Applied Linear Algebra Notes, Fall 2021

Contents

Fun Stuff

1. Feynman Method: `https://www.youtube.com/watch?v=FrNqSLPaZLc`
2. Bad math writing: `https://lionacademytutors.com/wp-content/uploads/2016/10/sat-math-section.jpg`
3. Google AI experiments: `https://experiments.withgoogle.com/ai`
4. Babylonian tablet: `https://www.maa.org/press/periodicals/convergence/the-best-known-old-baby`
5. Parabola in real world: `https://en.wikipedia.org/wiki/Parabola#Parabolas_in_the_physical_world`
6. Parabolic death ray: `https://www.youtube.com/watch?v=TtzRAjW6KO0`
7. Parabolic solar power: `https://www.youtube.com/watch?v=LMWIgwvbrcM`
8. Robots: `https://www.youtube.com/watch?v=mT3vfSQePcs`, riding bike, kicked dog, cheetah, backflip, box hockey stick
9. Cat or dog: `https://www.datasciencecentral.com/profiles/blogs/dogs-vs-cats-image-classificat`
10. History of logarithm: `https://en.wikipedia.org/wiki/History_of_logarithms`
11. Log transformation: `https://en.wikipedia.org/wiki/Data_transformation_(statistics)`

12. Log plot and population: https://www.google.com/publicdata/explore?ds=kf7tgg1uo9ude_&met_y=population&hl=en&dl=en#!ctype=l&strail=false&bcs=d&nselm=h&met_y=population&scale_y=lin&ind_y=false&rdim=country&idim=state:12000:06000:48000&ifdim=country&hl=en_US&dl=en&ind=false
13. Yelp and NLP: https://github.com/skipgram/modern-nlp-in-python/blob/master/executable/Modern_NLP_in_Python.ipynb https://www.yelp.com/dataset/challenge
14. Polynomials and splines: https://www.youtube.com/watch?v=O0kyDKu8K-k, Yoda / matlab, https://www.google.com/search?q=pixar+animation+math+spline&espv=2&source=lnms&tbm=isch&sa=X&ved=0ahUKEwj474fQja7TAhUB3YMKHY8nBGYQ_AUIBigB&biw=1527&bih=873#tbm=isch&q=pixar+animatio mesh+spline, http://graphics.pixar.com/library/
15. Polynomials and pi/taylor series: Matlab/machin https://en.wikipedia.org/wiki/Chronology_of_computation_of_%CF%80 https://en.wikipedia.org/wiki/Approximations_of_%CF%80#Machin-lik formula https://en.wikipedia.org/wiki/William_Shanks
16. Deepfake: face https://www.youtube.com/watch?v=ohmajJTcpNk
dancing https://www.youtube.com/watch?v=PCBTZh41Ris
17. Pi digit calculations: https://en.wikipedia.org/wiki/Chronology_of_computation_of_%CF%80, poor shanks...https://en.wikipedia.org/wiki/William_Shanks

Course Introduction

1. Syllabus highlights
   (a) Grades:
      i. Know the expectation / what you are getting into.
      ii. 15perc A (excellent), 35perc B (good), 35perc C (satisfactory),10perc D (passing), some F (failing)
      iii. Expect lower grades than you are used to. I was a student once upon a time. I know what it's like to give some effort in a class and still get an A/B. Night before study, good enough?
      iv. Turn in an exam / project. Did you do good work?
      v. Many will start off doing good / satisfactory work. Improve to something more. C is not the worst thing in existence. These letters say nothing of your capability.
   (b) What does good mean? Good means good. Good job! Excellent means you showed some flair.
   (c) Expect: More work, more expectation on good writing.
   (d) Math is a challenging subject. Not a natural thing to think or write in. It takes work and practice to be better. My goal is to train you to be better and give you ideas of where it can go.
   (e) Fact that you are here shows you are smart and capable. Your goal should be to improve.
   (f) Why do I do this? I do it out of respect for you. You are smart enough. I want you to gain something valuable here. I wouldn't do this job if I didn't think you were gaining something of value.
2. Grand scheme of things. Where does this class sit inside all of mathematics.
   (a) Basics: Algebra, arithmetic.
   (b) First steps: Geometry, functions. (us now)
   (c) Calculus: Math of change / infinity.
   (d) Linear algebra: Math of vectors. Anything with finite representation. Invention of computers fueled this one. Gateway to real math / applications.

