College Algebra Notes

Contents

Fun Stuff

1. Google AI experiments: `https://experiments.withgoogle.com/ai`

2. Babylonian tablet: `https://www.maa.org/press/periodicals/convergence/the-best-known-old-baby`

3. Parabola in real world: `https://en.wikipedia.org/wiki/Parabola#Parabolas_in_the_physical_world`

4. Parabolic death ray: `https://www.youtube.com/watch?v=TtzRAjW6KO0`

5. Parabolic solar power: `https://www.youtube.com/watch?v=LMWIgwvbrcM`

6. Robots: `https://www.youtube.com/watch?v=mT3vfSQePcs`, riding bike, kicked dog, cheetah, backflip, box hockey stick

7. Cat or dog: `https://www.datasciencecentral.com/profiles/blogs/dogs-vs-cats-image-classificat`

8. History of logarithm: `https://en.wikipedia.org/wiki/History_of_logarithms`

9. Log transformation: `https://en.wikipedia.org/wiki/Data_transformation_(statistics)`

10. Log plot and population: `https://www.google.com/publicdata/explore?ds=kf7tgg1uo9ude_&met_y=population&hl=en&dl=en#!ctype=l&strail=false&bcs=d&nselm=h&met_y=population&scale_y=lin&ind_y=false&rdim=country&idim=state:12000:06000:48000&ifdim=country&hl=en_US&dl=en&ind=false`

11. Yelp and NLP: `https://github.com/skipgram/modern-nlp-in-python/blob/master/executable/Modern_NLP_in_Python.ipynb` `https://www.yelp.com/dataset/challenge`

12. Polynomials and splines: `https://www.youtube.com/watch?v=O0kyDKu8K-k`, Yoda / matlab, `https://www.google.com/search?q=pixar+animation+math+spline&espv=2&source=lnms&tbm=isch&sa=X&ved=0ahUKEwj474fQja7TAhUB3YMKHY8nBGYQ_AUIBigB&biw=1527&bih=873#tbm=isch&q=pixar+animatio mesh+spline`, `http://graphics.pixar.com/library/`

13. Polynomials and pi/taylor series: Matlab/machin `https://en.wikipedia.org/wiki/Chronology_of_computation_of_%CF%80` `https://en.wikipedia.org/wiki/Approximations_of_%CF%80#Machin-like formula` `https://en.wikipedia.org/wiki/William_Shanks`

Course Introduction

1. What is algebra? Complete the sentence: Algebra is

   - the math of equations.
   - the study of math symbols.
   - literally translated as the "reunion of broken parts"

2. Most important use of algebra is the idea of a function.

Chapter 1 Equations and graphs

1. Motivation: Housing data and curve fitting.

   (a) R code. Plot of Ames housing data.

   (b) On own: Describe what you see. What are the key features of the graph? What conclusions can you draw?

   (c) Axis labels super important, axis scales can differ, title super important, can also be misleading

   (d) Intuition: add extreme dots and interpret

   (e) What if reversed x,y axis? Same data new meaning.

   (f) Often no equation (general curve) to fit real data. Not even a function in this case.

2. Linear regression: best fit line

   (a) How to interpret line? Slope is value of each square foot

   (b) How to tell if line is any good?

   (c) What if above line? Below? Distance from line is important.

   (d) Can use more complex curves (polynomials, others). Time series and forecasting.

3. Chapter outline: Will teach from scratch, fast pace, deeper understanding.

   (a) Understand 2 dimensional space (cartesian plane, distance)

   (b) Curves in 2 space (circles, lines, basics)

   (c) Regions in 2 space (inequalities)

   (d) Solving equations and inequalities compared to graphs.

.1   1.1 The coordinate plane

1. Rectangular (Cartesian) coordinate system

   (a) Draw coordinate plane

   (b) What does it represent? 2D space (width, height), time and quantity (temp), anything else

   (c) x, y axis

   (d) $P = (x, y)$ point in space, over and up, not same as (y, x). Example point. What does it represent?

   (e) Title, units and labels super important (housing example)

   (f) Important features: Origin, quadrants, axis

   (g) Note, other coordinate systems (polar coords for rotation)

2. Curves and regions in the plane

   (a) Efficient to define many points at once. Connect visualization (intuition) with equation (calculation)

      - Equations represent curves in space. $x = 4$ (all points $(4, y)$, $y = -2, x = y$.
      - Inequalities represent regions in space. $y \geq 0$, $-1 < x <= 3$, , $y > 1$ and $x <= 0$

   (b) Try on own, check in Desmos: $x = 4$ and $y >= 0, y = -2$ and $-1 <= x < 0, xy = 0, xy > 0, y = |x|, x = |y|$

3. Pythagoras and the distance formula.

   (a) Example: 2 random points $P_1 = (1, 2)$, $P_2 = (-3, 4)$, draw triangle separate and remind of Pythagorean theorem

(b) General formula for any 2 points. $d(P_1, P_2)$, $P_1 = (x_1, y_1)$, $P_2 = (x_2, y_2)$.

(c) Pythagorean theorem (picture proof of why is true). Wiki page.

(d) Example: Do the three points $(-1, -3), (6, 1), (2, -5)$ form a right triangle?

4. Averages and the midpoint formula

(a) Find the midpoint between $(1, 2)$ and $(-3, 4)$. Verify via the distance formula.

(b) General case for any two points.

5. Summary of two formulas to memorize.

6. Homework: 5, 7, 13, 15, 17, 19, 23, 27, 33, 35, 39, 45

.2   1.2 Graph of Equations

1. Motivation: Equations and graphs

(a) Gas at the pump.

(b) Plot points, generalize to equation of line (considering all points at once). $x-$int, $y-$int.

(c) What if: know tank capacity? Domain and range, add car wash?

2. Graph intuition: Try on own.

(a) Complete a table for $x = -3, -2, -1, 0, 1, 2, 3$. Generalize to graph.

(b) $y = x^2, 2x^2, 2x^2 - 4$ graph transformations, get to all parabolas $ax^2 + bx + c$.

3. Important graph features: $y = 2x^2 - 4$

(a) Domain, range, intercepts, symmetry (x, y, rotational, how to check?)

(b) Must there be intercepts / symmetry? No

(c) Increasing and decreasing.

(d) Trending: as x goes to $+ - \infty$, what happens to y? Asymptotes

4. Graph symmetry:

(a) Examples of each case.

(b) Find intercepts, check for symmetry: $\frac{x^2}{9} + \frac{y^2}{4} = 1$, ellipse.

(c) Show in desmos.

