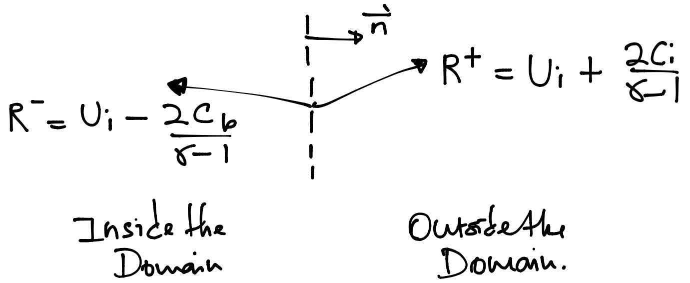
Riemann Invariants

- A further simplification of the Characteristic Boundary Conditions result in the Riemann Invariants
- One characteristic variable must be interpolated from the interior of the flow domain, while the other is interpolated from the exterior.



where R^- and R^+ are the incoming and outgoing characteristics.

- The invariants determine the local normal velocity and the speed of sound.
- The entropy, s and speed of sound, c are used to determine the density and pressure on the boundary
- From $R^+ = U_i + \frac{2C_i}{\gamma-1}$ and $R^- = U_o - \frac{2C_o}{\gamma-1}$, where $U_i = \vec{U_i}\cdot\vec{n}$ and $C_i^2 = \gamma p_i/\rho_i$
  $U_o = \vec{U_o}\cdot\vec{n}$ and $C_o^2 = \gamma p_o/\rho_o$

  If the flow leaving the domain is supersonic then <u>no</u> incoming characteristic exist and hence $R^- = U_i - \frac{2C_i}{\gamma-1}$

  If the flow entering the domain is supersonic then <u>no</u> outgoing characteristic exist and hence $R^+ = U_o + \frac{2C_o}{\gamma-1}$

  From the above equations, the velocity and speed of sound at the boundary can be evaluated purely as a function of the Riemann Invariants,

The sum of the equations are $R^+ + R^- = 2U_b + \frac{2}{\gamma-1}\underbrace{(C_b - C_b)} = 2U_b$

$$U_b = \frac{1}{2}(R^+ + R^-)$$

The difference o the equations are: $R^+ - R^- = U_b - U_b + \frac{4C_b}{\gamma-1}$

$$C_b = \frac{1}{4}(\gamma-1)(R^+ - R^-)$$

The density and pressure at the boundary can be updated as

$$\rho_b = \left(\frac{C_b^2}{\gamma S_b}\right)^{\frac{1}{\gamma-1}} \text{ and } p_b = \frac{\rho_b C_b^2}{\gamma}, \text{ where } S_b = \frac{C_i^2}{\gamma \rho_i^{\gamma-1}} \text{ if } U_b > 0$$

$$S_b = \frac{C_o^2}{\gamma \rho_o^{\gamma-1}} \text{ if } U_b < 0$$

Vortex Correction

- A lifting body induces perturbations at great distance from the surface. Hence the far-field boundary must be placed at great distances. If they are placed closer than the far-field boundary condition must be augmented with a single vortex whose strength is such that the correct circulation produced by a lifting body is recovered at the far-field boundary.

In this approach the far-field velocity, U_∞ is corrected by,

$$U_\infty^* = U_\infty + \left(\frac{\Gamma\sqrt{1-M_\infty^2}}{2\pi d}\right)\frac{1}{1-M_\infty^2\sin^2(\theta-\alpha)}\sin\theta$$

where Γ is the circulation
(d,θ) are the coordinates of the far-field point based on polar coordinates.
α is the angle of attack.

The circulation $\Gamma = \frac{1}{2}\|\vec{V}_\infty\|_2 \bar{c}\, C_L$, where $\bar{c}$ is the chord length

Subsequently the freestream pressure can be updated

$$p_\infty^* = \left[p_\infty^{(\gamma-1)/\gamma} + \left(\frac{\gamma-1}{\gamma}\right)\frac{\rho_\infty}{2p_\infty^{1/\gamma}}\left(\|\vec{V}_\infty\|_2^2 - \|\vec{V}_\infty^*\|_2^2\right)\right]^{\gamma/\gamma-1}$$

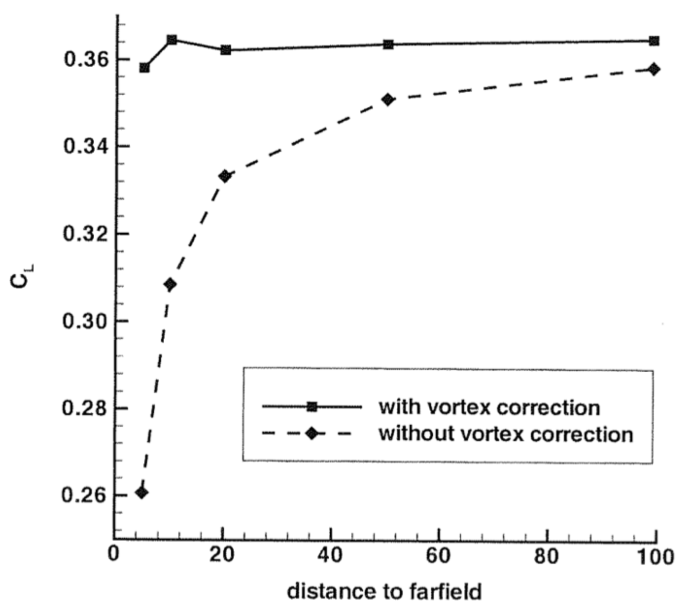


Figure 8.7: Effects of distance to the farfield boundary and of single vortex on the lift coefficient. NACA 0012 airfoil, $M_\infty = 0.8$, $\alpha = 1.25°$. (Taken from Computational Fluid Dynamics...., by Blazek)