

# Week 1: What is a network?

Daryl DeFord

Math 581.05:  
Computational Tools for Complex Networks  
Fall 2020



# Outline

① Introduction and Overview

② What is a network?

Ego Networks

People are complicated

③ First Networkx steps



# Caveat(s)



**Figure:** The elephants in the (Zoom) room



## Introductory Poll 1

How many of these answers will get no votes?

- 0
- 1
- 2
- 3



# Co-Instructors



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Izzy conquering the laptop



Skye investigating the camera



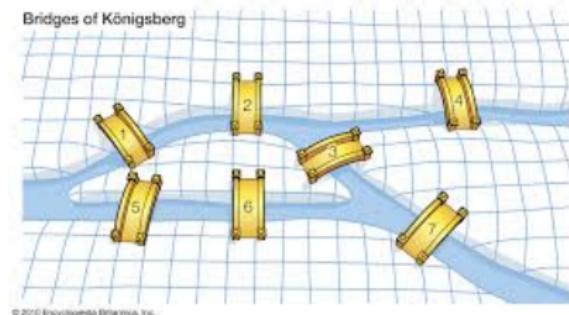
## Course Details

[https://github.com/  
drdeford/Math\\_581\\_05](https://github.com/drdeford/Math_581_05)



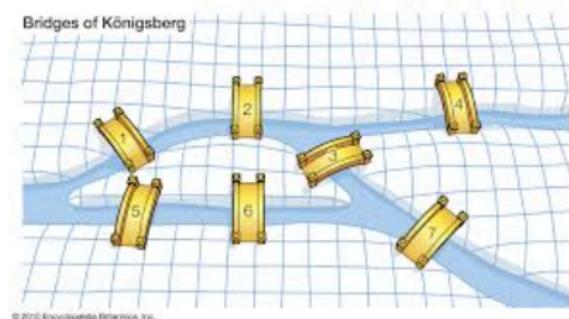
# Historical Overview

- Euler (bridges of Königsberg)



# Historical Overview

- Euler (bridges of Königsberg)
- Sociologists
- Physical Infrastructure

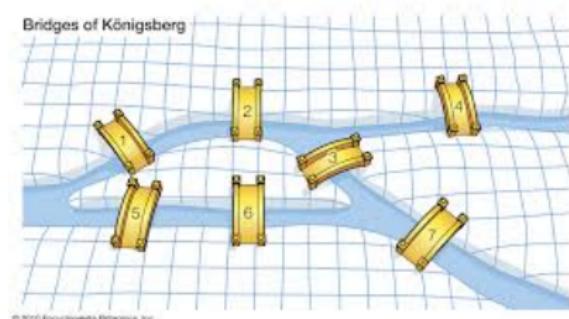


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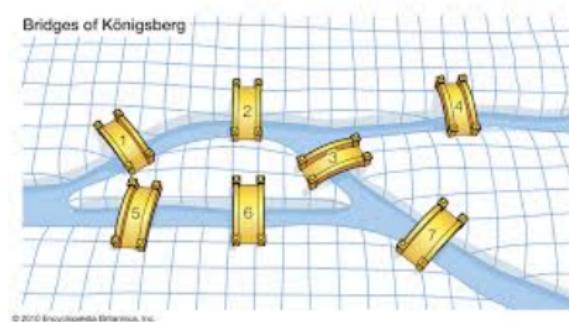
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- Euler (bridges of Königsberg)
  - Sociologists
  - Physical Infrastructure
  - The internet!
- 
- Directed
  - Weighted
  - Hypergraphs
  - Multiplex



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# Are networks graphs?

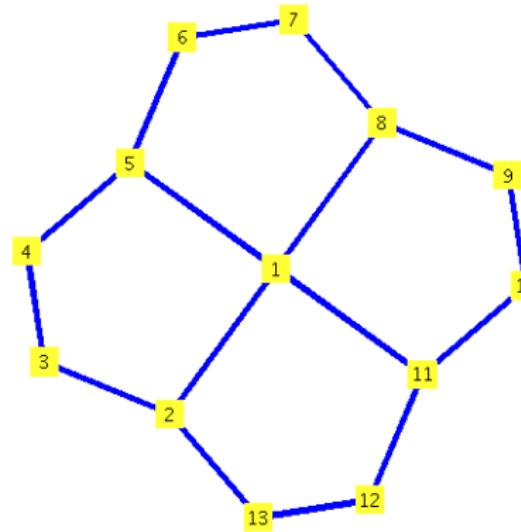
?



# Graph Theory

## Definition (Graph)

A Graph  $G = (V, E)$  is a set of nodes  $V$  and a set of edges  $E \subseteq V \times V$ .

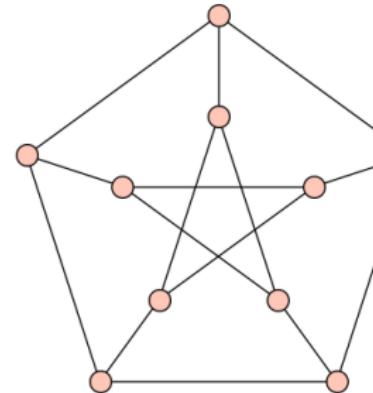
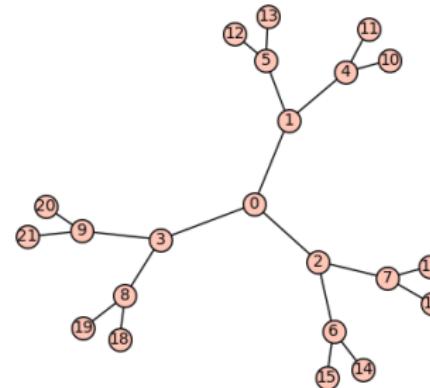
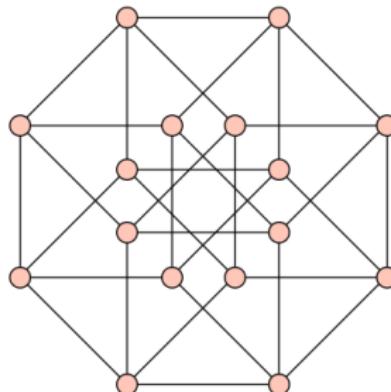


# Graph Questions

- Are there Hamiltonian/Eulerian Paths? If so, how many?
- Are there perfect matchings? If so, how many?
- Is it possible color the nodes with  $k$  colors so that no neighboring nodes are the same color?
- What is the largest set of nodes that share no edges?
- How many edges/vertices must be removed to disconnect the graph?
- Is it possible to embed the graph in the plane without any edges crossing?
- How many spanning trees does the graph have?
- What is the automorphism group of the graph?
- ...



