x\ y)x)y)y$

   (6 marks)

   (c) Write a λ calculus function for each of the following:

   i. Take three arguments, multiply the first and second together, then divides the answer by the product (multiplication) of the second and third arguments.
   ii. Take three arguments and apply the second to the first and the third.
   iii. Take two arguments and return whichever is larger.

   (9 marks)

   (d) Show, using ASTs, what happens when the following function is executed:
   (λ f. f (+ 2 3)) (λ z. (* z 2)) (7 marks)

5. (a) Given the following function definition:

```
(define f
  (lambda (x y)
     (if (= y 0)
         x
         (+ x (f x (- y 1)))
     )
   )
 )
```

What is the value of

i. (f 2 0)
ii. (f 1 3)
iii. (f 2 2)

(5 marks)

(b) Write a recursive function using lambda calculus that produces the following sequence

n	1	2	3	4	...
f (n)	1	4	7	10	...

(10 marks)

(c) Write a recursive function using lambda calculus that implements the **mod** function, i.e. the remainder function, that operates as follows:

```
(mod 4 2) = 0
(mod 4 3) = 1
(mod 12 5) = 2
```

(10 marks)