HW1

1.

min $\rho L A_s = \rho L \pi (R_o^2 - R_i^2)$

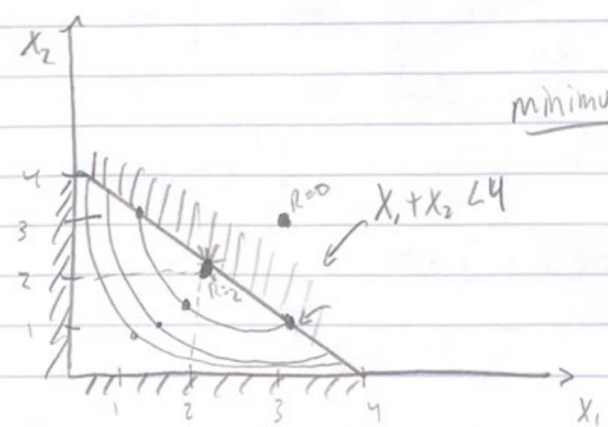
such that:

- $\frac{\rho L R_o}{I} \le \sigma_{max}$ where $I = \frac{1}{2} M (R_o^2 + R_i^2) = \frac{1}{2} \rho A (R_o^2 + R_i^2)$, $M = \rho A$
- $\frac{P}{3I} (R_o^2 + R_o R_i + R_i^2) \le \tau_{max}$
- $R_i \le R_o \le 40\,cm$

with $\rho = 7850\ kg/m^3$; $P = 14\,kN$; $L = 10\,m$; $\sigma_b = 165\,MPa$; $\tau_a = 50\,MPa$

$$\boxed{\begin{aligned} &\min \quad 78500\pi (R_o^2 - R_i^2) \quad \frac{kg\,kN}{m^3} \\ &\text{S.T.} \quad - \frac{10 R_{on}}{\frac{1}{2}\pi (R_o^2 - R_i^2)(R_o^2 + R_i^2)} - 165\,MPa \le 0 \\ &\quad - \frac{14\,kN}{3(\frac{1}{2}\rho A)(R_o^2 + R_i^2)} (R_o^2 + R_o R_i + R_i^2) - 50\,MPa \le 0 \\ &\quad - R_i \le R_o \le 40\,cm. \end{aligned}}$$

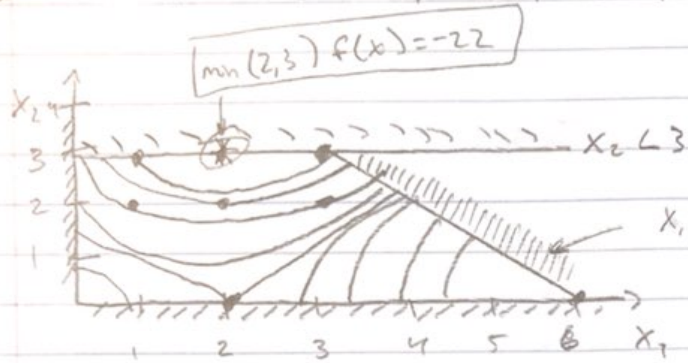
2. a. min $(x_1 - 3)^2 + (x_2 - 3)^2$ s.t. $x_1 + x_2 \le 4$; $-x_1, -x_2 < 0$



minimum →

x_1	x_2	R
2	2	2
3	3	0
1	3	4
3	1	4

b. min $x^2 - 2x_2^2 - 4x_1$ S.T. $x_1 + x_2 < 6$; $x_2 < 3$; $-x_1, -x_2 \le 0$



x_1	x_2	$f(x)$
3	3	-21
1	3	-21
2	3	-22
3	2	-11
2	2	-12
1	2	-11

3. 1. True 2. False 3. True 4. False 5. True
6. false 7. false 8. false 9. True 10. false
11. True 12. True 13. false 14. false 15. True
16. false 17. false 18. True 19. True 20. True.

4. a. e^x about $x^* = 2$ note: $f(x) \approx f(x^*) + \frac{\partial f(x^*)}{\partial x}(x - x^*) + \frac{1}{2}\frac{\partial^2 f(x^*)}{\partial x^2}(x - x^*)^2 + \dots$

$$f(x) = \frac{\partial f(x)}{\partial x} = \frac{\partial^2 f(x)}{\partial x^2} = e^x$$

So $\boxed{f(2) = e^2 \approx e^2 + e^2(x-2) + \frac{1}{2}e^2(x-2)^2 + e.}$

b. $f(x_1, x_2) = 10x_1^4 - 20x_1^2 x_2 + 10x_2^2 + x_1^2 - 2x_1 - 5$ @ point (1,1)

$$\nabla f(x) = \begin{bmatrix} 40x_1^3 - 40x_1 x_2 + 2x_1 - 2 \\ 0 - 20x_1^2 + 20x_2 \end{bmatrix} \text{ @ }(1,1) \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$Hf(x) = \begin{bmatrix} 120x_1^2 - 40x_2 + 2 & -40x_1 \\ -40x_2 & 20 \end{bmatrix} \text{ @ }(1,1) \begin{bmatrix} 82 & -40 \\ -40 & 20 \end{bmatrix}$$

$$\bar{f}(1,1) \approx 4 + \frac{1}{2}\left(\underset{\sim}{x} - \begin{pmatrix}1\\1\end{pmatrix}\right)^T \begin{bmatrix} -82 & -40 \\ -40 & 20 \end{bmatrix}\left[x - \begin{pmatrix}1\\1\end{pmatrix}\right]$$