# Graph Examples



# Graph Theorems

## Theorem

*Every graph has an even number of odd-degree vertices*

## Theorem

*The number of perfect matchings in an  $m \times n$  grid graph is:*

$$\sqrt[4]{\prod_{j=1}^m \prod_{k=1}^n \left( 4 \cos^2 \left( \frac{\pi j}{m+1} \right) + 4 \cos^2 \left( \frac{\pi k}{n+1} \right) \right)}$$

## Theorem

*Every graph where every node has even degree has an Eulerian path.*

## Theorem

*Determining whether an arbitrary graph has a Hamiltonian cycle is NP-Hard*

# Network Examples



# Network Examples

- Internet
  - Nodes: Hardware
  - Edges: Physical Connections
- WWW
  - Nodes: Webpages
  - Edges: Links
- Neuroscience
  - Nodes: Brain areas
  - Edges: Functional Connections
- Food webs
  - Nodes: Species
  - Edges: Predation
- World Trade
  - Nodes: Countries
  - Edges: Trade Agreements
- Banking
  - Nodes: Banks
  - Edges: Loans
- Genetics
  - Nodes: Genes/Proteins
  - Edges: Functional Interactions
- Recommendations
  - Nodes: People/Products
  - Edges: Ratings/Consumption



# What is a social network?

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Mathematically, a social network is represented by a collection of “nodes” representing individual actors and a set of “edges” representing a binary relationship between the actors.



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## Example

What kinds of systems can social networks describe?

- What could be represented by nodes?
  - **Academic Departments**
- What type of edges could connect them?
  - **Located in same building**
  - **Students who major in both**
  - **Crosslisted courses**



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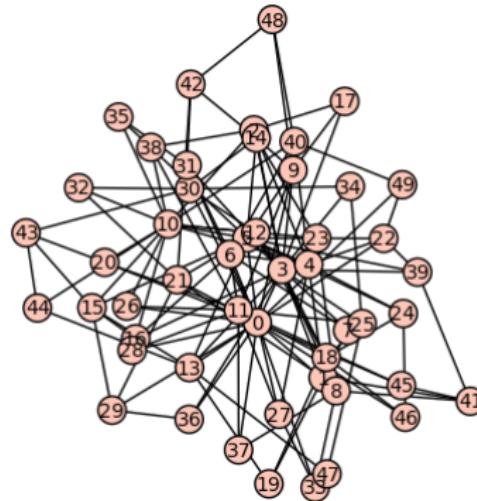
## Example

What kinds of systems can social networks describe?

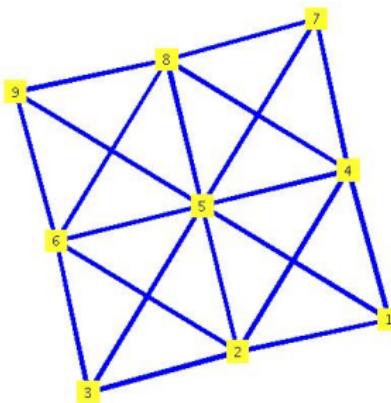
- What could be represented by nodes?
  - **WSU Students**
- What type of edges could connect them?
  - **In a class together**
  - **Facebook friends**
  - **Speak at least twice a week**



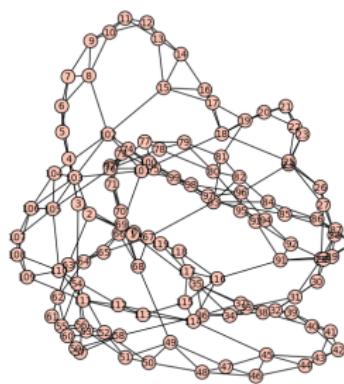
# Social Networks



# Examples



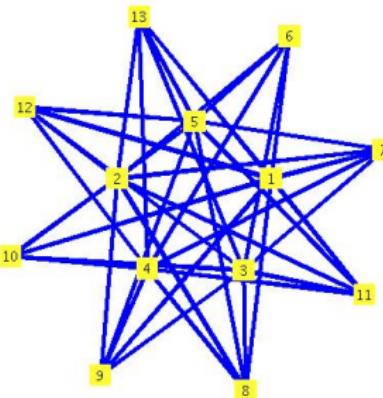
Graph



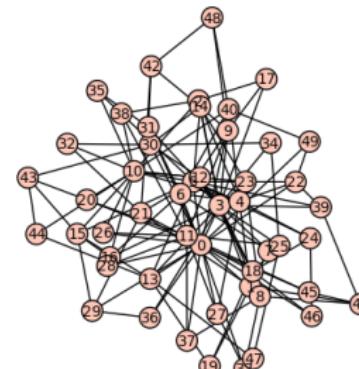
Network



# Examples



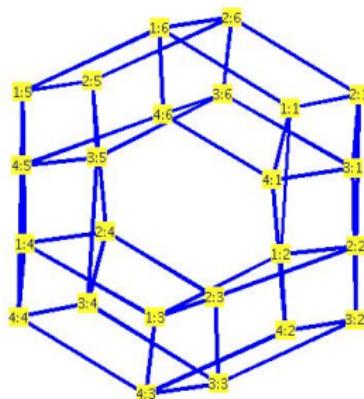
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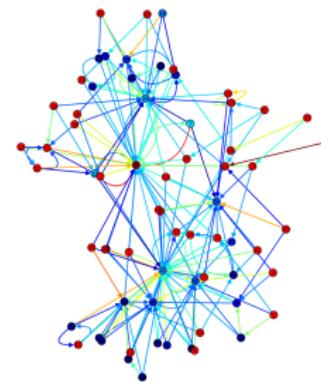
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# Examples



