Neutron star mergers simulations

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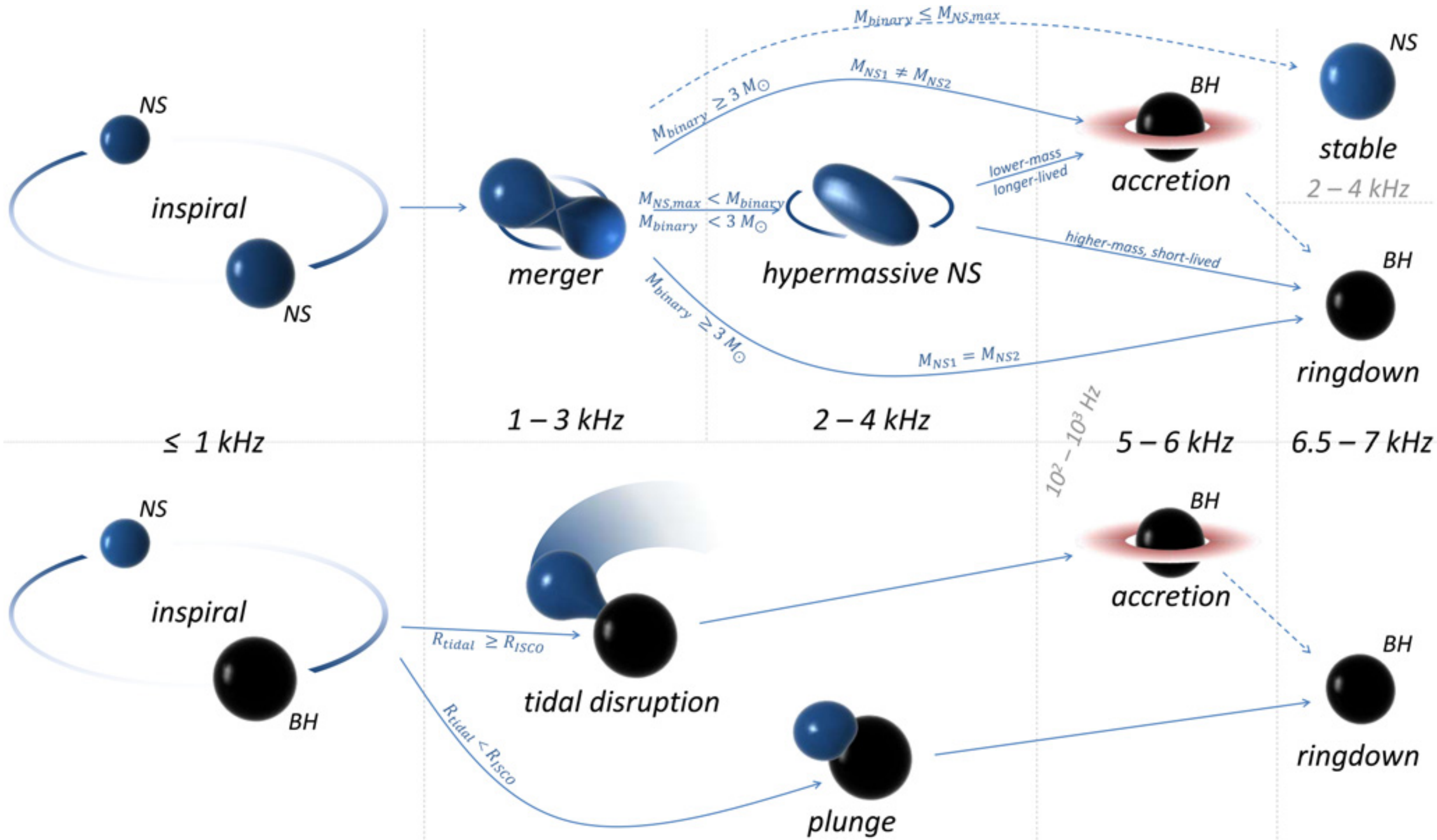
Neutron star mergers for non-experts:
GW170817 in the multi-messenger astronomy and FRIB eras

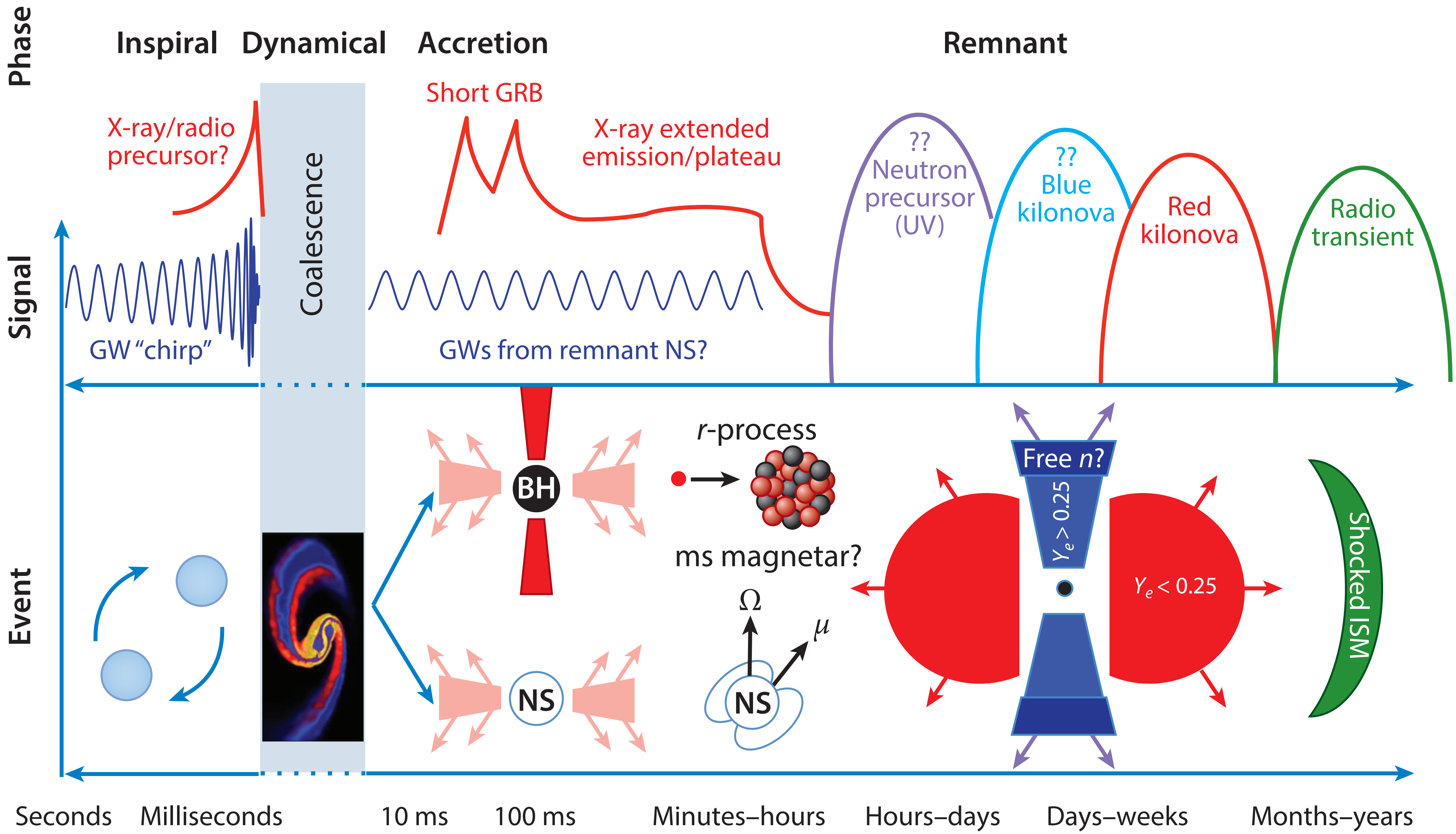
https://github.com/dradice/JINA_MSU_School_2018.git

Outcome of NS mergers



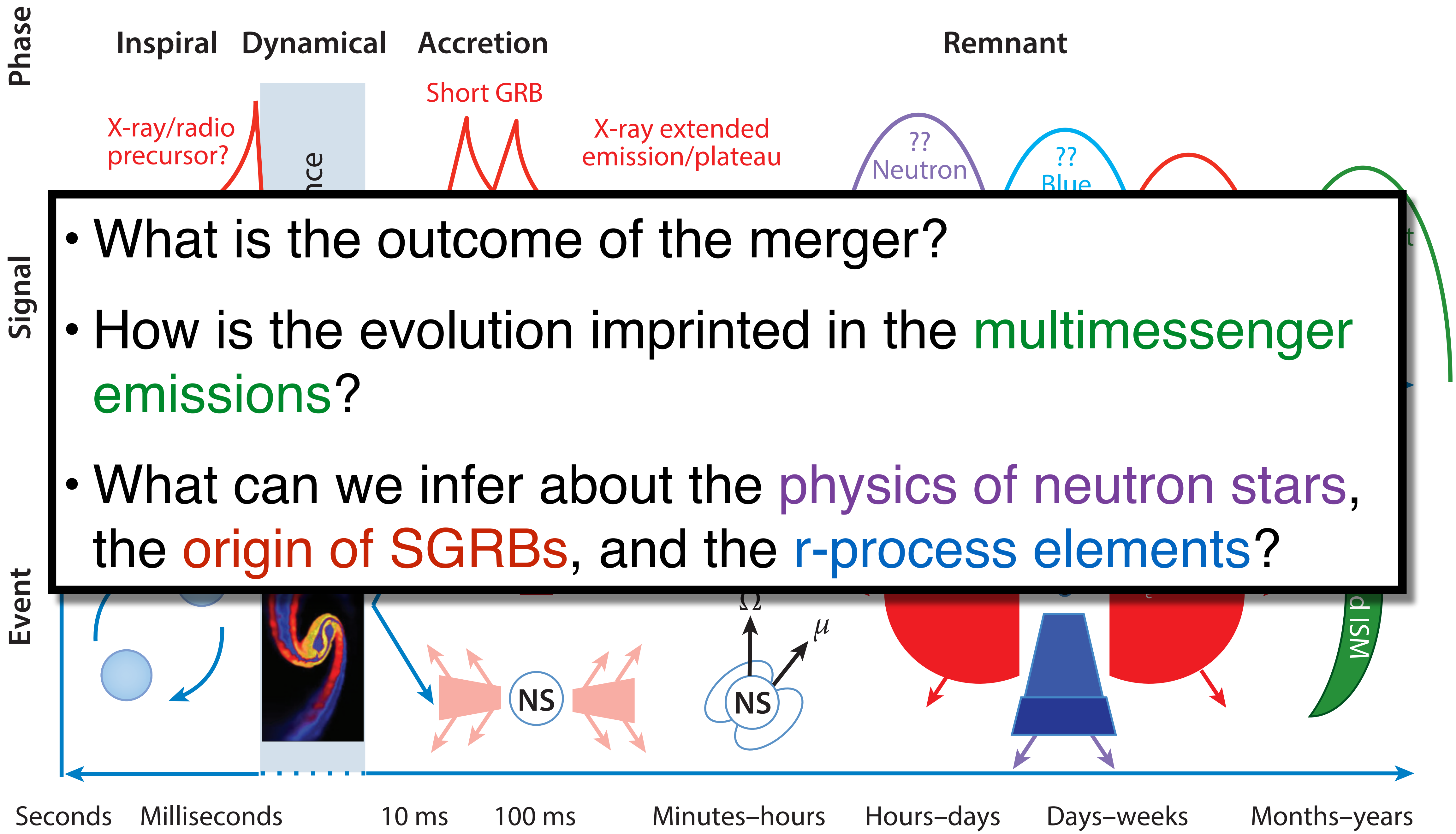
From Bartos, Brady, & Márka 2013

Multimessenger emissions



From Fernández & Metzger 2016

Multimessenger emissions



- What is the outcome of the merger?
- How is the evolution imprinted in the multimessenger emissions?
- What can we infer about the physics of neutron stars, the origin of SGRBs, and the r-process elements?

From Fernández & Metzger 2016

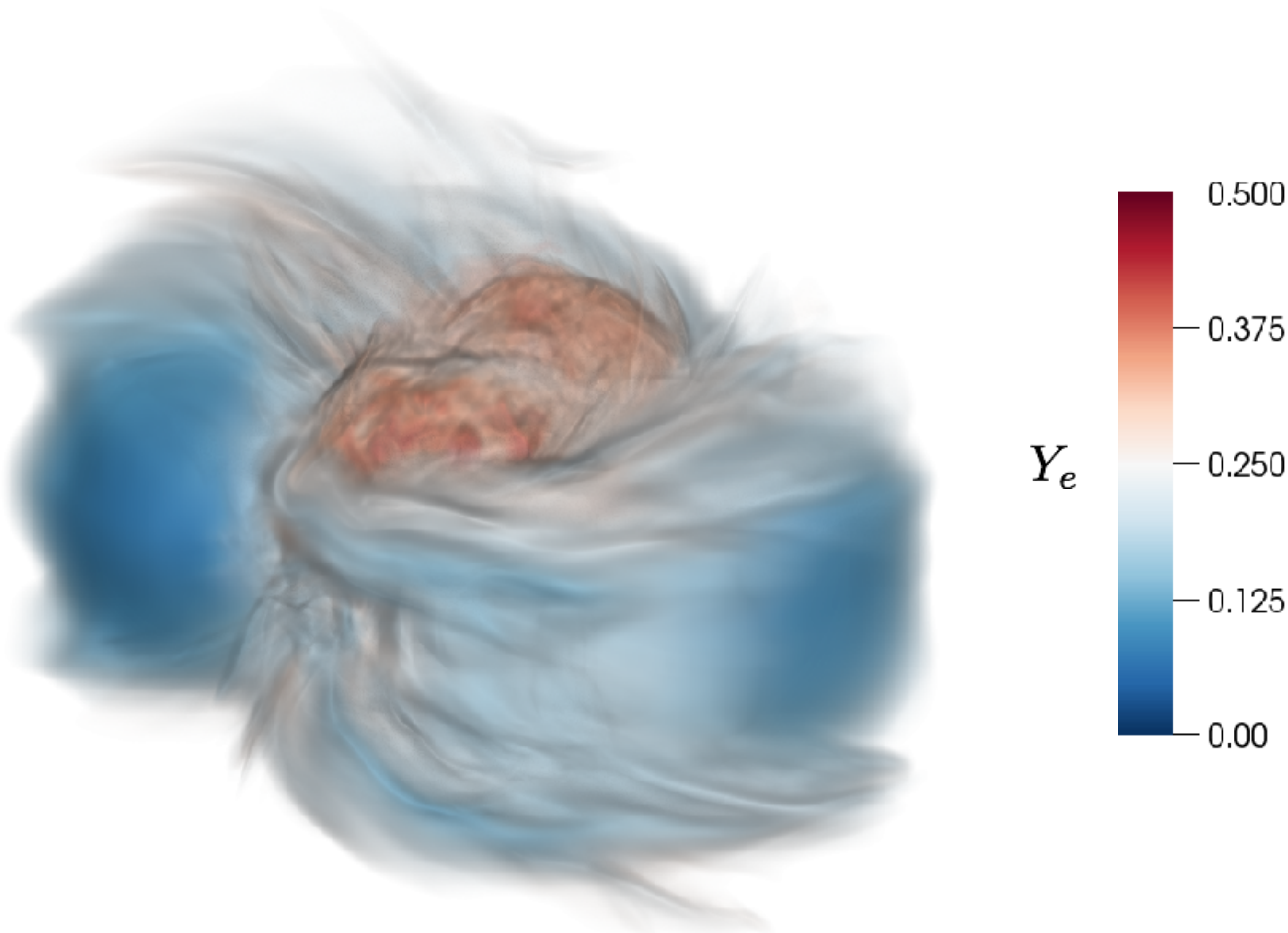
A multi-physics problem

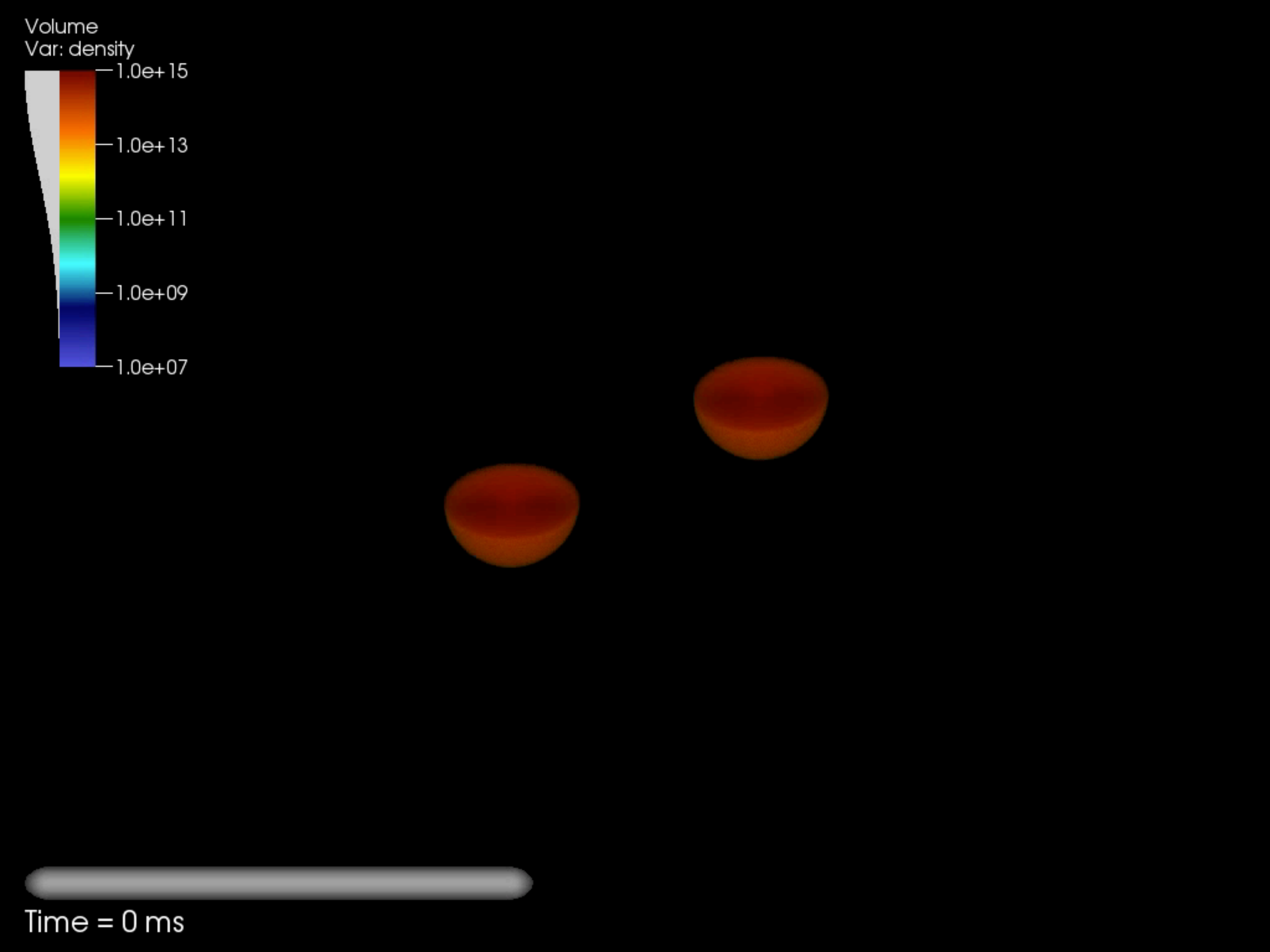
General relativity	Strong gravity
Nuclear EOS	Nuclear forces
Neutrino-matter interactions	Weak reactions
Relativistic MHD	Electromagnetic forces

Multiple time and spatial scales

Dynamical time of a NS	~20 μs
Postmerger GW emission	~10-20 ms
Viscous time	~0.01-1 sec
Neutrino cooling timescale	~2-3 sec
GRB lunching timescale	~1.7 sec
Secular mass ejection	~5 sec

Mass ejection

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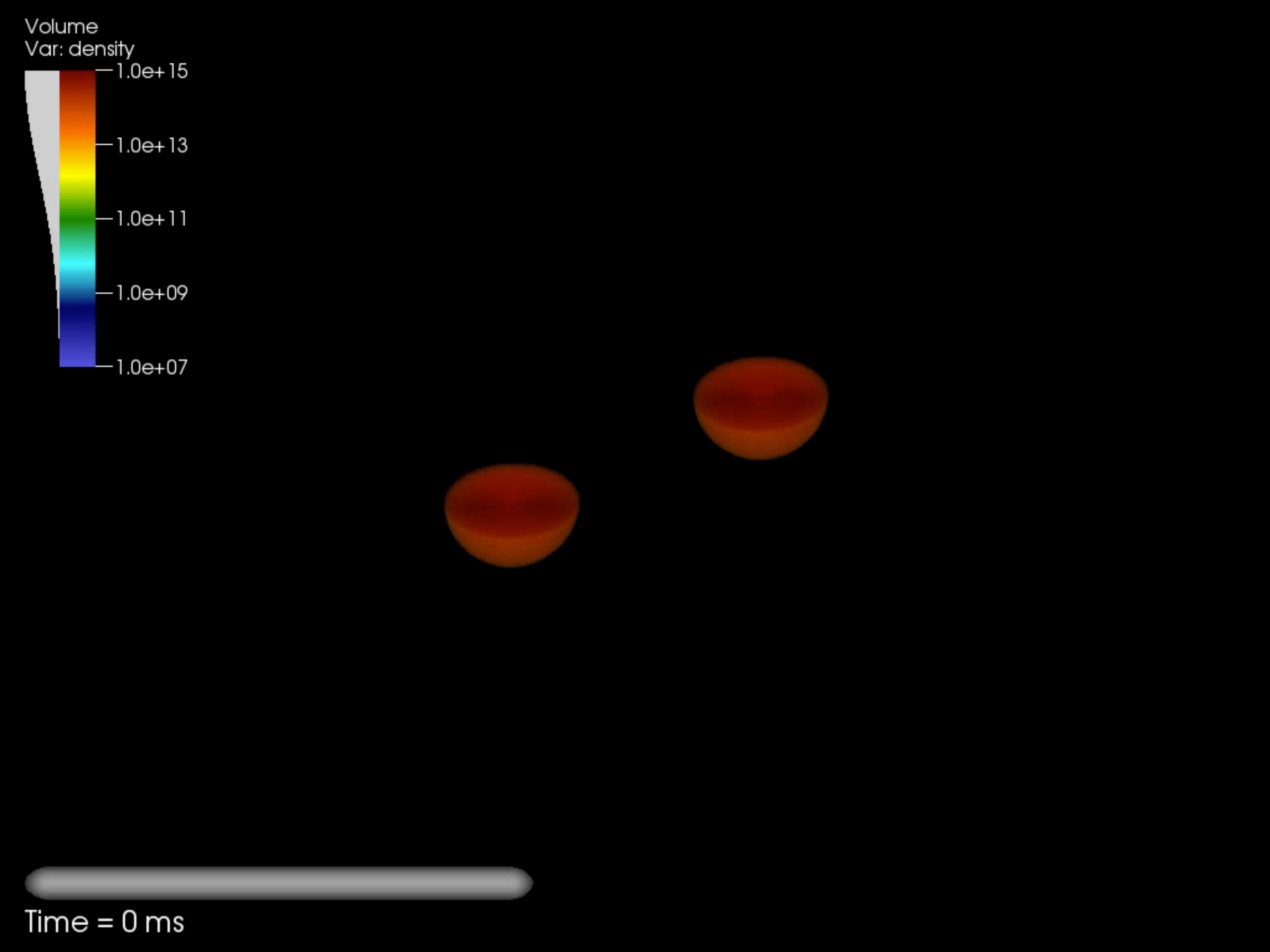
1.0e+13
1.0e+11
1.0e+09
1.0e+07
Time = 0 ms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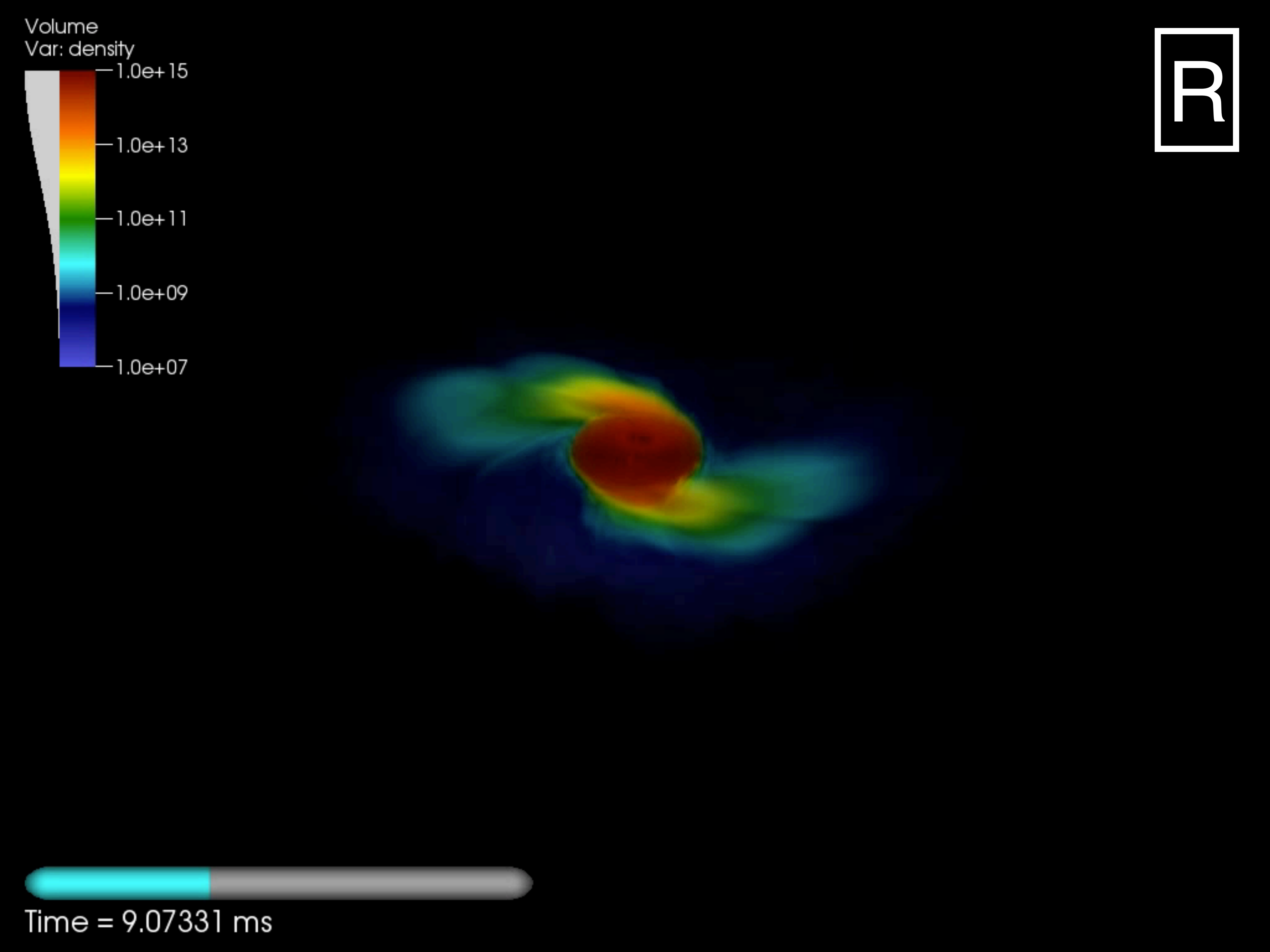
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1.0e+11
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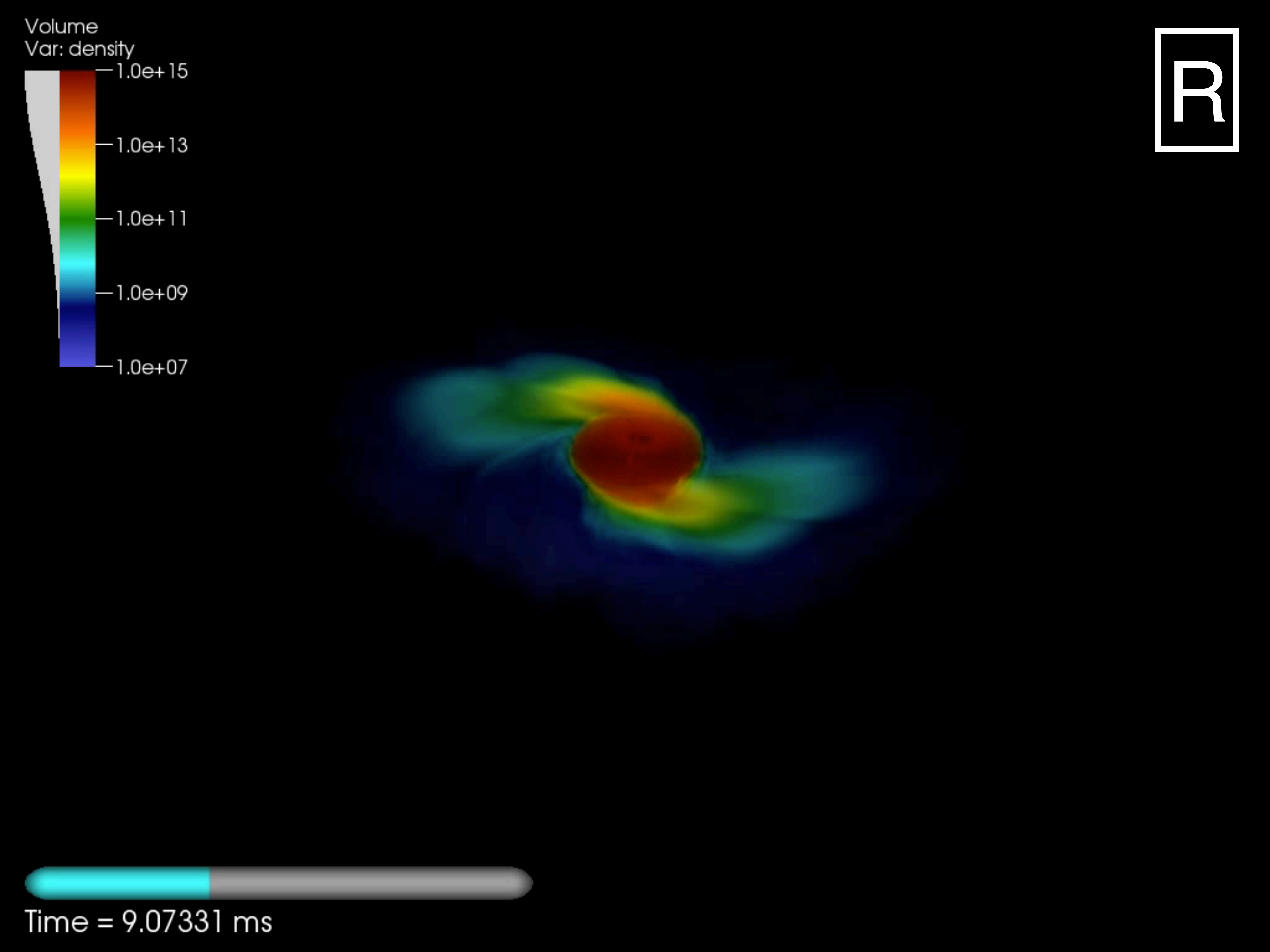
1.0e+07
R
Time = 9.07331 ms


Volume
Var: density
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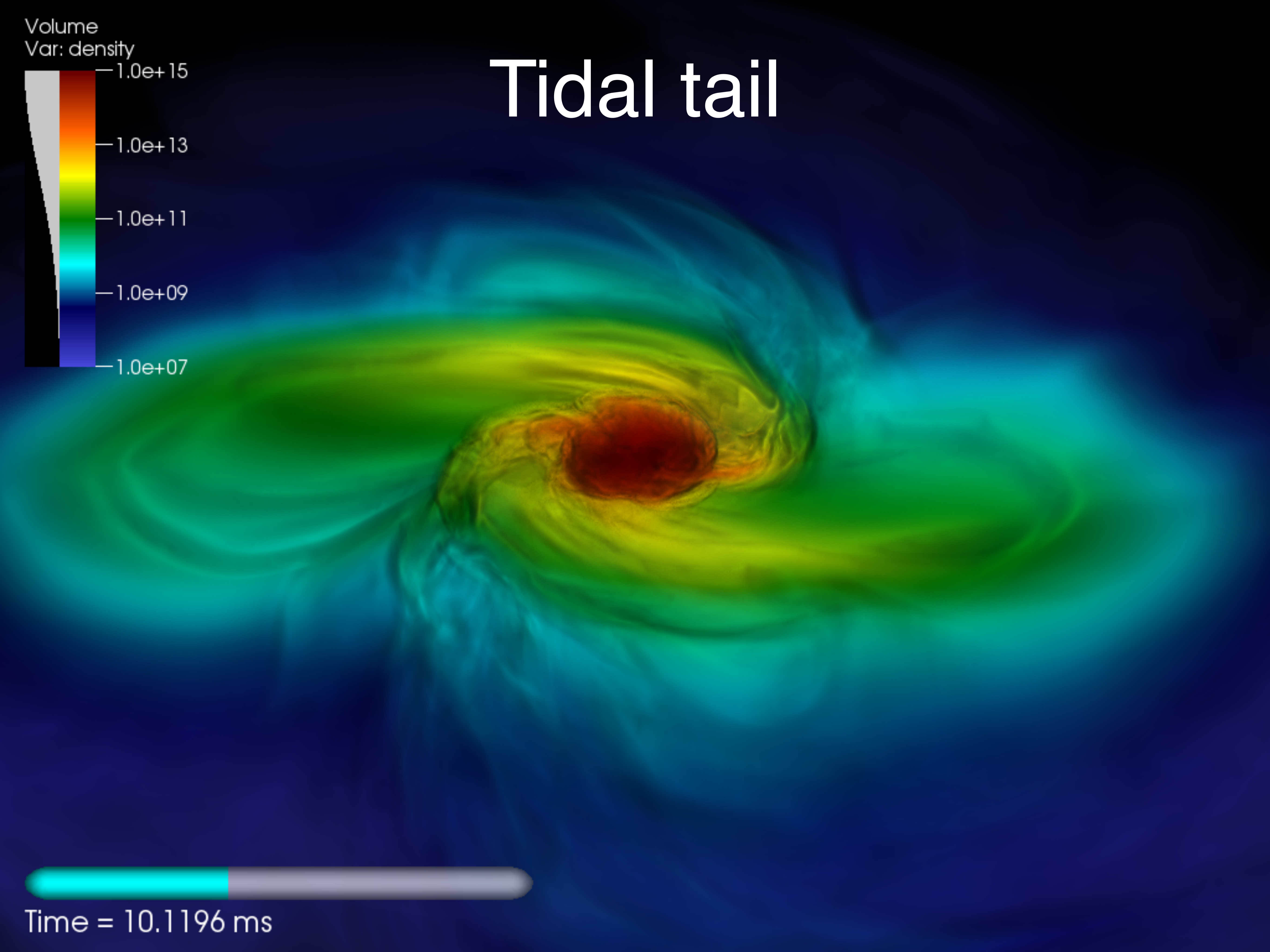
1.0e+13
1.0e+11
1.0e+09
1.0e+07
R
Time = 9.07331 ms

