- In this lecture:
  - Review of some discrete mathematics, sets relations, functions, cartesian products etc...
  - Why?
    - We wll equate computer programs with mathematical functions
    - We will write (simple) java programs to test some properties of sets, functions and relations. This will improve our programming skills and our understanding of the mathematics underlying computer programs
    - Discrete Maths is also the basis for database design which you will see later in the course

- A set is a well-defined collection of objects e.g. $A = \{Anne, Annette, Anthony\}$ or $L = \{a, b, c, \ldots, z\}$ or $Bool = \{True, False\}$
- The order of the elements in a set doesn't matter
  $\{Anne, Annette, Anthony\}$ is the same as $\{Anthony, Anne, Annette\}$
- Terminology: $a \in L$, a is an element of L
- A set is finite if all the elements of the set can be listed e.g. the set of all even numbers less than 20,
  $E = \{0, 2, 4, 6, 8, 10, 12, 14, 16, 18\}$

- The cardinality of a set=number of elements in the set:
  for the set E on previous slide $|E| = 10$
  (note the notation)

- A subset of a set: $R \subseteq P$ if and only if all the elements of R are also elements of P

- $A = \{cat, dog, mouse\}$; $B = \{mouse, cat\}$, $C = \{mouse, dog, cat\}$

- $B \subseteq A$, $C \subseteq A$ In fact $B \subset A$(B is a proper subset of A) and $C = A$

- If the elements of a set cannot all be listed(enumerated) then the set is infinite

- Infinite Sets

  - $Nat = \{0, 1, 2, 3, 4, \ldots\}$,
    $EvenNums = \{0, 2, 4, 6, 8, \ldots\}$

  - $OddNums =$
    $\{x \mid x \in Nat$ and x is not divisible by 2$\}$

- It is possible for one infinite set to be a proper subset of another infinite set as the example above shows...Explain

- The elements(objects) of a set could be sets e.g. $\{\{a\},\{a,b\},\{a,b,c\}\}$

- What is the cardinality of this set?

- The empty set: ϕ, $\{\}$

- $A = \{x|1 \leq x \leq 10\}$ NOT ADEQUATE, must say what x must be i.e. integer, rational natural number etc...

- A Singleton set: a set with a single element

- Suppose $A = \{a,b,c\}$ and $B = \{c,d,e\}$

- Union of sets

  - $A \cup B = \{a,b,c,d,e\}$

- Intersection of sets

  - $A \cap B = \{c\}$

- Note that sets are normally denoted by an uppercase(capital) letter whereas elements of sets are normally denoted by lowercase(small) letters

- Consider groupings of two elements
  - Unordered pair: $(a, b) = (b, a)$
  - Ordered pair $< a, b >=< c, d >$ iff $a = c$ and $b = d$
  - Ordered pair $< a, b >\neq< b, a >$ unless $a = b$
- Now consider a set of ordered pairs, i.e. a group which probably contains more than one

- Suppose $A = \{a, b\}$ and $B = \{c\}$ then $< a, c >$ and $< b, c >$ are ordered paiirs where the first element of each ordered pair comes from A and the second element of each ordered pair comes from B
- This leads us to the definition of the cartesian product of two sets:
  - denoted by $A \times B$
  - the set of *all* ordered pairs where the first element comes from A and the second element comes from B

- Cartesian product written mathematically as:
  - $A \times B = \{< a, b > | a \in A \text{ and } b \in B\}$
- Note: $|A \times B| = |B \times A|$. However $A \times B \neq B \times A$
- Do you understand what the notation means?

- Note the curly brackets { and } in the definition of $A \times B$. These denote a set.
- Therefore the Cartesian Product is a set
- A relation: any subset of $A \times B$ is a relation from A to B
  - ie. a relation is a subset of the cartesian product
- Therefore a relation is a set (a set of ordered pairs)

- A function is a special type of relation

- A function is a mapping from one set to another set

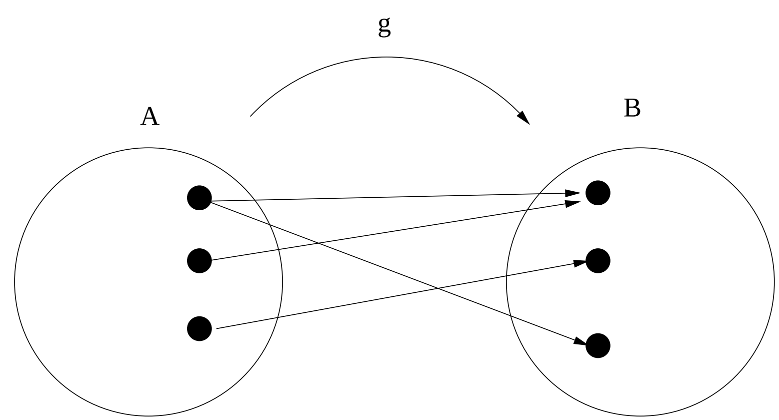
- A function is a set of *ordered* pairs of elements, where the elements of each pair are related in some way (i.e. a relation)

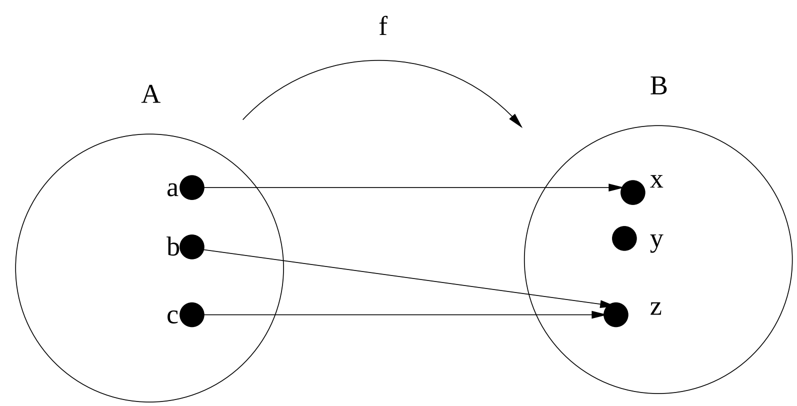
  – i.e. the order of the elements in each pair matters

- But the order in which the pairs are listed doesn't matter

- Explained by example...PTO

- Examples using Venn Diagrams





- In examples on previous page:
  - Are f and g functions? Explain
  - How would you change either (if necessary) to make a function
  - Can you represent f and g as sets of ordered pairs
- Definition: A function from a set A to a set B is a SET of ordered pairs, such that each element of A appears exactly once as the first component(element) of an ordered pair.
- function *type*: $f : A \to B$
- A is the *domain* of the function, B is the *codomain* of the function

- Example of function:
  - $f : \mathbb{R} \to \mathbb{Z}$ (the function *type*)
  - For $r \in \mathbb{R}$, $f(r)$= the greatest integer less than or r (the function definition)
- Example of function:
  - $g : Nat \to Bool$
  - $Is\_Even : Nat \to Bool$
- List some of the elements of these functions

- Summary
  - What is the link between discrete maths, computer science and software engineering?
  - What is a set? Cardinality? Can the elements of sets be other sets?
  - What is a Cartesian Product?
  - What is a Relation?
  - What is a Function?
  - Do you know how all these are linked?
  - Can you give examples of all these?