(e) Applied math. Any application you want. Physics, finance, marketing, material science, CFD, sports.

(f) Abstract math. Create your own world of ideas. Number theory, analysis, algebra, topology, more.

.1 Day 1

1.

Chapter 1: Linear Equations in Linear Algebra

.1 1.1 Systems of linear equations

1. Definition: A *linear equation* is of the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

where x_i are unknown variables with a_i known constant coefficients and b known constant. Only powers of 1 per variable. No other products or quotients.

2. Fundamental problem of linear algebra:

   - Solve a system of linear equations (rich theory can completely study).
   - Key questions: Existence and uniqueness.

3. Familiar example, new ideas.

   (a) Solve for x and y.

$$\begin{cases} 2x - y = 0 \\ -x + 2y = 3 \end{cases}$$

   Linear equations, graphs are lines in 2d.

   (b) Three perspectives of this class:

   - Row picture (familiar)
   - Column picture (new)
   - Matrix representation (maybe new)

   (c) Row picture:

   - Graph in xy-plane. Solution is intersection of two lines. How to find? Substitute or elimination.
   - In general, can see thee possibilities: Unique solution (lines differ in slope), infinite solutions (2 lines overlap), no solution (2 parallel non intersecting lines). No solution is called *inconsistent*. One or infinite many solutions called *consistent*.

   (d) Column picture: Vector representation

   - Remind of 2D vector geometry, scalar multiplication, vector addition, graph, and linear combination.
   - Rewrite in vector form. How to think of this? What linear combination of column vectors $\vec{v}_1$ and $\vec{v}_2$ result in vector $\vec{b}$? Draw in the plane and sketch solution.
   - Verify that solutions $x = 1, y = 2$ from before work.
   - Again, three possibilities. What are the vector analogies regarding column vectors and RHS vector?
   - Generalize: If we change the RHS vector, will we always have a solution? In this case yes since $\vec{v}_1$ and $\vec{v}_2$ span $\mathbb{R}^2$. Change for parallel column vectors to see not always.

(e) Matrix representation:

- Rewrite as coefficient matrix times unknown vector equal a RHS vector.
- Notation: Note text uses bold face letters for vectors.

$$A, \quad \vec{x}, \quad \vec{b}$$

- Can also write short hand as an augmented matrix.
- Solve using the same elimination strategy as with linear equations. Think of this as a computational view. Next section covers this.
- Matrix A can be thought of as an operator on solution vector $\vec{x}$ with resulting vector $\vec{b}$. Studying this linear system equations to studying properties of matrix A.

4. Higher dimensions:

(a) 3 equations, 3 unknowns:

$$\begin{cases} x + 2y + 3z = 5 \\ 2x + 5y + 2z = 7 \\ 6x - 3y + z = -2 \end{cases}$$

(b) Row picture

- Ask graph of each linear equation. Graph in Geogebra 3d to see. Can anyone solve? Plot solution point as well.
- Again 3 cases here, but a bit richer. 1 solution, infinite solutions (plane or line of intersection), no solution (2 planes parallel but not the same).
- Solve by row reduction and backwards substitution. Goal is to replace system with equivalent, though simpler system. Summarize 3 elementary row operations (swap, scale, replace with row plus multiple of another). Why bother swap or scale? Take advantage of zeros and nice numbers. Computers care for high dimension to avoid roundoff error. Mention could eliminate all the way to Gauss Jordan form.

(c) Column picture: Linear combination of three vectors giving RHS vector. Use Geogebra 3d again. Again, think of three cases. Key is all three vectors are linearly independent.

(d) Matrix picture: Easy to write down? Now what?

