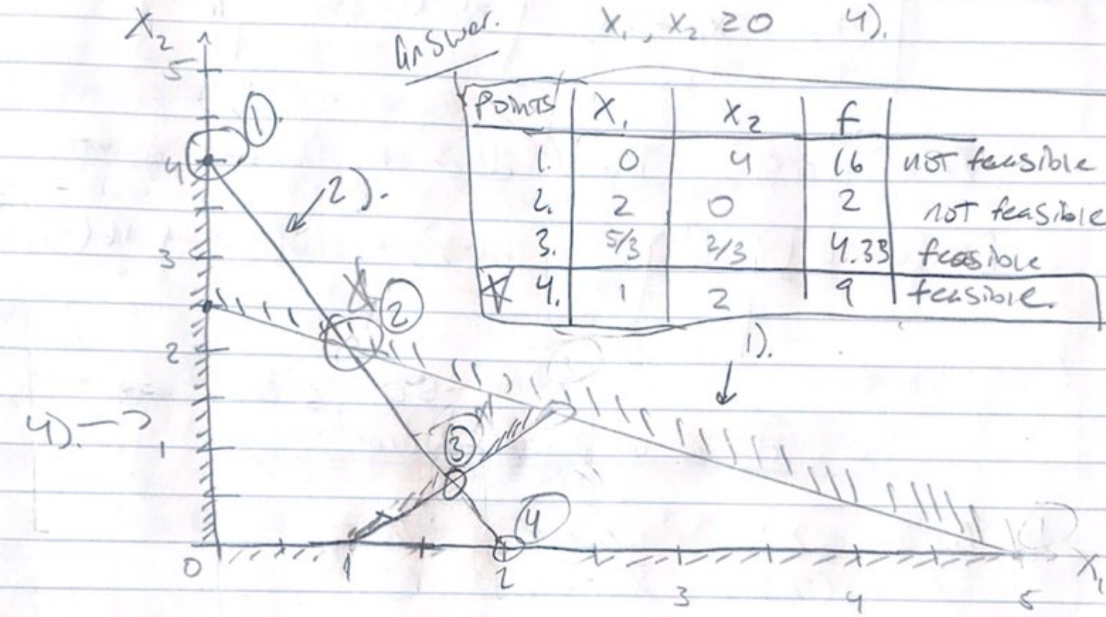
Hw3

1. Max: $z = x_1 + 4x_2$ S.T.

$$x_1 + 2x_2 \le 5 \quad 1).$$
$$2x_1 + x_2 = 4 \quad 2).$$
$$x_1 - x_2 \ge 1 \quad 3).$$
$$x_1, x_2 \ge 0 \quad 4).$$



Answer.

Points	x_1	x_2	f	
1.	0	4	16	not feasible
2.	2	0	2	not feasible
3.	5/3	2/3	4.33	feasible
✓ 4.	1	2	9	feasible.

Point ~~1~~ 4: $x = (2, 0)$

Point ~~2~~ 3: $2x_1 + x_2 = 4$

$$x_1 - x_2 = 1$$
$$3x_1 = 5$$
$$x_1 = 5/3$$
$$x_2 = 2/3$$
$$X = (5/3, 2/3)$$

$\boxed{\text{Point } (1, 2) \leftarrow \text{Max}}$

HW 3

2. Maximize $z = x_1 + 0.5x_2$ S.T. $6x_1 + 5x_2 \leq 30$

$3x_1 + x_2 \leq 12$

$x_1 + x_2 \leq 12$

$x_1, x_2 \geq 0$

Putting into Standard form:

min $z = -x_1 - 0.5x_2$ S.T. $6x_1 + 5x_2 + S_1 = 30$

$3x_1 + x_2 + S_2 = 12$

$x_1 + x_2 + S_3 = 12$

$x_1, x_2 \geq 0 \quad S_i \geq 0$

Table 1:

Basic	x_1	x_2	S_1	S_2	S_3	b	ratios
S_1	6	5	1	0	0	30	$30/6 = 5$
S_2	3	1	0	1	0	12	$12/3 = 4$
S_3	1	1	0	0	1	12	$12/1 = 12$
Cost f:	-1	-0.5	0	0	0	f	

initial basic feasible solution:
- basic variables: $S_1 = 30$, $S_2 = 12$, $S_3 = 12$
- non basic variables: $x_1, x_2 = 0$. $f = 0$

row w/ S_2 has smallest ratio so S_2 will become a non basic variable. (1. $\frac{1}{3} \cdot$ row2 ; 2. row1 − 6row2 ; 3. row3 − row2
4. ~~row~~ cost row + row2

- Creating a new table:

	x_1	x_2	S_1	S_2	S_3	b	ratio
S_1	0	3	1	-2	0	6	$6/3 = 2$
x_1	1	1/3	0	1/3	0	4	$4/\frac{1}{3} = 12$
S_3	0	2/3	0	-1/3	1	8	$8/\frac{2}{3} = 12$
Cost f:	0	-1/6	0	1/3	0	f + 4	

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now we have a new ~~...~~ Table to Look at.

