Assigned: Wednesday, April 16, 2014
Due: Week of April 21, 2014

MTH 371: Group Project 3
Gaussian Quadrature Rules

GENERAL GROUP PROJECT GUIDELINES:

- Group project assignments should be a collaborative effort. All should participate in discussion and solution writing.
- Each week, your group must meet with Dr. Vidden to discuss your findings. All members must be present. Your grade will be assigned at the end of the meeting.
- Each student should keep group project solutions in a dedicated notebook. Bring this notebook to your weekly meeting to discuss your findings. For coded solutions, bring a laptop to your weekly meeting. Have the laptop ready before the start of the meeting.

1. Below is a table of the zeros of the kth degree Legendre polynomial for $k = 2, 3, 4, 5$. These zeros give the nodes x_i for Gaussian quadrature rules on interval $[-1, 1]$. Using the method of undetermined coefficients, find the corresponding A_i values for each k via Scilab. Summarize your results in a table.

k	x_i
2	$\pm\sqrt{\frac{1}{3}}$
3	$0, \ \pm\sqrt{\frac{3}{5}}$
4	$\pm\sqrt{\frac{1}{7}(3-4\sqrt{0.3})}, \quad \pm\sqrt{\frac{1}{7}(3+4\sqrt{0.3})}$
5	$0, \quad \pm\sqrt{\frac{1}{9}\left(5-2\sqrt{\frac{10}{7}}\right)}, \quad \pm\sqrt{\frac{1}{9}\left(5+2\sqrt{\frac{10}{7}}\right)}$

2. With the transformation $t = \dfrac{2x-(a+b)}{b-a}$, a Gaussian quadrature rule of the form

$$\int_{-1}^{1} f(t)\ dt \approx \sum_{i=0}^{n} A_i f(t_i)$$

can be used over the interval $[a, b]$. Write a function `y = GaussianQuad(f,a,b,n)` which computes the the nth Gaussian quadrature rule

$$\int_{a}^{b} f(x)\ dx \approx \sum_{i=0}^{n} A_i f(x_i).$$

Use the x_i, A_i values found from problem 1 stored as a table as well as the above transformation. Assume input `f` is a defined Scilab function.

3. Use your code from problem 2 to compute the Gaussian quadrature approximations of the following functions for $n = 1, 2, 3, 4$.
   (a) $\int_0^1 \frac{1}{\sqrt{x}}\, dx$
   (b) $\int_0^1 \frac{\sin(x)}{x}\, dx$

4. Modify the function from problem 2 to create a composite Gaussian quadrature function `y = CompGaussianQuad(f,a,b,n,m)`. This function evaluates $\displaystyle\int_a^b f(x)\, dx$ by first dividing interval $[a, b]$ into m equally spaced subintervals, then applying the nth Gaussian quadrature on each subinterval.

5. Use the function from problem 4 to compute the composite Gaussian quadrature approximations of the following functions.
   (a) $\int_0^1 x^5\, dx$ using $n = 2$, $m = 1, 2, 10$.
   (b) $\int_0^1 \frac{\sin(x)}{x}\, dx$ using $n = 3$, $m = 1, 2, 3, 4$.