1 Preface

1.1 Brief history

CVM C++ class library[1] encapsulates concepts of vector and different kinds of matrices including square, band, symmetric and hermitian ones, in Euclidean space of real and complex numbers. The library is distributed under the Boost Software License, Version 1.0. First version of the library was released in 1992. Originally it allowed to simplify a code dealing with matrices and vectors in the following way:

```
rvector a(10), b(20);
rmatrix A(20,10);
......
b = A * a;
cout << "Norm of b equals " << b.norm() << endl;
```

By overloading of arithmetic operators and parentheses relative readability of source code was achieved.

In 1995[2] Russian office of Intel Corporation began to distribute a CD with last software products from Intel. Among others there was a freely (at that time) distributed library called "Intel BLAS Library". More than 20 years BLAS library (Basic Linear Algebra Subprograms) was known to experts in numerical algorithms programming in FORTRAN language. This library features common vector–matrix operations for data types `REAL` and `DOUBLE RECISION`. Both implementations of the BLAS are utilized since version 2.0 of CVM Class library. For example, operator of addition of two vectors utilizes subroutine `DAXPY`. Later Intel changed the name "BLAS" to "Intel Math Kernel Library". Since version 3.0 LAPACK subroutines were added to the Intel MKL library. Version 4.0 featured complex numbers. And since version 5.0 band, symmetric and hermitian matrices are implemented as well. Version 6.0 had many additional algorithms implemented, performance enhancements and bug fixes. Version 6.1 brought transposed versions of linear solvers as well some vector–matrix operators. Version 7.0 added functional classes including elementary functions, vectors and matrices of them and supporting simplification and derivatives computation (analytically, not numerically). Version 8.0 implemented new C++ 11 standard features like move semantics for better performance.

1.2 Libraries Supported

The last version of the Intel MKL library at the moment of writing of this document for Win32 and Linux is 11.3. Version 8.2 of the CVM library was built and tested with that MKL library implementation.

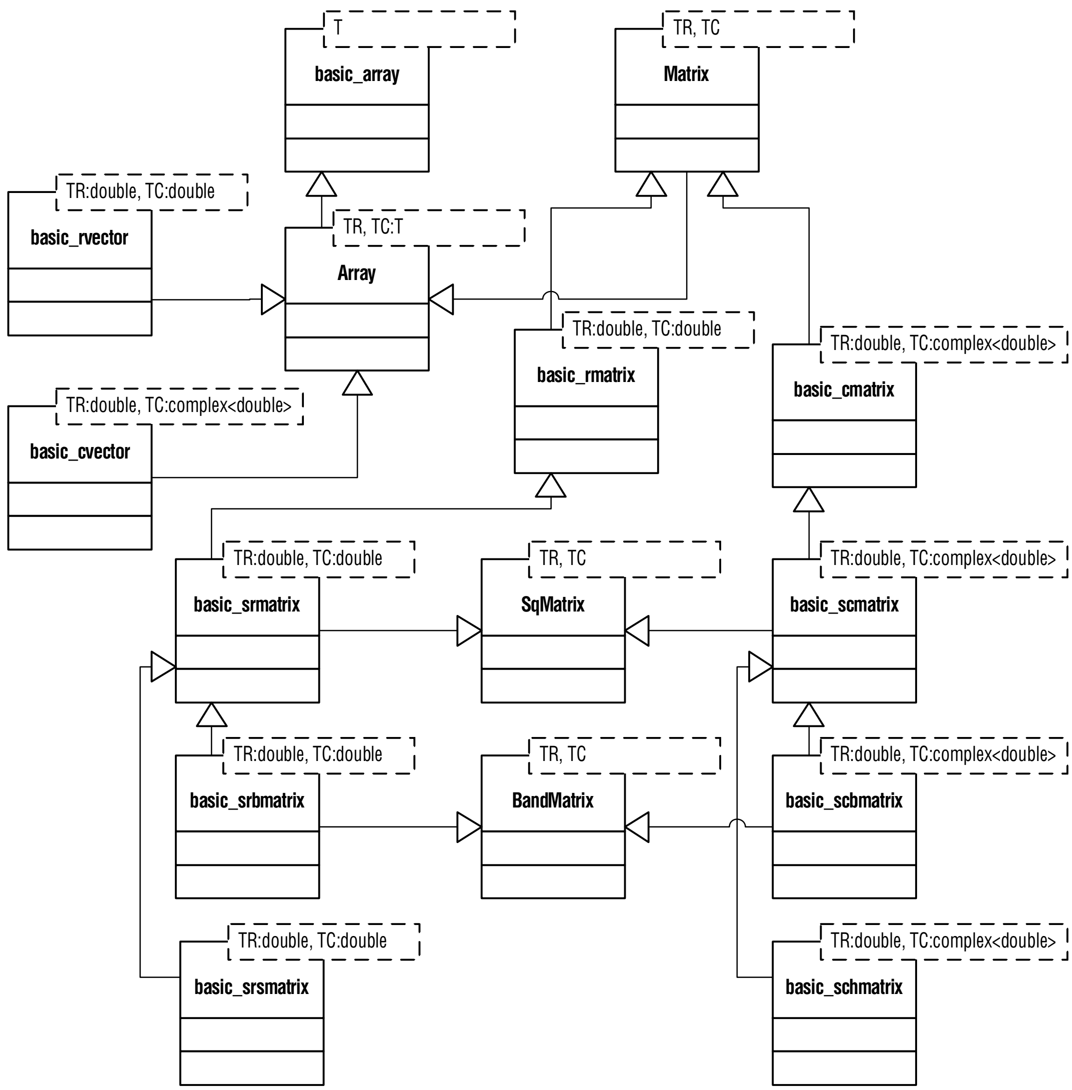
Finally, native BLAS and LAPACK libraries are supported and starting from CVM 5.7 they are part of binary packages as well.

[1] This document describes version 8.2. Copyright © Sergei Nikolaev, 1992–2016, http://cvmlib.com

[2] I'm not sure actually, may be that was 1994

1.3 Object Model

Object model of the CVM library is shown at the following picture



Base classes names are beginning with capital letters. They implement common interfaces and are *not* designed to be instantiated. This is the list of end-user classes:

- `basic_array` is an abstract array of elements of any type. It's used mostly as an array of integers like `basic_array<int>`, but you can use it in your projects within any other types as well.
- `basic_rvector` is a class encapsulating a vector of real numbers.
- `basic_cvector` is a class encapsulating a vector of complex numbers.
- `basic_rmatrix` is a class encapsulating a matrix of real numbers.
- `basic_cmatrix` is a class encapsulating a matrix of complex numbers.
- `basic_srmatrix` is a class encapsulating square matrix of real numbers.

- `basic_scmatrix` is a class encapsulating square matrix of complex numbers.
- `basic_srbmatrix` is a class encapsulating square band matrix of real numbers. Packed storage is used.
- `basic_scbmatrix` is a class encapsulating square band matrix of complex numbers. Packed storage is used.
- `basic_srsmatrix` is a class encapsulating symmetric matrix of real numbers.
- `basic_schmatrix` is a class encapsulating hermitian matrix of complex numbers.

