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College *of* Informatics *and* Electronics

Department of Computer Science
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Final Assessment Paper

Academic Year:	2007/2008	Semester:	Spring
Module Title:	Data Structures and Algorithms	Module Code:	CS4115
Duration of Exam:	$2\frac{1}{2}$ hours	Percent of Semester Marks:	65
Lecturer:	P. Healy	Paper marked out of:	100

Instructions to Candidates:

- There are three sections to the paper: Multiple Choice Questions, Short Questions and Long Questions
- The mark distribution is 40 marks for Multiple Choice Questions, 20 marks for Short Questions and 40 marks for the Long Questions
- Answer all questions in all sections

Section 1. Multiple Choice Answers (40 marks).

Use the machine-readable multiple-choice question grid that has been provided to answer these questions. Please completely mark in black exactly one circle on the grid for each answer. A penalty will be charged for wrong answers. Mark the **X** bubble for those questions you wish to skip.

1. The number of nodes in a *complete* binary tree of height h is
   (a) exactly $2^{h-1} - 1$
   (b) exactly $2^h - 1$
   (c) exactly $2^{h+1} - 1$
   (d) None of the above

2. How many nodes are on the bottom layer, h, of a *perfect* binary tree?
   (a) at least 2^h
   (b) at most 2^h
   (c) exactly 2^h
   (d) none of the above

3. Let $S_1 = \sum_{i=1}^{n} i^2$ and $S_2 = \left(\sum_{i=1}^{n} i\right)^3$. Which one of the following statements is true?
   (a) $S_1 = S_2$ for $1 \leq n \leq 30$ only
   (b) $S_1 = S_2$ for $1 \leq n \leq 100$ only
   (c) $S_1 = S_2$ for all n
   (d) None of the above

4. If $f(n) = O(g(n))$ which of the following statements cannot be true?
   (a) $g(n) = O(f(n))$
   (b) $g(n) = \Theta(f(n))$
   (c) $f(n) = o(g(n))$
   (d) $f(n) = \Theta(g(n))$

5. On the first day of Christmas,
   my true love sent to me
   A partridge in a pear tree.

   On the second day of Christmas,
   my true love sent to me
   Two Zetor tractors, and
   A partridge in a pear tree.

   On the third day of Christmas ...

   How many lines would be in such a “poem” if it ran for 365 days instead of the usual 12?
   (a) $\frac{365 \times 366}{2} + 2 * 365$
   (b) $\frac{367 \times 368}{2} - 3$
   (c) Neither of the above
   (d) Both of the above

6. What is the time-complexity of the following piece of code in “Big-Oh” notation?
   (a) $O(n^2)$
   (b) $O(n)$
   (c) $O(\log n)$
   (d) $O(n \log n)$

7. The worst-case performances of the heap operations `deleteMin()` and `insert()` are both $O(\log n)$. Given the two statements below, which of them are true?

   S1 The experimentally found average case performance of `deleteMin()` is $O(1)$

   S2 The experimentally found average case performance of `insert()` is $O(1)$

   (a) Both statements are true
   (b) **S1** is true, but **S2** is false
   (c) **S1** is false, but **S2** is true
   (d) Both statements are false

8. Given the two statements below, which of them are true?

   S1 In a strongly connected graph, every node connects to every other node by an edge

   S2 If a graph is strongly connected then it cannot have a cut vertex (articulation point)

   (a) Both statements are true
   (b) **S1** is true, but **S2** is false
   (c) **S1** is false, but **S2** is true
   (d) Both statements are false

9. Given the two statements below, which of them are true?

   S1 If an n-vertex graph has n articulation points then the graph must have a cycle

   S2 If the Depth-First Tree of a graph G has no back edges then G has no cycles

   (a) Both statements are true
   (b) **S1** is true, but **S2** is false
   (c) **S1** is false, but **S2** is true
   (d) Both statements are false

10. Given the two statements below, which of them are true?

    S1 Starting from vertex v_0 in a graph, the time required by Depth-First Search to find a path (if one exists) to some vertex v^* is *less* than that required by Breadth-First Search

    S2 The space required by Depth-First Search is *less* than that required by Breadth-First Search

    (a) Both statements are true
    (b) **S1** is true, but **S2** is false
    (c) **S1** is false, but **S2** is true
    (d) Both statements are false

Section 2. Short Questions (5×4 marks).

- Please put your answers to these questions in the answer book provided to you, labelling your answers 2.1, 2.2, etc.

1. The *unweighted shortest path* problem can be solved in __________ time.
2. With $O(n)$ calls to `percolate_down()`, a heap can be created in ____ time.
3. Give the recurrence relation for the *best-case* running time of `QuickSelect()`, the algorithm for finding the k^{th} largest element in an array: ________________
4. Recursion is to algorithm implementation as ________ is to proof techniques. That is, what is the proof technique analogue of recursion?
5. Ordinarily the most appropriate way to represent a graph internally is with ____________; however, if many queries are of the form "Is node u adjacant to node v?" then the most appropriate representation may be ________________

Section 3. Long Questions (40 marks).

- Please put your answers to these questions in the answer book provided to you
- Label your answers 3.1, 3.2, and 3.3 in your answer books

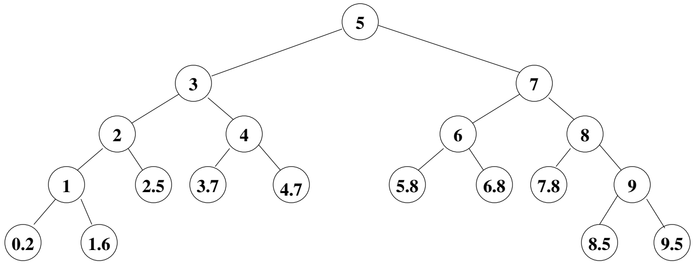

Figure 1: A Binary Search / AVL Tree

1. **(10 marks.)**

   Consider the tree shown in Figure 1.

   (a) Assuming that the tree is a Binary Search Tree, delete the node numbered **5**; (5 marks.)

   (b) Assuming that the tree is an AVL tree, insert the number **8.8**. (5 marks.)

2. **(15 marks.)**

   (a) In arguing that any sorting algorithm that relies on 2-way comparisons requires $\Omega(n \log n)$ comparisons we claimed that a binary tree with L leaves had to have depth at least $\lceil \log L \rceil$.

   Use induction to show that the number of leaves, $|L_d|$, of a binary tree of depth d is at most $|L_d| \leq 2^d$. (7 marks.)

   (b) Draw the height-4 AVL tree that has the minimum number of nodes. That is, draw the worst, most skewed tree that has depth, $d = 4$. (3 marks.)

   (c) Give the recurrence relation for n_d the smallest number of nodes possible in an AVL tree of depth d. What are the initial conditions? (5 marks.)

3. **(15 marks.)**

   The vertex connectivity problem that we have looked at in class has a wide variety of applications. The *vertex connectivity* problem is to determine the number of edge-disjoint paths that exist between pairs of nodes in G.

   (a) What is the vertex-connectivity of the graph shown in Figure 2 over? (4 marks.)

   (b) Give an *upper* bound, k_u, on the connectivity of a graph, G in terms of its nodes and edges; (4 marks.)

   (c) Prove that no graph can have higher connectivity than your bound k_u above. (7 marks.)

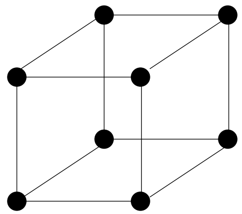

Figure 2: What is the vertex-connectivity of this graph?