ACMANTv5.2: SCIENTIFIC CONTENT

Supplementary material of "Domonkos, P. Homogenization of climatic time series with ACMANTv5.2"

Table of content

I INTRODUCTION

ACMANTv5.2 is a software for removing non-climatic biases from climatic time series, or with the more usual term: homogenizing climatic time series. ACMANT is a relative homogenization method, which means that it is based on the exploitation of the spatial redundancy existing in the observed climatic data. The use of the software requires the use of spatially sufficiently correlating networks of time series. After the preparation of input data and introduction of some parameters (e.g. number of time series, study period of time series, etc.), the run of ACMANTv5.2 can be fully automatic or interactive according to user choice. Automatic homogenization methods have three advantages in comparison with manual or partly manual methods: i) Automatic methods can be tested in large benchmark datasets, thus their performances are the best controlled; ii) Their use is feasible even for very large datasets; iii) Their use is relatively easy. Notwithstanding, user interactions may sometimes be beneficial, it is the case e.g., when synchronous or semi-synchronous inhomogeneities occur in several time series, and the evaluation of inhomogeneity detection results is supported by documented information so-called metadata of station histories.

The purpose of this document is to provide the full description of the scientific content of ACMANTv5.2 for interested readers. This method can be applied to the homogenization of various climatic variables either on daily or monthly time scales. The execution of homogenization may differ according to climatic variables, the spatial density and temporal resolution of the data, the presumed annual cycle of inhomogeneities and the size of the dataset. The software package includes 24 programs specific for homogenization tasks, and a high user-friendliness is provided by the inclusion of one more program which manages the homogenization procedure. The managing program performs some preparatory steps, then selects the most appropriate homogenization program of the 24 others and transmits the task to the selected program. The computational time demand of ACMANTv5.2 is relatively low.

ACMANT was constructed on the base of some earlier developed modern homogenization tools (ACMANT = Adapted Caussinus-Mestre Algorithm for the homogenization of Networks of climatic Time series), and during its development further statistical tools and other algorithm modifications have been added. The effect of method developments on the homogenization accuracy is continuously tested. This empirical control is necessary, as theoretically excellent statistical tools do not always provide the optimal solution for finite and noisy samples, like observed climatic time series. According to known method comparison tests, ACMANT most often provides the smallest residual root mean square error and trend bias among the tested methods.

One important novelty in ACMANTv5 in comparison with earlier ACMANT versions is that the metadata can be used together with the statistical homogenization either in automatic or interactive modes.

The description of the software is organised to eight main sections following this Introduction. Section II includes a list of concepts and definitions, as well as the explication of all symbols used in the document. The knowledge of concepts and definitions presented in Sec. II is necessary to understand the content of the later sections. Section III presents the most important operations of ACMANTv5.2, many of them are performed repeatedly during the homogenization procedure. The remaining sections (Sec. IV – IX) describe all the steps of the homogenization procedure in the temporal order of their execution. In Sec. IV the preparatory operations are presented, such as the construction of climatically semi-homogeneous station networks of large datasets, calculation of derived variables from the input data, etc. Sections V – VII describe the 3 main homogenization cycles. Section VIII presents the calculations for seasonal variations of inhomogeneity biases. This section is performed only when the shape of the annual cycle of inhomogeneity biases is presumed to be notably different from both of the sinusoid model and flat model. The last section (Sec. IX) presents the post-operation steps, e.g., final gap filling, and the preparation of the final output results. ACMANTv5.2 includes a few methods which were developed by other method developers, in such cases the sources are shown in the References.

Inhomogeneity detections are always performed on the difference series of a candidate series and a reference series, and such difference series are named relative time series. Each of the three main homogenization cycles includes the construction of relative time series, filtering of spatial outlier values (exceptions will be specified), gap filling, inhomogeneity detection and adjustments for the detected inhomogeneities, but the details of the execution often differ according to the specifics of the homogenization task (e.g. climatic variable, seasonality of inhomogeneities, etc.) and the phase of the homogenization procedure. With the repetition of the same kind routines, most steps are performed with gradually increasing accuracy during the homogenization procedure.

In spite of the easy-to-use construction of ACMANTv5.2, its use is recommended for skilled persons, as any error in the input data preparation or introduction of parameters may lead to serious errors in the results. The input data preparation, the properties of the output items, and the user options are described in the Manual which can be purchased together with the software from the creator (https://acmant.eu).

II BASIC CONCEPTS AND DEFINITIONS

A1 Concepts and definitions

1-network homogenization: An input dataset including no more than 22 time series and sufficient spatial correlations for all pairs of the time series is homogenized together, and no network construction is performed during the homogenization procedure (see also **Network**). In 1-network homogenization the **core network** is identical with the base network.

3-month overlapping seasons: 12 seasons constructed by merging 3 adjacent calendar month, i.e. JFM, FMA, MAM, etc. They are referred to as 3-month seasons.

Additive variables: In the modelling with ACMANT, the magnitude of inhomogeneities (i.e. the deviation of the observed values from the true climate) can be independent from the climatic value (additive) or proportional to the climatic value (multiplicative). The additive model is applied for temperature, relative humidity, sunshine duration, radiation, wind speed, wind gust and atmospheric pressure. Hereafter the latter group of climatic elements is referred as additive variables.

Bi-seasons for precipitation data (RR): The two seasons are rainy season and snowy season. The seasonal division must be uniform for a whole input dataset. A month belongs to the snowy season if more than 50% of the precipitation falls in form of solid precipitation, while it belongs to the rainy season in the reverse case. A seasonal value is the sum of the monthly values. Under a warm climate the full year is considered to be rainy season (1-season model). Note that “dry season” does not exist in ACMANT homogenization. The length of the snowy season can be 0 or between 3-9 months. If it was 1 month or 2 months by user definition, ACMANT adds the adjacent months to the defined months to lengthen the snowy season. If it was higher than 9 months by user definition, the 1-season model is applied.

Bivariate homogenization: The homogenization is performed for two different variables. In ACMANTv5.2 the two variables can be (a) the annual mean and the **summer – winter difference**, or (b) the precipitation total of rainy season and precipitation total of snowy season. In both cases, the two variables are derived from the data of a single climate variable. Several steps of the homogenization are performed separately for the two variables, but a peculiarity is that in bivariate break detection the break positions are searched with the joint optimization of two step functions (Sec. B7). Based on some introductory pieces of information introduced by the user at the beginning of running ACMANTv5.2, the software decides if univariate homogenization or bivariate homogenization will be performed.

Break: Sudden shift in the section means of **station effect**. Breaks are instantaneous changes, they take place between two consecutive values of a time series. The date (timing) of the break is the last date before the event. ACMANT detects only break type inhomogeneities, which means that the detection of other kind of inhomogeneities are approached with the detection of one or more breaks.

Breakpoint: The timing of the break can be referred to as breakpoint.

Candidate series: During the homogenization with ACMANT the time series of nearby observations are compared. The spatial comparisons are made for the homogenization of 1 series that is the candidate series. The concept of candidate series is also used in gap-filling with spatial interpolation, and in the calculation of adjustment terms with weighted ANOVA model.

Central series: In **multi-network homogenization** each network has a central series, and the purpose of the homogenization in network is the homogenization of the central series. The other time series of the network, so-called **neighbour series** for the central series, are gathered to the network principally according to their spatial correlation with the central series, but the data coverage between the central series and potential neighbour series is also considered. ACMANT functions in a way that all time series of a network are homogenized together, thus in spite of the aim is homogenizing only the central series, most steps are performed in the same way for every series. The main difference between the concepts of **candidate series** and central series is that during the homogenization of a network the role of candidate series varies, while the central series is fixed. In 1-network homogenization central series is not defined.

Core network: In large networks (number of time series > 20) often two different size networks are applied. In such cases the base network is applied only in some early steps of the homogenization procedure, while a smaller network, "core network" is used in most of the steps. Both the base network and core network are edited automatically, and they are editable in the interactive mode (see also **Network**).

Daily homogenization: Homogenization based on an input dataset of daily resolution.

Effective number of time series: In any network, the effective number of time series ($\overline{N'}$) is the temporal average of the annual number of time series (*N'*) whose **homogenized period** include a given year (*y*). The arithmetical average of the annual values over the **homogenized period for network** (*n**) is taken, and the integer part of this average defines the effective number of time series (1).

$$\overline{N'} = \mathrm{Int}\left\{\frac{N'(y)}{n^*}\right\} \qquad (1)$$

Effective partner series: It is a time dependent characteristic. A **partner series** is effective when it has observed data.

Ensemble homogenization: Some steps of homogenization are repeated with slightly differing setups for the ensemble members and statistics of the ensemble results are used for adjusting the time series. Steps of the homogenization procedure repeated within an ensemble cycle will be marked with "E" in the headline of the step.

Excluded year: When less than 3 stations have observed annual values for a year, that year is excluded from most steps of the homogenization procedure, exceptions will be indicated. An annual data is considered to be "observed" when in at least 9 months of the year the status of monthly data is "observed". Excluded years are not part of any homogenized period or treated period, except when text indicates it in other way.

External missing data: Missing observation out of the **treated period** of a time series. While common missing data are for gaps in data availability (for any reason), the principal source of external missing data is the relative shortness of the period of continuous observations. In ACMANTv5.2 external missing data are not substituted by interpolated data, except for the completion of the final output time series to the **target period** when user requires that.

Homogenized period for network: It lasts from the earliest starting year of **homogenized period**s **of time series** to the latest ending year of homogenized periods of time series.

Homogenized period of time series: ACMANT needs at least 4 time series of sufficiently high spatial correlations for performing homogenization, and this condition is also a requirement for sections of time series. The homogenized period includes entire years only, and it is shorter than or identical to the treated period. Time series without homogenized period are left out of consideration in most steps of the homogenization procedure.

Note that term “homogenized period” reflects the spatial-temporal coherence of the data and does not the status of the data, so that it can be applied either to series already homogenized or to series which will be homogenized in a later phase of the ACMANT procedure.

If a given year would be part of the homogenized periods of less than 4 station series of a network, this year will not be part of a homogenized period in any series. If more than one homogenized periods can be edited for a station series, only the last of them will be used as homogenized period.

Input dataset: Collection of time series of observed climatic values. Each time series must contain the values of the same climatic element and with the same temporal resolution, which can be monthly or daily. An input dataset may have 4 to 5000 time series which may cover varied periods.

Missing annual value: When no more than 3 monthly values are missing in a year, the status of annual data is observed, while it is missing or interpolated in the opposite case.

Missing monthly value in case of daily resolution of data: For additive variables: when no more than 7 observed daily values are missing in a month, the status of the monthly data is observed, while it is missing or interpolated in the opposite case. For precipitation: when any daily data is missing, the monthly data is set to missing or interpolated.

Monthly homogenization: Homogenization based on an input dataset of monthly resolution.

Monthly value in daily homogenization: It is monthly mean of daily values for most of the additive variables, except for sunshine duration and radiation. It is monthly total of daily values for precipitation, radiation and sunshine duration.

Multi-network homogenization: In case of large input datasets or datasets containing pairs of time series with insufficient spatial correlation between them, one specific network is constructed for each time series of the input dataset. (See also **Network**).

Multiplicative variable: In this model, inhomogeneity biases are proportional to the climatic values. This model is applied to precipitation amount (RR).

Neighbour series: All the time series within a network are neighbour series of each-other.

Network: If an input dataset has no more than 22 time series and all of the spatial correlations are higher or equal to 0.4, then the dataset forms one only network (**1-network homogenization**), while in other cases the dataset is divided to smaller networks and **multi-network homogenization** is performed. In ACMANTv5.2 the network formation is automatic, although in interactive mode user may modify the networks. The number of time series within a network is maximised by 99 and rarely higher than 50. Manual edition of too large networks is not recommended, since the time consumption of ACMANT increases fast with network size, while the accuracy of homogenization cannot be improved by them. The homogenizations in different networks are fully independent from each-other.

Outlier period: Short-term biases, examined in the homogenization of additive variables. These inhomogeneities can be detected only for relatively large size biases, and in this respect they are similar to spatial outliers. In ACMANTv5.2 the length of outlier periods varies between 10 days and 28 months (2 months and 28 months) in daily (monthly) homogenization.

Outlier value: Two kinds of outlier values are considered: physical outliers and spatial outliers. The thresholds for physical outliers can be user-defined values or default values. In monthly homogenization, spatial monthly outliers of the homogenized period are filtered (optional). Possible spatial daily outliers are not examined.

Partner series: Neighbour series taking part in the homogenization or gap-filling of a candidate series are referred to as partner series.

Reference series: A time series which is compared with the candidate series for inhomogeneity detection is referred to as reference series. Reference series are usually composed from several **partner series**.

Seasonal cycle: The modelled seasonal cycle of inhomogeneity biases is referred with this term, it can be sinusoid, irregular or flat (constant) according to user choice. In the sinusoid model the modes of the cycle is coincides with the two solstices. In the irregular model the shape of the annual cycle is calculated from the observed data.

Station effect: Non-climatic component (v) of observed data, influenced by the characteristics of station location, as well as the technical and personal conditions of climate observation.

$$\mathbf{X_s} = \mathbf{U_s} + \mathbf{V_s} + \boldsymbol{\varepsilon_s} \qquad (2)$$

In (2) **U** stands for the regional climate signal, **V** for the station effect, while in ε the effects of weather and occasional observation errors are summed up and the subscript s is the station index. ε usually can be modelled well with white noise or red noise. In case of a homogeneous series, the station effect is constant, while temporal changes of v are inhomogeneities.

Summer – winter difference: When the seasonal cycle is sinusoid, the summer – winter difference (denoted with upper wave) is used in the homogenization procedure. It is defined for any month h_0 of **X** by the values within a time window around h_0 (3).

$$\widetilde{x_{h_0}} = \frac{1}{3.5}\left(\sum_{h=h_0-5}^{h_0+5} \mu_m x_h + 0.5\mu_m (x_{h_0-6} + x_{h_0+6})\right) \quad (3)$$

For summer months: $\mu_5 = \mu_6 = \mu_7 = 1$ $\quad \mu_8 = 0.5$
For winter months: $\mu_1 = \mu_{11} = \mu_{12} = -1$ $\quad \mu_2 = -0.5$
For the other months: $\mu_3 = \mu_4 = \mu_9 = \mu_{10} = 0$

When h_0 is closer than 6 months to the starting or ending month of **X**, the time window around h_0 will be truncated and a logically fitting definition is provided (not shown). The relation between the serial number of month from the beginning of time series h, the serial number of year (y) from the beginning of time series and the serial number of calendar month m is defined by (4).

$$h = 12(y-1) + m \qquad y \in \{1,2,\ldots n\} \quad (4)$$

The definition of summer – winter difference for year y, coherent with (3) and (4), is shown by (5).

$$\widetilde{x_y} = \frac{1}{3.5}\left(\sum_{m=1}^{12} \mu_m x_{y,m}\right) \quad (5)$$

Target period of homogenization: While the periods of input time series may be varied, the target period is fixed for the entire dataset. A target period can be either shorter or longer than the period covered by individual time series of the input dataset. When an input time series is longer than the target period, the input data outside the target period will be dropped, and when an input time series is shorter than the target period, that series will be supplied automatically with missing data codes before the homogenization. The target period is defined by the user at the beginning of the homogenization. Its length is expected to be between 10 and 200 years, and ideally it reflects well both the purpose of homogenization and the data availability. After the definition of the target period, all the time series of the dataset have the same length. An example of N monthly series of n year period is shown by (6).

$$\mathbf{X_s} = x_{s,1}, x_{s,2}, \ldots x_{s,h} \ldots x_{s,H} \quad (s = 1,2 \ldots N),\; H = 12n \quad (6)$$

In (6), h stands for the serial number of month from the beginning of the time series.

Transformed precipitation (TR): The multiplicative variable precipitation (RR) is transformed to the additive variable TR (see Sec. B1) for most of the operations with precipitation data. When operations are performed with the untransformed RR, they will be indicated in the description. Note that in spite of TR behaves as an additive variable, it does not belong to the group of additive variables in this description.

Treated period of time series: ACMANT needs a certain amount and temporal compactness of the observed data for treating time series. The minimum amount of observed monthly values is 114 and the minimum compactness is 25% temporal density of observed values. Note, however, that between two adequately compact blocks with at least 60 observed monthly values in each, the extent of data gaps is unlimited.

A treated period includes entire years only, i.e. it starts with 01 January of its first year and ends with 31 December of its last year. The minimum length of treated period is 10 years without excluded years. Time series without acceptable treated period are left out of consideration during the homogenization procedure. Usually no or few observed data occur outside the treated period. Observed data outside the treated period are left out of consideration in most steps of the homogenization procedure, exceptions will be indicated.

A2 Mathematical symbols

Most ymbols, although not all of them, are used with the same meaning throughout this document. The meanings of letters *a*, *A*, *b*, *B*, *i* and *j* are varied and they are defined at the relevant section of the document. Versions of the same kind variable are often distinguished with apostrophe, asterisk or other supplements. The list of symbols shown here always includes the basic version of the symbols, and only for a few cases includes other versions. Most of the mathematical symbols are printed with italics, except for Y (which represents cluster) and vectors of time series, the latter ones are printed bold.