- Can see columns of A are column vectors.
- What about row vectors? Will develop this.
- Augmented matrix. Algorithm in next section.

(e) Advantages / disadvantages of each picture: Combined they offer a complete theory.

- Row picture: Lots of info and intuition, cannot extend beyond 3d, will think in analogies.
- Column picture: Easy to extend, hard to solve, lots of info and intuition.
- Easy to adapt as algorithm, little intuition.

5. Homework: 3, 7, 13, 18, 19, 23, 25, 33, 34

.2 1.2 Row reduction and echelon form

1. 2 algorithms for solving linear systems of equations:
   - Gaussian elimination and backwards substitution (saw last time).
   - Gauss-Jordan elimination.

2. Example, 2×2: Solve the system using equation form.

$$\begin{cases} x - 2y = 1 \ (R_1) \\ 3x + 2y = 11 \ (R_2) \end{cases}$$

(a) Use the same forward reduction and back substitution idea as in last section.

$$R_2 \to -3R_1 + R_2$$

Check solution works. Recall 3 elementary row operations.

(b) Generalize: Use augmented matrix and aim towards a standard form.

- Row echelon form (GE)

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 3 & 2 & 11 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 8 & 8 \end{array}\right]$$

- Reduced row echelon form (G-JE)

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 3 & 2 & 11 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 1 \end{array}\right]$$

- Pivot entries correspond to locations of 1's in RREF. Pivot columns are columns which contain a pivot entry.
- Note, for any matrix REF is not unique but RREF is. Will prove the latter later.

(c) What if...

- No solution:

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 3 & -6 & 11 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 0 & 8 \end{array}\right]$$

- Infinitely many solutions:

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 3 & -6 & 3 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

  Here y is a free variable and all solutions are

$$\begin{cases} x = 1 + 2y \\ y \text{ free} \end{cases}$$

  or written parametrically as

$$\begin{cases} x = 1 + 2t \\ y = t \end{cases}$$

  for parameter t.

3. Example: Higher dimension, try on own:

$$\begin{cases} 2x + 4y - 2z = 2 \\ 4x + 9y - 3z = 8 \\ -2x - 3y + 7z = 10 \end{cases}$$

   REF and backwards sub vs RREF.

4. Homework: 1, 3, 5, 7, 11, 13, 15, 17, 21, 23, 33-34

.3 1.3 Vector equations

1. 3 view of linear algebra:
   - Equation (row picture)
   - Matrix
   - Vector (column picture): This section, this is where we get geometric reasoning with math rigor.