Tidal tail


Volume
Var: density
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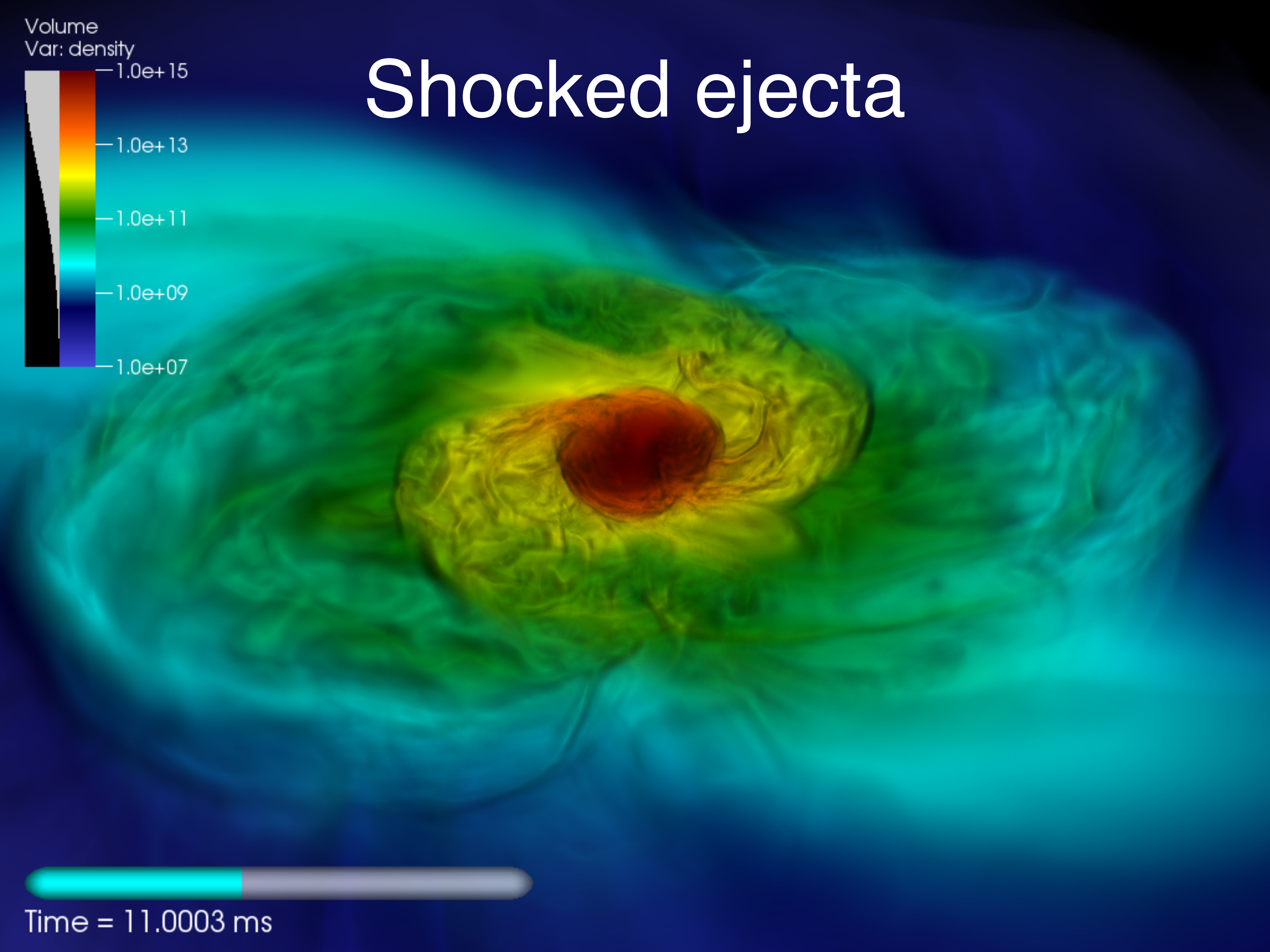
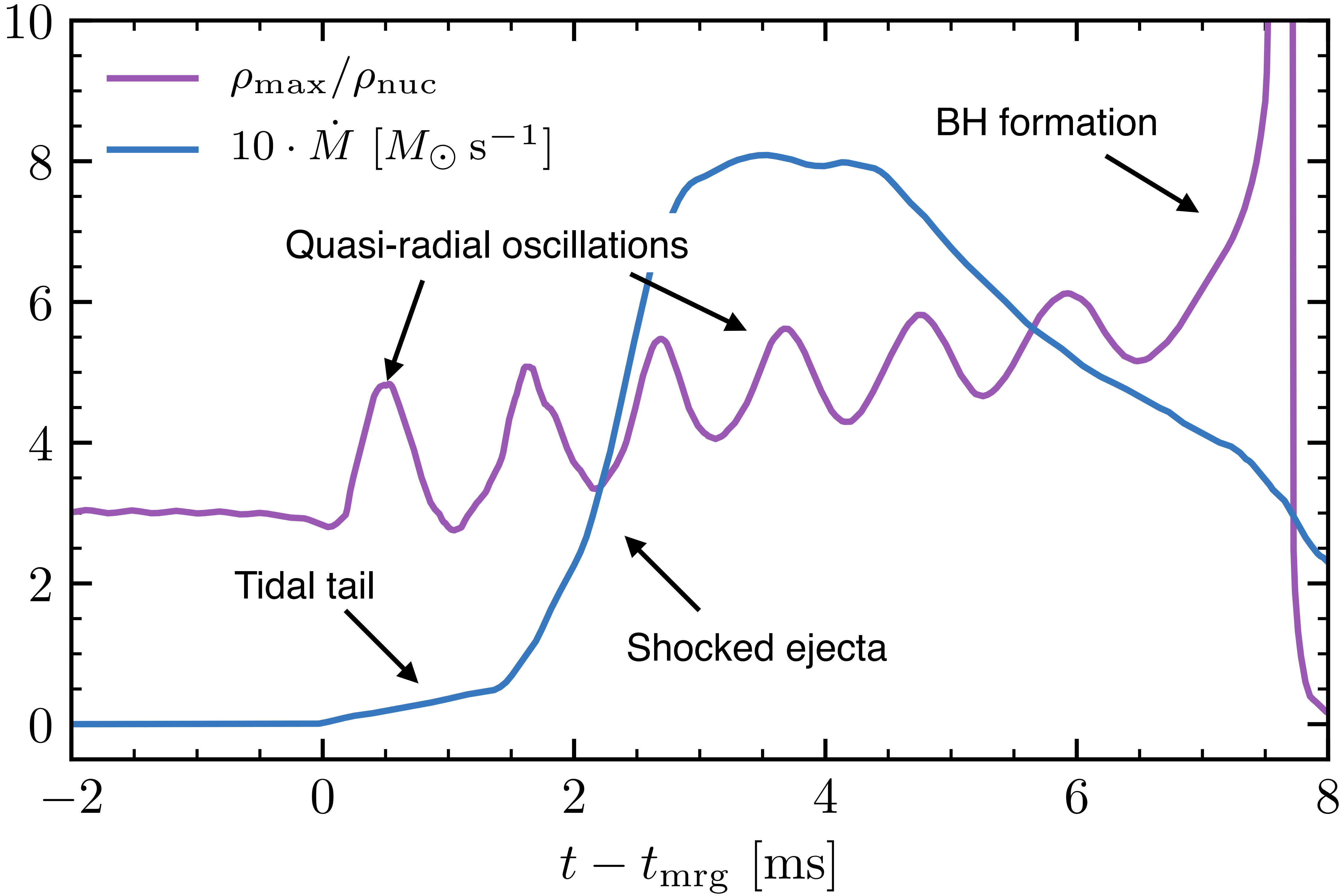
1.0e+11
1.0e+09
1.0e+07
Time = 10.1196 ms

Shocked ejecta

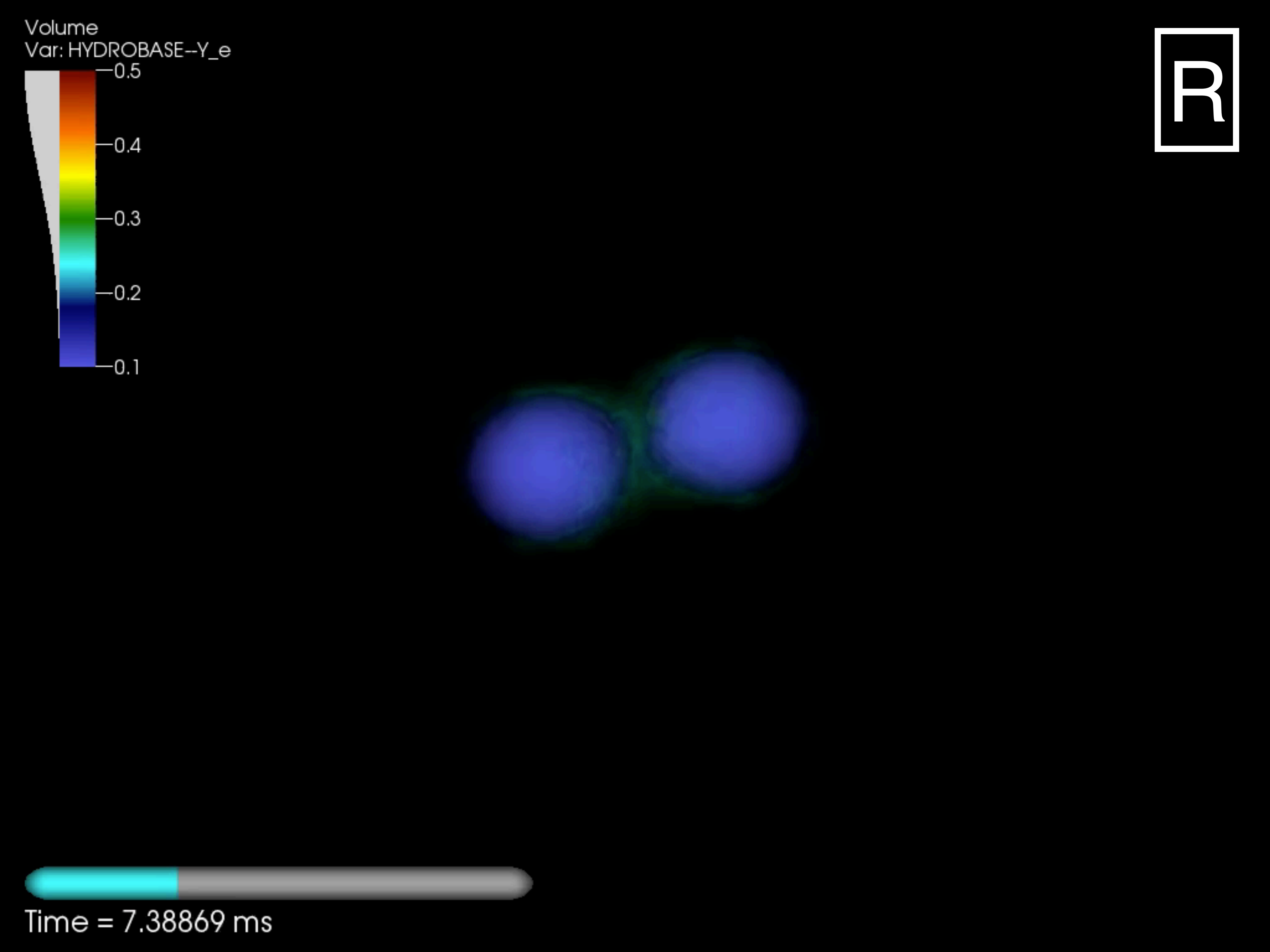


Origin of the shocked ejecta



See also Bauswein+ 2013

DR et al. (2018), in prep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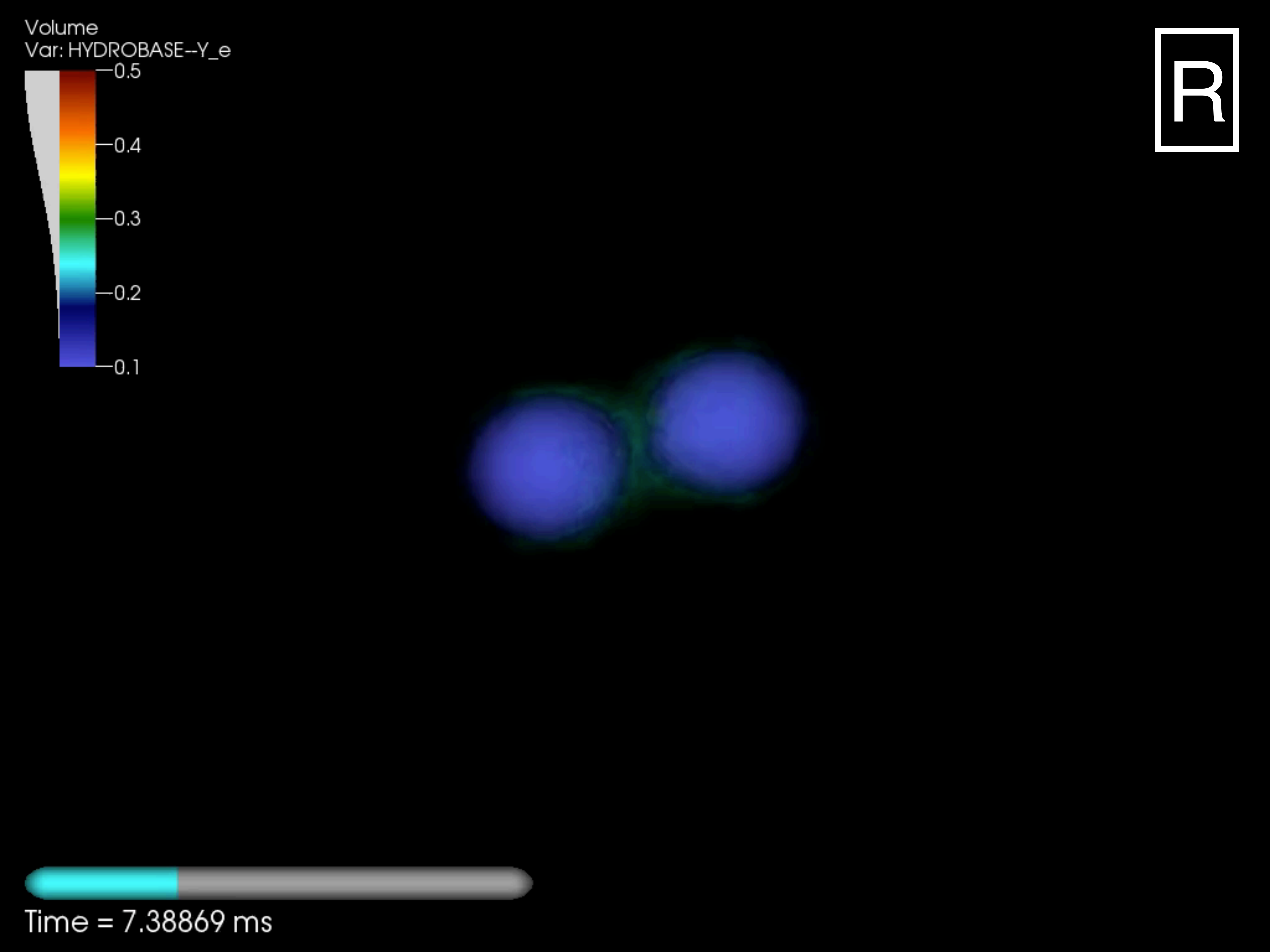
0.4
0.3
0.2
0.1
R
Time = 7.38869 ms


Volume
Var: HYDROBASE--Y_e
0.5
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0.3
0.2
0.1
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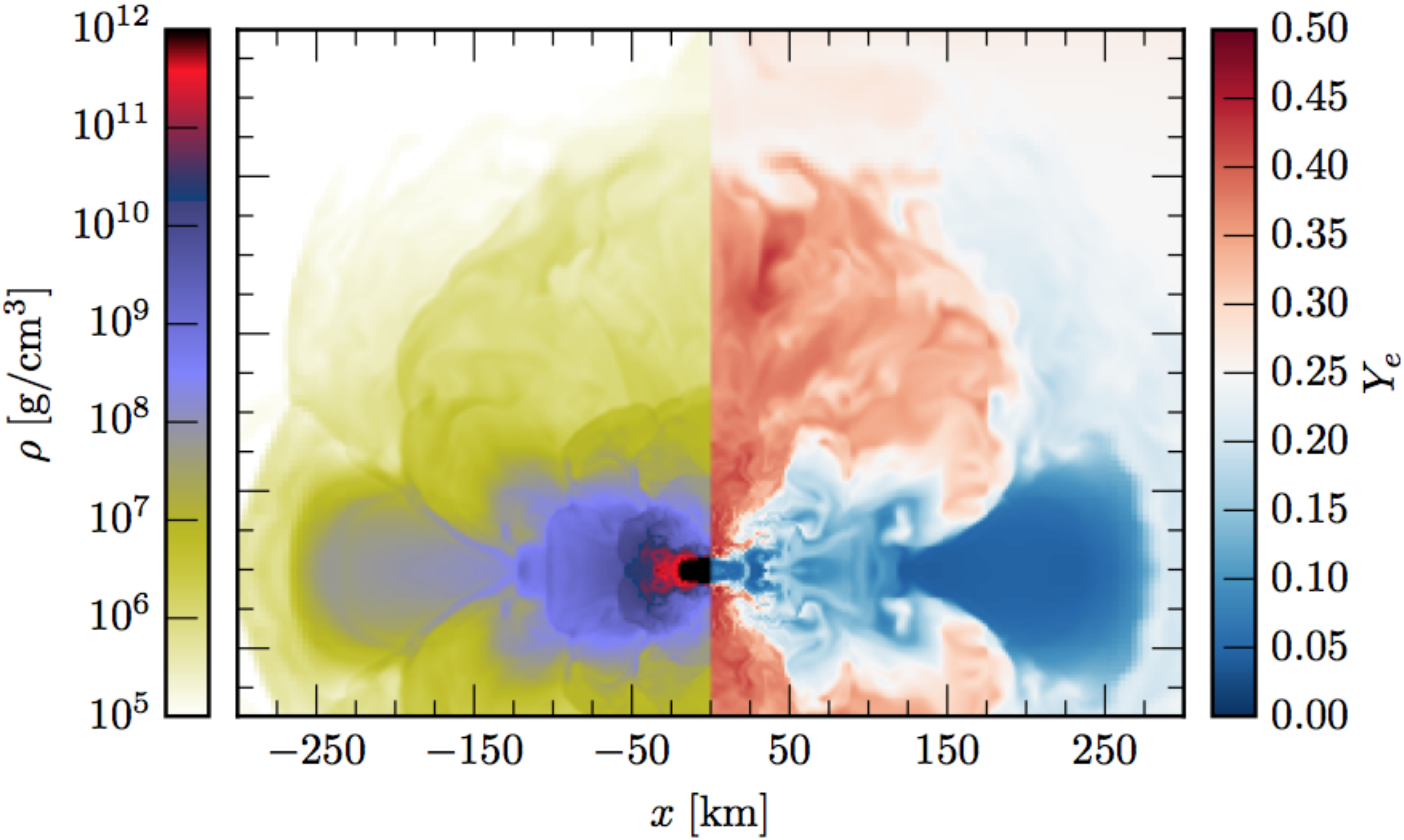
Time = 7.38869 ms

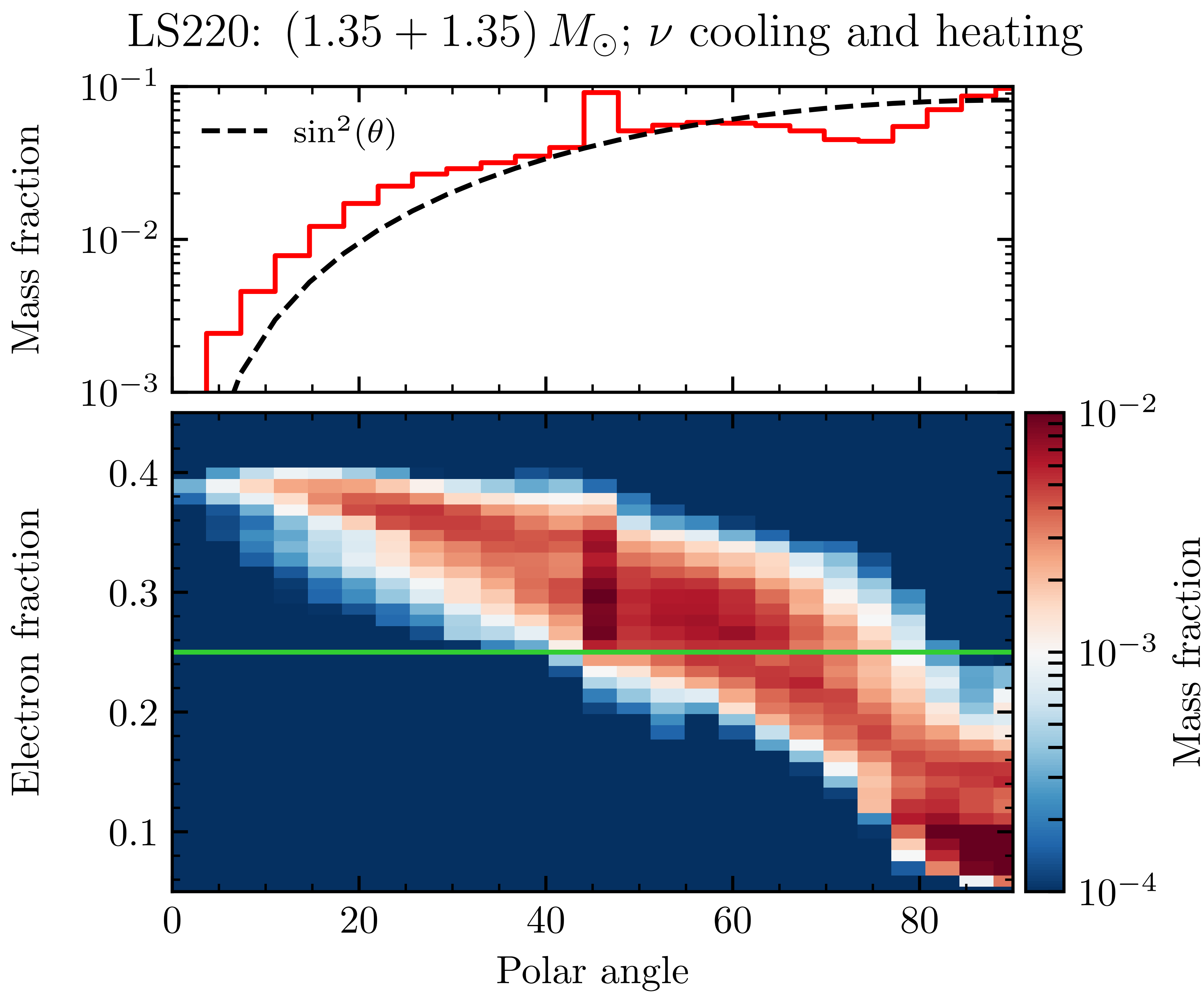
Dynamical ejecta



See also Wanajo+ 2014,
Sekiguchi+ 2015, 2016, Foucart+ 2016

DR, Galeazzi+ MRAS 460:3255 (2016)

Dynamical ejecta: composition



DR et al. (2018), in prep

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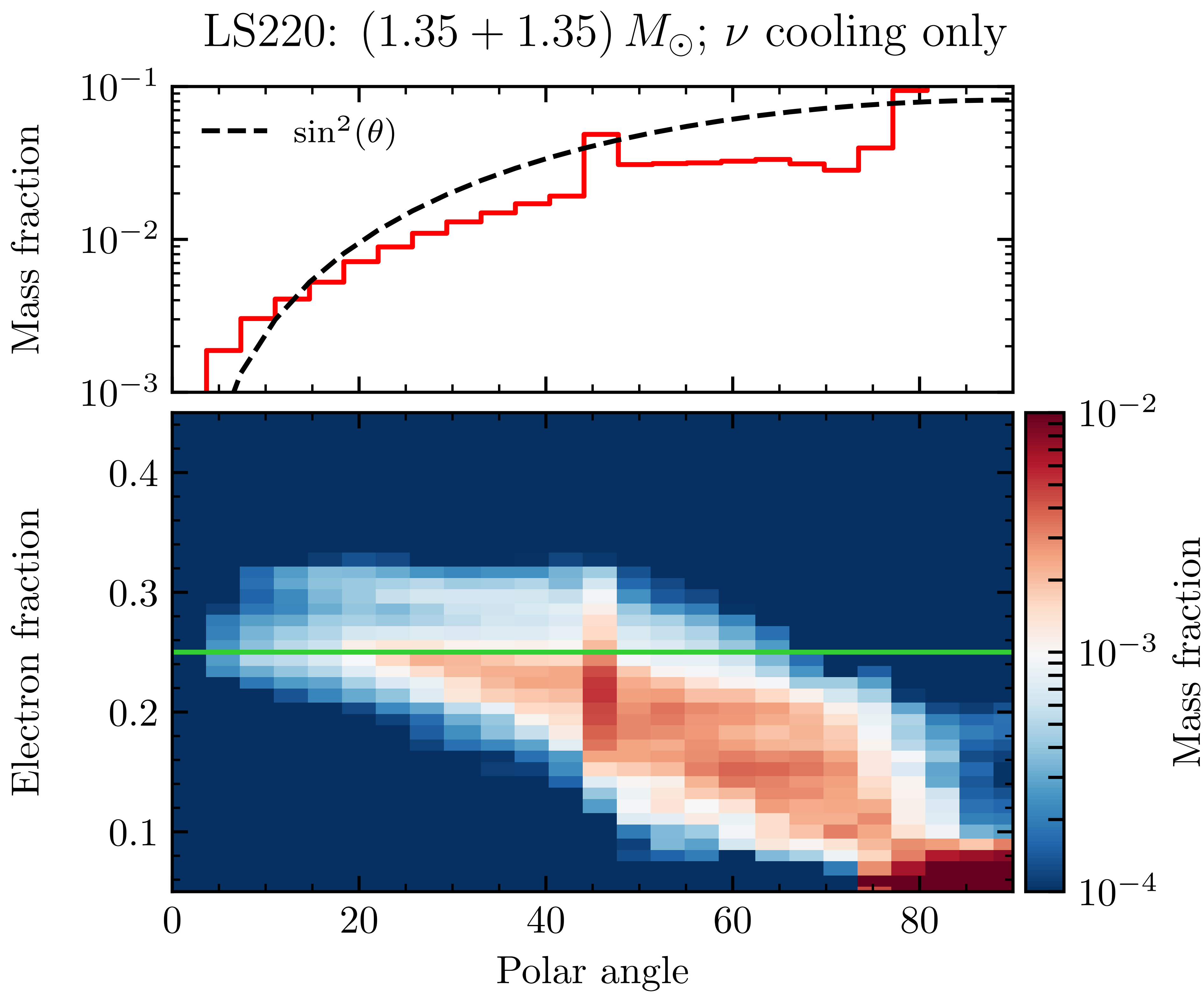
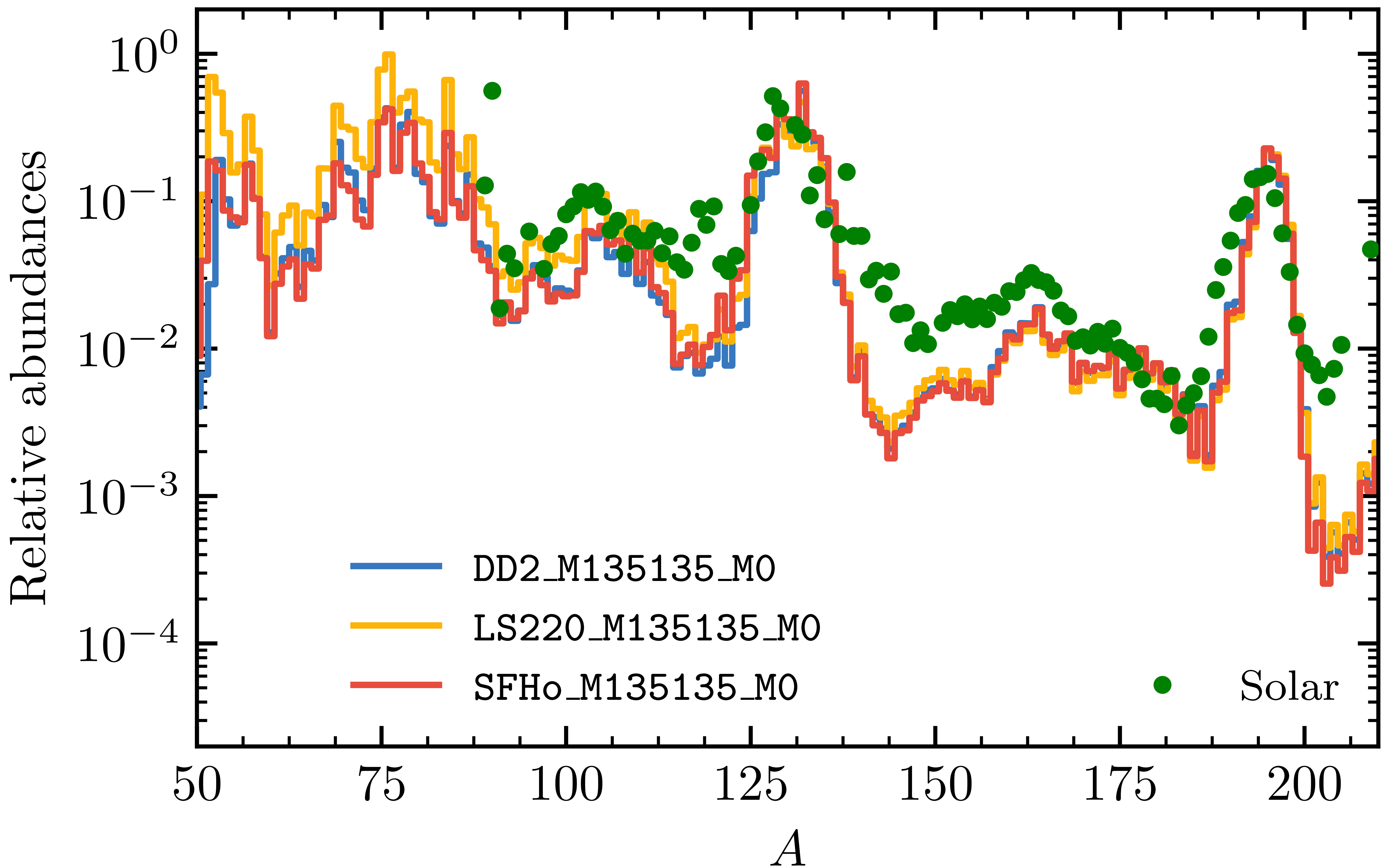


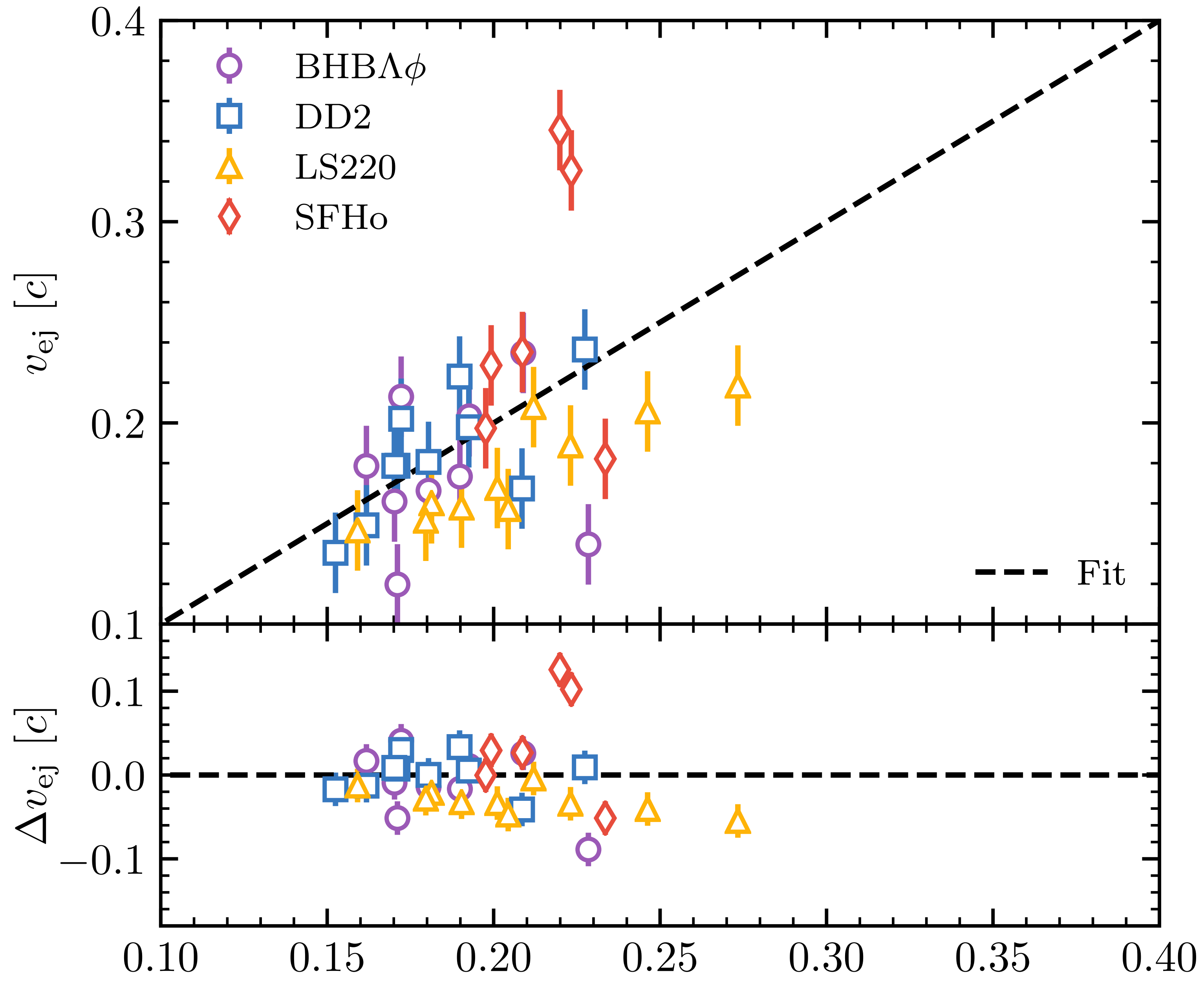
DR et al. (2018), in prep

Dynamical ejecta: yields



DR et al. (2018), in prep

Dynamical ejecta: rms velocity



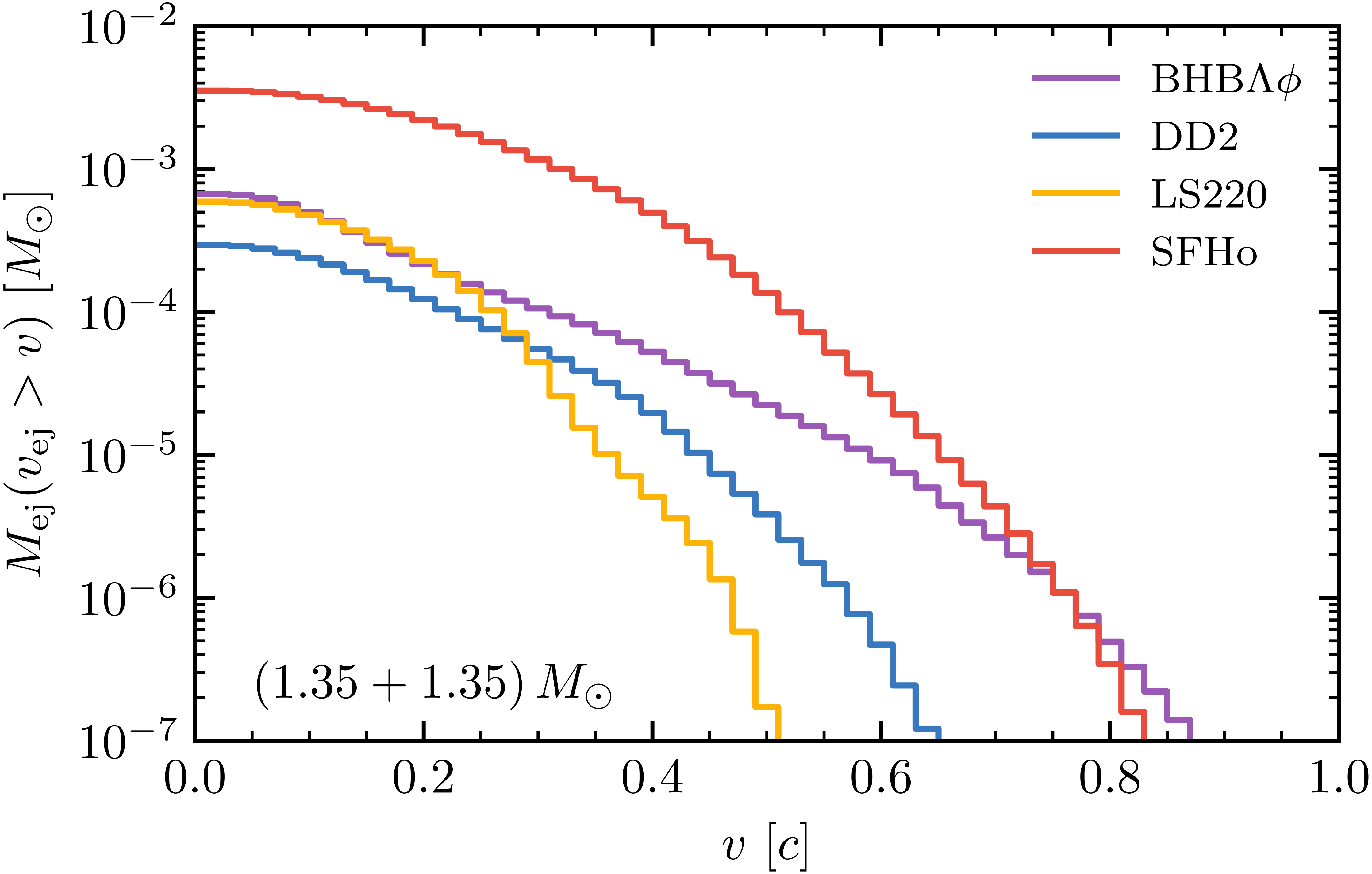
$$v_\rho = \left[a\left(\frac{M_1}{M_2}\right)(1 + c\, C_1)\right] + (1 \leftrightarrow 2) + b.$$

Fitting formula from Dietrich+ CQG 34:105014 (2017)

Warning: very different values for the fitting coefficients

DR et al. (2018), in prep

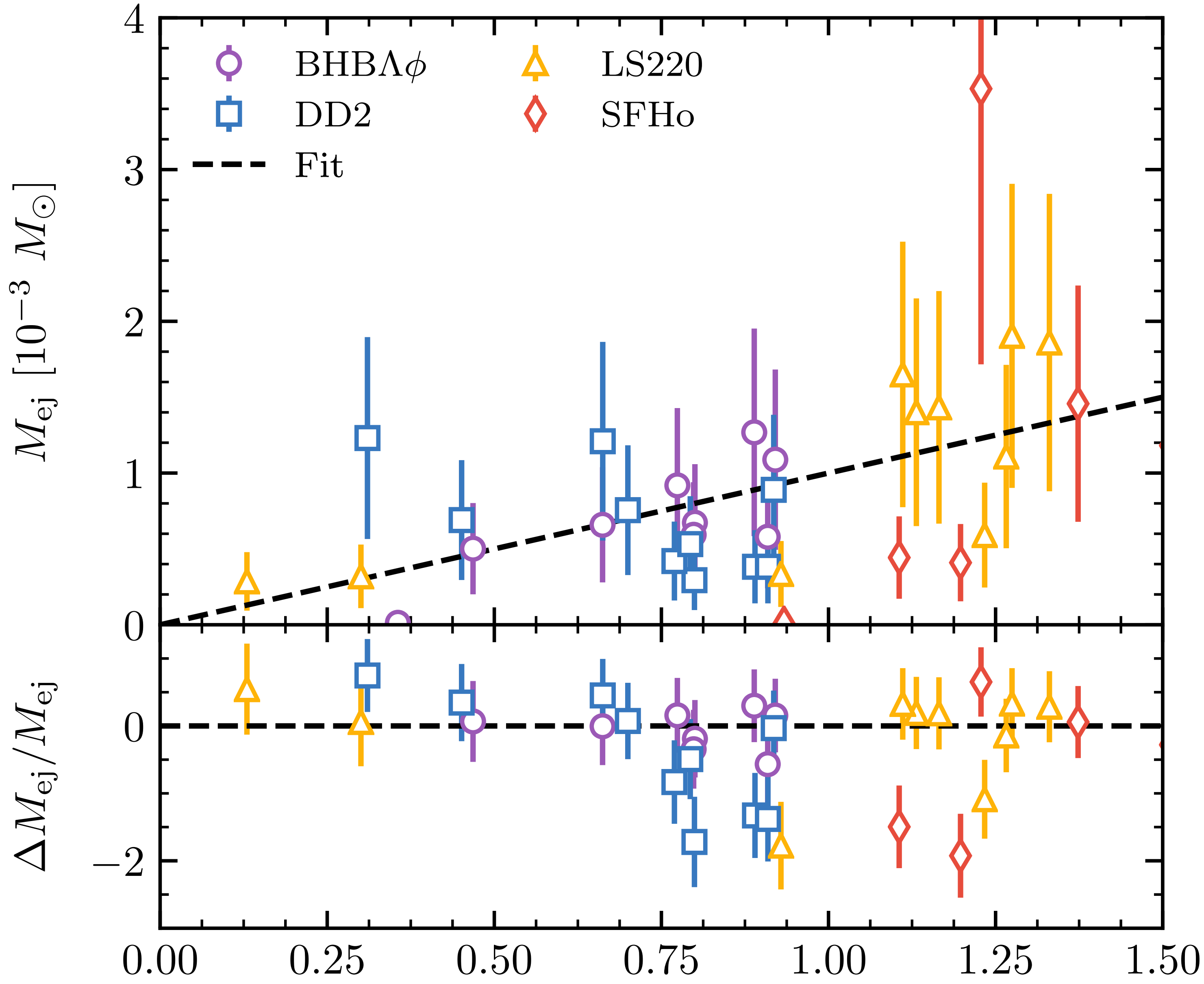
High-velocity tail



See Hotokezaka et al. (2018)