C – Caussinus-Lyazrhi statistic
d – calendar day
d' – serial number of the day from the beginning of the examined section of time series
D – number of days within a month
E – external variance
f, **F** – reference series
g, **G** – deseasonalised time series
gc, **Gc** – candidate series (deseasonalised)
h – serial number of the month from the beginning of the examined section of time series
H – number of months in time series
I – internal variance
J – number of effective partner series
k – serial number of break
K – number of breaks in a time series
l – length of period in days
L – length of period in months or years
m – calendar month
*m** – 3-month season
M – number of relative time series for a given candidate series
n – number of years in the treated period
n' – number of years in the homogenized period of a time series
*n** – number of years in the homogenized period of a network
N – number of stations in the dataset or in a network
N' – number of stations in a network whose homogenized periods include a given year
$\overline{N'}$ – effective number of time series

N^* – number of partner series
$N^{\#}$ – number of time series including synchronous breaks at a given time
p – parameter
P – penalty term
q – bias size for a section of time series
r – spatial correlation
r^* – spatial correlation of increment series
r_{th} – threshold correlation vale
s – station index of time series
S – score
t, **T** – relative time series
u, **U** – climate signal
v, **V** – station effect
w – weight
W – sum of the weights of the partner series
x, **X** – climate data series
xc, **Xc** – candidate series
$\tilde{x}, \widetilde{\mathrm{X}}$ – summer – winter difference
y – serial number of year from the beginning of the examined section of time series
Y – cluster
z, **Z** – adjustment term
α – score for break significance
$ß$ – usefulness of relative time series
δ – break size
Δ – difference, deviation
ε – noise
μ – season-coefficient
σ – standard deviation
ω, $\mathbf{\Omega}$ – adjustment term for the seasonal variation of biases
¯ [upper stroke] – arithmetical average
^ [cup over a letter] – estimated parameter
[a,b] section between a and b, in which a and b are included

A3 Quality indicators

Series **G**, **Gc**, **X**, **Xc** often hold quality indicators, which show that the time series have passed the outlier filtering and/or inhomogeneity adjustments, or not.

G – neither outlier filtered, nor homogenized
$\mathbf{G}^{+}$ – outlier filtered, but not homogenized
$\mathbf{G}^{\#}$ – outlier filtered for outlier periods shorter than 5 months. Longer outlier periods and inhomogeneities have not been treated.
$\mathbf{G}^{*}$ – pre-homogenized, but not outlier filtered
$\mathbf{G}^{+*}$ – outlier filtered and pre-homogenized
$\mathbf{G}^{**-}$ – completely homogenized, but not outlier filtered
$\mathbf{G}^{**}$ – completely homogenized and outlier filtered

III BASIC OPERATIONS

Characteristic operations of ACMANTv5.2 are picked out to this section. Most of them are executed repeatedly in different stages of the homogenization procedure, although some details of the execution may depend on the stage of the homogenization. Details dependent on the stage of the homogenization procedure are not shown in this section.

B1 Transformation of precipitation data

A quasi-logarithmic transformation is applied to RR data to create the additive version (TR) of this variable (7).

$$TR = \ln(RR) \qquad \text{when } RR \geq 30 \text{ mm}$$
$$TR = \ln(0.4RR + 0.01RR^2 + 9) \quad \text{when } RR < 30 \text{ mm} \tag{7}$$

Monthly, annual and bi-seasonal RR are subjected to this transformation. Annual and bi-seasonal values of TR are converted from annual and bi-seasonal RR, respectively, and never from summing up monthly TR. In most operations of the precipitation homogenization TR is used.

B2 Removal of climatic seasonality

For additive variables, as well as for monthly TR: As a preparatory step, the seasonal cycle is removed and it is added back only at the end of the homogenization procedure. The treatment for TR is partly different (see step 4.5).

Observed values of the treated period of series *s* are separated, and their cluster is denoted with Y_s, its sub-cluster for calendar month *m* with $Y_{s,m}$. The number of elements in Y_s and $Y_{s,m}$ are H_s^* and $H_{s,m}^*$, respectively. The monthly climatic normal ($\overline{X_{s,m}}$) is subtracted from the observed values (8-10).

$$\overline{X_{s,m}} = \frac{1}{H_{s,m}^*} \sum_{Y_{s,m}} x_{s,h} \tag{8}$$

$$g_{s,h} = x_{s,h} - \overline{X_{s,m}} \tag{9}$$

$$\mathbf{G_s} = g_{s,1}, g_{s,2}, \dots g_{s,h} \dots g_{s,H} \tag{10}$$

Note that (i) in daily homogenization (8) is unchanged, and in (9) $\overline{X_{s,m}}$ is subtracted from the daily values of **X**; (ii) in the daily homogenization of sunshine duration or radiation, $\overline{X_{s,m}}$ represents the monthly mean climatic value of the daily values.

B3 Spatial correlation

Two kinds of spatial correlations are used, both of them are based on deseasonalised monthly values. In precipitation homogenization, the transformed variable (TR) is used. In the first version (r) the Spearman correlation between simultaneous monthly values is calculated. In the other version (r^*) the increment series of monthly values are calculated, and then the Spearman correlation is computed from them. r^* is widely used in time series homogenization, since it is less affected by inhomogeneities than r. In ACMANTv5.2 mostly r^* is used, except for gap filling, because there data of limited time windows are used, hence the impact of possible inhomogeneities is different in gap filling than in the other steps of the homogenization procedure.

Both for r and r^*: Both values of a source data pair must be observed values (i.e. interpolated values are excluded), and they must be within the treated period. The minimum number of source data pairs is 50, otherwise the correlation is considered 0.

B4 Creation of relative time series

In break detection and outlier filtering steps the difference of a candidate series (**Gc**) and its reference series (**F**(gc)) is examined and it is named relative time series (**T**). Typically, **F** is a linear combination of some partner series (11-12), and referred to as composite reference series. The creation and use of composite reference series is based on the work of Peterson and Easterling (1994). Note that differing from the usual case, single series are used as reference series in pairwise comparisons (Sec. B8.1). Two kinds of composite reference series are used, type 1 is for the break detection on the annual scale, while type 2 is for the other operations. For this reason, we also use the concepts of type 1 relative time series and type 2 relative time series.

$$\mathbf{T} = \mathbf{Gc} - \mathbf{F} \tag{11}$$

$$\mathbf{F} = \frac{\sum_{s=1}^{N^*} w_s \mathbf{G_s}}{\sum_{s=1}^{N^*} w_s} \tag{12}$$

N^* is the number of partner series used to compute the composite reference series. ($3 \le N^* \le N - 1$). The selection of partner series to a given section of the candidate series has the following rules both for type 1 and type 2 reference series:
i) The minimum length of the examined section of the candidate series is 10 years;
ii) The treated period of a partner series cannot be shorter than the examined section of the candidate series;
iii) Possible data gaps in the treated period of the partner series are infilled with spatial interpolation before their use;
iv) Each partner series must contain at least 12 observed monthly values (the other values may be interpolated values).
v) The partner series are selected from the neighbour series of the base network or core network according to the stage of the homogenization.
vi) Only neighbour series with sufficient spatial correlation with the candidate series (default $r_{th} = 0.4$) are accepted to be partner series.
vii) The partner series are selected in the order of their spatial correlation (r^*) with the candidate series, starting from the best correlating series.

Only one difference exists between the generation of type 1 and type 2 reference series: while for type 1 series all the neighbour series are used which meet the conditions of ii) to v), for type 2 series the integer part of the mean effective partner series is maximised by 10.

B5 Multiple sets of reference series

When the input dataset is complete over the target period of homogenization, the treated period is identical for every time series. In this case, 1 relative time series is edited to each candidate series, and each of them covers the full target period. However, the temporal extent of the observed data in the time series of the input dataset often cover varied periods. In this case, the edition of suitable relative time series is less straightforward, and multiple sets of relative time series are used in ACMANT, and individual relative time series often cover only a section of the candidate series.

For a given candidate series **Gc**, the number of usable neighbour series may vary from year to year, but within a given year it is constant, since treated periods include entire years. The concept of "best fitting reference series" to year *y* of the candidate series will mean the composite reference series composed by the maximum possible number of partner series starting with year *y*, and whose length is the maximal allowed by the data availability of the partner series. A simplification is that here in Sec. B5 the term "reference series" is used for the unweighted compositions of partner series (denotation: **F'**). The rules of weighting vary according to the steps of the homogenization procedure and they will be detailed later.

Often, the same set of partner series is applicable for many years, hence the number of reference series is often lower than 10, in spite of their theoretical maximum number is as large as $\frac{(n'-10)^2}{2} \approx 18{,}000$ when n' = 200. A procedure is established (B5.1 and B5.2) to exclude less effective versions, and to limit the maximum number of reference series by 80.

B5.1 Covering the treated period with at least 1 reference series

The purpose of this step is to cover as much part of the treated period ($y = 1,2,\ldots n$) of the candidate series as possible with at least 1 reference series. The best fitting reference series to the first year is selected, and its period $[1,y_1]$ and the sum of squared spatial correlations with the candidate series (13) are retained.

$$W_i = \sum\nolimits_{\mathrm{Y}_i} r^{*2} \tag{13}$$

In (13), Y_i denotes the cluster of the selected partner series composing $\mathbf{F_i'}$, W_i is the sum of the weights, while index *i* is the serial number of the edited reference series. When no valid reference series can be edited to $y = 1$, then the following years are tried one-by-one, until a valid reference series can be edited. If $y_i = n$, this part of the procedure stops, while the best fitting reference series to year $y_i - 8$ is edited in the reverse case. This latter series $[y_i - 8, y_{i+1}]$ will include at least 1 year after y_i, since the minimum length of a relative time series is 10 years. The procedure stops when no further years of the treated period can be covered by reference series. In this part of the procedure $M' \leq n/10$ reference series are constructed and kept. The earliest starting year of these reference series defines the start of the homogenized period, while the latest ending year of the reference series defines the end of the homogenized period. Rarely, middle sections of the candidate series cannot be covered by reference series, while early and late sections yes. This may occur when the ratio of external missing data in a middle section of the target period is lower than in the starting and ending periods. In such rare cases only the latest homogenized period of the candidate series will be considered, so that a homogenized period is always one continuous period.

B5.2 Selecting the best fitting reference series for each year

In the continuation, the best fitting reference series are edited to each year of the homogenized period. In a given step, reference series $\mathbf{F_i'}$ is compared with each of the *M''* reference series those have already been retained (*M''* ≥ *M'*). Let $y_{i,1}$ $\left(y_{i,n_i}\right)$ stand for the first (last) year of $\mathbf{F_i'}$. $\mathbf{F_i'}$ will be retained if at least one of relations (14-16) (but not necessarily the same relation) is true for all of the earlier selected reference series (*j* = 1,2,…*M''*), while it is dropped in the reverse case.

$$0.95W_i > W_j \tag{14}$$

$$y_{i,1} \le y_{j,1} - p_1 \tag{15}$$

$$y_{i,n_i} \ge y_{j,n_j} + p_1 \tag{16}$$

Usually $p_1 = 0$, but in the unlikely case of *M''* > 80, the procedure of B5.2 is re-started with a new p_1 elevated with 1 in comparison to its previous value.

Finally, with fixing the weights of reference composites, the set of *M* reference series ($\mathbf{F_1}$, $\mathbf{F_2}$,… $\mathbf{F_M}$) and *M* relative time series ($\mathbf{T_1}$, $\mathbf{T_2}$,… $\mathbf{T_M}$) are constructed by Eqs. (11-12) to a given candidate series.

B5.3 Quality level of candidate series

ACMANTv5.2 includes 3 homogenization cycles, and in later cycles the previous operations are usually repeated with data of increased quality. However, the quality level of candidate series in break detection and outlier filtering is exception, as the same inhomogeneities and outliers are examined in the later homogenization cycles as in the earlier ones. In break detection, the candidate series is outlier filtered, but not inhomogeneity adjusted ($\mathbf{Gc^+}$). In outlier filtering, the candidate series includes the adjustments for inhomogeneities, but excludes any correction for outliers ($\mathbf{Gc^*}$). By contrast, once the program has proceeded the first iteration cycle, the reference composites are both outlier filtered and homogenized ($\mathbf{G^{+*}}$) .

B5.4 Use of the multiple sets

A particular year of **Gc** may belong to several relative time series (**T**), but the detection of a break or an outlier is based on one only **T**. In the selection of **T**, higher weights (*W*) and longer series (*n*) are preferred via indicator *ß* (17).

$$\beta_i = W_i \ln(6n_i) \quad i \epsilon (1,2, \dots M) \tag{17}$$

The set of M relative time series are ordered according to their *ß* scores. At any date of the candidate series, the **T** of the highest *ß* will be selected from those which covering the given date. Note, however, that for reducing tail-effects, sometimes not the **T** of the highest *ß* is used, but another one, in which a break or an outlier period is farer from the endpoints of the series. Close to the endpoints of **T** series often an overlap of some relative time series is applied, its specifics will be shown later.

B6 Break detection with step function fitting

A step function with K steps (breaks) splits the time series of {y = 1, 2,…n'} to K+1 constant sections. The method is an adaptation from Caussinus and Mestre (2004), and it is also referred to as optimal segmentation.

We introduce the concepts of internal distance (I) for values from their section mean and external distance (E) for section means from the mean of the whole time series (18, 19).

$$I_y = t_y - \overline{\mathbf{T_k}} \qquad k = 1,2, \dots K+1, \;\; y \in \text{section } k \tag{18}$$

$$E_k = \overline{\mathbf{T_k}} - \overline{\mathbf{T}} \tag{19}$$

For a given number of K, the step positions with minimum internal variance provides the best fitting step function (20).

$$\text{Optimal solution} \equiv \min_{y_1, y_2 \dots y_K} \sum_{k=1}^{K+1} \sum_{y=y_{k-1}+1}^{y_k} I_y^2 \tag{20}$$

In (20), y_1, y_2,… y_K are the estimated break positions, $y_0 = 0$, $y_{K+1} = n'$.

In time series homogenization, the optimal number of steps is unknown. Here, a modified version of the Caussinus – Lyazrhi criterion (C-L criterion, Caussinus and Lyazrhi, 1997) is used for setting K. In the C-L criterion the first expression monotonously decreases with increasing K, but this decrease may be balanced or overbalanced by the penalty term P which increases with increasing K (21,22).

$$C_{\text{opt}} = \min_K \{ \ln(1 - \frac{\sum_{k=1}^{K+1} L_k E_k^2}{\sum_{y=1}^{n'} (t_y - \overline{\mathbf{T}})^2}) + P \} \tag{21}$$

$$P = \frac{p_2 K}{n'-1} \ln(n') \tag{22}$$

L_k in (21) stands for the length of section k. Eqs. (19-22) show the example of annual resolution for the homogenized period (n' years), but note that these formulas can be applied for any time period and with any time resolution. Eqs. (21-22) differ only in parameter p_2 from the original C-L criterion where $p_2 = 2$. In ACMANT, p_2 depends on the stage of the homogenization procedure, and its values differ also between univariate break detection and bivariate break detection.

Eqs. (19-22) can be applied to the whole homogenized period of the candidate series, or to a previously defined section of that.

B7 Bivariate break detection

The step functions fitted to the relative time series of two variables (**T**$\mathbf{T_A}$ and $\mathbf{T_B}$) are jointly optimized by (23-24).

$$\min_{y_1, y_2 \ldots y_K} \{\textstyle\sum_{k=1}^{K+1} \sum_{y=y_{k-1}+1}^{y_k} (I_{A,y}^2 + p_3 I_{B,y}^2)\} \tag{23}$$

$$C_{\text{opt}} = \min_{K} \{ \ln(1 - \frac{\sum_{k=1}^{K+1} L_k (E_{A,k}^2 + p_3 E_{B,k}^2)}{\sum_{y=1}^{n'} ((t_{A,y} - \overline{\mathbf{T_A}})^2 + p_3 (t_{B,y} - \overline{\mathbf{T_B}})^2)}) + P\} \tag{24}$$

B8 Break detection with combined time series comparison

The break detection with combined time series comparison consists of one break detection step performed with pairwise comparisons of time series, and a further break detection step with the use of composite reference series. No adjustments for inhomogeneities is performed between the two break detection steps. The break positions detected in the first step of the procedure are fix break positions in the second break detection step, while the detection of further breaks is possible in the second step. This combined break detection is included in the first homogenization cycle of ACMANTv5.2.

B8.1 Relative time series in pairwise comparisons

Pairwise comparisons mean that the differences between a candidate series and its partner series are examined one-by-one. However, when individual pieces of break detection results are considered, they may point on inhomogeneities in the candidate series or inhomogeneities in the reference series with identical probability. Therefore, inhomogeneities of a candidate series are evaluated only when all pieces of the break detection results have been calculated. Further, as we homogenize all the time series of a network together, the assignation of breaks to individual time series is done only when all pieces of the break detection results for all series have been calculated.

In pairwise comparisons always the differences of two neighbour series are examined. The form of such difference series is the same as in Eq. (11), except that the roles of candidate series and reference series are undefined.

B8.2 Break detection with pairwise comparisons

Step function fitting is applied in univariate or bivariate mode similarly to B6 and B7, but with an important modification: when the differences (Δ) between two or more Caussinus-Lyazrhi statistics (*C*) belonging to different number of breaks (*K*) are small, more than one solutions are simultaneously considered. A difference between $C(K)$ of two different K is small when relation (25) is true.

$$\Delta < 0.5 p_2 \frac{\ln(n')}{n'-1} \tag{25}$$

The meaning of (25) is that the uncertainty of finding the optimal K is considered to be high when there exist at least two C-L statistics being closer to each-other than the half of the increment of the penalty term between two consecutive K. In such cases the break positions detected by differing K but with insignificantly differing C statistics are simultaneously used in ACMANTv5.2. This rule works in the same way in univariate and bivariate homogenization.

The break detection score for a given time point of a given relative time series is usually 0 or 1. However, other pairwise break detection scores (S_p) are possible when more solutions are simultaneously considered, yet scores are the same for all breaks belonging to a given K of a given relative time series. Eq. (26) shows the calculation of preliminary scores (S_p'), which are normalized to S_p by (27). In (27) Y stands for the cluster of the simultaneously considered K values. The aim of the normalization is to provide the same overall weight to any simultaneous solution as solutions with solely K have.

$$S_p'(K) = 1 - 2\Delta(K) \cdot \frac{n'-1}{p_2 \ln(n')} \tag{26}$$

$$S_p(K) = \frac{S_p'(K)}{\sum_Y S_p'} \tag{27}$$

B8.3 Evaluation of pairwise break detection results

When all pairs of time series have been examined, their results are evaluated to assess which time series and at which timings have significant breaks. In ACMANTv5.2 the method of Menne and Williams (2009) is applied with some small modifications.