2. Definition: The vector space $\mathbb{R}^n$ consists of all column vectors $\vec{u}$ with n real valued components.
   - Notation: $\vec{u} = [u_1, u_2, \ldots, u_n]^T$, each entry is called a component.
   - Special case: $\vec{0}$.
3. Examples: Geometry of vectors, imagine displacement.
   - $vecu = [1, 2]^T \in \mathbb{R}^2$. Note not the same as (1,2). Vectors are location independent. Other examples in 4 quadrants. Sad zero vector.
   - $vecu = [-3, 1, 2]^T \in \mathbb{R}^3$
4. Definitions: Vector operations
   - Addition: $\vec{u} + \vec{v} = [u_1 + v_1, \ldots, u_n + v_n]^T$ in $\mathbb{R}^n$. Note need vectors of same length.
   - Scalar multiplication: $c\vec{u} = [cu_1, \ldots, cu_n]^T$ for scalar c.
   - Subtraction (triangular law): $\vec{u} - \vec{v}$
   - Bonus (dot product to compare direction, more later): $\vec{u} \cdot \vec{v}$
   - Bonus (norm or length, more later): $\|\vec{u}\|_n = \sqrt{u_1^2 + \cdots + u_n^2}$
5. Examples: $\vec{u} = [1, 2]^T, \vec{v} = [3, 1]^T$
   - $2\vec{u}, -\vec{u}, 4\vec{u}, 0\vec{u}, c\vec{u}$, set of all scalar multiples results in a line (rescaling gives name to scalar)
   - $\vec{u} + \vec{v}, \vec{v} + \vec{u}$ (Parallelogram law)
   - $\vec{u} - \vec{v} = \vec{u} + (-\vec{v})$ (Triangular law)
6. Theorem (these mirror familiar algebraic properties, some proofs in HW): For all $\vec{u}, \vec{v} \in \mathbb{R}^n$ and scalars
   (a) $\vec{u} + \vec{v} = \vec{v} + \vec{u}$ (Commutative)
   (b) $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ (Associative)
   (c) $\vec{u} + \vec{0} = \vec{u}$ (Identity)
   (d) $\vec{u} + (-\vec{u}) = \vec{0}$ for $-\vec{u} = (-1)\vec{u}$ (Inverse)
   (e) $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$ (Distribution)
   (f) $(c + d)\vec{u} = c\vec{u} + d\vec{u}$ (Distribution)
   (g) $c(d\vec{u}) = (cd)\vec{u}$ (Compatibility)
   (h) $1\vec{u} = \vec{u}$ (Identity)
7. Definition (the linear of linear algebra): Vector $\vec{y} \in \mathbb{R}^n$ is a linear combination of vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ if there exists scalars $c_1, \ldots, c_n$ (called weights) such that
$$\vec{y} = c_1\vec{v}_1 + \cdots + c_n\vec{v}_n$$
8. Example (Vector equation): Show that $\vec{b} = [3, 1, -1]^T$ is a linear combination of vectors $\vec{a_1} = [2, 0, -1]^T$ and $\vec{a_2} = [-1, 1, 1]^T$.
   - This is equivalent to solving a linear system via GE.
   - Geogebra and geometric interpretation.
   - Is the same true for any $\vec{b}$? No, only if it lies in the plane generated by all linear combinations of $\vec{a_1}$ and $\vec{a_2}$. Consider a $\vec{b}$ which does not.
9. Definition: The collection of all linear combinations of $\vec{v_1}, \ldots, \vec{v_p} \in \mathbb{R}^n$ is called the Span$\{\vec{v_1}, \ldots, \vec{v_p}\}$ and is a subset of $\mathbb{R}^n$.
10. Homework: 1, 3, 5, 7, 9, 11, 13, 15, 17, 21, 23, 27

.4 1.4 The matrix equation $A\vec{x} = \vec{b}$

1. 3 views of linear algebra:
   - Row picture (lines and planes, done)
   - Column picture (vectors, done)
   - Matrix picture (now, idea is to capture linear combination as an operation)
2. Definition: For A a $m \times n$ matrix with columns $\vec{a_1}, \dots, \vec{a_n}$ and $\vec{x} \in \mathbb{R}^n$, the produce $A\vec{x}$ is the linear combination of the columns of A with weights as entries in $\vec{x}$. That is,
$$A\vec{x} = [\vec{a_1} \dots \vec{a_n}] \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = x_1\vec{a_1} + \dots x_n\vec{a_n}$$
   Note, the number of columns in A must match the number of entries of $\vec{x}$.
3. Example: Multiply a random $A_{2\times 3}$ matrix by a $\vec{x}_{3\times 1}$ vector.
   3 linear algebra POVs are here. For general $\vec{x}$, write
   - 2 equations (planes, geometry)
   - Linear combinations of 3 vectors (vectors, geometry)
   - Matrix equation $A\vec{x} = \vec{b}$ (operation on a vector, similar to idea of function). Important question is given A, can we solve $A\vec{x} = \vec{b}$ for any RHS vector $\vec{b}$.

   We will readily switch between these views to gain insight and perspective.
4. Example (entry-wise matrix multiplication): Multiply a random $A_{3\times 3}$ matrix by a $\vec{x}_{3\times 1}$ vector.
   - Linear combination of 3 row vectors. Important concept.
   - Dot product of rows and $\vec{x}$. This version is more convenient for hand calculation.

