Homework #1

1. A cantilever beam is subjected to the point load P (kN), as shown in Figure E2.23. The maximum bending moment in the beam is PL (kN·m) and the maximum shear is P (kN). Formulate the minimum-mass design problem using a hollow circular cross-section. The material should not fail under bending or shear stress. The maximum bending stress is calculated as

$$\sigma = \frac{PL}{I} R_o$$

where *I*=moment of inertia of the cross-section. The maximum shearing stress is calculated as

$$\tau = \frac{P}{3I}({R_o}^2 + R_o R_i + {R_i}^2)$$

Transcribe the problem into the standard design optimization model (also use $R_o \le$ 40.0 cm, $R_i \le$ 40.0 cm). Use this data: *P*=14 kN; *L*=10 m; mass density ρ=7850 kg/m^3; allowable bending stress σ_b=165 MPa; allowable shear stress τ_a=50 MPa.

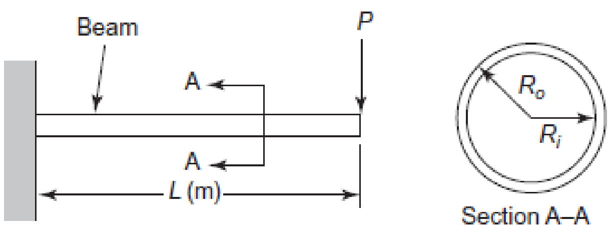

FIGURE E2.23 Cantilever beam.

2. Put the following programs in standard form, then solve them using the graphical method.

(a) Min $(x_1 - 3)^2 + (x_2 - 3)^2$

S.T. $x_1 + x_2 \ll 4$;

$x_1, x_2 \gg 0$;

(b) Min ${x_1}^2 - {x_2}^2 - 4x_1$

S.T. $x_1 + x_2 \ll 6$; $x_2 \ll 3$;

$x_1, x_2 \gg 0$;

3. Answer True or False.

1). A function can have several local minimum points in a small neighborhood of x*.

2). A function cannot have more than one global minimum point.

3). The value of the function having a global minimum at several points must be the same.

4). A function defined on an open set cannot have a global minimum.

5). The gradient of a function f(x) at a point is normal to the surface defined by the level surface f(x)=constant.

6). The gradient of a function at a point gives a local direction of maximum decrease in the function.

7). The Hessian matrix of a continuously differentiable function can be asymmetric.

8). The Hessian matrix for a function is calculated using only the first derivatives of the function.
9). Taylor series expansion for a function at a point uses the function value and its derivatives.
10). Taylor series expansion can be written at a point where the function is discontinuous.
11). Taylor series expansion of a complicated function replaces it with a polynomial function at the point.
12). Linear Taylor series expansion of a complicated function at a point is only a good local approximation for the function.
13). A quadratic form can have first-order terms in the variables.
14). For a given x, the quadratic form defines a vector.
15). Every quadratic form has a symmetric matrix associated with it.
16). A symmetric matrix is positive definite if its eigenvalues are non-negative.
17). A matrix is positive semidefinite if some of its eigenvalues are negative and others are non-negative.
18). All eigenvalues of a negative definite matrix are strictly negative.
19). The quadratic form appears as one of the terms in Taylor's expansion of a function.
20). A positive definite quadratic form must have positive value for any x ≠ 0.

4. Write the Taylor's expansion for the following functions up to quadratic terms.
   (a) e^x about the point $x^*=2$
   (b) $f(x_1, x_2) = 10x_1{}^4 - 20x_1{}^2x_2 + 10x_2{}^2 + x_1{}^2 - 2x_1 + 5$ about the point(1, 1).
   Compare approximate and exact values of the function at the point (1.2, 0.8).

Note: you may leave the approximation in vector form

5. Determine the form of the following quadratic function:

$$F(x) = x_1{}^2 + 4x_1x_2 + 2x_1x_3 - 7x_2{}^2 - 6x_2x_3 + 5x_3{}^2$$

6. Find stationary points (and the nature of each stationary point—min, max, indefinite) for the following functions:
   (a) $f(x_1, x_2) = x_1{}^2 - 2x_1 + 4x_2{}^2 - 8x_2 + 6$
   (b) $f(x_1, x_2) = 3x_1{}^2 - 2x_1x_2 + 5x_2{}^2 + 8x_2$
   (c) $f(x_1, x_2) = -4x_1 + 2x_2 + 4x_1{}^2 - 4x_1x_2 + 2x_2{}^2$

7. Consider the following program:
   Min $(x_1 - 1)^2 + (x_2 - 1)^2$
   S.T. $x_1 + x_2 - 4 = 0;$
   $x_1 - x_2 - 2 = 0$
   (a) Is this a "valid" optimization problem? Please comment and explain.
   (b) Do you need necessary conditions to solve it?

8. Solve the "beer can" problem (Lecture 2) using MATLAB (Arora S.3.3). Produce a plot that has your name on it.