5. Important classes of equations and graphs

(a) Lines (next section), parabolas (quadratic), polynomials (next chapters), absolute value (review now), root

(b) Circles and ellipses

(c) Rationals (chapter 3)

(d) Exponential, logarithm (chapter 4)

(e) Trigonometric (precalc)

(f) More...functions are an important case.https://en.wikipedia.org/wiki/List_of_mathematical_functions

(g) Serve 2 purposes:
- simplification of reality which allow us to better understanding
- examples which we can study completely to develop rich theory

6. Circles: Generalizing the distance formula.

   (a) Definition: A circle of radius $r > 0$ and center $C = (h, k)$ is the set of all points $P = (x, y)$ of distance r from C.

   (b) Draw picture. $d(P, C) = r$ gives

   $$(x - h)^2 + (y - k)^2 = r^2.$$

   (c) Example: Circle with center $C = (1, 2)$ and radius $r = 3$.
      - Write equation and verify 4 easy points work.
      - How to find intercepts, maybe only x in this case?
      - Expand out to show non-standard form. How to go back? Complete the square, later in course.

   (d) Try on own: Find equation of circle with two points on a diameter as $(-1, 3), (7, -5)$. Check equation in Desmos, tell to use Desmos in HW

7. Homework: 9, 13, 19, 23, 33, 49, 55, 59, 67, 69, 73, 77, 81, 95, 99

.3   1.3. Lines

1. Idea of a line

   (a) Draw line and curve.

   (b) How to carefully distinguish? Line has constant rate of change, known as the slope. (draw many cases of lines)

2. Example:

   (a) Line thru points $A = (1, 2)$ and $B = (3, -4)$.

   (b) What is the constant rate of change?

   $$m = \frac{y2 - y1}{x2 - x1} = -3 = \frac{\Delta y}{\Delta x} = \frac{rise}{run}$$

   (c) How to define in general for any point (x, y)? Slope still should hold.

   $$m = \frac{y - y1}{x - x1} = -3$$

   for any point on the line. Leads to point-slope form.

   (d) Find another point on line and double check slope is the same.

3. Equations and cases of lines:

   (a) Slopes cases and graphs: Positive, negative, zero, none

   (b) Equations of lines:
      - Point-slope form (reaffirms constant change)
      - Slope intercept (simple, know where you start)
      - Standard form $Ax + By + C = 0$ (most general, every line captured, many equations for same line, nice for theory)
      - Which form is best? Different intuition of each, different interpretation.

   (c) Find the equation of the line given two intercepts. Point and slope.

4. Example: Interpretation of lines.

(a) Getting gas and car wash

(b) General slope-intercept form: $y = mx + b$

(c) How much gas can you get if only have \$20?

5. Comparing lines

   (a) Parallel / perpendicular / intersecting lines.

      - Example: Find a line perpendicular to $y = 2x$. Show perpindicular.
      - General case for $y = mx + b$.

   (b) Example: Find the perpendicular bisector of segment AB where $A = (1, 4), B = (7 - 2)$. Find its x and y intercepts. Graph result. Check in Desmos.

6. Homework: 9, 17, 21, 23, 25, 29, 35, 37, 43, 47, 61, 63, 65, 67, 77, 81

.4   1.4 Solving quadratic equations

1. Equation solving:

   (a) Linear: $mx + b = 0$, $m \neq 0$ (easy to solve)

   (b) Quadratic: $ax^2 + bx + c = 0$, $a \neq 0$ (harder / richer)

      - Shows up in naturally many places: geometry, physics, optics, optimization, finance
      - Babylonian story and the solutions (YBC 7289)
      - Here we just focus on solving equations, graphs in Chapter 2-3

2. Solving quadratic equations $ax^2 + bx + c = 0$. Three methods:

   (a) Factor (always easiest, not always doable)

   (b) Complete the square (useful technique in surprising places)

   (c) Quadratic formula (memorize, can be tedious)

3. Example: Try on own. Solve $2x(x - 2) = x + 3$ for x.

   (a) 2 ways, factoring and quadratic formula

   (b) Check solutions work

   (c) Note why factoring separates into two equations ONLY if one side is zero.

   (d) Note factoring not always possible

4. Complete the square

   (a) Basic examples: $x^2 = 4$, $(x - 1)^2 = 4$, $3(x - 1)^2 = 4$, note the +- for two solutions.

   (b) What if not in this form? $3x^2 - 6x - 1 = a(x - h)^2 + k = 0$, needs some work to get there.

   (c) Idea: Rewrite $ax + bx + c = 0$ (standard form) into $a(x - h)^2 + k = 0$ (vertex form)

   $$3x^2 - 6x - 1 = 0 \leftrightarrow 3(x - 1)^2 = 4$$

   (d) First example: $x^2 + 6x - 7 = 0$. Note could have factored.

   (e) Less basic: $3x^2 - 6x - 1 = 0$.

   (f) Try on own: $4x^2 - 40x + 13 = 0$. Check via quadratic formula, doesn't factor easily. Note how similar to quadratic formula.

   (g) Quadratic formula derived: $ax + bx + c = 0$ into $a(x - h)^2 + k = 0$. Wikipedia animation: https://en.wikipedia.org/wiki/Completing_the_square

5. Number of solutions of a quadratic equation

(a) 0-2 solution: Investigate each and discuss why. ($x^2 = 0, 1, -1$ and compare to completing the square process).

   (b) Discriminant and the quadratic formula (give a table with number of solutions).

6. Homework: 7, 13, 17, 25, 25, 35, 37, 41, 57, 59, 61, 69, 75, 85

.5 1.5 Complex numbers

1. Quadratic equation with no real solution

   (a) $x^2 = -1$ not solvable with real numbers. Extending our real number systems allows for a solution.

   (b) Imaginary number: Define $i = \sqrt{-1}$.

   (c) Powers of $i, i^2, i^3, ...$ cycle. 4 cases.

2. Complex numbers and the complex plane.

   (a) Real numbers, imaginary numbers, add to get complex numbers in complex plane. Real and imaginary parts.

   (b) Graph in the complex plane.

   (c) Think of an extension of real numbers to allow more room.

   (d) Complex conjugate, pairs of numbers.

3. Complex arithmetic:

   (a) Add / subtract

   (b) Multiply

   (c) Divide, multiply by the conjugate

   (d) Note: Result is always of the form $a + bi$.

   (e) Example: Try on own
   $$\frac{(1 + 2i)(3 - 4i)}{5 + 6i}$$

   (f) Example: Try on own.
   - Solve for x: $x = 6 - 13/x$
   - Note a quadratic, check the discriminant for them to know expect complex.
   - Verify one of the solutions, can see where conjugate comes from

4. Applications of complex numbers: `https://en.wikipedia.org/wiki/Complex_number#Applications`

5. Homework: 7, 11, 19, 29, 35, 39, 43, 45, 47, 49, 55, 57, 63, 69

.6 1.6 Solving other types of equations

1. Ideas of this section: Handle equations with...

   (a) Factoring by grouping for polynomial equations

   (b) Fractional expressions

   (c) Mixed powers and radicals

   (d) Substitution

2. Factoring by grouping:

   (a) Example: $3x^3 - 5x^2 - 12x + 20 = 0$

(b) Note, solving cubics not easy compared to quadratics.