   Replace A with identity matrix $I_{3\times 3}$ and ask them to guess result.
5. Theorem (linearity of matrix multiplication): For matrix A $m \times n$, vectors $\vec{u}, \vec{v}$ $n \times 1$, and scalar c, we have
   (a) $A(\vec{u} + \vec{v}) = A\vec{u} + A\vec{v}$ (distributive)
   (b) $A(c\vec{u}) = c(A\vec{u})$ (associative)

   Proof (of (a), $n = 3$ case, (b) in text): All we need is the corresponding result from vectors in previous section.
$$\begin{aligned} A(\vec{u} + \vec{v}) &= A \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \\ u_3 + v_3 \end{bmatrix} \\ &= (u_1 + v_1)\vec{a_1} + (u_2 + v_2)\vec{a_2} + (u_3 + v_3)\vec{a_3} \\ &= (u_1\vec{a_1} + u_2\vec{a_2} + u_3\vec{a_3}) + (v_1\vec{a_1} + v_2\vec{a_2} + v_3\vec{a_3}) \\ &= A\vec{u} + A\vec{v} \end{aligned}$$
6. Theorem (big result for entire course, will grow this list): For A a $m \times n$ matrix, the following statements are either all true or all false.
   (a) For each $\vec{b} \in \mathbb{R}^n$, equation $A\vec{x} = \vec{b}$ has a solution.
   (b) Each $\vec{b} \in \mathbb{R}^n$ is a linear combination of the columns of A.
   (c) The columns of A span $\mathbb{R}^m$.
   (d) A has a pivot position in every row.
7. Homework: 5, 7, 9, 11, 13, 15, 17, 23, 29, 30

.5 1.5 Solution sets of linear equations

1. We want to characterize solutions to a linear system of equations $A\vec{x} = \vec{b}$ for A and $\vec{b}$ given and $\vec{x}$ unknown thru two perspectives:

   - Geometrically (picture, intuition)
   - Explicitly (formula, practical)

   Our approach will be to consider two related cases:

   - Homogeneous linear system: $A\vec{x} = \vec{0}$
   - Nonhomogeneous linear system: $A\vec{x} = \vec{b}$

2. Homogeneous linear system: $A\vec{x} = \vec{0}$

   (a) For any A, $\vec{x} = \vec{0}$ is always a solution (called the trivial solution). We seek nontrivial solutions $\vec{x} \neq \vec{0}$. Will there always be a nontrivial solution? Only if the GE solution has at least one free variable.

   (b) Solve the homogeneous linear system:

   $$\begin{bmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{bmatrix} \vec{x} = \vec{0}$$

   Solving by GE gives x_3 a free variable with

   $$\vec{x} = x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix} = x_3\vec{v} = span\{\vec{v}\}$$

   The set of these solutions are a line thru the origin parallel to $\vec{v}$.

   (c) Change above example so three rows are multiples of eachother giving 2 free variables.

   $$\begin{bmatrix} 1 & 3 & -5 \\ 1 & 3 & -5 \\ 1 & 3 & -5 \end{bmatrix} \vec{x} = \vec{0}$$

   Solving by GE gives x_2, x_3 free variables with

   $$\vec{x} = \begin{bmatrix} -3x_2 + 5x_3 \\ x_2 \\ x_3 \end{bmatrix} = x_2\vec{v_2} + x_3\vec{v_3} = span\{\vec{v_2}, \vec{v_3}\}$$

   generating a plane thru the origin. View in Geogebra.

3. Nonhomogenous linear system: $A\vec{x} = \vec{b}$

   (a) Example as from before:

   $$\begin{bmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{bmatrix} \vec{x} = \begin{bmatrix} 4 \\ 7 \\ 6 \end{bmatrix}$$

   gives

   $$\left[\begin{array}{ccc|c} 1 & 3 & -5 & 4 \\ 0 & 1 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Again x_3 is free and we have

$$\vec{x} = \begin{bmatrix} -4x_3 - 5 \\ 3x_3 + 3 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix} = \vec{p} + x_3\vec{v}$$

for the same $\vec{v}$ as in the homogenous case. Graph same lines as before but first shifted by vector $\vec{p}$ away from the origin.

(b) Solution to nonhomogenous equation is the same as the homogenous case but translated.

