Electronic information processing and cybernetics - An algebrasation of the synthesis problem for circuits

Günter Hotz

April 16, 2020

1 Introduction

The occasion for this work is a problem from automata theory: From a given set of building blocks an automaton, whose functionality is predetermined, shall be assembled. From the different, eventually existing solutions the cheapest shall be selected.

A building block $A \in \mathcal{U}$ is a physical, mostly electrical device with $Q(A)$ inputs and $Z(A)$ outputs. For each input a Set S of input signals is permitted, on which the building block reacts with output signals. We assume the following simplifications with regard to the issue, the following holds:

1. For each input of the elemnts of $\mathcal{U}$ a set of signals S is prescribed, and each element of S^n is allowed as input signal for A with $n = Q(A)$.
2. The set of output signals of $A \in \mathcal{U}$ lies in S^m with $m = Z(A)$.
3. If at time t the input signal $s \in S^n$ is applied to A, then the output signal at time t is uniquely determined by s. (We therefore neglect the finite propagation speed of signals).

Thus, the finite automaton is completely described by its function $\phi(A) : S^n \to S^m$. It is presumed that inputs and outputs of A are labeled with a fixed numbering from 1 to $Q(A)$, and repesctively from 1 to $Z(A)$. The i-th input (output) is assigned to the i-th component of S^n (S^m).

An element of $\mathcal{U}$ is a circuit. If A and B are circuits with $Q(A)$, or $Q(B)$ inputs and $Z(A)$, or repectivley $Z(B)$ outputs, then we build new circuits from A and B by integrating them to a new element $A \times B$ with $Q(A) + Q(B)$ inputs and $Z(A) + Z(B)$ outputs. We declare the i-th input of A as the i-th input of $A \times B$ and the i-th input of B as the $(Q(A) + i)$-th input of $A \times B$ (figure 1).

If $Z(A) = Q(B)$ we get from A and B a circuit $B \circ A$ by switching the i-th output of A to the i-th input of B.

A circuit of elements of $\mathcal{U}$ is a device, which is described inductively by the preceeding explanations.

If $\phi(A)$ ($\phi(B)$) is the function of circuit A (B), then $\phi(A) \times \phi(B)$ is the function of $A \times B$, and $\phi(B) \circ \phi(A)$ for $Q(B) = Z(A)$ is the function of $B \circ A$.

The costs for the building blocks in $\mathcal{U}$ shall be defiend by the function $L : \mathcal{U} \to N \cup 0$. We define:

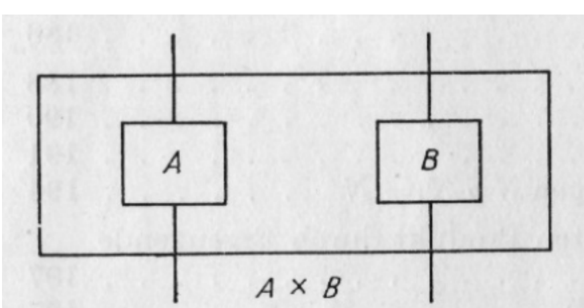

Figure 1:

$$
\begin{aligned}
L(A \times B) &= L(A) + L(B), \\
L(A \circ B) &= L(A) + L(B).
\end{aligned}
$$

Hereby a price is assigned to each circuit.

Now, the task is the following: Given $f : S^n \to S^m$, find a circuit A with $\phi(A) = f$ and

$$L(A) = \min_{B \in \phi^{-1}(f)} \{L(B)\}.$$

If f does not fully map to S^n, but only to $R \subset S^n$, then the optimium shall be searched on $\cup_{g|R=f} \phi^{-1}(g)$.

The task is generalized in an obvioius way, if $Q(f) = Z(F) = S^n$ and f, as it is often the case with finite automata, is determined just by a transformation of S^n,

In order to solve this problem it appears advantageous to know relations, which allow to generate from an element $A \in \phi^{-1}(f)$ all the elements from the class $\phi^{-1}(f)$.

In the first two sections of this work a theory of interconnection of automata will be developed, as already sketched out in the description of the task:

First, the toplological notion of a *flat* network is introduced. The binary operators "$\circ$" and "$\times$" will be explained for these networks. One obtains an algebraic structure $\mathcal{R}$, which is a catgory with respect to "$\circ$" and a semi-group with respect to "$\times$". $\mathcal{R}$ turns out to be a generalisation of the D-category (category with direct products), which we want to call *X-Kategorie.*