DR et al. (2018), in prep

Dynamical ejecta: mass



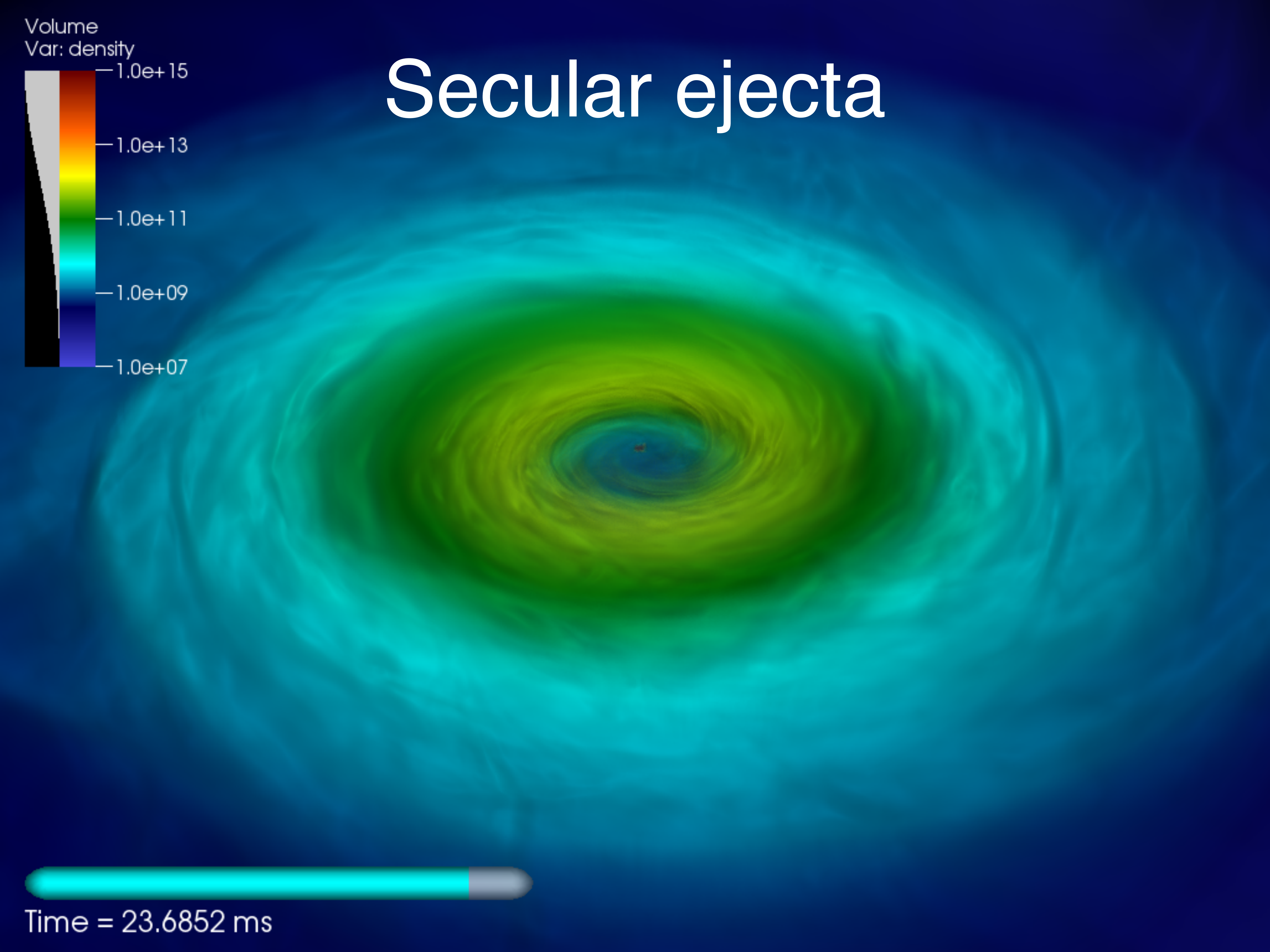
$$\frac{M_{\rm ej}^{\rm fit}}{10^{-3}M_\odot} = \left[a\left(\frac{M_2}{M_1}\right)^{1/3}\left(\frac{1-2C_1}{C_1}\right) + b\left(\frac{M_2}{M_1}\right)^{n} + c\left(1-\frac{M_1}{M_1^*}\right)\right]M_1^* + (1\leftrightarrow 2) + d.$$

Fitting formula from Dietrich+ CQG 34:105014 (2017)

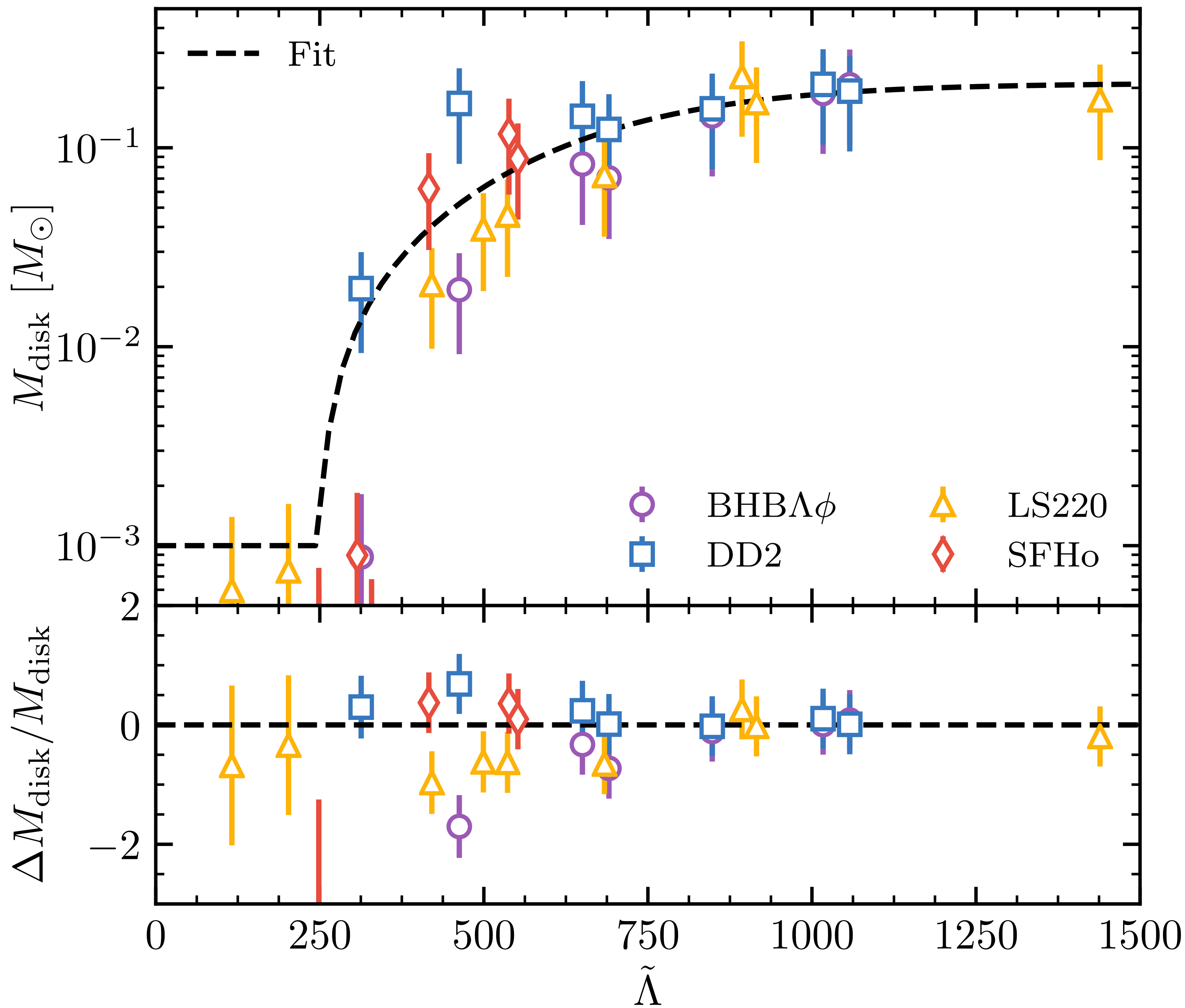
Warning: very different values for the fitting coefficients

DR et al. (2018), in prep

Secular ejecta



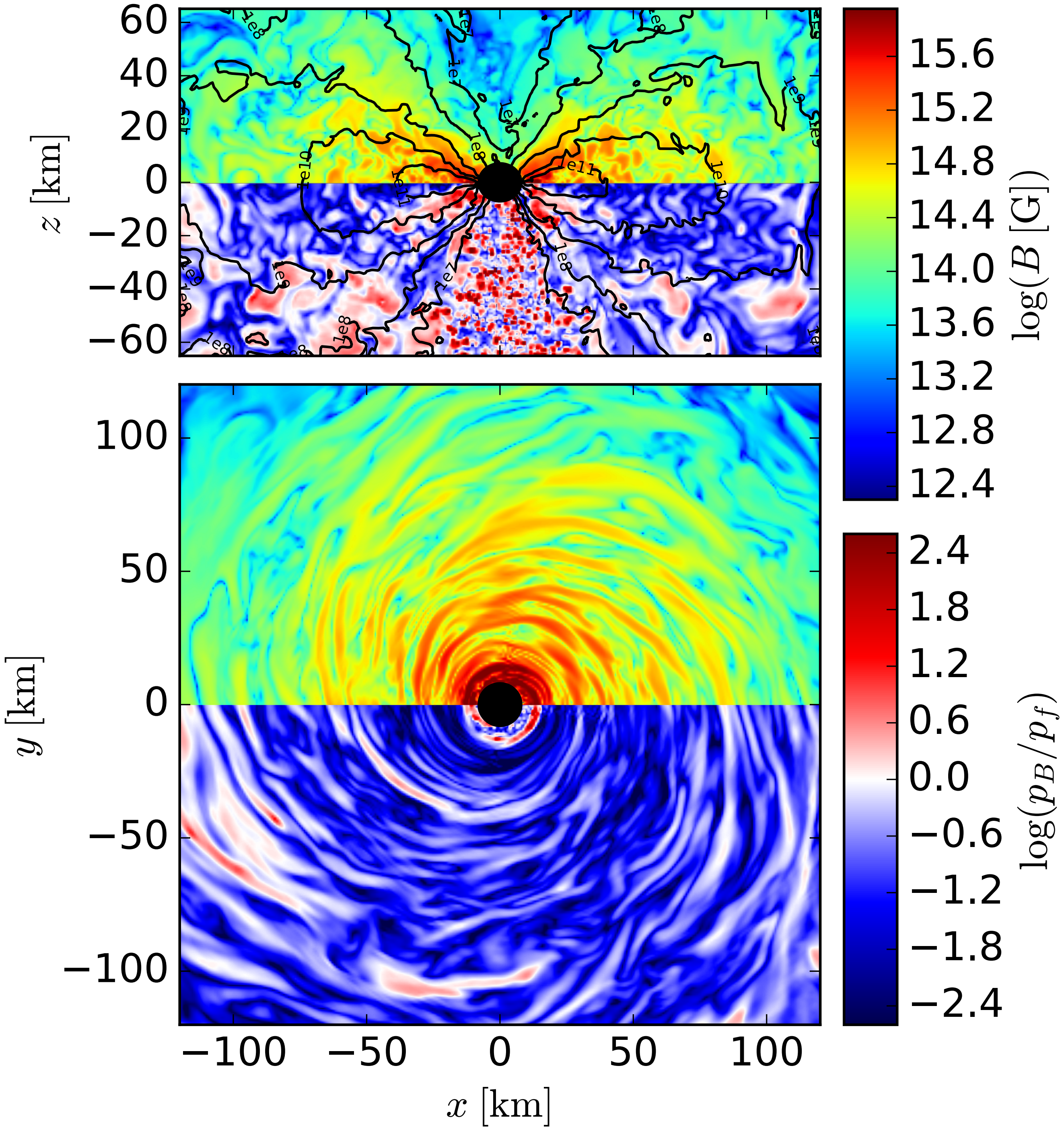
Typical disk masses

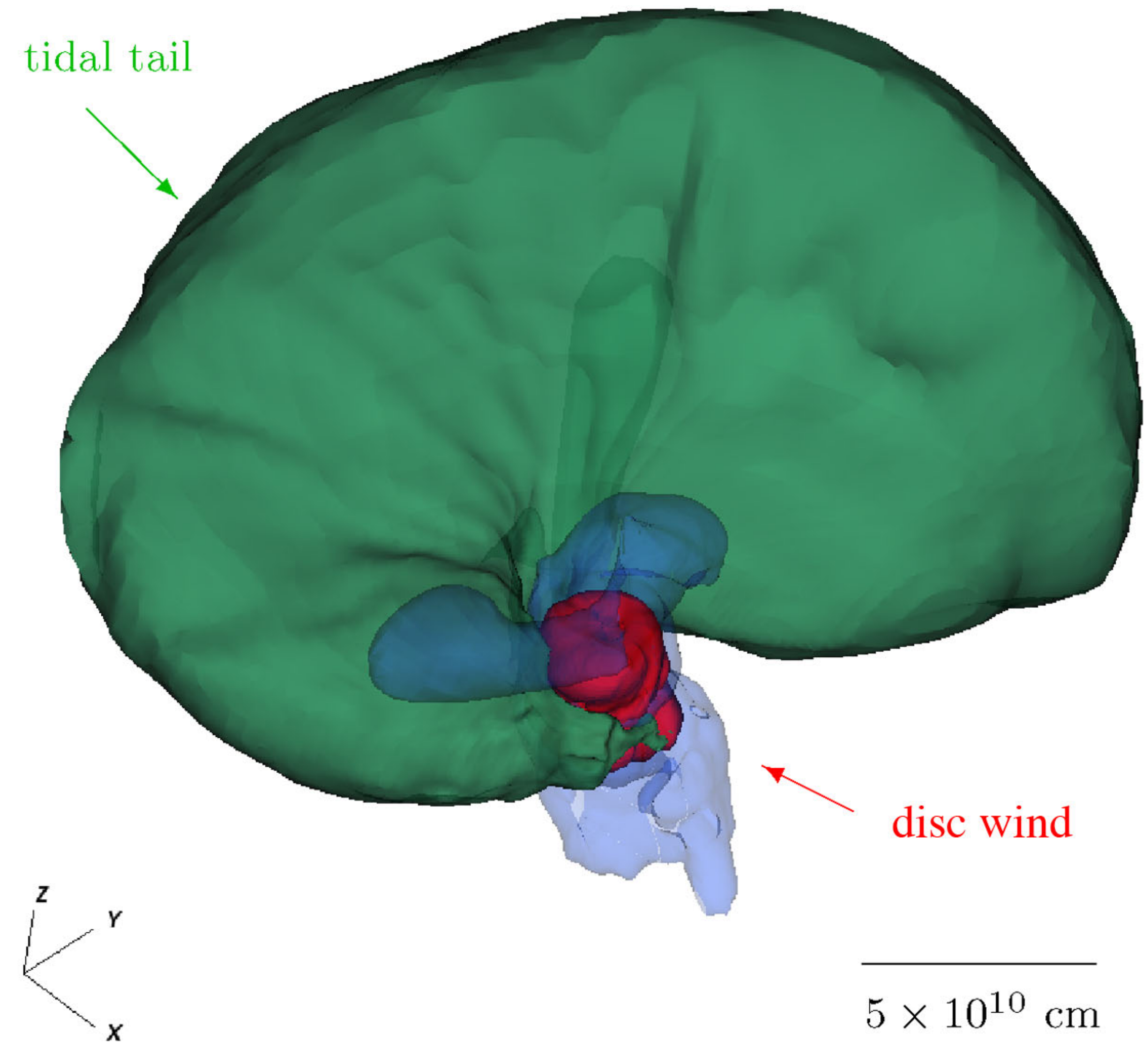


See also **DR**, Perego, Zappa, ApJL 852:L29 (2018)

DR et al. (2018), in prep

Secular ejecta

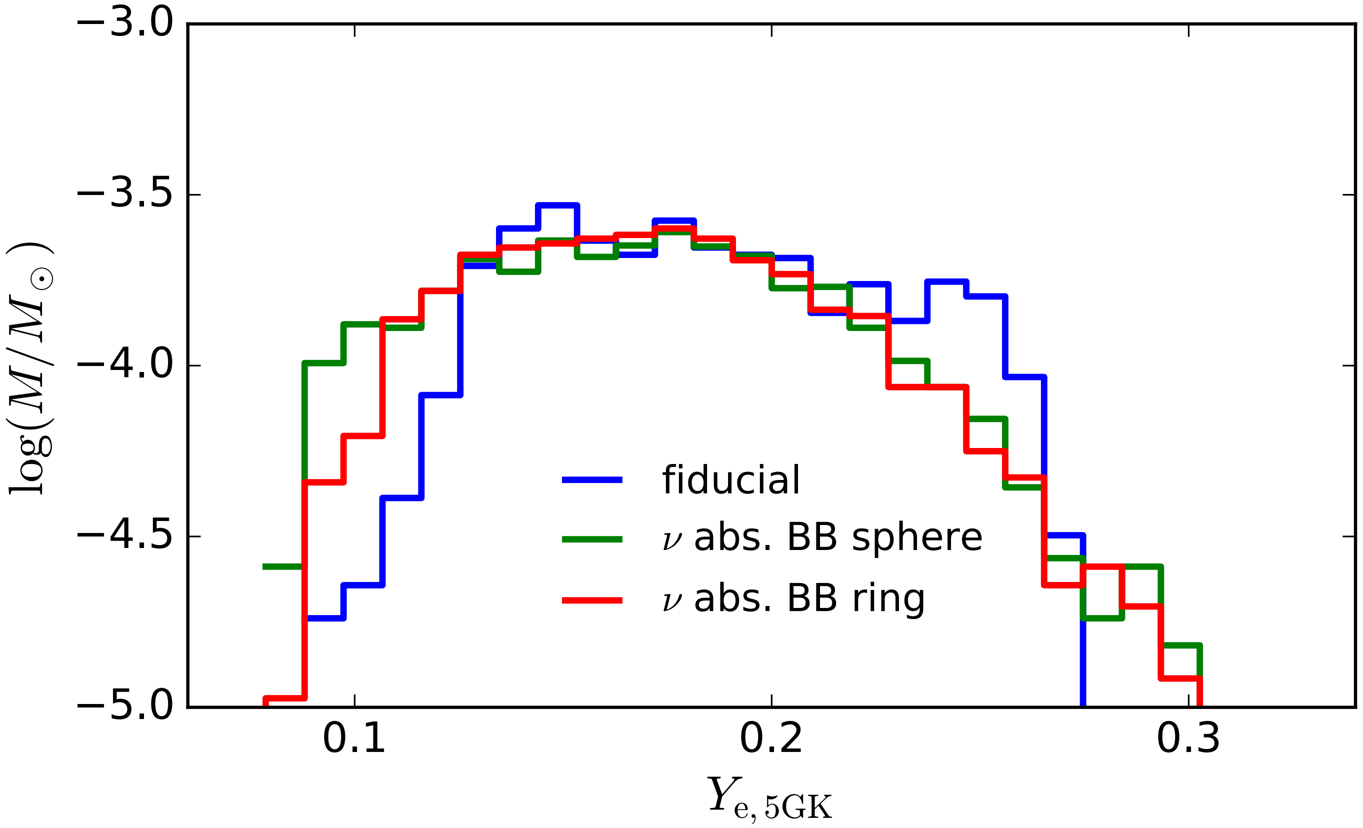




From Siegel & Metzger, arXiv:1711.00868

From Fernandez+, MNRAS 449:390 (2015)

Secular ejecta: properties

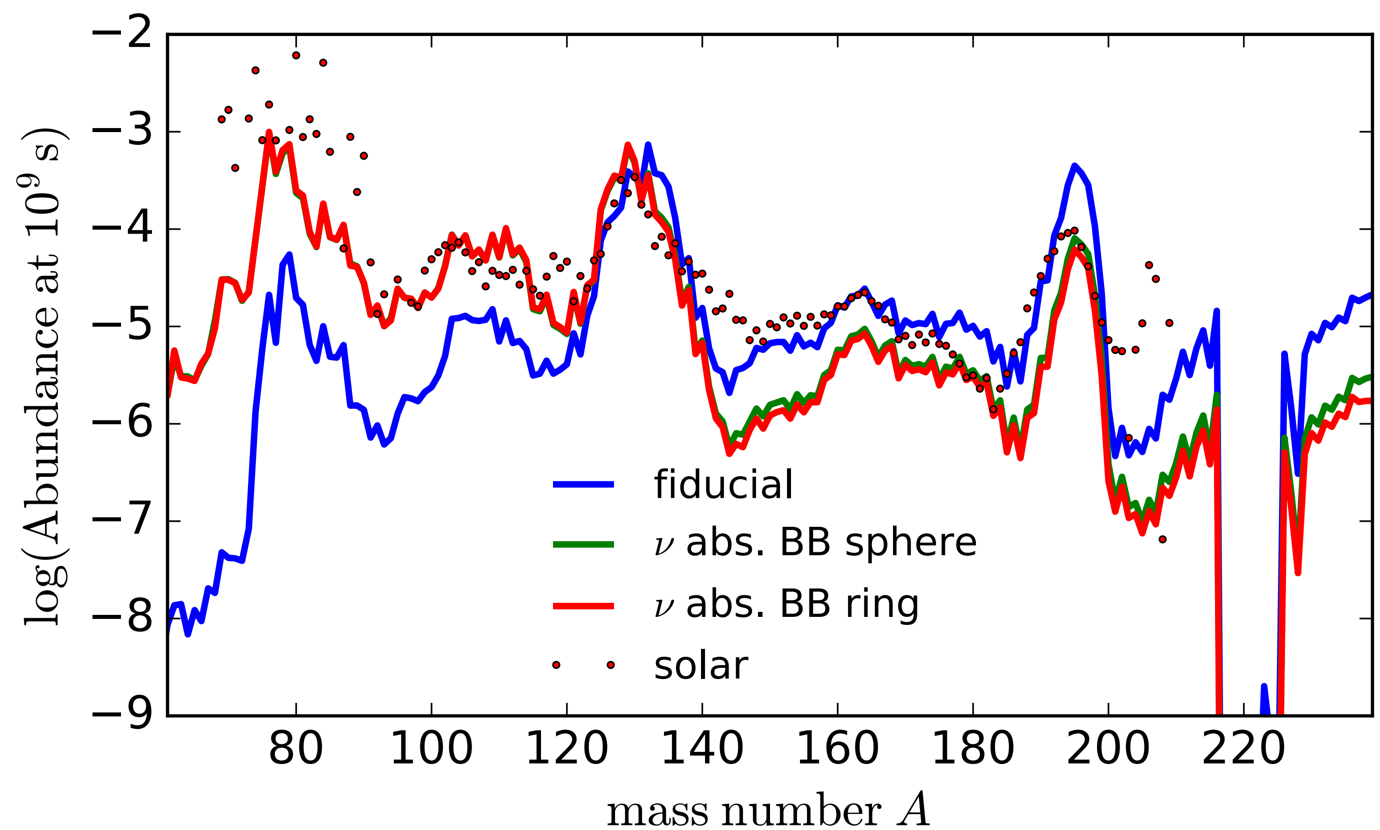


Mass ejection due to nuclear recombination. The characteristic energy gained by each baryon is:

$$\frac{1}{2}\left(\frac{v}{c}\right)^2 \sim \frac{8.8\,\mathrm{MeV}}{931.5\,\mathrm{MeV}}$$

This implies $v \sim 0.1c$

Up to 50% of the remnant accretion disk is unbound!



From Siegel & Metzger, arXiv:1711.00868

Summary

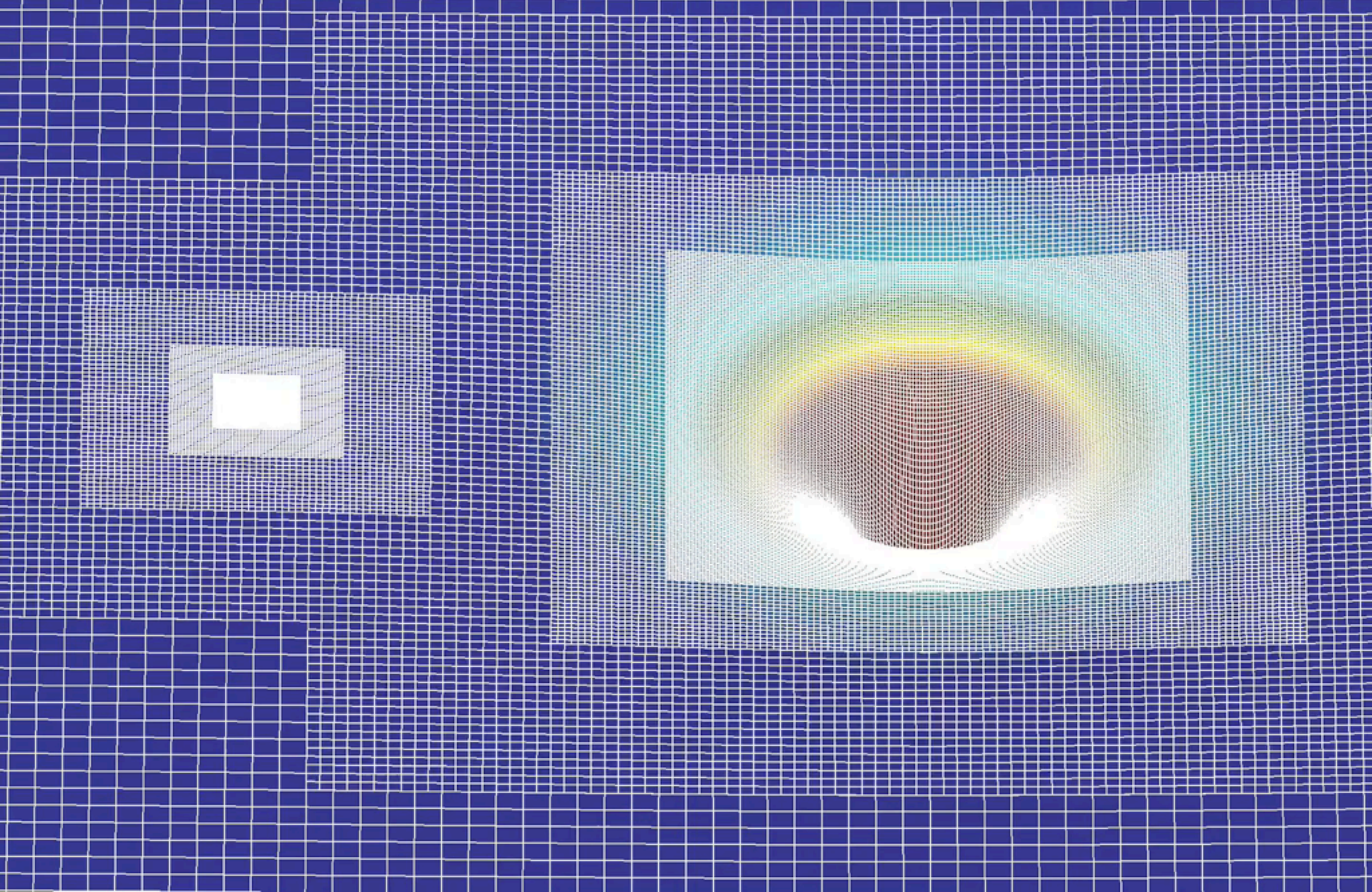
For moderate mass ratio NS-NS mergers:

- the dynamical ejecta determines the opacity of the outermost layer of the outflow;
- the disk wind dominates the nucleosynthetic yields.

The ejecta components from a NS-NS merger are:

- the tidal tail: neutron rich, fast;
- the shocked ejecta: moderate Ye ~ 0.2, fast;
- the neutrino-driven wind: high Ye > 0.25, fast;
- the viscous-driven wind: slower, wide Ye distribution, v ~ 0.1c

How do the simulations work?



Field equations

- Einstein tensor

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

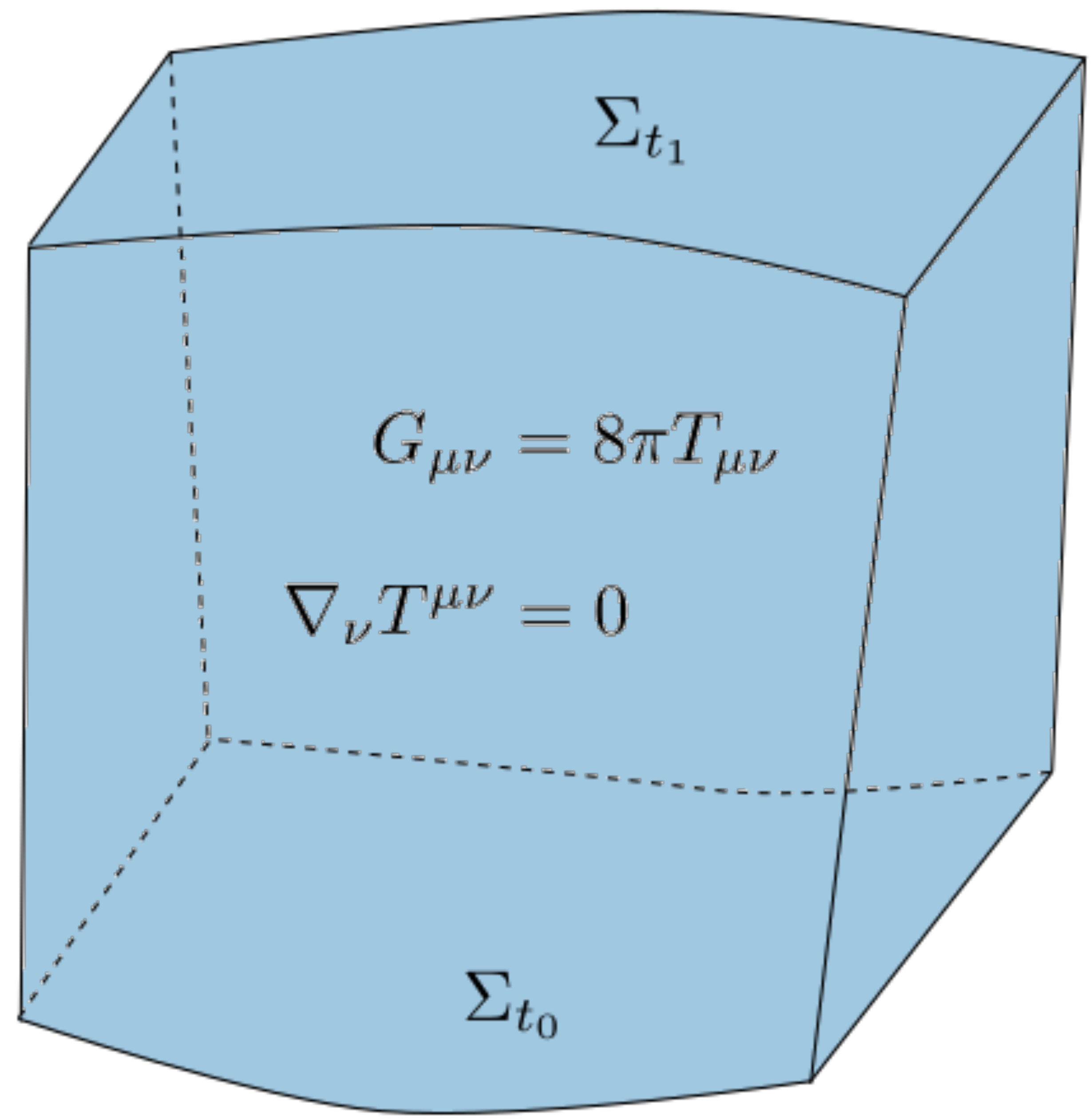
  describes the geometry of spacetime

- Stress energy tensor

$$T_{\mu\nu}$$

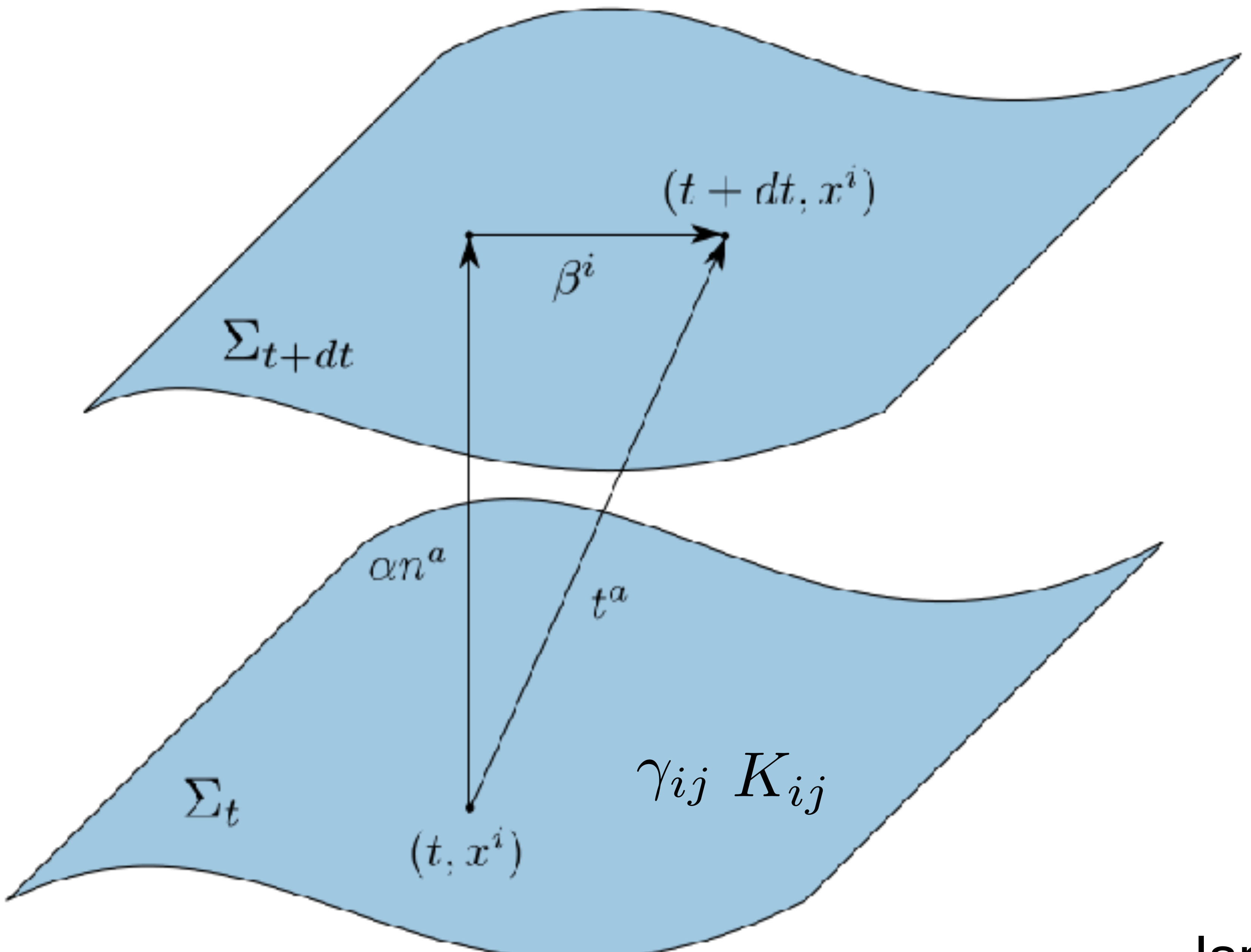
  describes matter and radiation content of the spacetime

- Field equations and Bianchi identities connect geometry and flow



Note G = c = 1 for us!

3+1 decomposition



γ_{ij} spatial metric

K_{ij} extrinsic curvature

$\vec{t}$ vector tangent to the constant x^i curves

$\vec{n}$ vector normal to the constant t hyper-surfaces

$$\underbrace{\vec{t} = \alpha}_{\text{lapse function}}\vec{n} + \underbrace{\vec{\beta}}_{\text{shift vector}}$$

Lapse and shift vector are gauge degrees of freedom

Spatial metric

The spatial metric describes the intrinsic geometry of the spatial slices

$$\mathrm{d}\ell^2 = \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

As a four-dimensional object $\gamma^\mu{}_\nu$ is used to project vectors tangentially to the spatial hypersurface

$$\gamma^\mu{}_\nu = \delta^\mu{}_\nu + n^\mu n_\nu$$

The spatial metric is prescribed as initial data and then evolved

We use the symbol D_i for the covariant derivative associated with γ_{ij}

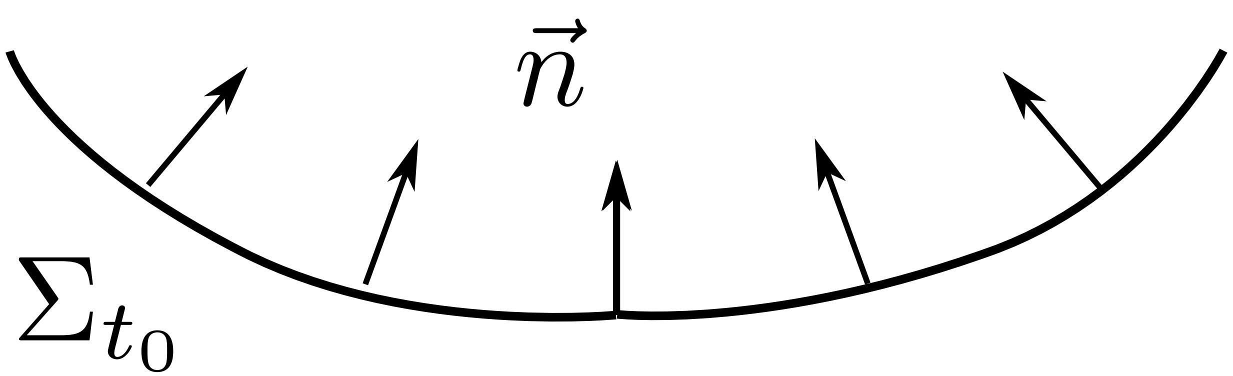
Extrinsic curvature (I)

$$K_{\mu\nu} = -\gamma^{\alpha}{}_{\mu} \nabla_{\alpha} n_{\nu}$$

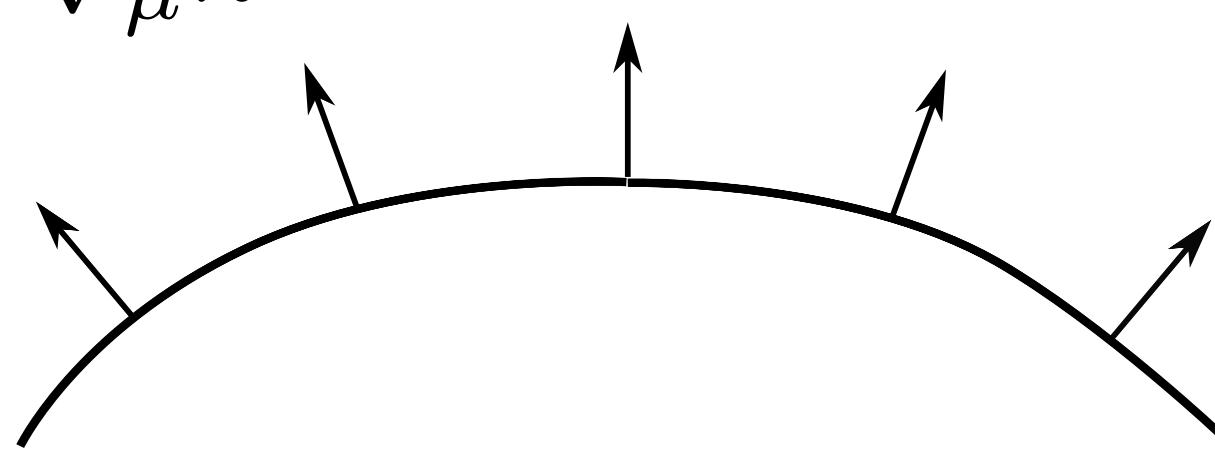
Tangent projection of the gradient of the normal vector

The trace of the extrinsic curvature is associated with the expansion of the world lines of the Eulerian observers:

$$K = -\nabla_{\mu} n^{\mu}$$



$K > 0$



$K < 0$

Extrinsic curvature (II)

$$K_{ij} = -\frac{1}{2}\mathcal{L}_{\vec{n}}\gamma_{ij}$$

The extrinsic curvature is the Lie derivative of the metric along the normal to the spatial hypersurface

$K_{ij} = 0 \implies$ three-metric is stationary!

