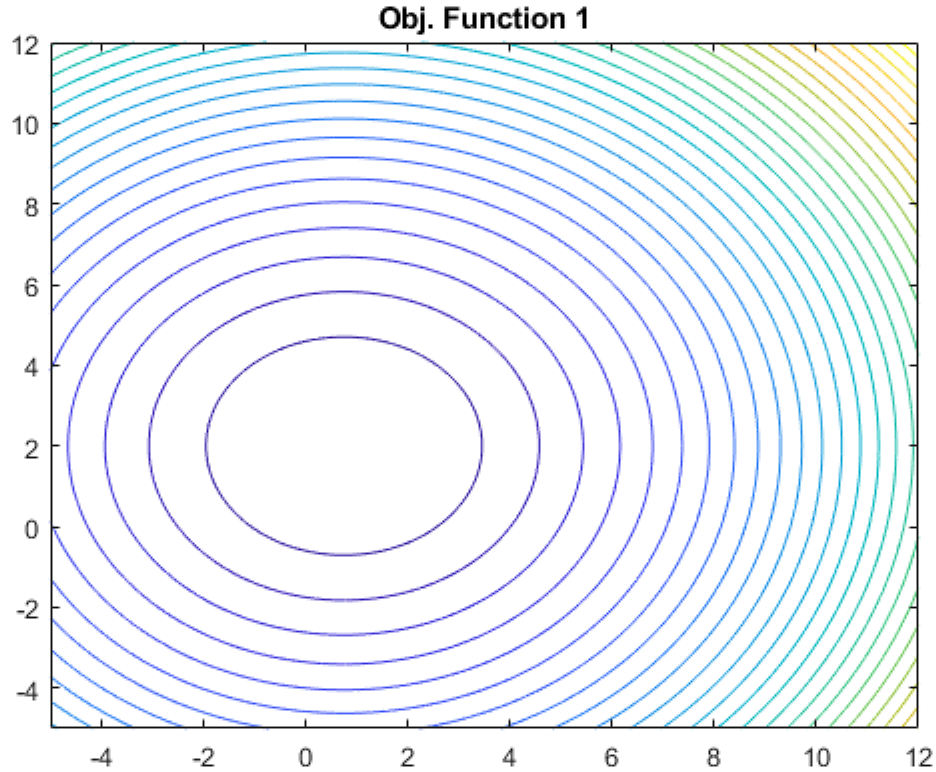
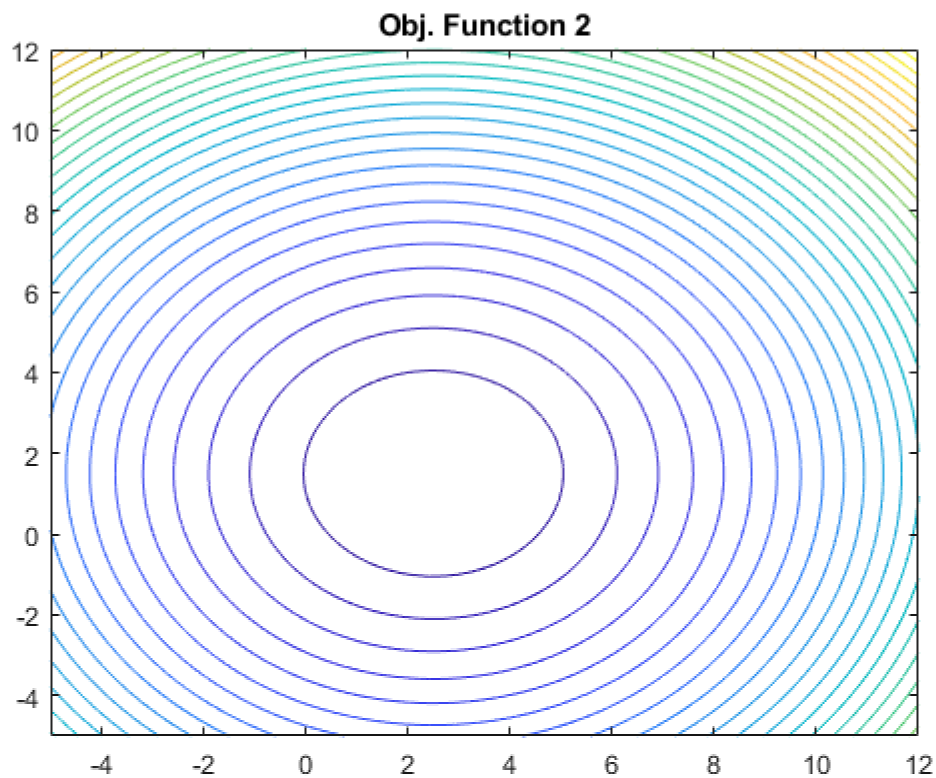
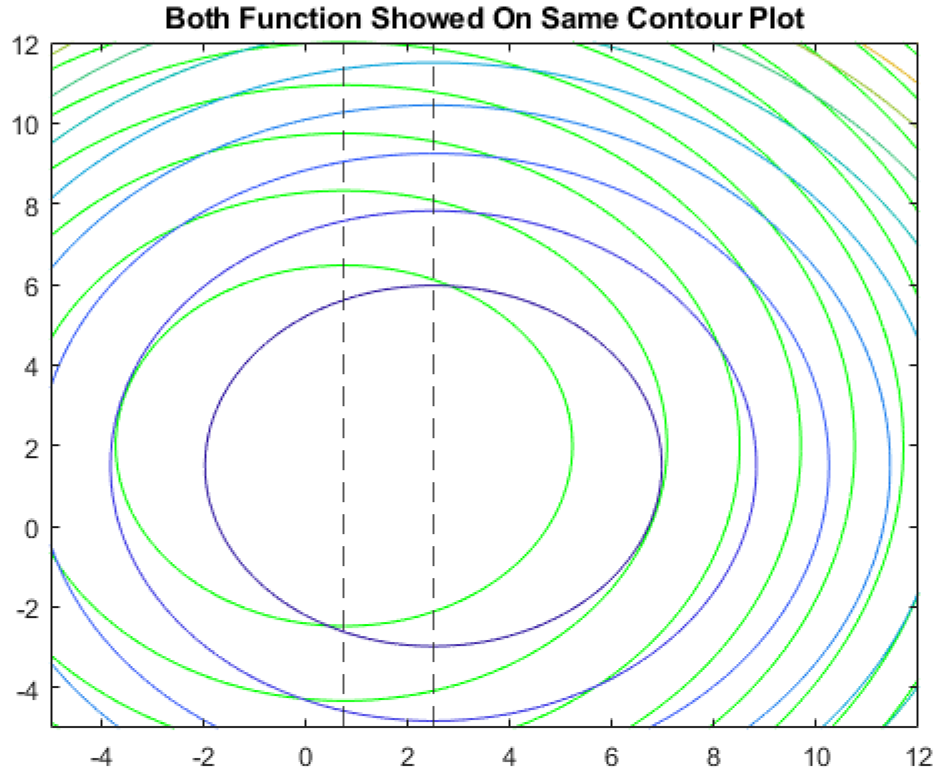
Table of Contents