~~...~~ Basic Variables : X_1 , S_1 , S_3

non basic Variables : X_2 , S_2

- Since there is still a negative # in the cost function row. ~~...~~ it is not an optimum solution and need to repeat the steps taken. Elimination will be "done" by X_2 column and basic Variable to become nonbasic is S_1 since it had lowest ratio. (Step 1: Divide row 1 by 3. ; Step 2: row 2 - 1/3 new row 1 Step 3: row 3 - 2/3 new row 1 ; Step 4: row 4 + 1/6 new row 1

basic	x_1	x_2	S_1	S_2	S_3	b
X_1	0	1	1/3	-2/3	0	2
X_2	1	0	-1/3	5/9	0	10/3
S_3	0	0	-1/9	-1/9	1	22/3
Cost function	0	0	1/18	2/9	0	f+4.33

all columns in cost function are now ≥ 0 so we found optimum.

$\boxed{\begin{array}{l} \text{basic Variables } X_1 = 2 \ ; \ X_2 = 10/3 \ ; \ S_3 = 22/3 \\ \text{Non basic Variables } S_1 = 0 \ , \ S_2 = 0 \\ \text{Cost function: } 0 = f + 4.33 \rightarrow f = -4.33 \end{array}}$

Hw 3.

3. $x_1 + x_2 + x_3 + x_4 = 2$
$2x_1 + x_2 - x_3 + x_4 = 2$
$-x_1 + 2x_2 + 3x_3 + x_4 = 1$
$3x_1 + 2x_2 - 2x_3 - x_4 = 8$

$$\longrightarrow \begin{array}{c|c} & \begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & b \end{array} \\ \begin{array}{c} \text{row1} \\ \text{row2} \\ \text{row3} \\ \text{row4} \end{array} & \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 2 & 1 & -1 & 1 & 2 \\ -1 & 2 & 3 & 1 & 1 \\ 3 & 2 & -2 & -1 & 8 \end{array}\right] \end{array}$$

Step 1. row 3 by -1

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 2 & 1 & -1 & 1 & 2 \\ -1 & 2 & 3 & 1 & 1 \\ 3 & 2 & -2 & -1 & 8 \end{array}\right]$$

Step 2. row 2 - 2·row1

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -1 & -3 & -1 & -2 \\ -1 & 2 & 3 & 1 & 1 \\ 3 & 2 & -2 & -1 & 8 \end{array}\right]$$

Step 3. add row 1 and 3

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -1 & -3 & -1 & -2 \\ 0 & 3 & 4 & 2 & 3 \\ 3 & 2 & -2 & -1 & 8 \end{array}\right]$$

Step 4. row4 - 3row1

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -1 & -3 & -1 & -2 \\ 0 & 3 & 4 & 2 & 3 \\ 0 & -1 & -5 & -4 & 2 \end{array}\right]$$

Step 5: multiply row 2 by -1

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 1 & 2 \\ 0 & 3 & 4 & 2 & 3 \\ 0 & -1 & -5 & -4 & 2 \end{array}\right]$$

Step 6: row 3 - 3 row 2

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 1 & 2 \\ 0 & 0 & -5 & -1 & -3 \\ 0 & -1 & -5 & -4 & 2 \end{array}\right]$$

Step 7: row 4 + row 2

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 1 & 2 \\ 0 & 0 & -5 & -1 & -3 \\ 0 & 0 & -2 & -3 & 4 \end{array}\right]$$

→ 2

Step 8: row 3 by -1/5

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 1 & 2 \\ 0 & 0 & 1 & 0.2 & 0.6 \\ 0 & 0 & -2 & -3 & 4 \end{array}\right]$$

Solution

$$\boxed{\begin{aligned} x_1 &= 2 \\ x_2 &= 1 \\ x_3 &= 1 \\ x_4 &= -2 \end{aligned}}$$