If $\beta^i = 0$ then $K_{ij} = -\frac{1}{2}\partial_t \gamma_{ij}$

The extrinsic curvature is also prescribed at the initial time and then evolved

ADM equations

$$(\partial_t - \mathcal{L}_{\vec{\beta}})\gamma_{ik} = -2\alpha K_{ik}$$

$$(\partial_t - \mathcal{L}_{\vec{\beta}})K_{ik} = -D_i D_k \alpha + \alpha\left({}^{(3)}R_{ik} - 2K_{ij}K^j{}_k + KK_{ik}\right) - 8\pi\alpha\left(S_{ik} - \frac{1}{2}\gamma_{ik}(S-E)\right)$$

$${}^{(3)}R + K^2 - K_{ik}K^{ik} = 16\pi E$$

$$D_k(K\gamma^k{}_i - K^k{}_i) = 8\pi j_i$$

$$S_{ik} = \gamma^\mu{}_i\gamma^\nu{}_k T_{\mu\nu} \qquad S = S^i{}_i \qquad j_i = -\gamma^\mu{}_i n^\nu T_{\mu\nu} \qquad E = T^{\mu\nu}n_\mu n_\nu$$

- Solve constraint equations to create initial data
- Evolve metric and extrinsic curvature
- Note: numerically stable evolution requires additional modification to the equations: BSSN, Z4c
- Another approach: generalized harmonics

Evolution equations

Constraints

Projections of the matter fields

Modeling neutron stars

1. Assemble stress-energy tensor of matter and radiation

$$T_{\mu\nu} = T_{\mu\nu}^{(\mathrm{HD})} + T_{\mu\nu}^{(\mathrm{EM})} + T_{\mu\nu}^{(\nu)} + \ldots$$

2. Solve for “conservation” of energy and momentum

$$\nabla_\nu T^{\mu\nu} = 0$$

3. Solve for conservation of baryon and lepton number

$$\nabla_\mu J^\mu = 0 \qquad\qquad u^\mu \nabla_\mu Y_e = R$$

4. Add equation of state and other constitute relations

$$p = p(n, T, Y_e)$$

5. Equations for neutrino radiation and EM fields

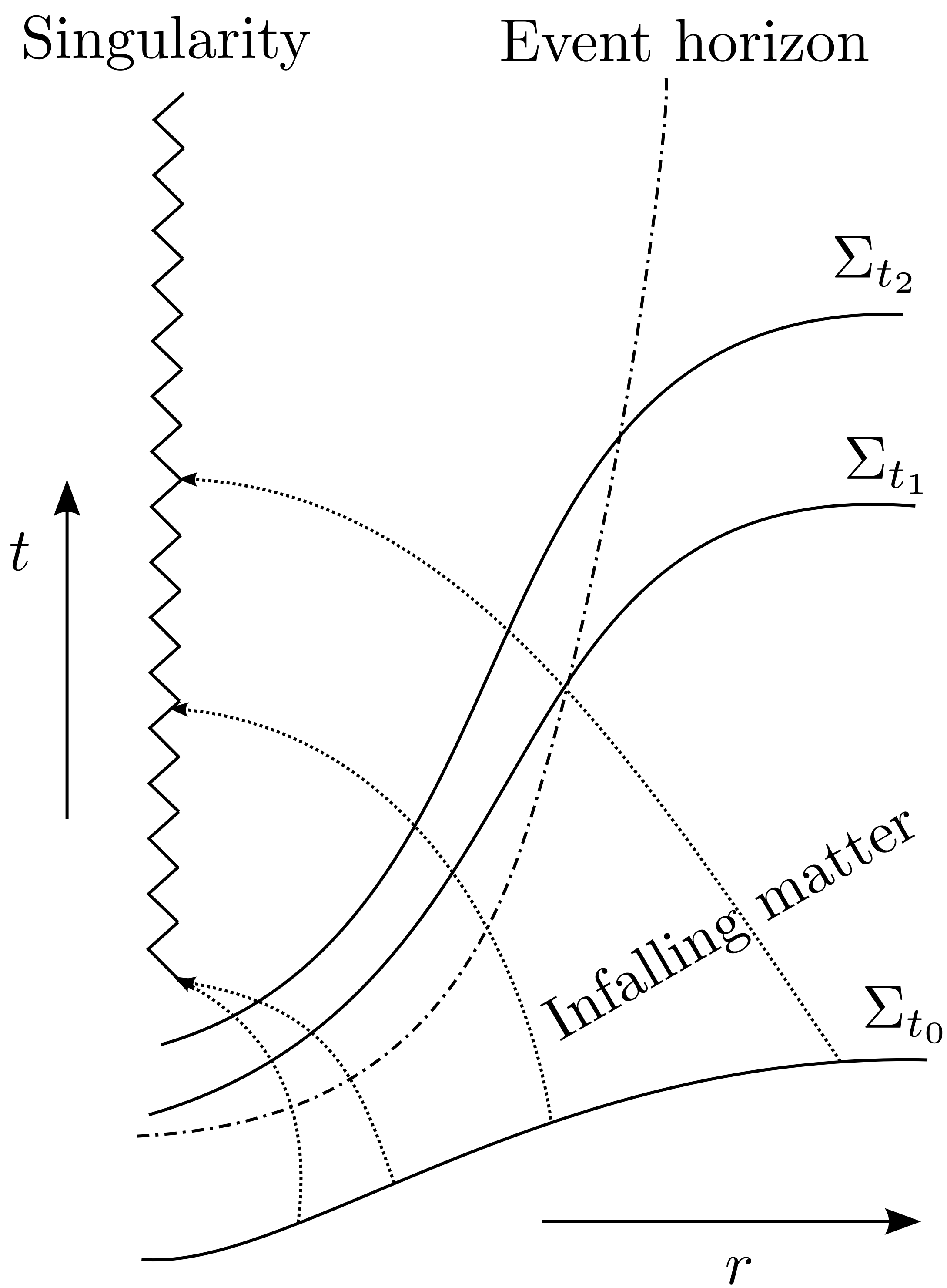
…

Valencia formulation

$$\nabla_\nu T_\mu{}^\nu = 0 \quad \longrightarrow \quad \boxed{\partial_t F_\mu{}^0 + \partial_i F_\mu{}^i = S_\mu}$$

- Projection of $\nabla_\nu T_\mu{}^\nu = 0$ on the space hypersurface
- Equations similar to the classical hydrodynamics equations
- Discretize with standard high-resolution shock-capturing (HRSC) schemes
- Note S_μ is in general not zero! Balance laws.

Slicing freedom



BH formation cartoon

Einstein equations do not tell us how to traverse the spacetime: we need to choose a slicing condition

Requirements:

- smoothness
- minimize grid distortion
- avoid singularities

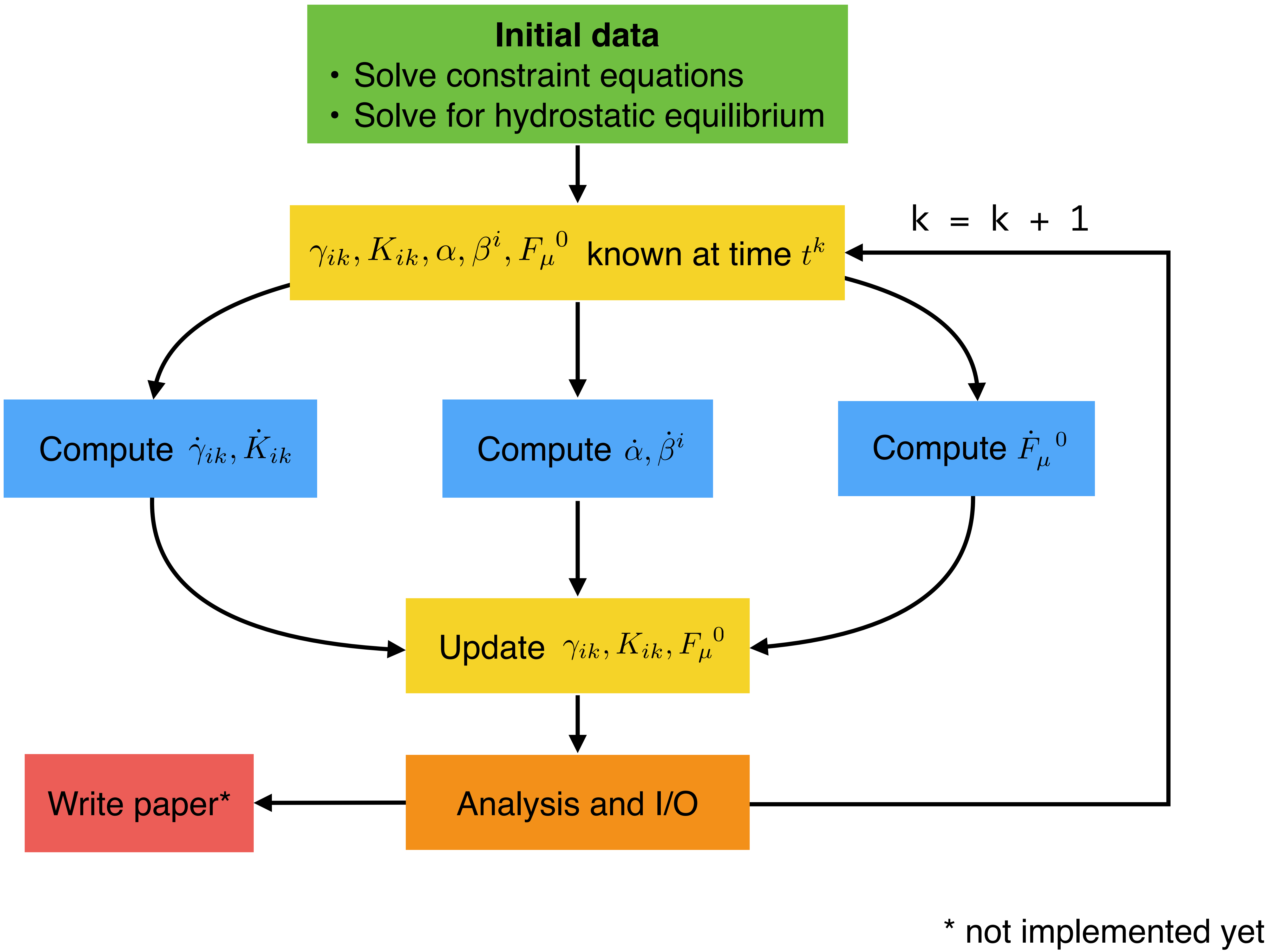
Two families that work in practice

- “puncture” gauge
- generalized harmonic

Moving puncture gauge

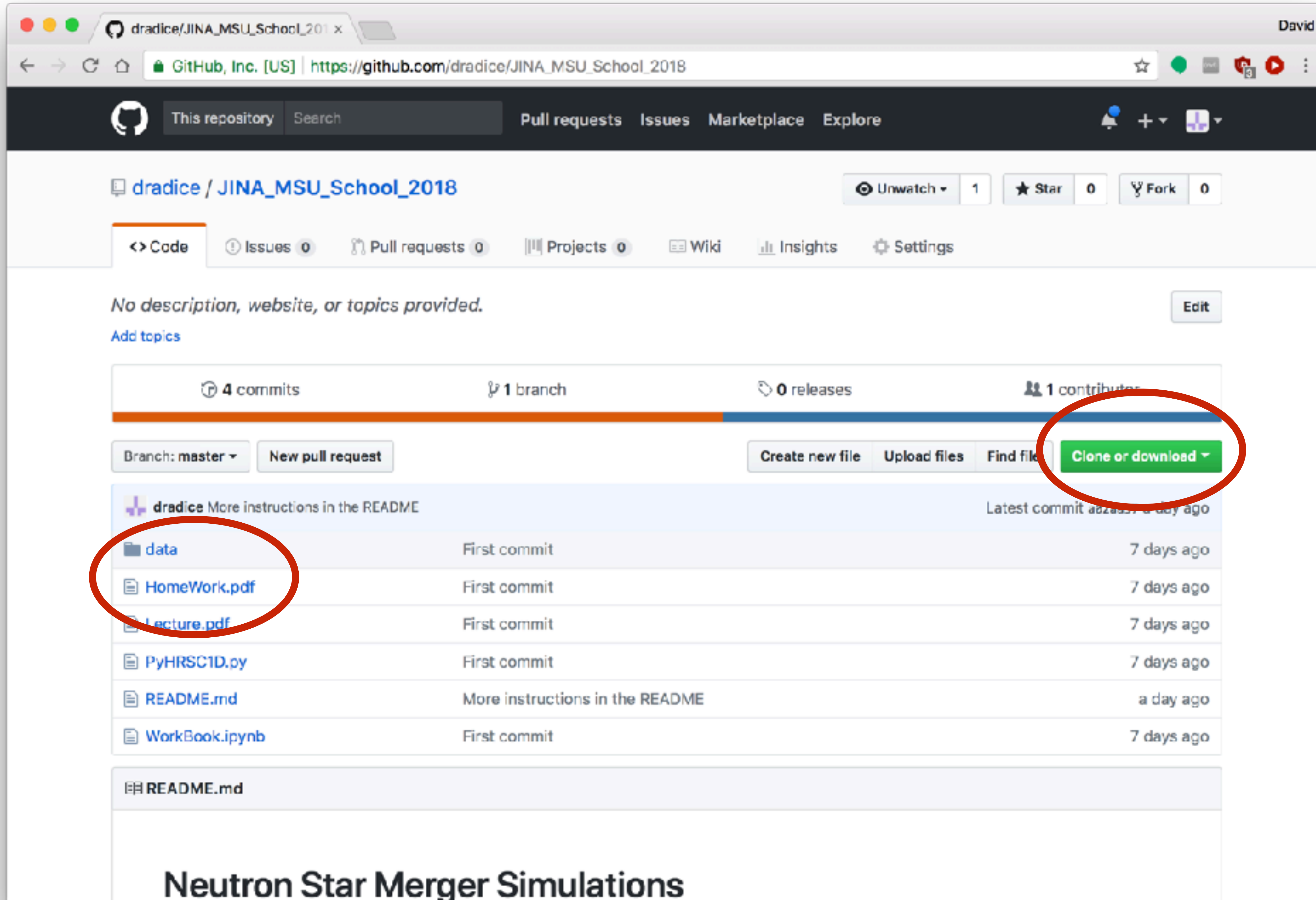
$$(\partial_t + \beta^i \partial_i)\alpha = -2\alpha K$$

- No caustics can be formed ($K = -\nabla_\mu n^\mu$)
- Singularity avoidance: the lapse drops to zero before the foliation hits a singularity (diverging *K*)
- If there is a time symmetry, the slicing tries to adapt to it
- Advection with β^i allows black-holes to move through the grid



Initial data
• Solve constraint equations
• Solve for hydrostatic equilibrium
$\gamma_{ik}, K_{ik}, \alpha, \beta^i, {F_\mu}^0$ known at time t^k
k = k + 1
Compute $\dot{\gamma}_{ik}, \dot{K}_{ik}$
Compute $\dot{\alpha}, \dot{\beta}^i$
Compute $\dot{F_\mu}^0$
Update $\gamma_{ik}, K_{ik}, {F_\mu}^0$
Write paper*
Analysis and I/O
* not implemented yet

https://github.com/dradice/JINA_MSU_School_2018.git



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HomeWork.pdf (page 1 of 4)

Relativistic Hydrodynamics: Fundamentals

David Radice

May 4, 2018

Introduction

The goal of this exercise sheet is to become familiar with the equations of relativistic hydrodynamics and their basic properties, and to learn about the most important building blocks required to assemble the simulations codes used in numerical relativity. We will write a small python code that can solve the equations of special-relativistic hydrodynamics in 1D (1+1) starting from a toy code for non-relativistic gasdynamics. We will use units where $c = 1$ throughout these notes.

Basic equations

The special relativistic hydrodynamics equation describe the conservation of energy and momentum of a fluid:

$$\partial_\nu T^{\mu\nu} = 0, \tag{1}$$

where we have used Einstein's convention of sum over repeated indices. We consider a perfect fluid, i.e., we neglect viscosity and heat transport inside of the fluid so that the stress energy tensor reads

$$T^{\mu\nu} = (\rho + p)\, u^\mu u^\nu + p\, \eta^{\mu\nu}\,, \tag{2}$$

where ρ is the energy density of the fluid measured in a comoving frame, p is the pressure, u^μ the fluid four-velocity and $\eta = \mathrm{diag}(-1, 1, 1, 1)$ is the metric of Minkowsky spacetime. We remind that the four-velocity can be written as

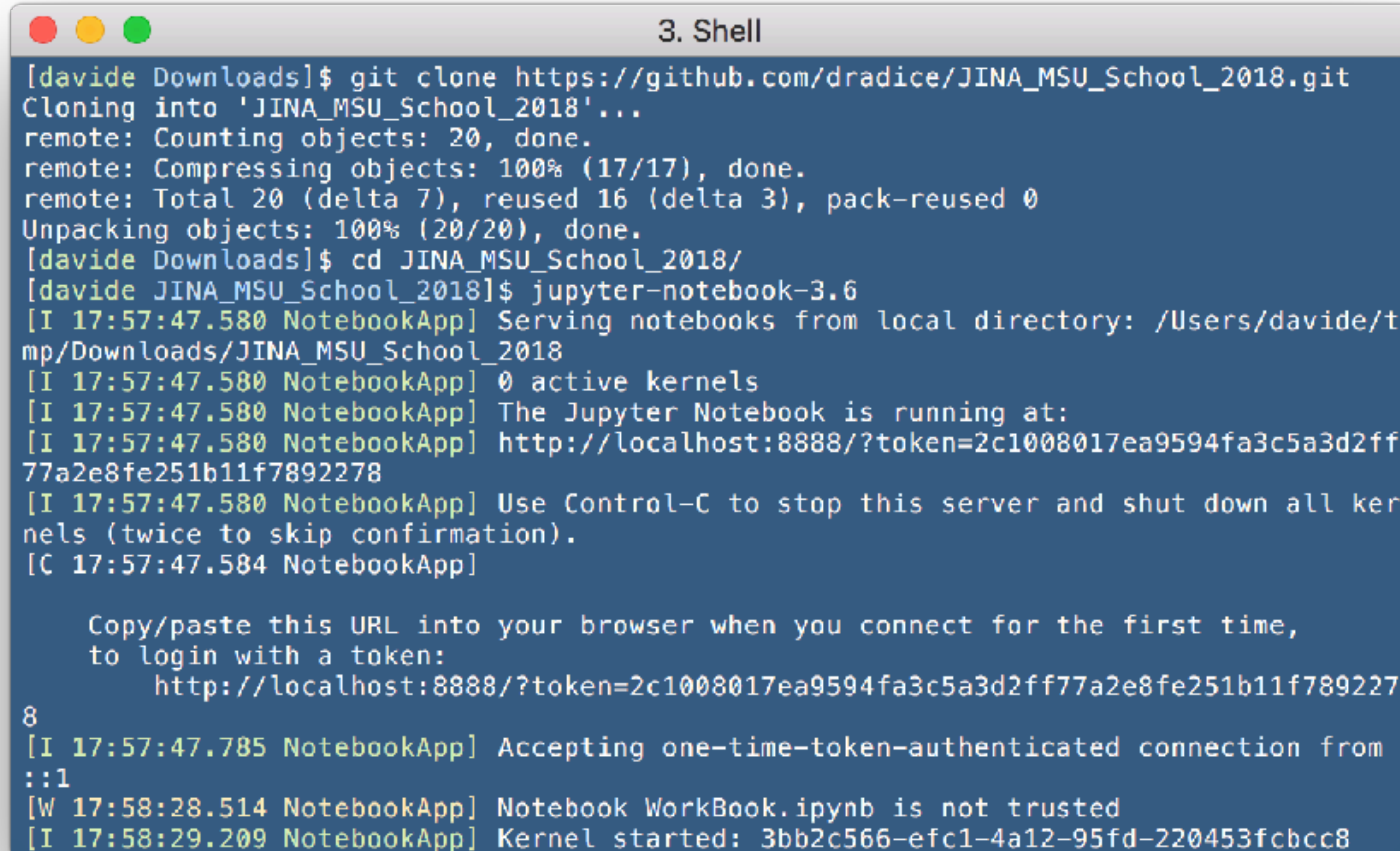
$$u^\mu = W(1, v^i)\,, \tag{3}$$

Where

$$W = (1 - \eta_{ij} v^i v^j)^{-1/2} \tag{4}$$

is the Lorentz factor and v^i the spatial velocity.

