Math 371
Quiz 9, April 11, 2014

Name: Solution

The trapezoidal rule applied to $\int_a^b f(x)\,dx$ gives value 5 and the midpoint rule gives value 4. What does Simpson's rule give? (Note: These are NOT composite quadrature rules.)

Midpoint Rule: $\int_a^b f(x)\,dx \approx (b-a) f\left(\frac{a+b}{2}\right) = 4$

Trapezoidal Rule: $\int_a^b f(x)\,dx \approx \frac{b-a}{2}\left(f(a)+f(b)\right) = 5$

Simpson's Rule: $\int_a^b f(x)\,dx \approx \frac{b-a}{6}\left(f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right)$

$$= \frac{1}{3}\left(\frac{b-a}{2}\left(f(a)+f(b)\right)\right) + \frac{2}{3}\left((b-a) f\left(\frac{a+b}{2}\right)\right)$$

$$= \frac{1}{3}\cdot 5 + \frac{2}{3}\cdot 4$$

$$= \frac{13}{3}$$