The scores of individual break detection results are summed at each time point in each time series, these are referred to as yearly summed scores (S_y). An S_y score calculated from the statistical break detection is elevated with 1.0 when metadata points on a likely break occurrence in the same time series and at the same year.

Scores of adjacent years are presumed to indicate the same break, therefore a summation of scores is performed also for adjacent years. Generally, 3-year sums of S_y scores (S3) are generated, and the middle years of the 3-year periods are characterized by them. However, when S_y of the first year or third year of a 3-year period is higher than the 65% of the 3-year total score, the middle year receives zero S3 score to support the break detection at the adjacent year where its true position is the most likely, based on the yearly score values.

When a time series at a given time point has high S3 score, it likely has a break there, while low scores may indicate the impact of the breaks of the neighbour series or noise. One important aspect of the correct evaluation is that any piece of the pairwise detection results is assigned to only 1 time series, or dropped when it seems to be noise effect. The evaluation goes from the highest S3 scores in the order of decreasing scores, and once a break has been assigned to a time series, its S_p score is subtracted from the relevant S3 scores of the other time series. The procedure terminates when the remaining S3 scores are smaller than a given threshold (p_{th}). In ACMANTv5.2 p_{th} = 2.1. In the present application the algorithm excludes the possibility of detecting two breaks in adjacent years.

B8.4 Break detection with the use of composite reference series

A step function fitting procedure is performed according to the rules of B6 or B7, but the optimal solution is selected from a reduced set of possible solutions. Let suppose that for series s K_P breaks have been detected in the step of pairwise comparisons, with break dates $\{y_1, y_2, \dots y_{KP}\}$. The optimum solution will be searched for the cases $K \geq K_P$ where break dates $\{y_1, y_2, \dots y_{KP}\}$ are included.

B9 ANOVA correction models

ACMANTv5.2 applies two versions of the ANOVA correction models. In the simpler model the climate is presumed to be spatially constant, while in the more complex “weighted ANOVA model” the spatial variation of climate is taken into account.

B9.1 Simple ANOVA model

The simple ANOVA model was introduced to time series homogenization by Caussinus and Mestre (2004). Using this model, the break sizes for all breaks of all time series of a given network are jointly estimated by an equation system. The equation system is based on the model of Eq. (2) and on the minimization of the variance of homogenized time series. In the practical solution, Eq. (2) is applied to the estimates of **U** and **V** with known break positions ($j_{s,k}$) and under the condition $\hat{\varepsilon} \equiv 0$. In this model, $\mathbf{\Delta U} \equiv 0$ within a given network of N time series. Based on these conditions, the equation system (28, 29) is constructed, from which the optimal break size estimates can be calculated.

$$\frac{1}{L_{s,k}} \sum_{i=j_{s,k-1}+1}^{j_{s,k}} \widehat{u_i} + \widehat{v_{s,k}} = \overline{\mathbf{x_{s,k}}} \quad (k = 1, 2, \dots K+1) \qquad (28)$$

$$\widehat{u_i} + \frac{1}{N} \sum_{s=1}^{N} \widehat{v_{s,k(i)}} = \frac{1}{N} \sum_{s=1}^{N} x_{s,i} \qquad (29)$$

Eq.(28) is applied to each homogeneous section (k) of each time series (s), while (29) is applied to each time point of the homogenized period of the network. $j_{s,0}$ and $j_{s,K(s)+1}$ point to the endpoints of the homogenized period of s, e.g., when it is [1n’], $j_{s,0}$ = 0 and $j_{s,K(s)+1} = n'$. Note, however, that the homogenized periods of individual time series may differ in a network, therefore the values of $j_{s,1}$ and $j_{s,K(s)+1}$ may depend on s, and N may vary according to i. The model can be applied in any time resolution.

B9.2 Weighted ANOVA model

In the weighted ANOVA model the spatial differences of climate within a network are taken into account as deviations from one selected station series, referred to as candidate series. In multi-network homogenization the central series is the candidate series, while in 1-network homogenization the weighted ANOVA is repeatedly performed giving once the role of candidate series to each time series. Eq. (29) of the simple ANOVA model is transformed to (30) which includes station depending weights (*w*). In ACMANTv5.2 the weights are determined by the squared spatial correlation (r^{*2}) with the candidate series. For the candidate series *w*=1. No other change occurs in comparison with the simple model.

$$\sum_{s=1}^{N} w_s \widehat{u_{s,i}} + \sum_{s=1}^{N} w_s \widehat{v_{s,k(i)}} = \sum_{s=1}^{N} w_s x_{s,i} \tag{30}$$

B10 Gap filling with spatial interpolation

B10.1 Gap filling for additive variables

A spatial interpolation for the monthly missing value $h0$ of candidate series **Gc** is performed using the weighted average of the observed values in a few partner series at the same time (31). Deseasonalised values are used, and the weighted average of the partner series values is tuned to the long-term mean of the candidate series (32).

$$gc_{h0} = \frac{1}{W'} \sum_{s=1}^{N^*} w_s g'_{s,h0} \tag{31}$$

$$g'_{s,h0} = g_{s,h0} + \frac{1}{H'} \sum_{h=h_1}^{h_2} (gc_h - g_{s,h}) \tag{32}$$

Neighbour series with observed value in $h0$ and with sufficient spatial correlation with the candidate series ($r \geq 0.4$) can be accepted as partner series. The maximum number of partner series depends on the suitability of partner series and the stage of the homogenization procedure. Weight w_s is a function of r_s, the frequency of missing data around $h0$ and the status of observed data, i.e. they are within the homogenized period or not. h_1 and h_2 are thresholds of the time window around $h0$, and H' denotes the number of observed values used within that window. The window width around $h0$ depends on the data availability around $h0$, and it may be different in the comparisons with different partner series *s*. The window width depends also on the stage of the homogenization procedure. Minimum threshold $p_4 = 0.4$ is set for the sum of weights (W') according to (33-34).

$$W' = \max(p_4, W) \tag{33}$$

$$W = \sum_{s=1}^{N^*} w_s \tag{34}$$

It follows from Eq. (33) that when *W* is very low, gc_{h0} is close to 0, which is the climatic normal value.

Following the logic of Eqs. (31,32), the interpolation for daily missing data *d*0' is performed by (35,36).

$$gc_{d0'} = \frac{1}{W'}\sum_{s=1}^{N^*} w_s g'_{s,d0'} \tag{35}$$

$$g'_{s,d0'} = g_{s,d0'} + \frac{1}{H'}\sum_{h=h_1}^{h_2}(gc_h - g_{s,h}) \tag{36}$$

Eq. (36) shows that monthly data are used in the tuning to the long-term mean of the candidate series also in daily data interpolation. Eqs. (33,34) are valid also for daily data interpolation.

B10.2 Gap filling for precipitation data

In this operation, data without logarithmic transformation (RR) are used, and the annual cycle of observed data is kept (37, 38).

$$\overline{xc_{[h_1,h_2]}} = \frac{1}{H'}\sum_{h=h_1}^{h_2} xc_h \tag{37}$$

$$\overline{x_{s[h_1,h_2]}} = \frac{1}{H'}\sum_{h=h_1}^{h_2} x_{s,h} \tag{38}$$

Omitting $[h_1, h_2]$ from the indexes of the long-term averages, and using *W* according to Eqs. (33-34), the interpolation for monthly missing data is shown by (39).

$$\begin{aligned} &\text{if } W \geq p_4, \quad xc_{h0} = \frac{1}{W}\sum_{s=1}^{N^*} \frac{w_s x_{s,h0}\overline{xc}}{\overline{x_s}} \\ &\text{if } W < p_4, \quad xc_{h0} = \frac{1}{p_4}\left(\sum_{s=1}^{N^*} \frac{w_s x_{s,h0}\overline{xc}}{\overline{x_s}} + (p_4 - W)\overline{xc_m}\right) \end{aligned} \tag{39}$$

In Eq. (39), *m* stands for the calendar month to which *h*0 belongs. In daily data interpolation (39) is valid with the only changes that daily value index *d*0'should stand in the places of monthly value index *h*0. The rules of partner series selection and parameter use are the same as in B10.1.

C Further important characteristics of ACMANTv5.2

Time scales of operations: The signal-to-noise ratio is the highest in the homogenization of the annual values, while it gradually worsens with the increase of time resolution. For this reason, ACMANT preforms several operations on annual scales, even when the input data and final homogenization products are of finer resolution. Given that the accurate estimations for annual mean trends and variability are of high practical importance in climatology, the preservation of the accuracy of the annual data is prioritized in downscaling the annual homogenization results to finer time scales. In the first two homogenization cycles most operations are performed in annual or monthly scales, while in Sec. VII the homogenization is performed in finer time resolutions.

Filtering of spatial outlier values and outlier periods: Possible occurrences of outlier values and outlier periods are examined in relative time series. Firstly the concept of outlier period is explained here. If the difference for a short section mean is large in the comparison with the average value of adjacent sections of the time series, it is an indication of an outlier period, and its statistical significance is examined. Confirmed outlier values and values belonging to a confirmed outlier periods are considered to be missing data, and they are substituted by spatially interpolated values. However, there is one important exception: In the last round of the filtering of outlier periods, those of longer than 4 months duration are considered to be short-term inhomogeneities, and they are adjusted in the same way as any other inhomogeneity bias.

Coincidental breaks: Coincidental breaks may occur accidentally or for organized and synchronously introduced technical changes in a group of observing stations. Coincidental breaks are a principal error source of time series homogenization, since the basic tool of creating the differences of the observed series may lose their efficiency when the same kind of inhomogeneities with the same timing present in several time series. The severity of the problem depends on the ratio of the time series influenced by the synchronous inhomogeneity, among other factors. The software sometimes automatically cancel some detected breaks of relatively low significance when more significant breaks are detected with the same timing in some other time series. ACMANTv5.2 sometimes send warning messages about the incidences of detected coincidental breaks. The purpose of such warning messages is to call the user's attention to the occurrence, timing and ratio of coincidental breaks. Nevertheless, coincidental breaks never impede the operation of the software, and the automatic programs still intend to minimize inhomogeneity biases.

IV PREPARATORY STEPS

From now the operation of ACMANTv5.2 is described in its logical and operational order. The operations are organised into 27 main steps of 5 sections. The main steps are often divided into sub-steps.

The sections describing the steps often start with basic information regarding the target variables, period of time series, time resolution, etc. However, such pieces of information are excluded when they are obvious from the section titles. Beyond this, target variables are not mentioned when a step is performed for all variables treated by ACMANTv5.2. For steps including the use of relative time series, the working period (i.e. the period for which the operations are performed) is not mentioned, as it must be the homogenized period.

1 Reading of input

Input dataset, metadata (if provided) and user defined parameters are read.

2 Basic control of input data

2.1 Control of date order and input data format, and justification to the target period

Data out of the target period are removed. For time series shorter than the target period, the missing sections are added with blocks of missing data codes. From this step, all the series have the same extent over the same period, which is the target period.

If the program finds a date order error or data formatting error, then it will stop with an error message.

2.2 Control of physical outlier values

Section of time series: whole series (target period).

If the program finds a physical outlier, it will be considered missing data in the continuation.

3 Construction of networks

The dataset is divided into smaller networks when the number of time series is higher than 22, or when the spatial correlation (r^*) is smaller than 0.4 for at least one pair of time series. In datasets of $N \leq 22$, the correlations are controlled only for time series pairs with at least 50 data pairs. Steps 3.1 – 3.4 are necessary to calculate the correlations in step 3.5.

3.1 Calculation of monthly values from daily values

Performed in daily homogenization. Section of time series: whole series.

Monthly values are calculated from daily values. The status of each monthly value is determined, they can be “observed” or “missing”.

3.2 Determination of the treated period

3.3 Transformation from monthly RR to monthly TR

Performed in precipitation homogenization. Section of time series: treated period.

Monthly RR is transformed to monthly TR (see B1).

3.4 Calculation of deseasonalised monthly values

Performed for additive variables and for TR. Section of time series: treated period. Time resolution: monthly.

Climatic mean monthly values are subtracted from the observed values (see B2).

3.5 Calculation of spatial correlation

Section of time series: treated period. Time resolution: monthly.

Spatial correlation (r^*) is calculated for each pair of series of the dataset, according to B3.

3.6 Construction of core networks

Performed when $N > 22$ or when $r^* < r_{th}$ for at least one pair of time series. The default value of r_{th} is 0.4.

A distinct network is constructed for each series, in which they will be central series. In the network construction the inclusion of highly correlated partner series and an even coverage of the central series with observed data of the partner series are favoured, while the increase of the number of time series in network over a limit is penalized. Data gaps or lapses in the periods of observed data can reduce the number of the truly comparable observed data even in large size networks, therefore the number of effective partner series (J) is controlled for every month of the central series. The building of a network starts with the construction of the core network.

3.6.1 Selection of the most highly correlated partner series up to 20 series.

Neighbour series are ordered according to their spatial correlations (r^*) with the central series, then they one-be-one selected to be partner series of the central series up to 20 partner series. When no more than 20 series with sufficient r^* are available, the networking procedure terminates here, and the size of the network is determined by the number of series with sufficient correlations. By contrast, when the number of potential partner series with $r^* \geq r_{th}$ is higher than 20, the following steps (3.6.2 and 3.6.3) are performed recursively, and the size of the core network may be larger than 21. However, the extension of network size above 20 depends principally on the possible unevenness of data coverage by partner series. When each partner series fully covers the treated period of the central series, the size of the core network remains 21.

3.6.2 Calculation of scores indicating the potential usefulness of further partner series

Period of time series: treated period.

In steps 3.6.2 and 3.6.3 further partner series are selected and added to the core network one-by-one. These steps are recurrently performed, as long as new suitable partner series can be found.

Scores are calculated for each series *s* correlating sufficiently with the central series and not yet has been selected to be partner series.

i) Possible occurrences of having less than 10 effective partner series ($J_h < 10$) is checked for every observed monthly value *h* of series *s*. If such cases are found, score S_1 is calculated by (40-41), otherwise $S_1 = 0$.

$$S_1(s) = \sum_{h=1}^{H} r_s^{*4} \left(12 - J_h^*(s)\right)^3 \tag{40}$$

$$J^* = \begin{cases} J & \text{if } J < 10 \\ 12 & \text{if } J \geq 10 \end{cases} \qquad \text{for every selected } h \tag{41}$$

ii) Ratios of cases $J_h < 20$ are checked for all possible overlapping 10-year periods starting on January (referred to as decades). If decades with higher than 25% ratio of $J_h < 20$ occur, score S_2 is calculated (42-43), otherwise $S_2 = 0$. The cluster of the dates belonging to at least one decade with >25% ratio of $J_h < 20$ and which have monthly observed values in *s* is denoted with Y.

$$S_2(s) = \sum_{h \in \mathrm{Y}} r_s^{*4} \left(20 - J_h^{**}(s)\right)^2 \tag{42}$$

$$J^{**} = \begin{cases} J & \text{if } J < 20 \\ 20 & \text{if } J \geq 20 \end{cases} \qquad \text{for every } h \text{ of Y} \tag{43}$$

iii) Network size is scored with S_3 (44).

$$S_3 = -(N'' - 20)^2 \tag{44}$$

In (44), N'' stands for the number of the already approved series in network.

iv) Overall score (S) is calculated by (45).

$$S(s) = S_1 + S_2 + S_3 \tag{45}$$

3.6.3 Selection of an additional partner series

The series with maximal score ($S^* = \max\{S(s)\}$) is selected. If $S^* > 0$, the series of S^* will be a partner series, and the procedure continues from step 3.6.2. If $S^* \leq 0$, no series is selected at this step, and the edition of the core network for the given central series terminates here.

3.7 Creation of the base network

The time series of the core network are kept, and further partner series are searched with lightening the conditions of step 3.6.

3.7.1 Selection of the most highly correlated partner series up to 30 series

This step differs from step 3.6.1 only in the threshold number of the time series. Note that no time series can be selected twice, therefore the number of newly added partner series by this step can be smaller than 10, even when the number of sufficiently correlating neighbour series is higher than 30.

3.7.2 Calculation of scores indicating the potential usefulness of further partner series

Steps 3.7.2 and 3.7.3 are recurrently performed with the same logic and similar content to steps 3.6.2 and 3.6.3.

The operations of step 3.6.2 are performed here with the only difference that the constant of (44) is modified, see (46).

$$S_3 = -(N'' - 30)^2 \tag{46}$$

3.7.3 Selection of an additional partner series

The same as step 3.6.3.

Finally, the size of a base network is larger with 0 to 10 time series than its core network. In base networks with no more than 20 time series, the base network and core network are identical. However, users may edit manually both the base network and core network.

This networking has an important impact on the rest of the homogenization tasks, namely the homogenization results of the central series must be provided only, as each series of the dataset is central series in one network. By contrast, in 1-network homogenization the homogenization results for all the series must be provided. In most steps of the homogenization procedure this difference does not appear, as ACMANT is based on the joint improvement of the data quality and homogeneity in all time series within network. The few exceptions will be indicated.

Note that once the network construction has been finished, data of different networks are never treated together within the homogenization procedure.

4 Preparatory operations in networks

The base network is used.

4.1 Calculation of monthly values from daily values

The same as step 3.1.

4.2 Determination of possible excluded years

Section of time series: whole series.

4.3 Determination of the treated period

The same as step 3.2.