https://github.com/dradice/JINA_MSU_School_2018.git



3. Shell

```
[davide Downloads]$ git clone https://github.com/dradice/JINA_MSU_School_2018.git
Cloning into 'JINA_MSU_School_2018'...
remote: Counting objects: 20, done.
remote: Compressing objects: 100% (17/17), done.
remote: Total 20 (delta 7), reused 16 (delta 3), pack-reused 0
Unpacking objects: 100% (20/20), done.
[davide Downloads]$ cd JINA_MSU_School_2018/
[davide JINA_MSU_School_2018]$ jupyter-notebook-3.6
[I 17:57:47.580 NotebookApp] Serving notebooks from local directory: /Users/davide/t
mp/Downloads/JINA_MSU_School_2018
[I 17:57:47.580 NotebookApp] 0 active kernels
[I 17:57:47.580 NotebookApp] The Jupyter Notebook is running at:
[I 17:57:47.580 NotebookApp] http://localhost:8888/?token=2c1008017ea9594fa3c5a3d2ff
77a2e8fe251b11f7892278
[I 17:57:47.580 NotebookApp] Use Control-C to stop this server and shut down all ker
nels (twice to skip confirmation).
[C 17:57:47.584 NotebookApp]

    Copy/paste this URL into your browser when you connect for the first time,
    to login with a token:
        http://localhost:8888/?token=2c1008017ea9594fa3c5a3d2ff77a2e8fe251b11f789227
8
[I 17:57:47.785 NotebookApp] Accepting one-time-token-authenticated connection from
::1
[W 17:58:28.514 NotebookApp] Notebook WorkBook.ipynb is not trusted
[I 17:58:29.209 NotebookApp] Kernel started: 3bb2c566-efc1-4a12-95fd-220453fcbcc8
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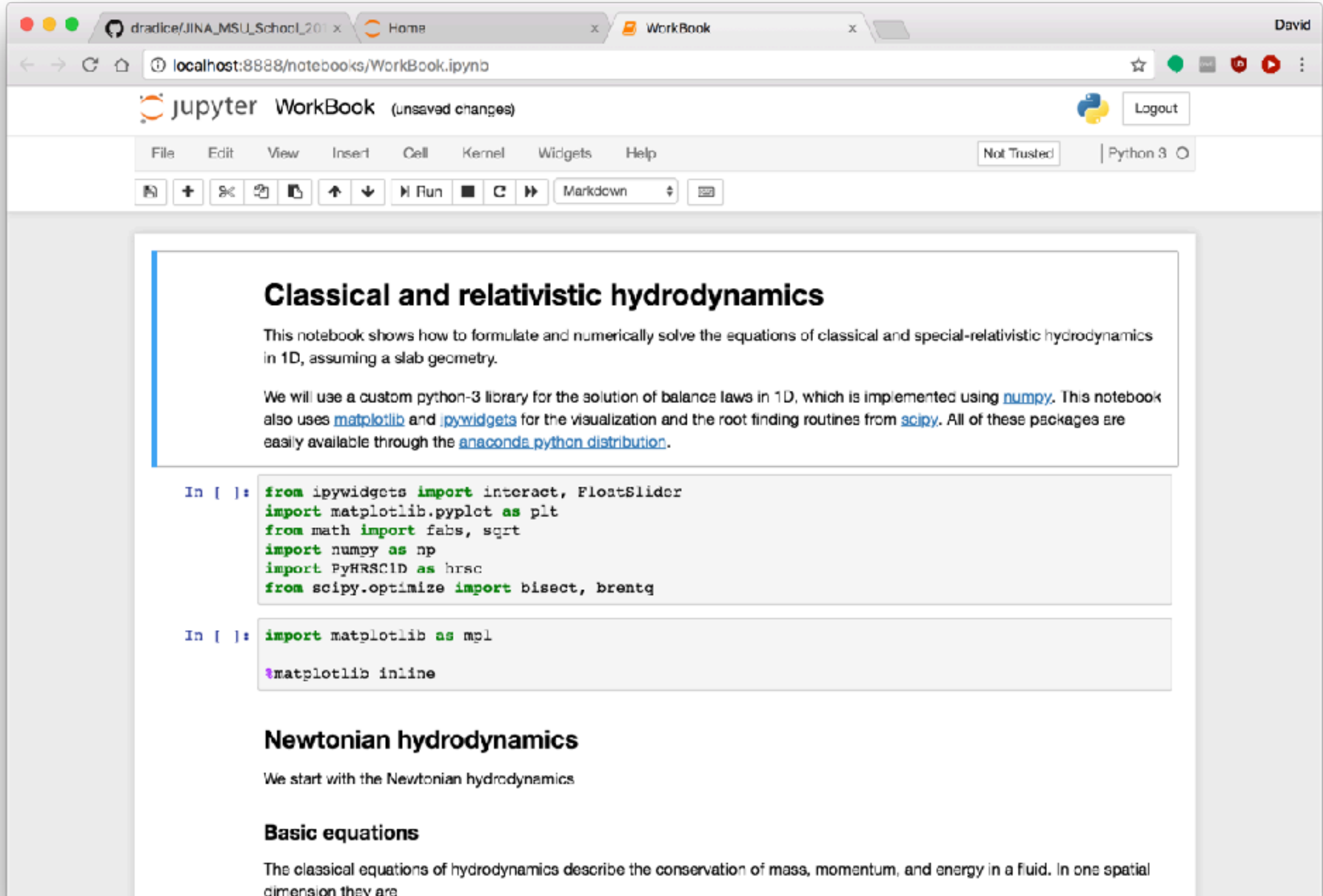
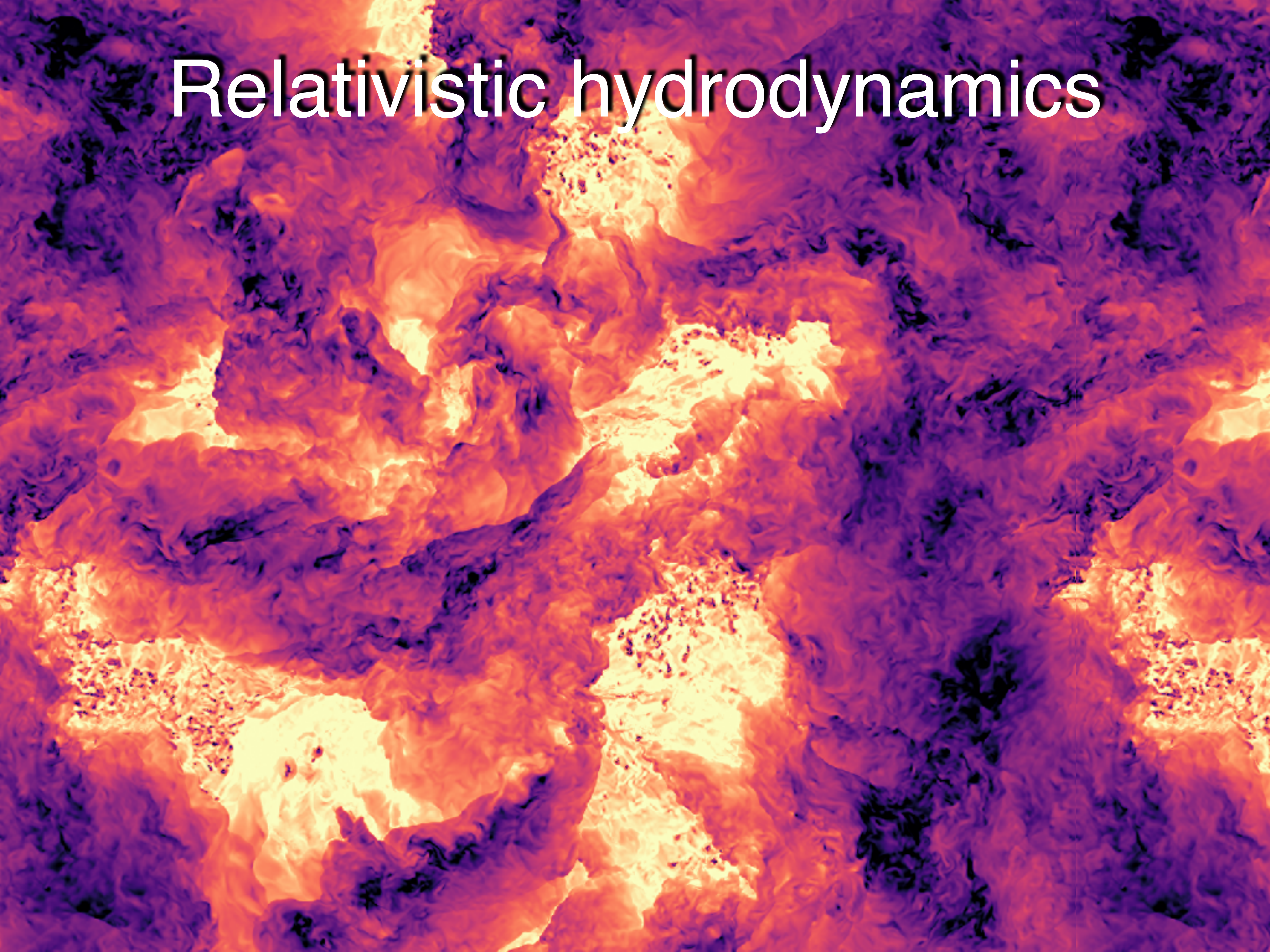
https://github.com/dradice/JINA_MSU_School_2018.git



Relativistic hydrodynamics



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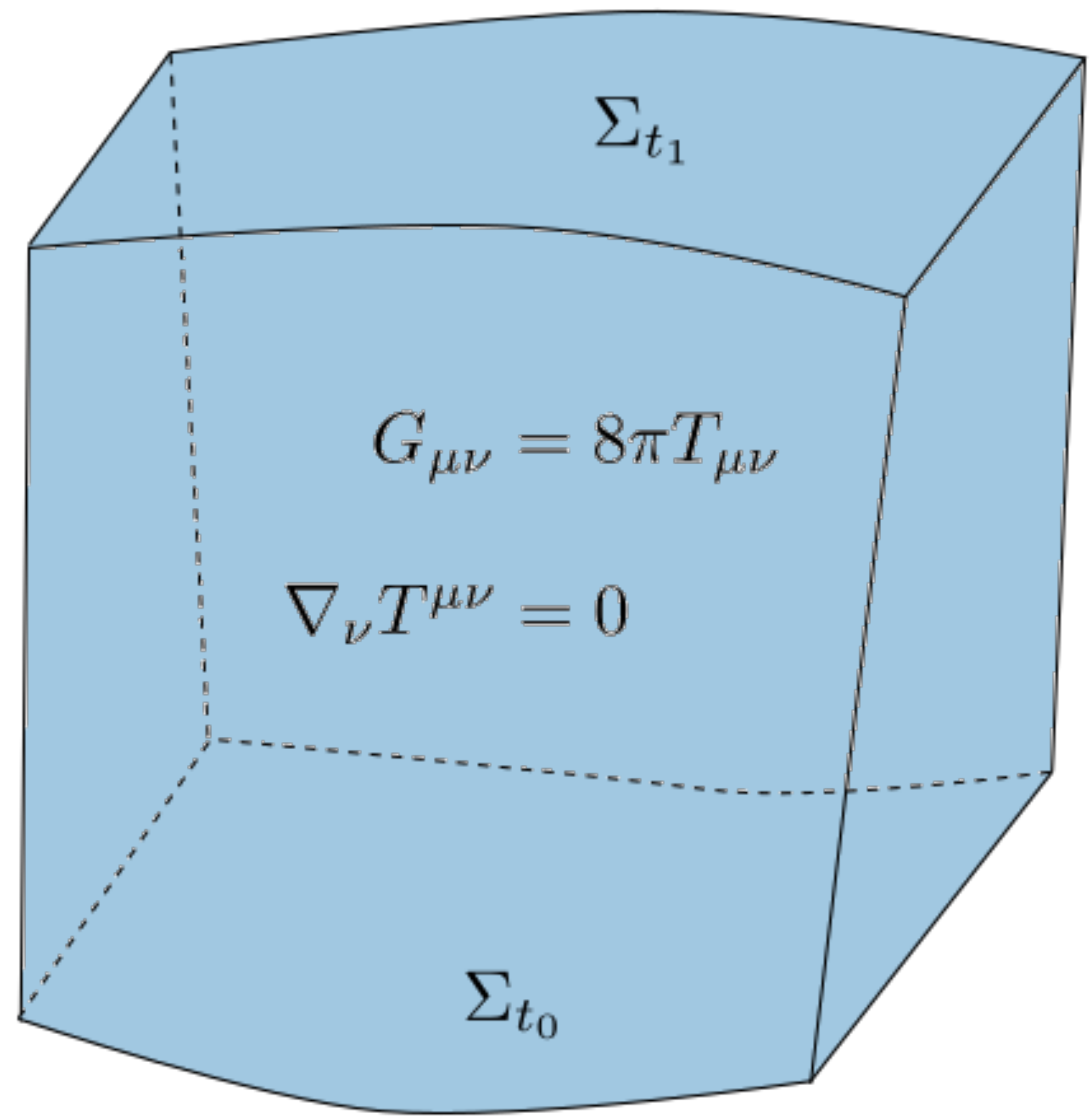
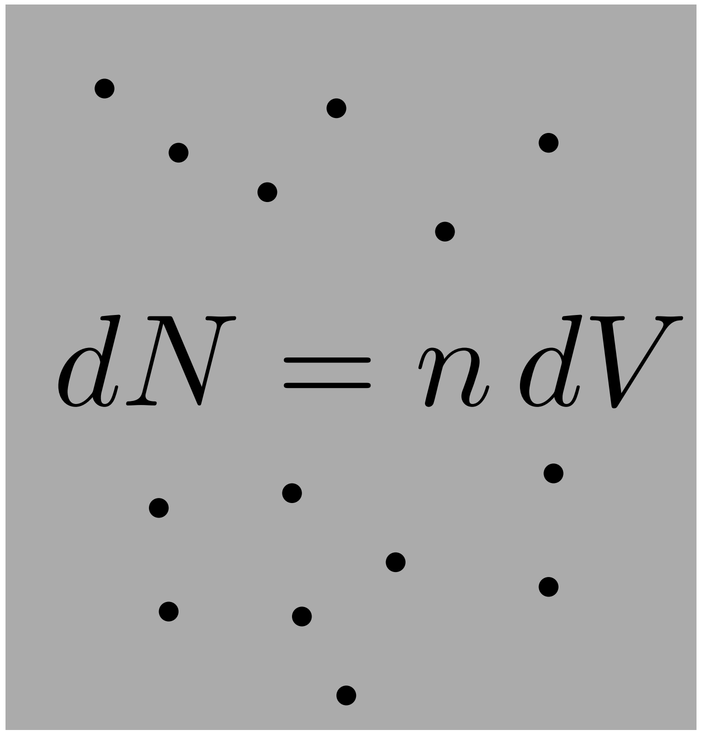


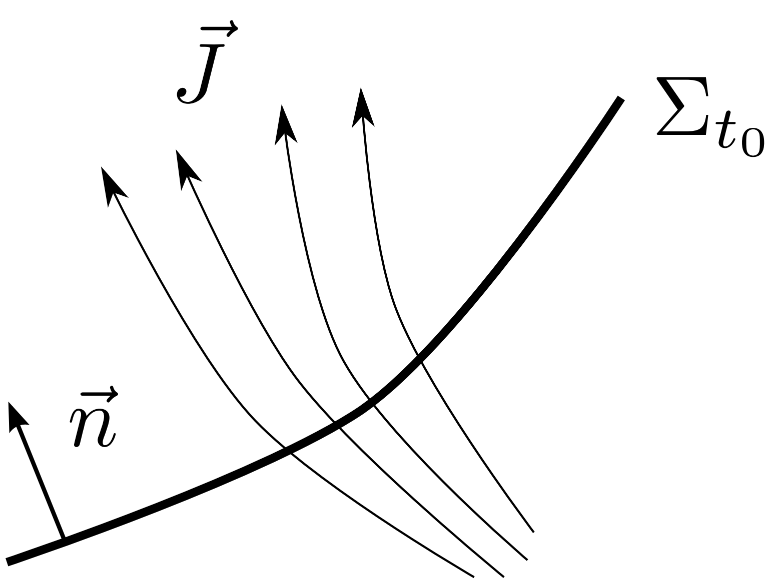
Note G = c = 1 for us!

Relativistic hydrodynamics



Classical picture

- Intrinsic variable: particle number density
- Equations describe the flux of matter through space as a function of time

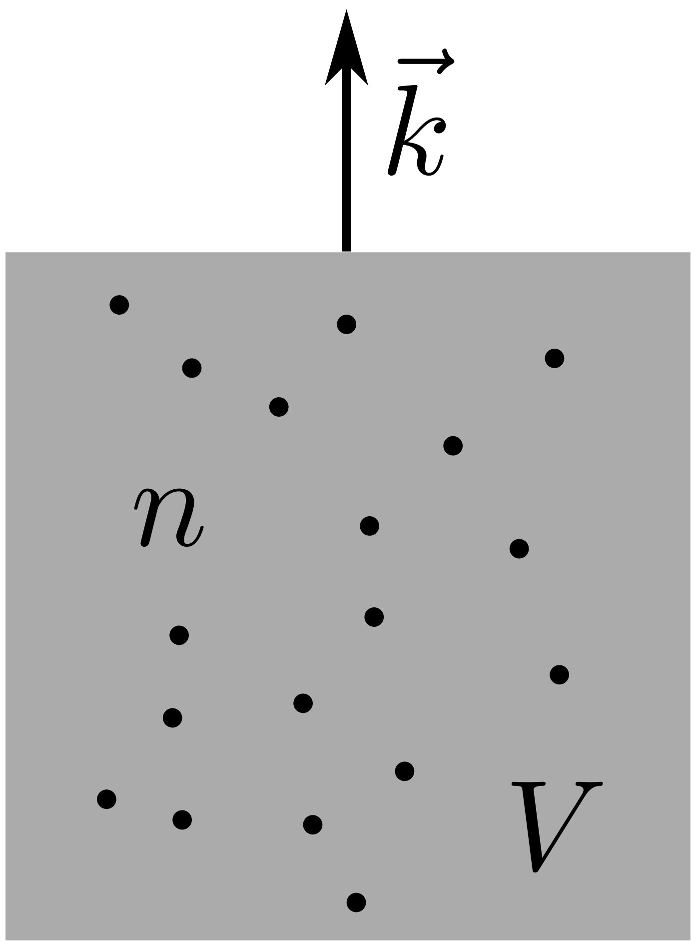


$$dN = -\vec{n} \cdot \vec{J}\,dV$$

Relativistic picture

- Intrinsic variable: flux of particles through spacetime
- Equations describe the density as measured by any observer

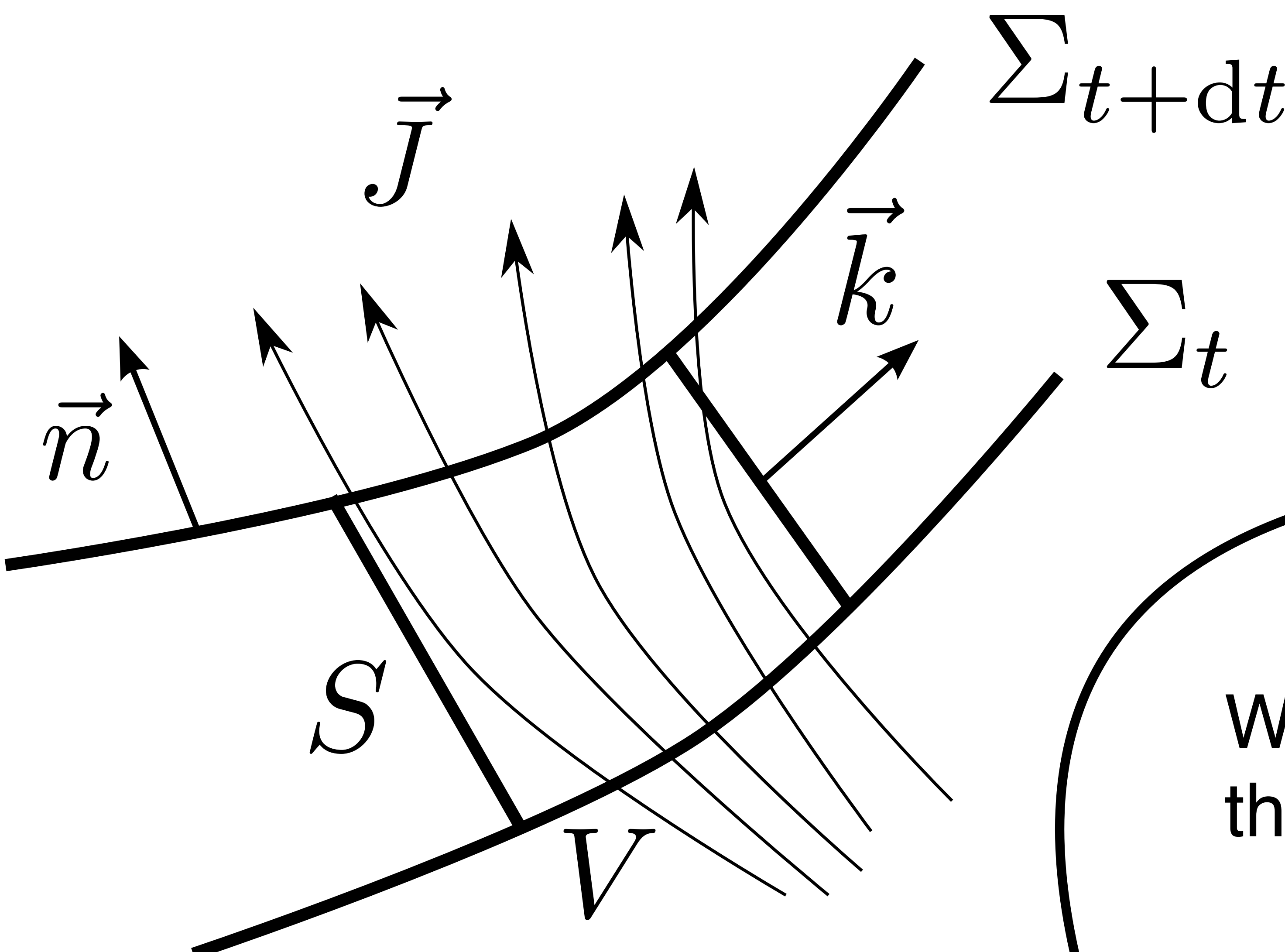
Classical continuity equation



$$\underbrace{\frac{\mathrm{d}}{\mathrm{d}t}\int_V n\,\mathrm{d}V}_{\text{Number of particles in a control volume}} + \underbrace{\int_{\partial V} n\vec{v}\cdot\vec{k}\,\mathrm{d}S}_{\text{Flux of particles across the boundaries of the control volume}} = 0$$

Differential form: $\boxed{\partial_t n + \nabla_i (n v^i) = 0}$

Relativistic continuity equation

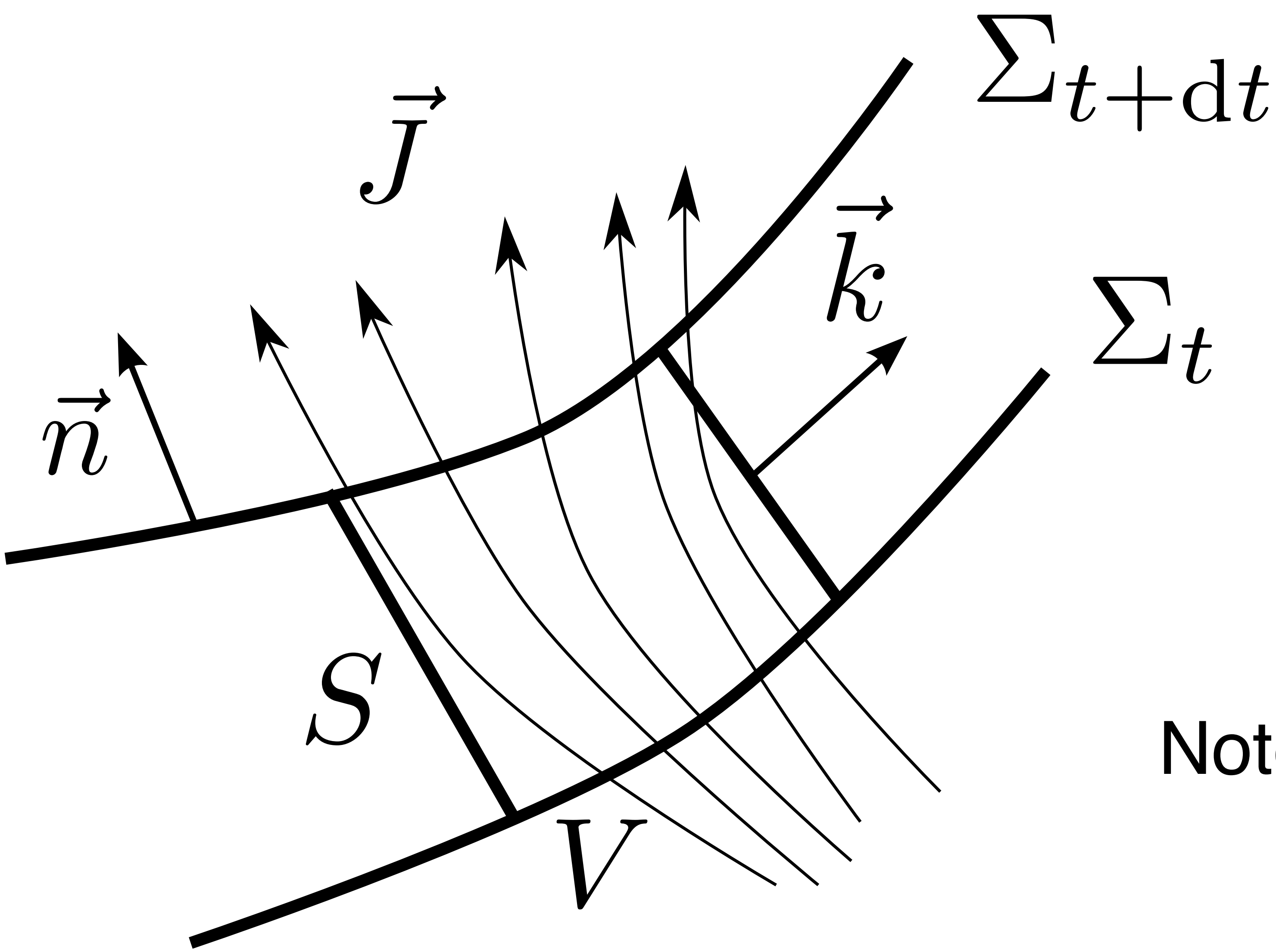


World lines passing through control volume

World lines leaving / entering through the time-like boundary

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \vec{J}\cdot\vec{n}\,\mathrm{d}V + \int_S \vec{J}\cdot\vec{k}\,\mathrm{d}S = 0$$

Relativistic continuity equation



4-velocity

Number current

$$J^\mu = n u^\mu$$

Note: $$-J^\mu u_\mu = n$$

Differential form: $$\boxed{\nabla_\mu J^\mu = 0}$$

Stress-energy tensor

The number current of a fluid is such that

$$-J_\mu n^\mu = \text{“Number density measured by a fiducial observer”}$$

In analogy the stress-energy tensor of a fluid

$$T_{\mu\nu} n^\mu n^\nu = \text{“Energy density measured by a fiducial observer”}$$

$$-T_{\mu\nu} n^\mu (\partial_i)^\nu = \text{“Energy flux measured by a fiducial observer”}$$

$$T_{\mu\nu} (\partial_i)^\mu (\partial_j)^\nu = \Sigma_{ij} = \text{“Stress forces measured by a fiducial observer”}$$

Where we have chosen $\{\vec{n}, \partial_i\}$ such that

$$n^\mu (\partial_i)_\mu = 0$$

Modeling neutron stars

1. Assemble stress-energy tensor of matter and radiation

$$T_{\mu\nu} = T_{\mu\nu}^{(\mathrm{HD})} + T_{\mu\nu}^{(\mathrm{EM})} + T_{\mu\nu}^{(\nu)} + \ldots$$

2. Solve for “conservation” of energy and momentum

$$\nabla_\nu T^{\mu\nu} = 0$$

3. Solve for conservation of baryon and lepton number

$$\nabla_\mu J^\mu = 0 \qquad\qquad u^\mu \nabla_\mu Y_e = R$$

4. Add equation of state and other constitute relations

$$p = p(n, T, Y_e)$$

5. Equations for neutrino radiation and EM fields

…

Perfect fluid

It is easy to write the stress energy tensor for NS matter in a locally-flat comoving frame $\{\vec{u}, \partial_i\}$ assuming a perfect fluid

We impose the following condition

$$T_{\mu\nu} u^\mu u^\nu = \rho$$ Energy density of the fluid

$$T_{\mu\nu} u^\mu (\partial_i)^\nu = 0$$ No heat conduction

$$T_{\mu\nu} (\partial_i)^\mu (\partial_j)^\nu = p\delta_{ij}$$ Only isotropic pressure

In this case the stress energy tensor takes the form

$$T_{\mu\nu} = \begin{pmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix} \quad u^\mu = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \xrightarrow{\text{General frame}} \begin{aligned} T_{\mu\nu} &= \rho u^\mu u^\nu + p \perp^{\mu\nu} \\ \perp_{\mu\nu} &= g_{\mu\nu} + u_\mu u_\nu \end{aligned}$$

“Conservation” laws (I)

The key part of the equations of motion is

$$\nabla_\nu T^{\mu\nu} = 0$$

what is really conserved?

“Conservation” laws (II)

Multiply $\nabla_\nu T^{\mu\nu} = 0$ by any smooth vector field p^μ to get

$$\nabla_\mu \left(T^{\mu\nu} p_\nu \right) = -T^{\mu\nu} \nabla_\mu p_\nu = -\frac{1}{2} T^{\mu\nu} \underbrace{\left(\nabla_\mu p_\nu + \nabla_\nu p_\mu \right)}_{= 0 \text{ for Killing vectors}}$$

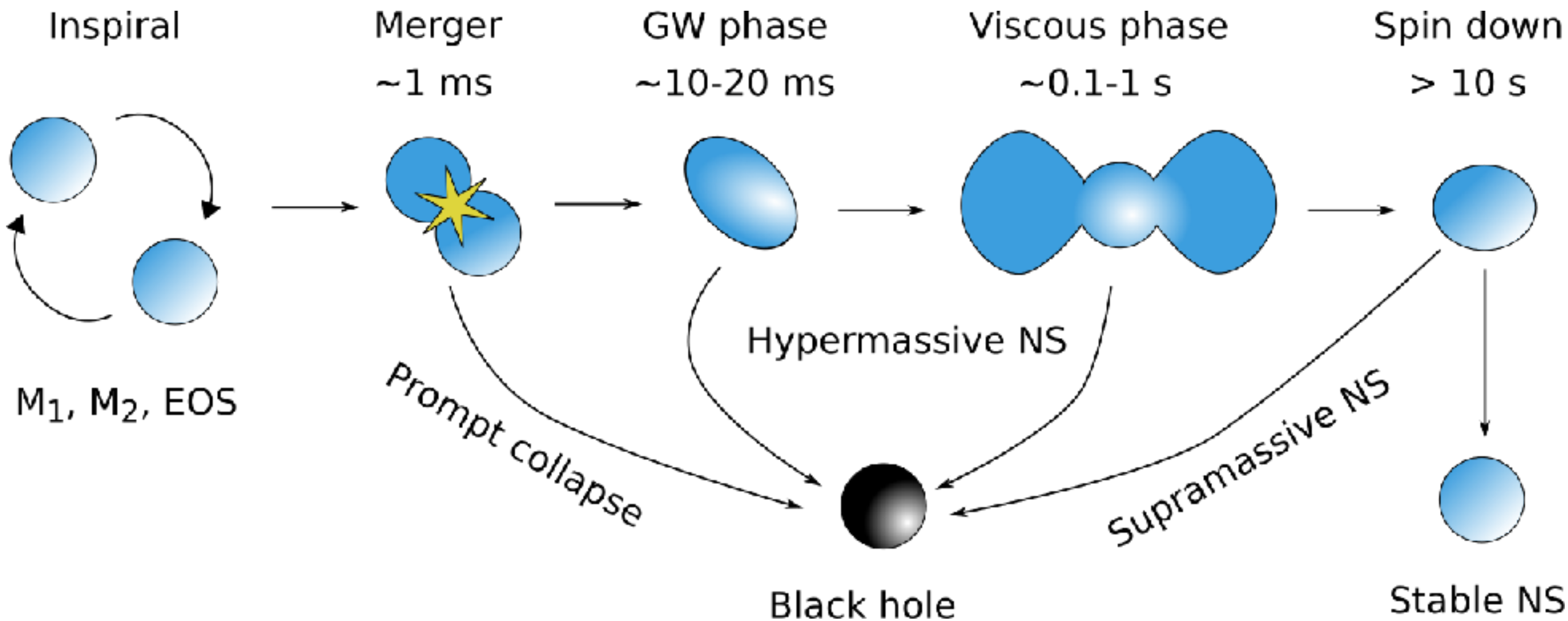
$$T^{\mu\nu} = T^{\nu\mu} = \frac{1}{2}\left(T^{\mu\nu} + T^{\nu\mu} \right)$$

Killing vectors are direction of symmetry for the metric

$$0 = \nabla_\mu p_\nu + \nabla_\nu p_\mu \implies \mathcal{L}_{\vec{p}}\, g_{\mu\nu} = 0$$

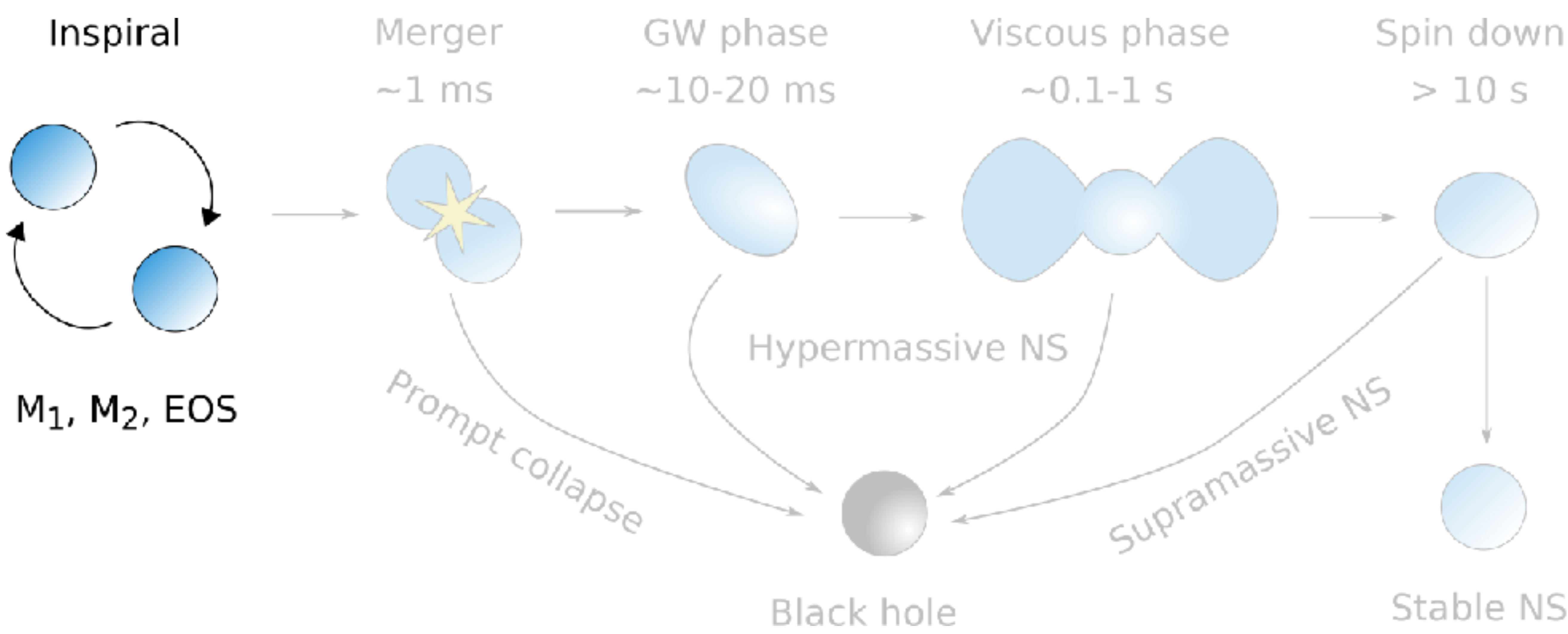
Noether’s theorem:
Spacetime symmetries generate conserved charges

p^μ Killing vector $\implies$ $C^\mu = T^\mu{}_\nu p^\nu$ conserved

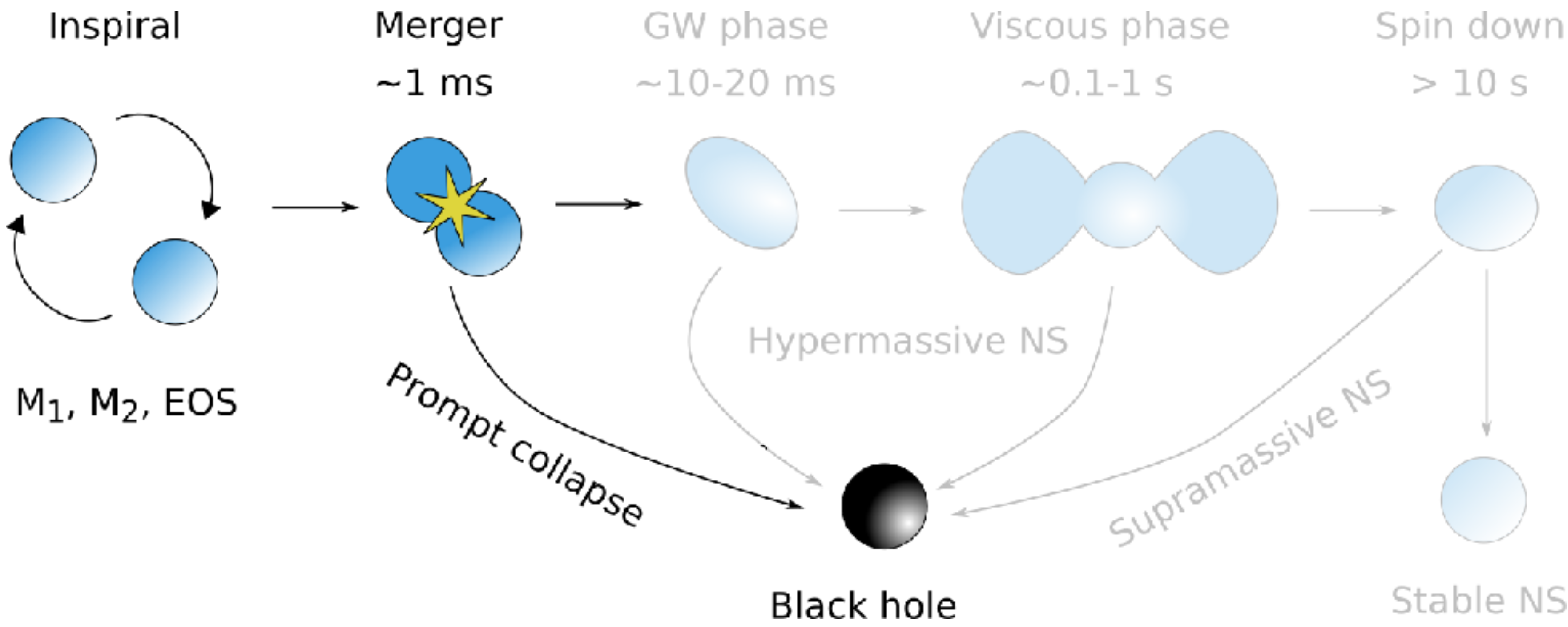


Neutron star mergers phases and possible outcomes

