Course Notes
for
MS4111
Discrete Mathematics 1

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CHAPTER 1 Review of sets

1.1 Some basic definitions

Definition 1.1 *A set is a collection of objects.*

Example 1.1

$$A = \{1, 2, 3, 4\}$$

describes a set A made up of the four elements $1, 2, 3, 4$.

Remark 1.1 *A set is determined by its elements and NOT BY a particular order in which the elements might be listed.*

Remark 1.2 *A set can be described by*

1. *listing its elements like in example 1.1*

$$A = \{1, 2, 3, 4\}$$

2. *by listing a property necessary for membership like*

$$A = \{x \in \mathbf{R} \mid 1 \leq x \leq 4,\ x \text{ is an integer}\}$$

3. *by the Veen diagram.*

Definition 1.2 *Let A be a set. If an object x belongs to A, we write*

$$x \in A$$

and we say that x belongs to A or that x is an element of A. If an object x does not belong to A, we write

$$x \notin A$$

and we say that x does not belong to A or that x is not an element of A.

Example 1.2 *If we consider the set*

$$A = \{1, 2, 3, 4\}\,,$$

then we can say

$$1 \in A, \quad 2 \in A, \quad 3 \in A, \quad 4 \in A$$

and for example

$$5 \notin A, \quad 10 \notin A.$$

Definition 1.3 *The set with no elements is called the empty set and it is denoted by* $\emptyset$*.*

Note: The empty set is unique!

It is important to notice that for any object x we can consider the set

$$\{x\},$$

which is different from x because $\{x\}$ is the set having x as the only element, where x is an element.

Definition 1.4 *We say that two sets A and B are equal and we write*

$$A = B$$

if and only if A and B have the same elements, i.e. whenever $x \in A$, then $x \in B$ and whenever $x \in B$, then $x \in A$.

Example 1.3 *The two sets*

$$A = \{1, 2, 3, 4\}, \qquad B = \{2, 4, 1, 3\}$$

are equal as they have the same elements and the order in which elements are listed does not matter.

Definition 1.5 *Given two sets A and B, we say that A is a subset of B and we write*

$$A \subseteq B$$

if and only if whenever $x \in A$ then $x \in B$. This relation between two sets is called inclusion.

Remark 1.3 *If two sets A and B are equal, i.e.*

$$A = B,$$

then

$$A \subseteq B$$

and

$$B \subseteq A.$$

Definition 1.6 *If a set A is a subset of B, but $A \neq B$ we will write*

$$A \subset B$$

and by that we will mean that there are elements of B that are not elements of A and we will say that A is a proper subset of B.

1.2 Sets operations

Given two sets A and B, there are different ways to combine A and B to form a new set. We are going to introduce some sets operations by making use of the relation

$$\in.$$

1.2.1 Union

The union of two sets A and B consists of all elements belonging to at least one of the two sets, i.e. consists of all elements belonging to either A or B (or both). It is denoted by

$$A \cup B = \{x \mid (x \in A) \text{ or } (x \in B)\}$$

and it is called the union of A and B.

1.2.2 Intersection

The intersection of two sets A and B consists of all elements belonging to both A and B. It is denoted by

$$A \cap B = \{x \mid (x \in A) \text{ and } (x \in B)\}$$

and it is called the intersection of A and B.

Definition 1.7 *We say that two sets A and B are disjoint if and only if*

$$A \cap B = \emptyset.$$

1.2.3 Difference or relative complement

The difference of two sets A and B consists of all elements belonging to A and do not belong to B. It is denoted by

$$A \setminus B = \{x \mid (x \in A) \text{ and } (x \notin B)\}$$

and it is called the difference of A and B or the relative complement of B with respect to A.

We also define

Definition 1.8 *Given two sets A and B we define the symmetric difference A, B the set of elements of A that are not elements of B and of elements of B that are not elements of A i.e. the set*

$$A \Delta B = (A \setminus B) \cup (B \setminus A)$$

Remark 1.4 *It can be easily verified that*

$$A \Delta B = (A \cup B) \setminus (A \cap B).$$

Definition 1.9 *Sometimes we can be dealing we sets that are all subsets of a set X. This set X is called the universal set or the universe.*

The universe set X must be explicitly given or clear from the context.

Definition 1.10 *Given a universal set X and a subset $A \subset X$, the set $X \setminus A$ is called the complement of A and it is often denoted by A'.*

Given a set, we want to define another set, which elements are sets, i.e.

Definition 1.11 *Let T be a set. We define the power set of T to be the set which elements are all the subsets (or parts) of T and we denote it by*

$$\mathcal{P}(T).$$

Remark 1.5 *If T is a set with $|T| = n$, then*

$$|\mathcal{P}(T)| = 2^n.$$

Example 1.4 *Let $T = \{a, b, c\}$, then*

$$\mathcal{P}(T) = \{\emptyset,\ \{a\},\ \{b\}\ \{c\}\ \{a,\ b\}\ \{a,\ c\}\ \{b,\ c\}\ \{a,\ b,\ c\}\}\,.$$

Let X be a universal set and let A, B and C be subsets of X. The following properties hold:

1. Associative laws

$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$(A \cap B) \cap C = A \cap (B \cap C)$$

2. Commutative laws

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

3. Distributive laws

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

4. Identity laws

$$A \cup \emptyset = A$$
$$A \cap X = A$$

5. Complement laws

$$A \cup A' = X$$
$$A \cap A' = \emptyset$$

6. Idempotent laws

$$A \cup A = A$$
$$A \cap A = A$$

7. Bound laws

$$A \cup X = X$$
$$A \cap \emptyset = \emptyset$$

8. Absorption laws

$$A \cup (A \cap B) = A$$
$$A \cap (A \cup B) = A$$

9. Involution law

$$(A')' = A$$

10. 0/1 laws

$$\emptyset' = X$$
$$X' = \emptyset$$

11. De Morgan's laws for sets

$$(A \cup B)' = A' \cap B'$$
$$(A \cap B)' = A' \cup B'.$$

The above properties can be visualized quite easily by making use of the Veen diagram of sets.