(c) Cubic formula for general solution.

3. Fractional expressions and extraneous solutions (solutions introduced thru alg operations, domain change, always check at end).

$$\frac{1}{x-6} + \frac{x}{x-2} = \frac{4}{x^2 - 8x + 12}$$

4. Fractional powers, remove the root, more extraneous solutions possible.

(a) Example: $x + \sqrt{5x - 19} = -1$

(b) Try on own: $\sqrt{2x - 3} - \sqrt{x + 7} + 2 = 0$

5. Substitution, simple but powerful technique.

$$x^6 - 3x^3 - 40 = 0$$

6. Homework: 7, 19, 21, 25, 27, 33, 35, 43, 47, 55, 57, 59, 67, 73

.7   1.7 Solving inequalities

1. Inequality basics:

(a) $<, >, \geq, \leq$

(b) Draw on number line, give interval notation: $x > -1$, $x \leq 2$, $-1 < x \leq 2$.

(c) Emphasize the difference between $[$ and $($.

(d) Intersection and union notation. And vs or.

2. Rules for inequalities: Add, subtract, multiply, divide

(a) $x > 1, x + 2 > 1 + 2, 2x > 2, x/3 > 1/3$, draw number lines.

(b) Negative multiplication / division exception: Reflection about zero. $2 > 1, -2 < -1$

3. Solving linear inequality:

(a) Example: $9 + x/3 > 4 - x/2$

(b) Use above operations. Avoid negative mult / division if possible.

(c) Not easy to check, can use Desmos to visualize.

(d) Double linear inequality. Think of as two inequalities (and), but can combine work into one.

$$2 < (6 - 5x)/3 \leq 5$$

Check via Desmos.

4. Solving nonlinear inequalities.

(a) Example: $x^2 - 3x <= 4$

(b) Factor and consider cases, sign chart on number line, write as both interval and inequality, RHS must be zero. Don't divide by expressions involving x since we don't know the sign.

(c) Try on own: $-3x^2 < -21x + 30$

5. Challenge problems.

(a) $(x - 5)(x - 2)(x + 1) > 0$. Zero on one side is key. Draw number line and sign chart.

(b) Try on own: $(1 + x)/(1 - x) >= 1$

(c) Try on own: $(x+2)/(x+3) < (x-1)/(x-2)$

6. Modeling

- Jobs: Number of employees x. I have 220 hours work to cover every week and each person works 40 hours per week. I pay 1000 per person per week and I have a budget for 7500 per week. What are the possible number of employees?

- Car rental: plan A, 30 per day, 0.1 per mile, plan B, 50 per day, 0.05 per mile. For what range of miles will plan B save your money?

- Projectile: A ball is thrown upward with an initial velocity of 20 ft/s from the top of a building 100 ft high. It's height h above the ground t seconds late will be $h = 100 + 20t - 20t^2$. During what time interval will the ball be at least 60 ft above the ground?

7. Homework: 19, 27, 33, 37, 39, 47, 51, 55, 59, 67, 81, 93

.8 1.8 Solving absolute value equations and inequalities

1. Idea of absolute value:

   (a) Distance from zero on the number line
   (b) Piecewise definition
   (c) Graph via PW definition
   (d) Important when thinking of size

2. Absolute value equations: Goal is to isolate and remove the absolute value.

   (a) Basic examples: $|x| = 2, |x - 3| = 1$. Number line version of each. Plug in and check.
   (b) $3|2x - 7| - 9 = 0$. Check via Desmos. What if +9? No solution possibility.
   (c) Try on own: $|x + 3| = |2x + 1|$. Check via Desmos.

3. Absolute value inequalities: Goal is to isolate and remove the absolute value.

   (a) Examples: $|x| < 2$, $|x| > 2$. Again explain by distance. Interval notation for the solution.
   (b) Example: $|3x + 1| + 4 > 15$
   (c) Example: Try on own: $1 < |x - 5| <= 3$
   (d) Nuance example: $(x - 1)^2 > 4$. Variation: $(x - 1)^2 < 4$
   (e) Challenge example: $|x + 1| + |x - 2| \geq 4$

4. Homework: 7, 9, 13, 17, 21, 25, 27, 31, 39, 45, 49, 51, 53, 55, 57

.9 Chapter 1 Review

Exam review problems:

1. Chapter 1 Review:

2. Concept check: 1-20

3. Exercises: 1-30, 35-94

4. Chapter 1 test: 1-14

Chapter 2 Functions

1. Function: the most important idea of this class.

   - Applications: Channel an action into equation
   - Math: Foundation of all math theory

2. Equations vs functions

   - Static: Solve $3 = 4x - 5$ for x
   - All cases: Study $y = 4x - 5$ for all (x, y), as x changes how is y influenced?
   - Key is generalization, thinking of this equation as an action performed on input x to generate output y.
   - Housing example and linear regression. Instead of understanding a single house instance, we can to generalize to the entire real estate situation.

3. Power of functions:

   - Practical view: Understanding and intuition. General rule (conceptual), formula (calculation), graph (intuition), super rich discussion
   - Math Theory foundation: Study many situations at once (classes of functions, lines, polynomials, exp; large groupings, smooth, continuous). Calculus is built on this, all smooth functions. More general makes for fuzzy understanding, but far reaching.

4. Chapter outline:

   - Idea of functions (concept, visualize)
   - First full function story (linear, quad/poly/rationals in chp 3, exp/logs in chp 4)
   - Combining / transforming simple functions to harness complexity
   - Reversing functions (inverses)

5. Application: Real world is complex, how to find a function that represents it? Machine learning is one way.

   - Classification: Cat, bird, bird song,
   - Forecasting: Stock market, covid
   - Many more

.1 2.1 Functions

1. Idea and definition of function:

   (a) Complete this sentence: A function $f(x)$ is....
   (b) What is the key idea? Why does it matter? Go as detailed as possible.
   (c) Definition: A function $f(x)$ is a rule which maps one input x to at most one output y, written $y = f(x)$.
   (d) Diagram to conceptualize: Domain, range, indep variable, dep variable
   (e) Most important part: One input to at most one output. Why? Want certainty in what will happen.
   (f) Think of as a machine: One thing in, at most one out.
   (g) Example: Email mapped to student. Want message to go to one person. Is reverse association possible?

