Macroeconometrics

Lecture 9 Forecasting with Bayesian VARs

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Predictive density: frequentist approach

Predictive density: Bayesian approach

Forecasting Australian real output and inflation

Useful reading:

Karlsson (2013) Forecasting with Bayesian Vector Autoregression, Handbook of Economic Forecasting

Materials:

An R file `L9 mcxs.R` for the reproduction the results

A data file `RGDP-RGDPDEF.csv`

Objectives.

- To introduce a full statistical characterisation of future unknown values of interest
- To present multivariate forecasting using Bayesian VARs
- To understand the difference between frequentist and Bayesian density forecasting

Learning outcomes.

- Deriving the joint density of forecasted values
- Working on advanced transformation of normal distributions
- Reporting useful characteristics of the predictive densities

The objective of economic forecasting

... is to use the available data to provide a statistical characterisation of the unknown future values of quantities of interest.

The full statistical characterisation of the unknown future values of random variables is given by their predictive density.

Simplified outcomes in a form of statistics summarising the predictive densities are usually used in decision-making processes.

Summary statistics are also communicated to general audiences.

Predictive density: frequentist approach

Predictive density: frequentist approach

VAR(p) model.

$$y_t = \mu_0 + A_1 y_{t-1} + \cdots + A_p y_{t-p} + \epsilon_t$$
$$\epsilon_t | Y_{t-1} \sim iid\mathcal{N}_N\left(\mathbf{0}_N, \Sigma\right)$$

Matrix notation.

$$Y = XA + E$$
$$E|X \sim \mathcal{MN}_{T\times N}\left(\mathbf{0}_{T\times N}, \Sigma, I_T\right)$$

$$\underset{(K\times N)}{A} = \begin{bmatrix} \mu_0{}' \\ A_1{}' \\ \vdots \\ A_p{}' \end{bmatrix} \quad \underset{(T\times N)}{Y} = \begin{bmatrix} y_1' \\ y_2' \\ \vdots \\ y_T' \end{bmatrix} \quad \underset{(K\times 1)}{x_t} = \begin{bmatrix} 1 \\ y_{t-1} \\ \vdots \\ y_{t-p} \end{bmatrix} \quad \underset{(T\times K)}{X} = \begin{bmatrix} x_1' \\ x_2' \\ \vdots \\ x_T' \end{bmatrix} \quad \underset{(T\times N)}{E} = \begin{bmatrix} \epsilon_1' \\ \epsilon_2' \\ \vdots \\ \epsilon_T' \end{bmatrix}$$

where $K = 1 + pN$

Predictive density: frequentist approach

Predictive density.

$$
\begin{aligned}
y_{t+1} &= \mu_0 + A_1 y_t + \cdots + A_p y_{t-p+1} + \epsilon_{t+1} \\
\epsilon_{t+1} | Y_t &\sim iid\mathcal{N}_N\left(\mathbf{0}_N, \Sigma\right)
\end{aligned}
$$

$$\downarrow$$

$$p(y_{t+1}|Y_t, A, \Sigma) = \mathcal{N}_N\left(\mu_0 + A_1 y_t + \cdots + A_p y_{t-p+1}, \Sigma\right)$$

$$
\begin{aligned}
Y &= XA + E \\
E|X &\sim \mathcal{MN}_{T\times N}\left(\mathbf{0}_{T\times N}, \Sigma, I_T\right)
\end{aligned}
$$

$$\downarrow$$

$$p(Y|X, A, \Sigma) = \mathcal{MN}_{T\times N}\left(XA, \Sigma, I_T\right)$$

Predictive density: 1-period ahead forecast

Data generating process.

$$y_{t+1} = \mu_0 + A_1 y_t + \cdots + A_p y_{t-p+1} + \epsilon_{t+1}$$

1-period ahead forecast.

$$\begin{aligned}
\mathbb{E}[y_{t+1}|Y_t] &= \mu_0 + A_1 \mathbb{E}[y_t|Y_t] + \cdots + A_p \mathbb{E}[y_{t-p+1}|Y_t] + \mathbb{E}[\epsilon_{t+1}|Y_t] \\
y_{t+1|t} &= \mu_0 + A_1 y_t + \cdots + A_p y_{t-p+1}
\end{aligned}$$

1-period ahead forecast error.

$$\mathbf{e}_{t+1|t} = y_{t+1} - y_{t+1|t} = \epsilon_{t+1}$$

1-period ahead forecast error variance.

