



UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

Faculty of Science and Engineering

Department of Mathematics and Statistics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MS4111

SEMESTER: Spring 2007-08

MODULE TITLE: Discrete Mathematics 1

DURATION OF EXAMINATION: 2 1/2 hours

LECTURER: E. MacAogáin

PERCENTAGE OF TOTAL MARKS: 80%

EXTERNAL EXAMINER: Prof. J. King

INSTRUCTIONS TO CANDIDATES: Answer four questions. All questions are weighted equally. Give the reasoning for your answers.

- 1 (a) Let $A = \{1, 2, 3, \dots, 10\}$ and $B = \{2, 4, 6, \dots, 12\}$. Find:
- (i) $A \cup B$ 1
 - (ii) $A \cap B$ 1
 - (iii) $A - B$ 2
 - (iv) $(A - B) \cup (B - A)$ 2
- (b) Let $A = \{a, b, c\}$ and $B = \{1, 2\}$. Find $A \times B$. 4
- (c) Let C , D and E be three sets. Simplify: $\overline{C \cap (D - E)} \cup C$ 5
- (d) Prove, using any method, that a set of order n has 2^n subsets. 5
- 2 (a) Find:
- (i) $\gcd(140, 48)$ 2
 - (ii) $\text{lcm}(35, 14)$ 2
- (b) Convert the following numbers to decimal numbers:
- (i) 11001_2 2
 - (ii) 104_8 2
 - (iii) $A2B_{16}$ 2
- (c) Using the Binomial Theorem, or otherwise, expand: $(2a - b)^4$ 5
- (d) Prove, using mathematical induction, that: 5

$$2 + 4 + 6 + \dots + 2n = n(n + 1), \forall n \in \mathbf{N}$$

(where \mathbf{N} is the set of natural numbers.)

- 3 (a) For each of the following relations $*$ on the given sets, which of the properties reflexivity, symmetry, transitivity do they have?
- (i) Set P of living persons: $x * y$ iff x is married to y . 3
 - (ii) Set \mathbf{Z} of integers: $x * y$ iff $x < y$. 3
 - (iii) Set \mathbf{Z} : $x * y$ iff $x + y$ is even. 3
- (b) Let $m \in \mathbf{N}$. Prove that $x \equiv y \pmod{m}$ is an equivalence relation on \mathbf{Z} . 5
- (c) Find the set of all integers between -12 and 12 such that:
- $$x \equiv 3 \pmod{7}$$
- 3
- (d) Find the set of all integers between -12 and 12 such that:
- $$2x \equiv 1 \pmod{3}$$
- 3
- 4 (a) Show that the following is a contradiction: $(P \wedge (Q \vee (\sim P))) \wedge (\sim Q)$ 3
- (b) Form the negation of the following statement: $3 < 5$ and 4 is even. 3
- (c) Write down the converse, inverse and contrapositive of the following proposition:
if he's old then he's wise. 6
- (d) For the following truth table:
- (i) write down the disjunctive normal form of the function 2
 - (ii) simplify algebraically 3
 - (iii) simplify using Karnaugh maps. 3

P	Q	f(P,Q)
T	T	T
T	F	F
F	T	T
F	F	T

- 5 (a) Prove directly that if x and y are even integers then xy is an even integer. 5
- (b) Use the contrapositive to prove:
if the square of an integer is odd then the integer is odd. 7
- (c) Prove by contradiction (i.e. "reductio ad absurdum") that $\sqrt{3}$ is irrational. 8

- 6 (a) Find the general solution of the following recurrence relation:

$$a_n = 5a_{n-1} - 6a_{n-2}$$

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- (b) Find the particular solution of the above recurrence relation which satisfies the initial conditions:

$$a_0 = 2, a_1 = 5$$

Hence evaluate a_5 .

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- (c) Find the general solution of the following recurrence relation:

$$a_n = 5a_{n-1} - 6a_{n-2} + 10$$

[Hint: see part(a)]

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