You don't need to use those class names directly unless you want to `typedef` your own ones. Otherwise, you should use the following pre-defined classes:

```
using iarray    = basic_array<tint,tint>;
using rvector   = basic_rvector  <treal>;
using rmatrix   = basic_rmatrix  <treal>;
using srmatrix  = basic_srmatrix <treal>;
using cvector   = basic_cvector  <treal,tcomplex>;
using cmatrix   = basic_cmatrix  <treal,tcomplex>;
using scmatrix  = basic_scmatrix <treal,tcomplex>;
using srbmatrix = basic_srbmatrix<treal>;
using scbmatrix = basic_scbmatrix<treal,tcomplex>;
using srsmatrix = basic_srsmatrix<treal>;
using schmatrix = basic_schmatrix<treal,tcomplex>;
```

The rest of this manual describes them in details. Since version 8.2 you can use both precisions simultaneously:

```
using rvector32   = basic_rvector  <float>;
using rmatrix32   = basic_rmatrix  <float>;
using srmatrix32  = basic_srmatrix <float>;
using cvector32   = basic_cvector  <float,std::complex<float>>;
using cmatrix32   = basic_cmatrix  <float,std::complex<float>>;
using scmatrix32  = basic_scmatrix <float,std::complex<float>>;
using srbmatrix32 = basic_srbmatrix<float>;
using scbmatrix32 = basic_scbmatrix<float,std::complex<float>>;
using srsmatrix32 = basic_srsmatrix<float>;
using schmatrix32 = basic_schmatrix<float,std::complex<float>>;

using rvector64   = basic_rvector  <double>;
using rmatrix64   = basic_rmatrix  <double>;
using srmatrix64  = basic_srmatrix <double>;
using cvector64   = basic_cvector  <double,std::complex<double>>;
using cmatrix64   = basic_cmatrix  <double,std::complex<double>>;
using scmatrix64  = basic_scmatrix <double,std::complex<double>>;
using srbmatrix64 = basic_srbmatrix<double>;
using scbmatrix64 = basic_scbmatrix<double,std::complex<double>>;
using srsmatrix64 = basic_srsmatrix<double>;
using schmatrix64 = basic_schmatrix<double,std::complex<double>>;
```

1.4 Installation

1.4.1 Directory Structure

The CVM library distribution has the following directory structure.

- `.\*2013.sln` MS Visual Studio 2013 solution files.
- `.\*2015.sln` MS Visual Studio 2015 solution files.
- `ftn` contains FORTRAN files. This source code is the part of the library: it implements some numerical algorithms.
- `lib` is the directory for 32-bit libraries.
- `lib64` is the directory for 64-bit libraries.
- `test` contains regression test code and projects.
- `src` contains source code of the library along with `cvm.h` and `cfun.h` header files.

1.4.2 Usage Notes

These are definitions and data types used in the library.

`CVM_ZERO_BASED`	define this macro to build the library with 0-based indexing
`CVM_FLOAT`	**(deprecated)** define this macro in order to build a float version
`CVM_NO_NAMESPACE`	define this macro if you don't want to use `cvm` namespace
`tint`	is `typedef`'ed as `long long int` (or as `__int64` for MSVC) if `ILP64` is defined and as `int` otherwise (by default)
`treal`	is `typedef`'ed as `float` if `CVM_FLOAT` is defined and as `double` otherwise (by default)
`tcomplex`	is `typedef`'ed as `std::complex<treal>`

In order to use the library just include its header file:

```
#include <cvm.h>
```

To use functional classes include this one:

```
#include <cfun.h>
```

You should also link your project with one of `cvm*.lib` for Microsoft's C++ compilers and `cvm*.so` for GNU C++ compilers (debug versions are `*_debug.lib` and `*_debug.so` respectively).

1.4.3 Installation – Windows

If you don't want to rebuild the library just download appropriate version of `cvm*.dll` and `cvm*.lib` files from binaries section of the site. If you want to rebuild the whole library you'll need Intel Fortran (and optionally Intel C++) along with MS Visual Studio 2013 or 2015. You can also use Intel MKL 10.2 (or higher) library. Open `.\cvmlib*.sln` solution and choose the library version you want to build.

1.4.4 Installation – Unix and MinGW

Use the Makefile provided in the root directory:

```
make [release|debug] [IFORT=1] [ICC=1] [MKL=1] [MKL_PATH=/opt/...]
     [EM64T=1] [ILP64=1] [CVM_FLOAT=1] [ICCT=1] [STATIC_ONLY=1]
     [IFORT_PATH=/opt/...] [MAC=1] [GF_BLAS=1]
```

Here

- `release|debug` is a target (by default it builds both)
- `IFORT=1` instructs to use `ifort` compiler (`gfortran` is the default)
- `ICC=1` instructs to use `icpc` (g++ is the default)
- `MKL=1` instructs to use Intel MKL library (it uses native BLAS and LAPACK libraries by default)
- `MKL_PATH=path` specifies the directory where the MKL is installed to. It's set to `/opt/intel/mkl/lib` by default.
- `EM64T=1` instructs to build EM64T version of the library.
- `ILP64=1` instructs to build INT64-based version of the library (EM64T must be turned on).
- `CVM_FLOAT=1` **(deprecated)** instructs to build `float` version of the library.
- `ICCT=1` instructs to use Intel C++ compiler for building the regression test utility (g++ is the default)
- `STATIC_ONLY=1` instructs to build static libraries only. Both .so and .a will be built otherwise.
- `IFORT_PATH=path` specifies the directory where the Intel Fortran is installed to. It's set to `/opt/intel` by default.
- `MAC=1` instructs to build Mac OS X version of the library.
- `GF_BLAS=1` instructs to link with BLAS and LAPACK libraries built by `gfortran` compiler. If not set, `ifort`-built libraries will be linked.

On Unix platforms Intel MKL and native BLAS/LAPACK libraries as well as both Intel's and GNU compilers are supported. On MinGW platform the native libraries and compilers only supported.

1.5 Storage

The way of storage of matrices units is the same as the FORTRAN's one. Units are stored by columns, see the following example:

```
cvm::rmatrix m(2,2);
m(1,1) = 1.;        // first  row, first  column
m(1,2) = 2.;        // first  row, second column
m(2,1) = 3.;        // second row, first  column
m(2,2) = 4.;        // second row, second column
double* p = m;
cout << p[0] << " " << p[1] << " " << p[2] << " " << p[3] << endl;
```

Output will be the following:

```
1 3 2 4
```

Since version 5.0 band matrices are introduced. Band storage can be described as follows (cited from MKL manual): *an* m *by* n *band matrix with* k_l *sub-diagonals and* k_u *super-diagonals is stored compactly in a two-dimensional array with* $k_l + k_u + 1$ *rows and* n *columns. Columns of the matrix are stored in the corresponding columns of the array, and diagonals of the matrix are stored in rows of the array*. This way of storage can be illustrated as follows (referenced elements are shown as "$*$", not referenced as "$-$", zeros are not stored):

$$m = n = 3, k_l = 0, k_u = 0 : \begin{bmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{bmatrix}$$

$$m = n = 4, k_l = 1, k_u = 0 : \begin{bmatrix} * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ 0 & * & * & 0 \\ 0 & 0 & * & * \\ & & & - \end{bmatrix}$$

$$m = n = 6, k_l = 1, k_u = 2 : \begin{bmatrix} - & & & & & \\ - & - & & & & \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ 0 & * & * & * & * & 0 \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \\ & & & & & - \end{bmatrix}$$

CVM library implements square band matrices only, therefore $m = n$ is satisfied for them.

1.6 Indexing

By default index numbering in CVM library corresponds to the FORTRAN's one: index of the first unit is equal to 1:

```
cvm::rector v(2);
cvm::rmatrix m (2,3);
v[1] = 1.3;        // first vector unit
v(2) = 2.1;        // second vector unit
m(1) = v;          // first column of matrix
m(1,2) = 3.7;      // element located on the first row
                   // and the second column of matrix
```

Starting from version 5.7 0-based indexing is supported as well. In order to use it you'd need to re-build the library with `CVM_ZERO_BASED` macro defined. To distinguish this fact, ℓ-based notation will be used below in this document, which means 1-based by default and 0-based if the macro is used.