2. Representations of functions:

   (a) Description (easy to understand, hard to use)

   (b) Table (practical and easy, not general)

   (c) Formula (good for calculation, not intuitive)

   (d) Graph (good for intuition, not practical)

   (e) Example: Line $y = 2x + 1$. 4 versions.

   (f) Best case is knowing all versions.

3. Example: Consider $y = f(x) = x^2 + 2x - 3$

   (a) Input to output: $f(2), f(a + 2)$.

   (b) Output to input: When is $f(x) = -3$? More at end of chapter.

   (c) Try on own: Simplify the difference quotient $(f(a + h) - f(a))/h$.

4. Piecewise functions:

   - Combining rules, three part example (const, $-2x + 1$, x^2).
   - Graph. Function evaluations.
   - Should remind of absolute value.

5. Finding domain and range:

   (a) $y = f(x) = x^2$, parabola, will study quadratics as transformations of this one in chapter 3.

   (b) Define domain and range. Draw diagram.

   (c) Find domain and range, check in desmos.

   (d) Domain usually easy, range take knowledge or intuition and often not doable.

   (e) Try on own: $f(x) = 3/\sqrt{4 - x}$. What can't x be? Check in desmos, graph inequality

   (f) Try on own: Find the domain of $f(x) = \frac{\sqrt{x^2 - 1}}{x + 2}$ and write in interval notation. Range not easy. Check for zero division and negative roots, same for most of this class with functions.

6. Homework: 1, 3, 5, 7, 11, 17, 19, 21, 25, 29, 31, 33, 37, 39, 43, 49, 51, 55, 59, 69

.2   2.2 Graphs of functions

1. Graph features and stock prices.

   (a) Google tesla stock

   (b) Graph labels important

   (c) Is this a function? What is x, y? Domain / range?

   (d) Key features: Inc / dec, max / min (local and global)

   (e) Net change: $f(b) - f(a)$, average change: $(f(b) - f(a))/(b - a)$

   (f) Hard to find precise values with graph, better to have a formula $f(x)$. Not doable here.

2. Graph features: Definitions

   (a) Draw $y = f(x)$, general graph on side

   (b) Vertical line test

   (c) Domain / range

   (d) Inc / dec

(e) Max / min, local and absolute

3. Example: Does the equation $x^2 - y = 7$ represent a function?

   (a) Is y a function of x? Is x a function of y? If yes write in function notation.
   (b) Graph function knowing basic $y = x^2$.
   (c) Domain and range
   (d) Try on own for $x^2 + y^2 = 9$. In some cases can solve for y as two separate functions. Easy if know the graph.

4. Graphs of functions: Visualizing a graph gives intuition of its behavior.

   (a) 2 ways to draw graphs:
     • Plot points, last resort, better left to computers
     • Understand classes of function (lines, parabolas, etc).
   (b) Important classes of functions:
     • Linear: $f(x) = mx + b$
     • Powers: x^n, even vs odd cases
     • Root: $\sqrt[n]{x}$
     • Reciprocal: $1/x^n$
     • Abs value: $|x|$
     • Greatest integer: $[x]$
     • Will modify these later to grow complexity.

5. Homework: 5, 7, 9, 15, 19, 23, 35, 37, 45, 49, 51, 53, 57, 61, 77

.3   2.3 Information from graphs of functions

1. Recall: Key graph features.

   (a) $y = f(x)$, general graph on side
   (b) Vertical line test
   (c) Domain / range
   (d) Inc / dec
   (e) Max / min, abs and local
   (f) Example: Find all important features of $f(x) = \sqrt{9 - x^2}$.
   (g) Example: Graph comparisons of $g(x) = x >, <, =, f(x) = \sqrt{9 - x^2}$.
   (h) Try on own: Graph comparisons of $g(x) = x^2 >, <, =, f(x) = x + 2$.

2. Homework: 7, 9, 11, 13, 15, 23, 25, 31, 33, 43, 45,

.4   2.4 Average rate of change of a function

1. AROC idea:

   (a) Dis $=$ rate (time), one decides the other, odometer vs spedometer.
   (b) What is the average velocity ($v(t)$, change in distance) if I go 20 miles in 15 mins? Note, we don't know a formula for v for all t.
   (c) Also called difference quotient.
   (d) Distinguish between net change and AROC.

2. AROC and function graphs:

   (a) $f(x) = x^2$ graph, AROC on [1,3]. Slope of secant line.

   (b) General case $f(x)$, 2 forms.

3. Lines redefined, the precise way.

   (a) Line $y = f(x) = mx + b$ is the function such that AROC is always constant.

   (b) $(f(b) - f(a))/(b - a) = \cdots = m$ for any a, b.

4. Calculus and the paradox of AROC:

   (a) First problem of calculus considers AROC approaching IROC. Secant lines approaching tangent lines.

   (b) Need a way around zero division. Invention of idea of limit is the key.

   (c) Desmos tangent and secant line

   (d) `https://en.wikipedia.org/wiki/Rate_(mathematics)#Rate_of_change`

   (e) 3blue1brown the essence of calculus

5. Homework: 7, 9, 11, 15, 19, 23, 25, 27,

.5 2.6 Transformation of functions

1. Motivation: Power of linear transformations.

   (a) List of 6 transformations for $f(x)$ and general combination.

   (b) Why bother? Can go from a basic function ($y = x^2$) to very complex transformation ($y = -2(x-3)^2 + 10$).

   (c) Grow tremendously the number of functions we can graph and understand.

   (d) Study each individually to gain intuition. Combine to understand complexity.

2. y-direction transformations for $f(x) = x^2$.

   (a) Summary: $y = f(x)$ vs $y = f(x) + k$ vs $y = af(x)$ vs $y == -f(x)$ vs all combined $y = -af(x) + k$.

   (b) Vertical shift:
       - $f(x) + k$, how is the output of f changed? Note, output changes, input location does not.
       - $y = f(x) = x^2, f(x) + 2, -3$.
       - Use Desmos tables. Draw +2 carefully on slide. Keep track of three points moving.
       - This balance of formula, table, graph is what grows your intuition in this class, rich diversity of perspective.

   (c) Try yourself:
       - Remaining 2, $af(x), -f(x)$
       - Try in Desmos for various choices. Explain what happens.
       - Y multiplied (vertical stretch). Y flipped in sign (x-axis reflection)

3. x-direction transformations for $f(x) = x^2$.

   (a) Summary: $y = f(x)$ vs $y = f(x-h)$ vs $y = f(bx)$ vs $y == f(-x)$ vs all combined $y = f(-b(x-h))$.

   (b) Horizontal shift:
       - $f(x - h)$, how is the output of f changed? Note, output stays same, input location changes. Why minus?
       - $y = f(x) = x^2, f(x - 2), +3$

- Use Desmos tables. Draw -2 carefully on slide. Keep track of three points moving.