Step 9: row 4 + 2 row 3

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 1 & 2 \\ 0 & 0 & 1 & 0.2 & 0.6 \\ 0 & 0 & 0 & -2.6 & 5.2 \end{array}\right]$$

Step 10: Divided row 4 by 2.6

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 1 & 2 \\ 0 & 0 & 1 & 0.2 & 0.6 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

row 4: So $x_4 = -2$

row 3: $x_3 + 0.2x_4 = 0.6 \rightarrow x_3 = 0.6 - 0.2(2) = 1$

row 2: $x_2 + 3x_3 + x_4 = 2 \rightarrow x_2 = 2 - 3(1) - -2) = 1$

row 1. $x_1 + x_2 + x_3 + x_4 = 2 \rightarrow x_1 = 2 - 1 - 1 - -2$

$x_1 = 2$

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3. $f(x) = x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_2 + 2x_2x_3$

$$\nabla f(x) = \begin{bmatrix} 2x_1 + 2x_2 \\ 4x_2 + 2x_1 + 2x_3 \\ 4x_3 + 2x_2 \end{bmatrix} = @(1,2,3) = \begin{bmatrix} 6 \\ 16 \\ 16 \end{bmatrix}$$

$\vec{\nabla f}(x)^T \vec{d} \leq 0$ Then d is a Descent direction.

$$[6\ 16\ 16] \begin{bmatrix} -3 \\ 10 \\ -12 \end{bmatrix} = 6(-3) + 16(10) + 16(-12)$$

$$= -50 \leq 0$$

So $\vec{d}$ is a descent Direction

4. $f(x) = \frac{1}{2}x_1^2 + x_2^2 - x_1x_2 - 7x_1 - 7x_2$
w/ $d = (7,6)$ $\vec{X} = (1,1)$

$$\bar{f}(\alpha) = f(x^{(k)} + \alpha_k d^{(k)})$$
$$= f\left(\begin{pmatrix}1\\1\end{pmatrix} + \alpha_k \begin{pmatrix}7\\6\end{pmatrix}\right) = f\begin{pmatrix}1+7\alpha_k \\ 1+6\alpha_k\end{pmatrix}$$

$$\bar{f}(\alpha) = \frac{1}{2}(1+7\alpha)^2 + (1+6\alpha)^2 - (1+7\alpha)(1+6\alpha) - 7(1+7\alpha) - 7(1+6\alpha)$$
$$= \frac{1}{2}(49\alpha^2 + 14\alpha + 1) + (36\alpha^2 + 12\alpha + 1) - (1 + 13\alpha + 42\alpha^2) - (7 + 49\alpha) - (7 + 42\alpha)$$
$$= \frac{49}{2}\alpha^2 + 7\alpha + \frac{1}{2} + 36\alpha^2 + 12\alpha + 1 - 1 - 13\alpha - 42\alpha^2 - 7 - 49\alpha - 7 - 42\alpha$$

$$\boxed{\bar{f}(\alpha) = 18.5\alpha^2 - 85\alpha - 13.5}$$

5. $F(x) = x_1^2 + 2x_2^2 - 4x_1 - 2x_1x_2 \qquad x^0 = (1, 1)$

$$\nabla f(x) = \begin{bmatrix} 2x_1 - 4 - 2x_2 \\ 4x_2 - 2x_1 \end{bmatrix}$$

$$C = -\nabla f(x_0) = \begin{bmatrix} 2x_1 - 4 - 2x_2 \\ 4x_2 - 2x_2 \end{bmatrix} \overset{@x_0}{=} \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$\bar{F}(\alpha_0) = f(x_0 + \alpha \underbrace{[\cancel{-\nabla f(x_0)}]}_{C}) = f\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \alpha \begin{pmatrix} 4 \\ -2 \end{pmatrix}\right) = f\begin{pmatrix} 1 + 4\alpha \\ 1 - 2\alpha \end{pmatrix}$$

$$= (1+4\alpha)^2 + 2(1-2\alpha)^2 - 4(1+4\alpha) - 2(1+4\alpha)(1-2\alpha)$$
$$= 16\alpha^2 + 8\alpha + 1 + 2(4\alpha^2 - 4\alpha + 1) - 4 - 16\alpha - 2(1 + 2\alpha - 8\alpha^2)$$
$$= 16\alpha^2 + 8\alpha + 1 + 8\alpha^2 - 8\alpha + 1 - 4 - 16\alpha - 2 - 4\alpha + 16\alpha^2$$