1.7 Polymorphism

The majority of CVM Class Library member functions are not declared as virtual, but it doesn't mean that the classes are not polymorphic. Those member functions just wrap virtual ones. For example, the following code

```
void print_solution (const srmatrix& a, const rvector& b)
{
    std::cout << a.solve(b);
}
...
rvector b(3);
srmatrix m(3);
srsmatrix ms(3);
...
print_solution(m, b);
print_solution(ms, b);
```

will use symmetric solver for symmetric matrix `ms`.

1.8 Multi-threading

The library fully supports multi-threading environments. Its binary files are linked with multi-threaded versions of appropriate run-time libraries.

1.9 Regression test utility

The library is shipped with regression utility testing almost all its functions and operators. It's strongly recommended to build it upon installation and verify (see `test` directory for workspace and make files):

```
$ ./regtest_cvm_em64t
[==========] Running 540 tests from 10 test cases.
[----------] Global test environment set-up.
[----------] 61 tests from InitializationTest/0, where TypeParam = float
[ RUN      ] InitializationTest/0.TestSizes
[       OK ] InitializationTest/0.TestSizes (0 ms)

...

[ RUN      ] BlasTest/1.TestHermitianMatrixExponent
[       OK ] BlasTest/1.TestHermitianMatrixExponent (0 ms)
[----------] 23 tests from BlasTest/1 (1 ms total)

[----------] Global test environment tear-down
[==========] 540 tests from 10 test cases ran. (813 ms total)
[  PASSED  ] 540 tests.
```

Since version 8.2 Google Test Library is used here, so you can use `--gtest_filter` option for filtering tests etc.

2 CVM Class Library Reference

2.1 basic_array

This class contains array-specific member functions inherited in other classes. It can be utilized as a standalone class too. It also provides STL-compatible functions and type definitions, so itself and derived classes can be used in the same way as `std::vector<T>`. Since version 5.0 the `iarray` class is defined as

```
typedef basic_array<int> iarray;
```

```
template <typename T>
class basic_array {
public:
    int basic_array ();
    explicit basic_array (int nSize, bool bZeroMemory = true);
    basic_array (T* p, int nSize);
    basic_array (const T* p, int nSize);
    basic_array (const T* first, const T* last);
    basic_array (const basic_array& a);
    int size () const;
    T* get ();
    const T* get () const;
    operator T* ();
    operator const T* () const;
    T& operator () (int i) throw (cvmexception);
    T operator () (int i) const throw (cvmexception);
    T& operator [] (size_type i) throw (cvmexception);
    T operator [] (size_type i) const throw (cvmexception);
    T& operator [] (int i) throw (cvmexception);
    T operator [] (int i) const throw (cvmexception);
    basic_array& operator = (const basic_array& a) throw (cvmexception);
    basic_array& assign (const T* p);
    basic_array& set (T x);
    basic_array& resize (int nNewSize) throw (cvmexception);
    // STL-specific type definitions
    typedef T value_type;
    typedef value_type* pointer;
    typedef const value_type* const_pointer;
    typedef value_type* iterator;
    typedef const value_type* const_iterator;
    typedef value_type& reference;
    typedef const value_type& const_reference;
    typedef size_t size_type;
    typedef ptrdiff_t difference_type;
    typedef std::reverse_iterator<const_iterator> const_reverse_iterator;
```

```
    typedef std::reverse_iterator<iterator> reverse_iterator;
    // STL-specific functions
    iterator begin ();
    const_iterator begin () const;
    iterator end ();
    const_iterator end () const;
    reverse_iterator rbegin ();
    const_reverse_iterator rbegin () const;
    reverse_iterator rend ();
    const_reverse_iterator rend () const;
    size_type max_size () const;
    size_type capacity () const;
    bool empty () const;
    reference front ();
    const_reference front ();
    reference back ();
    const_reference back () const;
    void assign (size_type n, const T& val) throw (cvmexception);
    void assign (const_iterator first,
                 const_iterator last) throw (cvmexception);
    void resize (size_type nNewSize) throw (cvmexception);
    void clear ();
    void swap (basic_array& v);
    reference at (size_type n) throw (cvmexception);
    const_reference at (size_type n) const throw (cvmexception);
    void push_back (const T& x) throw (cvmexception);
    void pop_back () throw (cvmexception);
    iterator insert (iterator position, const T& x) throw (cvmexception);
    iterator erase (iterator position) throw (cvmexception);

    template <typename T>
    friend std::istream& operator >> <> (const std::istream& is,
                                         basic_array<T>& aIn);

    template <typename T>
    friend std::ostream& operator << <> (std::ostream& os,
                                         const basic_array<T>& aOut);
};
```

2.1.1 basic_array()

Default constructor

```
basic_array<T>::basic_array();
```

creates empty `basic_array` object. See also `basic_array`. Example:

```
using namespace cvm;

iarray a;
std::cout << a.size() << std::endl;
a.resize(10);
std::cout << a.size() << std::endl;
```

prints

```
0
10
```

2.1.2 basic_array(int, bool)

Constructor

```
explicit basic_array<T>::basic_array(int nSize, bool bZeroMemory = true);
```

creates `basic_array` object of size equal to `nSize`. Allocated memory is initialized with zero values by default (you can pass `false` in second argument in order to avoid this initialization). Constructor throws `cvmexception` in case of non-positive size passed or memory allocation failure. See also `basic_array`. Example:

```
using namespace cvm;

iarray a(5);
std::cout << a.size() << " " << a[1] << std::endl;
```

prints

```
5 0
```

2.1.3 basic_array(T*, int)

Constructor

```
basic_array<T>::basic_array (T* p, int nSize);
```

creates `basic_array` object of size equal to `nSize` and shares memory (shallow copy) with an array pointed to by `p` parameter. Constructor throws `cvmexception` in case of non-positive size passed. See also `basic_array`. Example:

```
using namespace cvm;

int a[] = {1, 2, 3, 4};
iarray v (a, 3);
std::cout << v;
a[0] = 77;
std::cout << v;
```

prints

```
1 2 3
77 2 3
```

2.1.4 basic_array(const T*, int)

Constructor

```
basic_array<T>::basic_array (const T* p, int nSize);
```

creates `basic_array` object of size equal to `nSize` and copies `nSize` elements of an array `p` to it. Constructor throws `cvmexception` in case of non-positive size passed or memory allocation failure. See also `basic_array`. Example:

```
using namespace cvm;

const int a[] = {1, 2, 3, 4};
iarray v (a, 3);
std::cout << v;
```

prints

```
1 2 3
```

2.1.5 basic_array(const T*, const T*)

Constructor

```
basic_array<T>::basic_array (const T* first, const T* last);
```

creates `basic_array` object of size equal to `last-first` and copies all elements in the range of `[first,last)` to it. Constructor throws `cvmexception` in case of wrong range passed or memory allocation failure. See also `basic_array`. Example:

```
using namespace cvm;

const int a[] = {1, 2, 3, 4};
const iarray v (a+1, a+3);
std::cout << v << std::endl;
```

prints

```
2 3
```

2.1.6 basic_array(const basic_array&)

Copy constructor

```
basic_array<T>::basic_array (const basic_array& a);
```

creates `basic_array` object of size equal to size of vector `a` and sets every element of created array to a value of appropriate element of `a`. See also `basic_array`. Example:

```
using namespace cvm;

iarray a(5);
a.set(3);
iarray b(a);
std::cout << b;
```

prints

```
3 3 3 3 3
```

2.1.7 size()