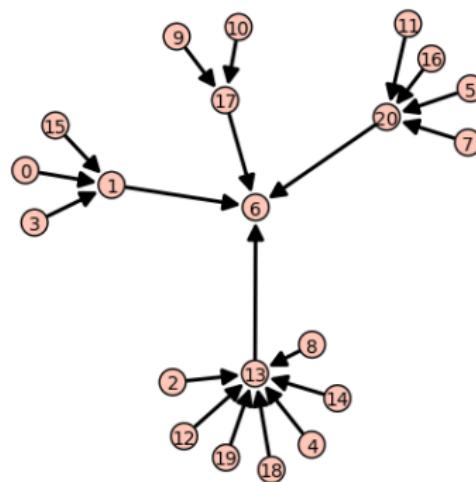
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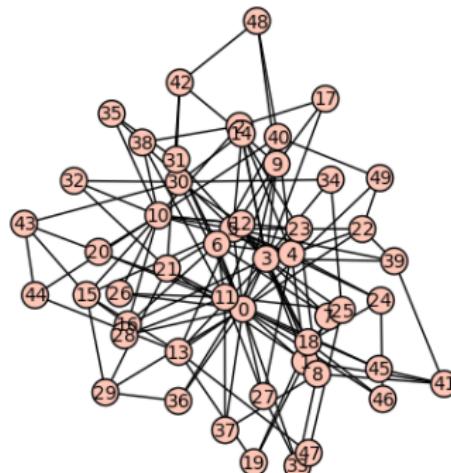
Network



# Centrality



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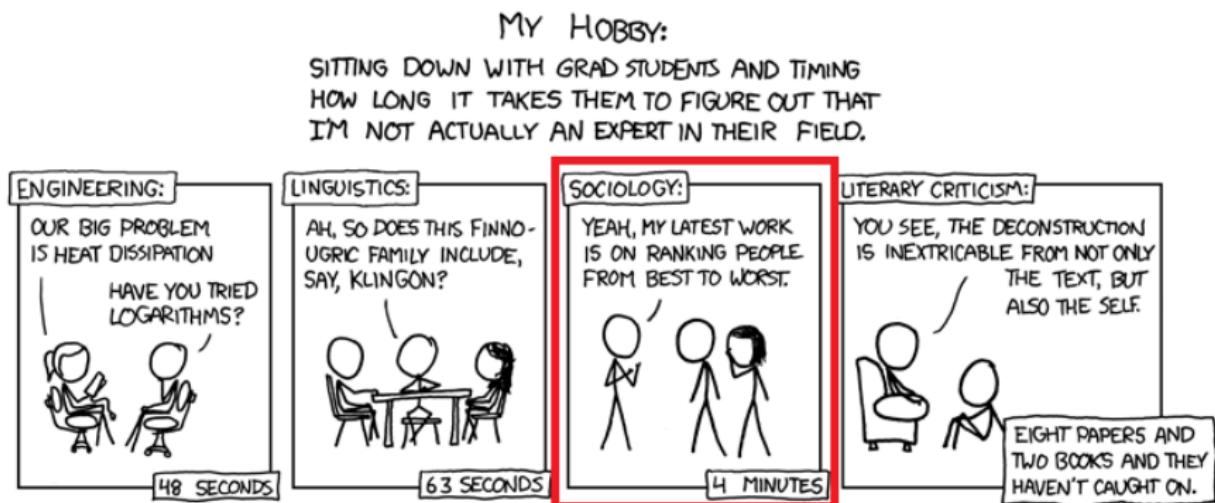
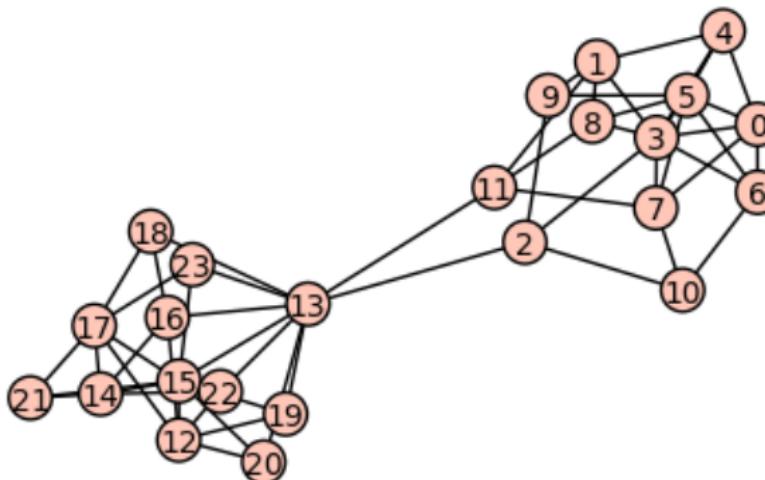


Figure: Relevant comic by Randall Munroe<sup>1</sup> (**emphasis** mine).