(c) Try yourself:

- Remaining 2, $f(bx)$, $f(-x)$
- Try in Desmos for various choices. Explain what happens.
- Note the reversing of horiz stretch
- Trace points

4. Combining transformations:

   (a) Example: Knowing the graph of $y = x^2$, graph instead $y = -3(x-1)^2 + 2$.

   (b) Break into many steps. Order of steps matters. Track three main points. Show how to check in end.

   (c) Trace points

   (d) Standard form $ax^2 + bx + c$ to vertex form $a(x-h)^2 + k$. Complete the square, use transformations of x^2.

5. Graph symmetry:

   (a) 3 types for general graphs: x-axis, y-axis, rotational. How to describe carefully? For all (x,y) on the graph, what other point must be on?

   (b) 2 cases apply to functions. Name even and odd for power functions x^n, give careful definition, criteria to check. Goal is to identify and leverage pattern in a function

   (c) Check if even, odd , or neither.

   - $f(x) = x^4 - 2|x|$
   - $f(x) = x + 1/x$
   - $f(x) = x + |x|$
   - Note we don't need to know the graph (shouldn't trust anyways).Check via Desmos.

6. Homework: 1, 3, 7, 9, 11, 15, 23, 25, 27, 29, 33, 41, 45, 47, 51, 69, 71, 83, 85, 87

.6   2.7 Combining functions

1. Idea: 2 ways to grow complexity of functions to understand.

   (a) Graph transformations

   (b) Combine functions

2. Definitions: Arithmetic combinations of functions $f(x)$ and $g(x)$.

   (a) $f + g, f - g, f \cdot g, f/g$

   (b) Only new thing is notation, take care about how you write.

   (c) Domain is $D_f \cap D_g$, avoid zero division for last case.

   (d) Example: $f(x) = \sqrt{x+2}, g(x) = x/(x+1)$, compute $(f+g)(1), (f/g)(0)$.

   (e) Example: Compute $(f/g)(x)$ in general. Keep track of domain changes and compare to $D_f \cap D_g$, avoid zero division.

   (f) Basic arithmetic operations bit boring. Simple, notation just shorthand.

3. Function composition definition and idea.

   (a) Doing functions in order more interesting.

   (b) $(f \circ g)(x) = f(g(x))$, draw diagram, relay race.

(c) Do an operation first, apply to the result.

(d) Domain, all x in D_g such that $g(x)$ is in D_f.

(e) Note, $(f \circ g)(x) \neq (g \circ f)(x)$.

(f) Example: Savy shopper. 10\$ off. 7% discount. Which to use first? Let x be the cost of the thing. Write as two functions: $f(x) = x - 10, g(x) = 0.97x$, $f(g(x))$ vs $g(f(x))$.

(g) Example:

- $f(x) = x/(x - 2), g(x) = 1/x$
- $(f \circ g)(x)$ at $x = 3, 1/2$
- Domain is the key discussion, $1/2$ not in the domain even though $g(1/2)$ makes sense.
- Find the domain 2 ways: need x in domain of g, $g(x)$ in domain of f, simplify keeping track of domain changes.

(h) Example: Try on own. $f(x) = x/(x^2 - 1)$, $g(x) = 2x - 1$, $f \circ g$ versus $g \circ f$. Domain of each, note not the same.

4. Example: Decomposing composition.

(a) $h(x) = (2x + 5)^3$, many ways to write as composition, no uniquiness.

5. Example: Next section teaser.

(a) $f(x) = 3x - 5, g(x) = 1/3x + 5/3$

(b) $f \circ g = g \circ f = x$

(c) Draw diagram. g action undoes f action.

(d) This is the idea of inverse, next section.

6. Iteration and chaos: `https://en.wikipedia.org/wiki/Chaos_theory`

7. Homework: 1, 9, 11, 15, 17, 21, 27, 29, 31, 33, 35, 37, 49, 55, 59, 63, 73, 81, 83

.7   2.8 One to one functions and their inverse

1. Key idea: Reversing a function.

(a) Thinking of a function as an action, how to reverse this action? Reverse is called the inverse function.

(b) Domain/range diagram. Notation.

(c) Question: Can you always reverse a function? No. Need each output to belong to a single input.

(d) Examples: Student to id, encrypt and ecrypt, currency exchange, feet/meters Need certainty for reversing. Not always possible. Student to class.

(e) Example: $f(x) = -2x + 1$. Given and output, find the input. $y = -3, 5$. Draw graph. General y. New function $x = f^{-1}(y)$ is the reverse mapping of f

2. Definitions:

(a) Careful definitions: One-to-one function, inverse function, diagram, function composition identities, flip of domain and range.

(b) How to tell in invertible (one-to-one)?

- $f(x) = -2x + 1, g(x) = x^2$
- 2 ways, definition (if outputs match, show inputs match or give contradiction), graph (HLT, not precise).

3. Examples

(a) $f(x) = x^2 - 1, x >= 0$ domain restriction to make invertible

(b) Graph, show one-to-one

(c) Find the inverse. Check composition identity.

(d) Graph both, relate the domain and range. Note reflective symmetry.

(e) Graph in Desmos, hint at general case.

4. Graphs of a function and its inverse

(a) General picture. Domain and range flip.

(b) Reflective symmetry across y=x.

(c) All different views of the same basic definition.

5. Example: Try on own.

(a) F(x) =(x+2)/(2x-1)

(b) Use Desmos to show invertible

(c) Find inverse, verify composition identity.

(d) Verify domain / range relation

6. Homework: 5, 7, 9, 13, 15, 17, 23, 25, 29, 31, 39, 43, 47, 49, 55, 61, 69, 73

.8 Chapter 2 Review

1. Concept Check: 1-9, 11-14

2. Exercises: 1-42, 51-56, 69-74, 85-92, 95-102

3. Chapter test: 1-4, 6-21

Chapter 3 Polynomial and rational functions

1. Goals of the chapter:

(a) Study an important class of function: Polynomials

- Application importance: Approximation by polynomials (Computer graphics, math modeling, physics, splines, FE method, parabolas)
- Math importance: Represent any function $f(x)$ as a power series.

(b) Study infinity (asymptotics) via rational functions.

.1 3.1 Quadratic functions and models

1. Why quadratic functions?

(a) Applications: `https://en.wikipedia.org/wiki/Parabola`, death ray, etc

(b) Math theory: Rich example compared to lines, balance equation solving and alg techniques with graphs and intuition.

2. Basics:

(a) Definition: Standard form and vertex form

(b) Connect to what we know via graph transformations

(c) Vertex, axis of symmetry (max/min location), concave up / down

(d) Example: $f(x) = x^2 - 5x - 6$.

