UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

College of Informatics and Electronics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MS4111

SEMESTER: Spring 2005-06

MODULE TITLE: Discrete Mathematics 1

DURATION OF EXAMINATION: 2 1/2 hours

LECTURER: E. MacAogáin

PERCENTAGE OF TOTAL MARKS: 80%

EXTERNAL EXAMINER: Prof. J. King

INSTRUCTIONS TO CANDIDATES: **Answer four questions. All questions are weighted equally. Give the reasoning for your answers.**

1 (a) Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{-2, 0, 2, 4\}$. Find:

(i) $A \cap B$ 2

(ii) $A - B$ 2

(iii) $(A - B) \cup (B - A)$. 2

(b) Let $C = \{a, b\}$. Find its power set 2^C. 4

(c) Let D, E and F be three sets. Simplify: $\overline{D \cap (E - F)} \cup D$ 5

(d) Prove that a set of order n has 2^n subsets. 5

2 (a) Find:

(i) gcd(132,60) 2

(ii) lcm(105,42). 3

(b) Express in the form $\frac{p}{q}$ where $p,\ q\ \in$ **Z**, the set of integers, $q \neq 0$:

(i) 5.478 2

(ii) $4.\dot{2}\dot{1}$ 3

(c) Expand: $(2x + y)^4$ 5

(d) Prove, using mathematical induction, that: 5

$$3 + 6 + 9 + \cdots + 3n = \frac{3n(n+1)}{2}, \forall n \in \mathbf{N}$$

(**N**: the set of positive integers)

3 (a) For each of the following relations $*$ on the given sets, which of the properties reflexivity, symmetry, transitivity do they have?

(i) set **Z** of integers: $x * y$ iff $x < y$ 3

(ii) set **Z**: $x * y$ iff $x + y$ is odd. 3

(b) Show that $x^2 + y^2 = 2007$ has no solution in integers.
[Hint: look mod 4] 5

(c) Let $m \in \mathbf{N}$. Prove that $x \equiv y\ (mod\ m)$ is an equivalence relation on **Z**. 5

(d) Find the set of all integers between -9 and 9 such that:

$$2x \equiv 2\ (mod\ 3)$$

4

4 (a) Write down the converse, inverse and contrapositive of the following proposition:
if he's old then he's wise. 6

(b) For the following truth table:

(i) write down the disjunctive normal form of the function 2

(ii) simplify algebraically 3

(iii) simplify using Karnaugh maps. 3

P	Q	f(P,Q)
T	T	T
T	F	F
F	T	T
F	F	T

(c) (i) Show that the following is a contradiction: $(P \wedge Q) \wedge (\sim P)$ 3

(ii) Show that the following is a tautology: $(\sim Q) \vee (P \rightarrow Q)$ 3

5 (a) Prove directly:
if x and y are odd integers then xy is an odd integer. 5

(b) Use the contrapositive to prove:
if the square of an integer is even then the integer is even. 7

(c) Prove by contradiction (i.e. "reductio ad absurdum") that $\sqrt{3}$ is irrational. 8

6 (a) Find the general solution of the following recurrence relation:

$$a_n = 5a_{n-1} - 6a_{n-2}$$

7

(b) Find the particular solution of the above recurrence relation which satisfies the initial conditions:

$$a_0 = 2, \; a_1 = 5$$

Hence evaluate a_5. 7

(c) Find the general solution of the following recurrence relation:

$$a_n = 5a_{n-1} - 6a_{n-2} + 10$$

[Hint: see part(a)] 6