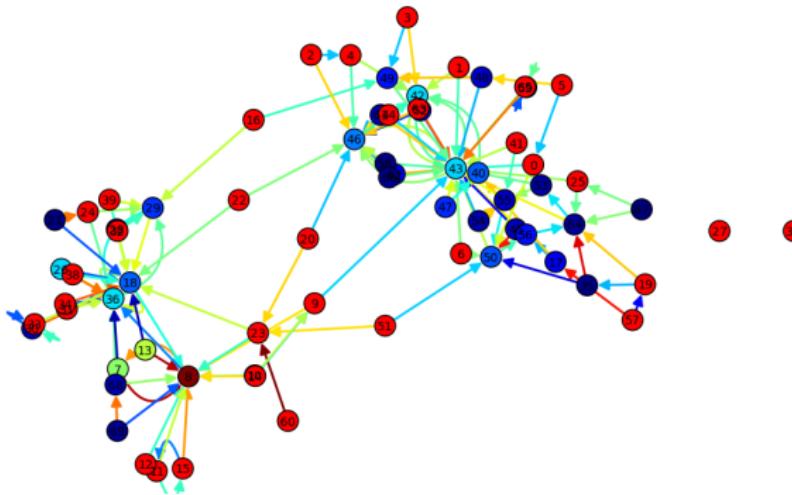
<sup>1</sup> <https://xkcd.com/451/>



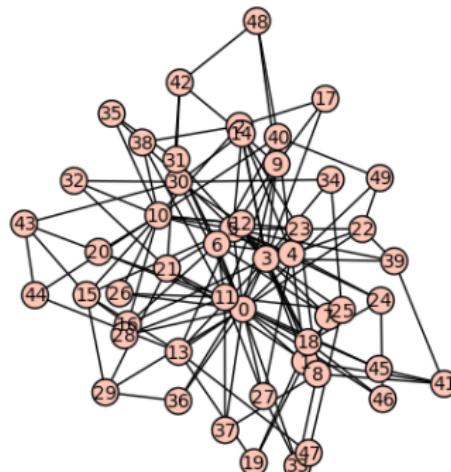
## Clustering



# Clustering



# Clustering



# Common Properties of Social Networks

Example (What features distinguish social networks?)

- ?



# Common Properties of Social Networks

Example (What features distinguish social networks?)

- ?
- Transitivity
- Community structure
- Small average path length
- Long-tailed degree distribution
- Hubs
- ...



# Ego Networks

## Definition (Ego Network)

An ego network is a social network centered at a particular individual containing their connections and the connections between their “friends.”



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## Example (Draw your ego network)

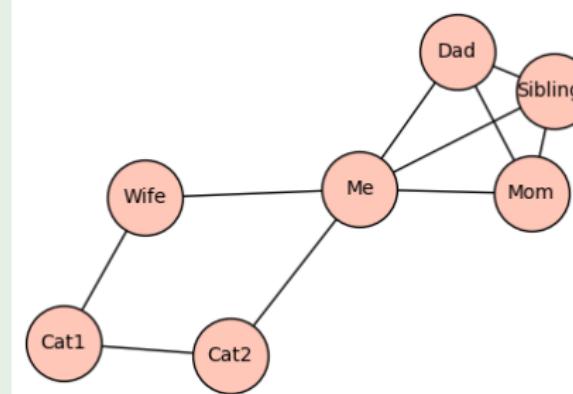


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## Example (Draw your ego network)



# How to construct networks?

Example (Which edges to add?)

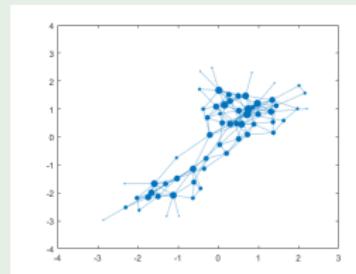
- ?



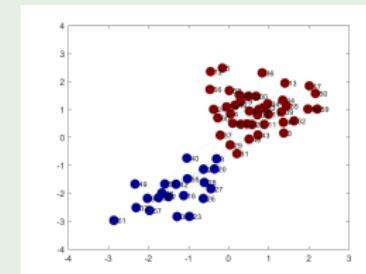
# How to construct networks?

Example (Which edges to add?)

- ?
- Proximity



Centrality

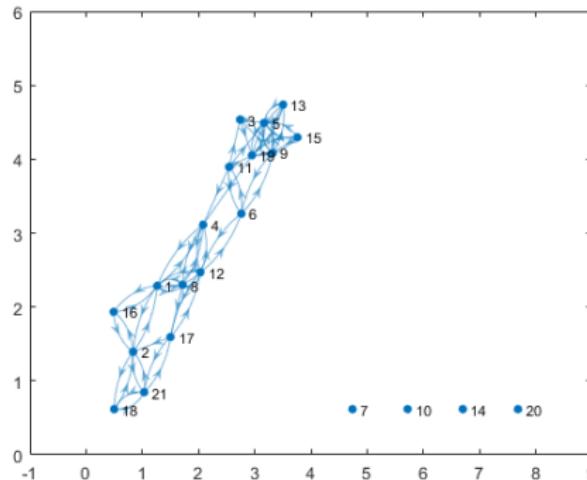


Clustering

Figure: Dolphin social network<sup>1</sup>

<sup>1</sup> D. Lusseau, K. Schneider, O. Boisseau, Patti Haase, E. Slooten, and S. Dawson, The bottlenose dolphin community of Doubtful Sound features a large proportion of long-lasting associations, *Behavioral Ecology and Sociobiology* 54 (2003), no. 4, 396–405.