(c) Theorem: For $A\vec{x} = \vec{b}$ consistent and $\vec{p}$ a particular solution, then the solution set of all $A\vec{x} = \vec{b}$ is all vectors of the form

$$w = \vec{p} + \vec{v_h}$$

where $\vec{v_h}$ is any solution to the homogeneous equation $A\vec{x} = \vec{0}$. (sketch the plane case in $\mathbb{R}^3$)

4. Homework: 1, 5, 7, 9, 11, 13, 17, 19, 21, 23, 27, 29, 31

.6 1.6 Applications of linear systems

1. Skip. Possible lab material.

.7 1.7 Linear independence

1.
2. Homework: 9-20, 23-30

.8 1.8 Introduction to the linear transformation

1. Homework: 17-20, 25, 31

.9 1.9 The matrix of a linear transformation

1. Homework: 25-28, 31-34

.10 1.10 Linear models in business, science, and engineering

1. Homework:

Chapter 2: Matrix algebra

.1 2.1 Matrix operations

1. Homework: 13, 17-26

.2 2.2 The inverse of a matrix

1. Homework: 11-24, 35

.3 2.3 Characterizations of invertible matrices

1. Homework: 15-24

.4 2.4 Partitioned matrices

1. Homework: 1-10, 13, 14, 16

.5 2.5 Matrix factorizations

1. Homework: 22-26

.6 2.6 The Leontief input-output model

1. Homework:

.7 2.7 Applications to computer graphics

1. Homework:

.8 2.8 Subspaces of $\mathbb{R}^n$

1. Homework: 5-20, 23-26

.9 2.9 Dimension and rank

1. Homework: 9-16

Chapter 3: Determinants

.1 3.1 Introduction to determinants

1. Homework:

.2 3.2 Properties of determinants

1. Homework:

.3 3.3 Cramer's rule, volume, and linear transformations

1. Homework:

Chapter 4: Vector spaces

.1 4.1 Vector spaces and subspaces

1. Homework: 1-18, 23, 24

.2 4.2 Null spaces, column spaces, and linear transformations

1. Homework: 3-6, 17-26

.3 4.3 Linearly independent sets, bases

1. Homework: 21-25

.4 4.4 Coordinate systems

1. Homework: 25

.5 4.5 The dimension of a vector space

1. Homework:

.6 4.6 Rank

1. Homework:

.7 4.7 Change of basis

1. Homework:

.8 4.8 Applications to difference equations

1. Homework:

.9 4.9 Applications to Markov chains

1. Homework:

Chapter 5: Eigenvalues and eigenvectors

.1 5.1 Eigenvectors and eigenvalues

1. Homework:

.2 5.2 The characteristic equation

1. Homework: 25, 27

.3 5.3 Diagonalization

1. Homework: 18

.4 5.4 Eigenvectors and linear transformations

1. Homework:

.5 5.5 Complex eigenvalues

1. Homework:

.6 5.6 Discrete dynamical systems

1. Homework:

.7 5.7 Applications to differential equations

1. Homework:

.8 5.8 Iterative estimates to eigenvalues

1. Homework:

Chapter 6: Orthogonality and least squares

.1 6.1 Inner product, length, and orthogonality

1. Homework:

.2 6.2 Orthogonal sets

1. Homework:

.3 6.3 Orthogonal projections

1. Homework: 19, 20

.4 6.4 The Gram-Schmidt process

1. Homework:

.5 6.5 Least-squares problems

1. Homework:

.6 6.6 Applications to linear models

1. Homework:

.7 6.7 Inner product spaces

1. Homework:

.8 6.8 Applications of inner product spaces

1. Homework:

Chapter 7: Symmetric matrices and quadratic forms

.1 7.1 Diagonalization of symmetric matrices

1. Homework:

.2 7.2 Quadratic forms

1. Homework:

.3 7.3 Constrained optimization

1. Homework:

.4 7.4 The singular value decomposition

1. Homework:

.5 7.5 Applications to image processing and statistics

1. Homework: