DISCRETE MATHEMATICS 1 (MS4111): TUTORIAL 4

1. Give the definition of a **valid** and **invalid argument**.

2. Check whether the following arguments are valid or not.

   (a)
   $$\begin{array}{l} p \Rightarrow q \\ \overline{p} \\ \hline \therefore \overline{q} \end{array}$$

   (b)
   $$\begin{array}{l} p \Rightarrow (q \Rightarrow r) \\ q \Rightarrow (p \Rightarrow r) \\ \hline \therefore (p \vee q) \Rightarrow r \end{array}$$

3. Formulate the arguments below symbolically and determine whether they are valid or not.

   (a)
   $$\begin{array}{l} \text{If I study hard, then I get A's} \\ \text{I study hard} \\ \hline \therefore \text{I get A's.} \end{array}$$

   (b)
   $$\begin{array}{l} \text{If I study hard or I get rich, then I get A's} \\ \text{I get A's} \\ \hline \therefore \text{If I don't study hard, then I get rich.} \end{array}$$

4. Define the concepts of **relation from a set X to a set Y** and of **relation on a set X**.

5. Let $X = \{1, 2, 3\}$ and consider the relation $\mathcal{R}$ on X given by

   $$\mathcal{R}\{(1,2), (2,1), (3,3), (1,1), (2,2)\}$$

   (a) What are the domain and the range of $\mathcal{R}$?

   (b) What can you say about $\mathcal{R}$?

   (c) Draw the digraph of $\mathcal{R}$.

   (d) List the elements of $\mathcal{R}^{-1}$.

6. Let $X = \{1, 2, 3, 4\}$ and let $\mathcal{R}_1$ and $\mathcal{R}_2$ be the relations on X given by

$$\begin{aligned} \mathcal{R}_1 &= \{(1,1), (1,2), (3,4), (4,2)\} \\ \mathcal{R}_2 &= \{(1,1), (2,1), (3,1), (4,4), (2,2)\}. \end{aligned}$$

   List the elements of $\mathcal{R}_1 \circ \mathcal{R}_2$ and $\mathcal{R}_2 \circ \mathcal{R}_1$.

7. Consider the set of non-negative integers

$$\mathbb{N} = \{0, 1, 2, \dots\}$$

   and consider the following relations on $\mathbb{N}$

   (a) $(x, y) \in \mathcal{R}$ if $x = y^2$;
   (b) $(x, y) \in \mathcal{R}$ if $x > y$.

   Determine whether relations (a) and (b) are an order and if that is the case then specify whether the order is partial or total.

8. Let $X = \{1, 2, 3\}$ and consider the relation $\mathcal{R}$ on X defined by

$$(x, y) \in \mathcal{R} \qquad \text{if} \quad x^2 \geq y.$$

   (a) List the elements of $\mathcal{R}$.
   (b) What are the domain and the range of $\mathcal{R}$?
   (c) What can you say about $\mathcal{R}$?
   (d) Draw the digraph of $\mathcal{R}$.