$$\mathbb{V}\mathrm{ar}[\mathbf{e}_{t+1|t}] = \mathbb{E}\left[\mathbb{E}_t[\epsilon_{t+1}\epsilon'_{t+1}]\right] = \Sigma$$

Predictive density: 2-period ahead forecast

Data generating process.

$$y_{t+2} = \mu_0 + A_1 y_{t+1} + \cdots + A_p y_{t-p+2} + \epsilon_{t+2}$$

2-period ahead forecast.

$$\begin{aligned}
\mathbb{E}[y_{t+2}|Y_t] &= \mu_0 + A_1 \mathbb{E}[y_{t+1}|Y_t] + \cdots + A_p \mathbb{E}[y_{t-p+2}|Y_t] + \mathbb{E}[\epsilon_{t+2}|Y_t] \\
y_{t+2|t} &= \mu_0 + A_1 y_{t+1|t} + A_2 y_t + \cdots + A_p y_{t-p+2}
\end{aligned}$$

2-period ahead forecast error.

$$\mathbf{e}_{t+2|t} = y_{t+2} - y_{t+2|t} = \epsilon_{t+2} + A_1 (y_{t+1} - y_{t+1|t}) = \epsilon_{t+2} + A_1 \epsilon_{t+1}$$

2-period ahead forecast error variance.

$$\begin{aligned}
\mathbb{V}\text{ar}[\mathbf{e}_{t+2|t}] &= \mathbb{E}[\mathbf{e}_{t+2|t} \mathbf{e}'_{t+2|t}] \\
&= \mathbb{E}[(\epsilon_{t+2} + A_1 \epsilon_{t+1})(\epsilon_{t+2} + A_1 \epsilon_{t+1})'] \\
&= \mathbb{E}\left[\mathbb{E}_{t+1}[\epsilon_{t+2} \epsilon'_{t+2}]\right] + A_1 \mathbb{E}\left[\mathbb{E}_t[\epsilon_{t+1} \epsilon'_{t+1}]\right] A'_1 \\
&= \Sigma + A_1 \Sigma A'_1
\end{aligned}$$

Predictive density: frequentist approach

Covariance.

$$
\begin{aligned}
\mathbb{C}\mathrm{ov}[y_{t+1}, y_{t+2}] &= \mathbb{E}\left[(y_{t+1} - y_{t+1|t})(y_{t+2} - y_{t+2|t})'\right] \\
&= \mathbb{E}\left[(\epsilon_{t+1})(\epsilon_{t+2} + A_1\epsilon_{t+1})'\right] \\
&= \mathbb{E}\left[\mathbb{E}_t[\epsilon_{t+1}\epsilon_{t+1}']\right] A_1' \\
&= \Sigma A_1'
\end{aligned}
$$

Predictive density: joint density

Joint predictive density of y_{t+1} and y_{t+2} given Y_t, A, Σ.

$$p(y_{t+1}, y_{t+2}|Y_t, A, \Sigma) = p(y_{t+2}|y_{t+1}, Y_t, A, \Sigma)p(y_{t+1}|Y_t, A, \Sigma)$$

$$p(y_{t+2}|y_{t+1}, Y_t, A, \Sigma) = \mathcal{N}_N\left(y_{t+2|t}, \Sigma + A_1\Sigma A_1{}'\right)$$

$$p(y_{t+1}|Y_t, A, \Sigma) = \mathcal{N}_N\left(y_{t+1|t}, \Sigma\right)$$

$$\downarrow$$

$$p\left(\begin{bmatrix} y_{t+1} \\ y_{t+2} \end{bmatrix}\middle| Y_t, A, \Sigma\right) = \mathcal{N}_{2N}\left(\begin{bmatrix} y_{t+1|t} \\ y_{t+2|t} \end{bmatrix}, \begin{bmatrix} \Sigma & \Sigma A_1{}' \\ A_1\Sigma & \Sigma + A_1\Sigma A_1{}' \end{bmatrix}\right)$$

Predictive density: joint density

Joint predictive density of $y_{t+1}, y_{t+2}, \dots, y_{t+h}$ given Y_t, A, Σ.