$f(1.2, 0.8) = 8.136$ $\bar{f}(1.2, 0.8) = 7.64$

error = 0.496

5. $F(x) = x_1^2 + 4x_1x_2 + 2x_1x_3 - 7x_2^2 - 6x_2x_3 + 5x_3^2$

$$\nabla f(x) = \begin{bmatrix} 2x_1 + 4x_2 + 2x_3 \\ 4x_1 - 14x_2 - 6x_3 \\ 2x_1 - -6x_2 + 10x_3 \end{bmatrix}$$

$$Hf(x) = \begin{bmatrix} 2 & 4 & 2 \\ 4 & -14 & -6 \\ 2 & -6 & 10 \end{bmatrix} \longrightarrow \lambda = -16.45,\ 2.91,\ 11.54$$

since There is a neghtive eigen value Then $F(x)$ is indefinite.

6. a. $f(x_1, x_2) = x_1^2 + 2x_1 + 4x_2^2 - 8x_2 + 6$

$$\nabla f(\underset{\sim}{x}) = \begin{bmatrix} 2x_1 - 2 \\ 8x_2 - 8 \end{bmatrix} \qquad Hf(\underset{\sim}{x}) = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}$$

$\nabla f(\underset{\sim}{x}) = 0$

$2x_1 - 2 = 0 \longrightarrow x_1 = 1$

$8x_2 = 8 = 0 \longrightarrow x_2 = 1$

$\boxed{x^* = (1,1) \leftarrow \text{local min b/c. } Hf(\underset{\sim}{x}) \geq 0}$

b. $f(x_1, x_2) = 3x_1^2 - 2x_1x_2 + 5x_2^2 + 8x_2$

$$\nabla f(\underset{\sim}{x}) = \begin{bmatrix} 6x_1 - 2x_2 \\ -2x_1 + 10x_2 + 8 \end{bmatrix} \qquad Hf(x) = \begin{bmatrix} 6 & -2 \\ -2 & 10 \end{bmatrix}$$

$\nabla f(\underset{\sim}{x}) = 0$

$6x_1 - 2x_2 = 0 \longrightarrow 6x_1 = 2x_2 \longrightarrow x_2 = 3x_1$

$-2x_1 + 10x_2 + 8 = 0 \longrightarrow -2x_1 + 10(3x_1) + 8 = 0 \longrightarrow 28x_1 + 8 = 0$

$x_1 = -2/7 \longrightarrow x_2 = -6/7$ $\qquad x_1 = -2/7$

$$\det(Hf(x)) = \begin{vmatrix} 6-\lambda & -2 \\ -2 & 10-\lambda \end{vmatrix} = \lambda^2 - 16\lambda + 56 = 0 \longrightarrow \lambda_i = 5.17,\ 10.8$$

$\lambda_1, \lambda_2 > 0$

So it is positive Definite and will be a Local min.

$\boxed{x^* = \left(-2/7,\ -6/7\right) - \text{Local minimum}}$

6 c. $f(x_1, x_2) = -4x_1 + 2x_2 + 4x_1^2 - 4x_1x_2 + 2x_2^2$

$\nabla f(\underset{\sim}{x}) = \begin{bmatrix} -4 + 8x_1 - 4x_2 \\ 2 - 4x_1 + 4x_2 \end{bmatrix}$ $\quad Hf(x) = \begin{bmatrix} 8 & -4 \\ -4 & 4 \end{bmatrix}$

$-4 + 8x_1 - 4x_2 = 0 \qquad x_2 = 0$

$2 - 4x_1 + 4x_2 = 0 \qquad x_1 = 1/2$

$\det(Hf(x)) = \begin{vmatrix} 8-\lambda & -4 \\ -4 & 4-\lambda \end{vmatrix} = \lambda^2 - 12\lambda + 16 = 0$

$\lambda = 1.527, \quad \lambda = 10.47$

Since $\lambda > 0$ the $Hf(x)$ is positive Definite
So $x^* = (1/2, 0)$ – Local minimum

7. $\min (x_1 - 1)^2 + (x_2 - 1)^2$

S.T. $x_1 + x_2 - 4 = 0$

$x_1 - x_2 - 2 = 0$

a. This is not a valid optimization problem because there is only one solution @ (3, 1).

b. you Do not need Necessary Conditions

8. In matLab, graph attached TO HW.