Function

```
int basic_array<T>::size () const;
```

returns number of elements of a calling array. This function is *inherited* in all classes of the library: rvector, cvector, rmatrix, cmatrix, srmatrix, scmatrix, srbmatrix, scbmatrix, srsmatrix and schmatrix. See also basic_array. Example:

```
using namespace cvm;

rvector v(3);
cmatrix m(10,20);
cout << v.size() << " " << m.size() << endl;
```

prints

```
3 200
```

2.1.8 get(), operator T*()

Functions and operators

```
T* basic_array<T>::get ();
const T* basic_array<T>::get () const;
basic_array<T>::operator T* ();
basic_array<T>::operator const T* () const;
```

return pointer to beginning (first element) of a calling array. These functions and operators are *inherited* in all classes of the library: rvector, cvector, rmatrix, cmatrix, srmatrix, scmatrix, srbmatrix, scbmatrix, srsmatrix and schmatrix. See also basic_array. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

void cprint (const std::complex<double>* p, int size)
{
    for (int i = 0; i < size; ++i)
        std::cout << p[i] << " ";
    std::cout << std::endl;
}
...
iarray a(10);
scmatrix m(3);
a[2] = 1;
m(3,1) = std::complex<double>(1., 2.);
std::cout << a.get()[1] << std::endl;
cprint(m, 3);
```

prints

```
1
(0.00e+00,0.00e+00) (0.00e+00,0.00e+00) (1.00e+00,2.00e+00)
```

2.1.9 Indexing operators

Indexing operators

```
T& basic_array<T>::operator () (int i) throw (cvmexception);
T  basic_array<T>::operator () (int i) const throw (cvmexception);
T& basic_array<T>::operator [] (size_type i) throw (cvmexception);
T  basic_array<T>::operator [] (size_type i) const throw (cvmexception);
T& basic_array<T>::operator [] (int i) throw (cvmexception);
T  basic_array<T>::operator [] (int i) const throw (cvmexception);
```

return reference (or value for constant versions) to `i`-th element of a calling array. Please note that *all indexing operators of the library are* *ℓ-based*. These operators are *inherited* in `rvector` and `cvector` classes of the library but *reimplemented* in other ones: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `basic_array`. Example:

```
using namespace cvm;

try {
    rvector v (10);
    v[1] = 1.;
    v(2) = 2.;
    std::cout << v;

    double a[] = {1., 2., 3., 4.};
    const rvector vc (a, 4);
    std::cout << vc(1) << " " << vc[2] << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 2 0 0 0 0 0 0 0 0
1 2
```

2.1.10 operator = (const basic_array&)

Assignment operator

```
basic_array<T>&
basic_array<T>::operator = (const basic_array& a) throw (cvmexception);
```

sets every element of a calling array to a value of appropriate element of an array a and returns a reference to the object changed. This operator is *reimplemented* in all classes of the library: `rvector`, `cvector` , `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `basic_array`. Operator throws `cvmexception` in case of different sizes of the arrays. Example:

```
using namespace cvm;

iarray a(5), b(5);
a.set(3);
b = a;
std::cout << b;
```

prints

```
3 3 3 3 3
```

2.1.11 assign(const T*)

Function

```
basic_array<T>& basic_array<T>::assign (const T* p);
```

sets every element of a calling array to a value of appropriate element of an array pointed to by parameter p and returns a reference to the object changed. This function is *reimplemented* in all classes of the library: rvector, cvector , rmatrix, cmatrix, srmatrix, scmatrix, srbmatrix, scbmatrix, srsmatrix and schmatrix. See also basic_array. Example:

```
using namespace cvm;

const int a[] = {1, 2, 3, 4, 5, 6, 7};
iarray v (5);

v.assign(a);
std::cout << v;
```

prints

```
1 2 3 4 5
```

2.1.12 set(T)

Function

```
basic_array<T>& basic_array<T>::set (T x);
```

sets every element of a calling array to a value of parameter `x` and returns a reference to the object changed. This function is *reimplemented* in all classes of the library: `rvector`, `cvector` , `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `basic_array`. Example:

```
using namespace cvm;

iarray a(5);
a.set(3);
std::cout << a;
```

prints

```
3 3 3 3 3
```

2.1.13 resize

Function

```
basic_array<T>&
basic_array<T>::resize (int nNewSize) throw (cvmexception);
```

changes size of a calling array to be equal to `nNewSize` and returns a reference to the array changed. Calling array will be filled with zeroes in case of increasing of its size. This function is *reimplemented* in all classes of the library: `rvector`, `cvector` , `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. Function throws `cvmexception` in case of non-positive size passed or memory allocation failure. See also `basic_array`. Example:

```
using namespace cvm;

try {
    const int a[] = {1, 2, 3, 4};
    iarray v (a, 3);
    std::cout << v;
    v.resize(2);
    std::cout << v;
    v.resize(4);
    std::cout << v;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 2 3
1 2
1 2 0 0
```

2.1.14 STL-specific type definitions

Type definitions

```
typedef T value_type;
typedef value_type* pointer;
typedef const value_type* const_pointer;
typedef value_type* iterator;
typedef const value_type* const_iterator;
typedef value_type& reference;
typedef const value_type& const_reference;
typedef size_t size_type;
typedef ptrdiff_t difference_type;
typedef std::reverse_iterator<const_iterator> const_reverse_iterator;
typedef std::reverse_iterator<iterator> reverse_iterator;
```

are provided for every class of the library to be compatible with STL algorithms and methods. See also basic_array. Example:

```
using namespace cvm;

rvector vs1(5);
vs1[1] = 1.; vs1[2] = 2.; vs1[3] = 3.; vs1[4] = 4.; vs1[5] = 5.;
std::cout << vs1;

rvector::iterator it = vs1.begin() + 1;
rvector::iterator ite = vs1.erase(it);
std::cout << vs1;
std::cout << *ite << std::endl;

ite = vs1.insert(ite, 10.);
std::cout << vs1;
std::cout << *ite << std::endl;

vs1.push_back(11.);
std::cout << vs1;

vs1.randomize(0., 2.);
std::cout << vs1;
std::sort(vs1.begin(), vs1.end());
std::cout << vs1;
std::cout << *std::max_element(vs1.begin(), vs1.end()) << std::endl;

std::reverse(vs1.begin(), vs1.end());
std::cout << vs1;
```

prints

```
1.00e+000 2.00e+000 3.00e+000 4.00e+000 5.00e+000
1.00e+000 3.00e+000 4.00e+000 5.00e+000
3.00e+000
1.00e+000 1.00e+001 3.00e+000 4.00e+000 5.00e+000
1.00e+001
1.00e+000 1.00e+001 3.00e+000 4.00e+000 5.00e+000 1.10e+001
1.11e-001 4.96e-001 1.70e+000 1.91e+000 1.19e-001 1.11e+000
1.11e-001 1.19e-001 4.96e-001 1.11e+000 1.70e+000 1.91e+000
1.91e+000
1.91e+000 1.70e+000 1.11e+000 4.96e-001 1.19e-001 1.11e-001
```

2.1.15 STL-specific functions: begin(), end(), rbegin(), rend(), max_size(), capacity(), empty(), front(), back(), assign(), resize(), clear(), swap()

Functions