$$\mathcal{N}_{hN}\left(\begin{bmatrix} y_{t+1|t} \\ y_{t+2|t} \\ \vdots \\ y_{t+h|t} \end{bmatrix}, \begin{bmatrix} \Sigma & \Sigma\Phi_1{}' & \dots & \Sigma\Phi_{h-1}{}' \\ & \Sigma + \Phi_1\Sigma\Phi_1{}' & \dots & \Sigma\Phi_1{}' + \Phi_1\Sigma\Phi_2{}' + \dots + \Phi_{h-1}\Sigma\Phi_h{}' \\ & & \ddots & \vdots \\ & & & \Sigma + \Phi_1\Sigma\Phi_1{}' + \dots + \Phi_h\Sigma\Phi_h{}' \end{bmatrix}\right)$$

$\Phi_i = J\mathbf{A}^i J'$ – parameters of VMA(∞) representation of VAR(p)

$\mathbf{A}$ – parameter matrix of VAR(1) representation of VAR(p)

Elements below the main diagonal of the covariance matrix are equal to the transpose of the corresponding elements above the main diagonal

Predictive density: frequentist approach

Predictive density using parameter estimates $\hat{A}, \hat{\Sigma}$.

$$p\left(\begin{bmatrix} y_{t+1} \\ y_{t+2} \end{bmatrix} \middle| Y_t, A, \Sigma\right)\Bigg|_{\substack{A = \hat{A} \\ \Sigma = \hat{\Sigma}}} = \mathcal{N}_{2N}\left(\begin{bmatrix} \hat{y}_{t+1|t} \\ \hat{y}_{t+2|t} \end{bmatrix}, \begin{bmatrix} \hat{\Sigma} & \hat{\Sigma}\hat{A}_1' \\ \hat{A}_1\hat{\Sigma} & \hat{\Sigma} + \hat{A}_1\hat{\Sigma}\hat{A}_1' \end{bmatrix}\right)$$

Predictive density is evaluated by plugging in the estimates in the place of the parameters

Estimates of parameters in forecasting applications are treated as not random

Estimation uncertainty is ignored in forecasting applications

Forecast error variances are underestimated

Predictive densities might be inaccurate

Solution: Bayesian forecasting

Predictive density: frequentist approach

1-period ahead predictive density.

$$p(y_{t+1}|x_{t+1}, A, \Sigma) = \mathcal{N}_N\left(x'_{t+1}A, \Sigma\right)$$

$$p\left(y_{t+1}\middle|x_{t+1}, A, \Sigma\right) = \mathcal{MN}_{1\times N}\left(x'_{t+1}A, \Sigma, 1\right)$$

$$p(y_{t+1}|x_{t+1}, Y, X, A, \Sigma)\Big|_{\substack{A=\hat{A}\\ \Sigma=\hat{\Sigma}}} = \mathcal{N}_N\left(x'_{t+1}\hat{A}, \hat{\Sigma}\right)$$

$$p\left(y_{t+1}|x_{t+1}, Y, X, A, \Sigma\right)\Big|_{\substack{A=\hat{A}\\ \Sigma=\hat{\Sigma}}} = \mathcal{MN}_{1\times N}\left(x'_{t+1}\hat{A}, \hat{\Sigma}, 1\right)$$

Predictive density: Bayesian approach

Predictive density: Bayesian approach

Posterior density.

$$p\left(A, \Sigma | Y, X\right) = p(A|Y, X, \Sigma)p\left(\Sigma | Y, X\right)$$

$$p(A|Y, X, \Sigma) = \mathcal{MN}_{K\times N}\left(\overline{A}, \Sigma, \overline{V}\right)$$
$$p(\Sigma|Y, X) = \mathcal{IW}_{N}\left(\overline{S}, \overline{\nu}\right)$$

$$\overline{V} = \left(X'X + \underline{V}^{-1}\right)^{-1}$$
$$\overline{A} = \overline{V}\left(X'Y + \underline{V}^{-1}\underline{A}\right)$$
$$\overline{\nu} = T + \underline{\nu}$$
$$\overline{S} = \underline{S} + Y'Y + \underline{A}'\underline{V}^{-1}\underline{A} - \overline{A}'\overline{V}^{-1}\overline{A}$$

Predictive density: Bayesian approach

1-period ahead predictive density.

$$p(y_{t+1}|x_{t+1}, Y, X) = \iint p(y_{t+1}|x_{t+1}, Y, X, A, \Sigma)p\left(A, \Sigma|Y, X\right) dAd\Sigma$$

Predictive density is evaluated by integrating out the parameters from a joint distribution of the forecasted values y_{t+1} and the parameters A, Σ

