Math 371
Quiz 7,   April 10, 2015   Name: ___Solution___

1. Consider the table of function values below.

x	1	3/2	0
$f(x)$	3	13/4	3

(a) Fill out the Newton divided difference table provided below for the above function values.

x	$f[\,\cdot\,]$	$f[\,\cdot\,,\,\cdot\,]$	$f[\,\cdot\,,\,\cdot\,,\,\cdot\,]$
1	$f[1] = 3$	$f[1,3/2] = 1/2$	$f[1,3/2,0] = 1/3$
3/2	$f[3/2] = 13/4$	$f[3/2,0] = 1/6$	
0	$f[0] = 3$		

Scratch space:

$$f[1,3/2] = \frac{f[3/2] - f[1]}{3/2 - 1} = \frac{13/4 - 3}{1/2} = \frac{1}{4} \cdot \frac{2}{1} = \frac{1}{2}$$

$$f[3/2,0] = \frac{f[0] - f[3/2]}{0 - 3/2} = \frac{3 - 13/4}{0 - 3/2} = \frac{1}{4} \cdot \frac{2}{3} = \frac{1}{6}$$

$$f[1,3/2,0] = \frac{f[3/2,0] - f[1,3/2]}{0 - 1} = \frac{1/6 - 1/2}{-1} = \frac{1}{3}$$

(b) Write down, but do not simplify, the polynomial interpolant P in *Newton form* of f through points $(1,3)$, $(3/2, 13/4)$, $(0,3)$.

$$P(x) = f[1] + f[1,3/2](x-1) + f[1,3/2,0](x-1)(x-3/2)$$
$$= 3 + \frac{1}{2}(x-1) + \frac{1}{3}(x-1)(x-3/2) .$$

(c) Write down, but do not simplify, the polynomial interpolant P in *Lagrange form* of f through points $(1,3)$, $(3/2, 13/4)$, $(0,3)$.

$$P(x) = 3 \frac{(x-3/2)(x-0)}{(1-3/2)(1-0)} + \frac{13}{4} \frac{(x-1)(x-0)}{(3/2-1)(3/2-0)} + 3 \frac{(x-1)(x-3/2)}{(0-1)(0-3/2)} .$$