- Complete the square to vertex form.
- Graph via transformations and label details.
- Complete the square to vertex form.
- Can also find the vertex midway between the x intercepts and skip graph transformations. Vertex also appears in quadratic formula.

(e) Example: Try on own. $f(x) = -2x^2 - 12x - 8$.

3. Quadratic function summary

(a) Recap main ideas.

(b) Hint at applications with max / min values (optimization).

4. Applications of quadratics.

(a) Chicken run, 200ft of fence. How to maximize area enclosed?

5. Homework: 5, 7, 15, 17, 23, 27, 29, 47, 51, 57, 63

.2 3.2 Polynomial functions and their graphs

1. Why polynomials?

(a) Applications: Computers, splines and yoda, approximation in general.

(b) Math: Taylor series and theory of all functions

(c) This class: Important class of functions, think in terms of generalization (constant, to linear, to quadratic, and so on)

2. Definition:

(a) General definition, coefficients, degree, domain, range, standard form

(b) Graphs: linear, quad, cubic, quart, ... graph to dev intuition

(c) Degree n can have at most $(n - 1)$ extrema

3. Examples:

(a) $p(x) = -2(x - 4)^2(x + 1)$ as factored form. Find degree, leading coefficient, zeros, multiplicity, real / complex zeros, Graph in Desmos, end behavior same as high order term.

(b) $p(x) = x^4 + x^3 - x^2$. Factor completely and sketch the graph. Domain, range, end behaviour, multiplicity of zeros. Some features need calculus to get all details. Take advantage of multiplicity on sign chart.

(c) Try on own: $p(x) = 2(x + 3)(x - 1)^2(x - 2)$.

4. Homework: 5, 7, 9, 11, 13, 17, 23, 25, 29, 35, 37

.3 3.3 Dividing polynomials

1. Motivation:

(a) Graphing polynomials is doable if you have factored form. How to get factored form?

(b) Find a zero, and divide by the corresponding factor.

(c) Find any factor and divide.

2. Recap of integer long division: Same as long division for polynomials.

(a) Try on own: $1234/8 = 161 + 6/8$. Dividend, divisor, quotient, remainder.

(b) Remainder. Can rewrite as $1234 = 8 * 161 + 6$

(c) If remainder is zero, division is really factoring: $1228/8 = 161$ is same as $1228 = 8 * 161$. This is our target.

3. Polynomial long division:

(a) Example: $(2x^3 - 7x^2 + 5)/(x - 3)$. Check via multiplication. Note, if higher degree divisor, can do just as easy $(2x^3 - 7x^2 + 5)/(x^2 - 3)$.

(b) Example: Try on own. Solve $x^3 - 7x + 6 = 0$. Note, $x = 1$ is a solution, meaning $(x - 1)$ is a factor. Do long division. Expect quotient to be a quadratic. Check solution via desmos.

(c) Overview:
   - $p(x)/d(x) = q(x) + r(x)/d(x)$ terminolgy
   - $p(x) = q(x)d(x) + r(x)$
   - Compare to previous example
   - Note, if $r(x) = 0$ then $d(x)$ is a factor of $p(x)$
   - Challenge of factoring, need to know a zero first. Many techniques.
   - Synthetic division vs long division. Is short hand better?

4. Homework: 3, 7, 9, 13, 17, 19, 57, 61, 63, 67, 71, 73

.4   3.4 Real zeros fo polynomials

SKIP

1. Rational zeros of polynomial

   - Rational zeros theorem: if the polynomial $P(x) = c_n x^n + \ldots c_1 x + c_0$ has integer coefficients (where $c_n \neq 0$ and $c_0 \neq 0$), then every rational zero of P is of the form p/q (fraction in lowest terms) where p and q are integers and p is a factor of a_0, q is a factor of a_n.
   - Proof: Assume p/q is a rational zero. Then $P(p/q) = 0$ and rearranging yeilds

   $$p(a_n p^{n-1} + a_{n-1} p^{n-2} q + \ldots a_1 q^{n-1}) = -a_0 q^n$$

   So p is a factor of the number on the left and since p/q is in lowest terms, a_0 must have a factor of p.
   - Process:
     (a) List all possible zeros and check if they work.
     (b) Once you find a zero. Divide and find zero remainder.
     (c) Repeat.
   - Example: finding rational zeros of $P(x) = 2x^3 + x^2 - 13x + 6$, $P(x) = 12x^3 - 20x^2 - 13x - 6$, $p(x) = x^4 - 5x^3 - 5x^2 + 23x + 10$.

2. Decartes' rule of signs: OMIT

   - the number of positive real zeros of $P(x)$ is equal to the number of variations in sign in P(x) or is less than that by an even whole number
   - the number of negative real zeros of $P(x)$ is equal to the number of variations in sign in P(-x) or is less than that by an even whole number

3. Upper and lower bounds theorem: OMIT

- If we divide $P(x)$ by $x - b$ with $b > 0$ using synthetic division and if the row that contains the quotient and remainder has no negative entry then b is and upper bound for the real zeros of $P(x)$

- If we divide $P(x)$ by $x - a$ with $a < 0$ using synthetic division and if the row that contains the quotient and remainder has entries that are alternately nonpositive and nonnegative, then a is a lower bound for the real zeros of P

- Show that all the zeros of the polynomial $P(x) = x^4 - 3x^2 + 2x - 5$ lie between -3 and 2

- Does it make sense? Try take a big upper bound and small lower bound

4. Factoring any polynomial and graph the polynomial: OMIT

$$x^4 - 6x^3 + 3x^2 + 26x - 24$$

- Possible zeros
- Decartes rule
- Graph the polynomial

.5 3.5 Complex zeros and the fundamental theorem of algebra

1. Motivation:

    (a) Factor $p(x) = x^4 + x^3 + 7x^2 + 9x - 18$ knowing that 1 and –2 are both zeros.
    (b) Result is linear factor and irreducible quadratic (with complex roots).
    (c) Still, roots are factors, just need complex numbers to make sense of it.
    (d) Note conjugate pair relationships.
    (e) Every polynomial can be factored completely.
    (f) Check in Desmos.

2. Fundamental Theorem of Algebra

    (a) Every degree n polynomial has n roots (real or complex) up to multiplicity.
    (b) Huge math impact. Connects two big areas: algebra and geometry. Complex plane is a elegant place to do geometry.
    (c) Search FTOA, complex plane, fourier series
    (d) Fundamental theorem of calculus is your chance to see such a thing in action.

3. Example: Find a degree 5 polynomial with zeros –3 repeated twice, $2i$, and 1 with y-intercept 5. Check your result in Desmos.

