

DISCRETE MATHEMATICS 1 (MS4111): TUTORIAL 5

1. Given the set

$$X = \{1, 2, 3, 4, 5, 6\},$$

consider the set S of sets of X defined by

$$S = \{\{1, 3, 5\}, \{2, 6\}, \{4\}\}.$$

- (a) Prove that S is a partition of X ;
- (b) Prove that the relation \mathcal{R} on X defined by

$$x\mathcal{R}y \quad \text{if } x \text{ and } y \text{ belong to the same set of } S$$

is an equivalence relation.

- (c) Draw the digraph of \mathcal{R} .

2. Given the set

$$X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\},$$

consider the relation \mathcal{R} on X defined by

$$x\mathcal{R}y \quad \text{if } 2 \text{ divides } x - y.$$

- (a) Prove that \mathcal{R} is an equivalence relation.
- (b) List the equivalence classes of \mathcal{R} and their elements.
- (c) Draw the digraph of \mathcal{R} .

3. Given the set

$$X = \{2, 5, 7\},$$

consider the relation \mathcal{R} on X given by

$$\mathcal{R} = \{(2, 2), (2, 5), (2, 7), (5, 2), (5, 7), (5, 5), (7, 2), (7, 5), (7, 7)\}.$$

- (a) Prove that \mathcal{R} is an equivalence relation on X .
- (b) List the equivalence classes of \mathcal{R} and their elements.
- (c) Draw the digraph of \mathcal{R} .

4. Consider the set X of question 3 and the relation \mathcal{R} on X given by

$$\mathcal{R} = \{(2, 2), (2, 5), (2, 7), (5, 2), (5, 7), (7, 2), (7, 5), (7, 7)\}.$$

Is the above relation an equivalence relation? Justify your answer.

5. Consider the set X of question 3 and the relation \mathcal{R} on X given by

$$\mathcal{R} = \{(2, 2), (2, 5), (2, 7), (5, 2), (5, 5), (5, 7), (7, 5), (7, 7)\}.$$

Is the above relation an equivalence relation? Justify your answer.

6. Given the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix},$$

write the relation \mathcal{R} from $X = \{a, b, c, d\}$ to $Y = \{x, y, z, w\}$, given by A , where A is relative to the orderings a, b, c, d and x, y, z, w ;

7. Given the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

- (a) write the relation \mathcal{R} on $X = \{a, b, c, d\}$ given by A , where A is relative to the ordering a, b, c, d .
- (b) state whether \mathcal{R} is an equivalence relation by looking at matrix A and justify your answer;
- (c) state whether \mathcal{R} is antisymmetric by looking at matrix A and justify your answer.