UNIVERSITY *of* LIMERICK

OLLSCOIL LUIMNIGH

COLLEGE *of* INFORMATICS *and* ELECTRONICS

Department of Computer Science
and Information Systems

End-of-Semester Assessment Paper

Academic Year:	1998/99	Semester:	Autumn
Module Title:	Data Structures and Algorithms	Module Code:	CS4115
Duration of Exam:	$2\frac{1}{2}$ hours	Percent of Total Marks:	65
Lecturer:	P. Healy	Paper marked out of:	100

Instructions to Candidates:

- Answer *all* questions
- All questions carry equal marks
- Please keep your answers *precise* and *concise*

Q1. Short questions: (5 @ 4 marks each) (20 marks)

(i) What is the worst-case running time for the *find, insert or delete* operations on an AVL tree?

(ii) Given a graph, $G = (V, E)$, what is the largest number of edges exactly a graph can have in terms of $|V|$, the number of nodes?

(iii) Show the multiplications performed by the *fast exponentiation* algorithm to compute x^{15}

(iv) Evaluate or, give a closed form expression for, $\sum_{i=0}^{k-1} 2^i$

(v) As a function of n, what is the running-time (in Big-Oh notation) of the section of the following code?

```
sum = 0;
for (int i = 1; i <= n; i++)
  for (j = 1; j <= i*i; j++)
    for (int k = 0; k < j; k++)
      sum++;
```

Q2. (20 marks)

(a) Insert the integers $1, \ldots, 7$ (in that order) into an AVL tree, showing the resulting trees after each insertion (7 marks)

(b) Now insert the integers $15, 14, 13$ and 12 in that order in the tree, showing the resulting trees after the insertions of 14 and 12. (7 marks)

(c) A *full* node in a binary tree is one which has two children. Prove *by induction* that the number of full nodes in a binary tree is equal to one less than the number of leaves.
(6 marks)

Q3. (20 marks)

(a) Given the input **41, 58, 26, 59, 53, 97** show the intermediate steps of how the input is non-decreasing sorted by: (12 marks)

1. heapsort
2. mergesort

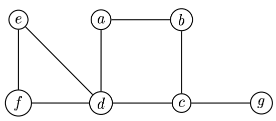
(b) Given a list like **1, 2, 3, 4, 5, 6, 3, 2, 4** that is, a list where the first n elements are non-decreasing sorted (1 to 6) and the remaining elements are unsorted, explain how would you sort the *entire* list if the number of unsorted elements, $f(n)$, is:

1. $f(n) = O(1)$ (a constant)
2. $f(n) = O(\log n)$
3. $f(n) = O(\sqrt{n})$

State the running times for each. (8 marks)

Q4. (20 marks)

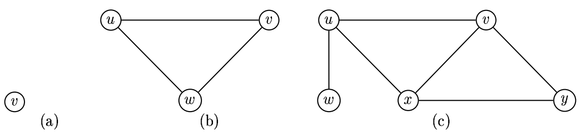
(a) Find all the articulation points in the graph below. Show the depth-first spanning tree and the values of *num* and *low* for each vertex starting your search from vertex a. When you have a choice of vertex to visit next in your search, visit the lexicographically smallest one. (14 marks)



(b) Prove by induction that for a connected graph $G = (V, E)$, with no crossing edges, the number of regions in the graph, $|R|$, is related to the number of edges, $|E|$, and the number of vertices, $|V|$, by the following formula: (6 marks)

$$|E| = |V| + |R| - 2$$

In a graph, the "outside world" is considered to be a region. The graph in the previous question has $|E| = 8, |V| = 7$ and $|R| = 3$. Here are some more examples of graphs, including the graph comprising a single vertex:



(a) (b) (c)

(a)	(b)	(c)
$\lvert R\rvert = 1$	$\lvert R\rvert = 2$	$\lvert R\rvert = 3$
$\lvert V\rvert = 1$	$\lvert V\rvert = 3$	$\lvert V\rvert = 5$
$\lvert E\rvert = 0$	$\lvert E\rvert = 3$	$\lvert E\rvert = 6$

Q5. (20 marks)

Suppose you want to perform an experiment to investigate the problems that can arise with the insertion and deletion of random elements in a binary search tree. Here is a strategy that is not perfect but will be close enough to give a good idea of what goes on behind the scenes: build a tree with n elements by inserting n elements chosen at random from the range $[1, \ldots, m]$, where $m = \alpha n$. You then perform some very large number – say n^2 – of insert/delete operations. An insert/delete operation firstly inserts a random number into the tree that was not in the tree prior to the insert and then deletes a random element from the tree.

Assume the existence of a function `rand_int(a,b)` which returns a random integer in the range $[a, \ldots, b]$ (between a and b, inclusive).

(a) Explain how to generate a random integer between 1 and m that is *not* already in the tree (so that a random insert can be performed). Does the running time depend on α? If so, in terms of n and α, what is the running time of this operation? (7 marks)

(b) Explain how to generate a random integer between 1 and m that *is* already in the tree (so that a random deletion can be performed). Does the running time depend on α? In terms of n and α, what is the running time of this operation? (7 marks)

(c) What is a good or a bad choice of α? Why? (6 marks)