4. Homework: 7, 9, 13, 29, 31, 37, 41, 47, 67

.6 3.6 Rational functions

1.

2.

3. Motivitation and infinity twice.

    (a) Basic case: $r(x) = 1/x$
    (b) Reason out graph. Domain and range.
    (c) Label VAs and HAs.

4. Definitions:

   (a) $r(x) = p(x)/q(x)$ for p, q polynomials. Domain requires $q(x)$ not zero. Show up places, especially in math theory. Good setting to think about infinity.

   (b) Vertical and horizontal asymptotes. Asymptotics and end behavior.

5. Example:

   (a) Graph $r(x) = (3x + 6)/(x - 1)$

   (b) Zeros, asymptotes, sign chart helps.

   (c)

   (d) Think of end behavior by dividing by the highest order term. $1/x$ is easy to understand.

   (e) Check with Desmos.

   (f) Try on own: Same for $r(x) = (2x^2 + 7x - 4)/(x^2 + x - 2)$.

6. Other situations.

   (a) Does there always have to be a VA? No can be a hole: $x/x, (x^2 - 2)/(x + 2)$

   (b) Does there always have to be a HA? No degree doesn't have to match for p and q. Zero HA, slant asymp, etc. Long division gives the slant asymptote.

7. Homework: 9, 11, 13, 15, 21, 23, 29, 33, 35, 45, 53, 55, 63, 73

Chapter 10 Systems of equations and inequalities

.1 10.1-10.2 Systems of linear equations in two variables

1. Motivation: Building a shed.

   - One company charges $2000 plus $15 per square foot.
   - One company charges $5000 plus $10 per square foot.
   - For what square footage will the companies match?

2. Motivation: Bottle feed a goat.

   - Formula 1 contains 5 mlg of calcium per ounce and 10 mlg of vitamin A per ounce.
   - Formula 2 contains 8 mlg of calcium per ounce and 2 mlg of vitamin A per ounce.
   - The goat needs 100 mlg of calcium and 60 mlg of vitamin A per day.
   - How much of each formula should we use without wasing?

3. System of linear equations

   (a) Definition

   (b) Solution by graph: intersection of lines

4. Solving system of linear equations

   (a) Substitution

   (b) Elimination

5. The number of solution

   (a) One solution

   (b) No solution

   (c) Infinitely many solutions

.2   10.2 Systems of linear equations in several variables

1. General linear system

    (a) Definition
    (b) Method of substitution
    (c) Method of elimination

        i. Triangular system
        ii. Method of elimination: transfer all system to an equivalent triangular system
            A. Equivalent system
            B. Steps
                - Add a nonzero multiple of one equation to another
                - Multiply an equation by a nonzero constant
                - Interchange the positions of two equations

2. Number of solutions of a linear system: count number of equations and number of variables

    (a) No solution: inconsistent
    (b) The system has exactly one solution
    (c) Infinitely many solution:

.3   10.4 Systems of nonlinear equations

1. System of nonlinear equations: definition and graph $y = x^2$ and $y = x_1$

2. Solving system of nonlinear equations

    (a) Substitution
    (b) Elimination: limited

$$y = x^2, \quad y = 2 - x^2$$

.4   10.5 System of inequalities

1. Graphing a (single) inequality

    (a) Move y on one side
    (b) (linear, quadratic, circle)

2. Graph the solution set of a system of inequalities

    (a) Nonlinear system
    (b) Linear system
    (c) Vertex
    (d) Bounded, bounded

3. Optimization: give one example, don't test

Chapter 4 Exponential and Logarithmic functions

.1   4.1 Exponential functions

1. Motivation: Compound interest example

   (a) Quick example

   (b) General formula and explanation of each variable

   $$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

   (c) Applied problem to find the amount given principal, compounding period, and rate.

2. Basic: Review laws of exponents! Refresher examples.

   (a) LoE: $a^0, a^1, a^m a^n, a^m/a^n, a^n b^n, (a/b)^n, a^{-n}$.

   (b) **Student Examples**: Simplify $\dfrac{\sqrt[3]{ab} \cdot b^2}{a^3 \cdot b^{1/2}}; (-27)^{2/3}(4)^{-5/2}; \left(\dfrac{2x^{2/3}}{y^{1/2}}\right)\left(\dfrac{3x^{-5/6}}{y^{1/3}}\right)$

   (c) What exponent means: $2^3, 2^{-1}, 2^{1/2}, 2^{-4/3}$, good for any rational number, $2^\pi, 2^i$ needs calculus, but we have faith..

   (d) Solving exponential equations

      - **Student Examples**: Solve for x: $2^{-x} = 8$; $8^{2x} = \dfrac{1}{2^{2-x}}$; $3(3^x) + 9(3^{-x}) = 28$ (rewrite as same base and hidden quadratic)

3. Exponential function: $f(x) = a^x$

   (a) Definition: why $a > 0$ and $a \neq 1$

   (b) Graphs

      - Concrete examples: $f(x) = 2^x, 5^x, (1/3)^x = 3^{-x}$
      - Domain and range
      - $a^0 = 1$
      - Increasing/decreasing
      - Shape: depends on the a
      - Horizontal Asymptote
      - Note they are all one-to-one

4. Reading exponential function

   - Comparing base
   - General format: $b \cdot a^x$
   - Identify graphs with points and shift

5. Intuition / examples:

   - Exponential function grows fast (mark pen example)
   - Application: Student loan interest calculation, mortgage payment calculator.

.2   4.2 The Natural exponential functions

1. Motivation: Need for a single, uniform base.

   - Which one is bigger? (3^4 or 4^3)
   - The idea of a uniform base(base is not unique 2^{3x}, 4^x)

2. The natural base e

   (a) Rather than lots of bases a, we would like a uniform base with nice properties (the natural exponential). Called natural since it shows up in interesting way (instantaneous, large populations and reproduction, many times, many things, life isn't always discrete).

   (b) Continuous compound interest:

       - Invest \$1000 at 5% per year.
       $$1000 + (0.05)1000 = 1050$$

       - Same, twice a year, $\frac{5\%}{2}$ each time.
       $$1000 + (0.025)1000 + (0.025)(1000 + (0.025)1000) = 1000(1 + 0.05/2)^2 = 1050.625$$

       - Quarterly, $\frac{5\%}{4}$ each time.
       $$1000(1 + 0.05/4)^4 = 1050.945$$

       - Daily: 1051.267 (let students choose and guess here, per day second etc)
       - This seems to approach a limit / max.
       - Desmos: $(1 + \frac{0.05}{n})^{n/0.05}$.

   (c) Fact: modify above desmos, sort of growth rate 1.

   $$\left(1 + \frac{1}{n}\right)^n \to e, \quad \text{when } n \to \infty$$

   where $e \approx 2.72$, Euler's number. Can show e is irrational as important as π, if not more. Shows up in applications all the time.