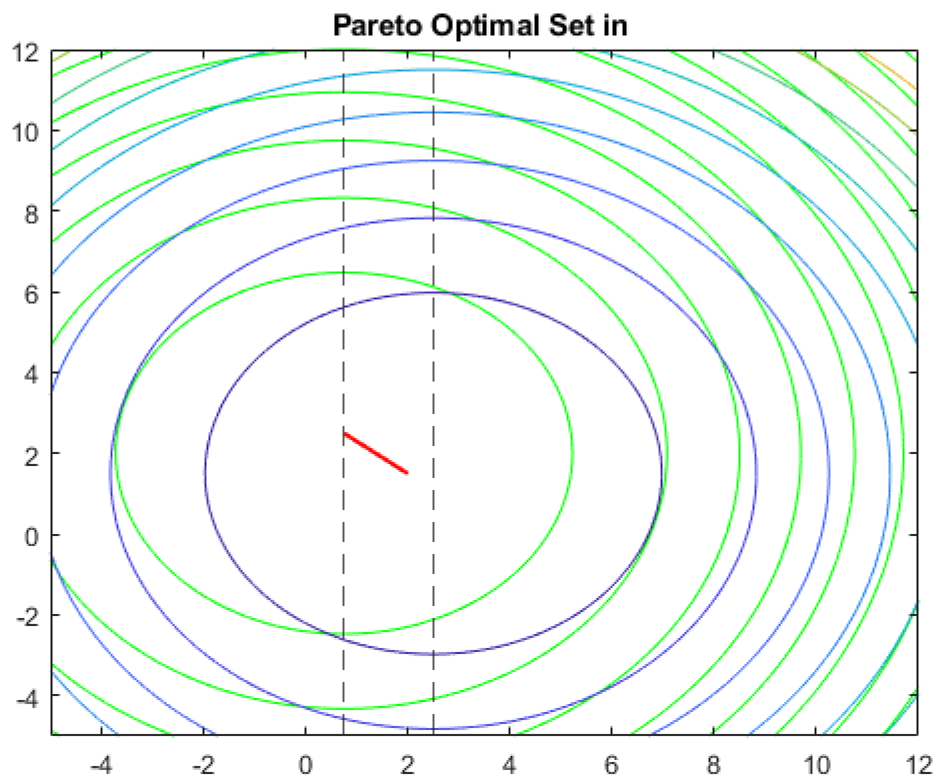
Problem 1

```
[x1, x2] = meshgrid(-5:0.01:12, -5:0.01:12);

f1 = (x1 - 0.75).^2 + (x2 - 2).^2;
f2 = (x1 - 2.5).^2 + (x2 - 1.5).^2;

figure(1)
contour(x1,x2,f1,30)
title('Obj. Function 1')

figure(2)
contour(x1,x2,f2,30)
title('Obj. Function 2')

figure(3)
contour(x1,x2,f1,'color','g')
hold on
contour(x1,x2,f2)
hold on
xline(0.75, '--')
hold on
xline(2.5, '--')
title('Both Function Showed On Same Contour Plot')
hold off

figure(4)
contour(x1,x2,f1,'color','g')
hold on
contour(x1,x2,f2)
hold on
xline(0.75, '--')
hold on
xline(2.5, '--')
hold on
plot([0.75 2], [2.5 1.5], 'linewidth', 1.5, 'color', 'r')
title('Pareto Optimal Set in ')
hold off
```


