Math 371
Quiz 5, March 27, 2015

Name: Solution.

1. Show for all vectors $\vec{v}$ in $\mathbb{R}^n$, $\|\vec{v}\|_1 \le n\|\vec{v}\|_\infty$.

$$\|\vec{v}\|_1 = \sum_{i=1}^{n} |v_i| = |v_1| + |v_2| + \ldots + |v_n|$$

$$\le \max_{1\le i\le n} |v_i| + \ldots + \max_{1\le i\le n} |v_i| \quad \left(\begin{array}{c} n\text{-times} \\ \text{repeated} \end{array}\right)$$

$$= n\|\vec{v}\|_\infty.$$

2. Verify the above inequality for vector $\vec{v} = [1, -2, 3]^T$.

$$\|\vec{v}\|_1 = \sum_{i=1}^{3} |v_i| = |1| + |-2| + |3| = 6$$

$$\|\vec{v}\|_\infty = \max_{1\le i\le 3} |v_i| = 3.$$

$$6 = \|\vec{v}\|_1 \le 3\|\vec{v}\|_\infty = 3\cdot 3 = 9 \quad \checkmark$$