   (d) The natural exponential function f
   $$f(x) = e^x$$

3. Law of continuous growth formula
   $$q = q_0 e^{rt}$$

   - q_0: initial quantity
   - r: the growth rate
   - t: time
   - e: natural base

   (a) Note:

       i. $r > 0$: growth rate
       ii. $r < 0$: decay rate
       iii. r is better in terms of identifying the increasing and decreasing rate, no longer have cases with the base
       iv. "real" base: e^r

   (b) Continuous compound interest.

   (c) When to apply:

       i. grows/decays proportional to its current value

   ii. continuously (instantaneously) changing

 (d) Uniform base: transform $y = ae^{kt}$ to ab^t (still need logs to get here)

4. Applications

- Continuous compound interest
- Population growth
- Radioactive decay (half life)
- Anything that grow/decays at a percentage
- How to understand continuous (not all the time, but can happen any time)
- `https://www.google.com/publicdata/explore?ds=kf7tgg1uo9ude_&met_y=population&idim=`
  `state:06000:48000&hl=en&dl=en#!ctype=l&strail=false&bcs=d&nselm=h&met_y=population&`
  `scale_y=lin&ind_y=false&rdim=country&idim=state:06000:48000:12000&ifdim=country&hl=`
  `en_US&dl=en&ind=false`

.3   4.3-4.4 Logarithmic functions and log properties

1. Basics

 (a) Finding growth rate involves finding an input corresponding to a known output. The inverse of exponential function (all one-to-one here).

 (b) Graph $f(x) = a^x$ for $a > 1$ and $0 < a < 1$, automatically can draw f^{-1}. Name $f^{-1}(x) = \log_a(x)$.

 (c) Careful definition of logarithm (defined to be inverse).

$$y = \log_a x \quad \text{if and only if} \quad x = a^y$$

 (d) The log as a function:
- i. Domain, range
- ii. Special point (1,0)
- iii. Special bases
- iv. Function composition of a^x and $\log_a(x)$.
  - A. The logarithmic function with natural base: $\ln x$
  - B. The common logarithmic function: $y = \log x$.

 (e) **Examples:**
- i. Compute $\log(1/100), \log_4(2), \log_5(1/5), \log^3(1), \log_8(4), \log_9(sqrt3)$ (easier to look at exponential form.
- ii. Solve for x: $\log_3(x+4) = \log_3(1-x)$ (one-to-one), $e^{2\ln(x)} = 9$ (inverses and domain restriction).
- iii. Find the domain and range: $\ln(\ln x)$.

2. Applications:

 (a) Originally for hand calculation because of log properties below. (Napier, slide rule, revolution of calculation)

 (b) Astronomical distance `https://en.wikipedia.org/wiki/Astronomical_system_of_units`

 (c) The Benford's law (first digit law) `https://en.wikipedia.org/wiki/Benford%27s_law`

 (d) Logarithmic transformation in data science: `https://en.wikipedia.org/wiki/Data_transformation_` `(statistics)`

 (e) Nature: `https://en.wikipedia.org/wiki/Logarithmic_spiral`

 (f) Solve exponential equation: $23x = 10$, $e^{2x} - 3e^x + 2 = 0$

3. Log properties:

   (a) $\log_a(xy)$, $\log_a(x/y)$

   (b) $\log_a(x^p)$

   (c) $\log_a x = \frac{\log_b(x)}{\log_b(x)}$ change of base

   (d) $a^{\log_a x} = x$, $\log_a a^x = x$ inverse relation

   (e) These are just the laws of exponents written in logarithmic form. Write a^{s+t}, a^{st}, a^{-s} and draw parallels.

      • Prod to sum: Let $\log_a(x) = s, \log_a(y) = t$, then $a^s = x, a^t = y$.
      • $xy = a^s a^t = a^{x+t}$, rewrite in log form
      • $\log_a(xy) = s + t = \log_a(x) + \log_a(y)$
      • Rest are same idea.

   (f) As mentioned before, make calculation easier (product to sum, power to product, etc).

4. Typical problems

   (a) Express $\log_a \frac{x^3 \sqrt{y}}{z^2}$ in terms of $\log x$, $\log y$, $\log z$
      i. Split $\cdot$ and /
      ii. Bring down the power

   (b) Express as one logarithm, opposite direction

   (c) Why are we doing this? Solving equations? Solve real life problem.
      i. The population of La Crosse 50000 in 2000, 55000 in 2010, what will it be in 2020 assuming continuous growth?
      ii. Which would you choose and why? Invest $100 at 4% or $500 at 3%? When do they equal? Depends on length of investment.
      iii. Google population of Florida, Cali, and Texas. Which is growing faster? Let them guess and explain why. Care about growth rate here, use log plot instead. Care about slope of this new line. Not a realistic fit globally though! Population of sad North Dakota
      $$y = Pe^{rt}, \ \ \ln(y) = \ln(P) + rt, \ \ z = c + rt$$
      `https://www.google.com/publicdata/directory` Possible project here, fit exponential, logistic growth, etc

5. Solving equations examples, these main ideas are all there is.

   (a) $8^{2x}(\frac{1}{4})^{x-2} = 4^{-x}$. Rewrite in same base.

   (b) $2^x = 3^{1-x}$, cannot rewrite in same base, use logarithm of any base. Many equivalent but different looking solutions. Nice bases to choose are 2,3.

   (c) $\log_3(-x) + \log_3(8 - x) = 2$. Beware of domain changes. Always need to check solution. Only $x = -1$ works here.

6. Transfer anything to base e: $y = 2^x$

   (a) Connection between continuous and discrete cases

   (b) Everything is continuous

   (c) One formula but restrict your x to be integer.

   (d) Groupwork handout, treat as take home quiz.
      i. Tips:
      ii. Remove the log
      iii. Check the domain

.4   4.5 Exponential and logarithmic equations

1. Exponential function

   - Basic: $3^{x+2} = 7$
   - Different base
   - Quadratic: $e^{2x} - e^x - 2 = 0$
   - Factor: $xe^x + x^2 e^x = 0$

2. Logarithmic function

   (a) $\log_6(4x - 5) = \log_6(2x + 1)$
   (b) $\log_2(5 + x) = 4$
   (c) $e^x = 4$
   (d) $\log(2x + 3) = \log x + 1$, $\log_2 x + \log_2(x + 2) = 3$
   (e) $2^x = 3^{2x-1}$
   (f) $\log_4(x) + \log_8(x) = 1$, change of basis formula.
   (g) $\ln(x^2) = (\ln x)^2$

3. Application problem (la crosse population, radiactive decay)