Assigned: Monday, January 26, 2015
Due: Wednesday, February 4, 2015

MTH 371: Homework 1
Scilab and Programming Introduction

GENERAL HOMEWORK GUIDELINES:

- On the very first page of your homework, provide your name, date, and homework number.
- Homework will be graded in part on neatness, organization, and completeness of solutions. Multiple pages MUST BE STAPLED.
- Attach all Scilab code, output, and plots to the page *immediately following* each problem.
- Clearly label all plots (title, x-axis, y-axis, legend). Use the "subplot" when needed

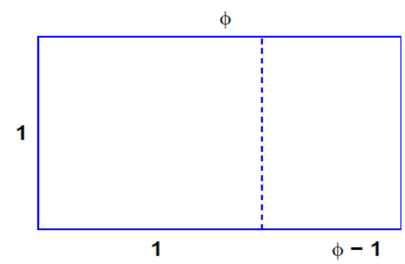
1. With matrices and vectors
$$A = \begin{pmatrix} 10 & -3 \\ 4 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \vec{w} = \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$
compute the following *both* by hand and in Scilab. For the Scilab computations, use the "diary" command to record your session. Here, T denotes the matrix or vector transpose.

   (a) $\vec{v}^T\vec{w}$
   (b) $\vec{v}\vec{w}^T$
   (c) $A\vec{v}$
   (d) $A^T\vec{v}$
   (e) AB
   (f) BA
   (g) $A^2 (= AA)$
   (h) the vector $\vec{y}$ for which $B\vec{y} = \vec{w}$
   (i) the vector $\vec{x}$ for which $A\vec{x} = \vec{v}$

2. A magic square is a $n \times n$ matrix constructed with integers 1 through n^2 (each number used exactly once) with equal row, column, and diagonal sum.
   (a) What do you expect all of the row, column, and diagonal sums to be for a $n \times n$ magic square? Explain why.
   (b) Using the "diary" command to record your session, complete the following steps in *as few commands as possible.* Make sure to take advantage of matrix indexing and the `sum` command.
      i. Create a 4×4 magic square matrix via the command `A=testmatrix('magi',4)`.
      ii. Check all the row sums.
      iii. Check all the column sums.
      iv. Check both the diagonal sums.
   (c) Visualize the $n \times n$ magic squares via the command `surf(A)` for $n = 8, 9, 10, 11$. Use the subplot commands to include these four plots within a single figure.

3. (a) Write a script file which constructs a vector x consisting of 200 evenly spaced points on the interval $[0, 2\pi]$, then plot $y = \sin(kx)$ for $k = 1, 2, 3, 4, 5$ all on the same plot. Be sure to give your graph a title, label the x and y axis, display a legend, and distinguish each curve.
   (b) Using the "subplot" command, create a single figure including plots $y = \sin(k)$ for k integers ranging from 0 to 1000, and also $y = \sin(k)$ for k integers ranging from 0 to 10000. (For more, see Richert's paper posted on the D2L website).

4. The golden ratio ϕ shows up in many places in mathematics (and nature!). This ratio gets its name from the golden rectangle shown below. This rectangle has the property that removing a square leaves a smaller rectangle of the same proportions as the original.



Taking the ratio of corresponding sides gives $\frac{1}{\phi} = \frac{\phi - 1}{1}$. Rearranging, we have the quadratic equation $\phi^2 - \phi - 1 = 0$.

(a) Find the two roots of this quadratic by hand. The positive root is the golden ratio.

(b) Write a Scilab script which plots this quadratic function. Find and plot its zeros via the "roots" command.

5. The Fibonacci Sequence is defined recursively as

$$f_n = f_{n-1} + f_{n-2}, \quad f_1 = 1, f_2 = 2.$$

(a) Write a Scilab function `f = fibonacci(n)` returns array `f` of the first n Fibonacci numbers. (Hint: Use a for loop.)

(b) Write a Scilab script which computes the term-by-term growth rate of the Fibonacci sequence. To do this, compute the first 40 Fibonacci numbers via the function in part (a), then compute ratios $\phi_n = \frac{f_{n+1}}{f_n}$. You can compute all these ratios in a single line of code. What does ϕ_n seem to approach?

(c) Record the following session in Scilab via the "diary" command.

i. Enter the statements

$$A = [1\ 1; 1\ 0]; \ \ X = [1\ 0; 0\ 1];$$

then the statement

$$X = A * X$$

Now, repeatedly press the up arrow key, followed by the enter key. What happens? Explain why.

ii. Use Scilab to compute the eigenvalues of matrix A. What is the result and why?

6. A famous unsolved problem in number theory the $3n+1$ problem and is as follows. Start with positive integer n. Repeat the following steps.

   - If $n = 1$, stop.
   - If n is even, replace it with $\frac{n}{2}$.
   - If n is odd, replace it with $3n+1$.

   For example, starting with $n = 7$ produces

   $$7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1.$$

   The sequence terminates after 17 steps. The unanswered question is, does this process always terminate?

   (a) Write a Scilab function `y = threenplus1(n)` returning array `y` which is the entire sequence generated by positive integer `n`. (Hint: Use a while loop and if statements.)

   (b) Write a Scilab script which computes the sequences generated by integers 2 through 10, and display them all on the same plot.

   (c) The $3n+1$ sequence has a particular shape for n starting at $5, 10, 20, 40, 80, \ldots$ Why is this?

   (d) The graphs of $3n+1$ sequences are all quite similar for $n = 108, 109, 110$. Why?

7. OPTIONAL challenge problem. Consider organizing the positive integers in an $n \times n$ array in a spiral fashion as illustrated in the below picture. Note the prime numbers are highlighted in red. The location of these primes forms what is called an Ulam prime spiral. By plotting points, this spiral is hilighted in the next image for the 200×200 case. Write a Scilab script which replicates the second image. Generate your own image for the 400×400 and 800×800 cases. For more on Ulam prime spirals, see `http://blogs.mathworks.com/cleve/2015/01/05/prime-spiral/`.

73	74	75	76	77	78	79	80	81	82
72	43	44	45	46	47	48	49	50	83
71	42	21	22	23	24	25	26	51	84
70	41	20	7	8	9	10	27	52	85
69	40	19	6	1	2	11	28	53	86
68	39	18	5	4	3	12	29	54	87
67	38	17	16	15	14	13	30	55	88
66	37	36	35	34	33	32	31	56	89
65	64	63	62	61	60	59	58	57	90
100	99	98	97	96	95	94	93	92	91