University of Limerick

College of Informatics and Electronics
Department of Computer Science and Information Systems

Spring 2008

Semester : Spring	Academic Year : 2007/08
Module Code : CS4112	Module Title : Computer Science 2
Duration of Exam : 2.5 hours	% of total: 70%
Lecturer(s) : Dr. Michael English	Marked out of: 125

Instructions to Candidates :- Question 1 is compulsory. Attempt Question 1 and three other questions. Question 1 carries 50 marks. Questions 2 to 5 inclusive carry 25 marks each. Note that the examiner can take into account the quality of presentation and exposition as well as the content. Calculators are not allowed.

Q1

(a) What is the cardinality of each of the following sets?

- $\{< 1, 2 >\}$
- $\{<< 1, 2 >, 1 >\}$
- $\{1, < 1, 2 >\}\}$
- $\{1, < 1, 2 >, < 1, < 1, 2 >>\}$

(5 marks)

(b) Construct a truth table for each of the following compound propositions:

(a) $(p \Rightarrow q) \Rightarrow r$
(b) $(p \Leftrightarrow q)$
(c) $(p \wedge q) \vee r$

(5 marks)

(c) State whether the folowing Hoare Triples are correct or incorrect? You can assume that x and y are integer values.

(a) $\{x > 3\}x = x + 5\{x > 8\}$
(b) $\{x > 4\}x = x + 5\{x \geq 10\}$
(c) $\{x \neq y\}x = y\{x = y\}$

(5 marks)

(d) Using the appropriate rule, show that the following Hoare Triple is correct: $\{a > 5\}$ if $(b < 0)$ $a = 10$ $\{a > 5\}$. (5 marks)

(e) Given two integer arrays A and B which store bits (either a 0 or 1 is stored in each position in an array) that represent subsets of a set with n elements, complete the Java method definition given below to determine $A \cap B$. The method should output the result(a series of 1's and 0's) to the screen.

```
public void IntersectionOfSets(int A[], int B[])
{
//insert the appropriate code in your answer book
}
```

(5 marks)

(f) Complete the recursive implementation of the factorial function (by replacing the '?' symbols with appropriate Java code) given the following inductive definition.
fac(0)=1
fac(n)=n * fac(n-1)

```
int fac(int n)
{
if (n==0) return ? ; else return ? ;
}
```

(5 marks)

(g) Given the following recursive definition of the function *Power* which calculates x^n for some natural number n, demonstrate how $Power(5, 3)$ is evaluated using this definition.
$Power(x, 0) = 1$
$Power(x, n) = x * Power(x, n - 1)$ (5 marks)

(h) Provide a recursive definition of the function *sum* which calculates the sum of the first n natural numbers for all $n \geq 0$. (5 marks)

(i) Give an iterative implementation of the *sum* function defined recursively in the previous part of this question. (5 marks)

(j) Suppose $A \subseteq \{1, \ldots, 10\}$, where $\{1, \ldots, 10\}$ is the universal set. Write a Java method which takes a bit string representation of A (stored as an array of integers) and determines the complement of A. (5 marks)

Q2

(a) What is a Singleton set? Give an example. (2 marks)

(b) Is it possible to have a situation where one infinite set is a proper subset of another? If so give an example. (3 marks)

(c) List the members of the following sets:

- $\{x \mid x$ is a real number such that $x^4 = 16\}$
- $\{x \mid x$ is a positive integer less than $12\}$
- $\{x \mid x$ is an integer and $x < 1\}$
- $\{x \mid x$ is an integer such that $x^2 = 2\}$

(4 marks)

(d) Suppose $A = \{a, b, c\}$ and $B = \{1, 2\}$. How many relations exist between A and B? How many of these relations are functions that have both a and b as the first component of ordered pairs? Explain. (6 marks)

(e) Show using truth tables that the following statements form a valid argument. (4 marks)

if P then Q
if Q then R
---
if P then R

(f) Identify propositions, P, Q in the following argument and then formalise it using the logical symbols. Confirm your answer using truth tables:
If the book is a recommended text, then the book is in the library. The book is not in the library. Therefore the book is not a recommended text.
(6 marks)

Q3

(a) What is an assertion? (2 marks)

(b) What is an invariant? (2 marks)

(c) Give an example of a condition which is stronger than $x < 4$ and another condition that is weaker than $x < 4$ (4 marks)

(d) Is it possible to strengthen or weaken a precondition or postcondition and still ensure the correctness of a Hoare triple? Explain. (4 marks)

(e) The rule $\frac{P \Rightarrow Q^E_V}{\{P\}V=E\{Q\}}$ can be used to prove the correctness of an assignment statement. Use it to prove the correctness of the following Hoare triple: $\{x > 5\}x = x + 10\{x > 15\}$ (3 marks)

(f) Draw a flowchart to illustrate an if-then-else statement. (2 marks)

(g) Based on the paths in this flowchart provide a rule which can be applied to demonstrate the correctness of if-then-else statements. (4 marks)

(h) Apply this rule to demonstrate the correctness of the following Hoare Triple:
$\{\}$
if $a > b$ $\{max = a\}$ else $\{max = b\}$
$\{(max \geq a) \wedge (max \geq b)\}$ (4 marks)

Q4

(a) Define a function $isin$ which returns the truth value $true$ if there is a value between 1 and N for which the function f returns the value a. i.e. if there is at least one value k such that $f(k) = a$, where $f : \{1, \ldots, N\} \rightarrow Nat$. Otherwise the function should return $false$. (5 marks)

(b) Give a recursive implementation (a Java method) of the $isin$ function. (5 marks)

(c) Give an iterative implementation (a Java method) of the $isin$ function. (5 marks)

(d) Construct a flowchart for the iterative implementation of $isin$ and annotate the flowchart with assertions. (5 marks)

(e) Show that if the invariant and the loop condition hold before execution of the loop in the iterative implementation then the invariant still holds after execution of the body of the loop. (5 marks)

Q5

(a) Explain Structural Induction in your own words. When is it used? (5 marks)

(b) A binary tree is a special type of graph. Give an example of a binary tree labelling its components. What does the notation (A, c, B) mean in relation to binary trees. (5 marks)

(c) Give an inductive definition of the set of binary trees. (5 marks)

(d) Define a function h recursively which specifies the height of a binary tree. Remember an empty tree has height 0. (5 marks)

(e) Prove that the maximum number of nodes in a tree of height n is $2^n - 1$, where a tree of height 0 is the empty tree. (5 marks)