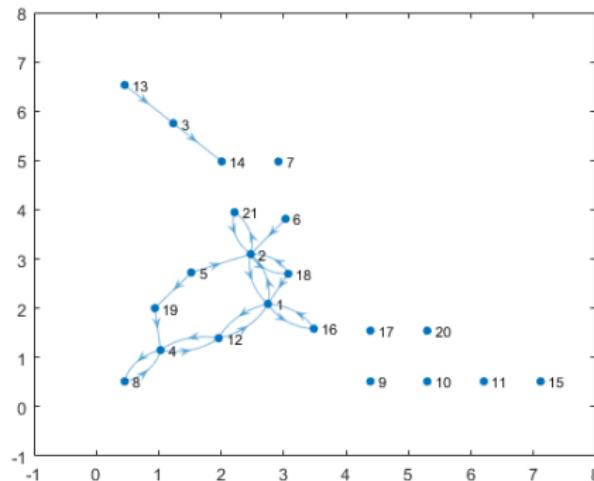
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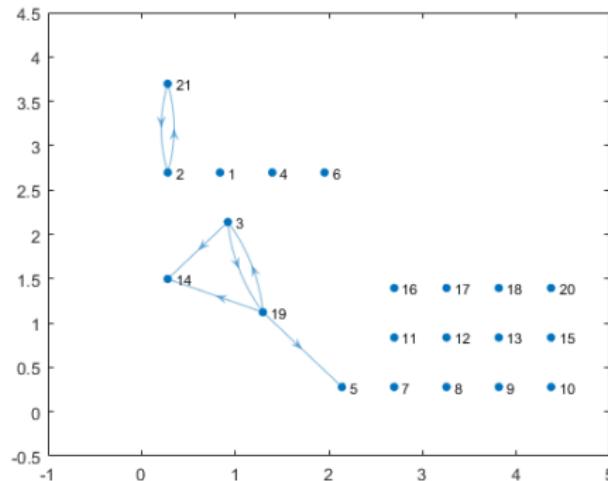
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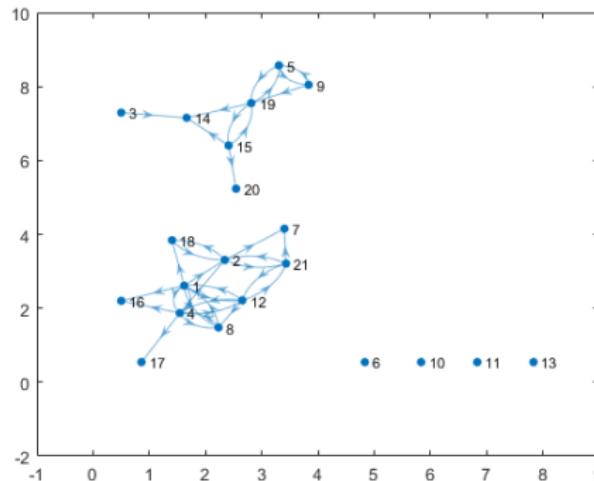
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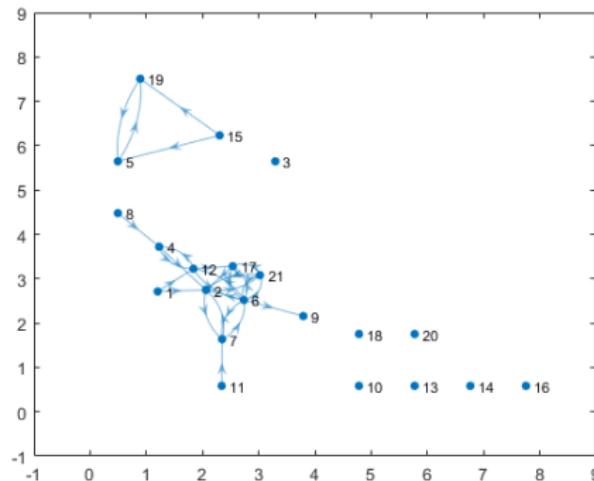
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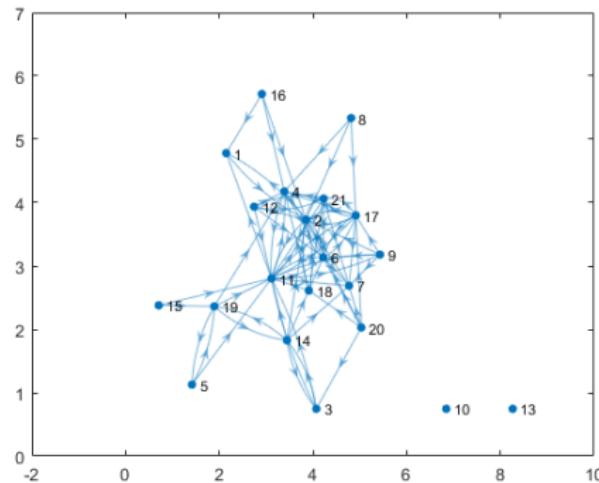
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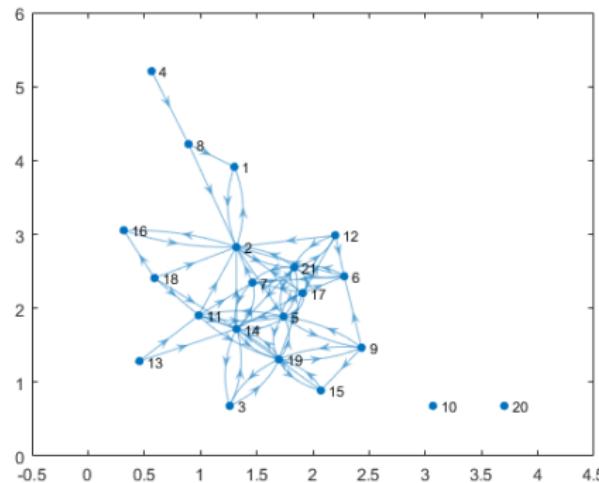
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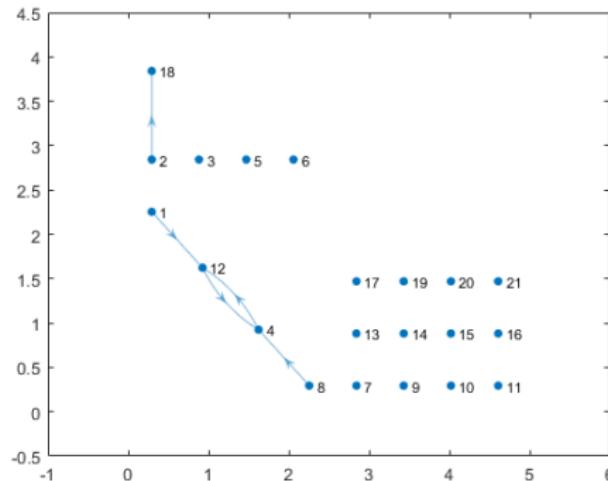
