Assigned: Monday, April 28, 2014
Due: Week of May 5, 2014

MTH 371: Group Project 5
Systems of Differential Equations

GENERAL GROUP PROJECT GUIDELINES:

- Group project assignments should be a collaborative effort. All should participate in discussion and solution writing.
- Each week, your group must meet with Dr. Vidden to discuss your findings. All members must be present. Your grade will be assigned at the end of the meeting.
- Each student should keep group project solutions in a dedicated notebook. Bring this notebook to your weekly meeting to discuss your findings. For coded solutions, bring a laptop to your weekly meeting. Have the laptop ready before the start of the meeting.

Much of our discussion of numerical methods for differential equations applies to systems of differential equations! Let vector $\vec{y}(t)$ denote a vector of n functions, $\vec{y}(t) = [y_1(t), y_2(t), \ldots, y_n(t)]^T$ and let $\vec{f}(t, \vec{y}(t))$ denote a vector valued function, $\vec{f}(t, \vec{y}(t)) = [f_1(t, \vec{y}(t)), f_2(t, \vec{y}(t)), \ldots, f_n(t, \vec{y}(t))]^T$. Then, the initial value problem

$$\vec{y}\,' = f_n\left(t, \vec{y}\right), \quad \vec{y}(t_0) = \vec{y}_0$$

represents the system

$$\begin{cases} y_1' = f_1(t, y_1, y_2, \ldots, y_n) \\ y_2' = f_2(t, y_1, y_2, \ldots, y_n) \\ \vdots \\ y_n' = f_n(t, y_1, y_2, \ldots, y_n) \end{cases}$$

with the initial conditions

$$y_1(t_0) = y_{10},\ y_2(t_0) = y_{20}, \ldots, y_n(t_0) = y_{n0}.$$

For Euler's method, we have the form

$$\vec{y}_{k+1} = \vec{y}_k + h\vec{f}(t_k, \vec{y}_k),$$

which componentwise means,

$$y_{i,k+1} = y_{ik} + hf_i(t_k, y_{1k}, \ldots, y_{nk}), \quad i = 1, \ldots, n.$$

Use this framework to answer the questions on the backside of this handout.

Here we will model the romantic love of Romeo and Juliet whose affection for eachother is quantified on a scale from -5 to 5 described below.

hysterical hatred	disgust	indifference	sweet affection	ecstatic love
-5	-2.5	0	2.5	5

These characters struggle with frustrated love due to the lack of reciprocity of their feelings. Mathematically, they might say:

ROMEO: "My feelings for Juliet decrease in proportion to her love for me"'
JULIET: "My love for Romeo grows in proportion to his love for me"'

We will measure time in days beginning at $t = 0$ and ending at $t = 60$. This ill-fated love affair might be modeled by the differential equations

$$\begin{cases} \frac{dx}{dt} = -0.2y, \\ \frac{dy}{dt} = 0.8x, \end{cases}$$

where x and y are Romeo's and Juliet's love, respectively.

1. Explain how the above differential equations model Romeo and Juliet's affections.
2. Write a Scilab function `y=EulerSys(f,a,b,y0)` which solves a general system of first order equations as outline on the previous page. Here, `f=f(t,y)` is a predefined Scilab function where `t` denote the time variable and `y` denotes a vector of function values. Also, `y0` denotes a vector of initial values and `a,b` are the interval endpoints.
3. Solve the above system using your function from problem 2. For endpoints, take $t = 0$ to $t = 60$ and for initial values take $\vec{y}_0 = [2, 0]^T$. Also, use $N = 10,000$ nodes for this calculation. To view your results, plot the following three graphs all on the same figure using the `subplot` command.
   (a) Plot Romeo and Juliet's love (y-axis) against time t (x-axis) for the above time interval.
   (b) Plot Juliet's love (y-axis) against Romeo's love (x-axis) for the above time interval.
   (c) Use the Scilab `plot3d3` command to view Juliet's love (y-axis) against Romeo's love (x-axis) along with the time t (z-axis).

   What will be the fate of Romeo and Juliet's romance?
4. Repeat problem 3 for the refined model

$$\begin{cases} \frac{dx}{dt} = -0.2y, \\ \frac{dy}{dt} = 0.8x - 0.1y. \end{cases}$$

   What is the interpretation of this model?
5. (Bonus!) Solve the two systems of differential equations EXACTLY. Compare the exact solution to your results from Euler's method.
6. (Bonus!) Modify the above models to achieve a more desirable outcome for Romeo and Juliet. Analyze your results.