Obj. Function 1
12
10
8
6
4
2
0
-2
-4
-4
-2
0
2
4
6
8
10
12


Obj. Function 2
12
10
8
6
4
2
0
-2
-4
-4
-2
0
2
4
6
8
10
12

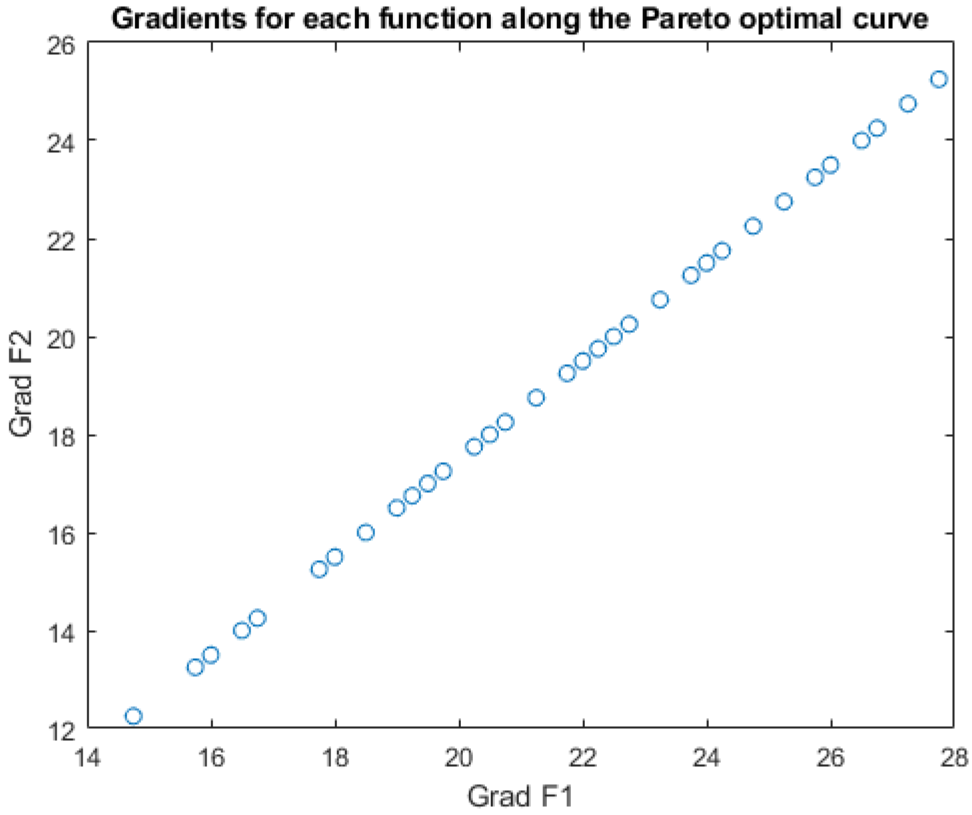

Both Function Showed On Same Contour Plot
12
10
8
6
4
2
0
-2
-4
-4
-2
0
2
4
6
8
10
12


Pareto Optimal Set in
12
10
8
6
4
2
0
-2
-4
-4
-2
0
2
4
6
8
10
12

Plotting gradients of each function in the pareto set

```
f1_grad = @(x) 2.*(x(1) - 0.75) + 2.*(x(2) - 2);
f2_grad = @(x) 2.*(x(1) - 2.5)  + 2.*(x(2) - 1.5);

for i = 1:length(x_ps2(:,1))
     x = x_ps2(i,:);
     grad1(i) = f1_grad(x);
     grad2(i) = f2_grad(x_ps2(i,:));

end

disp('As seen from the plot below, the relationship between the gradient of f1
 and f2 are linear.')
disp('We can see that within the Pareto optimal curve, when f1 has a 0
 gradient(meaning it is at the min')
disp('then f2 has a gradient of -2.5. While when the gratdient of f2 is at 0,
 f1 has a positive gradient of a little of 2.5.')
disp('And inbetween one is positive while the other negative.')

plot(grad1,grad2,'o')
title('Gradients for each function along the Pareto optimal curve')
xlabel('Grad F1')
ylabel('Grad F2')
```

```
As seen from the plot below, the relationship between the gradient of f1 and
 f2 are linear.
We can see that within the Pareto optimal curve, when f1 has a 0
 gradient(meaning it is at the min
then f2 has a gradient of -2.5. While when the gratdient of f2 is at 0, f1 has
 a positive gradient of a little of 2.5.
And inbetween one is positive while the other negative.
```



