- Logic: what is it:
  - A language which is unambiguous
  - A formal approach to providing a proof of a given situation

- The logic of mathematics is applied to decide whether one statement is a logical consequence of one or more other statements

- We use some of the syntax and semantics of logic.

- A proposition(assertion) is a primitive statement which is either true or false

- Examples:

  1. Computing students learn Java in t 1st year

  2. John drives a porsche

  3. There are 4 administrative staff in the CSIS dept.

- There are some sentences which cannot be considered propositions
  e.g. Please switch off all mobile phones

- Combining 2 or more primitive propositions into compound propositions

  1. Negation of p is denoted by $\neg p$ and is read 'not p'

  2. Conjunction of p, q is denoted by $p \wedge q$ is read 'p and q'

  3. Disjunction of p, q is denoted by $p \vee q$ is read 'p or q'

  4. Conditional of p, q is denoted by $p \Rightarrow q$ is read 'p implies q'

  5. Biconditional of p, q is denoted by $p \Leftrightarrow q$ is read 'p if and only if q'

- Note: The symbol $\neg$ is a Unary Prefix operator called not.

- The symbols $\wedge, \vee, \Rightarrow, \Leftrightarrow$ are Binary Infix operators.

- The effects of these operators on truth values and combinations of truth values can be displayed using a table or a function

- $\neg$ (not)

  1. $\neg T = F$, $\neg F = T$
  2. $\neg(\neg T) = \neg F = T$
  3. $\neg(\neg F) = \neg T = F$
  4. $not = \neg = \{T \mapsto F, F \mapsto T\}$
  5. $not = \neg = \{\langle T, F\rangle, \langle F, T\rangle\}$

- $\wedge$ (and)

  1. $T \wedge T = T, T \wedge F = F, F \wedge T = F, F \wedge F = T$

  2. $p \wedge q = T$ only if both p and q take the value T

  3. Note: $p \wedge T = T \wedge p = p$, T is the identity element for $\wedge$

  4. $and = \wedge = \{\langle T, T\rangle \mapsto T, \langle T, F\rangle \mapsto F, \langle F, T\rangle \mapsto F, \langle F, F\rangle \mapsto F\}$

  5. Using a slightly different notation: $and = \wedge = \{\langle\langle T, T\rangle, T\rangle, \langle\langle T, F\rangle, F\rangle, \langle\langle F, T\rangle, F\rangle, \langle\langle F, F\rangle\, F\rangle\}$

- $\vee$ (or)

  1. $T \vee T = T, T \vee F = T, F \vee T = T, F \vee F = F$

  2. $p \vee q = T$ if either p or q take the value T

  3. Note: $p \vee F = F \vee p = p$, F is the identity element for $\vee$

  4. $or = \vee = \{\langle T, T\rangle \mapsto T, \langle T, F\rangle \mapsto T, \langle F, T\rangle \mapsto T, \langle F, F\rangle \mapsto F\}$

  5. Using a slightly different notation: $or = \vee = \{\langle\langle T, T\rangle, T\rangle, \langle\langle T, F\rangle, T\rangle, \langle\langle F, T\rangle, T\rangle, \langle\langle F, F\rangle\, F\rangle\}$

- Truth value of compound statements

  1. $\neg p$ is true if p is false

  2. $p \wedge q$ is true iff p is true and q is true

  3. $p \vee q$ is true if either p is true or q is true

  4. $p \Rightarrow q$ is true in all cases except where p is true and q is false

  5. $p \Leftrightarrow q$ is true when p, q have the same truth value and false otherwise

- Consider the following statements

  1. p: Mary drives to college

  2. q: Mary lives in Cork

- If Mary drives to college then Mary lives in Cork

- Suppose p is true and q is false. You do not want a true statement to lead you into believeing something is false.

- Precedence Rules: $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$

- If you succeed you will have worked hard
  - However even if you have not succeeded you may still have worked hard

- If the book is a recommended text a copy will be held in the library
  - The book is a recommended text
  - Therefore a copy will be held in the library

- In general:
  if A then B
  A
  therefore B

- How about: if P then Q
  - not P
  - therefore, not Q ....valid argument????

- if P then Q
  - if Q then R
  - therefore, if P then R

- Equate the following textual statements with their mathematical equivalent.

  1. x less than 4

  2. The sum of a and b is less than 5

  3. All the elements of the set A are contained in the set B

  4. the value of the function g is p for arguments less than 5