Integration is performed with respect to posterior distribution

Parameters are treated as unknown random variables

Estimation uncertainty is incorporated into forecasting

Predictive densities are accurate

Predictive density: Bayesian approach

1-period ahead predictive density.

$$p(y_{t+1}|x_{t+1}, Y, X) = \iint p(y_{t+1}|x_{t+1}, Y, X, A, \Sigma)p\left(A, \Sigma|Y, X\right) dAd\Sigma$$

$$p\left(y_{t+1}\middle|x_{t+1} Y, X, A, \Sigma\right) = \mathcal{MN}_{1\times N}\left(x_{t+1}'A, \Sigma, 1\right)$$

$$\begin{aligned}
p\left(A, \Sigma|Y, X\right) &= p(A|Y, X, \Sigma)p\left(\Sigma|Y, X\right) \\
p(A|Y, X, \Sigma) &= \mathcal{MN}_{K\times N}\left(\overline{A}, \Sigma, \overline{V}\right) \\
p(\Sigma|Y, X) &= \mathcal{IW}_N\left(\overline{S}, \overline{\nu}\right)
\end{aligned}$$

Derive the solution step-by-step

Step 1 Integrate out A

Step 2 Integrate out Σ

Useful distribution transformations

Linear combination of matrix-variate normal random variable.

$$X|\Sigma \sim \mathcal{MN}_{K\times N}(M, \Sigma, V)$$
$$AX + a|\Sigma \sim \mathcal{MN}_{L\times N}(AM + a, \Sigma, AVA')$$
$$XB + b|\Sigma \sim \mathcal{MN}_{K\times L}(MB + b, B\Sigma B', V)$$

Matrix-variate normal compound distribution.

$$X|C, \Sigma \sim \mathcal{MN}_{K\times N}\left(\overline{M}C, \Sigma, V\right)$$
$$C|\Sigma \sim \mathcal{MN}_{L\times N}\left(\overline{C}, \Sigma, W\right)$$

$$\begin{aligned} p(X|\Sigma) &= \int p(X|C, \Sigma)p(C|\Sigma)dC \\ &= \mathcal{MN}_{K\times N}\left(\overline{MC}, \Sigma, V + \overline{M}W\overline{M}'\right) \end{aligned}$$

$\underset{(K\times N)}{X}, \underset{(K\times N)}{M}, \underset{(K\times L)}{\overline{M}}, \underset{(L\times N)}{C}, \underset{(L\times N)}{\overline{C}}, \underset{(L\times K)}{A}, \underset{(L\times N)}{a}, \underset{(N\times L)}{B}, \underset{(K\times L)}{b}, \underset{(N\times N)}{\Sigma}, \underset{(K\times K)}{V}, \underset{(L\times L)}{W}$

Useful distributions: matric-variate t

Matric-variate t as a marginal distribution of X.

$$
\begin{aligned}
X|\Sigma &\sim \mathcal{MN}_{K\times N}(M, \Sigma, V) \\
\Sigma &\sim \mathcal{IW}_{N\times N}(S, \nu) \\
&\downarrow \\
p(X) &= \int p(X|\Sigma)p(\Sigma)d\Sigma \\
&= \mathcal{M}t_{K\times N}(M, V, S, \nu)
\end{aligned}
$$

Density function.

$$
\mathcal{M}t_{K\times N}(M, V, S, \nu) = c_{mt}^{-1}\det\left[S + (X-M)'V^{-1}(X-M)\right]^{-\frac{\nu+K}{2}}
$$

$$
c_{mt} = \pi^{\frac{KN}{2}}\det(V)^{\frac{N}{2}}\det(S)^{-\frac{\nu}{2}}\left(\prod_{n=1}^{N}\frac{\Gamma\left(\frac{\nu+1-n}{2}\right)}{\Gamma\left(\frac{\nu+K+1-n}{2}\right)}\right)
$$

Useful distributions: matrix-variate t

$$X \sim \mathcal{M}t_{K\times N}(M, V, S, \nu)$$

Moments.

$$\begin{aligned} \mathbb{E}[X] &= M && \text{for } \nu > N \\ \mathbb{V}\text{ar}[\text{vec}(X)] &= \frac{1}{\nu - N - 1} S \otimes V && \text{for } \nu > N + 1 \end{aligned}$$

Predictive density: Bayesian approach

1-period ahead predictive density.