Problem 2

```
rng default % For reproducibility
fun = @objval1;
opts_ps.ParetoSetSize =
 optimoptions('fmincon','MaxFunctionEvaluations',1e4,'PlotFcn','psplotparetof');
[x_ps2,fval_ps1,~,psoutput2] = paretosearch(fun,2);
disp("Total Function Count: " + psoutput2.funccount);x = paretosearch(fun,2);
plot(fval_ps1(:,1),fval_ps1(:,2),'ko');
title('Pareto Optimal Set')
xlabel('f_1')
ylabel('f_2')
```

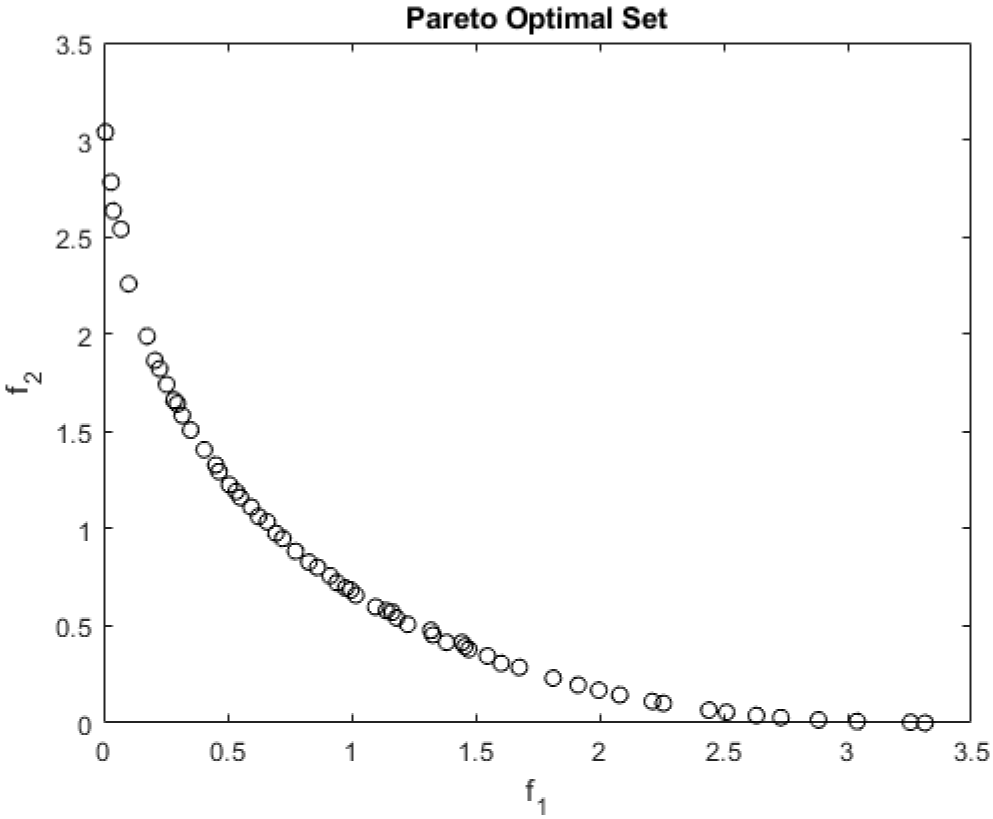
Pareto set found that satisfies the constraints.

Optimization completed because the relative change in the volume of the Pareto set is less than 'options.ParetoSetChangeTolerance' and constraints are satisfied to within 'options.ConstraintTolerance'.

Total Function Count: 1762

Pareto set found that satisfies the constraints.