```
basic_array<T>::iterator basic_array<T>::begin();
basic_array<T>::const_iterator basic_array<T>::begin() const;
basic_array<T>::iterator basic_array<T>::end();
basic_array<T>::const_iterator basic_array<T>::end() const;
basic_array<T>::reverse_iterator basic_array<T>::rbegin();
basic_array<T>::const_reverse_iterator basic_array<T>::rbegin() const;
basic_array<T>::reverse_iterator basic_array<T>::rend();
basic_array<T>::const_reverse_iterator basic_array<T>::rend();
basic_array<T>::size_type basic_array<T>::max_size() const;
basic_array<T>::size_type basic_array<T>::capacity() const;
bool basic_array<T>::empty() const;
basic_array<T>::reference basic_array<T>::front();
basic_array<T>::const_reference basic_array<T>::front();
basic_array<T>::reference basic_array<T>::back();
basic_array<T>::const_reference basic_array<T>::back();
void basic_array<T>::assign (size_type n,
                             const T& val) throw (cvmexception);
void basic_array<T>::assign (const_iterator first,
                             const_iterator last) throw (cvmexception);
void basic_array<T>::resize (size_type nNewSize) throw (cvmexception);
void basic_array<T>::clear();
void basic_array<T>::swap (basic_array& v) throw (cvmexception);
```

are provided for every class of the library to be compatible with STL algorithms and methods. See also basic_array and STL documentation for further details. Example:

```
using namespace cvm;

iarray a(5);
a[1] = 1; a[2] = 2; a[3] = 3; a[4] = 4; a[5] = 5;

for (iarray::reverse_iterator it = a.rbegin(); it != a.rend(); ++it)
{
    std::cout << *it << " ";
}
std::cout << std::endl;
std::cout << a.front() << std::endl;
std::cout << a.back() << std::endl;
```

prints

```
5 4 3 2 1
1
5
```

2.1.16 at()

Functions

```
basic_array<T>::reference
basic_array<T>::at(size_type n) throw (cvmexception);

basic_array<T>::const_reference
basic_array<T>::at(size_type n) throw (cvmexception);
```

return reference to `(n-1)`-th element of a calling array, i.e., unlike indexing operators, these functions are *0-based*. They are provided for every class of the library to be compatible with STL algorithms and methods. Functions throw `cvmexception` in case of negative parameter passed. See also `basic_array` and STL documentation for further details. Example:

```
using namespace cvm;

iarray a(5);
a[1] = 1; a[2] = 2; a[3] = 3; a[4] = 4; a[5] = 5;
std::cout << a.at(0) << " " << a.at(1) << std::endl;
```

prints

```
1 2
```

2.1.17 push_back(const T&), pop_back()

Functions

```
void basic_array<T>::push_back (const T& x) throw (cvmexception);
void basic_array<T>::pop_back () throw (cvmexception);
```

add and remove an element to/from calling array. They are provided for every class of the library to be compatible with STL algorithms and methods. Since CVM doesn't pre-allocate memory for extra storage, these functions *will require memory reallocation every time they are being executed* and may slow down your application. Please consider usage of `std::vector<T>` in such cases. Functions throw `cvmexception` in case of memory allocation failure. See also `basic_array` and STL documentation for further details. Example:

```
using namespace cvm;

iarray a(5);
a.push_back(77);
std::cout << a;
a.pop_back();
std::cout << a;
```

prints

```
0 0 0 0 0 77
0 0 0 0 0
```

2.1.18 insert (iterator, const T&), erase (iterator)

Functions

```
basic_array<T>::iterator
basic_array<T>::insert (iterator pos, const T& x) throw (cvmexception);

basic_array<T>::iterator
basic_array<T>::erase (iterator pos) throw (cvmexception);
```

insert and remove an element to/from calling array at given position `pos`. They are provided for every class of the library to be compatible with STL algorithms and methods. Since CVM doesn't pre-allocate memory for extra storage, these functions *will require memory reallocation every time they are executed* and may slow down your application. Please consider usage of `std::vector<T>` in such cases. Functions throw `cvmexception` in case of memory allocation failure. See also `basic_array` and STL documentation for further details. Example:

```
using namespace cvm;

iarray a(5);
iarray::iterator pos = a.begin() + 2;
a.insert(pos, 77);
std::cout << a;
pos = a.begin() + 1;
a.erase(pos);
std::cout << a;
```

prints

```
0 0 77 0 0 0
0 77 0 0 0
```

2.1.19 operator >> <> (std::istream& is, basic_array<T>& aIn)

Friend template operator

```
template <typename T>
friend std::istream& operator >> <> (std::istream& is,
                                     basic_array<T>& aIn);
```

fills object referenced by aIn with numbers from is stream. Operator is redefined in the class Array. See also basic_array Example:

```
using namespace cvm;

try {
    std::ofstream os;
    os.open ("in.txt");
    os << 1 << " " << 2 << std::endl << 3;
    os.close ();

    std::ifstream is("in.txt");
    iarray v(5);
    is >> v;

    std::cout << v;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 2 3 0 0
```

2.1.20 operator << <> (std::ostream& os, const basic_array<T>& aOut)

Friend template operator

```
template <typename T>
friend std::ostream& operator << <> (std::ostream& os,
                                     const basic_array<T>& aOut);
```

outputs object referenced by aOut into os stream. Operator is redefined in the class Array. See also basic_array Example:

```
using namespace cvm;

iarray v(5);
v(1) = 1;
v(2) = 2;

std::cout << v;
```

prints

```
1 2 0 0 0
```

2.2 Array

This class contains a couple of common for arrays member functions inherited in user classes. This class is not designed to be instantiated.

```
template <typename TR, typename TC>
class Array : public basic_array<TC> {
public:
    int incr () const;
    int indofmax () const;
    int indofmin () const;
    virtual TR norm () const;
    virtual TR norminf () const;
    virtual TR norm1 () const;
    virtual TR norm2 () const;

    <typename TR, typename TC>
    friend std::istream& operator >> <> (std::istream& is,
                                         Array<TR,TC>& aIn);

    <typename TR, typename TC>
    friend std::ostream& operator << <> (std::ostream& os,
                                         const Array<TR,TC>& aOut);
};
```

2.2.1 incr

Function

```
int Array<TR,TC>::incr () const;
```

returns increment between elements of a calling array. This function is *inherited* in all classes of the library: `rvector`, `cvector`, `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. It always returns 1 for matrices. See also `Array`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.};
rvector v1 (a, 3, 2);
rvector v2(10);

std::cout << v1 << v1.incr () << std::endl;
std::cout << v2.incr () << std::endl;
```

prints

```
1 3 5
2
1
```

2.2.2 indofmax

Function

```
int Array<TR,TC>::indofmax () const;
```

returns 1-based index of a calling array's element with the maximum absolute value. Function is *inherited*[3] in all classes of the library: rvector, cvector, rmatrix, cmatrix, srmatrix, scmatrix, srbmatrix, scbmatrix, srsmatrix and schmatrix. See also Array. Example:

```
using namespace cvm;

double a[] = {3., 2., -5., -4., 5., -6.};
const rvector v (a, 4);
const rmatrix m (a, 2, 3);

std::cout << v << v.indofmax () << std::endl << std::endl;
std::cout << m << m.indofmax () << std::endl;
```

prints

```
3 2 -5 -4
3

3 -5 5
2 -4 -6
6
```

[3]Calls virtual function inside

2.2.3 indofmin

Function

```
int Array<TR,TC>::indofmin () const;
```

returns ℓ-based index of a calling array's element with the minimum absolute value. Function is *inherited*[4] in all classes of the library: rvector, cvector, rmatrix, cmatrix, srmatrix, scmatrix, srbmatrix, scbmatrix, srsmatrix and schmatrix. See also Array. Example:

```
using namespace cvm;

double a[] = {3., 2., -5., 0., 0., -6.};
const rvector v (a, 4);
const rmatrix m (a, 2, 3);

std::cout << v << v.indofmin () << std::endl << std::endl;
std::cout << m << m.indofmin () << std::endl;
```

