Math 371
Quiz 2, February 18, 2014

Name: Solution

1. Let $f(x) = x^3$.

(a) (3 points) Find the second Taylor polynomial $P_2(x)$ of f with center $a = 1$.

$f'(x) = 3x^2$

$f''(x) = 6x$

$$P_2(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2$$
$$= 1 + 3(x-1) + 6/2\,(x-1)^2$$
$$= 1 + 3x - 3 + 3x^2 - 6x + 3$$
$$= 3x^2 - 3x + 1.$$

(b) (3 points) Use Taylor's theorem to bound the error $|f(0.5) - P_2(0.5)|$. That is, bound $|R_2(0.5)|$.

$$|f(1/2) - P_2(1/2)| = |R_2(1/2)| = \left|\frac{f^{(3)}(\xi)}{3!}(1/2 - 1)^3\right|, \quad 1/2 \le \xi \le 1.$$
$$= \left|\frac{6}{6}(1/2-1)^3\right| = |(-1/2)^3| = 1/8.$$

(c) (2 points) Compute the true error $|f(0.5) - P_2(0.5)|$.

$f(1/2) = 1/8$, $\quad P_2(1/2) = 3(1/2)^2 - 3\cdot 1/2 + 1 = 3/4 - 3/2 + 1 = 1/4.$

Thus, $|f(0.5) - P_2(0.5)| = |1/8 - 1/4| = 1/8.$

(d) (2 points) What is $P_3(x)$ and why? You answer here should be very short!

$P_3(x) = x^3$ since f is a degree 3 polynomial.