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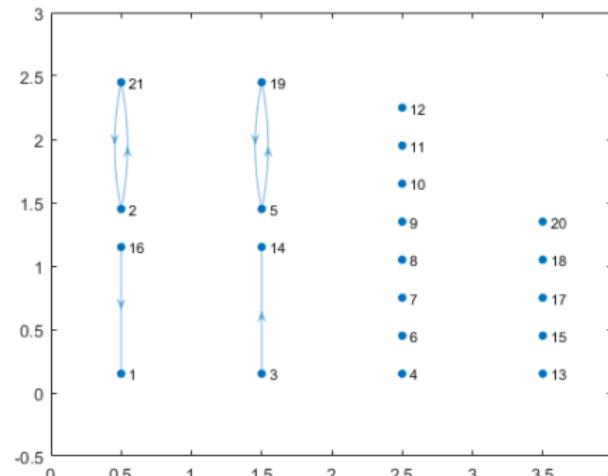
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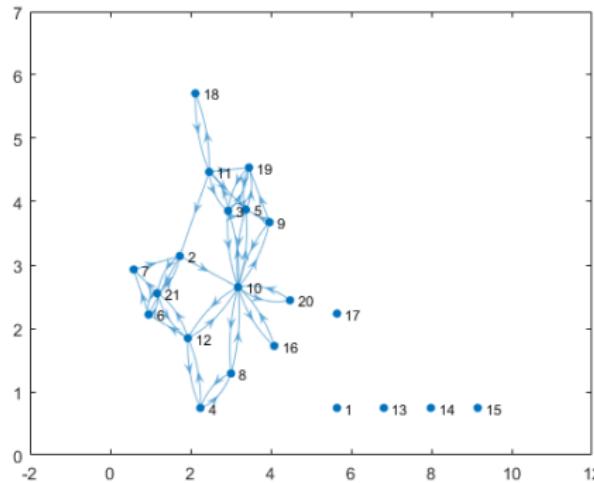
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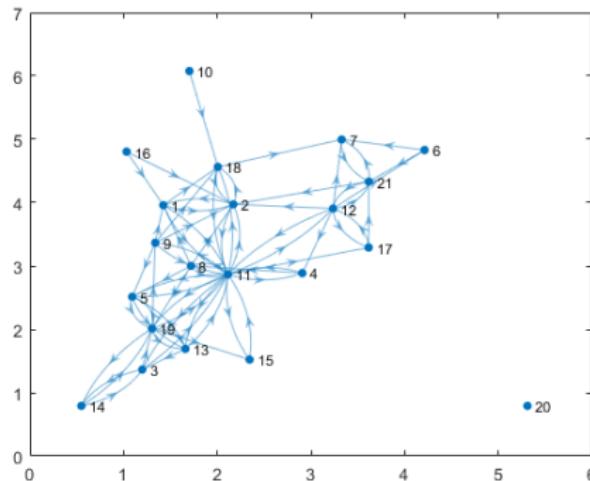
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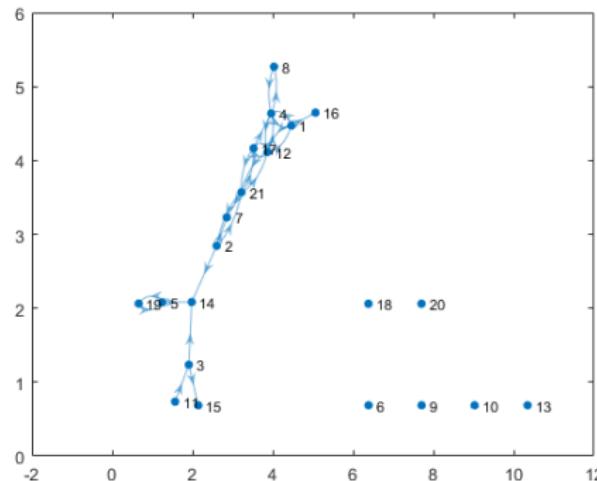
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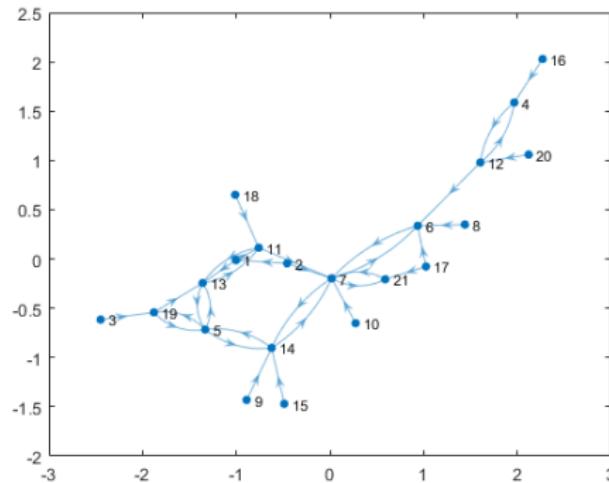
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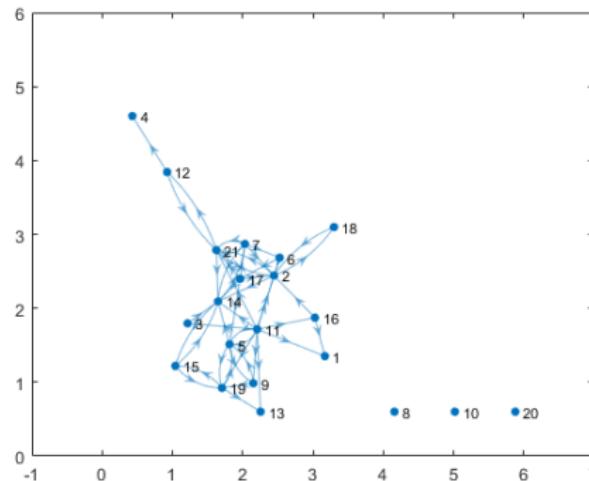
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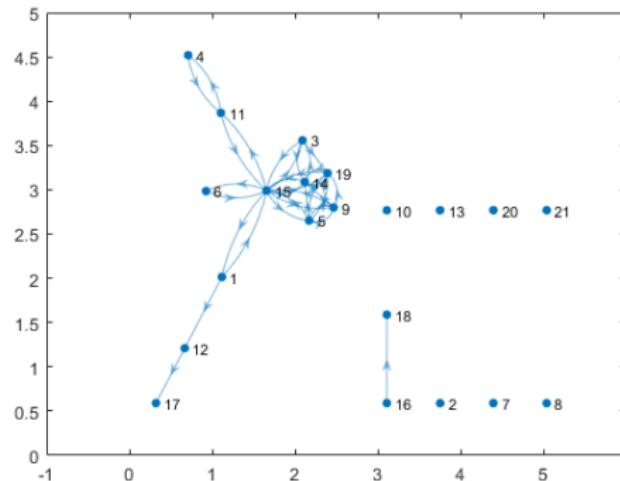