prints

```
3 2 -5 0
4

3 -5 0
2 0 -6
4
```

[4]Calls virtual function inside

2.2.4 norm

Virtual function

```
virtual TR Array<TR,TC>::norm() const;
```

returns Euclidean norm of a calling array that for vectors is defined as

$$\|x\|_E = \left(\sum_{i=1}^{n} |x_i|^2\right)^{1/2}$$

and for matrices as

$$\|A\|_E = \left(\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2\right)^{1/2},$$

where A is $m \times n$ matrix. Function is *inherited* in the following classes of the library: rvector, cvector, rmatrix, cmatrix, srmatrix, scmatrix, srsmatrix and schmatrix. It's *redefined* in srbmatrix and scbmatrix. See also rvector::normalize, cvector::normalize, rmatrix::normalize, cmatrix::normalize and Array. Example:

```
using namespace cvm;
std::cout.setf (ios::scientific | ios::showpos);
std::cout.precision (12);

double a[] = {1., 2., 3., -4., 5., -6.};
const rvector v (a, 3);
const rmatrix m (a, 2, 3);

std::cout << v << v.norm () << std::endl << std::endl;
std::cout << m << m.norm () << std::endl;
```

prints

```
+1.000000000000e+000 +2.000000000000e+000 +3.000000000000e+000
+3.741657386774e+000

+1.000000000000e+000 +3.000000000000e+000 +5.000000000000e+000
+2.000000000000e+000 -4.000000000000e+000 -6.000000000000e+000
+9.539392014169e+000
```

2.2.5 norminf

Virtual function

```
virtual TR Array<TR,TC>::norminf () const;
```

returns infinity norm of a calling array that for vectors is defined as

$$\|x\|_\infty = \max_{i=1,\dots,n} |x_i|$$

and for matrices as

$$\|A\|_\infty = \max_{i=1,\dots,m} \sum_{j=1}^{n} |a_{ij}|,$$

where A is $m \times n$ matrix. Function is *inherited* in rvector and cvector classes of the library. It's *redefined* in Matrix, srbmatrix, scbmatrix, srsmatrix and schmatrix. See also Matrix::norm1, Array. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., -4., 5., -6.};
const rvector v (a, 3);
const rmatrix m (a, 2, 3);

std::cout << v << v.norminf () << std::endl;
std::cout << m << m.norminf () << std::endl;
```

prints

```
1 2 3
3

1 3 5
2 -4 -6
12
```

2.2.6 norm1

Virtual function

```
virtual TR Array<TR,TC>::norm1 () const;
```

returns 1-norm of a calling array that for vectors is defined as

$$\|x\|_1 = \sum_{i=1}^{n} |x_i|$$

and for matrices as

$$\|A\|_1 = \max_{j=1,\dots,n} \sum_{i=1}^{m} |a_{ij}|,$$

where x is vector of size n and A is $m \times n$ matrix. Function is *inherited* in rvector and cvector classes. It's *redefined* in rmatrix and cmatrix and inherited thereafter. See also Array. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (12);

double a[] = {1., 2., 3., -4., 5., -6.};
const rvector v (a, 3);
const rmatrix m (a, 2, 3);

std::cout << v << v.norm1 () << std::endl << std::endl;
std::cout << m << m.norm1 () << std::endl;
```

prints

```
+1.000000000000e+000 +2.000000000000e+000 +3.000000000000e+000
+6.000000000000e+000

+1.000000000000e+000 +3.000000000000e+000 +5.000000000000e+000
+2.000000000000e+000 -4.000000000000e+000 -6.000000000000e+000
+1.100000000000e+001
```

2.2.7 norm2

Virtual function

```
virtual TR Array<TR,TC>::norm2 () const;
```

returns 2-norm of a calling array that for vectors is defined as

$$\|x\|_2 = \|x\|_E = \left(\sum_{i=1}^{n} |x_i|^2\right)^{1/2}$$

and for matrices as

$$\|A\|_2 = \max_i \sigma_i = \left(\max_{|x|=1} (Ax \cdot Ax)\right)^{1/2},$$

where σ_i is i-th singular value of $m \times n$ matrix A, $i = 1, \ldots, \min(m, n)$. Function is *inherited* in rvector and cvector classes. It's *redefined* in rmatrix and cmatrix and inherited thereafter. See also Array. Example:

```
using namespace cvm;
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (12);

double a[] = {1., 2., 3., -4., 5., -6.};
const rvector v (a, 3);
const rmatrix m (a, 2, 3);

std::cout << v << v.norm2 () << std::endl << std::endl;
std::cout << m << m.norm2 () << std::endl;
```

prints

```
+1.000000000000e+000 +2.000000000000e+000 +3.000000000000e+000
+3.741657386774e+000

+1.000000000000e+000 +3.000000000000e+000 +5.000000000000e+000
+2.000000000000e+000 -4.000000000000e+000 -6.000000000000e+000
+9.319612060784e+000
```

2.2.8 operator >> <> (std::istream& is, Array<TR,TC>& aIn)

Friend template operator

```
template <typename TR, typename TC>
friend std::istream& operator >> <> (std::istream& is,
                                      Array<TR,TC>& aIn);
```

fills calling object referenced by parameter aIn with numbers from is stream. See also basic_array::operator >> , Array. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    std::ofstream os;
    os.open ("in.txt");
    os << 1.2 << " " << 2.3 << std::endl << 3.4;
    os.close ();

    std::ifstream is("in.txt");
    rvector v(5);
    is >> v;

    std::cout << v;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.20e+000 2.30e+000 3.40e+000 0.00e+000 0.00e+000
```

2.2.9 operator << <> (std::ostream& os, const Array<TR,TC>& aOut)

Friend template operator

```
template <typename TR, typename TC>
friend std::ostream& operator << <> (std::ostream& os,
                                      const Array<TR,TC>& aOut);
```

outputs calling object referenced by aOut into os stream. Operator is redefined in the class Matrix. See also basic_array::operator << , Array. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v(3);
v(1) = tcomplex (1., 2.);
v(2) = tcomplex (3., 4.);

std::cout << v;
```

prints

```
(1.00e+000,2.00e+000) (3.00e+000,4.00e+000) (0.00e+000,0.00e+000)
```