4.4 Transformation from monthly RR to monthly TR

The same as step 3.3.

4.5 Deseasonalisation of observed values

Performed for additive variables and for TR. Section of time series: whole series. Time resolution: monthly and daily.

The monthly climatic means are removed from the observed data according to B2 (**X** → **G**).

Note that in precipitation homogenization the RR → TR transformation of monthly, annual and bi-seasonal values is repeated several times during the homogenization procedure. After such transformations the monthly TR values are always deseasonalised, while annual and bi-seasonal TR values are never.

4.6 Calculation of spatial correlations

Section of time series: treated period.

Both kinds of spatial correlations (r and r*) are calculated for each pair of monthly **G** series. The calculations are performed according to B3.

5 Interpolations for missing daily data in months with observed data

Performed in daily homogenization of additive variables.

The base network is used. Section of time series: treated period.

In the daily homogenization of additive variables a monthly data is classified to be observed when no more than 7 missing data occur in the month. However, missing data might cause biases in the calculation of monthly values. To reduce such biases estimations are made for the missing daily values by spatial interpolation.

The interpolation performed here differs in some details from the other interpolation steps of the homogenization procedure. In the estimation of a daily value, only other daily values of the same month are used, thus Eqs. (35,36) of B10.1 are modified to (47,48).

$$gc_{d0} = \frac{1}{W'} \sum_{s=1}^{N^*} w_s g'_{s,d0} \tag{47}$$

$$g'_{s,d0} = g_{s,d0} + \frac{1}{D'(s)} \sum_{d=1}^{D} (gc_d - g_{s,d}) \tag{48}$$

In (48) D and D' stand for the total number of calendar days and the number of days with observed value pairs in the month of $d0$, respectively. The number of observed value pairs depends on partner series, and only partner series with at least 15 observed value pairs with the candidate series are accepted, and the partner series must have observed value on $d0$, as well. When a missing daily value occurs either in the candidate series or in partner series s, the difference of daily values in Eq. (48) is considered to be zero. In the selection of partner series only neighbour series with sufficient spatial correlations are considered (r_{th} = 0.4), the partner series are selected in the order of their spatial correlation with the candidate series, and the maximum number of partner series is 15.

6 Infilling data gaps on the monthly time scale

The base network is used. Section of time series: treated period.

6.1 Gap filling

It is performed for each piece of missing monthly data of each time series in network, one-by-one, applying the method of B10.

In the selection of partner series two basic requisites are that they must have observed value synchronous with the target piece of the missing data, and they must have sufficient spatial correlation with the candidate series ($r_{gc,s} \geq 0.4$). Potential partner series are ordered according to their $r_{gc,s}$.

For using Eqs. (32), (37) and (38), the collection of a set of synchronous observed data for the candidate series and a partner series is needed. In collecting the necessary amount of data pairs, the use of relatively narrow time windows are preferred aiming to reduce the effect of possible inhomogeneities. However, the use of wider windows is allowed when there are not enough comparable data in narrower windows. For this reason, the window width around a piece of missing data is specific both for the date of the piece of the missing data and the candidate series – partner series pair.

Let the date of a piece of missing data in the candidate series be h_0. Firstly a relatively narrow symmetric time window is used around the year of h_0 denoted with y_0. Pairs of observed data for series *gc* and a potential partner series *s* (*xc* and *s* in precipitation homogenization) are searched first in y_0, then in gradually increasing distance from y_0. Time windows for gap filling include entire years, and differing from the other operations of ACMANT, they may include excluded years. For instance, a 7-year window around h_0 includes the entire period of $[y_0 - 3, y_0 + 3]$. The use of a given time window (p_5) terminates when either the borders of the window are reached, or the required number of monthly value pairs have been found. If the number of pairs of observed data (*H'*) is low within a given time window, then a wider p_5 will be applied. When the required number of data pairs have been found, their required statistics are retained, and the procedure continues with another potential partner series until the correlations are sufficient.

In the calculation of the interpolated value, the statistics of partner series determined by the use of narrower windows are preferred in two ways: a) Potential partner series are ordered according to the used p_5, starting from the narrowest window. Partner series related to wider time windows are considered only when the number of partner series of narrower p_5 is lower than threshold p_6, while the consideration of further partner series terminates when the number of partner series reaches threshold p_7. b) Weight (w_s) of Eqs. (31) and (39) depends on the window width related to a given *s*.

Four time windows can be applied at this step. The related parameters and the weighting of partner series are shown in Table S1.

Table S1. Time windows and weights in the use of partner series in the spatial interpolations of step 6.1. p_5 – width of time window around; p_6 – minimum number of partner series to accept a given time window; p_7 – number of partner series at which the search of further partner series is finished; *r* – spatial correlation; w_s – weight of the observed values in series *s*.

p_5 (years)	Required *H'*	p_6	p_7	w_s
7	60	15	15	r^2
13	30	10	15	$0.9r^2$
25	30	5	15	$0.8r^2$
Unlimited	30	-	15	$0.5r^2$

Note: If $30 \leq H' < 60$ monthly data pairs were found with p_5 = 7 years, p_5 = 13 years is not applied, as the number of data pairs fulfils the requisite for p_5 = 13.

6.2 Basic data treatments

Gap-filled monthly series are prepared for the next steps of the homogenization procedure. Annual and seasonal values are calculated from monthly values, and the RR → TR transformation is performed in precipitation homogenization.

V FIRST HOMOGENIZATION CYCLE

In the first cycle of homogenization he aim is to detect and remove the relatively large breaks, and to keep low the risk of unnecessary adjustments. For this purpose, stricter significance thresholds are applied than in the later homogenization cycles.

7 Creation of relative time series for outlier filtering

Performed in monthly homogenization always and in daily homogenization for additive variables. Not performed in the daily homogenization of precipitation.

The base network is used. Section of time series: treated period. Time resolution: monthly. Type of candidate series: **Gc**. Type of partner series: **G**.

7.1 Determination of sets of partner series

The rules of B5 are applied, and type 2 reference series are created.

7.2 Weighting of partner series

7.2.1 Core method of weighting

The weights of partner series are calculated by ordinary kriging for sets of partner series of $N^* > 5$, while the weights are the squared spatial correlations (r^{*2}) for smaller sets ($N^* \leq 5$) of partner series. These weights (w_s') are not always the final weights (see step 7.2.2).

7.2.2 Modifications of primarily calculated weights

In most cases the weights calculated by step 7.2.1 are the final weights, i.e., $w_s = w_s$'. However, the primarily calculated weights are modified when i) $w_{s'} < 0$ for any s, or ii) $w_{s'} > 0.4W$ for any s where W stands for the sum of the weights for all partner series.

i) if w_{s^*}'< 0 for series s^*, then $w_{s^*} = 0$.

ii) if $w_{s^*}' > 0.4W$, then w_s' values are increased by $0.01w_{s^*}$' for all the partner series of $w_s' < 0.4W$. Then W is recalculated from the larger w_s' values. If w_{s^*}' is still higher than $0.4W$, the increase of the other weights will be repeated as long as $w_s' \leq 0.4W$ does not become true for every s.

The reasoning of (i) is that a negative weight does not have climatological interpretation within an area of spatially similar climate, while that of (ii) is that too large individual weights are undesired, as any time series might include undetected errors or inhomogeneities.

7.3 Calculation of relative time series

Once the reference series (**F**) has been created, the calculation of **T** is generally straightforward by B4, and also at this step, for additive variables. However, for monthly precipitation homogenization 3 kinds of relative time series are generated with 3 versions of the same candidate series. The relation between the data of the original candidate series and those of the other versions (indexed with *a* and *b*) is shown by (49-50) for the RR data before transformation.

$$xc_{a,h} = \max(0, xc_h - 30) \quad \text{for every } h \tag{49}$$

$$xc_{b,h} = xc_h + 30 \qquad \text{for every } h \tag{50}$$

Before the calculation of relative time series, all RR values are transformed to TR, and with this $\mathbf{Gc_a}$ and $\mathbf{Gc_b}$ are obtained. Finally, the relative time series will be denoted with **Ta** (**Tb)** for $\mathbf{Gc_a}$ ($\mathbf{Gc_b}$).

Sets of relative time series for each candidate series are created according to B5. For monthly precipitation, such sets are created also for every **Ta** and **Tb** series.

8. Basic operations with metadata

The base network is used

8.1 Metadata dates in the annual and monthly time scales

Section of time series: treated period.

Metadata dates must be introduced with daily preciseness to ACMANTv5.2, even when the climate data are of monthly resolution. Notwithstanding, ACMANT interprets the metadata also in rougher time scales. The interpretations take into account that in statistical break detection on the annual scale a break in year y_0 means a break in the last day of year y_0, while that in the monthly scale in month h_0 means a break in the last day of month h_0.

i) On the monthly scale, the interpretation of metadata (j) is binary: $j = 1$ when metadata is given, and $j = 0$ in the reverse case. Let we see metadata date d_0 (day) of month m_0 of year y_0 for a given station series. When $d_0 \leq 20$, $j(m_{-1},y_0) = 1$ (or, if m_0 January, $j(m,y_{0-1}) = 1$). When $d_0 > 10$, $j(m_0,y_0) = 1$. As a consequence, metadata is considered to be given in two consecutive months when $10 < d_0 \leq 20$.

ii) On the annual scale, the interpretation of a piece of metadata is usually fragments for two consecutive years: Let the serial number of the day of the metadata date in year y_0 is d^*. This piece of metadata adds $d^*/365$ value to $j(y_0)$ and $1– d^*/365$ value to $j(y_{-1})$. In case of multiple metadata occurrences within one year for the same station, the annual scores are summed, but 3-year average scores of a given station are limited by 1.0.

8.2 Control of synchronous break indications in metadata

Section of time series: homogenized period or treated period. Time resolution: annual.

Users may introduce a list of metadata dates which ACMANT treats as break dates in its programs. The ratio of the number of time series with synchronous breaks (*$N^{\#}$*) compared to the total number of time series with values within the homogenized period (*N'*) is controlled for each year of the homogenized period for network. When this ratio equals or is higher than 0.33 (using the annual score described in step 8.1), the software sends warning message to the user. In daily precipitation homogenization homogenized periods have not been calculated yet, and the treated periods are used instead.

ACMANTv5.2 works with any ratio of synchronous metadata breaks, but note that that synchronous metadata breaks might hinder the consideration of other statistically significant breaks, since metadata breaks have higher priority than statistically detected breaks.

Note that in later ACMANT versions this step will likely be moved after step 13 when relative time series are ready for all kinds of variables.

9 Filtering of spatial outliers of individual monthly values

Performed in monthly homogenization. Not performed when user has switched off outlier filtering.

The base network is used. Section of time series: homogenized period.

The aim is to remove spatial outliers, but keep very low the risk of removing true extreme values. Therefore this routine is never performed for individual daily values. In precipitation homogenization the application of outlier filtering is more restricted than for the other climatic variables, as the frequent occurrence of dry days reduces the effective sample size influencing the signal-to-noise ratio.

Outlier filtered series (outlier filtered and deseasonalised series) will be referred to as $\mathbf{X}^{+}$ ($\mathbf{G}^{+}$). For sections out of the homogenized period, or when this step is not applied, $x^{+} = x$ and $g^{+} = g$ for all the relevant dates.

Here, a few of the operations will be performed for all of the *M* relative time series of a candidate series. Therefore, the best fitting relative time series will be denoted with index *i* ($\mathbf{T_i}$) for making it easily distinguishable from the other relative time series. The best fitting relative time series are selected according to B5.2.

9.1 Flagging likely outlier values

Performed for additive variables.

Standard deviation (σ) is calculated for 3-month seasons of $\mathbf{T_i}$. Dates, for which (51) is true, are flagged as positions of possible outliers.

$$\left|t_{i,y,m} - \overline{\mathbf{T_{i,m^*}}}\right| > 5\sigma_{m^*} \tag{51}$$

9.2 Confirmation of an outlier value

Performed for additive variables.

A 19-month symmetric time window is edited around the flagged outlier $t_{i,h}$. The mean and standard deviation within this window are calculated excluding the flagged date, and the qualification as outlier is confirmed if (52) is true, while it is withdrawn in the opposite case.

$$\left|t_{i,h} - \overline{\mathbf{T}_{\mathbf{1},[\boldsymbol{h}-\mathbf{9},\boldsymbol{h}+\mathbf{9}]}}\right| > 4\sigma_{[h-9,h+9]} \tag{52}$$

The exceedance of the starting or ending date of the homogenized period by a time window is not allowed. Therefore, when h is closer than 9 months to the first or last month of the homogenized period, the relevant half of the window will be truncated, while the other half remains 9-month wide.

9.3 Temporary adjustments for monthly outliers in relative time series

Performed for additive variables.

Confirmed outliers will be substituted with climatic averages in all of the relative time series (53).

If y and m define the year and month of an outlier, then

$$t'_{j,y,m} = t_{j,y,m} + \overline{\mathbf{T}_{\mathbf{1},\boldsymbol{m}^*}} - t_{i,y,m} \text{ for every } j \in (1,2, \dots M) \tag{53}$$

The purpose of this adjustment is to provide the set of **T'** without the detected monthly outliers for the next step (step 9: filtering of outlier periods).

9.4 Detection of outliers in precipitation series

$t_{i,y,m}$ is an outlier if any of relations (54-55) is true.

$$ta_{i,y,m} > 5\sigma_{m^*} \tag{54}$$

$$tb_{i,y,m} < -5\sigma_{m^*} \tag{55}$$

Ta and **Tb** are used according to the definitions of step 7.3. The comparison of Eqs. (54-55) with Eq. (51) shows that the possibility of detecting precipitation outliers is restricted, particularly for low precipitation totals.

10 Filtering of outlier periods

Performed for additive variables. The base network is used. Section of time series: homogenized period. Time resolution: monthly.

The detection of outlier periods is a step-by-step procedure, only the most significant outlier period is detected in a specific step.

10.1 Normalization of relative time series

Normalized relative time series (**Tᵢ''**) are used to reduce the possible effects of seasonally varying variance. Averages and standard deviations of 3-month seasons (denotation: *m**) are calculated, than the normalization is performed for each monthly value (*h*) of each relative time series (*j*) according to (56).

$$t''_{j,h} = \frac{t'_{j,h} - \overline{\mathbf{T}'_{\mathbf{m}^*}}}{\sigma_{m^*}} \tag{56}$$

In (56) $\mathbf{T_i}'$ is used in the meaning introduced in step 8.3.

10.2 Calculation of anomalies for sections of **T''**

Performed for all relative time series. Index *j* is omitted.
Let h_1, h_2, h_3 and h_4 be four time-ordered points of series **T''**. The size of the anomaly (*q'*) of the central section $[h_2+1,h_3]$ is the difference of its mean from the mean of the adjacent sections (57).

$$q' = \overline{\mathbf{T}''_{[\mathbf{h_2+1,h_3}]}} - \frac{(h_2-h_1)\overline{\mathbf{T}''_{[\mathbf{h_1+1,h_2}]}}+(h_4-h_3)\overline{\mathbf{T}''_{[\mathbf{h_3+1,h_4}]}}}{h_2-h_1+h_4-h_3} \tag{57}$$

Let the length of the period is defined by (58).

$$L' = h_3 - h_2 \tag{58}$$

Note that $[h_2+1,h_3]$ is not always the final position of the detected outlier period, that is why the symbols of the calculated statistics hold apostrophes.

Generally, *q'* is examined for all pairs of h_2 and h_3 for which $0 < L' \leq 36$, with three exceptions, a) in monthly homogenization $1 < L' \leq 36$, b) in mode outlier filtering switched off of monthly homogenization $4 < L' \leq 36$. While *L'* varies between 1 and 36 months, the length of the two adjacent sections (L^*) is 24 months for each, at least when the distance from the endpoints of **T''** allows that (see also steps 10.3 and 10.4).

10.3 Selection of optimal **Tᵢ''** to each year of the candidate series

Usually the selection of **Tᵢ''** is performed according to B5.2. However, if the closeness of one endpoint of series **Tᵢ''** to a potential outlier period does not allow to edit 24-month sections in both sides of the period, two cases are possible:

a) If period [h_2+1,h_3] can be examined with a **T''** whose endpoints are both at least 24-month distance from [h_2+1,h_3], then a relative time series completing this condition will be used. If more **T''** complete the condition, **T$_i$''** is selected from them according to B5.2. b) When period [h_2+1,h_3] is closer than 24 months to an endpoint of the homogenized period of the candidate series, one of the adjacent sections around [h_2+1,h_3] will be truncated, see next step.

10.4 Truncation of time window around the period examined

If the distance of h_2 from the starting point of the homogenized period is shorter than 24 months (12 months), L*(h_1,h_2) equals 12 months (zero), and the changes follow the same logic when h_3 is closer to the endpoint of the homogenized period than 24 months. If $L^* = 0$ for one of the adjacent sections, then the other adjacent section will be extended to 36 months, and the data of a possible outlier period are compared only to the outer section with available data (59-60).

$$q'_A = \overline{\mathbf{T}''_{[\mathbf{h_2+1,h_3}]}} - \overline{\mathbf{T}''_{[\mathbf{h_1+1,h_2}]}} \qquad (59)$$

$$q'_B = \overline{\mathbf{T}''_{[\mathbf{h_2+1,h_3}]}} - \overline{\mathbf{T}''_{[\mathbf{h_3+1,h_4}]}} \qquad (60)$$

10.5 Significance of anomalies

i) Significance of anomalies without consideration of seasonal cycle
The significance (α) is a function of the anomaly and the length of the period (61).

$$\alpha = (L')^{0.8}(q')^2 \qquad (61)$$

Note that the theoretical solution for temporally independent variables would be similar to Eq. (61) but without exponent over L'. With the exponent < 1, the possible autocorrelation of relative time series is taken into consideration.

ii) Significance of anomaly with consideration of seasonal cycle
The significance increases with duration (61), however, in case of sinusoid model, it is taken into consideration that the same season deviations from the mean of the adjacent years might indicate seasonal variations instead of altered values of the means. Therefore, in the use of sinusoid inhomogeneity model $L^\#$ ($L^\# \leq L'$) is applied in Eq. (61) instead of L', and $L^\#$ is calculated according to (62):

$$L^\# = \max\{1, L' - \frac{0.75}{3.5}\sum_{h=h_2+1}^{h_3} \mu_m\} \qquad (62)$$

Values of μ are given in the basic definitions (Sec. II) under "summer – winter difference".