Step 1: Integrate out A:

$$p(y_{t+1}|x_{t+1}, Y, X, \Sigma) = \int p(y_{t+1}|x_{t+1}, Y, X, A, \Sigma) p(A|Y, X, \Sigma) dA$$

$$p\left(y_{t+1}\middle| x_{t+1} Y, X, A, \Sigma\right) = \mathcal{MN}_{1\times N}\left(x'_{t+1}A, \Sigma, 1\right)$$

$$p(A|Y, X, \Sigma) = \mathcal{MN}_{K\times N}\left(\overline{A}, \Sigma, \overline{V}\right)$$

$$\downarrow$$

$$p(y_{t+1}|x_{t+1}, Y, X, \Sigma) = \mathcal{MN}_{1\times N}\left(x'_{t+1}\overline{A}, \Sigma, 1 + x'_{t+1}\overline{V}x_{t+1}\right)$$

Predictive density: Bayesian approach

1-period ahead predictive density.

Step 2: Integrate out Σ:

$$p(y_{t+1}|x_{t+1}, Y, X) = \int p(y_{t+1}|x_{t+1}, Y, X, \Sigma) p(\Sigma|Y, X) d\Sigma$$

$$\begin{aligned} p(y_{t+1}|x_{t+1}, Y, X, \Sigma) &= \mathcal{MN}_{1\times N}\left(x'_{t+1}\overline{A}, \Sigma, 1 + x'_{t+1}\overline{V}x_{t+1}\right) \\ p(\Sigma|Y, X) &= \mathcal{IW}_N\left(\overline{S}, \overline{\nu}\right) \\ &\downarrow \\ p(y_{t+1}|x_{t+1}, Y, X) &= \mathcal{M}t_{1\times N}\left(x'_{t+1}\overline{A}, 1 + x'_{t+1}\overline{V}x_{t+1}, \overline{S}, \overline{\nu}\right) \end{aligned}$$

Predictive density: Bayesian vs. frequentist approach

1-period ahead predictive density.

$$
\begin{aligned}
p(y_{t+1}|x_{t+1}, Y, X) &= \mathcal{M}t_{1\times N}\left(x'_{t+1}\overline{A}, 1 + x'_{t+1}\overline{V}x_{t+1}, \overline{S}, \overline{\nu}\right) \\
\mathbb{E}[y_{t+1}|x_{t+1}, Y, X] &= x'_{t+1}\overline{A} \\
\mathbb{V}\mathrm{ar}[y_{t+1}|x_{t+1}, Y, X] &= \frac{1 + x'_{t+1}\overline{V}x_{t+1}}{\overline{\nu} - N - 1}\overline{S}
\end{aligned}
$$

$$
\begin{aligned}
p\left(y_{t+1}|x_{t+1}, Y, X, A, \Sigma\right)\Big|_{\substack{A=\hat{A} \\ \Sigma=\hat{\Sigma}}} &= \mathcal{MN}_{1\times N}\left(x'_{t+1}\hat{A}, \hat{\Sigma}, 1\right) \\
\mathbb{E}[y_{t+1}|x_{t+1}, Y, X] &= x'_{t+1}\hat{A} \\
\mathbb{V}\mathrm{ar}[y_{t+1}|x_{t+1}, Y, X] &= \hat{\Sigma}
\end{aligned}
$$

Predictive density: Bayesian approach

Joint predictive density.

$$p\left(Y_{t+h} \middle| Y_t, A, \Sigma\right) = \mathcal{N}_{hN}\left(Y_{t+h|t}(A), \mathbb{V}\text{ar}\left[Y_{t+h|t} \middle| A, \Sigma\right]\right)$$

$$\underset{(hN\times 1)}{Y_{t+h}} = \begin{bmatrix} y_{t+1} \\ y_{t+2} \\ \vdots \\ y_{t+h} \end{bmatrix} \qquad \underset{(hN\times 1)}{Y_{t+h|t}(A)} = \begin{bmatrix} y_{t+1|t} \\ y_{t+2|t} \\ \vdots \\ y_{t+h|t} \end{bmatrix}$$

$$\mathbb{V}\text{ar}\left[Y_{t+h|t} \middle| A, \Sigma\right] = \begin{bmatrix} \Sigma & \Sigma\Phi_1{}' & \dots & \Sigma\Phi_{h-1}{}' \\ & \Sigma + \Phi_1\Sigma\Phi_1{}' & \dots & \Sigma\Phi_1{}' + \Phi_1\Sigma\Phi_2{}' + \dots + \Phi_{h-1}\Sigma\Phi_h{}' \\ & & \ddots & \vdots \\ & & & \Sigma + \Phi_1\Sigma\Phi_1{}' + \dots + \Phi_h\Sigma\Phi_h{}' \end{bmatrix}$$