2.3 rvector

This is end-user class encapsulating vector of real numbers.

```
template <typename TR>
class rvector : public Array<TR,TR> {
public:
    rvector ();
    explicit rvector (int nSize);
    rvector (std::initializer_list<TR> list);
    rvector (int nSize, TR d);
    rvector (TR* pD, int nSize, int nIncr = 1);
    rvector (const TR* pD, int nSize, int nIncr = 1);
    rvector (const rvector& v);
    rvector& operator = (const rvector& v) throw (cvmexception);
    rvector& assign (const TR* p, int nIncr = 1);
    rvector& assign (int n, const TR* p, int nIncr = 1);
    rvector& assign (int n, const TR* p, int nSize, int nIncr);
    rvector& assign (int n, const rvector& v) throw (cvmexception);
    rvector& set (TR x);
    rvector& resize (int nNewSize) throw (cvmexception);
    bool operator == (const rvector& v) const;
    bool operator != (const rvector& v) const;
    rvector& operator << (const rvector& v) throw (cvmexception);
    rvector operator + (const rvector& v) const throw (cvmexception);
    rvector operator - (const rvector& v) const throw (cvmexception);
    rvector& sum (const rvector& v1,
                  const rvector& v2) const throw (cvmexception);
    rvector& diff (const rvector& v1,
                   const rvector& v2) const throw (cvmexception);
    rvector& operator += (const rvector& v) throw (cvmexception);
    rvector& operator -= (const rvector& v) throw (cvmexception);
    rvector operator - () const;
    rvector operator * (TR d) const throw (cvmexception);
    rvector operator / (TR d) const throw (cvmexception);
    rvector& operator *= (TR d);
    rvector& operator /= (TR d) throw (cvmexception);
    rvector& normalize ();
    TR operator * (const rvector& v) const throw (cvmexception);
    rvector operator * (const rmatrix& m) const
                          throw (cvmexception);
    rvector& mult (const rvector& v, const rmatrix& m)
                   throw (cvmexception);
    rvector& mult (const rmatrix& m, const rvector& v)
                   throw (cvmexception);
```

```
rmatrix rank1update (const rvector& v) const;
rvector& solve (const srmatrix& mA,
                const rvector& vB, TR& dErr)
                throw (cvmexception);
rvector& solve (const srmatrix& mA,
                const rvector& vB) throw (cvmexception);
rvector& solve_tran (const srmatrix& mA,
                     const rvector& vB, TR& dErr)
                     throw (cvmexception);
rvector& solve_tran (const srmatrix& mA,
                     const rvector& vB) throw (cvmexception);
rvector operator / (const srmatrix& mA) const throw (cvmexception);
rvector operator % (const srmatrix& mA) const throw (cvmexception);
rvector& solve_lu (const srmatrix& mA, const srmatrix& mLU,
                   const int* pPivots, const rvector& vB, TR& dErr)
                   throw (cvmexception);
rvector& solve_lu (const srmatrix& mA, const srmatrix& mLU,
                   const int* pPivots, const rvector& vB)
                   throw (cvmexception);
rvector& gels (bool transpose, const rmatrix& mA, const rvector& mB,
               TR& dErr) throw (cvmexception);
rvector& gelsy (const rmatrix& mA, const rvector& mB,
                int& rank, TR tol = cvmMachSp())
                throw (cvmexception);
rvector& gelss (const rmatrix& mA, const rvector& mB,
                rvector& sv, int& rank, TR tol = cvmMachSp())
                throw (cvmexception);
rvector& gelsd (const rmatrix& mA, const rvector& mB,
                rvector& sv, int& rank, TR tol = cvmMachSp())
                throw (cvmexception);
rvector& svd (const rmatrix& mArg) throw (cvmexception);
rvector& svd (const cmatrix& mArg) throw (cvmexception);
rvector& svd (const rmatrix& mArg,
              srmatrix& mU, srmatrix& mVH) throw (cvmexception);
rvector& svd (const cmatrix& mArg,
              scmatrix& mU, scmatrix& mVH) throw (cvmexception);
rvector& eig (const srsmatrix& mArg) throw (cvmexception);
rvector& eig (const srsmatrix& mArg,
              srmatrix& mEigVect) throw (cvmexception);
rvector& eig (const schmatrix& mArg) throw (cvmexception);
rvector& eig (const schmatrix& mArg,
              scmatrix& mEigVect) throw (cvmexception);
rvector& gemv (bool bLeft, const rmatrix& m, TR dAlpha,
               const rvector& v, TR dBeta) throw (cvmexception);
rvector& gbmv (bool bLeft, const srbmatrix& m, TR dAlpha,
```

```
                   const rvector& v, TR dBeta) throw (cvmexception);
    rvector& randomize (TR dFrom, TR dTo);
};
```

2.3.1 rvector ()

Constructor

```
rvector::rvector ();
```

creates empty `rvector` object. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v;
std::cout << v.size() << std::endl;

v.resize (5);
v(1) = 1.5;
std::cout << v;
```

prints

```
0
1.50e+000 0.00e+000 0.00e+000 0.00e+000 0.00e+0000
```

2.3.2 rvector (int)

Constructor

```
explicit rvector::rvector (int nSize);
```

creates `rvector` object of size equal to `nSize`. It throws `cvmexception` in case of non-positive size passed or memory allocation failure. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v (5);
std::cout << v;
```

prints

```
0.00e+000 0.00e+000 0.00e+000 0.00e+000 0.00e+000
```

2.3.3 rvector (std::initializer_list<TC>)

Constructor

```
rvector::rvector (std::initializer_list<TR> list);
```

creates `rvector` object and fills it with values provided in the initializer list. This constructor is available only if your compiler supports initializer lists (GCC 4.4 and higher, MS Visual Studio 2013 and higher, Apple LLVM 5.0 and higher). It throws `cvmexception` in case of memory allocation failure. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf(std::ios::scientific | std::ios::left);
std::cout.precision(3);

rvector v = { 1, -2, 3.456, 99.99 };
std::cout << v;
```

prints

```
1.000e+000 - 2.000e+000 3.456e+000 9.999e+001
```

2.3.4 rvector (int, TR)

Constructor

```
rvector::rvector (int nSize, TR d);
```

creates `rvector` object of size equal to `nSize` and fills it with a value of `d`. It throws `cvmexception` in case of non-positive size passed or memory allocation failure. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v (5, 1.5);
std::cout << v;
```

prints

```
1.50e+00 1.50e+00 1.50e+00 1.50e+00 1.50e+00
```

2.3.5 rvector (TR*,int,int)

Constructor

```
rvector::rvector (TR* pD, int nSize, int nIncr = 1);
```

creates `rvector` object of size equal to `nSize`. Unlike others, this constructor *does not allocate memory*. It just shares memory with an array pointed to by `pD` using distance between elements equal to `nIncr`. Constructor throws `cvmexception` in case of non-positive size passed. See also `rvector`, `rvector (const TR*,int,int)`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {7., 7., 7., 7., 7., 7., 7.,};

rvector v1 (a, 4, 2);

std::cout << v1;
v1(2) = 1.23;
std::cout << v1 << std::endl;

for (int i = 0; i < 3; i++) {
    std::cout << a[i] << " ";
}
std::cout << std::endl;

rvector v2 (a, 5);
std::cout << v2;
```

prints

```
7.00e+000 7.00e+000 7.00e+000 7.00e+000
7.00e+000 1.23e+000 7.00e+000 7.00e+000

7.00e+000 7.00e+000 1.23e+000
7.00e+000 7.00e+000 1.23e+000 7.00e+000 7.00e+000
```

2.3.6 rvector (const TR*,int,int)

Constructor

```
rvector::rvector (const TR* pD, int nSize, int nIncr = 1);
```

creates `rvector` object of size equal to `nSize` and copies `nSize` elements of array `pD` to it using increment `nIncr` (1 by default). Object created has an increment set to 1. Constructor throws `cvmexception` in case of non-positive size passed. See also `rvector`, `rvector (TR*,int,int)`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

const double a[] = {7., 7., 7., 7., 7., 7., 7.,};

rvector v1 (a, 4, 2);

std::cout << v1;
v1(2) = 1.23;
std::cout << v1 << std::endl;

for (int i = 0; i < 3; i++) {
    std::cout << a[i] << " ";
}
std::cout << std::endl;

rvector v2 (a, 5);
std::cout << v2;
```

prints

```
7.00e+000 7.00e+000 7.00e+000 7.00e+000
7.00e+000 1.23e+000 7.00e+000 7.00e+000

7.00e+000 7.00e+000 7.00e+000
7.00e+000 7.00e+000 7.00e+000 7.00e+000 7.00e+000
```