10.6 Flagging potential outlier periods

The detection of outlier periods is iterative, and only one outlier period is detected in one iterative step. Potential outlier periods are flagged in each iteration.

Periods with $\alpha \geq 25$ of formula (61) are pre-selected. When $L^* > 0$ both for $[h_1+1, h_2]$ and $[h_3+1, h_4]$, a potential outlier period must complete further conditions: Writing any of q_A’ of (59) or q_B’ of (60) into (61), $\alpha \geq 25$ relation is expected to remain true, in addition, the signs of q_A’ and q_B’ must be identical.

When $L^* = 0$ for any of $[h_1+1, h_2]$ and $[h_3+1, h_4]$, an outlier period can be pre-selected relying only one of the comparisons shown by (59) and (60).

10.7 Selection of the outlier period with the most significant anomaly

When all the possible periods have been examined, the one with the maximal α is selected from those which have been retained in step 10.6. If no potential outlier period has remained flagged after steps 10.5 and 10.6, the procedure of outlier period filtering terminates for a given candidate series.

10.8 Refinement of the starting end ending months of outlier period

Performed for sinusoid cycle of inhomogeneities.
Data within and around the outlier period selected at 10.7 is examined further. The optimal modified step function is fitted to the t_i’’ values of $[h_1+1, h_4]$.

When the model of inhomogeneities is sinusoid, the unbiased values can be approximated by a modified step function including sections with no change in the annual mean, but of sinusoid seasonal changes for monthly values. For climatological reasons, the modes of the oscillation are at the solstices. The transformation of (20) of B6 to (63-64) provides the solution for modified step function fitting to data of monthly resolution.

$$\text{Optimal solution} \equiv \min_{h_1, h_2 \ldots h_K, a, b} \sum_{k=1}^{K+1} \sum_{h=h_{k-1}+1}^{h_k} I_h'^2 \qquad (63)$$

$$I_h' = t_h - a_k - b_k \sin\left(\frac{2\pi(m-3.2)}{12}\right) \quad k = \{1, 2, \ldots K+1\},\ h \in \text{section } k \qquad (64)$$

Using this operation the length of the outlier period is allowed to extend, while it is not allowed to shorten. More precisely, h_2 and h_3 are allowed to change to h_2^* and h_3^* according to (65-66).

$$h_2 - 14 < h_2^* \leq h_2 \qquad (65)$$

$$h_3 \leq h_3^* < h_3 + 14 \qquad (66)$$

As the timings of the two endpoints of the outlier period are searched, usually the number of breakpoints $K = 2$, but if $L^* = 0$ either for $[h_1+1,h_2]$ or for $[h_3+1,h_4]$, then $K = 1$. When $h_3^* - h_2^* < 10$, the new estimations are discarded and $[h_2^*+1,h_3^*] \equiv [h_2+1,h_3]$.

If $[h_2^*+1,h_3^*]$ differs from $[h_2+1,h_3]$, then the mean bias and length are recalculated (Eqs. 57-58) with the new parameters, and they are denoted with q and L, respectively. If $[h_2^*+1,h_3^*] \equiv [h_2+1,h_3]$, then $q = q'$ and $L = L'$.

10.9 Temporal adjustments

In step 9.8. h_2^*, h_3^*, L and q were determined for sinusoid cycle of inhomogeneities. If the model seasonality is not sinusoid, then $[h_2^*+1,h_3^*] \equiv [h_2+1,h_3]$, $q = q'$ and $L = L'$.

q is subtracted from all the values of $\mathbf{T}''_{[\mathbf{h_2+1},\mathbf{h_3}]}$ in all relative time series belonging to the given candidate series. This is necessary for the search of further outlier periods, but these adjusted values are not transmitted to other subroutines of the homogenization procedure. Note that detected outlier periods of $L > 28$ are not treated in the further steps of the ACMANT procedure. The only purpose of the search and temporary correction of inhomogeneities of 29-36 months duration in the procedure of outlier filtering is to provide a more accurate detection of the shorter outlier periods.

10.10 Exclusion of flip-flop

Rarely it happens that after the temporal adjustment of a section, its adjacent section (or a part thereof) seems to be outlier, and after the adjustment of the latter, the former becomes outlier again. This could lead to an infinite cycle, and to avoid it, L' of further outlier periods (L^+) is maximised by $0.8L'$ for any outlier period could be detected later around h_2 (67).

$$L^+(h_2^+) < 0.8L'(h_2) \text{ for any } h_2^+ \in [h_2 - 0.8L, h_2 + 0.8L] \qquad (67)$$

11 Calculation of spatial correlations

This step is performed when any of steps 9 and 10 is performed.

The base network is used. Section of time series: treated period.

Both kinds of spatial correlations (r and r^*) are calculated for each pair of monthly $\mathbf{G}^+$ series. The calculations are performed according to B3. The only difference in comparison with step 4.6 is that now the outliers and outlier period values are excluded from the calculations.

12 Infilling data gaps

This step is performed when any of steps 9 and 10 is performed. The base network is used. Section of time series: treated period. Time resolution: monthly.

The gap filling is repeated for missing data with renewed spatial correlations, and the exclusion of the detected outliers in the partner series. With this step the gap filling for the raw **G** series is improved. In addition, the detected outliers or values belonging to a detected outlier period in the candidate series are treated in the same way as missing data, with which the $\mathbf{G}^{+}$ series are created.

12.1 Gap filling

The gap filling is performed similarly to step 6.1, except for some modifications:

i) Time windows for gap filling still may include excluded years, but here and from this step onwards, possible pairs of observed values in excluded years are left out of consideration, because such data are not controlled by outlier filtering;

ii) Using the narrowest window (p_5 = 7), observed values collected to the use of Eqs. (32), (37), (38) must lie within the homogenized period of the time series to which they belong to.

iii) The parameterization for p_6 and p_7 differs from that of step 6.1, see Table S2.

Table S2. Time windows and weights in the use of partner series in the spatial interpolations of step 12.1. p_5 – width of time window around the central year; p_6 – minimum number of partner series to accept a given time window; p_7 – number of partner series at which the search of further partner series is finished; r – spatial correlation; w_s – weight of the observed values in series s.

p_5 (years)	Required H'	p_6	p_7	w_s
7	60	7	10	r^2
13	30	5	10	$0.9r^2$
25	30	2	10	$0.8r^2$
Unlimited	30	-	10	$0.5r^2$

12.2 Basic data treatments

Annual and seasonal values are calculated from monthly values, and the RR → TR transformation is performed in precipitation homogenization.

13 Creation of relative time series for break detection

The base network is used. Section of time series: treated period. Time resolution: annual. Type of candidate series: $\mathbf{Gc}^{+}$. Type of partner series: $\mathbf{G}^{+}$.

13.1 Determination of sets of partner series

The rules of B5 are applied, and type 1 reference series are created.

13.2 Weighting of partner series

The effective number of time series in network ($\overline{N'}$) is calculated. When $\overline{N'} \leq 15$, the partner series are equally weighted ($w \equiv 1$), while the weights are the squared spatial correlations ($w_s = r_s^{*2}$) in the reverse case.

13.3 Creation of relative time series

B4 is applied.

14 Break detection with combined time series comparison

The core network is used in the part of pairwise comparisons, while the base network is used in the part of the composite reference use. Time resolution: annual.

14.1 Calculation of homogenization periods in the core network

Performed in multi-network homogenization.

In the pairwise comparisons of the combined time series comparisons only the core network is used. Therefore, although the homogenized periods have been calculated in step 12, those homogenization periods are valid in the base network.

The calculation of the homogenization periods is performed according to B5.1.

14.2 Break detection with pairwise comparisons

The core network is used. Period of time series: homogenized period.

B8.1-B8.3 are applied. The break detection is performed according to B6 (B7) in univariate (bivariate) homogenization and with the modification of B8.2. In univariate break detection $p_2 = 2.8$. When the model seasonality of inhomogeneity biases is sinusoid, bivariate break detection is applied with $p_2 = 2.0$ and $p_3 = 0.2$. In the bivariate version of precipitation homogenization parameter p_3 depends on the length of the snowy season, ($L^{(S)}$, in months) according to (68).

$$p_3 = \left(\frac{L^{(S)}}{12-L^{(S)}}\right)^2 \tag{68}$$

Within a given homogenization procedure p_3 is constant.

The number of time series with synchronous breaks ($N^{\#}$) in a given year is not allowed to reach the half of the total number of time series with values within their homogenized period (N') for that year. If the break detection with pairwise comparisons would result in a synchronous break in which a higher ratio of time series are involved, ACMANT cancels the least significant breaks of the synchronous occurrence. Here the S3 statistics of B8.3 are used to determine the order of break significances.

14.3 Break detection with the use of composite reference series

The base network is used.

The break detection is performed according to B6 (B7) in univariate (bivariate) homogenization, but some details of the calculations need the clarification presented here.

14.3.1 Selection of relative time series

The principal rule is to select the series with the highest *ß*, as it is described in B5.4. However, to reduce edge effects, 15-year overlaps are applied. An example: Let suppose that the break detection for section [y_1,y_2] of the candidate series has been performed with $\mathbf{T_A}$, and sections before y_1 and after y_2 can be examined with other relative time series. In the continuation, $\mathbf{T_B}$ is involved for the break detection in section before y_1+15 or in section after y_2–15 or in both. However, if a break has been detected with $\mathbf{T_A}$ before y_1+15 or after y_2–15, the length of overlapping is reduced until the date of the detected break.

14.3.2 Parameterization and some special conditions

Following B8.4, the breaks which have been detected by pairwise comparisons are included the results of this step. Regarding the newly detected breaks, the minimal distance between two consecutive breaks is 3 years. Further, the detection of new breaks is not allowed close to any detected break of the pairwise comparisons. More precisely, if a break has been detected at $y0$ by the pairwise comparisons, no break can be detected by the use of composite reference series over the period [$y0$-4,$y0$+4]. p_2 = 3.92 (2.8) in univariate (bivariate) detection, while p_3 is the same as in step 14.2.

The changes at the breaks of the fitted step functions provide preliminary estimations for the break sizes. For break k of series s the break size estimate δ is calculated by (69).

$$\delta_{s,k} = \overline{\mathbf{T_{s,k+1}}} - \overline{\mathbf{T_{s,k}}} \tag{69}$$

(A more correct denotation would be $\hat{\delta}$ for the estimated variable, but we omit the cap.) These estimates are kept for later operations.

14.4 Control with t-test

When bivariate homogenization is applied, this step is performed separately for the two variables.

Common t-test is applied for checking the significance of breaks detected in step 14.3. The break sizes (δ) are estimated by (69), while the standard deviation (σ) of the t-test is the σ for the entire homogenized period of $\mathbf{T_j}$ in order to reduce estimation errors which could come from statistics of short sections. Under these condition the t-test statistic (S_t) for break k of series s can be calculated by (70-71).

$$S_t(k) = \frac{|\delta_k|\sqrt{(L-2)L_k L_{k+1}}}{L\sigma} \tag{70}$$

$$L = L_k + L_{k+1} \tag{71}$$

In (70) and (71) L_k denotes the length of the k-th section of the step function model, and the station index is omitted from the formulas. The significance threshold for S_t is given by parameter p_8 whose value depends on climatic variable and the phase of the homogenization procedure. At this step p_8 = 2.296, except for the breaks of summer – winter differences, there p_8 = 2.8. When $S_t < p_8$ for a given break, then it is considered to be insignificant, and is removed from the break list.

14.5. Limitation of the number of synchronous breaks

When bivariate homogenization is applied, this step is performed separately for the two variables. The base network is used.

The ratio of synchronous occurrences of breaks, i.e. the ratio between $N^{\#}$ and N' is controlled. $N^{\#} < 0.5N'$ must be true for each year, otherwise ACMANT step-by-step withdraws the least significant break of the synchronously detected breaks, as long as the expected relation between $N^{\#}$ and N' does not become true. On the other hand, at this stage ACMANT sends warning message about every occurrence of $N^{\#} \geq 0.5N' - 1$.

When $N^{\#}$ is higher than expected, the significance of the breaks is ordered, and one or more breaks are omitted starting from the least significant break. Here the break significance is related to its likely impact on the homogenization results, which is different from the concept of t-test where the risk of including false breaks is tested. At this step the significance (α) for break k of series s is calculated by (72).

$$\alpha_{s,k} = \frac{(L_k)(L_{k-1})\delta_{s,k}^2}{L} \tag{72}$$

15. Partial results

The base network is used.

ACMANT may send partial results depending on user's request. In interactive mode, partial results are always output. Partial results include
i) the list of the detected breaks in step 14;
ii) a summary table of pairwise comparison results;
iii) the list of the detected outlier values and outlier periods (when any of step 9 or step 10 has been performed);
iv) the table of the outlier filtered and normalized annual values of the time series in network. In performing the calculations, daily and monthly values are subjected to the filtering of physical outliers, they are normalized by B2, the monthly values are subjected to the outlier filtering of step 9 and step 10 (when these operations are applied), and the annual values are created from the daily or monthly values. No homogenization is performed for these values.

In bivariate homogenization two break detection lists are produced, one for each of the two variables. Similarly, two separate data tables are provided for the annual values of time series.

Users may edit the break list(s) output at this step.

16 Adjustments for inhomogeneities

The base network is used. Section of time series: treated period. Time resolution: monthly.

16.1 Application of the simple ANOVA correction model

When bivariate homogenization is applied, this step is performed separately for the two variables. Section of time series: homogenized period. Type of input series: $\mathbf{G}^{+}$. Time resolution: annual.

The simple ANOVA model (B9.1) is applied for the variable(s) examined in step 14. The result will be a vector of annual adjustment terms (**Z***) (or vectors **Z*** and **Z**** in bivariate cases) for each time series of the network.

Note that ANOVA models can be applied to variables like summer – winter difference in the same way as for annual, monthly or daily means.

16.2 Calculation of adjustment terms backwards from the beginning of the homogenized period

Section of time series: From the starting of the treated period until the starting of the homogenized period. Time resolution: annual.

This step is performed when the homogenized period starts later than the treated period, and the ANOVA correction model results in non-zero adjustment term for the first year of the homogenized period (x_{y0}+1). The idea behind the backwards adjustments is that a bias for inhomogeneities in the homogenised period may indicate a non-zero bias of the earlier sections of the time series, although with less certainty and accuracy.

Term "long-term adjustment term" (z_L) is introduced here as the minimum of the adjustment term for x_{y0}+1 (z_A) on the one hand, and of the average adjustment term for the first 30 years of the homogenized period (z_B) on the other hand, as it is shown by (73).

$$z_L = \begin{cases} \min(|z_A|, |z_B|) & \text{if } \operatorname{sign}(z_A) = \operatorname{sign}(z_B) \\ 0 & \text{if } \operatorname{sign}(z_A) \neq \operatorname{sign}(z_B) \end{cases} \tag{73}$$

The adjustment term (z^*) for the section before x_{y0} is identical with z_L, with the exception that when $z_L \neq z_A$, z^* gradually changes from z_A to z_L over a 3-year period, going backwards from y_0 to y_0–2. Note when the homogenized period is shorter than 30 years, $z_B = 0$, hence $z_L = 0$.

16.3 Monthly adjustment terms

Section of time series: treated period.

In univariate homogenization the monthly adjustment terms (z) are identical with the annual adjustment terms, while in bivariate homogenization z depends on both of the annual variables involved, they are denoted with z' and z''. The calculation of monthly adjustment terms in the two different cases of bivariate homogenization are as follows:

i) Additive variables with sinusoid cycle, monthly adjustment terms (74)

$$z_{y,m} = z'_y + 0.55\sin\left(\frac{2\pi(m-2.7)}{12}\right)z''_y \tag{74}$$

Here z' (z'') denote the adjustment term for annual means (summer – winter differences). Note: The constant in the numerator differs from that in Eq. (64), since for timings of annual (monthly) preciseness the timing is the last day of the year (month) by definition, while for adjustments the middle of the month represents best a month.

ii) Precipitation with rainy season and snowy season (75)

$$z_{y,m} = \begin{cases} z'_y & \text{if } m \in \text{rainy season} \\ z''_y & \text{if } m \in \text{snowy season} \end{cases} \tag{75}$$

16.4 Execution of adjustments

Section of time series: treated period. Time resolution: monthly

Both of **G** and $\mathbf{G}^+$ (**X** and $\mathbf{X}^+$) are adjusted for additive variables (for precipitation), and the adjusted series will be referred to as $\mathbf{G}^*$ and $\mathbf{G}^{+*}$ ($\mathbf{X}^*$ and $\mathbf{X}^{+*}$), respectively.

i) The execution of adjustments for additive variables is simple (76-77)

$$\mathbf{G}^{+*} = \mathbf{G}^{+} + \mathbf{Z} \tag{76}$$
$$\mathbf{G}^{*} = \mathbf{G} + \mathbf{Z} \tag{77}$$

ii) In precipitation homogenization the monthly RR data are adjusted according to (78-79), then the annual values are calculated as the sums of the monthly values.

$$x_{y,m}^{+*} = x_{y,m}^{+} e^{z_{y,m}} \quad \text{for every } y,m \tag{78}$$

$$x_{y,m}^{*} = x_{y,m} e^{z_{y,m}} \quad \text{for every } y,m \tag{79}$$

16.5 Basic data treatments

Annual and seasonal values are calculated from monthly values, and the RR → TR transformation is performed in precipitation homogenization

VI SECOND HOMOGENIZATION CYCLE

The purpose of this homogenization cycle is twofold: a) improve accuracy with the use of pre-homogenized reference series, and b) find the range of instability of the homogenization results, i.e. when small changes in the relative time series construction or parameterization yield different homogenization results, and this latter purpose is the most important here. In this homogenization cycle ensemble experiments are performed with the use of lighter significance thresholds than usual to allow higher dispersion of the results. The instability range will appear as the difference between the minimum and average adjustment terms of the ensemble homogenization.

