University of Limerick

College of Informatics and Electronics
Department of Computer Science and Information Systems

Spring 2007

Semester : Spring	Academic Year : 2006/07
Module Code : CS4112	Module Title : Computer Science 2
Duration of Exam : 2.5 hours	% of total: 70%
Lecturer(s) : Michael English	Marked out of: 125

Instructions to Candidates :- Question 1 is compulsory. Attempt Question 1 and three other questions. Question 1 carries 50 marks. Questions 2 to 5 inclusive carry 25 marks each. Note that the examiner can take into account the quality of presentation and exposition as well as the content. Calculators are not allowed.

Q1

(a) What is the cardinality of each of the following sets?

- $\{a\}$
- $\{\{a\}\}$
- $\{a, \{a\}\}$
- $\{a, \{a\}, \{a, \{a\}\}\}$

(5 marks)

(b) Construct a truth table for each of the following compound propositions:

(a) $(p \vee q) \vee r$
(b) $(p \vee q) \wedge r$
(c) $(p \wedge q) \vee r$

(5 marks)

(c) Which of the following Hoare Triples are correct? You can assume that x and y are integer values.

(a) $\{X < 5\}x = x + 4\{X < 8\}$
(b) $\{X < 4\}x = x + 5\{X \leq 8\}$
(c) $\{X < Y\}x = y\{X \leq Y\}$

(5 marks)

(d) Using the appropriate rule, show that the following Hoare Triple is correct: $\{a < 2\}$ if $(b < 10)$ $a = 0$ $\{a < 2\}$. (5 marks)

(e) Given two arrays A and B which store bit strings that represent subsets of a set with n elements, complete the Java method definition given below to determine $A \cup B$. The method should output the result(a series of 1's and 0's) to the screen.

```
public void UnionOfSets(int A[], int B[], int size)
{
//insert the appropriate code in your answer book
}
```

(5 marks)

(f) Provide a recursive definition of the function fac which calculates the factorial of a natural number. (5 marks)

(g) Given the following recursive definition of the function Add, demonstrate how $Add(5, 3)$ is evaluated using this definition.
$Add(x, 0) = x$
$Add(x, Sy) = S(Add(x, y))$ (5 marks)

(h) Give an iterative implementation of the Add function defined recursively in the previous part of this question. (5 marks)

(i) Consider a mathematical relation $R_1 \subseteq A \times B$. Suppose $A = B = \{1, \ldots, 10\}$. If the first element of each ordered pair of R_1 is stored in one array in a java program and the second element of each ordered pair of R_1 is stored in another array, then write a java method which will determine if the relation r_1 is a function. (5 marks)

(j) Suppose $A \subseteq \{1, \ldots, 10\}$, where $\{1, \ldots, 10\}$ is the universal set. Write a java method which takes a bit string representation of A and determines the complement of A. (5 marks)

Q2

(a) List the members of the following sets:

- $\{x \mid x$ is a real number such that $x^2 = 1\}$
- $\{x \mid x$ is a positive integer less than $12\}$
- $\{x \mid x$ is an integer and $x < 100\}$
- $\{x \mid x$ is an integer such that $x^2 = 2\}$

(4 marks)

(b) Suppose $A = \{a, b\}$ and $B = \{1, 2\}$. Compute the Cartesian Product of $A \times B$ and $B \times A$. Is $A \times B = B \times A$? Explain. (6 marks)

(c) Suppose $A = \{a, b\}$ and $B = \{1, 2\}$. How many relations exist between A and B? How many of these relations are functions that have both a and b as the first component of ordered pairs? Explain. (6 marks)

(d) If $f : Nat \times Nat \to Nat$, given by $f(x, y) = x * y$, show that f is onto. Is f one-to-one? (3 marks)

(e) Identify propositions, P, Q in the following argument and then formalise it using the logical symbols. Confirm your answer using truth tables:
If 6 is a prime, 6 cannot be equal to 2 times 3. 6 is equal to 2 times 3. Therefore 6 cannot be prime. (6 marks)

Q3

(a) What is an assertion? (2 marks)

(b) What does it mean to strengthen an assertion? (2 marks)

(c) Give an example of a condition which is stronger than $x > 5$ and another condition that is weaker than $x > 5$ (4 marks)

(d) Is it possible to strengthen a postcondition or weaken a precondition and still ensure the correctness of a Hoare triple? Explain. (4 marks)

(e) State a rule which can be applied to prove the correctness of an assignment statement. (3 marks)

(f) Draw a flowchart to illustrate an if-then-else statement. (2 marks)

(g) Based on the paths in this flowchart provide a rule which can be applied to demonstrate the correctness of if-then-else statements. (4 marks)

(h) Apply this rule to demonstrate the correctness of the following Hoare Triple:
$\{i \geq j - 1\}$
if $i > j$ $\{j = j + 1\}$ else $\{i = i + 1\}$
$\{i \geq j\}$ (4 marks)

Q4

(a) Give an inductive definition of the sq function which calculates x^2 for $x \in Nat$. (4 marks)

(b) Give a recursive implementation (a java method) of the sq function. (5 marks)

(c) Give an iterative implementation (a java method) of the sq function. (5 marks)

(d) Construct a flowchart for the iterative implementation and annotate the flowchart with assertions. (6 marks)

(e) Using the assertions and the inductive definition of Sum prove that the iterative implementation is correct. In other words show that if the loop is never executed the result is correct and that if the loop executes correctly i times and is executed once more that it is till correct. (5 marks)

Q5

(a) Given the sets A and B of size n:

- Write a java method that will store these sets as arrays. (5 marks)
- Write another java method that will take the 2 arrays as parameters and output the intersection of these sets. (5 marks)
- Write another java method that stores $A \cap B$ in a separate array. (3 marks)

(b) A binary tree can be described as (A, c, B), where c is the root node, A is its left subtree and B is its right subtree. Using this notation define the set of binary trees recursively. (4 marks)

(c) Define a function h recursively which specifies the height of a binary tree. Remember an empty tree has height 0. (4 marks)

(d) Prove, using structural induction that the maximum number of nodes in a tree of height h is $2^h - 1$. (4 marks)