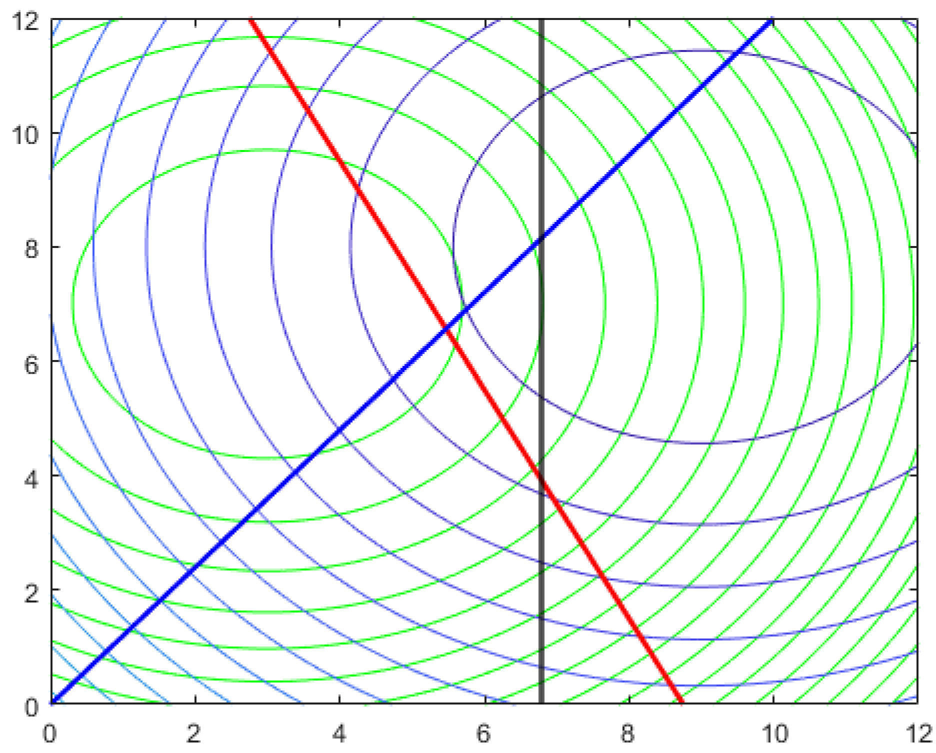
Optimization completed because the relative change in the volume of the Pareto set
is less than 'options.ParetoSetChangeTolerance' and constraints are satisfied to within
'options.ConstraintTolerance'.



Problem 3

```
X1 = -10:0.01:10;
X2 = -10:0.01:10;

f1 = (x1 - 3).^2 + (x2 - 7).^2;
f2 = (x1 - 9).^2 + (x2 - 8).^2;

%g1 = 70 - 4*X2 - 8*X1;
x2g1 = (70 - 8*X1)/4;
%g2 = -2.5*x2 + 3*x1;
x2g2 = 3/2.5*X1;
%g3 = -6.8 + x1; --> xline(6.8)

% Plotting constraints and and contours of objective function
contour(x1,x2,f1,30,'color','g')
hold on
contour(x1,x2,f2,30)
hold on
```

```
plot(X1,x2g1, 'color', 'r','linewidth', 2)
hold on
plot(X1,x2g2, 'color', 'b','linewidth', 2)
hold on
xline(6.8, 'color', 'k','linewidth', 2)
axis([0 12 0 12])
hold off
```



Finding pareto front

```
rng default % For reproducibility
fun = @objval3;
A = [];
b =[];
Aineq = [];
bineq = [];
lb = [];
ub = [];
nonlcon = @nonlcon3;
opts_ps.ParetoSetSize =
 optimoptions('fmincon','MaxFunctionEvaluations',1e4,'PlotFcn','psplotparetof');
[x_ps2,fval_ps1,~,psoutput2] = paretosearch(fun,2);
disp("Total Function Count: " + psoutput2.funccount);x =
 paretosearch(fun,2,A,b,Aineq,bineq,lb,ub,nonlcon);
plot(fval_ps1(:,1),fval_ps1(:,2),'ko');
title('Pareto Optimal Set')
xlabel('f_1')
```