The evolution and outcome of mergers depend on the NS masses, the NS EOS, and (possibly) spins.

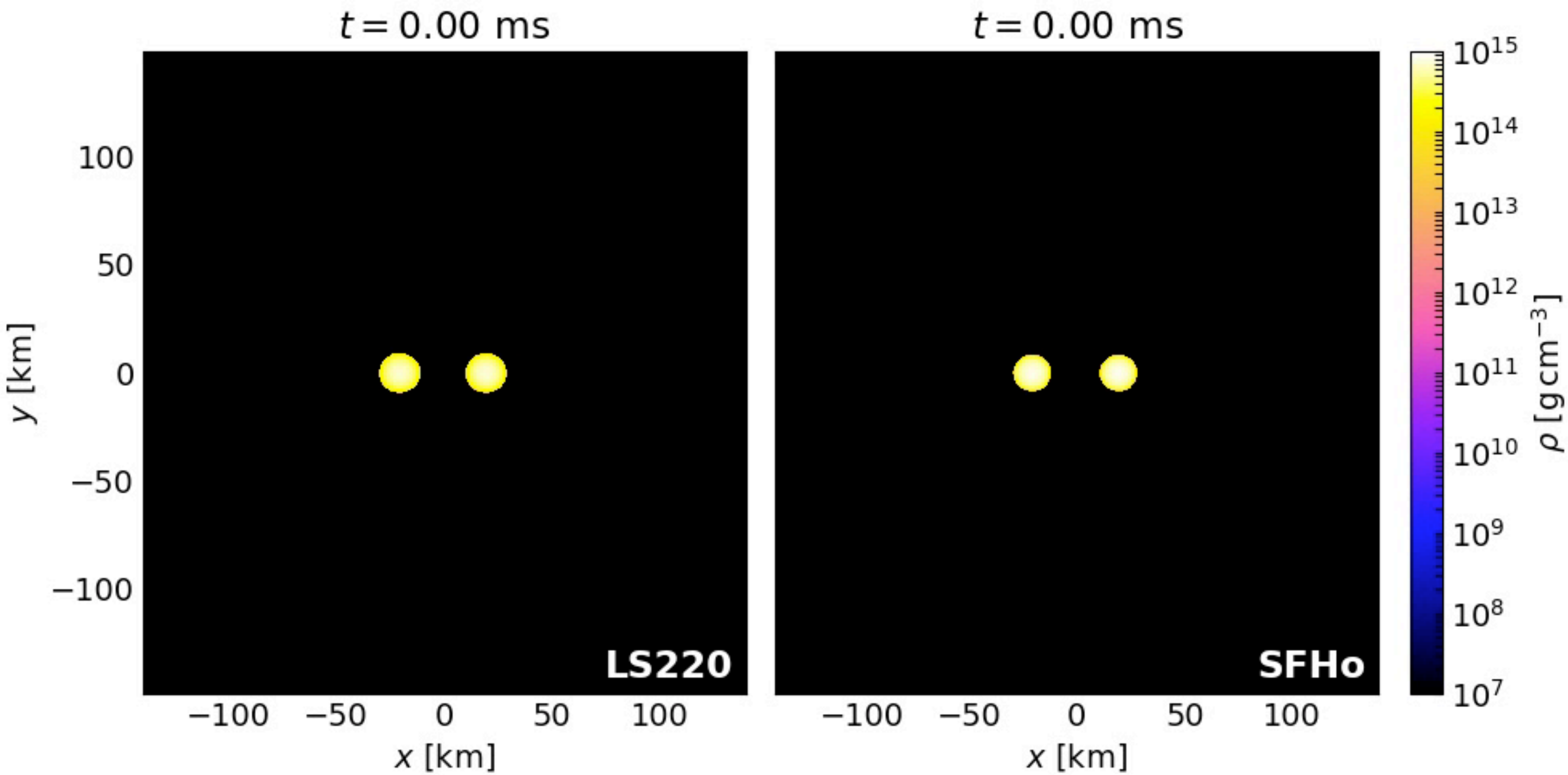


- The binary is in the LIGO/Virgo frequency band in the last few minutes of the inspiral ($O(10^3)$ orbits).
- Tidal effects in this phase can be used to constrain the EOS of NSs
- Best understood phase of the binary evolution



- Merger is the time when the GW signal reaches the maximum amplitude. Marks the end of the inspiral.
- If the total binary mass is larger than (1.3 — 1.6) M_{max} then a BH is formed promptly during merger.
- Prompt BH collapse leaves little debris: probably EM-quiet

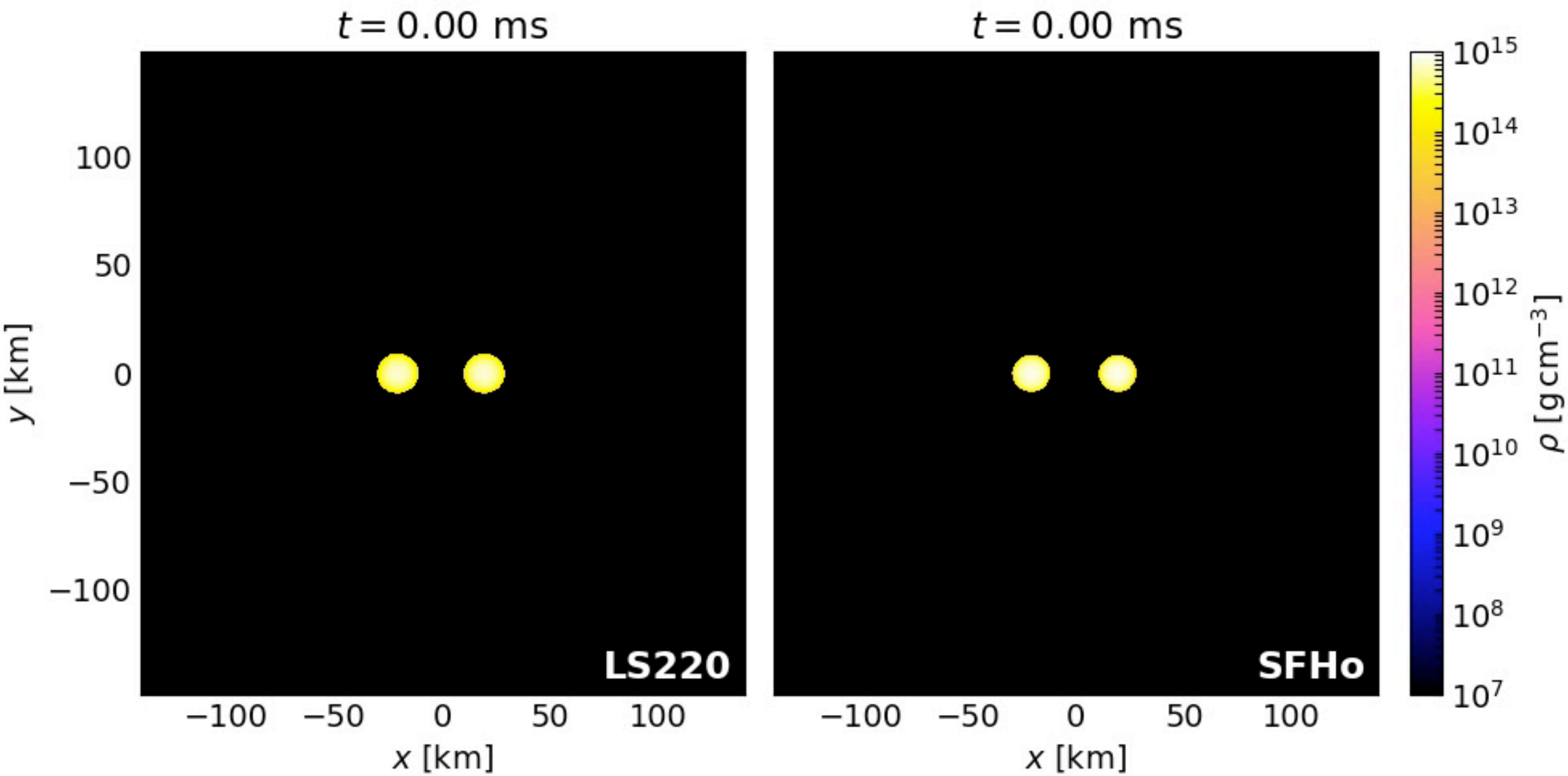
Prompt BH formation



$(1.44 + 1.39)\ M_\odot$ — B1913 + 13

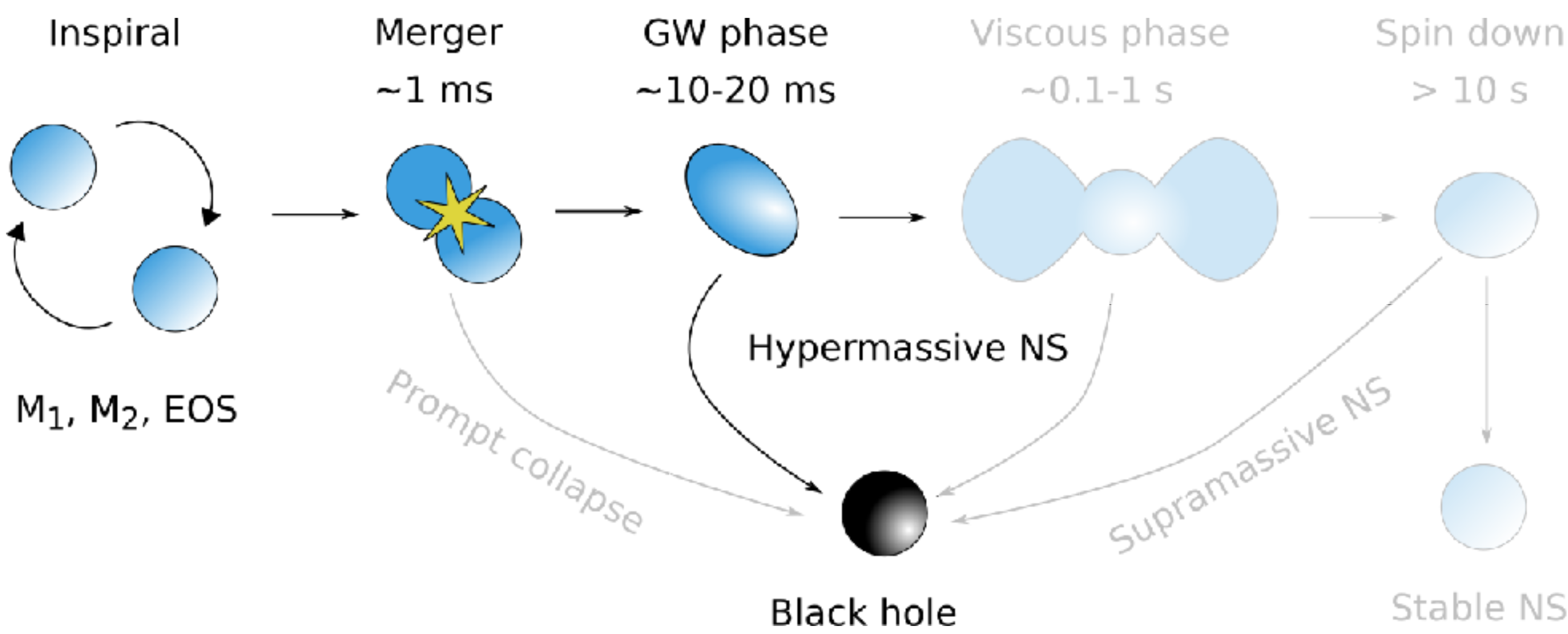
DR, Perego, Zappa, ApJL 852:L29 (2018)

Prompt BH formation



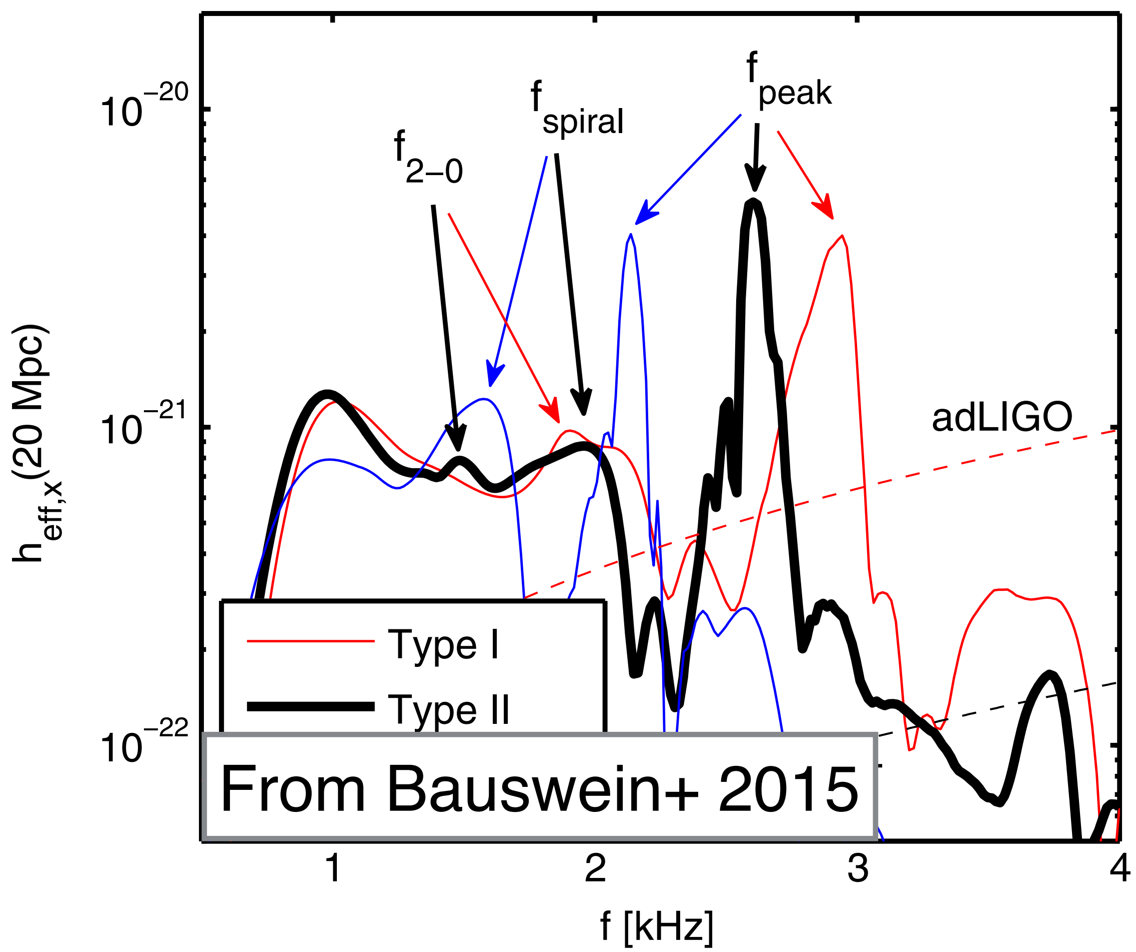
$(1.44 + 1.39)\ M_\odot$ — B1913 + 13

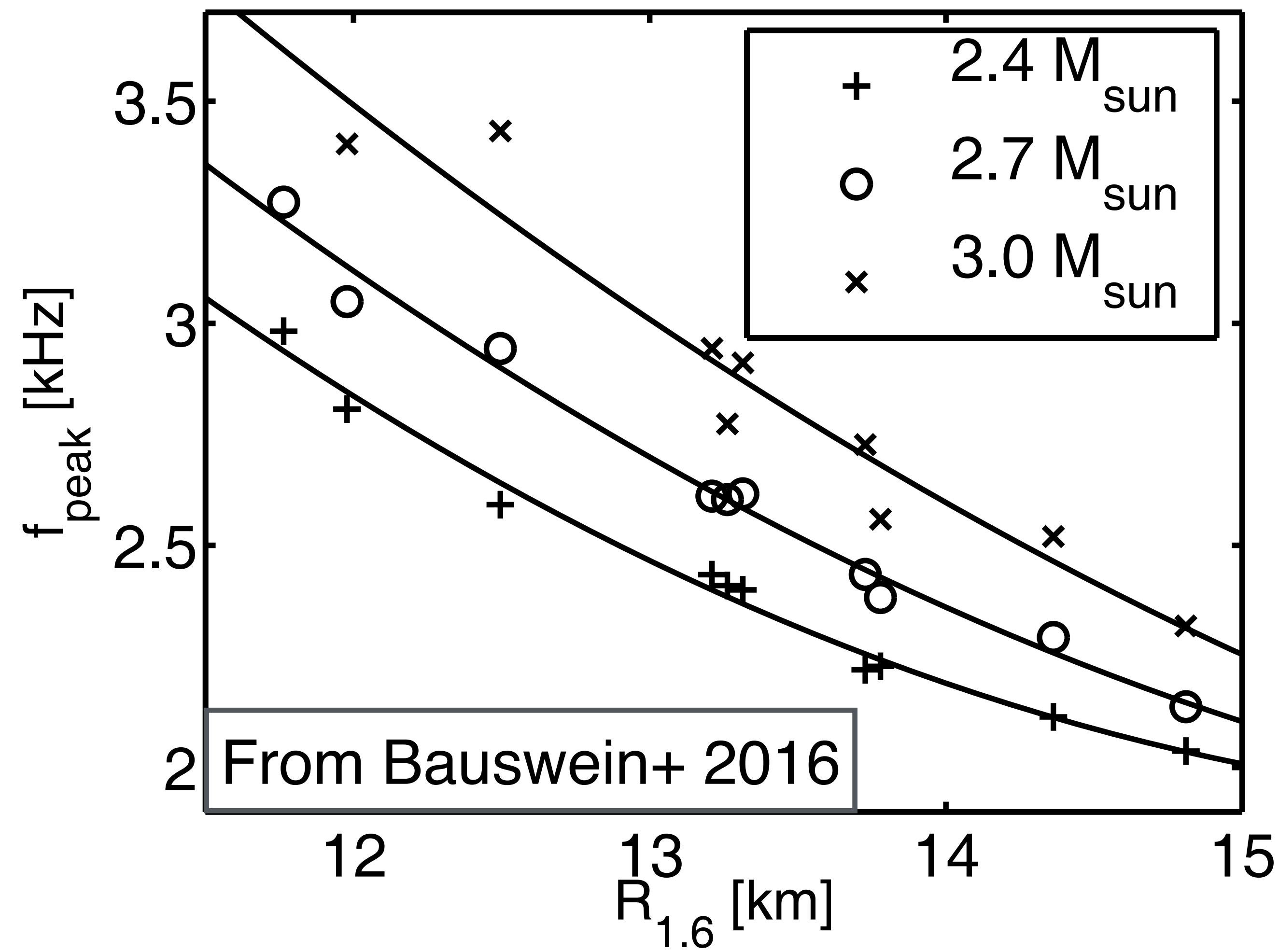
DR, Perego, Zappa, ApJL 852:L29 (2018)



- The merger remnant is highly deformed and radiates GWs
- Most luminous phase of the evolution, but outside of LIGO band
- Postmerger GW could also constrain the NS EOS
- GW losses decay within ~10-20 ms
- The merger remnant might collapse because of the loss of angular momentum during this phase (hypermassive NS)

Postmerger GWs signal (I)

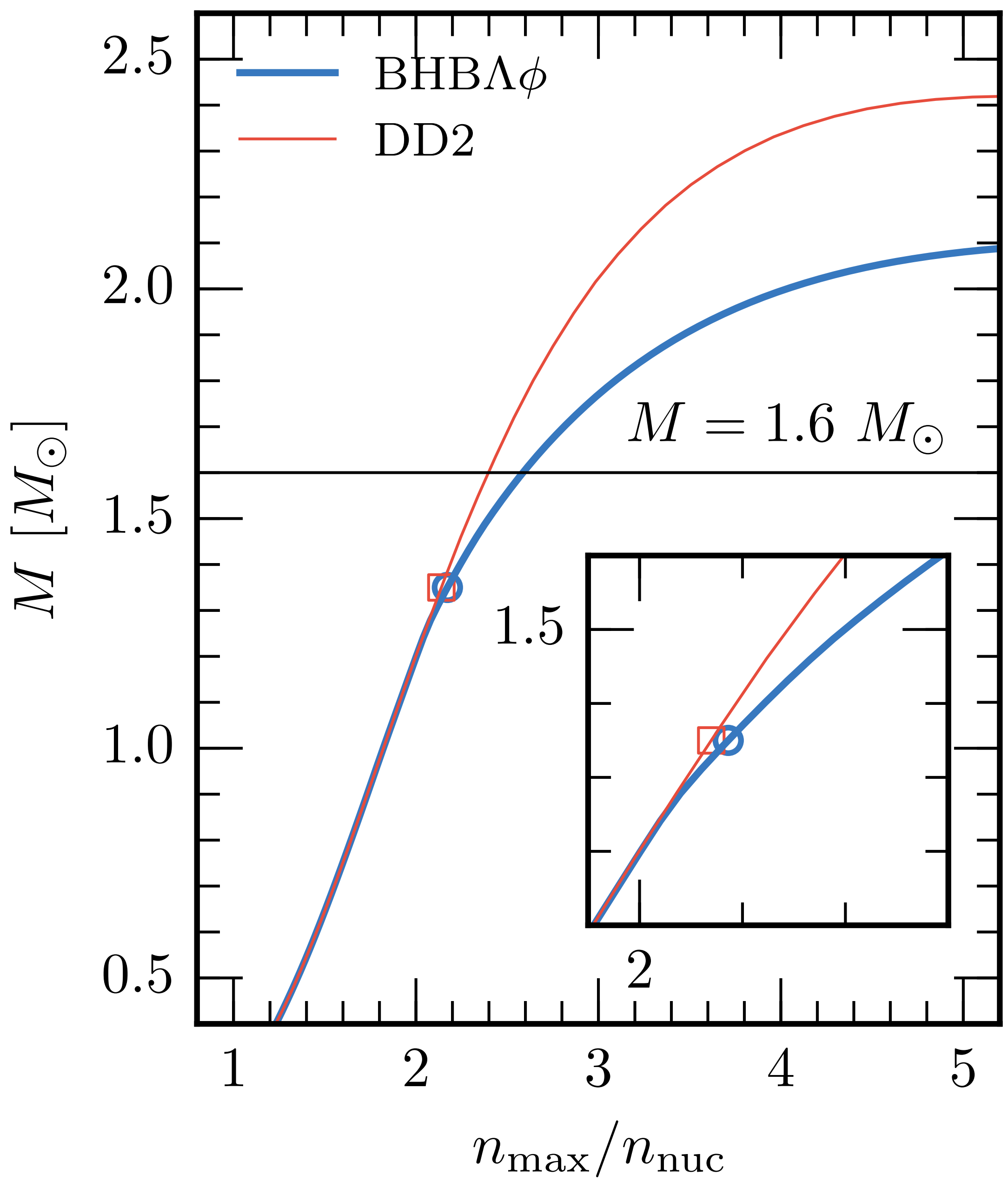


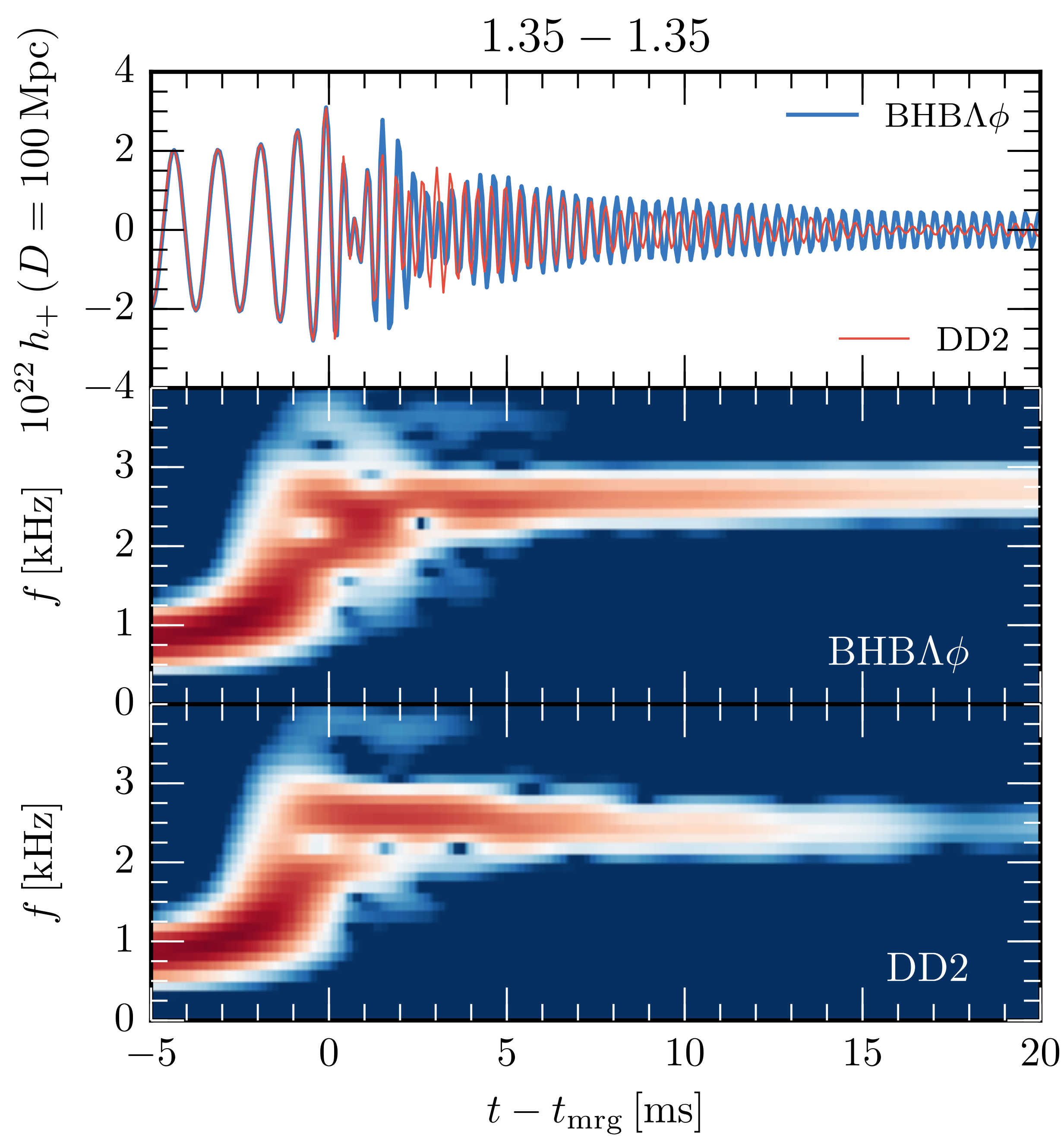


- Post-merger signal has a characteristic peak frequency
- f_{peak} correlates with the NS radius
- Small statistical uncertainty, systematics not understood yet

See also Takami+ 2014; Rezzolla & Takami 2016; Dietrich+ 2016; Bose+ 2017

Postmerger GW signal (II)

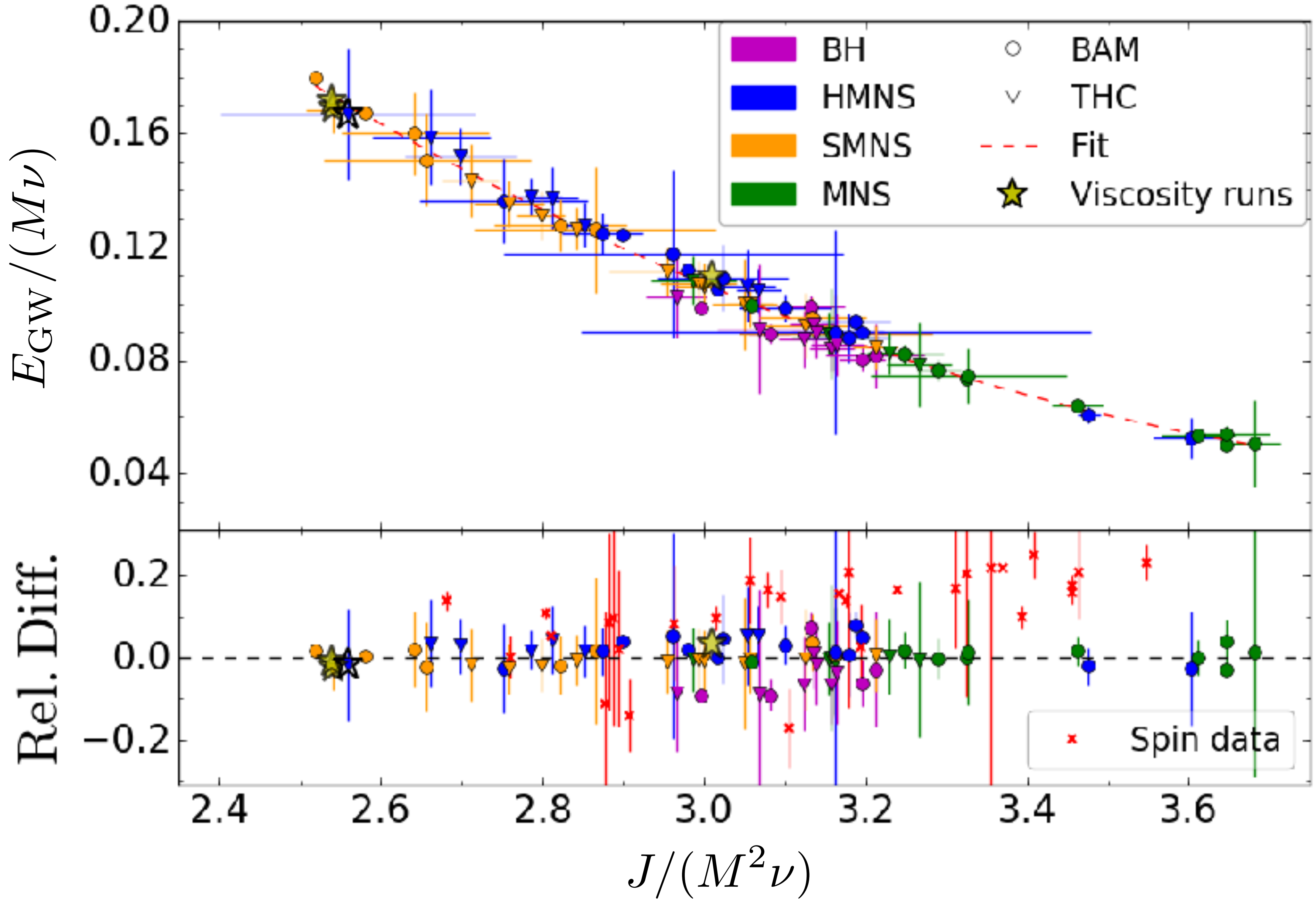




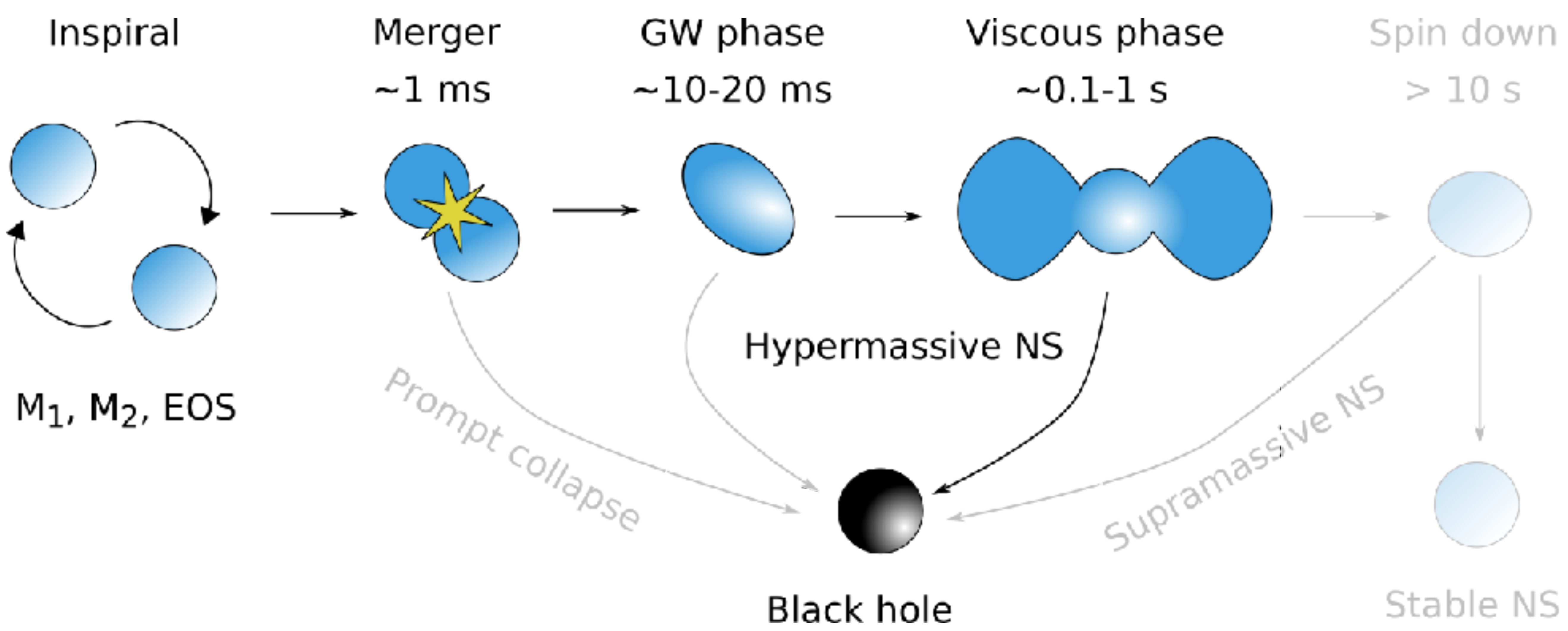
The GW amplitude could reveal the outcome of the merger and constraint the properties of matter at extreme densities

DR, Bernuzzi, Del Pozzo+, ApJL 842:L10 (2017)

Quasi-universal relations



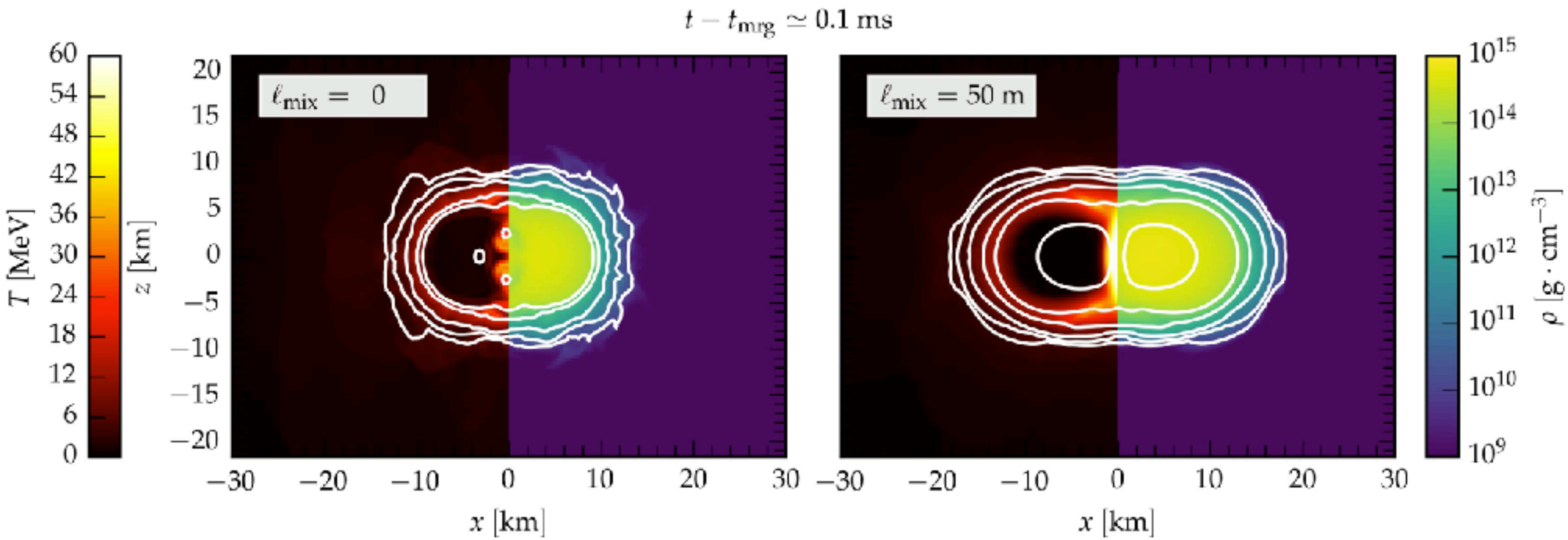
Zappa, Bernuzzi, **DR**+, Phys. Rev. Lett. 120, 111101 (2018)



- After the first ~10-20 ms, the remnant evolves because of the effective viscosity due to MHD turbulence, and neutrino cooling
- The remnant might still be hypermassive and undergo accretion-induced collapse during this phase

Angular momentum transport

$t_{\rm visc} = \infty$ $\qquad$ $t_{\rm visc} \sim 15\,{\rm ms}$

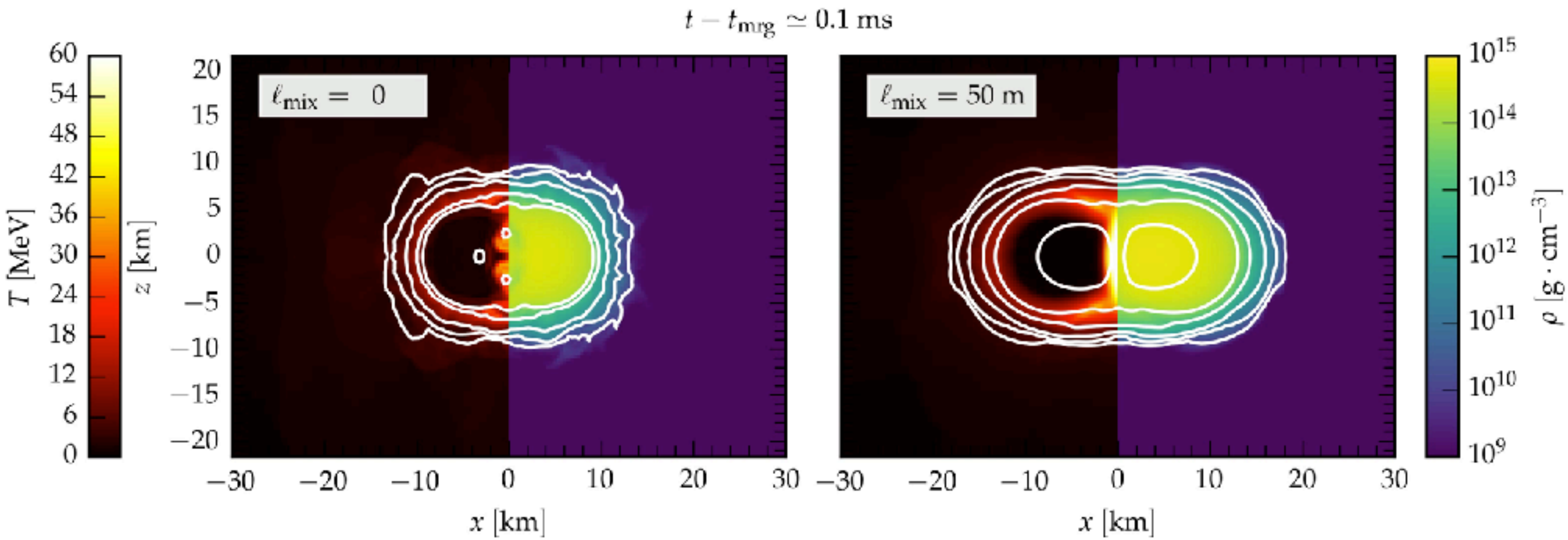


See also: Shibata & Kiuchi 2017; Kiuchi, Kyotoku+ 2017

DR ApJL:838 L2 (2017)

Angular momentum transport

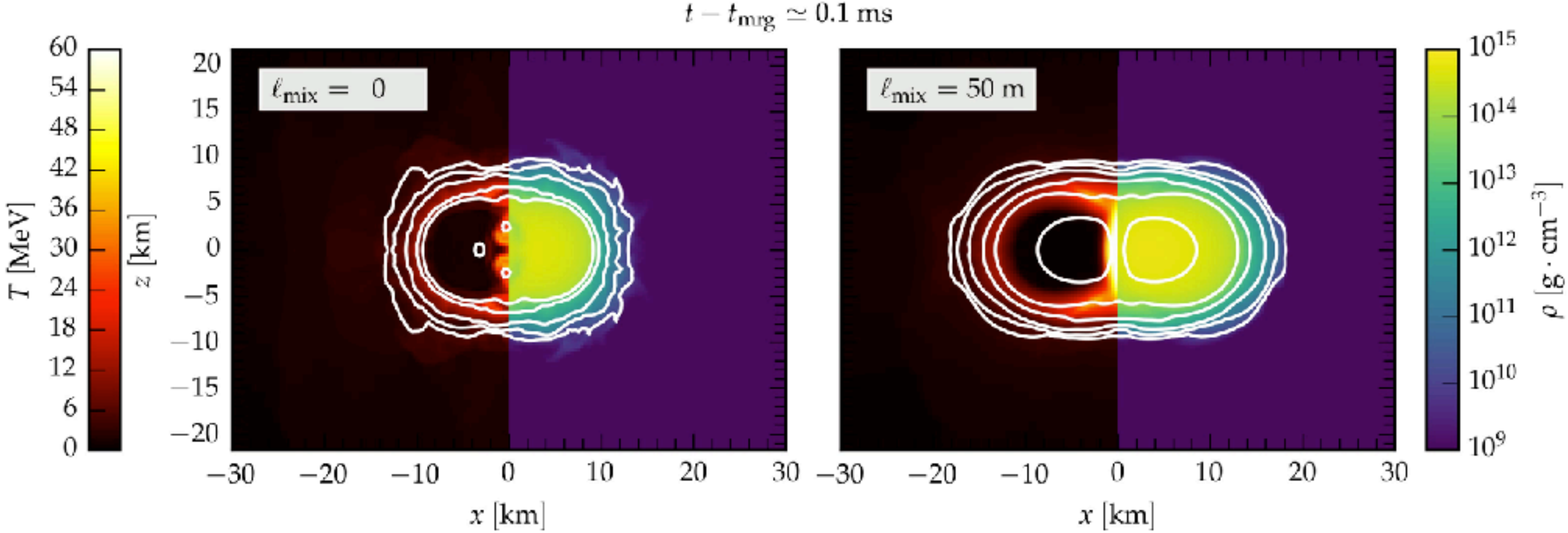
$t_{\rm visc} = \infty$ $\qquad\qquad$ $t_{\rm visc} \sim 15\,{\rm ms}$



See also: Shibata & Kiuchi 2017; Kiuchi, Kyotoku+ 2017 **DR** ApJL:838 L2 (2017)

Angular momentum transport

$t_{\rm visc} = \infty$ $\quad$ $t_{\rm visc} \sim 15\,{\rm ms}$

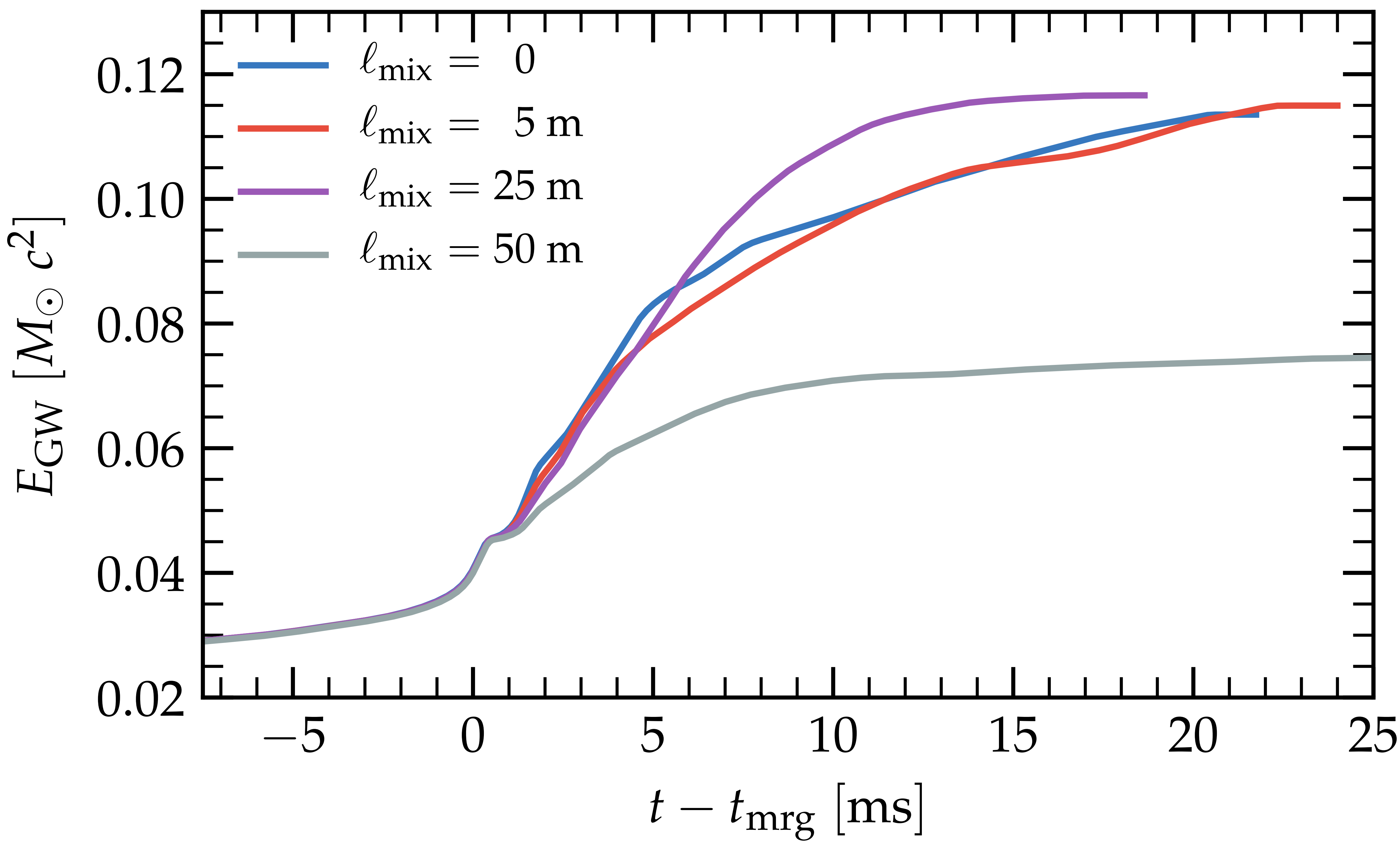


Delayed collapse!

See also: Shibata & Kiuchi 2017; Kiuchi, Kyotoku+ 2017

DR ApJL:838 L2 (2017)

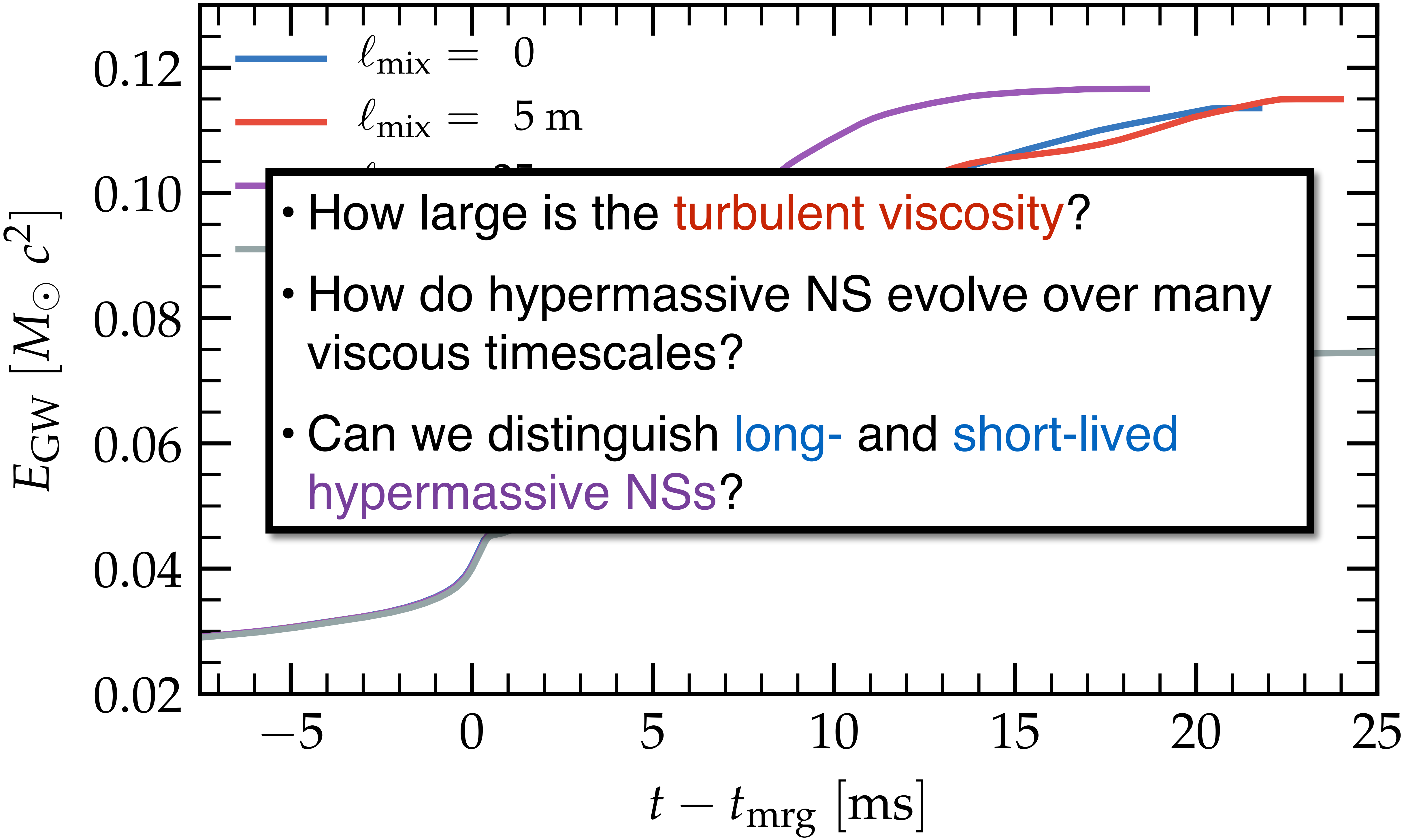
Gravitational waves



See also: Shibata & Kiuchi 2017; Kiuchi, Kyotoku+ 2017

DR ApJL:838 L2 (2017)

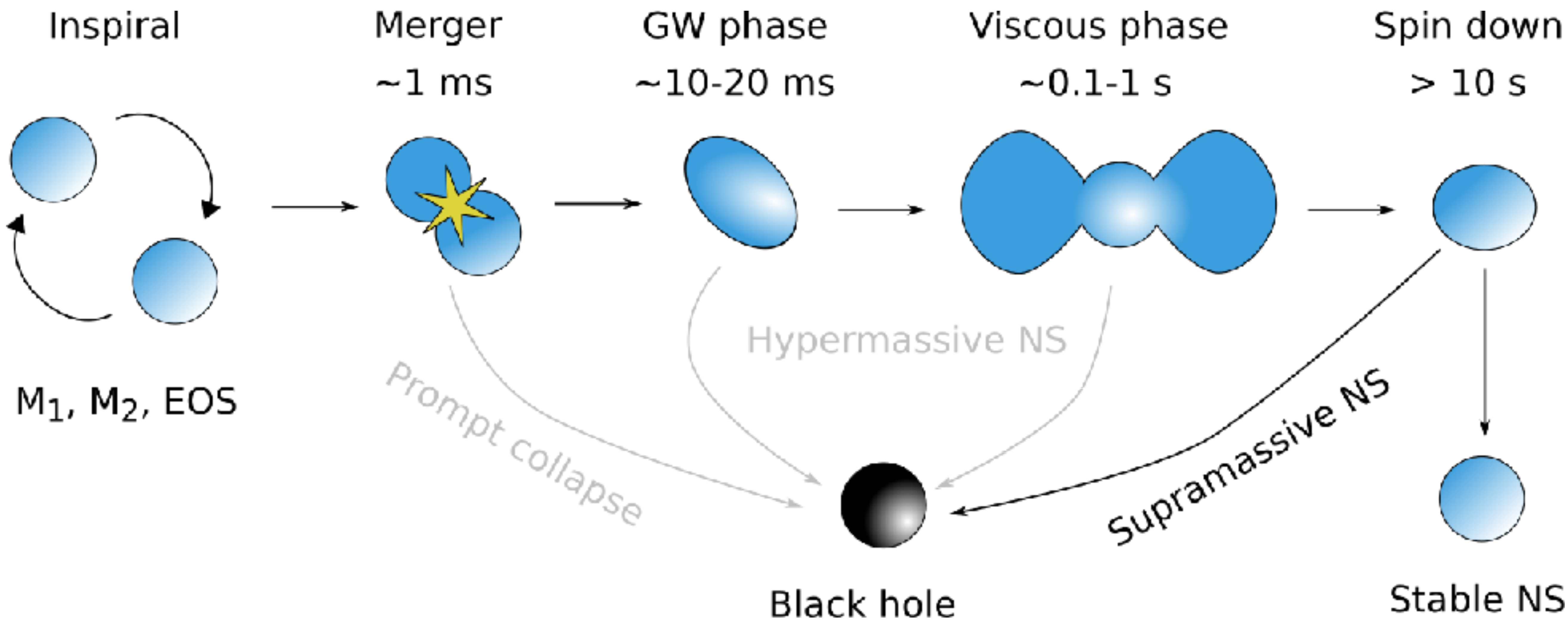
Gravitational waves



- How large is the turbulent viscosity?
- How do hypermassive NS evolve over many viscous timescales?
- Can we distinguish long- and short-lived hypermassive NSs?

See also: Shibata & Kiuchi 2017; Kiuchi, Kyotoku+ 2017

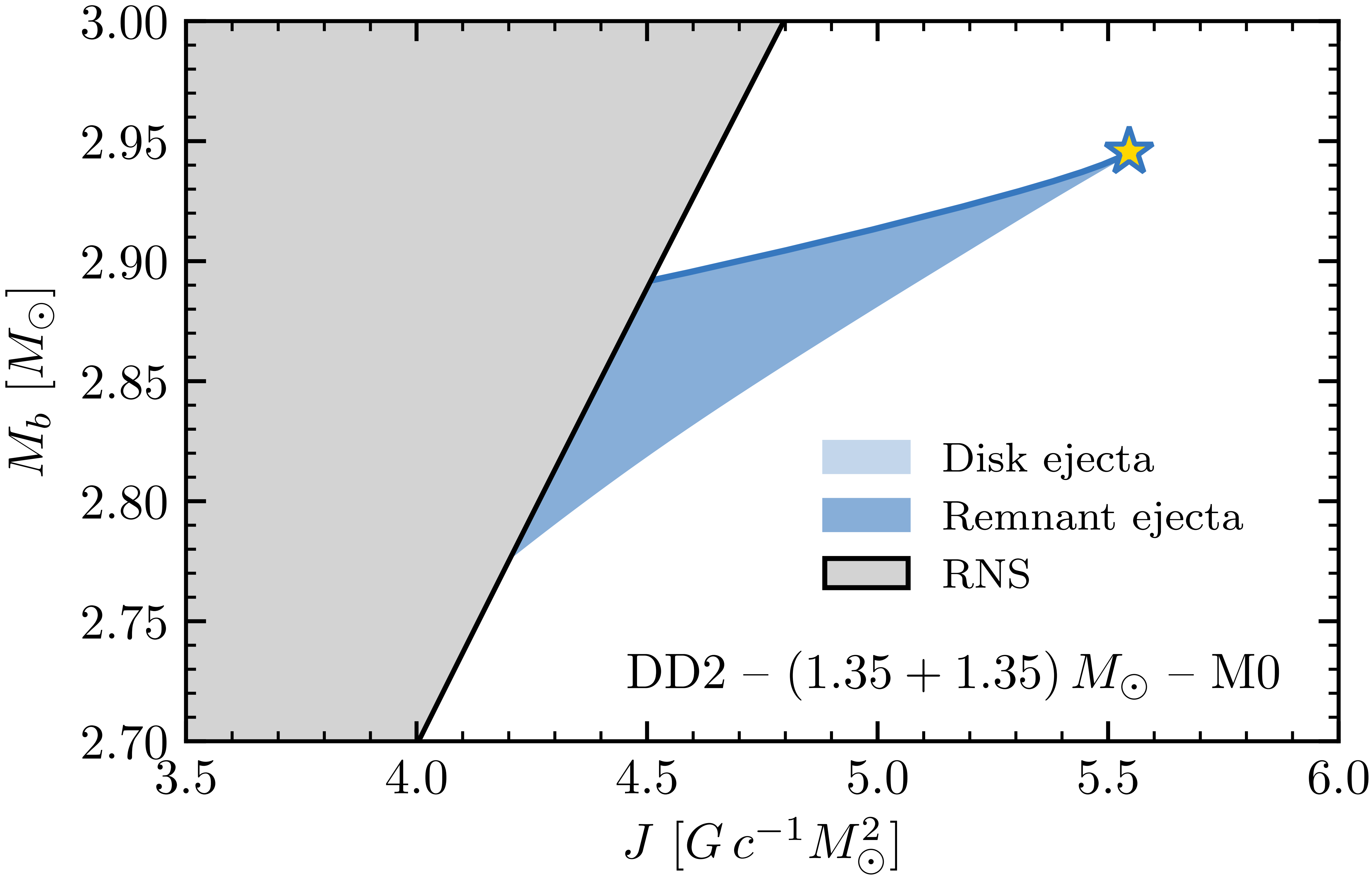
DR ApJL:838 L2 (2017)



- If the binary mass is less than ~1.2 M_{max} then it will be stable even after solid body rotation has been achieved
- Might collapse as it spins down due to magnetic torques (supramassive remnants) or might be stable forever
- How would a long-lived remnant look like in EM?

See e.g., Margalit & Metzger (2017) and DR, Perego, Bernuzzi, Zhang (2018)

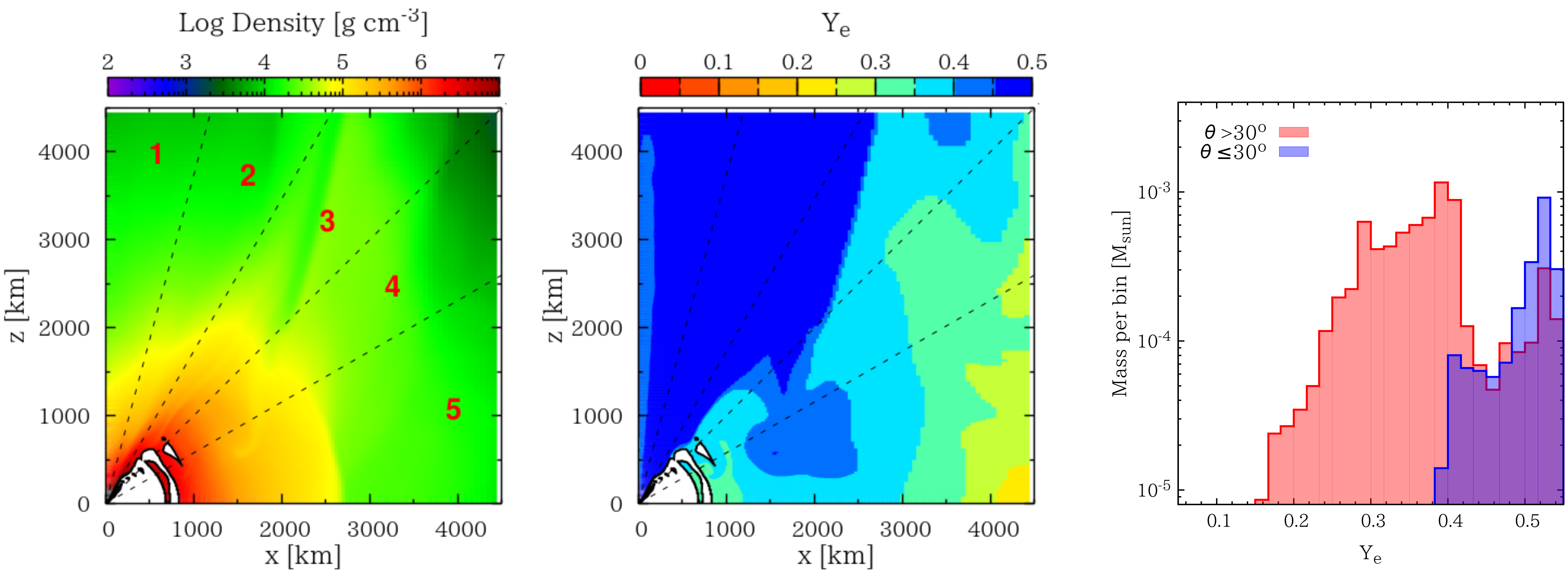
Viscous evolution (I)



See also Fujibayashi, Kiuchi+ (2017)

DR, Perego, Bernuzzi, Zhang, in prep.

Viscous evolution (II)



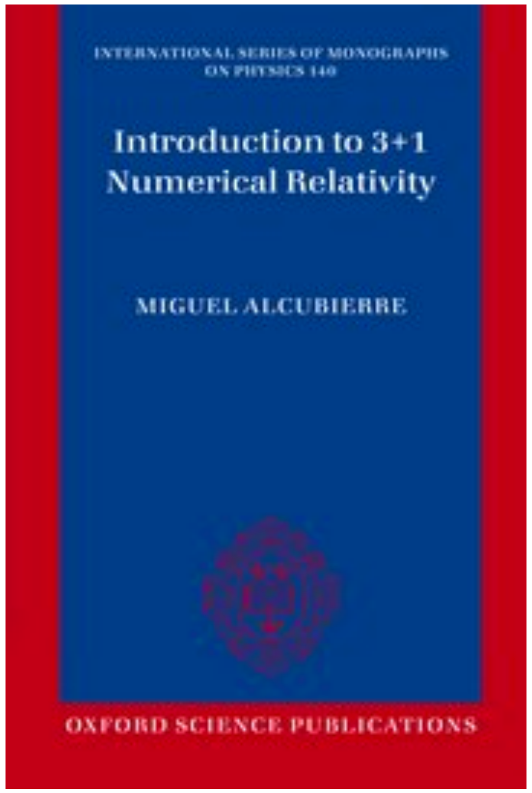
From Fujibayashi, Kiuchi+ (2017)

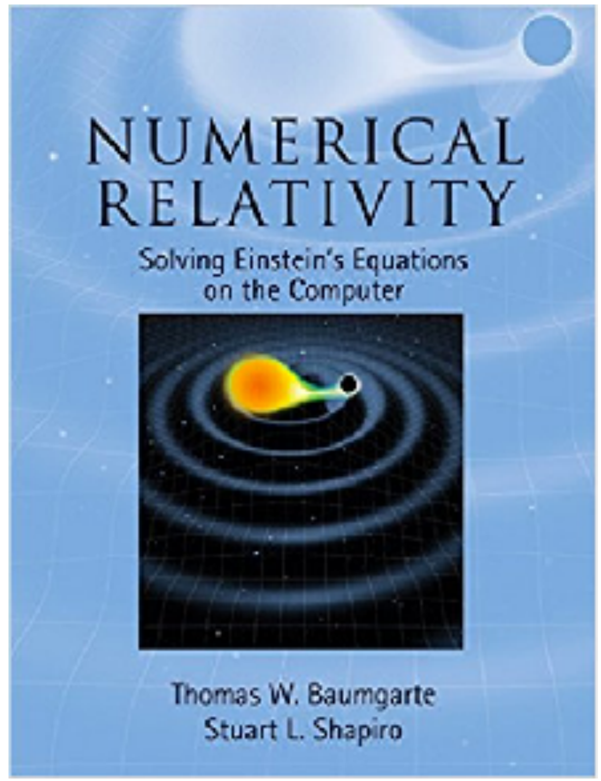
- Low-mass NS binaries exist* and likely form stable remnants
- Viscous evolution of stable remnants NS is not trivial
- Smoking gun: a very bright kilonova with a blue component

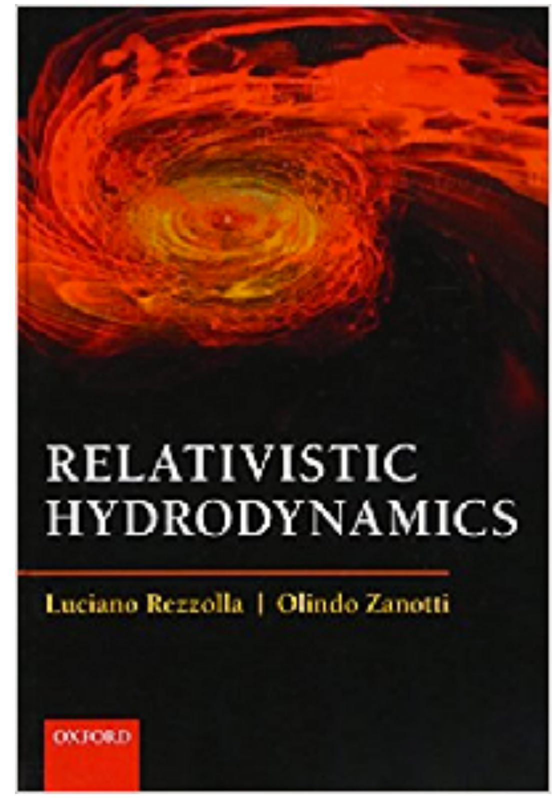
Summary

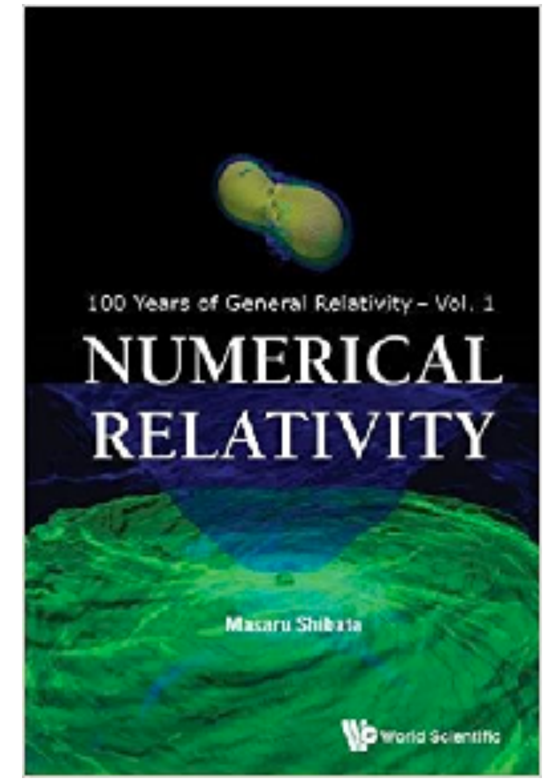
- The outcome of NS mergers depends on the NS masses and the (unknown) NS EOS
- The long term evolution of the remnants from NS mergers is not yet fully understood
- Future NS merger events might look very different from GW170817

Further readings









- Review articles on relativistic hydrodynamics
  Martí, Müller, Living Rev. Relativ. 6:7 (2003)
  Font, Living Rev. Relativ. 11:7 (2008)

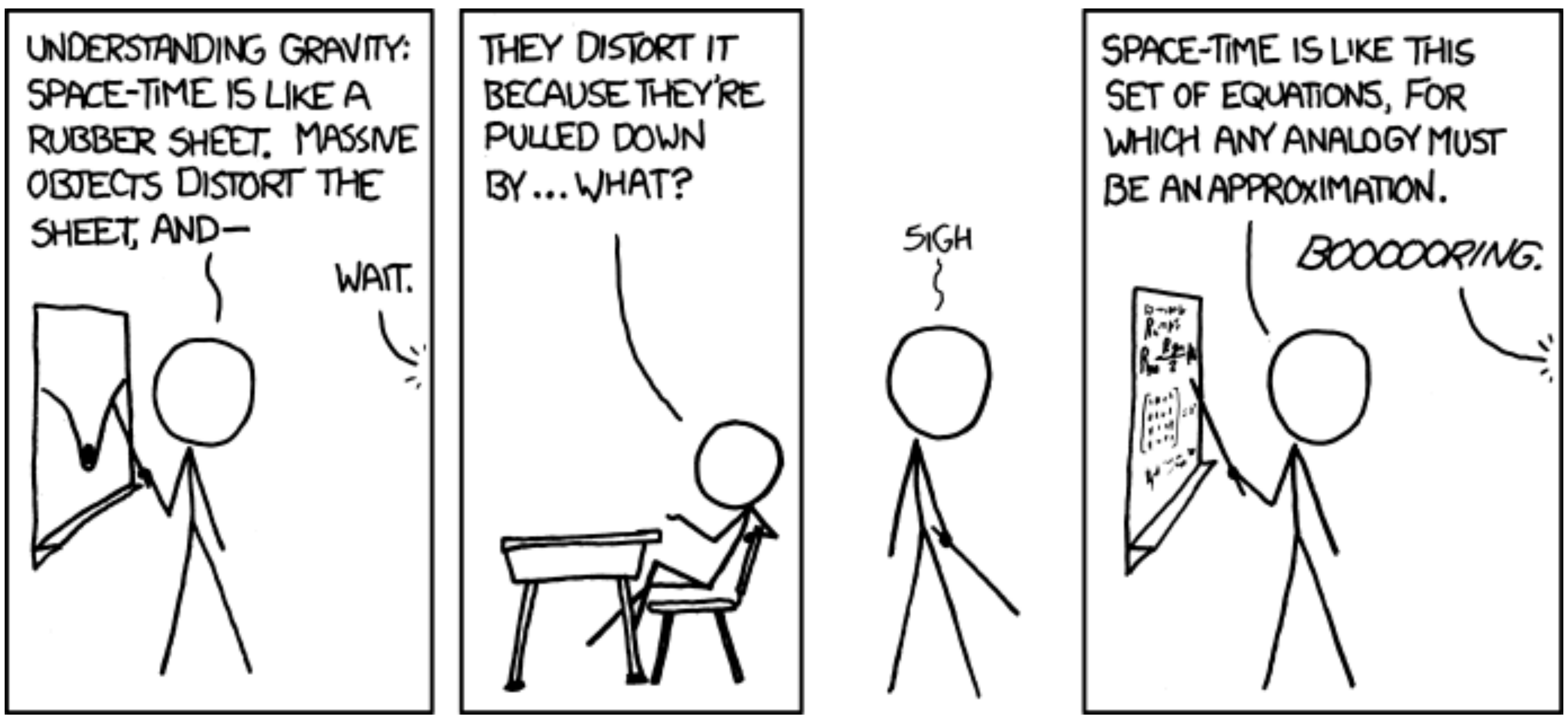
- Review articles on binaries
  Shibata & Taniguchi, Living Rev. Relativ. 14:6 (2011) — NS+BH
  Baiotti & Rezzolla, Rep. Prog. Phys. 80 096901 (2017) — NS+NS

Extra slides

General relativity

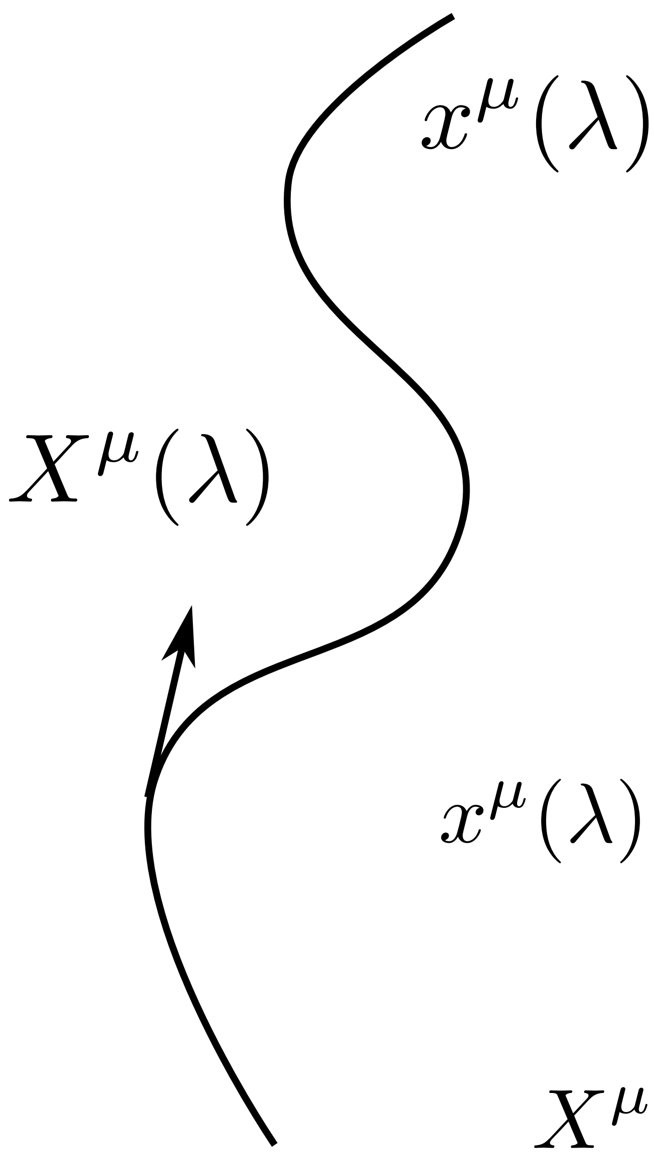
$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu \qquad x^\mu = \{t, x, y, z\}$$

Gravity is encoded in the geometry of spacetime. The metric tensor defines the “distance” between events and the causal structure of the spacetime.



https://xkcd.com/895

Four-vectors



Vectors fields are generated as the tangents to families of curves

$$\frac{\mathrm{d}x^\mu}{\mathrm{d}\lambda} = X^\mu$$

Special case, coordinate basis vectors

$$x^\mu(\lambda) = \lambda \delta^\mu{}_0 \implies \vec{X} = \vec{\partial_\mu}$$

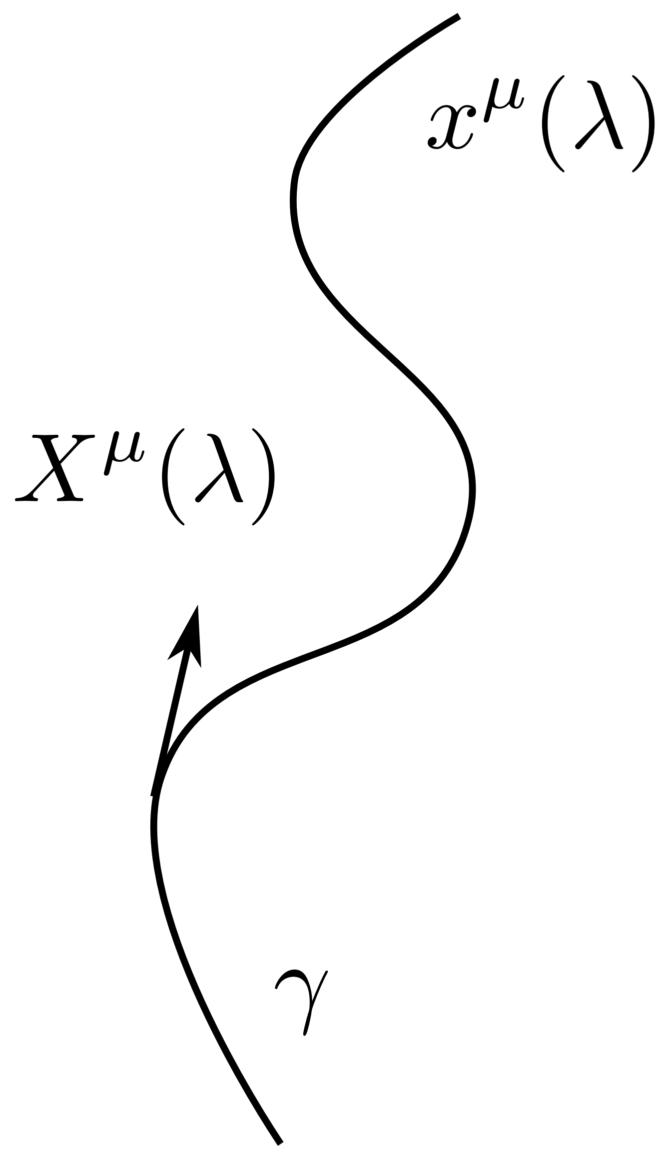
$$\delta^\mu{}_0 = \begin{cases} 1 & \text{if } \mu = 0, \\ 0 & \text{otherwise.} \end{cases}$$

$X^\mu X_\mu = g_{\mu\nu} X^\mu X^\nu > 0$ space-like vectors

$X^\mu X_\mu = g_{\mu\nu} X^\mu X^\nu < 0$ time-like vectors

$X^\mu X_\mu = g_{\mu\nu} X^\mu X^\nu = 0$ null vectors

Distances



Let γ be a curve with tangent vector X^μ

If X^μ is always space-like

