MTH 371: Homework 10
Hermite and Spline Interpolation

GENERAL HOMEWORK GUIDELINES:

- On the very first page of your homework, provide your name, date, and homework number.

- Homework will be graded in part on neatness, organization, and completeness of solutions. Multiple pages MUST BE STAPLED.

- Attach all Scilab code, output, and plots to the page immediately following each problem. Also, clearly indicate the problem they correspond to.

1. Use Scilab to fit various piecewise polynomials to the Runge function $f(x) = \frac{1}{1+x^2}$ on the interval $[-5, 5]$ using 13 equally spaced points.

   (a) Use either your `LagrangeInt.sci` or `NewtonInt.sci` to find the degree 12 polynomial through these points. Plot f and this polynomial on the same graph.

   (b) Write your own code to find the piecewise linear interpolant of f through these points. Plot f and this piecewise polynomial on the same graph.

   (c) Write your own code to find the piecewise cubic Hermite interpolant of f through these points. Plot f and this Hermite interpolant on the same graph.

   (d) Use the Scilab `splin` command to find the natural cubic spline of f through these points. Plot f and this spline interpolant on the same graph.

   Feel free to use the `subplot` command to have all 4 of these plots on the same figure. Include your Scilab code with your submission.

2. Determine the piecewise polynomial function

$$P(x) = \begin{cases} P_1(x), & \text{if } 0 \le x \le 1 \\ P_2(x), & \text{if } 1 \le x \le 2 \end{cases}$$

   such that

   (a) $P_1(x)$ is linear,

   (b) $P_2(x)$ is quadratic,

   (c) $P(x)$ and $P'(x)$ are continuous at $x = 1$,

   (d) $P(0) = 1$, $P(1) = -1$, and $P(2) = 0$.

   Graph this function.

3. Let f be a given function satisfying $f(0) = 1$, $f(1) = 2$, and $f(2) = 0$. A *quadratic* spline interpolant $r(x)$ is defined as a piecewise quadratic that interpolates f at the nodes $x_0 = 0, x_1 = 1, x_2 = 2$ and whose first derivative is continuous throughout the interval. Find the quadratic spline interpolant of f which also satisfies $r'(0) = 0$. [Hint: Start from the left subinterval.] Plot your result in Scilab and compare to the cubic spline resulting from the Scilab `splin` command.

4. Let $f(x) = x^2(x-1)^2(x-2)^2(x-3)^2$. What is the piecewise cubic Hermite interpolant of f on the grid $x_0 = 0, x_1 = 1, x_2 = 2, x_3 = 3$? Let $g(x) = ax^3 + bx^2 + cx + d$ for some parameters a, b, c, d. What is the piecewise cubic Hermite interpolant of g on the same grid? [Hint: You should not need to do any arithmetic or use any formulas for either of these problems.]

5. Show that the following function is a natural cubic spline through the points $(0, 1), (1, 1), (2, 0)$, and $(3, 10)$.

$$s(x) = \begin{cases} 1 + x - x^3, & \text{if } 0 \leq x < 1 \\ 1 - 2(x-1) - 3(x-1)^2 + 4(x-1)^3, & \text{if } 1 \leq x < 2 \\ 4(x-2) + 9(x-2)^2 - 3(x-2)^3, & \text{if } 2 \leq x \leq 3 \end{cases}$$