DISCRETE MATHEMATICS 1 (MS4111): TUTORIAL 6

1. Let $X = \{1, 2, 3\}$ and $Y = \{a, b, c, d\}$ and $\mathcal{R}$ be the relation from X to Y given by

$$\mathcal{R} = \{(1, a), (1, c), (2, a), (2, b), (3, d)\}.$$

   Find the matrix of $\mathcal{R}$ relative to the orderings given below:

   (a) Ordering of X: $1, 2, 3$;
   Ordering of Y: a, b, c, d.

   (b) Ordering of X: $3, 2, 1$;
   Ordering of Y: c, b, a, d.

2. Let $X = \{1, 2, 3, 4, 5\}$ and $\mathcal{R}$ be a relation on X given by

$$\mathcal{R} = \{(x, y) \mid \quad x < y\}.$$

   Find the matrix of $\mathcal{R}$ relative o the orderings given below:

   (a) Ordering of X: $1, 2, 3, 4, 5$;

   (b) Ordering of X: $5, 3, 1, 2, 4$.

3. Let $X = \{1, 2, 3, 4, 5\}$ and $\mathcal{R}$ be a relation on X given in question 2.

   (a) Analyze the matrix found in question 2 (a) and derive all the information you can about the relation $\mathcal{R}$ from it;

   (b) Analyze the matrix found in question 2 (b) and derive all the information you can about the relation $\mathcal{R}$ from it.

   (c) Explain results found in part (a) and (b) of this question.

4. Given the sets $X = \{1, 2, 3, 4\}$, $Y = \{x, y, z\}$ and $Z = \{a, b\}$ consider the relations $\mathcal{R}_1$ and $\mathcal{R}_2$ from X to Y and Y to Z respectively, given by

$$\begin{aligned} \mathcal{R}_1 &= \{(1, x), (1, y), (2, x), (3, x), (4, z)\} \\ \mathcal{R}_2 &= \{(x, b), (y, b), (y, a), (z, a)\}. \end{aligned}$$

Given the orderings:

ordering of X: $1, 2, 3, 4$; ordering of Y: x, y, z; ordering of Z: a, b, find

(a) the matrix A_1 of the relation $\mathcal{R}_1$;

(b) the matrix A_2 of the relation $\mathcal{R}_2$;

(c) the matrix of the relation $\mathcal{R}_2 \circ \mathcal{R}_1$;

(d) the relation $\mathcal{R}_2 \circ \mathcal{R}_1$ (as a set of orders pairs).