$$\ell = \int_\gamma \sqrt{g_{\mu\nu} X^\mu X^\nu} \mathrm{d}\lambda$$

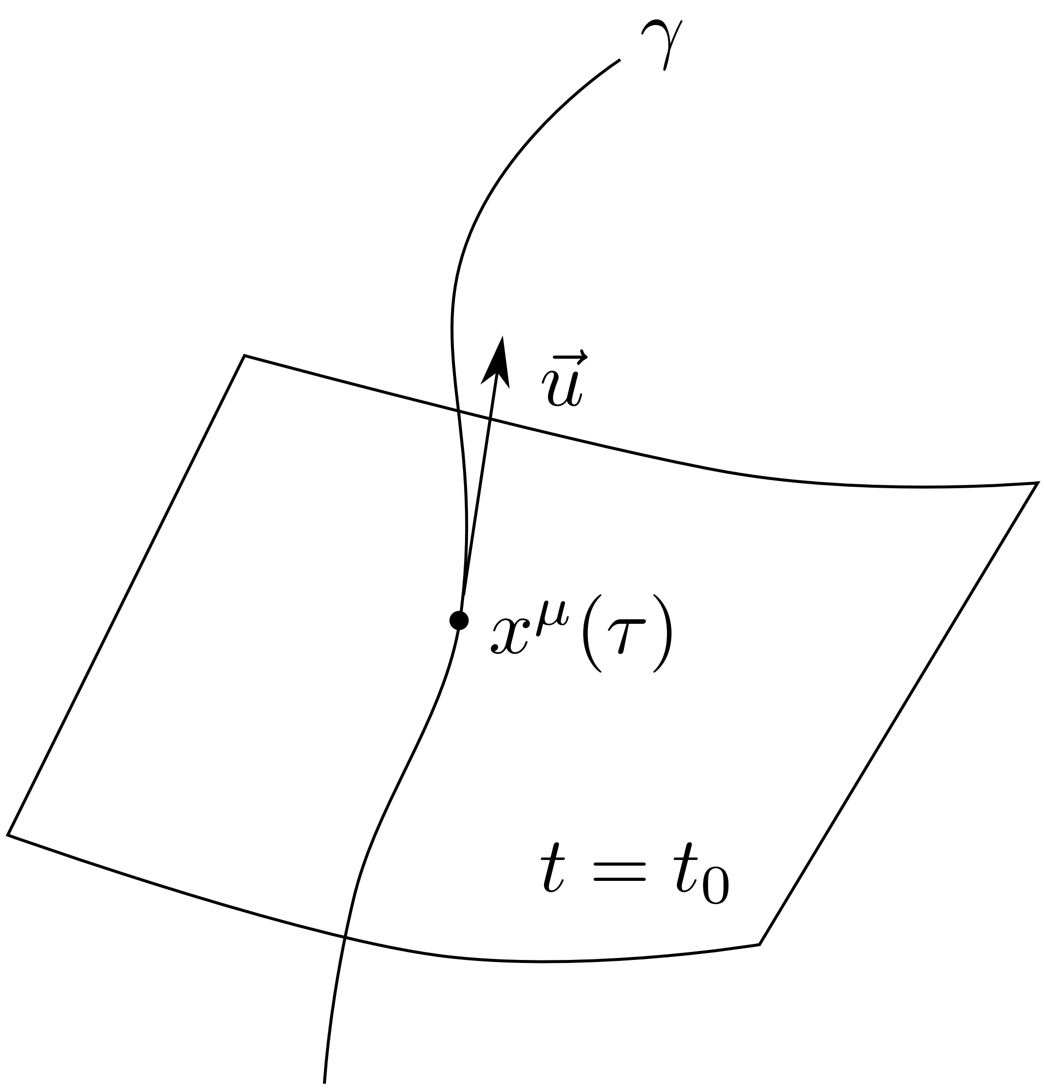
is the proper length of the along the curve γ

If X^μ is always time-like

$$\tau = \int_\gamma \sqrt{-g_{\mu\nu} X^\mu X^\nu} \mathrm{d}\lambda$$

is the proper time experienced by an observer moving along the curve γ

Four-velocity



The rate of change of the position of a point in spacetime as a function of its proper time

$$\frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = u^\mu$$

Four-vectors also define the derivative of functions along the world line of a particle

$$u^\mu \nabla_\mu f = \frac{\mathrm{d}f}{\mathrm{d}\tau}$$

Since τ is the proper time: $$g_{\mu\nu} u^\mu u^\nu = u^\mu u_\mu = -1$$

Geodesic

In GR gravity is an “apparent force”. Test masses in a gravitational field do not feel any force and proceed in their motion on “straight lines” (geodesics).

Straight line: the tangent vector remains parallel to itself

$$0 = u^\mu \nabla_\mu u^\nu = u^\mu \partial_\mu u^\nu + \Gamma^\nu{}_{\mu\alpha} u^\mu u^\alpha$$

Remembering the definition of the 4-velocity we have

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2} + \Gamma^\mu{}_{\alpha\beta} \frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau} \frac{\mathrm{d}x^\beta}{\mathrm{d}\tau} = 0$$

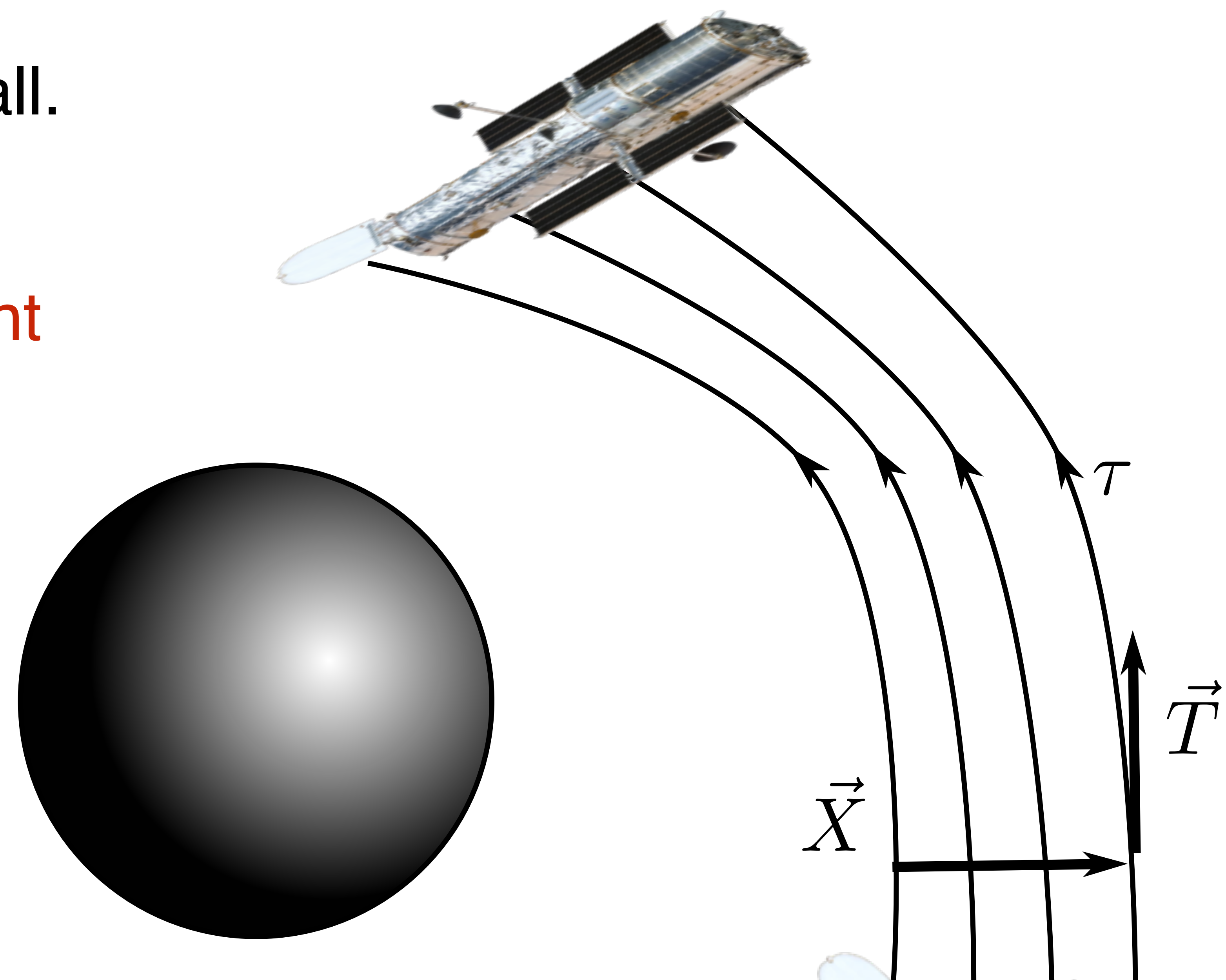
$\Gamma^\alpha{}_{\beta\gamma}$ are the Christoffel symbols

$$\Gamma^\alpha{}_{\beta\gamma} = \frac{1}{2} g^{\alpha\mu} \left(\partial_\gamma g_{\mu\beta} + \partial_\beta g_{\mu\gamma} - \partial_\mu g_{\beta\gamma} \right)$$

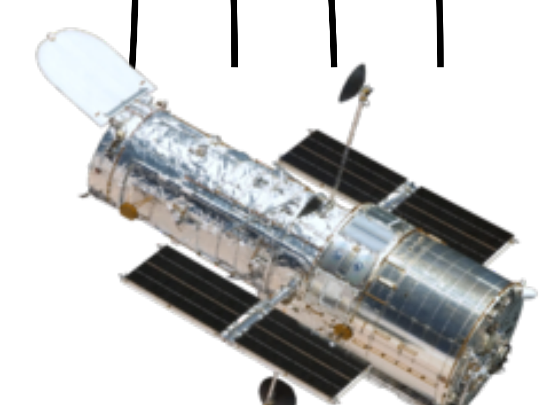
Geodesic deviation

Each particle is in free fall.

Problem: each particle wants to fall on a different geodesic: tidal effects!



$$\frac{D^2 X^\mu}{D\tau^2} + R^\mu{}_{\alpha\nu\beta} T^\alpha X^\nu T^\beta = 0$$



Weak field limit

Consider the geodesic deviation equation in the case of slow motion

$$\frac{D^2 X^\mu}{D\tau^2} + R^\mu{}_{\alpha\nu\beta} T^\alpha X^\nu T^\beta = 0 \qquad T^\alpha = \delta^\alpha{}_0$$

The spatial part of the relative acceleration reads

$$\frac{\mathrm{d}^2 X^i}{\mathrm{d}t^2} = -R^i{}_{0j0} X^j$$

Note that this is very similar to the Newtonian tidal field

$$\frac{\mathrm{d}^2 X^i}{\mathrm{d}t^2} = -\frac{\partial^2 \phi}{\partial x_i \partial x^j} X^j \qquad \Longrightarrow \qquad R^i{}_{0j0} \simeq \frac{\partial^2 \phi}{\partial x_i \partial x^j}$$

Field equations

Our previous discussion suggest that Newton field equation could be generalized as

$$R_{00} = R^{i}{}_{0i0} \simeq \triangle\phi = 4\pi\rho \simeq 4\pi T_{00}$$

The Einstein field equations are only a couple of tweaks away

$$R_{\mu\nu} + \underbrace{\frac{1}{2}g_{\mu\nu}R}_{\text{This term is needed to ensure conservation of energy}} = 8\pi T_{\mu\nu}$$

Use 4D objects

This fixes the Newtonian limit

Field equations

- Einstein tensor

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

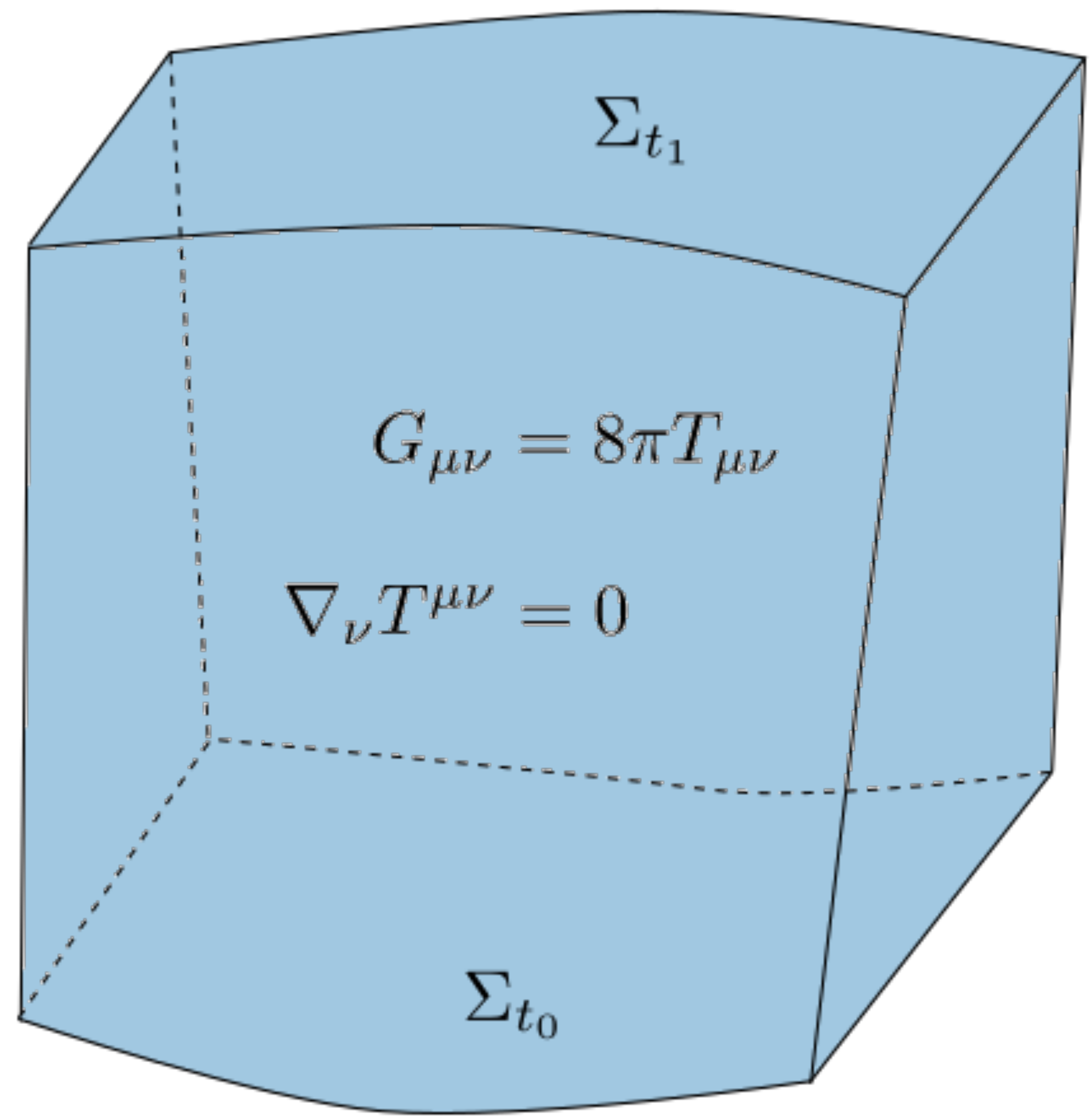
  describes the geometry of spacetime

- Stress energy tensor

$$T_{\mu\nu}$$

  describes matter and radiation content of the spacetime

- Field equations and Bianchi identities connect geometry and flow



Note G = c = 1 for us!

Vacuum spacetimes (I)

Inverse of the metric

$$R_{\mu\nu} = \underbrace{-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{\mu\nu}}_{\text{Wave operator acting on the spacetime metric}} + \ldots = 0$$

Ricci tensor

Wave operator acting on
the spacetime metric

$T_{\mu\nu} = 0$

Non-linear wave equation: non-trivial curvature even in vacuum (field can source itself)

Gravitational waves

Split the metric in a flat background part and a small perturbation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\eta_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} = -c^2\mathrm{d}t^2 + dx^2 + dy^2 + dz^2 \qquad h_{\mu\nu} \ll 1$$

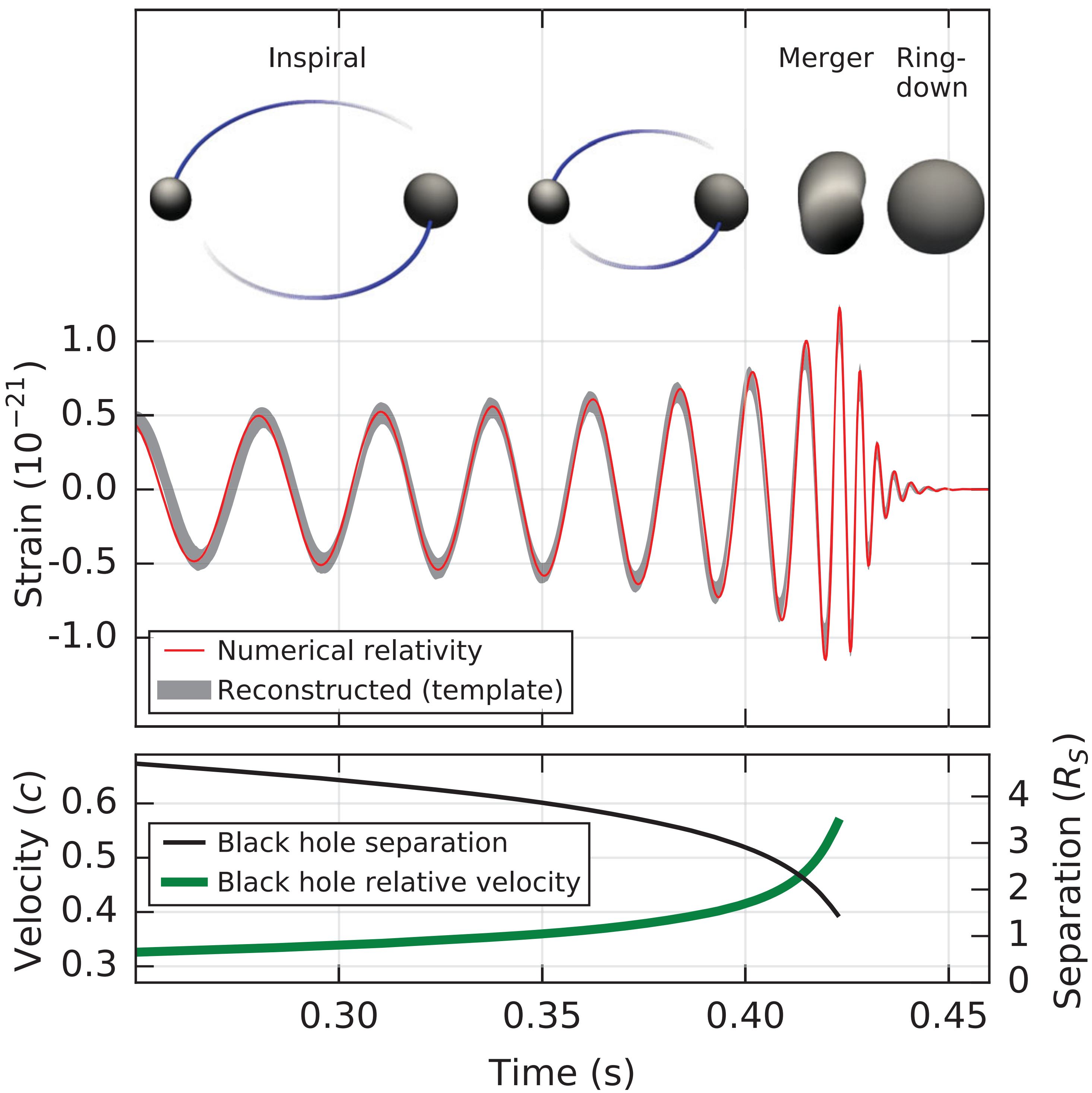
Linearize Einstein field equations

$$0 = R_{\mu\nu} \approx -\frac{1}{2}\eta^{\alpha\beta}\partial_\alpha\partial_\beta h_{\mu\nu} = 0$$

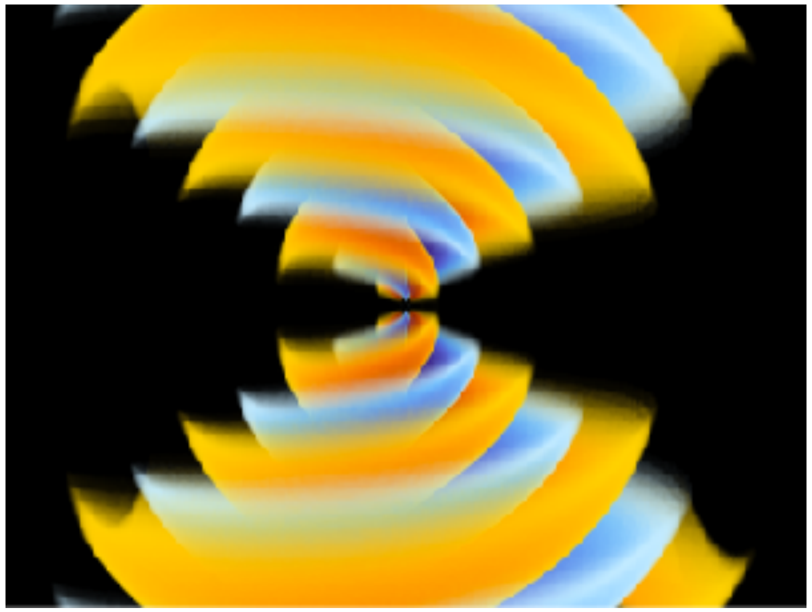
Perturbations satisfy the tensor wave equation: gravitational waves

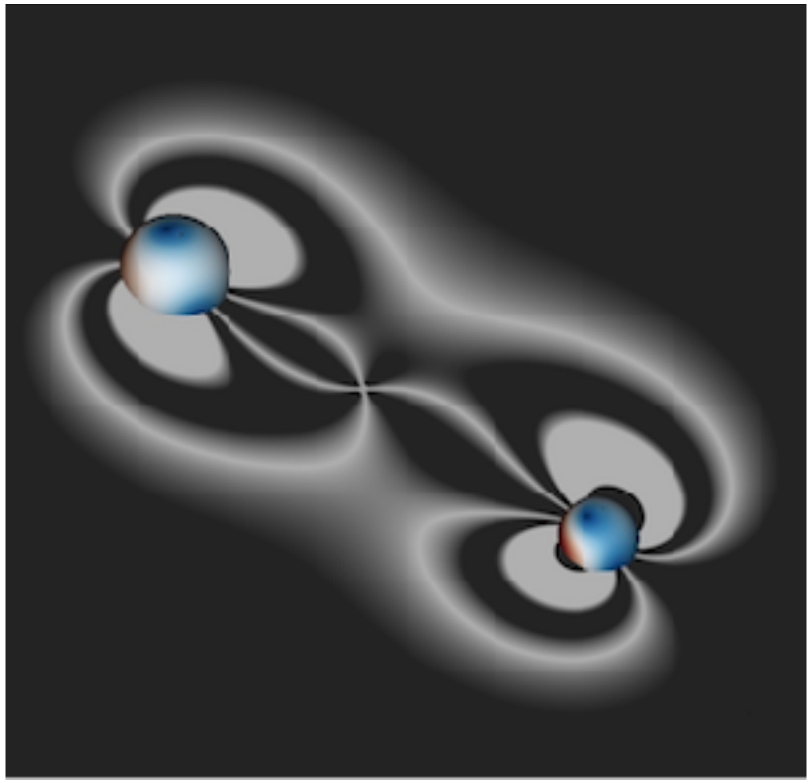
$$\partial_t^2 h_{\mu\nu} - c^2(\partial_x^2 + \partial_y^2 + \partial_z^2)h_{\mu\nu} = \Box h_{\mu\nu} = 0$$

Vacuum spacetimes (II)



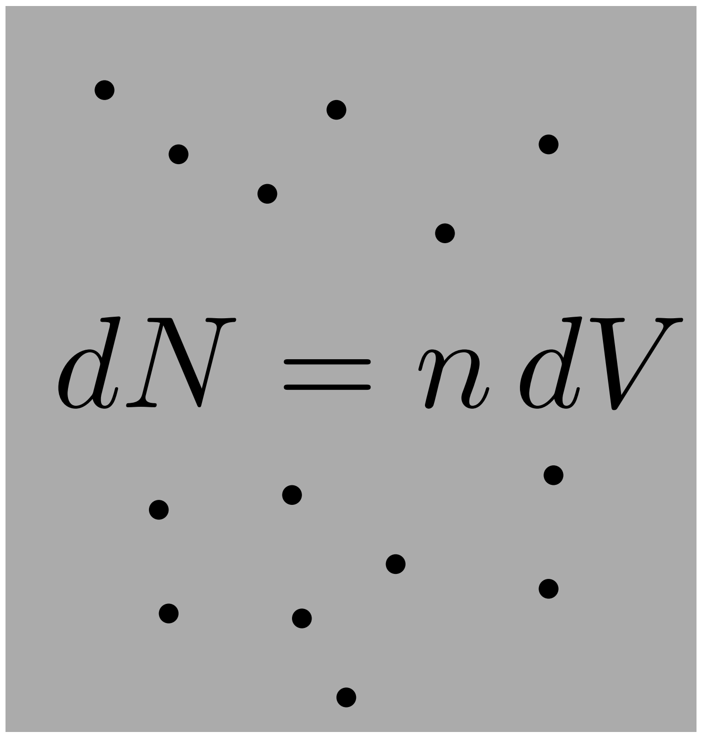
From Abbott+ PRL 116, 061102 (2016)





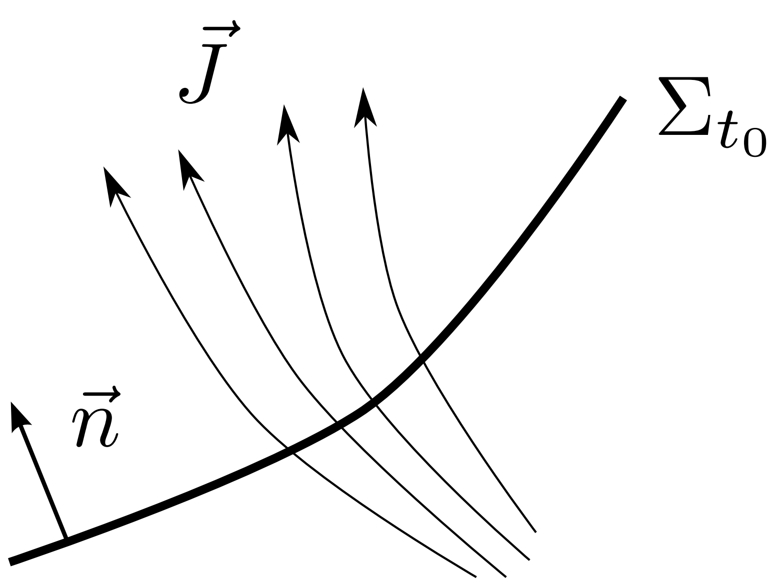
GW150914 simulations with ET

Relativistic hydrodynamics



Classical picture

- Intrinsic variable: particle number density
- Equations describe the flux of matter through space as a function of time

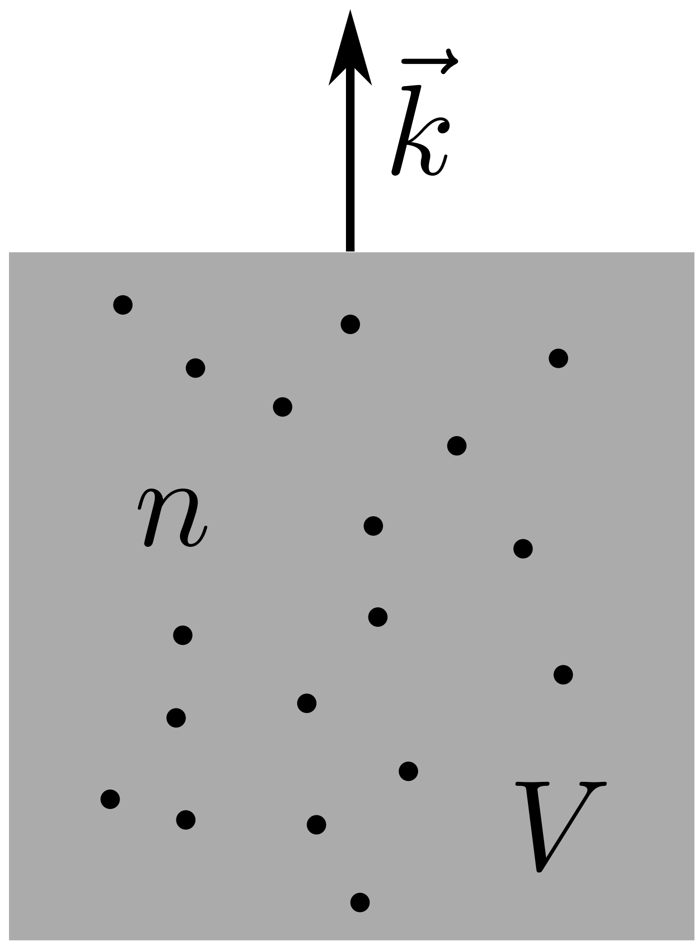


$$dN = -\vec{n} \cdot \vec{J}\,dV$$

Relativistic picture

- Intrinsic variable: flux of particles through spacetime
- Equations describe the density as measured by any observer

Classical continuity equation

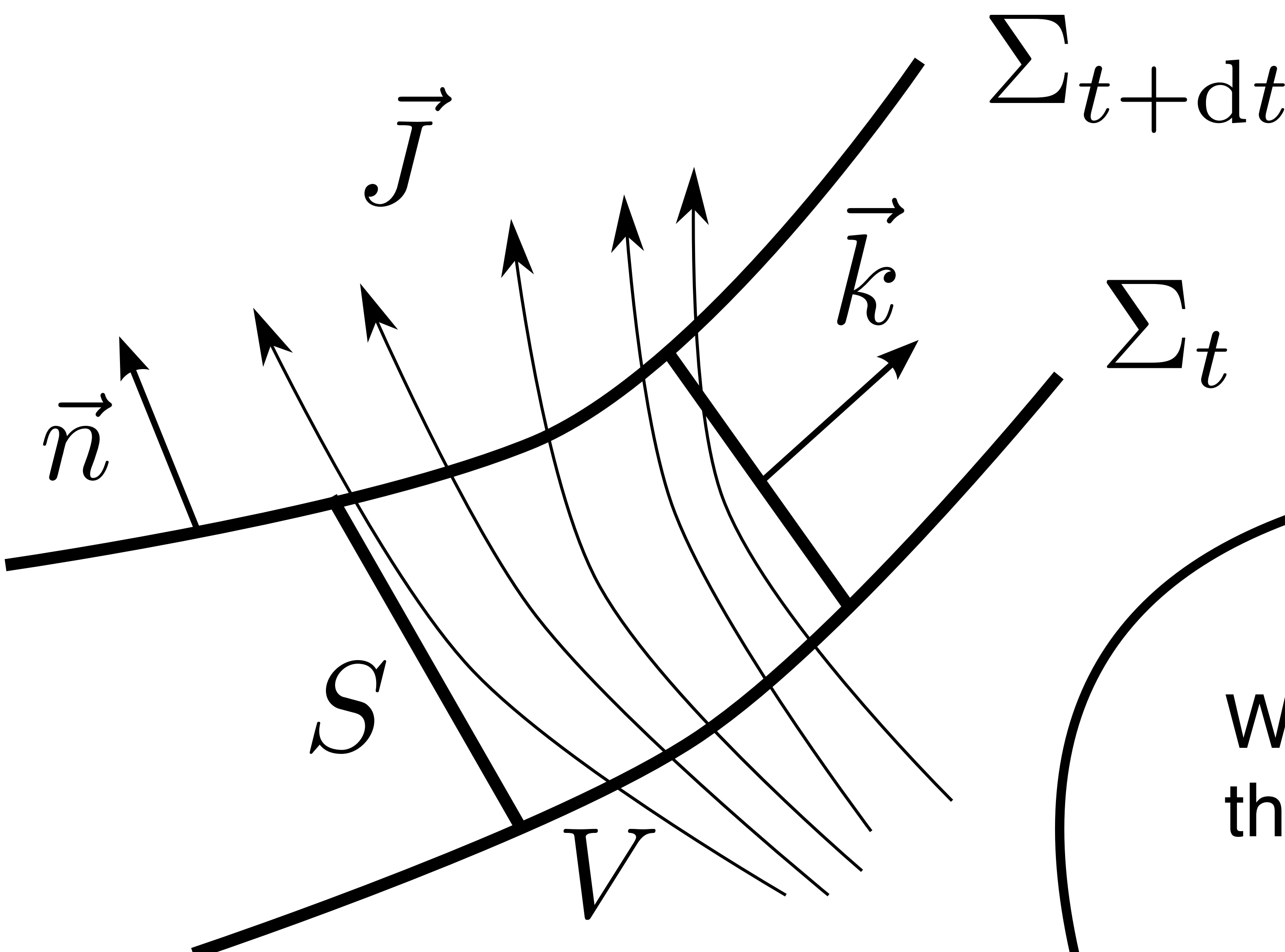


$$\underbrace{\frac{\mathrm{d}}{\mathrm{d}t}\int_V n\,\mathrm{d}V}_{\text{Number of particles in a control volume}} + \underbrace{\int_{\partial V} n\vec{v}\cdot\vec{k}\,\mathrm{d}S}_{\text{Flux of particles across the boundaries of the control volume}} = 0$$

Differential form:

$$\boxed{\partial_t n + \nabla_i(nv^i) = 0}$$

Relativistic continuity equation

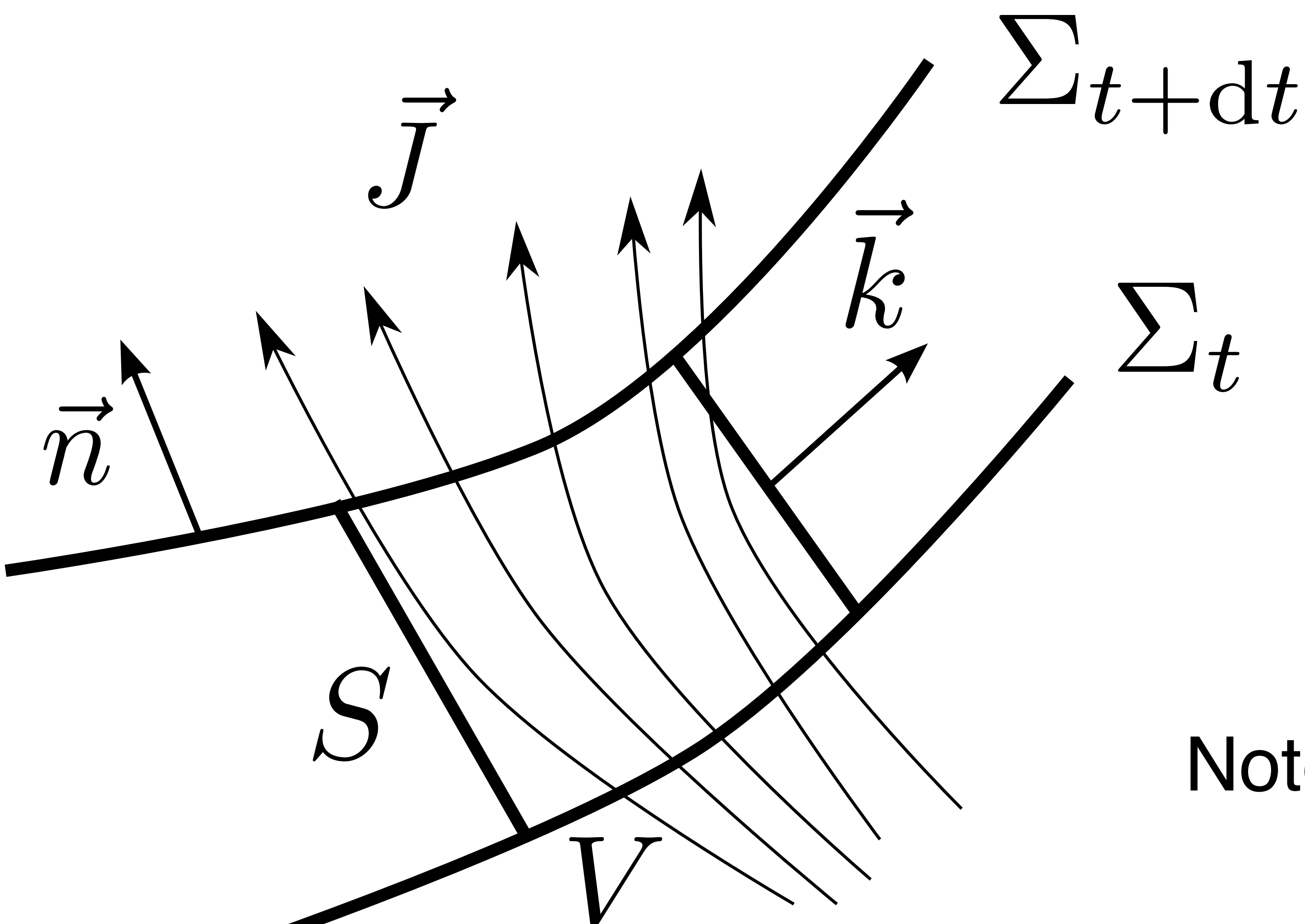


World lines passing through control volume

World lines leaving / entering through the time-like boundary

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \vec{J}\cdot\vec{n}\,\mathrm{d}V + \int_S \vec{J}\cdot\vec{k}\,\mathrm{d}S = 0$$

Relativistic continuity equation



Number current

4-velocity

$$J^\mu = nu^\mu$$

Note: $$-J^\mu u_\mu = n$$

Differential form: $$\boxed{\nabla_\mu J^\mu = 0}$$

Stress-energy tensor

The number current of a fluid is such that

$$-J_\mu n^\mu = \text{“Number density measured by a fiducial observer”}$$

In analogy the stress-energy tensor of a fluid

$$T_{\mu\nu} n^\mu n^\nu = \text{“Energy density measured by a fiducial observer”}$$