$$= 40\alpha^2 - 20\alpha + 4 = \bar{f}(\alpha)$$

$$\frac{\partial \bar{F}(\alpha_0)}{\partial \alpha} = 80\alpha - 20 \longrightarrow \alpha_0 = 20/80 = 1/4$$

$$X^1 = X^0 + \alpha \begin{pmatrix} 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 + 1 \\ 1 - 1/2 \end{pmatrix} = \boxed{\begin{pmatrix} 2 \\ 1/2 \end{pmatrix} = X^{(1)}}$$

$$C = -\nabla f(x_1) = -\begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\bar{f}(\alpha_1) = \bar{f}(x_1 + \alpha C) = f\left(\begin{matrix} 2 \\ 1/2 \end{matrix} + \alpha \begin{pmatrix} 1 \\ 2 \end{pmatrix}\right) = f\begin{pmatrix} 2 + \alpha \\ 1/2 + 2\alpha \end{pmatrix}$$

$$= (2+\alpha)^2 + 2(1/2 + 2\alpha)^2 - 4(2+\alpha) - 2(2+\alpha)(1/2 + 2\alpha)$$
$$= \alpha^2 + 4\alpha + 4 + 2(4\alpha^2 + 2\alpha + 1/4) - 4 - 4\alpha - 2(2\alpha^2 + 4.5\alpha + 1)$$
$$= \alpha^2 + 4\alpha + 4 + 8\alpha^2 + 4\alpha + 1/2 - 4 - 4\alpha - 4\alpha^2 - 9\alpha - 2$$

$$\bar{f}(\alpha_1) = 5\alpha^2 - 5\alpha - 3/2$$

$$\frac{\partial \bar{f}(\alpha)}{\partial \alpha} = 10\alpha - 5 \longrightarrow \alpha = 1/2$$

$$X^2 = X^1 + \alpha C = \begin{pmatrix} 2 \\ 1/2 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{matrix} 2 + 1/2 \\ 1/2 + 1 \end{matrix} = \boxed{\begin{pmatrix} 2.5 \\ 1.5 \end{pmatrix} = X^{(2)}}$$

6. Standard form: Point $= (6,15) \rightarrow (r,h)$

$\min -V = -\pi r^2 h$ S.T. $g_1 = -2\pi r h - 900 \le 0$

$g_2 = r \le 20 \rightarrow r - 20 \le 0$

$g_3 = -r \le 5 \rightarrow -r - 5 \le 0$

$g_4 = h \le 20 \rightarrow h - 20 \le 0$

$g_5 = -h \le 0 \rightarrow -h \le 0$

$$\nabla f(r,h) = \begin{bmatrix} -2\pi r h \\ -\pi r^2 \end{bmatrix}$$

$$\nabla g_1 = \begin{bmatrix} -2\pi h \\ -2\pi r \end{bmatrix} \quad \nabla g_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\nabla g_3 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad \nabla g_4 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \nabla g_5 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$\bar{f}(r,h) \cong f(6,15) + \nabla f(6,15)^T \left(X - \begin{pmatrix} r \\ h \end{pmatrix}\right)$$

$$= -540\pi + \begin{bmatrix} -90\pi & -36\pi \end{bmatrix} \left[\begin{matrix} r \\ h \end{matrix} - \begin{pmatrix} 6 \\ 15 \end{pmatrix}\right]$$

$$= -540\pi - 90\pi(r-6) - 36\pi(h-15)$$

$$= -90\pi r - 36\pi h + 540\pi$$

$$\bar{g}_1(r,h) = g_1(6,15) + \nabla g_1(6,15)^T \left(\begin{pmatrix} r \\ h \end{pmatrix} - \begin{pmatrix} 6 \\ 15 \end{pmatrix}\right)$$

$$= -180\pi - 900 + \begin{bmatrix} -30\pi & -12\pi \end{bmatrix} \begin{pmatrix} r-6 \\ h-15 \end{pmatrix}$$

$$= -180\pi - 900 - 30\pi(r-6) - 12\pi(h-15)$$

$\bar{g}_2(r,h) = g_2$; $\bar{g}_3(r,h) = g_3$ $\bar{g}_4 = g_4$ $\bar{g}_5 = g_5$

- Plot is graphed in matlab.
- The g_1 constraint is off the plotted area.