From this homogenization cycle onwards only the core network is used.

17 Creation of relative time series for outlier filtering

Performed in monthly homogenization always and in daily homogenization for additive variables. Candidate series: $\mathbf{Gc}^*$ (for precipitation $\mathbf{Gc_a}^*$ and $\mathbf{Gc_b}^*$). Partner series: $\mathbf{G}^{+*}$. Section of time series: homogenized period. Time resolution: monthly.

It is performed in the same way as step 7, but here the initial sections for time series construction are the lately used homogenized periods. This means that here, and from this step onwards, newly determined homogenized periods cannot stretch over the endpoints of the previously determined homogenized periods. Note that the extent of homogenized periods for a given time series is constant during the homogenization procedure in most practical cases. However, changes sometimes occur for changes in the calculated spatial correlations or changes in data availability for the exclusion of outlier values.

18 Filtering of spatial outliers of individual monthly values

Performed in monthly homogenization. Not performed when user has switched off outlier filtering. Section of time series: homogenized period.

Flags of outlier values and outlier periods from the previous homogenization cycle are cancelled before the execution of this step. Then the outlier filtering is performed in the same way as in step 9.

19 Filtering of outlier periods

Performed for additive variables. Section of time series: homogenized period. Time resolution: monthly.

It is performed in the same way as step 10.

20 Calculation of spatial correlations

Performed for all variables. Quality level of time series: $\mathbf{G^{+*}}$. Section of time series: homogenized period.

The r and r^* values are calculated according to B3.

21 Infilling data gaps

Performed for all variables. Quality level of time series: $\mathbf{G^{+*}}$. Section of time series: treated period. Time resolution: monthly.

21.1 Gap filling

It is performed similarly to the gap filling of step 6.1, with some modifications. The principal reason of the modifications is that after the first homogenization cycle has been completed, the difference of the reliability between observed values within the homogenized period and those out of the homogenized period is considered. Another reason is that the impact of window width is considered with reduced weights, as any error for the time distance is expected to be reduced by the accomplished pre-homogenization.

Data of the homogenized period are prioritized by the following modifications:

i) In Eqs. (32), (37) and (38) both of the candidate series values and partner series values must fall within the respective homogenized period, even when the piece of the missing data lies out of the homogenized period of the candidate series.

ii) For missing data within the homogenized period, only partner series with observed data synchronous with the missing data can be considered. It is a general rule, but here one more condition is added: the synchronous observed data must lie within the homogenized period of the partner series. However, when less than 3 partner series meet with this condition, or when the sum of the weights of the partner series (W of (34) and (39)) would less than p_4 (0.4), partner series with synchronous observed values out of their homogenized periods may also be incorporated to the interpolation.

iii) When the synchronous piece of observed data of a contributing partner series lies out of the homogenized period, its weight is reduced in comparison to the other partner series. The reduced weights are marked with w_s* in Table S3.

iv) For missing data lying out of the homogenized period of the candidate series, partner series with synchronous observed data out of their homogenized period can be considered without restriction, but the reduced weights (w_s*) are applied to them.

21.2 Basic data treatments

i) Although the interpolations are performed for $\mathbf{G^{+*}}$ series, monthly data of $\mathbf{G^{+}}$ series are also created by applying the inverse function of inhomogeneity bias removal after the interpolations; **G** and **G*** series are also renewed: In such series the gap filling for missing data is renewed, but no outlier filtering result is considered to them.
ii) Annual and seasonal values are calculated from monthly values, and the RR → TR transformation is performed in precipitation homogenization.

Table S3. Time windows and weights for partner series *s* at step 21.1. w_s*– the date of the piece of missing data lies out of the homogenized period of the partner series; number in brackets: threshold number of partner series with synchronous observed value within its homogenized period; the other symbols are explained in Table S1.

p_5 (years)	Required *H’*	p_6	p_7	w_s	w_s*
7	60	7	10	r^2	$0.5r^2$
21	30	7	10	$0.92r^2$	$0.46r^2$
41	30	5	10	$0.85r^2$	$0.425r^2$
Unlimited	30	(3)	10	$0.7r^2$	$0.35r^2$

22 Creation of relative time series

Type of candidate series: **Gc**$^+$. Type of partner series: **G**$^{+*}$. Section of time series: homogenized period. Time resolution: both annual and monthly.

22.1 Determination of sets of partner series

Time resolution: Both annual and monthly.

The rules of B5 are applied, and both of type 1 and type 2 reference series are created.

Sets of relative time series created at this step will be used in several later steps of the homogenization procedure, with the only differences that the values of the series may change for newly calculated adjustments for inhomogeneities, or for newly performed gap filling.

E22.2 Weighting of partner series

Type 1 reference series are used. Time resolution: annual.

With this step, an ensemble homogenization cycle starts, which lasts up to step 24.2. In each ensemble member, one partner series is excluded from the homogenization. For this reason, the number of partner series (*N**) can be as low as 2.

When *N** = 2, the two partner series are uniformly weighted, while for any other *N** the weighting rules are the same as in step 7.2.

E22.3 Calculation of relative time series

Type 1 reference series are used. Time resolution: annual.

B4 is applied.

E23 Break detection

Time resolution: annual.

E23.1 Selection of relative time series

The same as 14.3.1.

E23.2 Break detection with step function fitting

B6 is applied in univariate homogenization, while B7 is applied in bivariate homogenization. No metadata, either statistical break detection results of previous steps are considered. $p_2 = 1.4$ ($p_2 = 1.0$) in univariate (bivariate) detection, while p_3 is the same as in 14.2. The minimum time distance between two consecutive breaks is 3 years.

E23.3 Control with t-test

When bivariate homogenization is applied, this step is performed separately for the two variables.

The same as 14.4, except that the significance threshold (p_8) is lighter. At this step $p_8 = 0.82$, except for the breaks of summer – winter differences, there $p_8 = 1.0$.

E23.4 Limitation of the number of synchronous breaks

When bivariate homogenization is applied, this step is performed separately for the two variables.

The number of synchronous occurrences of breaks ($N^{\#}$) in a given year is compared to the number of time series (N') whose homogenized periods include that year, and an upper limit of $N^{\#}$ is set by (80).

$$N^{\#} \leq \max\{1, N' - 3\} \tag{80}$$

When relation (80) is not true for a detected synchronous break, the least significant breaks are omitted one-by-one as long as the relation is not true. For the necessary break cancellations the procedure shown in step 14.5 is applied.

24 Calculation of adjustment terms

Section of time series: treated period. Time resolution: monthly.

E24.1 Application of the simple ANOVA correction model

When bivariate homogenization is applied, this step is performed separately for the two variables. Section of time series: homogenized period. Type of input series: $\mathbf{G}^{+}$. Time resolution: annual.

The same as 16.1.

E24.2 Calculation of adjustment terms backwards from the beginning of the homogenized period

Section of time series: From the starting of the treated period until the starting of the homogenized period. Time resolution: annual.

The same as 16.2.

24.3 Key adjustment terms derived from ensemble results

Section of time series: treated period. Time resolution: annual.

Two kinds of key adjustment terms are calculated here: one is the arithmetical average of the ensemble member adjustment terms (denotation: $\mathbf{Z}^{+}$, in case of bivariate homogenization $\mathbf{Z}^{++}$) for the second variable), while the other is the minimum absolute value of the ensemble member adjustment terms (denotation: $\mathbf{Z}^{-}$ and $\mathbf{Z}^{--}$). When the sign of the ensemble member adjustment terms (z*) for a given year is varied, then z' will be zero, as it is shown by (81).

$$z_y^{-} = \begin{cases} \mathrm{sign}\left(z_{1,y}^{*}\right)\min\left(\left|z_{1,y}^{*}\right|, \left|z_{2,y}^{*}\right| \ldots \left|z_{N',y}^{*}\right|\right) \text{ if } \mathrm{sign}\left(z_{a,y}^{*}\right) = \mathrm{sign}\left(z_{b,y}^{*}\right) \\ \text{for every } a, b \in [1,2 \ldots N']; \\ 0 \text{ if } \mathrm{sign}\left(z_{a,y}^{*}\right) \neq \mathrm{sign}\left(z_{b,y}^{*}\right) \text{ for any pair of } a, b \end{cases} \tag{81}$$

24.4 Annual adjustment terms for 9 scenarios

Time resolution: annual.

In the third homogenization cycle an ensemble homogenization will be performed with 9 different pre-adjustment of partner series (while the candidate series are never adjusted). Here the creation of the 9 set of adjustment terms are shown. Note that one of the 9 scenarios is considered to be principal scenario, and in some operations only the principal scenario will be used.

Nine scenarios are created by the linear combination of $\mathbf{Z}^{+}$ and $\mathbf{Z}^{-}$ type adjustment terms. In (82) the serial number of ensemble member is denoted by upper index in brackets.

$$\mathbf{Z}^{(\mathbf{i})\prime} = p^{(i)}\mathbf{Z}^{+} + (1 - p^{(i)})\mathbf{Z}^{-}, \quad i \in (1,2, \ldots 9) \tag{82}$$

$p^{(i)}$ is of a Gaussian distribution with 0 expected value. The standard deviation is based on experiments of efficiency tests. $p^{(1)} = -3.5$, $p^{(2)} = -2.3$, $p^{(3)} = -1.44$, $p^{(4)} = -0.69$, $p^{(5)} = 0$, $p^{(6)} = 0.69$, $p^{(7)} = 1.44$, $p^{(8)} = 2.3$, $p^{(9)} = 3.5$. It can be seen that scenario 5 is identical with $\mathbf{Z}^{-}$ ($\mathbf{Z}^{-}$ and $\mathbf{Z}^{--}$ in case of bivariate homogenization). This scenario is the principal scenario.

24.5 Adjustment terms for monthly and daily data

Section of time series: treated period.

The monthly adjustment terms are calculated according to step 16.3. In univariate detection and in precipitation homogenization the adjustment terms of daily values are identical with the relevant monthly adjustment term. In daily homogenization with sinusoid inhomogeneity model, the monthly adjustment terms are considered to be valid for the middle day of the month, and the daily adjustment terms for the other days are determined with linear interpolation between adjacent mid-monthly values.

VII THIRD HOMOGENIZATION CYCLE

Based on the instability range calculated in the previous homogenization cycle, 9 scenarios of pre-homogenization results are taken, and the homogenization is done for each of these 9 scenarios. The homogenization result of this cycle will be the mean of the 9-member ensemble homogenization.

In this cycle some new steps appear (e.g. downscaling break detection results to monthly and daily time scales), since such steps can be done with fair reliability only when the homogeneity of the annual values is advanced.

25 Creation of relative time series for outlier filtering

Performed for additive variables. Candidate series: $\mathbf{Gc}^{*}$, partner series: $\mathbf{G}^{+*}$. Section of time series: homogenized period. Time resolution: monthly.

$\mathbf{G}^{*}$ and $\mathbf{G}^{+*}$ series are created by applying the adjustment terms of the principal scenario (defined in step 24.4). The adjustments are performed with the formulas (76-77) of step 16.4.

The sets of partner series are the same as those of the type 2 reference series in step 22.1, despite the values in time series have been changed since then. The weights of the partner series are calculated in the same way as in step 7.2, while the relative time series are calculated from the candidate series and reference series according to B4.

26 Filtering of outlier values and outlier periods

Performed for additive variables. Section of time series: homogenized period.

Flags of outlier values and outlier periods from the previous homogenization cycle are cancelled before the execution of this step.

26.1 Filtering of spatial outliers of individual monthly values

Performed in the monthly homogenization of additive variables. Not performed when user has switched off outlier filtering.

The same as step 9.

26.2 Filtering of outlier periods

The same as step 10. Hereafter outlier periods of 5 to 28 months duration will be treated as short-term inhomogeneities.

E27 Application of the ensemble adjustments

Section of time series: treated period. Time resolution: monthly in the monthly homogenization of additive variables and in precipitation homogenization always; daily in the daily homogenization of additive variables.

The ensemble adjustment terms described in step 24.4 are applied. The adjustments are performed by (76-77) for additive variables and by (78-79) for precipitation data. In adjusting daily data the rules of step 24.5 are applied.

E28 Infilling data gaps

Section of time series: whole series. Time resolution: monthly in the monthly homogenization of additive variables and in precipitation homogenization always; daily in the daily homogenization of additive variables.

E28.1 Gap filling

Section of time series: homogenized period.

This step is performed similarly to the gap filling in step 21, but with the following modifications:

i) The gap filling is performed only for the homogenized period of time series.

ii) In the daily homogenization of additive variables the gap filling is performed for daily data. There, Eqs. (35-36) of B10.1 are used.

iii) The parameters for window edition and weighting are changed (Table S4).

Table S4. Time windows and weights for potential partner series *s* at step 28.1. w_s*– the date of the piece of missing data lies out of the homogenized period of the partner series; number in brackets: threshold number of partner series with synchronous observed value in its homogenized period; the other symbols are explained in Table S1.

p_5 (years)	Required *H’*	p_6	p_7	w_s	w_s*
25	100	7	10	r^2	$0.5r^2$
51	30	5	10	$0.9r^2$	$0.45r^2$
Unlimited	30	(3)	10	$0.7r^2$	$0.35r^2$

E28.2 Basic data treatments

Section of time series: whole series.

i) Although the interpolations are performed for $\mathbf{G}^{+*}$ series, $\mathbf{G}^{+}$ series are also created by applying the inverse function of inhomogeneity bias removal after the interpolations.

In addition, in the homogenization of additive variables, $\mathbf{G}^{\#}$ series are also created, in such series only the outlier values and short outlier periods (L < 5 months) are considered to be quality issues. $\mathbf{G}^{\#}$ series are created for the whole target period, although for sections out of the treated period raw observed values and possible missing data codes of **G** are simply copied to $\mathbf{G}^{\#}$. Given that in precipitation homogenization filtering of outlier periods is not included, for TR data $\mathbf{G}^{\#} \equiv \mathbf{G}^{+}$. All the series of $\mathbf{G}^{+*}$, $\mathbf{G}^{+}$ and $\mathbf{G}^{\#}$ are created or renewed in daily resolution when the daily homogenization of an additive variable is performed, while they are created only in monthly resolution in other cases.

ii) Monthly data are recalculated from daily data when the interpolation has been performed in daily resolution; annual and seasonal values are calculated from monthly values; and the RR → TR transformation is performed in precipitation homogenization.

E29 Creation of relative time series

Type of candidate series: $\mathbf{Gc}^{+}$ and **G**. Type of partner series: $\mathbf{G}^{+*}$. Time resolution: annual and monthly.

The sets of partner series are the same as those of the reference series in step 22.1, despite the values in time series have been changed since then. Annual series of type 1 reference series and monthly series of type 2 reference series are created. The other details of the relative time series construction are the same as in step 7.

In most cases of the following steps, relative time series for outlier filtered candidate series ($\mathbf{Gc}^{+}$) will be used, and they are denoted with **T**. Notwithstanding, sometimes raw candidate series (**Gc**) will be used, and their relative time series are denoted with **T***.

E30 Break detection on the annual scale

E30.1 Selection of relative time series

The same as 14.3.1.

E30.2 Break detection with step function fitting

B6 is applied in univariate homogenization, while B7 is applied in bivariate homogenization. No metadata, either statistical break detection results of previous steps are considered. $p_2 = 2.8$ ($p_2 = 2.0$) in univariate (bivariate) detection, while p_3 is the same as in 14.2. The minimum time distance between two consecutive breaks is 3 years.

E31 Audit of outliers connected to breaks

Performed for additive variables and in the monthly homogenization of precipitation. Time resolution: monthly.

When a detected breakpoint is connected to adjacent detected monthly outlier values, it is controlled if these months are a part of the detected long-term inhomogeneity, or they are true outliers. This control is performed for each detected breakpoint.

E31.1 Selection of relative time series

Let the year of the detected break is y_0. As the break detection of step 30.1 was in annual scale, the month of the first estimated break position is the December of y_0, by definition.

For auditing possible connected outliers, a relative time series including the 4-year period of $[y_0–1,y_0+2]$ is used. B5.4 is applied to select the best relative time series covering the defined period. Type 2 relative time series will be used, in monthly resolution.

E31.2 Flagging outliers

Months of detected outliers and outlier periods are flagged if they have non-interrupted temporal connection with a detected break. Months of outlier periods of $L < 5$ are considered, while longer outlier periods are excluded. A connection with the break is interrupted when at least 1 non-outlier monthly value with status "observed" separates the break and the outlier value. Note that months of status "interpolated" do not produce interruption. The maximal time difference between the date of the detected break and that of a flagged month is 11 months.

