

## CS4112 Tutorial Exercises 2

1. Let  $A = \{1, 2, 3, 4\}$  and  $B = \{a, b, c\}$ . State which of the following functions,  $f_i : A \rightarrow B$  are injective and surjective.
 
$$f_1 = \{\langle 1, a \rangle, \langle 2, b \rangle, \langle 3, c \rangle, \langle 4, c \rangle\}$$

$$f_2 = \{\langle 1, b \rangle, \langle 2, a \rangle, \langle 3, a \rangle, \langle 4, c \rangle\}$$

$$f_3 = \{\langle 1, b \rangle, \langle 2, b \rangle, \langle 3, b \rangle, \langle 4, b \rangle\}$$
  
2. Given  $A = \{1, 2, 3\}$  and  $B = \{2, 4, 5\}$ 
  - Write a java code segment that will store these sets as arrays
  - Extend this code segment to output the union of these sets
  - Write a separate java code function that will take the 2 arrays as parameters and output the intersection of these sets
  - Extend this function such that  $A \cap B$  is stored in a separate array
  
3. Do the following sets define functions? If so state whether or not the functions are injective:
  - (a)  $\{\langle 1, \langle 2, 3 \rangle \rangle, \langle 2, \langle 3, 4 \rangle \rangle, \langle 3, \langle 1, 4 \rangle \rangle, \langle 4, \langle 2, 4 \rangle \rangle\}$
  - (b)  $\{\langle \langle 1, 2 \rangle 3 \rangle, \langle \langle 2, 3 \rangle 4 \rangle, \langle \langle 3, 3 \rangle 4 \rangle\}$
  - (c)  $\{\langle 1, \langle 2, 3 \rangle \rangle, \langle 2, \langle 3, 4 \rangle \rangle, \langle 1, \langle 2, 4 \rangle \rangle\}$
  - (d)  $\{\langle 1, \langle 2, 3 \rangle \rangle, \langle 2, \langle 2, 3 \rangle \rangle, \langle 3, \langle 2, 3 \rangle \rangle\}$
  
4. Which of the following functions are one-to-one and onto?
  - (a)  $f : \text{Nat} \rightarrow \text{Nat}$  and  $f(i) = i^2 + 4$
  - (b)  $f : \mathbb{Z} \rightarrow \mathbb{Z}$  and  $f(i) = i^2 + 4$
  - (c)  $f : \text{Nat} \rightarrow \text{Nat}$  and  $f(i) = i \bmod 5$
  - (d)  $\text{is\_even} : \text{Nat} \rightarrow \text{Bool}$  and  $\text{is\_even}(i) = \text{true}$  if  $i$  is an even number and  $\text{false}$  otherwise
  - (e)  $g : \text{Nat} \rightarrow \text{Nat}$  and  $g(i) = \begin{cases} 1 & \text{if } i \text{ is odd} \\ 0 & \text{if } i \text{ is even} \end{cases}$
  
5. How many functions exist from  $A = \{a, b, c\}$  to  $B = \{0, 1, 2\}$ ? Explain.
  
6. If  $f : \text{Nat} \times \text{Nat} \rightarrow \text{Nat}$ , given by  $f(x, y) = x + y$ , show that  $f$  is onto. Is  $f$  one-to-one?