



UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

College of Informatics and Electronics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MS4111

SEMESTER: Spring 2004-05

MODULE TITLE: Discrete Mathematics 1

DURATION OF EXAMINATION: 2 1/2 hours

LECTURER: E. MacAogáin

PERCENTAGE OF TOTAL MARKS: 80%

EXTERNAL EXAMINER: Prof. J. King

INSTRUCTIONS TO CANDIDATES: Answer four questions. All questions are weighted equally. Give the reasoning for your answers.

- 1 (a) Given the following sets:

$$A = \{2n : n \in \mathbf{Z}\}$$

$$B = \{3n : n \in \mathbf{Z} \text{ and } 0 \leq n \leq 3\}$$

$$C = \{n : n \in \mathbf{Z} \text{ and } n^2 \leq 5\}$$

where \mathbf{Z} is the set of integers, express each of the following in terms of A , B and C :

- (i) The set of all odd integers. 2
- (ii) $\{0, 6\}$ 2
- (iii) $\{3, 9\}$ 2
- (iv) $\{-2, 0, 2\}$ 2
- (v) $\{0\}$ 2

Find:

- (vi) $B \cup C$ 2
- (vii) $\overline{A} \cap C$ 2

- (b) Given two sets D and E , simplify: 6

$$D \cup \overline{D \cup E}.$$

- 2 (a) Find:

- (i) $\gcd(124, 60)$ 2
- (ii) $\text{lcm}(42, 105)$. 2
- (b) Expand: $(-2a + b)^4$ 4
- (c) (i) How many 8-bit binary strings begin and end with 1? 3
- (ii) How many 8-bit binary strings begin or end with 1? 3
- (d) Prove that the number of primes is infinite. 6

- 3 (a) For each of the following relations $*$ on the given sets, which of the properties reflexivity, symmetry, transitivity do they have?
- (i) set \mathbf{Z} of integers: $x * y$ iff $x \geq y$ 3
- (ii) set \mathbf{Z} : $x * y$ iff xy is even. 3
- (b) Let $A = \{a, b, c\}$. Write down all partitions of A . 4
- (c) Show that $x^2 + y^2 = 2003$ has no solution in integers.
[Hint: look mod 4] 5
- (d) Find the set of all integers between -10 and 10 such that: 5

$$2x \equiv 1 \pmod{5}$$

- 4 (a) (i) Show by any method that the following is a tautology: 3
- $$(\sim P) \vee (P \vee Q)$$
- (ii) Show by any method that the following is a contradiction: 3
- $$(\sim P) \wedge (P \wedge Q)$$
- (b) Form the negation of the following statement:
The weather is cloudy and windy. 3
- (c) Given the following truth table:
- (i) write down the disjunctive normal form of the function 2
- (ii) simplify algebraically 2
- (iii) simplify using Karnaugh maps. 2

P	Q	f(P,Q)
T	T	F
T	F	T
F	T	T
F	F	T

- (d) Given the following Karnaugh map, write down the corresponding truth table and an expression for the corresponding proposition: 5

T	T	T	T
F	F	T	F

- 5 (a) Write down the truth tables for an implication ($P \rightarrow Q$), its converse ($Q \rightarrow P$), its inverse ($\sim P \rightarrow \sim Q$) and its contrapositive ($\sim Q \rightarrow \sim P$). 8
- (b) Let x, y be integers. Prove that:
 $xy \text{ even} \rightarrow x \text{ even or } y \text{ even}$
- (i) using a direct proof, and 6
- (ii) using the contrapositive. 6

- 6 (a) Find the general solution of the following recurrence relation:

$$a_n = 4a_{n-1} - 4a_{n-2}$$

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- (b) Find the particular solution of the above recurrence relation which satisfies the initial conditions:

$$a_0 = 1, a_1 = 4$$

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- (c) Find the general solution of the following recurrence relation:

$$a_n = 4a_{n-1} - 4a_{n-2} + 6$$

[Hint: see part(a)]

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