Predictive density: Bayesian approach

Joint predictive density.

$$p\left(Y_{t+h}\middle|Y_t\right) = \int p\left(Y_{t+h}\middle|Y_t, A, \Sigma\right) p\left(A, \Sigma | Y, X\right) d(A, \Sigma)$$

$$p\left(y_{t+h}\middle|Y, X, A, \Sigma\right) = \mathcal{N}_{hN}\left(Y_{t+h|t}(A), \mathbb{V}\text{ar}\left[Y_{t+h|t}\middle|A, \Sigma\right]\right)$$
$$p\left(A, \Sigma | Y, X\right) = \mathcal{NIW}_{K\times N}\left(\overline{A}, \overline{V}, \overline{S}, \overline{\nu}\right)$$

The analytical solution to the problem cannot be found.
Solution: use numerical integration.

Predictive density: Bayesian approach

Sampling the joint predictive density (Algorithm 1).

Sample draws from $p(A, \Sigma | Y, X)$ and

Obtain $\left\{A^{(s)}, \Sigma^{(s)}\right\}_{s=1}^{S}$

Sample draws from $\hat{p}\left(Y_{t+h} \middle| Y_t\right)$ by:

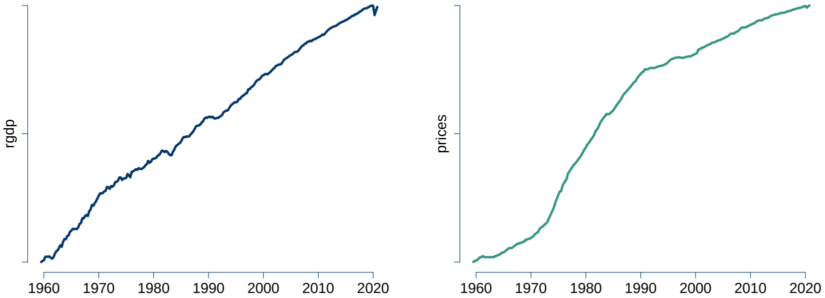
$$Y_{t+h}^{(s)} \sim \mathcal{N}_{hN}\left(Y_{t+h|t}\left(A^{(s)}\right), \mathbb{V}\text{ar}\left[Y_{t+h|t} \middle| A^{(s)}, \Sigma^{(s)}\right]\right)$$

Obtain $\left\{Y_{t+h}^{(s)}\right\}_{s=1}^{S}$

Characterise of the predictive density using $\left\{Y_{t+h}^{(s)}\right\}_{s=1}^{S}$

Forecasting Australian **real output and prices**

Australian real output and prices



Logarithms of real GDP and the CPI

Sample period: 1959Q3 – 2020Q4, T=246

Data source: Australian Macro Database: ausmacrodata.org

Australian real output and prices

Posterior estimates of the parameters of the VAR(4) model

$A|Y,X$

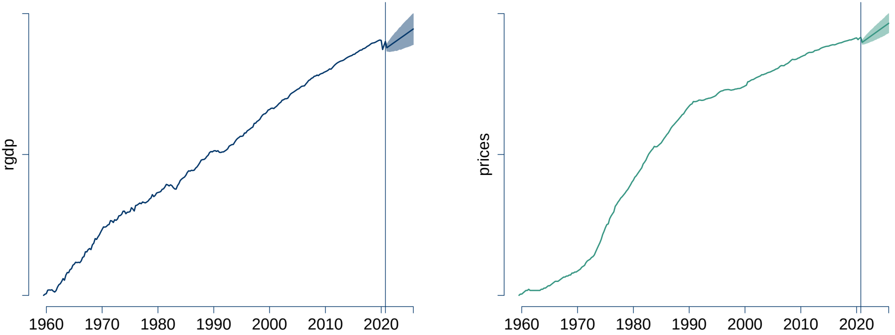
μ_2	A_1		A_2		A_3		A_4	
.010029	.999925	-.000114	-.000019	-.000030	-.000008	-.000012	-.000005	-.000007
[.00473]	[.00031]	[.00031]	[.00016]	[.00016]	[.00011]	[.00011]	[.00008]	[.00008]
.015123	-.000150	.999762	-.000036	-.000061	-.000016	-.000027	-.000009	-.000016
[.00398]	[.00026]	[.00026]	[.00013]	[.00013]	[.00009]	[.00009]	[.00007]	[.00007]

$\Sigma|Y,X$

0.000253	-0.000005
[0.00002]	[0.00001]
-0.000005	0.000177
[0.00001]	[0.00002]

The table reports posterior means and [standard deviations] based on 50,000 draws from the posterior distribution.
Minnesota prior specified as in the slides is used.

