Course Notes
for
MS4111
Discrete Mathematics 1

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CHAPTER 9 Number systems

9.1 Decimal (base 10) number system

We use ten symbols

$$0, 1, 2, 3, 4, 5, 6, 7, 8, 9$$

to represent any number in the decimal system.

Example 9.1 *When we write the number*

$$3854$$

in base 10 we mean that

$$3854_{10} = 4 \cdot 10^0 + 5 \cdot 10^1 + 8 \cdot 10^2 + 3 \cdot 10^3.$$

9.2 Binary (base 2) number system

We use two symbols (called BITS)

$$0, 1$$

to represent any number in the Binary system.

Example 9.2 *When we write the number*

$$101101$$

in base 2 we mean that

$$101101_2 = 1 \cdot 2^0 + 0 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^3 + 0 \cdot 2^4 + 1 \cdot 2^5.$$

9.2.0.11 Binary to Decimal

If we consider the above example, we can write 101101_2 in base 10 in the following way

$$\begin{aligned} 101101_2 &= 1 \cdot 2^0 + 0 \cdot 2^1 + 1 \cdot 2^2 + 1 \cdot 2^3 + 0 \cdot 2^4 + 1 \cdot 2^5 \\ &= 1 + 0 + 4 + 8 + 0 + 32 \\ &= 45_{10}. \end{aligned}$$

Note: We need to specify the base we are working in , i.e. the number system used when we write a number to avoid ambiguity since for example

$$101101_2 \neq 101101_{10}$$

9.2.0.12 Decimal to Binary

How can we write 91_{10} in base 2?

Divide 91 by 2:

$$
\begin{aligned}
91_{10} = \quad &= 2 \cdot 45 + 1 \\
&= 2(2 \cdot 22 + 1) + 1 \\
&= 2^2 \cdot 22 + 2 \cdot 1 + 1 \\
&= 2^2(11 \cdot 2) + 2 \cdot 1 + 1 \\
&= 2^3 \cdot 11 + 2^1 \cdot 1 + 2^0 \cdot 1 \\
&= (2 \cdot 5 + 1) \cdot 2^3 + 2^1 \cdot 1 + 2^0 \cdot 1 \\
&= 2^4 \cdot 5 + 2^3 \cdot 1 + 2^1 \cdot 1 + 2^0 \cdot 1 \\
&= 2^4(2 \cdot 2 + 1) + 2^3 \cdot 1 + 2^1 \cdot 1 + 2^0 \cdot 1 \\
&= 2^5 \cdot 2 + 2^4 \cdot 1 + 2^3 \cdot 1 + 2^1 \cdot 1 + 2^0 \cdot 1 \\
&= 2^6 + 2^4 \cdot 1 + 2^3 \cdot 1 + 2^1 \cdot 1 + 2^0 \cdot 1 \\
&= 1011011_2
\end{aligned}
$$

Note: We can add numbers in the binary system by noticing that

$$\begin{aligned} 0+0 &= 0 \\ 0+1 &= 1+0=1 \\ 1+1 &= 10. \end{aligned}$$

9.3 Octal (base 8) number system

We use eight symbols

$$0, 1, 2, 3, 4, 5, 6, 7$$

to represent any number in the Octal system.

Example 9.3 *When we write the number*

$$63$$

in base 8 we mean that

$$63_8 = 3 \cdot 8^0 + 6 \cdot 8^1.$$

9.3.0.13 Octal to Decimal

If we consider the above example, we can write 63_8 in base 10 in the following way

$$\begin{aligned} 63_8 &= 3 \cdot 8^0 + 6 \cdot 8^1 \\ &= 3 \cdot 1 + 6 \cdot 8 = 3 + 48 = 51_{10} \end{aligned}$$

9.3.0.14 Decimal to Octal

How can we write 400_{10} in base 8?

Divide 400 by 8:

$$
\begin{aligned}
400_{10} = \quad & = \quad 8 \cdot 50 + 0 \\
& = \quad 8(8\cot 6 + 2) + 0 \\
& = \quad 8^2 \cdot 6 + 8^1 \cdot 1 + 8^0 \cdot 0 \\
& = \quad 620_8.
\end{aligned}
$$

9.4 Hexadecimal (base 16) (or Hex) number system

We use sixteen symbols

$$0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F$$

to represent any number in the Hex system.

Note:

$$A = 10; \quad B = 11; \quad C = 12; \quad D = 13; \quad E = 14; \quad F = 15.$$

Example 9.4 *When we write the number*

$$B4F$$

in base 16 we mean that

$$B4F_{16} = F \cdot 16^0 + 4 \cdot 16^1 + B \cdot 16^2.$$

9.4.0.15 Hex to Decimal

If we consider the above example, we can write $B4F_{16}$ in base 10 in the following way

$$\begin{aligned} B4F_{16} &= F \cdot 16^0 + 4 \cdot 16^1 + B \cdot 16^2 \\ &= 15 \cdot 16^0 + 4 \cdot 16^1 + 11 \cdot 16^2 = 2895_{10} \end{aligned}$$

9.4.0.16 Decimal to Hex

How can we write 20385_{10} in base 16?

Divide 20385 by 16:

$$
\begin{aligned}
20385_{10} = \quad &= (16 \cdot 1274) + 1 \\
&= 16(16 \cdot 79 + 10) + 1 \\
&= 16^2 \cdot 79 + 16 \cdot 10 + 1 \\
&= 16^2(16 \cdot 4 + 15) + 16 \cdot 10 + 1 \\
&= 16^3 \cdot 4 + 16^2 \cdot 15 + 16 \cdot 10 + 1 \\
&= 4FA1_{16}.
\end{aligned}
$$

9.5 Fields

Here we only give the following definition.

Definition 9.1 *A field is a set F together with two operations called addition (" + ") and multiplication (" $\cdot$ ") such that*

1) $a+b \in F$, *for all* $a, b \in F$ *(F is closed with respect to "+");*

2) $a+(b+c)=(a+b)+c$, *for all* $a, b, c \in F$ *(associative law);*

3) $a+b=b+a$, *for all* $a, b, \in F$ *(commutative law);*

4) $\exists 0 \in F$ *such that* $a+0=a$, *for all* $a \in F$*;*

5) $\forall a \in F$, $\exists -a \in F$ *such that* $a+(-a)=0$*;*

6) $a \cdot b \in F$, *for all* $a, b \in F$ *(F is closed with respect to "$\cdot$");*

7) $a \cdot (b \cdot c)=(a \cdot b) \cdot c$, *for all* $a, b, c \in F$ *(associative law);*

8) $a + \cdot b=b \cdot a$, *for all* $a, b, \in F$ *(commutative law);*

9) $\exists 1 \in F$ *such that* $a \cdot 1=a$, *for all* $a \in F$*;*

10) $\forall a \in F$, $\exists a^{-1} \in F$ *such that* $a \cdot a^{-1} = 1$*;*

11) $a \cdot (b + c) = a \cdot b + a \cdot c$, *for all* $a, b, c \in F$ *(distributivity of multiplication over addition).*

Note: The set of real numbers $\mathbb{R}$, together with the usual addition and multiplication of numbers, is an example of field.