Assigned: Monday, March 24, 2014
Due: Week of March 31, 2014

MTH 371: Group Project 1
Interpolation

GENERAL GROUP PROJECT GUIDELINES:

- Group project assignments should be a collaborative effort. All should participate in discussion and solution writing.
- Each week, your group must meet with Dr. Vidden to discuss your findings. All members must be present. Your grade will be assigned at the end of the meeting.
- Each student should keep group project solutions in a dedicated notebook. Bring this notebook to your weekly meeting to discuss your findings. For coded solutions, bring a laptop to your weekly meeting. Have the laptop ready before the start of the meeting.

1. Prove that a polynomial interpolant of degree at most n through the $(n+1)$ points $\{(x_0, f(x_0)), (x_1, f(x_1)), \dots, (x_n, f(x_n))\}$ must be unique.

   Hint: Assume that there are two such polynomials, P and Q, and argue that they must be identical. Consider the function $R(x) = P(x) - Q(x)$.

2. Prove the recursion formula for computing Newton divided differences.

$$f[x_0, x_1, \dots, x_k] = \frac{f[x_1, x_2, \dots, x_k] - f[x_0, x_1, \dots, x_{k-1}]}{x_k - x_0}$$

   To do this, let P be the interpolating polynomial for $\{(x_0, f(x_0)), (x_1, f(x_1)), \dots, (x_{k-1}, f(x_{k-1}))\}$ and Q the interpolating polynomial for $\{(x_1, f(x_1)), (x_2, f(x_2)), \dots, (x_k, f(x_k))\}$ and consider the polynomial

$$R(x) = \frac{x_k - x}{x_k - x_0} P(x) + \frac{x - x_0}{x_k - x_0} Q(x).$$

   (a) Prove R is the unique polynomial of at most degree k which interpolates points $\{(x_0, f(x_0)), (x_1, f(x_1)), \dots, (x_k, f(x_k))\}$.

   (b) Determine the coefficient of x^k on each side of the equation.

3. Consider the Runge function $f(x) = \dfrac{1}{1 + 25x^2}$ on $[-1, 1]$. Graph the following all on the same plot. Be sure to label your graph and include a legend.

   (a) Graph $y = f(x)$ on $[-1, 1]$ using 100 data points (in Scilab: `x = linspace(-1, 1, 100)`).

   (b) Graph the degree 10 polynomial interpolant of function f through 11 equally spaced nodes in $[-1, 1]$ (in Scilab: `x = linspace(-1, 1, 11)`) using the same data points as in (a).

   (c) Graph the degree 10 polynomial interpolant of function f through the 11 non-equally spaced Chebyshev points $x_j = \cos\left(\dfrac{\pi j}{10}\right)$, $j = 0, 1, \dots, 10$ using the same data points as in (a).