E31.3 Audit of flagged values

Flagged months after the break of year y_0 are presented as $m^{\#}$ of year y_0+1 and their total number is $H^{\#}$. **T** and **T*** are used according to their definition in step 29. The outliers are retained if (83) is true, while their outlier status is cancelled in the opposite case.

$$\left|\overline{\mathbf{T}_{[\mathbf{y_0-1},\mathbf{y_0}]}} - \frac{1}{H^{\#}}\sum_{m^{\#}} t_{y_0+1,m^{\#}}\right| < \left|\overline{\mathbf{T}_{[\mathbf{y_0-1},\mathbf{y_0}]}} - \frac{1}{H^{\#}}\sum_{m^{\#}} t^{*}_{y_0+1,m^{\#}}\right| \tag{83}$$

Similarly, if the flagged months are before the break, the outliers are retained if (84) is true, while their outlier status is cancelled in the opposite case.

$$\left|\overline{\mathbf{T}_{[\mathbf{y_0+1},\mathbf{y_0+2}]}} - \frac{1}{H^{\#}}\sum_{m^{\#}} t_{y_0,m^{\#}}\right| < \left|\overline{\mathbf{T}_{[\mathbf{y_0+1},\mathbf{y_0+2}]}} - \frac{1}{H^{\#}}\sum_{m^{\#}} t^{*}_{y_0,m^{\#}}\right| \quad (84)$$

When the status of outlier is cancelled for a month, the original observed value is replaced in the candidate series, and the related corrections are made also in its relative time series.

E32 Monthly precision

For break in year y_0, the period $[y_0–1, y_0+2]$ of **T** is examined to find the timing with monthly precision. The break position is expected to be in a narrower, 29-month wide window, i.e. between October of $y_0–1$ and February of y_0+2. When metadata indicates the break date within the 29-month window, likely the date shown by the metadata will be accepted, but the decision depends on the results of some statistical examinations.

E32.1 Selection of relative time series

The same as 31.1.

E32.2 Search of break position with fitting 1-step functions

i) In all kinds of homogenization procedures except for the use of sinusoid model for the annual cycle of inhomogeneity biases: Optimal step function (see B6) of $K = 1$ is fitted to the 48-month period of **T**.
ii) Sinusoid seasonal cycle of inhomogeneity biases: Modified step function including sinusoid changes within sections (see step 10.8) of $K = 1$ is fitted to the 48-month period of **T**.

E32.3 Consideration of metadata

When metadata indicates the date of a break, the metadata date is accepted, except when the function fitting according to 32.2 shows significantly poorer result for the metadata date (h_{meta}) than for the statistically found optimal date (h_{opt}). Sum of squared errors (SSE) of function fitting results are examined for this purpose (85). The model function is denoted with **B**.

$$\mathrm{SSE}(h_0) = \sum_{h=1}^{48}\left(t_h - b(h_0)_h\right)^2 \quad (85)$$

A metadata date is accepted when relation (86) is true.

$$\mathrm{SSE}(h_{\text{meta}}) \le \mathrm{SSE}\left(h_{\text{opt}}\right) + p_9\sigma \quad (86)$$

In (86) σ stands for the standard deviation of the monthly values in **T**. In case of step function fitting p_9 = 2.0, while for modified step functions p_9 = 1.5.

E33 Preparation of unified break list

Time resolution: monthly

Breaks detected in time series of annual resolution in step 30 represent only one group of breaks for consideration. Other groups are the short-term inhomogeneities detected in 26.2, as well as metadata which are unpaired with statistical break detection results. On the other hand, the ratio of synchronously occurring breaks must be limited. The operations presented in this section are related to the preparation of the list of the breaks, outlier values and outlier periods, all of them in monthly resolution.

E.33.1 Separation of break positions for bi-seasonal detection results

Performed for bi-seasonal precipitation homogenization.

In spite of the purpose is to produce a unified break list, this separation is unavoidable. Let suppose that a common break for the two variables is detected at h_0. However, a break of rain precipitation does not have sense within the snowy season, and vice versa. Therefore, when h_0 is out of the season for one of the variables, the last month belonging to that season before h_0 will be the break position for that variable.

E33.2 Unifying break lists

Three break lists are unified here: one of them includes the breaks detected in step 30, another one includes the breaks detected in step 26.2, and the third list include those metadata dates which have remained unpaired with any statistically detected break. Of course, it sometimes occur that some of the lists or all lists are empty.

All breaks are put to one common list and they are according to their dates. Sometimes a break of the same date occurs in more than one source break list. Breaks of two or three break lists are considered to be identical when their dates are closer than 5 months to each-other. The one only remaining break date is determined by the following rank order: (1) estimate of monthly precision; (2) metadata; (3) estimate from the step of filtering of outlier periods.

E33.3 Limitation of the number of synchronous breaks

The operations performed here are similar to those of 14.5 and 23.4. The number of synchronous occurrences of breaks ($N^{\#}$) in every 5-month wide time window is compared to the number of time series (N') whose homogenized periods include the examined period, and an upper limit of $N^{\#}$ is set by (87).

$$N^{\#} \leq \max\{2, N' - 3\} \tag{87}$$

When relation (87) is not true for a detected synchronous break, the least significant breaks are omitted one-by-one as long as the relation is not true. In the ordering of break significances (α), the following rules are applied:

i) Any piece of metadata is more significant than a statistically detected break;

ii) Any break detected with step function fitting is more significant that breaks originated from outlier filtering;

iii) Between breaks detected in univariate homogenization with step function fitting, the significance is determined by (72) of step 14.5. This formula is applied with small modifications in other cases when breaks detected in the same step of the homogenization procedure are compared.

iv) When the significance order is examined between breaks of short-term inhomogeneities detected in the filtering of outlier periods, break size estimate δ of Eq. (72) is substituted by $q*$, as it is shown in (88).

$$\alpha_{s,k} = \frac{(L_k)(L_{k-1})q_{s,k}^{*2}}{L} \tag{88}$$

For breaks at the start of an outlier period $q* \equiv q_A$ of (59), while for those at the end of an outlier period $q* \equiv q_B$ of (60). The other denotations are the same as in (72).

v) When the significance order is examined between breaks detected in bivariate homogenization with step function fitting, squared break sizes are character by $\delta_A^2 + \delta_B^2$ where indexes A and B stand for the two variables involved. Note that in spite of the break dates are different for rainy season and snowy season in the bivariate homogenization of precipitation, the two variables have common significance scores.

33.4 Sending warning message

Performed only in the principal scenario

ACMANTv5.2 sends a warning message to user when the number of time series involved in a synchronous break occurrence ($N^{\#}$) exceeds a limit depending on N' using 5-month time windows (89).

$$N^{\#} > \max\{1, N' - 4\} \tag{89}$$

E34 Daily precision

Performed in daily homogenization.

Daily precision of detected breaks can be done by statistical examinations or using metadata. However, ACMANT often leaves the default solution, i.e., the last calendar day of the month of the break. One reason of this is that only metadata dates being close to the date of the default solution can be used; another reason is that the signal-to-noise ratio is often unfavourable for finding break dates with daily preciseness, but it might still be a question why ACMANT does not always give the most likely day of the break, while it gives the most likely month of the break under any signal-to-noise ratio conditions. There is one more reason for often leaving the default day for break dates: managers of observing stations or observing networks more frequently introduce technical changes in the first day of a month than in other days.

E34.1 Daily precision using metadata

When the distance of a metadata date is not longer than 25 days from the default day, the metadata date will be break date. When more metadata dates occur in a symmetric 51-day window around the default day (note that using high frequency of metadata dates is generally not recommended), the one with the shortest distance from the default day is selected.

E34.2 Construction of relative time series of daily data

Performed for additive variables.

A 12-month wide symmetric window is edited around the default position of the break. The section between its 5^{th} and 8^{th} months (both included) is called central section. Type of candidate series: $\mathbf{Gc}^{+}$, type of partner series: $\mathbf{G}^{+*}$.

A composite reference series is built following (12) of B4. Every partner series must cover the used time window, and they must have at least 0.4 spatial correlation (r*) with the candidate series. Appropriate neighbour series are ordered according to r*, and maximum 10 of them are included in the construction of the reference series (**F**). The partner series are weighted by r^{*2}, then the relative time series (**T**) is generated according to (11) of B4.

E34.3 Signal-to-noise ratio

Performed for additive variables.

Standard deviation of daily values ($\sigma^{(d)}$) of **T** is calculated for the whole **T** series ($\sigma_A^{(d)}$) and also for its central section ($\sigma_B^{(d)}$). The signal-to-noise ratio is considered to be sufficient when relation (90) is true.

$$|\delta| \geq 0.75(\max(\sigma_A^{(d)}, \sigma_B^{(d)}))^2 \qquad (90)$$

In (90), δ stands for any kind of break size estimate used in step 33.3.

E34.4 Calculation of break position with daily preciseness

Performed for additive variables when relation (90) is true.

Optimal step function of $K = 1$ is fitted (see B6) to the data of the central section of **T**. The break position is expected to be within the 45-day wide symmetric time window edited around the default day.

E35 Adjustments for inhomogeneities

From this step, the multi-network homogenization sometimes differ from the 1-network homogenization. Time resolution is monthly in the monthly homogenization of additive variables and in precipitation homogenization always, daily in the daily homogenization of additive variables.

E35.1 Application of the simple ANOVA correction model

Performed in multi-network homogenization. Type of input series: $\mathbf{G}^{\#}$.

The simple ANOVA model (B9.1) is applied, and its results are a vector of adjustment terms (**Z***) (or vectors **Z*** and **Z**** in bivariate cases) for each time series of the examined network.

E35.2 Calculation of adjustment terms backwards from the beginning of the homogenized period

Performed in multi-network homogenization. Section of time series: from the first year of the target period until the last year before the homogenized period.

This step is performed similarly to step 16.2, with two modifications: a) Here the time resolution is monthly or daily; b) the calculation of adjustment terms goes back until the first year of the time series.

Long-term adjustment term (z_L) is defined by (73) of step 16.2, and when the absolute value of the adjustment term at the beginning of the homogenization period (z_A) is higher than that of z_L, z^* gradually changes from z_A to z_L over a 3-year period. Differing from step 16.2, here the transition from z_A to z_L is of monthly resolution, going backwards from January of the first year of the homogenized period. In daily homogenization, the daily adjustment terms of a given month are uniform for this period.

E35.3 Application of the weighted ANOVA model

Type of input series: $\mathbf{G}^{\#}$.

i) 1-network homogenization
The model described in B9.2 is applied *N* times, each time with a different candidate series. The results of the candidate series are retained, and finally **Z*** (**Z*** and **Z**** in bivariate cases) is produced for each time series.

ii) Multi-network homogenization
The model is applied only once in the homogenization of a given network. The candidate series is the central series of the network. Results generated in step 35.1 for **Z*** (**Z*** and **Z**** in bivariate cases) of the central series are overwritten with the results of this section.

E35.4 Calculation of adjustment terms backwards from the beginning of the homogenized period

Performed for every time series in 1-network homogenization, while only for the central series in multi-network homogenization.

It is the same as 35.2.

35.5 Calculation of ensemble averages

Section of time series: whole series.

The ensemble cycle has been terminated with the previous step. The mean adjustment terms (**Z’**, or **Z’** and **Z’’** in the bivariate cases) are the arithmetic averages of the 9 ensemble results of $\mathbf{Z}^{*}$ ($\mathbf{Z}^{*}$ and $\mathbf{Z}^{**}$). For the correct adjustments, not only the adjustment terms, but also the gap filled $\mathbf{G}^{\#}$ ($\mathbf{X}^{+}$ for precipitation) series are retained from each ensemble cycle, and their ensemble averages will be used in the adjustments.

35.6 Adjustment terms for application

Section of time series: whole series including excluded years. Temporal resolution: monthly and daily.

The relations between **Z** and **Z’** (**Z** on the one hand and **Z’** and **Z’’** on the other hand in bivariate homogenization) are similar to the ones described in step 16.3.

i) Univariate homogenization
$z = z'$ for each month and day of the treated period. Adjustment terms out of the treated period will be presented in paragraphs v) – viii).

ii) Sinusoid cycle of inhomogeneities, monthly resolution
Eq. (74) can be applied, with the only difference that *z'* and *z''* hold both annual and monthly indexes.

iii) Sinusoid cycle of inhomogeneities, daily resolution

$$z_{y,m,d} = z'_{y,m,d} + 0.55\sin(\frac{2\pi\left(m-3.2+\frac{d}{D_m}\right)}{12})z''_y \qquad (91)$$

In (91), D_m stands for the number of days in month *m*.

iv) Bi-seasonal RR homogenization
Eqs (75) can be applied with the only difference that *z'* and *z''* hold both annual and monthly indexes.

v) Missing data in the climate series
Gap-filling was applied only within the treated period, hence data gaps may exist in $\mathbf{G}^{\#}$ and $\mathbf{X}^{+}$. For dates without data in series $\mathbf{G}^{\#}$ ($\mathbf{X}^{+}$ in precipitation homogenization), $z = 0$.

vi) Observed monthly values in excluded years
Let y_0 be an excluded year. The adjustment term is the same as for the first month of the first not excluded year after y_0.

$$z_{y_0,m} = z_{y_0+j,1} \qquad (92)$$

In Eq. (92) *j* denotes de lowest natural number for which y_0+j is not an excluded year.

vii) Observed daily values in excluded years
The adjustment term is the same as for the first day of the first not excluded year after y_0 (93).

$$z_{y_0,m,d} = z_{y_0+j,1,1} \qquad (93)$$

viii) Observed value of excluded year in bi-seasonal precipitation homogenization (94).

$$z_{y_0,m,d} = z_{y_0,m} = z_{y_0+j,m_1} \qquad (94)$$

In Eq. (94) m_1 is the first month of the season including *m*.

35.7 Execution of adjustments

Section of time series: whole series including excluded years. Time resolution: monthly in monthly homogenization and daily in daily homogenization.

i) Additive variables

$$\mathbf{G}^{**} = \overline{\overline{\mathbf{G}^{\#}}} + \mathbf{Z} \qquad (95)$$

In Eq. (95) double stroke denotes ensemble average. **G**** is close to the final homogenization result, but gap fillings for missing data and outliers will be performed in later steps, and in case of irregular shaped annual cycle of inhomogeneities, adjustments for seasonal variations (Sec. VIII) will modify **G****.

In daily homogenization, adjusted values of no outlier filtered series (96) will also be used in some later steps.

$$\mathbf{G}^{**-} = \mathbf{G} + \mathbf{Z} \qquad (96)$$

ii) Precipitation

$$x^{**}_{y,m,d} = \overline{\overline{x^{+}_{y,m,d}}} e^{z_{y,m,d}} \quad \text{for every } y,m,d \qquad (97)$$

Eq. (97) can be applied either in monthly homogenization (without index *d*) or daily homogenization.

VIII HOMOGENIZATION OF ANNUAL CYCLES

This section is performed only when the model of irregular annual cycle of inhomogeneity biases is applied. The homogenization of annual cycles is a shorter procedure than that of the annual means. Its reason is that the signal-to-noise ratio is less favourable for the assessment of the seasonal cycle of inhomogeneity biases than for that of the inhomogeneity biases of the annual means, hence in the homogenization of annual cycles it is generally more difficult to distinguish true inhomogeneities from natural variation.

Until now, **G**** meant data homogenized by the 3 main homogenization cycle. For simplicity, data which have not been subjected to the homogenization of the annual cycle are marked with one star less, only in this section. This means transformations of the denotations used in Sec. VII, from **G**** to **G*** and from **G**⁻** to **G*⁻**. After the homogenization of the annual cycle the data receive back the second star.

36 Break detection on the annual scale

Time resolution: annual. Note that in this section ACMANTv5.2 performs break detection only for the annual means.

36.1 Selection of relative time series

The relative time series used here are identical with the ones used in step 30.1 of the principal scenario of the ensemble homogenization performed there.

36.2 Break detection with step function fitting

Univariate detection (B6) is applied with p_2 = 3.36. The minimal time distance between two consecutive breaks is 5 years.

37 Monthly precision

The same as step 32.

38. Preparation of unified break list

The list of the dates of statistically detected breaks are unified with that of the metadata dates.

38.1 Unifying break lists

Here the concept is that the distance between two consecutive breaks in any time series should not be shorter than 5 years, except when monthly precision drags closer statistically detected breaks. For this reason, a part of the metadata dates can be excluded here.

Metadata dates of a given time series are selected by an iterative procedure for including them in the unified break list. In one step of the iteration one metadata date is selected only. For each metadata date their distance score (S_L) from the other breaks are calculated (98).

$$S_L = L_1 L_2 \tag{98}$$

In (98) L_1 (L_2) denotes the time distance between the examined date and the closest break date before (after) the examined date, in months. When a time distance is shorter than 60 months, it is set 0. The metadata dates selected in earlier iteration steps and all statistically detected breaks are considered in the calculation of L_1 and L_2, and the metadata date with maximal S_L is selected. The iteration ends when no more metadata date can be found with $S_L > 0$.

38.2 Limitation of the number of synchronous breaks

It is the same as 33.3, except that breaks have only two sources here, i.e. breaks detected with univariate step function fitting and metadata breaks.

In setting the significance order of breaks, any piece of metadata is considered to be more significant than statistically detected breaks, and Eq. (87) determines the significance order between statistically detected breaks.

39 Calculation of monthly adjustment terms

E39.1 Application of the simple ANOVA correction model

Type of input series: annual series of monthly $g+$ values. Section of time series: homogenized period.

Annual series for the monthly values of fixed calendar months are taken for all the time series in network and for all the 12 months of the year. The simple ANOVA correction model (B9.1) is applied. An ensemble is generated in the way that 1 time series is excluded for each ensemble member, hence a similar ensemble is produced to the one performed in the Sec. VI. The results of this step are N^*–1 vectors of adjustment terms (denoted by $\mathbf{\Omega^*}$) for each calendar month where N^* means the total number of partner series for a given candidate series.

E39.2 Calculation of adjustment terms backwards from the beginning of the homogenized period

Section of time series: from the first year of the target period until the starting of the homogenized period. Time resolution: annual.

The same as step 16.2, except that the calculations go back until the first year of the time series.

39.3 Monthly adjustment terms based on the ensemble calculations

Section of time series: whole series. Time resolution: monthly.