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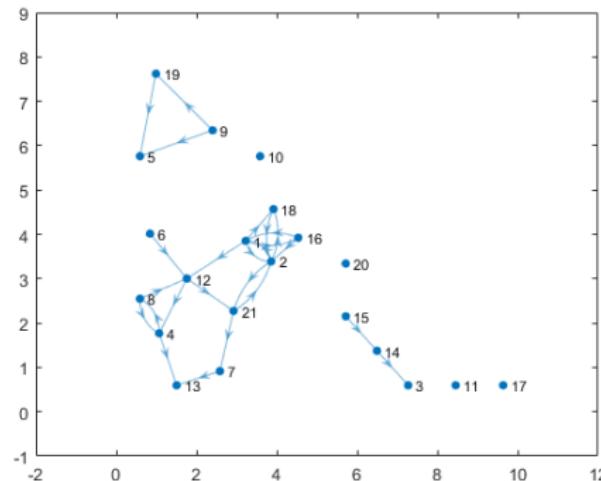
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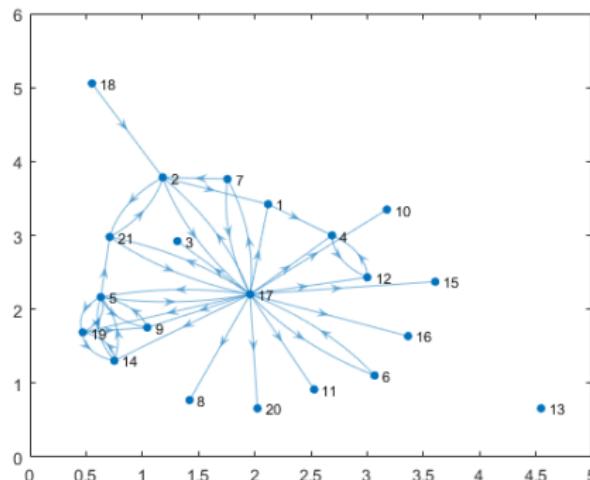
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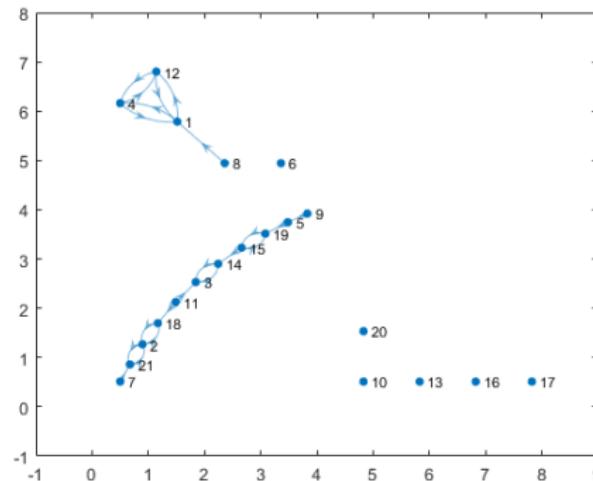
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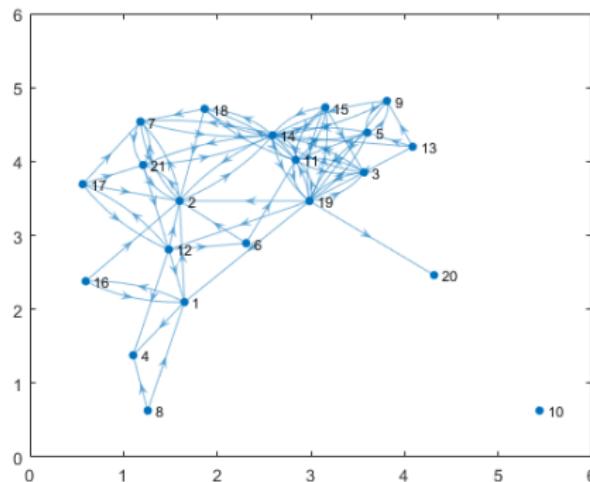
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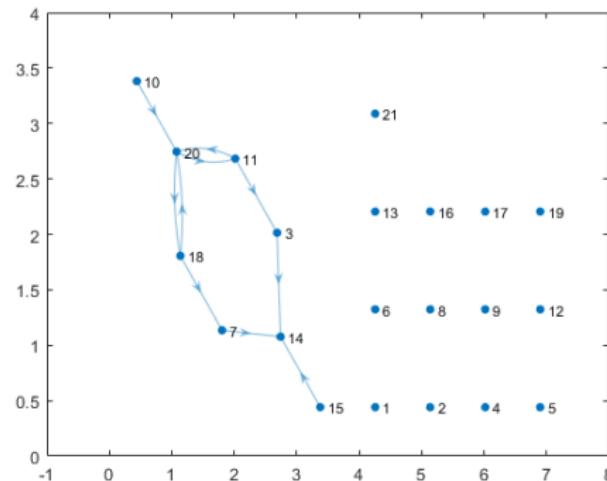
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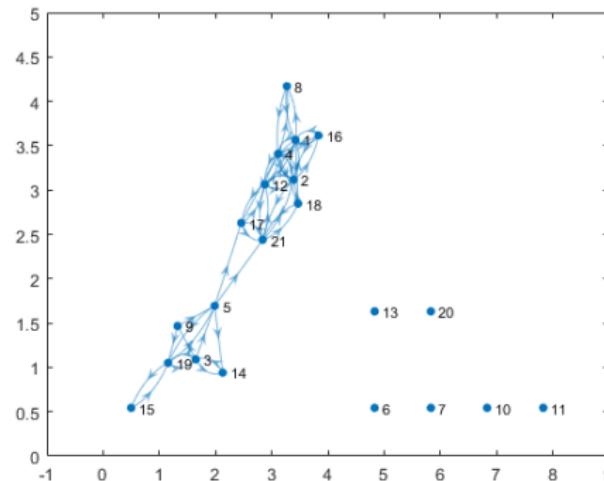
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Washington Produce