$$-T_{\mu\nu} n^\mu (\partial_i)^\nu = \text{“Energy flux measured by a fiducial observer”}$$

$$T_{\mu\nu} (\partial_i)^\mu (\partial_j)^\nu = \Sigma_{ij} = \text{“Stress forces measured by a fiducial observer”}$$

Where we have chosen $\{\vec{n}, \partial_i\}$ such that

$$n^\mu (\partial_i)_\mu = 0$$

Modeling neutron stars

1. Assemble stress-energy tensor of matter and radiation

$$T_{\mu\nu} = T_{\mu\nu}^{(\mathrm{HD})} + T_{\mu\nu}^{(\mathrm{EM})} + T_{\mu\nu}^{(\nu)} + \ldots$$

2. Solve for “conservation” of energy and momentum

$$\nabla_\nu T^{\mu\nu} = 0$$

3. Solve for conservation of baryon and lepton number

$$\nabla_\mu J^\mu = 0 \qquad\qquad u^\mu \nabla_\mu Y_e = R$$

4. Add equation of state and other constitute relations

$$p = p(n, T, Y_e)$$

5. Equations for neutrino radiation and EM fields

…

Perfect fluid

It is easy to write the stress energy tensor for NS matter in a locally-flat comoving frame $\{\vec{u}, \partial_i\}$ assuming a perfect fluid

We impose the following condition

$$T_{\mu\nu} u^\mu u^\nu = \rho$$ Energy density of the fluid

$$T_{\mu\nu} u^\mu (\partial_i)^\nu = 0$$ No heat conduction

$$T_{\mu\nu} (\partial_i)^\mu (\partial_j)^\nu = p\delta_{ij}$$ Only isotropic pressure

In this case the stress energy tensor takes the form

$$T_{\mu\nu} = \begin{pmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix} \quad u^\mu = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \xrightarrow{\text{General frame}} \quad \begin{aligned} T_{\mu\nu} &= \rho u^\mu u^\nu + p \perp^{\mu\nu} \\ \perp_{\mu\nu} &= g_{\mu\nu} + u_\mu u_\nu \end{aligned}$$

“Conservation” laws (I)

The key part of the equations of motion is

$$\nabla_\nu T^{\mu\nu} = 0$$

what is really conserved?

“Conservation” laws (II)

Multiply $\nabla_\nu T^{\mu\nu} = 0$ by any smooth vector field p^μ to get

$$\nabla_\mu \left(T^{\mu\nu} p_\nu\right) = -T^{\mu\nu}\nabla_\mu p_\nu = -\frac{1}{2} T^{\mu\nu} \underbrace{\left(\nabla_\mu p_\nu + \nabla_\nu p_\mu\right)}_{= 0 \text{ for Killing vectors}}$$

$$T^{\mu\nu} = T^{\nu\mu} = \frac{1}{2}\left(T^{\mu\nu} + T^{\nu\mu}\right)$$

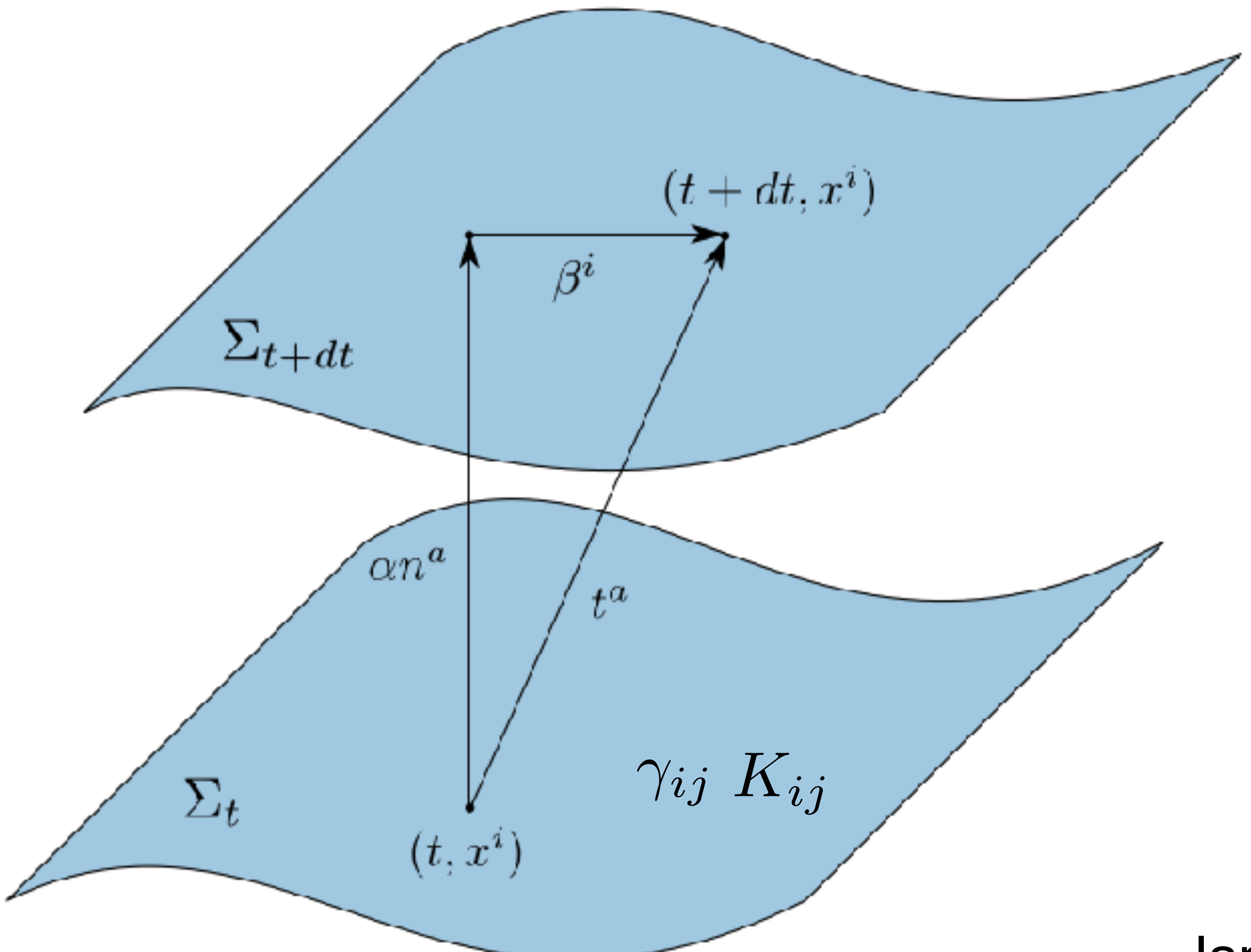
Killing vectors are direction of symmetry for the metric

$$0 = \nabla_\mu p_\nu + \nabla_\nu p_\mu \implies \mathcal{L}_{\vec{p}}\, g_{\mu\nu} = 0$$

Noether’s theorem:
Spacetime symmetries generate conserved charges

p^μ Killing vector $\implies$ $C^\mu = T^\mu{}_\nu p^\nu$ conserved

3+1 decomposition



γ_{ij} spatial metric

K_{ij} extrinsic curvature

$\vec{t}$ vector tangent to the constant x^i curves

$\vec{n}$ vector normal to the constant t hyper-surfaces

$$\vec{t} = \alpha\vec{n} + \vec{\beta}$$

lapse function shift vector

Lapse and shift vector are gauge degrees of freedom

Spatial metric

The spatial metric describes the intrinsic geometry of the spatial slices

$$\mathrm{d}\ell^2 = \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

As a four-dimensional object $\gamma^\mu{}_\nu$ is used to project vectors tangentially to the spatial hypersurface

$$\gamma^\mu{}_\nu = \delta^\mu{}_\nu + n^\mu n_\nu$$

The spatial metric is prescribed as initial data and then evolved

We use the symbol D_i for the covariant derivative associated with γ_{ij}

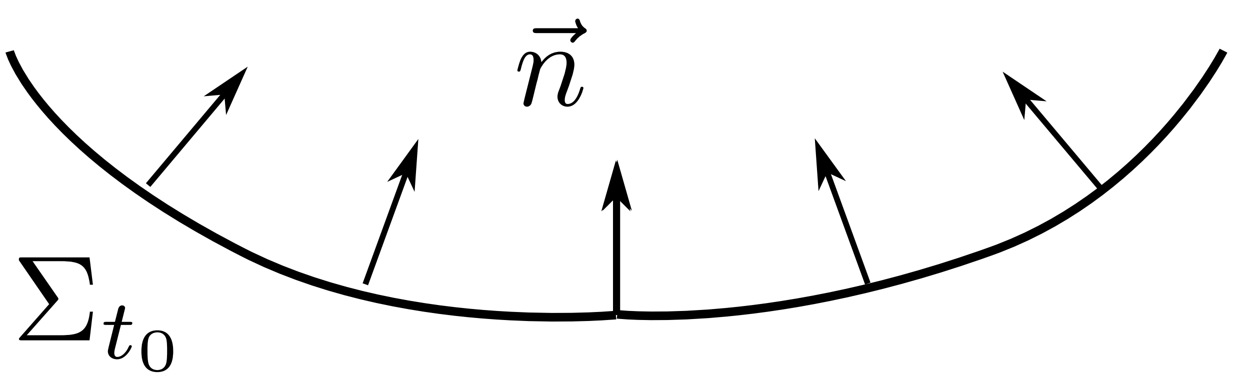
Extrinsic curvature (I)

$$K_{\mu\nu} = -\gamma^{\alpha}{}_{\mu}\nabla_{\alpha}n_{\nu}$$

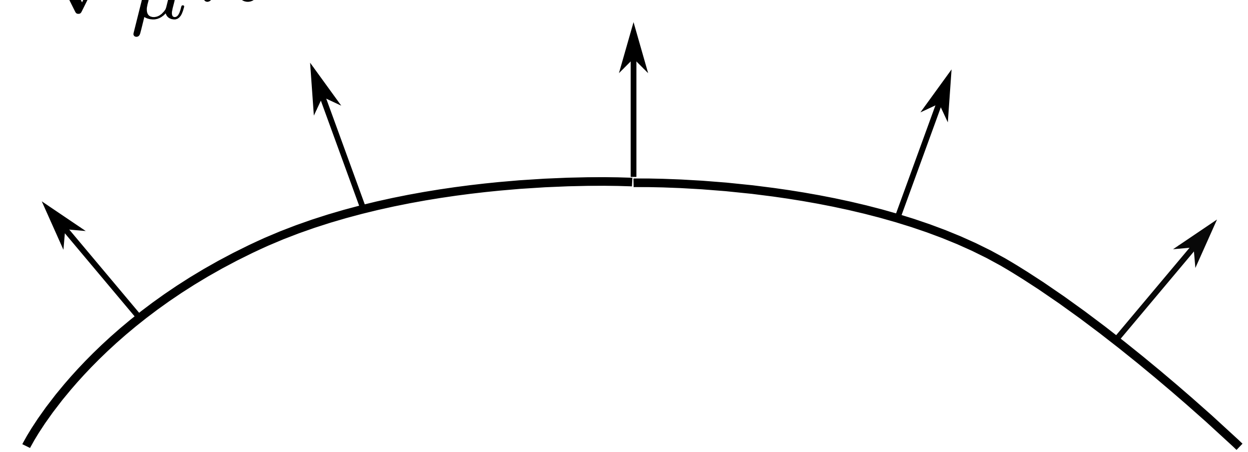
Tangent projection of the gradient of the normal vector

The trace of the extrinsic curvature is associated with the expansion of the world lines of the Eulerian observers:

$$K = -\nabla_{\mu}n^{\mu}$$



$K > 0$



$K < 0$

Extrinsic curvature (II)

$$K_{ij} = -\frac{1}{2}\mathcal{L}_{\vec{n}}\gamma_{ij}$$

The extrinsic curvature is the Lie derivative of the metric along the normal to the spatial hypersurface

$K_{ij} = 0 \implies$ three-metric is stationary!

If $\beta^i = 0$ then $K_{ij} = -\frac{1}{2}\partial_t \gamma_{ij}$

The extrinsic curvature is also prescribed at the initial time and then evolved

ADM equations

$$(\partial_t - \mathcal{L}_{\vec{\beta}})\gamma_{ik} = -2\alpha K_{ik}$$

$$(\partial_t - \mathcal{L}_{\vec{\beta}})K_{ik} = -D_i D_k \alpha + \alpha \left({}^{(3)}R_{ik} - 2K_{ij}K^j{}_k + K K_{ik} \right) - 8\pi\alpha \left(S_{ik} - \frac{1}{2}\gamma_{ik}(S - E) \right)$$

$${}^{(3)}R + K^2 - K_{ik}K^{ik} = 16\pi E$$

$$D_k(K\gamma^k{}_i - K^k{}_i) = 8\pi j_i$$

$$S_{ik} = \gamma^\mu{}_i \gamma^\nu{}_k T_{\mu\nu} \qquad S = S^i{}_i \qquad j_i = -\gamma^\mu{}_i n^\nu T_{\mu\nu} \qquad E = T^{\mu\nu} n_\mu n_\nu$$

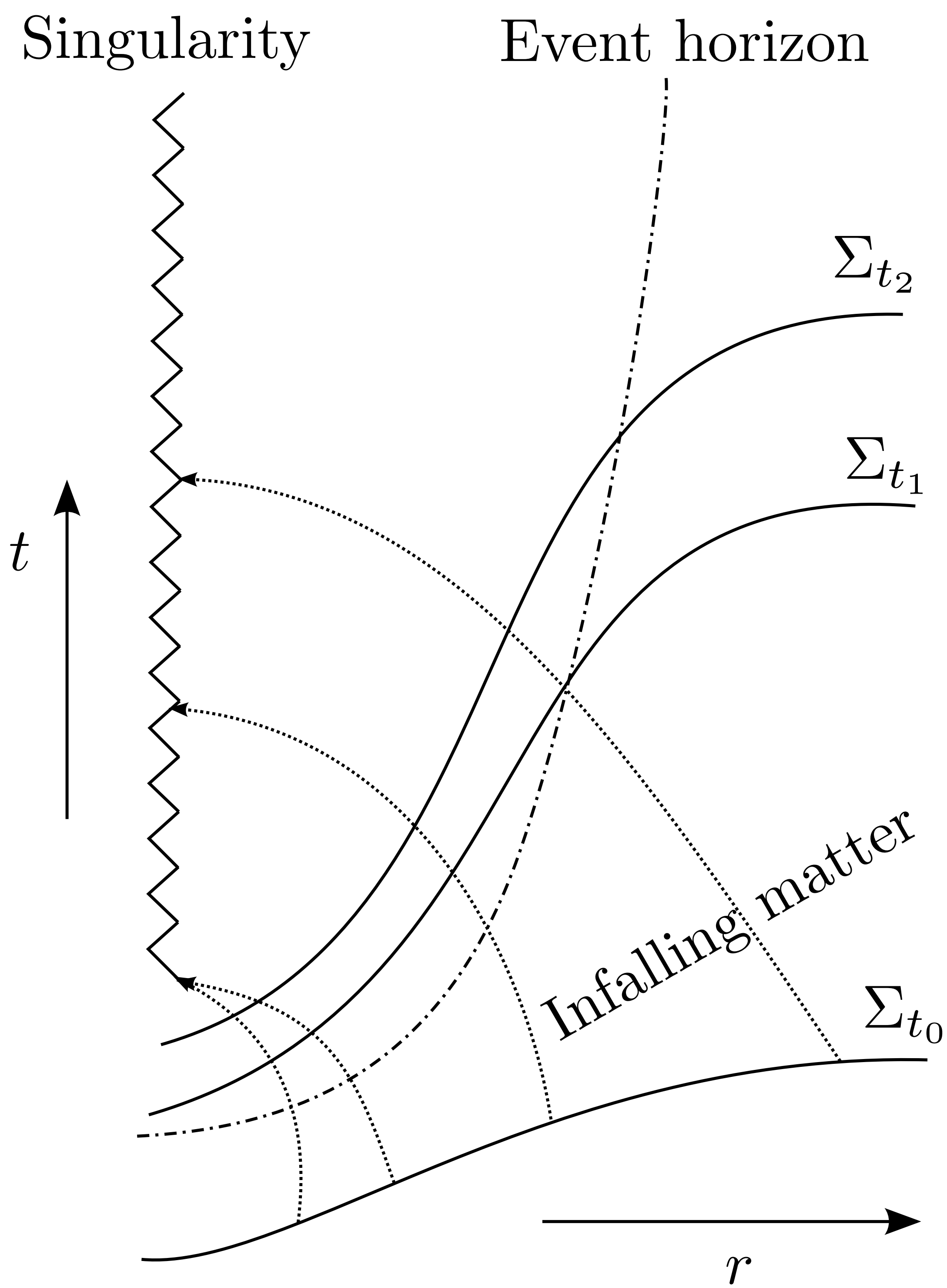
- Solve constraint equations to create initial data
- Evolve metric and extrinsic curvature
- Note: numerically stable evolution requires additional modification to the equations: BSSN, Z4c
- Another approach: generalized harmonics

Evolution equations

Constraints

Projections of the matter fields

Slicing freedom



BH formation cartoon

Einstein equations do not tell us how to traverse the spacetime: we need to choose a slicing condition

Requirements:

- smoothness
- minimize grid distortion
- avoid singularities

Two families that work in practice

- “puncture” gauge
- generalized harmonic

Moving puncture gauge

$$(\partial_t + \beta^i \partial_i)\alpha = -2\alpha K$$

- No caustics can be formed ($K = -\nabla_\mu n^\mu$)
- Singularity avoidance: the lapse drops to zero before the foliation hits a singularity (diverging *K*)
- If there is a time symmetry, the slicing tries to adapt to it
- Advection with β^i allows black-holes to move through the grid

Valencia formulation

$$\nabla_\nu T_\mu{}^\nu = 0 \quad \longrightarrow \quad \boxed{\partial_t F_\mu{}^0 + \partial_i F_\mu{}^i = S_\mu}$$

- Projection of $\nabla_\nu T_\mu{}^\nu = 0$ on the space hypersurface
- Equations similar to the classical hydrodynamics equations
- Discretize with standard high-resolution shock-capturing (HRSC) schemes
- Note S_μ is in general not zero! Balance laws.