The ensemble calculations are evaluated by Eq. (81), in the same way as in step 24.3.

40. Smoothing and postcorrections

Section of time series: whole series. Time resolution: monthly

40.1 Smoothing between adjacent months

Here, monthly adjustment terms are indexed by the serial number of month (h) from the starting of the time series. A refinement of adjustment terms is provided by the smoothing with (99).

$$\omega_h^{\#} = 0.3\omega_{h-1}^{*} + 0.4\omega_h^{*} + 0.3\omega_{h+1}^{*} \text{ for all } h \in (2,3, \ldots H-1) \qquad (99)$$

40.2 Removal of annual changes resulted by the seasonal adjustments

The previous operations might have resulted in undesired changes in the annual values. The seasonal adjustment terms are modified here to remove such annual changes.

i) Within the homogenized period: The mean of the seasonal adjustment terms for homogeneous section (k) detected by steps 36-37 is subtracted from every monthly adjustment term for each section (100).

$$\omega'_{y,m} = \omega^{\#}_{y,m} - \overline{\mathbf{\Omega}^{\#}_{\mathbf{k}}} \quad \text{for all } y, m \in k,\ \ k \in (1,2, \dots K+1) \qquad (100)$$

ii) Between the first year of the time series and the starting of the homogenized period: from the seasonal adjustment terms their annual mean is subtracted for each year.

40.3 Adjustment terms in excluded years

Let y_0 be an excluded year, and j is the lowest natural number for which y_0+j is not an excluded year. Then (101) shows the determination of *ω'* for any *m* of year y_0.

$$\omega'_{y_0,m} = \omega'_{y_0+j,m} \qquad (101)$$

In monthly homogenization, the final adjustment terms (*ω*) have already been produced, i.e., **Ω** ≡ **Ω'**, while in daily homogenization **Ω'** will be refined with further operations.

41 Calculation of daily adjustment terms

Section of time series: whole series, including excluded years.

41.1 Methods of downscaling monthly adjustment terms

Two kinds of downscaling methods are applied in ACMANTv5.2.

i) Simple linear interpolation
In the simple linear interpolation monthly values, in our case ω_h', are considered to be valid in the middle of month *h*, and daily values are calculated between linear interpolation between consecutive middle month values, according to (102). *D* stands for the number of days in month *h*.

$$\omega_{h,d} = \left(0.5 - \frac{d^*}{D}\right)\omega'_{h-1} + \left(1 - \frac{|0.5D - d|}{D}\right)\omega'_h + \left(0.5 - \frac{D - d^{**}}{D}\right)\omega'_{h+1} \qquad (102)$$

$$d^* = d \ \text{ for } d < 0.5D,\ \ d^* = 0.5D \ \text{ for } d \geq 0.5D \qquad (103)$$

$$d^{**} = d \ \text{ for } d > 0.5D,\ \ d^{**} = 0.5D \ \text{ for } d \leq 0.5D \qquad (104)$$

A drawback of this simple method is that the average of daily values generally does not give back the monthly values, although in most practical cases biases for this reason are insignificant.

ii) Vincent method
When the shape of the annual cycle is a priori not known, the best way of downscaling monthly adjustment terms to daily terms is the edition of linear changes of daily adjustment terms between two consecutive middle-of-months, in a way that monthly adjustment terms remain unchanged. This method was used first by Vincent et al. (2002) in time series homogenization, and it is often referred to as Vincent method.

When the simple linear interpolation between middle month values is applied according to (102-104), the result monthly values are determined by the input monthly value of the actual month in 75% and by those of the two adjacent months in 12.5% for each. Introducing auxiliary variable **A**, the monthly adjustment terms **Ω'** of a *H* months long series can be kept unchanged during the downscaling by (105-107).

$$0.125a_{h-1} + 0.75a_h + 0.125a_{h+1} = \omega'_h \text{ for all } h \in (2,3, \ldots H-1) \quad (105)$$

$$0.875a_1 + 0.125a_2 = \omega'_1 \quad (106)$$
$$0.125a_{H-1} + 0.875a_H = \omega'_H \quad (107)$$

Once **A** has been calculated, (102-104) can be applied putting *a* into the place of *ω'*.

41.2 Method selection

i) In 1-network homogenization
The Vincent method is applied to all series.

ii) In multi-network homogenization
The Vincent method is applied to the central series. Simple linear interpolation (with input monthly values of **Ω'**) is applied to the other time series.

42 Application of adjustments

Section of time series: whole series including excluded years. Time resolution: monthly and daily.

Equations (108-109) are valid in any time resolution.

$$\mathbf{G}^{**} = \mathbf{G}^{*} + \mathbf{\Omega} \quad (108)$$
$$\mathbf{G}^{**-} = \mathbf{G}^{*-} + \mathbf{\Omega} \quad (109)$$

Note that at this phase of the procedure, $\mathbf{G}^{**-}$ is used only in daily homogenization.

IX FINAL OPERATIONS

43 Refinement of outlier periods

Performed in the daily homogenization of additive variables.

The subject of the examinations are the outlier periods of 1-4 month length according to the detection results of step 26. Outlier periods of any time series are examined one-by-one here. The goal is to provide daily preciseness for the positions of these periods, and to assess their mean bias from data adjusted in the previous steps. Note that the latter is only for providing information in the output results, since the mean bias of an outlier period is not used for adjustments.

For the examinations, an outlier period is supplied with their adjacent 4-month long periods in its both sides. These supplement sections may not stretch out of the homogenized period, and may not contain detected outliers. When a supplement section does not meet with these conditions, it is shortened as far as the conditions are completed, and in the extreme case its extent can be 0. Time *d'* is defined as the distance in days from the starting point of the lengthened period including supplement sections. The pre-estimated dates of the borders of the outlier period (d_1^* and d_2^*) are identical with the first and last days of the detected outlier period in step 26. Dates d_1^* and d_2^* are also the default solution, for cases when the assessment with daily preciseness is denied by the program for the lack of required conditions.

43.1 Construction of relative time series

Section of time series: the lengthened outlier period. Type of candidate series: $\mathbf{Gc}^{**-}$, type of reference composites: $\mathbf{G}^{**}$.

Partner series must cover the examined period, may not contain detected outliers within the examined section, and must have at least 0.4 spatial correlation (r^*) with the candidate series. The other details are the same as in step 34.2.

43.2 Positions of outlier periods in daily scale

Section of time series: the lengthened outlier period.

Optimal step function (see B6) of K = 2 is fitted to the data. The final borders of the outlier period (d_1' and d_2') may have maximum 20 days distance from the default dates (d_1^* and d_2^*). The minimum length of an outlier period is 10 days.

The effects of these results are that a) interpolated values will be provided in step 44 for all the dates within the outlier period, b) the homogenized values for the dates out of the outlier period will be $gc + z$ (or $gc + z + \omega$, in case of irregular seasonality).

43.3 Mean bias of the outlier period

The mean bias (q) is the difference between the mean of the values within the outlier period and that within the supplement sections. In (110) l_1 is the length of the outlier period $[d_1', d_2']$, and l_2 is the length of the outlier period together with its supplement sections.

$$q = l_1 \overline{\mathbf{G}^{**-}_{[\boldsymbol{d_1'}+\mathbf{1},\boldsymbol{d_2'}]}} - \frac{d_1' \overline{\mathbf{G}^{**-}_{[\mathbf{1},\boldsymbol{d_1'}]}} + (l_2 - d_2') \overline{\mathbf{G}^{**-}_{[\boldsymbol{d_2'}+\mathbf{1},\boldsymbol{l_2}]}}}{d_1' + l_2 - d_2'} \qquad (110)$$

43.4 Required conditions

i) The two supplement sections together must contain at least 3 months of status “observed”, otherwise neither the refinement of the temporal position, nor the assessment of the mean bias is done. When the refinement on daily scale is denied by the program, the default position of the outlier period will be its final position, and mean bias is not calculated.

ii) At least 3 appropriate partner series are needed to construct the relative time series at step 43.1. If this condition is not completed, neither the refinement of the temporal position, nor the assessment of the mean bias are done.

iii) The standard deviation of daily values ($\sigma^{(d)}$) within the outlier period may not be too large in comparison to the standard deviation in the supplement sections (111).

$$\sigma^{(d)}_{d_1'+1,d_2']} < 2\sigma^{(d)}_{[1,d_1']\cup[d_2'+1,l_2]} \qquad (111)$$

Symbol ∪ in (111) means unification of data. If relation (111) is not true, mean bias is not assessed, but it does not affect the refinement of the temporal position.

44 Infilling data gaps

In 1-network homogenization performed for all series. In multi-network homogenization performed only for the central series. Section of time series in the default mode: homogenized period including possible excluded years, i.e, the exclusion of excluded years ends here. Section of time series on user’s request: target period. Time resolution: monthly or daily.

Interpolated values are provided a) for missing data; b) for detected monthly outlier values (in monthly homogenization); c) for values belonging to shorter than 5-months outlier periods according to step 26.2 and confirmed by step 31. In daily homogenization the results of step 43 are taken into account regarding the position and extent of outlier periods. Spatial correlations (r) of step 20 are used here.

The methods of B10 are used, but several details differ from the earlier gap filling steps of the homogenization procedure. Let suppose that the candidate series (**Gc**) have a missing value at date d_0-m_0-y_0. A neighbour series ($\mathbf{G_s}$) is considered to be accepted as partner series when it has an observed value at d_0-m_0-y_0, which may not belong to an outlier period. Pre-selected neighbour series are ordered according to their correlations ($r_{gc,s}$) with the candidate series, and the weights of accepted partner series in the interpolation will depend on these correlations. Selected series can be accepted to be partner series when they have sufficient number of observed monthly value pairs with the candidate series for calculating (36) for additive variables or (37-38) for precipitation. So far the selection and weighting of partner series is similar to the procedures in the earlier gap filling steps. However, there are three new aspects in the present procedure: a) Pairs of observed values are considered only from a given season of the year, e.g., in the interpolation for d_0-m_0-y_0 value pairs belonging to m_0^* (i.e., the 3-month season lasting from m–1 to m+1) are considered only; b) Although observed value pairs are intended to be gathered from relatively nearby dates, time window around y_0 is not applied, and time distance does not affect the weights of the partner series; c) Partner series are accepted one-by-one, and the reception of further partner series ends when pre-selected neighbour series hold lower weights than a threshold w_{th} calculated from the weights (w) of previously accepted partner series.

Synchronous observed values for **Gc** and $\mathbf{G_s}$ are searched first in year y_0, then in gradually increasing distances from y_0, as long as the number of value pairs does not reach 60. Value pairs within the homogenization periods of both series can be considered only, and minimum 25 value pairs are needed for the acceptance of a neighbour series.

The first estimation of the weight (w_s') of a pre-selected neighbour series equals to r^2 ($r = r_{gc,s}$). If the homogenized period of $\mathbf{G_s}$ includes date d_0-m_0-y_0, then the final weight is the same, i.e. $w_s = w_s'$. If the homogenized period does not include it, but the treated period yes, then $w_s = 0.5w_s'$, while if it is out of the treated period of $\mathbf{G_s}$, then $w_s = 0.33w_s'$.

When the weights have been calculated for at least 4 partner series, the weights are ordered, and the minimum threshold of weights is set by (112).

$$w_{th} = \frac{w^{(1)}+w^{(2)}+w^{(3)}}{3} - 0.2 \qquad (112)$$

In (112), the upper index of w denotes the serial number of weight in the rank order. Neighbour series with $w_s < w_{th}$ are excluded, and in any case the number of partner series cannot be higher than 10.

The procedure of selecting partner series and determining their weights in the interpolation is performed independently for each piece of missing data.

45 Preparation and writing of output items

In 1-network homogenization output is edited to all time series, while in multi-network homogenization, the output includes only items relating to the central series. In multi-network homogenization the homogenization procedure continues with the homogenization of a following network, so that after this step the homogenization procedure goes on from step 4 with the homogenization of a new network, as long as not all the networks have been homogenized.

The default output package includes: a) homogenized time series in the same time resolution as that of the input dataset, b) list of detected breaks and outliers, c) the list of partner series and their spatial correlations with the candidate series. In the default output time series gap filling is completed for the homogenized period of time series.

Users may opt for other output items: they may require monthly series or both of daily series and monthly series when the input dataset is of daily resolution; users may require gap-filled results for the entire target period, they may require the time series with leaving the data gaps of the input data unchanged, and they may ask tables of confidence indicators whose meaning will be explained in step 45.3.

45.1 Giving back mean seasonal cycle

Performed for additive variables.

Monthly climatic normal values defined by Eq. (8) are added to the homogenized deseasonalised series (113).

$$x_{y,m,d}^{**} = g_{y,m,d}^{**} + \overline{X_{s,m}} \qquad (113)$$

Eq. (113) is performed for every value of the homogenized time series, except for missing data codes.

45.2 Calculation of monthly values

Performed in daily homogenization.

The monthly values are the arithmetical averages of the daily values in the homogenization of the additive variables, except for sunshine duration and radiation. The monthly values are the sums of the daily values in the homogenization of precipitation, sunshine duration and radiation.

In a few output items, data gaps may occur with missing data code on the places of homogenized observed daily data. In such cases, monthly precipitation will be shown with missing data code. In case of additive variables, the calculation of monthly values up to 7 missing daily data is based on the observed data, while monthly data are shown with missing data code when the number of missing data in month is higher than 7.

When monthly totals of sunshine duration or radiation are calculated from an incomplete set of daily values, the unbiased solution takes into account the effect of the number of days with observed data on the monthly total (114).

$$x_{y,m}^{**} = \frac{D_m}{D'_{y,m}} \sum_{\mathrm{Y}} x_{y,m,d}^{**} \qquad (114)$$

In Eq. (114), Y stands for the cluster of days with observed values in month *m* of year *y*, while *D'* presents the number of days with observed data in cluster Y.

45.3 Reliability indicators

Reliability indicators do not have role in the accuracy of homogenization, but they give information about the data. Lower codes generally indicate higher reliability except for code 0 whose value falls between code 3 and code 2.

Each of codes 3…7 indicates interpolated data, and lower codes indicate higher quality, i.e. interpolation with the use of more or better correlating station series. The number of partner series (N^*) and the total weight of partner series (W) determine the codes.

Code 1 – Homogenized observed data
Code 2 – Observed data out of the homogenized period, but within the treated period
Code 3 – Interpolated data, $W \geq 3$
Code 4 – Interpolated data, $2 \leq W < 3$
Code 5 – Interpolated data, $N^* > 2$ and $1 \leq W < 2$
Code 6 – Interpolated data, ($N^* = 2$ and $W \geq 0.3$) or ($N^* > 2$ and $0.3 \leq W < 1$)
Code 7 – Interpolated data, $N^* = 1$ or $W < 0.3$
Code 8 – Long-term climatic mean value, as spatial interpolation is not possible
Code 9 – Missing data untreated by interpolation
Code 0 – Observed data out of the treated period

In daily homogenization, a monthly reliability indicator code shows that at least 75% of the daily data hold the same or lower code.

45.4 Elimination of physical outliers

When values near to the threshold(s) of the range of a climatic element frequently occur, physical outliers might occur in the homogenized values. For instance, a daily sunshine duration can be 0, or a daily mean relative humidity can be 100 (%), and after the adjustments the new values might fall out of the physically possible ranges. Rarely, but this problem still might occur with monthly data.

The solution is simple: When a value falls out of its defined physical range, the value is substituted with the closest physically acceptable value. For instance, if in the homogenization of relative humidity at day d_0 $x_{d0} = 102$, it is adjusted to $x_{d0} = 100$[%].

45.5 Adjustments for keeping monthly values unchanged

Performed in daily homogenization.

As in other parts of the homogenization procedure, daily adjustments are not allowed to change the monthly values. Let *b* be the monthly bias of $x_{y,m}$ caused by the adjustments of daily outliers in step 45.4. Before the adjustments will be done at this step, $\Delta x_{y,m} = b$. A step-by-step adjustment of daily values starts here to reduce $\Delta x_{y,m}$. For $b > 0$, the individual adjustments in the homogenization of sunshine duration or radiation are shown by (115), while those for the homogenization of other variables are shown by (116).

$$x_{y,m,d}^{(i+1)} = \max(x_{\min}, x_{y,m,d}^{(i)} - 0.01b) \tag{115}$$

$$x_{y,m,d}^{(i+1)} = \max(x_{\min}, x_{y,m,d}^{(i)} - 0.01bD_m) \tag{116}$$

The upper index of *x* denotes the serial number of the iteration. The adjustments of Eqs. (115-116) are performed from the first day until the last day of month *m* of year *y* as many times as they are necessary, and they are finished at any day if $|\Delta x_{y,m}| < 0.01b$. When $b < 0$, the daily values are raised by the same gradualness as the reductions are done for $b > 0$.

References

Caussinus, H. and Lyazrhi, F. (1997) Choosing a linear model with a random number of change-points and outliers. Ann. Inst. Statist. Math. 49(4):761-775.

Caussinus, H. and Mestre, O. (2004) Detection and correction of artificial shifts in climate series. J. Roy. Stat. Soc. C 53:405-425. http://doi.org/10.1111/j.1467-9876.2004.05155.x

Menne MJ, Williams Jr CN: Homogenization of temperature series via pairwise comparisons. *J Clim* 22:1700–1717, 2009. https://doi.org/10.1175/2008JCLI2263.1

Peterson, T.C. and Easterling, D.R. (1994) Creation of homogeneous composite climatological reference series. Int. J. Climatol. 14:671–679.

Vincent, L.A., Zhang, X., Bonsal, B.R. and Hogg, W.D. (2002) Homogenization of daily temperatures over Canada. J. Clim. 15:1322–1334. http://doi.org/10.1175/1520-0442(2002)015<1322:HODTOC>2.0.CO;2