Math 371
Quiz 8, April 17, 2015 Name: Solution.

1. Consider the following function.

$$S(x) = \begin{cases} x+1, & -2 \le x \le -1 \\ x^3 - 2x + 1, & -1 \le x \le 1 \\ x - 1, & 1 \le x \le 2 \end{cases}$$

(a) List the needed conditions for $S(x)$ to be a natural cubic spline.

$S(-1^-) = S(-1^+)$, $S(1^-) = S(1^+)$

$S'(-1^-) = S'(-1^+)$, $S'(1^-) = S'(1^+)$

$S''(-1^-) = S''(-1^+)$, $S''(1^-) = S''(1^+)$

$S''(-2) = 0$, $S''(2) = 0$.

8 conditions total.

(b) List all ways in which $S(x)$ fails to be a natural cubic spline.

$$S'(x) = \begin{cases} 1, & -2 \le x \le -1 \\ 3x^2 - 2, & -1 \le x \le 1 \\ 1, & 1 \le x \le 2 \end{cases}, \quad S''(x) = \begin{cases} 0, & -2 \le x \le -1 \\ 6x, & -1 \le x \le 1 \\ 0, & 1 \le x \le 2 \end{cases}.$$

$S(-1^-) = 0$, $S(-1^+) = 2$, $S(1^-) = 0$, $S(1^+) = 0$

$S'(-1^-) = 1$, $S'(-1^+) = 1$, $S'(1^-) = 1$, $S'(1^+) = 1$

$S''(-1^-) = 0$, $S''(-1^+) = -6$, $S''(1^-) = 6$, $S''(1^+) = 0$

$S''(-2) = 0$, $S''(2) = 0$.

So, only $S''(-1^-) \neq S''(-1^+)$, $S''(1^-) \neq S''(1^+)$ & $S(-1^-) \neq S(-1^+)$.