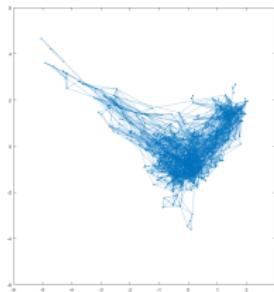
What is a network?

People are complicated

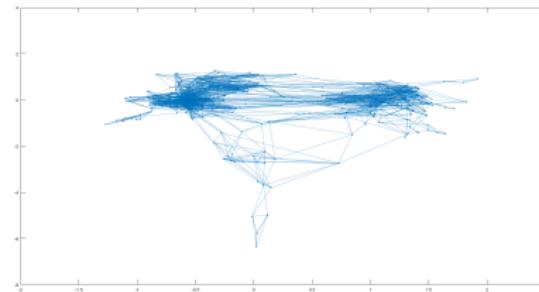
# Information Flow



# Karnataka Village Data



Village 5



Village 61

**Figure:** Two of the Karnataka Village networks<sup>1</sup>

<sup>1</sup> A. Banerjee, A.G. Chandrasekhar, E. Duflo, and M.O. Jackson, The Diffusion of Microfinance. *Science*, (2013).



# Village Layers

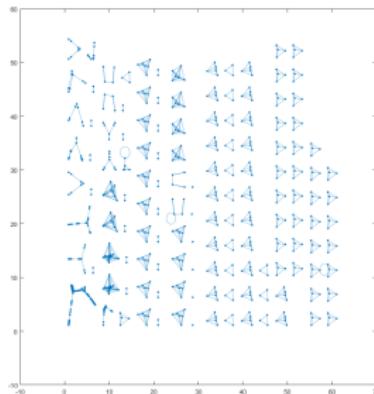
Layer	Village 4			Village 61		
Description	Density	Comp.	Giant %	Density	Comp.	Giant %
Borrow Money	.0082	26	.8354	.0108	15	.9188
Give Advice	.0077	49	.5892	.0098	34	.7377
Help Make Decisions	.0076	61	.1277	.0100	24	.8562
Borrow Kerosene or Rice	.0085	21	.8338	.0113	14	.9171
Lend Kerosene or Rice	.0086	22	.8308	.0113	14	.9255
Lend Money	.0081	14	.7908	.0107	17	.9036
Medical Advice	.0075	84	.2938	.0106	14	.9306
Friends	.0089	15	.9277	.0105	22	.8714
Relatives	.0085	29	.7231	.0105	26	.5448
Attend Temple With	.0073	117	.0462	.0089	108	.0372
Visit Their Home	.0087	15	.9185	.0116	11	.9475
Visit Your Home	.0088	16	.9108	.0117	11	.9492
Aggregate	.0121	3	.9862	.0155	8	.9679

Table: Layer information for two of the Karnataka Villages <sup>1</sup>.

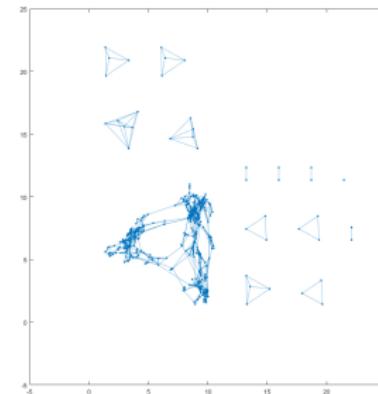
<sup>1</sup> D. DeFord and S. Pauls, A new framework for dynamical models on multiplex networks, Journal of Complex Networks, 6(3), 353–381  
2018.



# Medical Advice



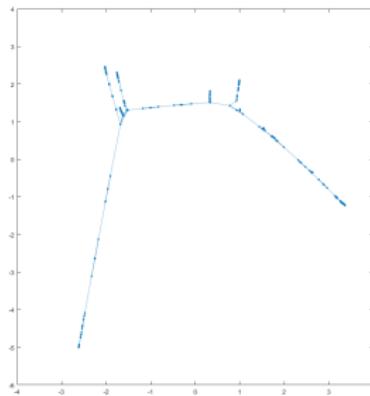
Village 5



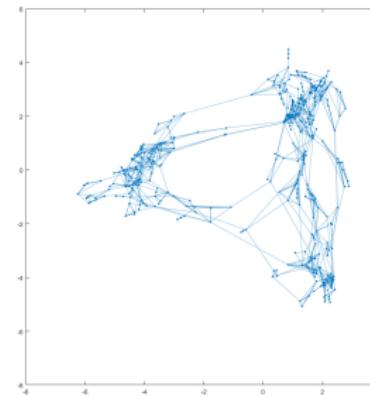
Village 61



# Medical Advice



Village 5



Village 61



What is a network?

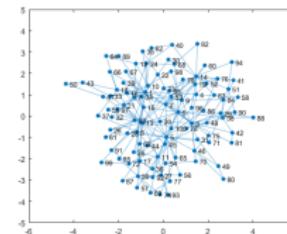
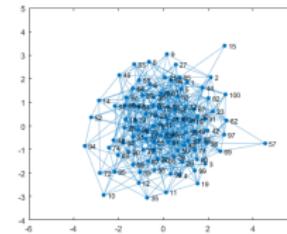
People are complicated

# Network Questions

## Dynamics



## Randomness



# Python Poll

- <https://cocalc.com/>
- <https://colab.research.google.com>
- [http://people.csail.mit.edu/ddeford/sage\\_cell](http://people.csail.mit.edu/ddeford/sage_cell)
- ...?

