Math 371
Quiz 10, April 25, 2014

Name: Solution.

Find A and B such that the below quadrature rule is exact for as high of degree polynomial as possible.

$$\int_a^b f(x)\,dx \approx Af(a) + Bf'(b)$$

Check the basis elements $1, x, x^2, \ldots$

$f(x) = 1$: $\int_a^b 1\,dx = \boxed{b - a = A}$

$f(x) = x$: $\int_a^b x\,dx = \frac{b^2 - a^2}{2} = A \cdot a + B = (b-a)a + B$

$\Rightarrow B = \frac{b^2 - a^2}{2} - a(b-a) = (b-a)\left(\frac{a+b}{2} - a\right)$

$\Rightarrow \boxed{B = \frac{(b-a)^2}{2}}$

So, $\int_a^b f(x)\,dx \approx (b-a)f(a) + \frac{(b-a)^2}{2} f'(b)$.

Curiosity check:

$f(x) = x^2$: $\int_a^b x^2\,dx = \frac{b^3 - a^3}{3}$

$(b-a)f(a) + \frac{(b-a)^2}{2} f'(b) = a^2(b-a) + b(b-a)^2$

$= (b-a)(a^2 - ab + b^2)$ Not exact!