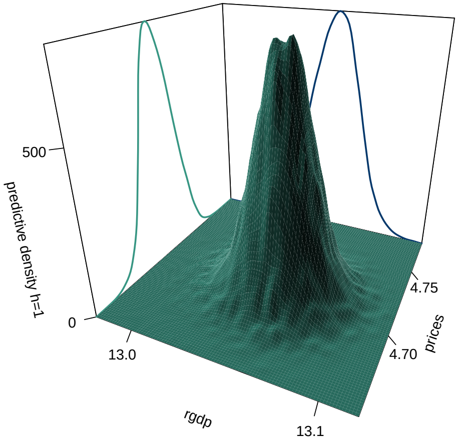
Australian real output and prices forecasts



Data and forecasts plot

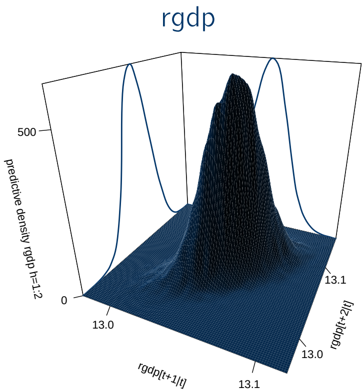
The forecasts are presented using predictive density means and 90% highest density intervals

Joint predictive density 1-period ahead

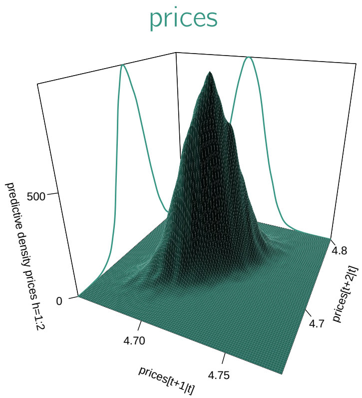


$$\mathbb{C}\text{or}(rgdp_{t+1}, p_{t+1}|Y_t) = -0.24$$

Joint predictive density 1 and 2 periods ahead

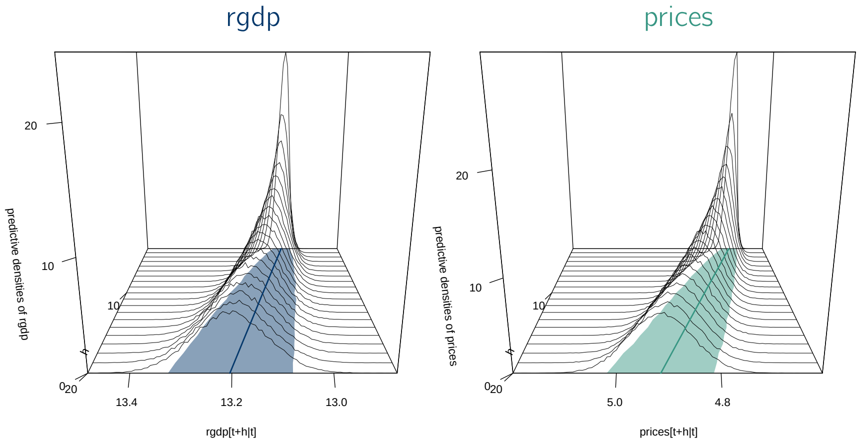


$\mathbb{C}\text{or}(rgdp_{t+1}, rgdp_{t+2}|Y_t) = 0.71$



$\mathbb{C}\text{or}(p_{t+1}, p_{t+2}|Y_t) = 0.71$

Predictive densities at all horizons



Forecasting with Bayesian VARs

Predictive densities contain the full statistical characterisation of future unknown values of interest

Bayesian predictive densities incorporate estimation uncertainty and differ with that respect from frequentist ones

Forecasts inherit the properties of the stochastic process on which they are based