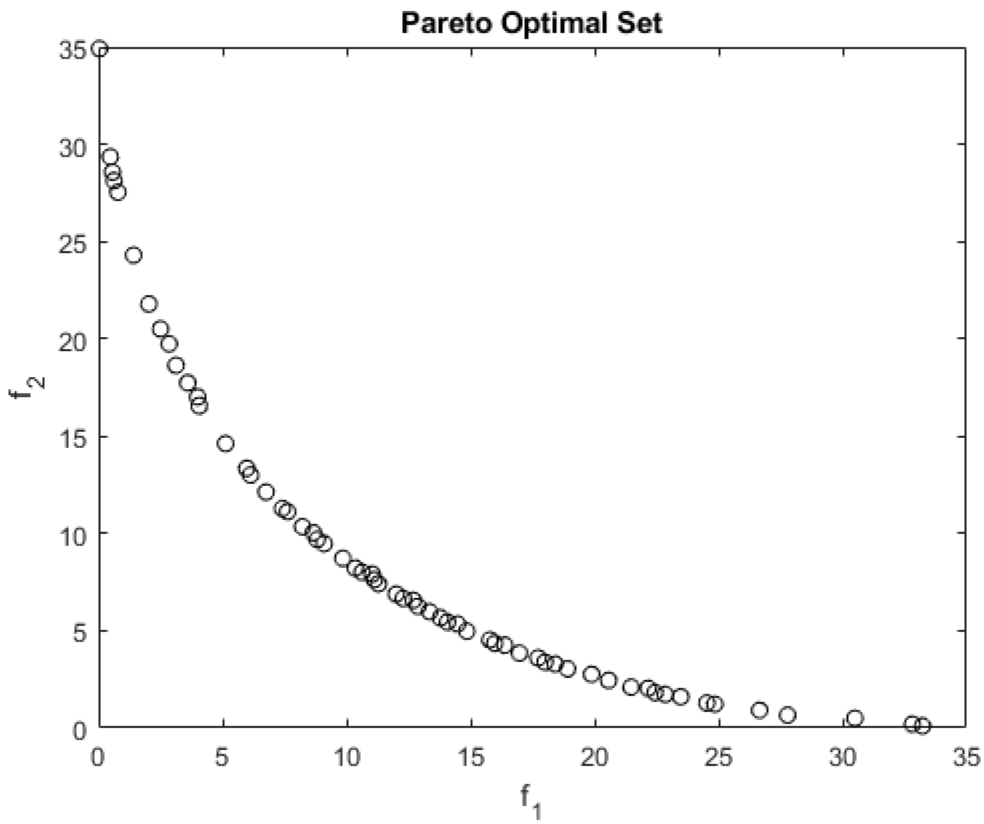
```
ylabel('f_2')
```

Pareto set found that satisfies the constraints.

Optimization completed because the relative change in the volume of the Pareto set
is less than 'options.ParetoSetChangeTolerance' and constraints are satisfied to within
'options.ConstraintTolerance'.

Total Function Count: 1629

Unable to find a feasible point.

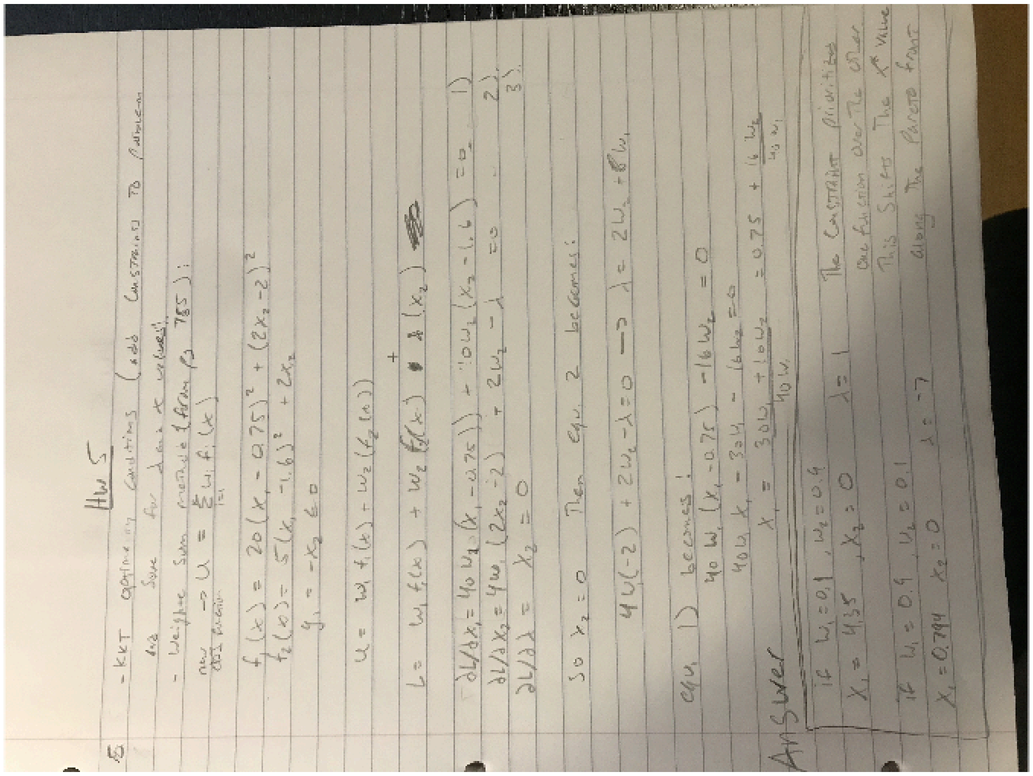


Problem 4 ANSWER

Weak pareto points are when there is point such that f(x) < f(x*). So looking at the graph, the weak pareto point is between f1 = 2 and f1 = 3.5 since it platoes there so f2 is not getting any better during the time.

