UNIVERSITY *of* LIMERICK

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COLLEGE *of* INFORMATICS *and* ELECTRONICS

Department of Computer Science
and Information Systems

Final Assessment Paper

Academic Year:	2001/2002	Semester:	Autumn
Module Title:	Data Structures and Algorithms	Module Code:	CS4115
Duration of Exam:	$2\frac{1}{2}$ hours	Percent of Semester Marks:	65
Lecturer:	P. Healy	Paper marked out of:	100

Instructions to Candidates:

- There are three sections to the paper: Multiple Choice Questions, Short Questions and Long Questions
- The mark distribution is 40 marks for Multiple Choice Questions, 20 marks for Short Questions and 40 marks for the Long Questions
- Answer all questions in all sections
- **You *must* return this paper with your answer book and bubble sheet**

Section 1. Multiple Choice Answers (40 marks).

Use the machine-readable multiple-choice question grid that has been provided to answer these questions. Please completely mark in black exactly one circle on the grid for each answer. A penalty will be charged for wrong answers. Mark the **X** bubble for those questions you wish to skip.

1. The number of nodes in a *complete* binary tree of height h is
   (a) exactly $2^{h-1}-1$
   (b) exactly 2^h-1
   (c) exactly $2^{h+1}-1$
   (d) None of the above

2. How many nodes are on the bottom layer, h, of a *perfect* binary tree?
   (a) at least 2^h
   (b) at most 2^h
   (c) exactly 2^h
   (d) none of the above

3. Let $S_1 = \sum_{i=1}^{n} i^2$ and $S_2 = (\sum_{i=1}^{n} i)^3$. Which one of the following statements is true?
   (a) $S_1 = S_2$ for $1 \leq n \leq 30$ only
   (b) $S_1 = S_2$ for $1 \leq n \leq 100$ only
   (c) $S_1 = S_2$ for all n
   (d) None of the above

4. If $f(n) = O(g(n))$ which of the following statements cannot be true?
   (a) $g(n) = O(f(n))$
   (b) $g(n) = \Theta(f(n))$
   (c) $f(n) = o(g(n))$
   (d) $f(n) = \Theta(g(n))$

(please turn over)

5. On the first day of Christmas,
   my true love sent to me
   A partridge in a pear tree.
   On the second day of Christmas,
   my true love sent to me
   Two Zetor tractors, and
   A partridge in a pear tree.
   On the third day of Christmas ...

   How many lines would be in such a "poem" if it ran for 365 days instead of the usual 12?
   (a) $\frac{365\times366}{2} + 2*365$
   (b) $\frac{367\times368}{2} - 3$
   (c) Neither of the above
   (d) Both of the above

6. What is the time-complexity of the following piece of code in "Big-Oh" notation?

```
sum = 0;
for (int i = 0; i < n; i++)
  for (j = 1; j < n; j = j*2)
    sum = sum + n;
```

   (a) $O(n^2)$
   (b) $O(n)$
   (c) $O(\log n)$
   (d) $O(n \log n)$

7. The worst-case performances of the heap operations `deleteMin()` and `insert()` are both $O(\log n)$. Given the two statements below, which of them are true?
   S1 The experimentally found average case performance of `deleteMin()` is $O(1)$
   S2 The experimentally found average case performance of `insert()` is $O(1)$
   (a) Both statements are true
   (b) **S1** is true, but **S2** is false
   (c) **S1** is false, but **S2** is true
   (d) Both statements are false

8. FIX THIS!!! Given the two statements below, which of them are true?
   S1 In a strongly connected graph, every node connects to every other node by an edge
   S2 If a graph is strongly connected then it cannot have a cut vertex (articulation point)
   (a) Both statements are true
   (b) **S1** is true, but **S2** is false
   (c) **S1** is false, but **S2** is true
   (d) Both statements are false

9. FIX THIS!!! Given the two statements below, which of them are true?
   S1 If an n-vertex graph has n articulation points then the graph must have a cycle
   S2 If the Depth-First Tree of a graph G has no back edges then G has no cycles
   (a) Both statements are true
   (b) **S1** is true, but **S2** is false
   (c) **S1** is false, but **S2** is true
   (d) Both statements are false

10. Given the two statements below, which of them are true?
    S1 Starting from vertex v_0 in a graph, the time required by Depth-First Search to find a path (if one exists) to some vertex v^* is *less* than that required by Breadth-First Search
    S2 The space required by Depth-First Search is *less* than that required by Breadth-First Search
    (a) Both statements are true
    (b) **S1** is true, but **S2** is false
    (c) **S1** is false, but **S2** is true
    (d) Both statements are false

Section 2. Short Questions (5 × 4 marks).

- Please put your answers to these questions in the answer book provided to you, labelling your answers 2.1, 2.2, etc.

1. The height of an AVL tree is no worse than _____ times the optimal height.
2. Give the recurrence relation for N_h, the number of nodes in the worst possible AVL tree of height h _____?
3. In a d-heap (a heap where each node can have at most d children), what are the locations of a node's children? The root node of the heap is at location 1. _____
4. Sorting is possible in $o(n \log n)$-time with _____ sort.
5. If a graph has negative edge costs then the "Big-Oh" running time of the shortest path algorithm increases to _____.

Section 3. Long Questions (40 marks).

- Please put your answers to these questions in the answer book provided to you
- Label your answers 3.1, 3.2, and 3.3 in your answer books

1. **(10 marks.)**
   (a) Suppose you have two polynomials P and Q with, respectively, n and m terms. Give an $O(n \times m^2)$-time algorithm for computing the polynomials that is the product of P and Q. The polynomials are to be represented by linked lists and the lists should store the non-zero terms ordered from largest power to smallest power. A picture describing the data structures your algorithm uses will probably help me understand your strategy.
   (b) Weiss considers an algorithm that performs this multiplication in $O(nm \log nm)$-time. Discuss the relative merits of the two algorithms. Support your argument with examples.

2. **(15 marks.)**
   A spelling checker reads an input file and prints out all words not in some on-line dictionary. Suppose the dictionary contains 30,000 words and the file to be checked is large, so that the algorithm can only make one pass over the input file.
   (a) A simple strategy would be to read the dictionary into a hash table and look for each inout word as it is read. Assuming that an average word is seven characters and that is it poissible to store words of length l in $l+1$ bytes (so wasted space is not much of an issue), and assuming a quadratic probing hash table, how much space will this strategy require? (5 marks.)
   (b) If memory is limited and the entire doctionary cannot be stored in a hash table, we can still get an efficient algorithm that almost always works. We declare an array, `table`, of `bool` (initialized to `false`) from 0 to *TableSize-1*. As we read in a word, we set `table[hash(word)] = true`. Suppose we choose *TableSize* = 300,007. Which of the following is true?
      i. If a word hashes to a location with value `false`, the word is not in the dictionary (2 marks.)
      ii. If a word hashes to a location with value `true`, the word is in the dictionary (2 marks.)
      iii. How much memory does this require? (2 marks.)
      iv. What is the probability of an error in this algorithm? (4 marks.)

3. **(15 marks.)**
   Prove that if every vertex in a graph $G = (V, E)$ has degree greater than 1, then there must be some cycle in the graph.