

$f(x,y,z) = x^2 + y^2 + z^2 - r^2 = 0$ being the equation for a sphere of radius r we want to find some vector on the tangent plane to the sphere at (x,y,z) to use for an up vector, however there are infinitely many such vectors. Projecting the “standard” up vector onto the tangent plane seems like a reasonable compromise.

The gradient $\vec{n} = \nabla f = (2x, 2y, 2z)$ is normal to the sphere at (x,y,z) . To find the projection Y_p of the Y axis $Y = (0,1,0)$ onto the tangent to the sphere at point (x,y,z) we have

$$Y_p = Y - \frac{(Y \cdot \vec{n})}{\|\vec{n}\|^2} \vec{n} = \left(-\frac{xy}{r^2}, -\frac{y^2}{r^2} + 1, -\frac{yz}{r^2}\right)$$