Problem 5

```
p5 = imread('hw5_p5.jpg');
imshow(p5)
```


HW 5
- KKT optimality conditions (add constraint to problem and solve for λ and x values)
- Weighted sum method (from p 785):
$U = \sum w_i f_i(x)$
$f_1(x) = 20(x_1 - 0.75)^2 + (2x_2 - 2)^2$
$f_2(x) = 5(x_1 - 1.6)^2 + 2x_2$
$g_1 = -x_2 \le 0$
$U = w_1 f_1(x) + w_2 f_2(x)$
$L = w_1 f_1(x) + w_2 f_2(x) + \lambda(x_2)$
$\partial L/\partial x_1 = 40w_1(x_1 - 0.75) + 10w_2(x_1 - 1.6) = 0$ 1)
$\partial L/\partial x_2 = 4w_1(2x_2 - 2) + 2w_2 - \lambda = 0$ 2)
$\partial L/\partial \lambda = x_2 = 0$ 3)
So $x_2 = 0$ Then equ. 2 becomes:
$4w_1(-2) + 2w_2 - \lambda = 0 \rightarrow \lambda = 2w_2 - 8w_1$
equ. 1) becomes:
$40w_1(x_1 - 0.75) - 16w_2 = 0$
$40w_1 x_1 - 30w_1 - 16w_2 = 0$
$x_1 = \frac{30w_1 + 16w_2}{40w_1} = 0.75 + \frac{16w_2}{40w_1}$
Answer
if $w_1 = 0.1$, $w_2 = 0.9$
$x_1 = 4.35$, $x_2 = 0$, $\lambda = 1$
if $w_1 = 0.9$, $w_2 = 0.1$
$x_1 = 0.794$, $x_2 = 0$, $\lambda = -7$
The constraint prioritizes one function over the other. This shifts the x^* value along the Pareto front.

Problem 6

```
k = 9; % number of sample points
f = @(x) 2.*x(1).^3 + 15.*x(2).^2 - 8.*x(1).*x(2) - 4.*x(1);

% Constructing x_table
        % x1  x2  f
x_table = [ -1.5, -3, f([-1.5,-3]);
          -1.5,  0, f([-1.5,0]);
          -1.5,  3, f([-1.5,3]);
           1.5, -3, f([1.5,-3]);
           1.5,  0, f([-1.5,0]);
           1.5,  3, f([-1.5,3]);
             3, -3, f([-1.5,-3]);
             3,  0, f([-1.5,0]);
             3,  3, f([-1.5,3]);];

% test (Not needed to run)
% x_table = [ -1.5, -3, -1.022;
%             -1.5,  0, 4.503;
%             -1.5,  3, 31.997;
%              1.25, -3, 8.704;
%              1.25,  0, 1.636;
```

```
%            1.25,  3, 8.793;
%              4, -3, 37.341;
%              4,  0, 10.243;
%              4,  3, 4.157];

% Constructing Zeta Table
zeta_table = zeros(9,3);
for i = 1:9
zeta_table(i,1) = x_table(i,1);
zeta_table(i,2) = x_table(i,2);
zeta_table(i,3) = x_table(i,1)^2;
zeta_table(i,4) = x_table(i,2)^2;
zeta_table(i,5) = x_table(i,1)*x_table(i,2);

end
% Quadratic Approximation:
% f = d0 + d1x1 + d2x2 + d3x1^2 + d4x2^2 + d5x1x2 + epsilon
% We set each x variable to zeta;
% f = d0 + d1zeta1 + d2zeta2 + d3 zeta3 + d4zeta4 + d5zeta5 + epsilon

% Now to approx. the coefficeints, need to use a loss function. In this
% case it is the OLS one (sum of squared error). The optimallity condition
% for this is that e' = 0. (e' is the derivative of the loss function with
% repsect to each coefficient).


% This A matrix is constructed as seen in 20.16 from the book (3rd
% edition).

A = [k, sum(zeta_table(:,1)), sum(zeta_table(:,2)), sum(zeta_table(:,3)),...
                              sum(zeta_table(:,4)), sum(zeta_table(:,5));

     sum(zeta_table(:,1)), sum(zeta_table(:,1).^2),
 sum(zeta_table(:,1)'*zeta_table(:,2)),...
     sum(zeta_table(:,3)'*zeta_table(:,1)),
 sum(zeta_table(:,4)'*zeta_table(:,1)), sum(zeta_table(:,5)'*zeta_table(:,1))

     sum(zeta_table(:,2)), sum(zeta_table(:,1)'*zeta_table(:,2)),
 sum(zeta_table(:,2).^2),...
     sum(zeta_table(:,3)'*zeta_table(:,2)),
 sum(zeta_table(:,4)'*zeta_table(:,2)), sum(zeta_table(:,5)'*zeta_table(:,2));

     sum(zeta_table(:,3)), sum(zeta_table(:,1)'*zeta_table(:,3)),
 sum(zeta_table(:,2)'*zeta_table(:,3)),...
     sum(zeta_table(:,3).^2), sum(zeta_table(:,4)'*zeta_table(:,3)),
 sum(zeta_table(:,5)'*zeta_table(:,3));

     sum(zeta_table(:,4)), sum(zeta_table(:,1)'*zeta_table(:,4)),
 sum(zeta_table(:,2)'*zeta_table(:,4)),...
     sum(zeta_table(:,3)'*zeta_table(:,4)), sum(zeta_table(:,4).^2),
 sum(zeta_table(:,5)'*zeta_table(:,4));

     sum(zeta_table(:,5)), sum(zeta_table(:,1)'*zeta_table(:,5)),
 sum(zeta_table(:,2)'*zeta_table(:,5)),...
```

```
    sum(zeta_table(:,3)'*zeta_table(:,5)),
 sum(zeta_table(:,4)'*zeta_table(:,5)), sum(zeta_table(:,5).^2);];

% This b matrix is constructed as seen in 20.16 from the book (3rd
% edition).
b = [sum(x_table(:,3)), sum(zeta_table(:,1)'*x_table(:,3)),
 sum(zeta_table(:,2)'*x_table(:,3)),...
    sum(zeta_table(:,3)'*x_table(:,3)),sum(zeta_table(:,4)'*x_table(:,3)),
 sum(zeta_table(:,5)'*x_table(:,3))];

% Now Solving the system of linear equations using matrix algebra
% Ad = b --> d = inv(A)*b
d = inv(A)*b';

disp('Table of 9 sample design points:')
x_table

disp('Table of zeta values at 9 samples points:')
zeta_table

disp('A matrix:')
A
disp('b matrix:')
b
disp('estimated coefficients (d matrix):')
d

func = '15.5833 + 8.1667*x1 + 8.5*x2  -5.4444*x1^2+ 16.3611*x2^2 -
 0.5833*x1*x2';

disp('From the outputs above the response surface model for the function f(x)
 is:')
fprintf('%f + %f*x1 + %f*x2 + %f*x1^2+ %f*x2^2 +
 %f*x1*x2',d(1),d(2),d(3),d(4),d(5),d(6))




function F = objval1(x)

f1 = (x(:,1) - 0.75).^2 + (x(:,2) - 2).^2;
f2 = (x(:,1) - 2.5).^2 + (x(:,2) - 1.5).^2;

F = [f1,f2];
end

function F = objval3(x)

f1 = (x(:,1) - 3).^2 + (x(:,2) - 7).^2;
f2 = (x(:,1) - 9).^2 + (x(:,2) - 8).^2;

F = [f1,f2];
```

```
end

function [Cineq,Ceq] = nonlcon3(x)

Cineq = [70 - 4*x(:,2) - 8*x(:,1); -2.5*x(:,2) + 3*x(:,1); -6.8 + x(:,1)];
Ceq = [];
end
```

Table of 9 sample design points:

x_table =

```
   -1.5000   -3.0000   98.2500
   -1.5000         0   -0.7500
   -1.5000    3.0000  170.2500
    1.5000   -3.0000  171.7500
    1.5000         0   -0.7500
    1.5000    3.0000  170.2500
    3.0000   -3.0000   98.2500
    3.0000         0   -0.7500
    3.0000    3.0000  170.2500
```

Table of zeta values at 9 samples points:

zeta_table =

```
   -1.5000   -3.0000    2.2500    9.0000    4.5000
   -1.5000         0    2.2500         0         0
   -1.5000    3.0000    2.2500    9.0000   -4.5000
    1.5000   -3.0000    2.2500    9.0000   -4.5000
    1.5000         0    2.2500         0         0
    1.5000    3.0000    2.2500    9.0000    4.5000
    3.0000   -3.0000    9.0000    9.0000   -9.0000
    3.0000         0    9.0000         0         0
    3.0000    3.0000    9.0000    9.0000    9.0000
```

A matrix:

A =

```
    9.0000    9.0000         0   40.5000   54.0000         0
    9.0000   40.5000         0   81.0000   54.0000         0
         0         0   54.0000         0         0   54.0000
   40.5000   81.0000         0  273.3750  243.0000         0
   54.0000   54.0000         0  243.0000  486.0000         0
         0         0   54.0000         0         0  243.0000
```

b matrix:

b =

```
   1.0e+03 *

    0.8768    0.9135    0.4275    3.7800    7.9110    0.3172
```

```
estimated coefficients (d matrix):

d =

   15.5833
    8.1667
    8.5000
   -5.4444
   16.3611
   -0.5833

From the outputs above the response surface model for the function f(x) is:
15.583333 + 8.166667*x1 + 8.500000*x2 + -5.444444*x1^2+ 16.361111*x2^2 +
 -0.583333*x1*x2
```

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