2.3.7 rvector (const rvector&)

Copy constructor

```
rvector::rvector (const rvector& v);
```

creates `rvector` object of size equal to size of vector `v` and sets every element of created vector to be equal to appropriate element of `v`. Constructor throws `cvmexception` in case of memory allocation failure. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4., 5., 6., 7.,};
const rvector v (a, 4, 2);
rvector vc (v);

vc(1) = 77.;
std::cout << vc;
std::cout << v;
```

prints

```
7.70e+001 3.00e+000 5.00e+000 7.00e+000
1.00e+000 3.00e+000 5.00e+000 7.00e+000
```

2.3.8 operator = (const rvector&)

Operator

```
rvector& rvector::operator = (const rvector& v) throw (cvmexception);
```

sets every element of a calling vector to be equal to appropriate element of a vector v and returns a reference to the vector changed. Operator throws cvmexception in case of different vector sizes. See also rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4., 5.};
    const rvector v (a, 5);
    rvector vc(5);

    vc = v;
    std::cout << vc;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+000 2.00e+000 3.00e+000 4.00e+000 5.00e+000
```

2.3.9 assign(const TR*, int)

Function

```
rvector& rvector::assign (const TR* p, int nIncr = 1);
```

sets every element of a calling vector to be equal to every `nIncr`-th element of an array pointed to by parameter `p` and returns a reference to the vector changed. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

const double a[] = {1., 2., 3., 4., 5., 6., 7.,};
rvector v (5);
rvector v2 (4);

v.assign(a);
v2.assign(a, 2);
std::cout << v;
std::cout << v2;
```

prints

```
1.00e+000 2.00e+000 3.00e+000 4.00e+000 5.00e+000
1.00e+000 3.00e+000 5.00e+000 7.00e+000
```

2.3.10 assign(int, const TR*, int)

Function

```
rvector& rvector::assign (int n, const TR* p, int nIncr = 1);
```

sets every element of a calling vector, starting from ℓ-based `n`-th one, to be equal to every `nIncr`-th element of an array pointed to by parameter `p` and returns a reference to the vector changed. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

const double a[] = {1., 2., 3., 4., 5.};
rvector v (5);
rvector v2 (5);

v.assign(2, a);
v2.assign(3, a, 2);
std::cout << v;
std::cout << v2;
```

prints

```
0.00e+000 1.00e+000 2.00e+000 3.00e+000 4.00e+000
0.00e+000 0.00e+000 1.00e+000 3.00e+000 5.00e+000
```

2.3.11 assign(int, const TR*, int, int)

Function

```
rvector& rvector::assign (int n, const TR* p, int nSize, tint nIncr);
```

sets every element of a calling vector, starting from ℓ-based `n`-th one, up to `nSize` total, to be equal to every `nIncr`-th element of an array pointed to by parameter `p` and returns a reference to the vector changed. If `n + nSize` goes beyond vector boundaries assignment stops at the last element. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

const double a[] = {1., 2., 3., 4., 5., 6., 7.};
rvector v (5);

v.assign(2, a, 5, 2);
std::cout << v;
v.assign(2, a, 3, 1);
std::cout << v;
```

prints

```
0.00e+000 1.00e+000 3.00e+000 5.00e+000 7.00e+000
0.00e+000 1.00e+000 2.00e+000 3.00e+000 7.00e+000
```

2.3.12 assign (int, const rvector&)

Function

```
rvector& rvector::assign (int n, const rvector& v) throw (cvmexception);
```

sets every element of a calling vector's sub-vector beginning with ℓ-based index `n` to a vector `v` and returns a reference to the vector changed. Function throws `cvmexception` if `n` is not positive or `v.size()+n-1` is greater than calling vector's size. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v1(5);
rvector v2(2);
v1.set(1.);
v2.set(2.);
v1.assign(3, v2);
std::cout << v1;
```

prints

```
1.00e+00 1.00e+00 2.00e+00 2.00e+00 1.00e+00
```

2.3.13 set(TR)

Function

```
rvector& rvector::set (TR x);
```

sets every element of a calling vector to be equal to parameter `x` and returns a reference to the vector changed. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v(5);
v.set(3.);
std::cout << v;
```

prints

```
3.00e+000 3.00e+000 3.00e+000 3.00e+000 3.00e+000
```

2.3.14 resize

Function

```
rvector& rvector::resize (int nNewSize) throw (cvmexception);
```

changes size of a calling vector to be equal to `nNewSize` and returns a reference to the vector changed. In case of increasing of its size, the vector is filled up with zeroes. Function throws `cvmexception` in case of negative size passed or memory allocation failure. See also `basic_array::resize` and `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    rvector v (a, 3);
    std::cout << v;
    v.resize(2);
    std::cout << v;
    v.resize(4);
    std::cout << v;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+00 2.00e+00 3.00e+00
1.00e+00 2.00e+00
1.00e+00 2.00e+00 0.00e+00 0.00e+00
```

2.3.15 operator ==

Operator

```
bool rvector::operator == (const rvector& v) const;
```

compares calling vector with a vector `v` and returns `true` if they have same sizes and their appropriate elements differ by not more than the smallest normalized positive number. Returns `false` otherwise. See also `rvector`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4.};
rvector v1 (a, 4);
rvector v2 (4);

v2 (1) = 1.; v2 (2) = 2.;
v2 (3) = 3.; v2 (4) = 4.;

cout << (v1 == v2) << endl;
```

prints

```
1
```

2.3.16 operator !=

Operator

```
bool rvector::operator != (const rvector& v) const;
```

compares calling vector with a vector `v` and returns `true` if they have different sizes or at least one of their appropriate elements differs by more than the smallest normalized positive number. Returns `false` otherwise. See also `rvector`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4.};
rvector v1 (a, 4);
rvector v2 (4);

v2 (1) = 1.; v2 (2) = 2.;
v2 (3) = 3.; v2 (4) = 4.;

cout << (v1 != v2) << endl;
```

prints

```
0
```

2.3.17 operator <<

Operator

```
rvector& rvector::operator << (const rvector& v) throw (cvmexception);
```

destroys calling vector, creates a new one as a copy of `v` and returns a reference to the vector changed. Operator throws `cvmexception` in case of memory allocation failure. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    rvector v (5);
    rvector vc (3);
    v(1) = 1.;
    v(2) = 2.;
    std::cout << v << vc << std::endl;

    vc << v;
    std::cout << vc;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+000 2.00e+000 0.00e+000 0.00e+000 0.00e+000
0.00e+000 0.00e+000 0.00e+000

1.00e+000 2.00e+000 0.00e+000 0.00e+000 0.00e+000
```

2.3.18 operator +

Operator

```
rvector rvector::operator + (const rvector& v) const throw (cvmexception);
```

creates an object of type `rvector` as a sum of a calling vector and a vector `v`. Operator throws `cvmexception` in case of different sizes of the operands or memory allocation failure. See also `rvector`, `rvector::sum`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    const double b[] = {3., 5., 7., 9.};
    const rvector va (a, 4);
    rvector vb (4);
    vb.assign(b);

    std::cout << va + vb;
    std::cout << va + va;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
4.00e+000 7.00e+000 1.00e+001 1.30e+001
2.00e+000 4.00e+000 6.00e+000 8.00e+000
```

2.3.19 operator -

Operator

```
rvector rvector::operator - (const rvector& v) const throw (cvmexception);
```

creates an object of type `rvector` as a difference of a calling vector and a vector `v`. It throws `cvmexception` in case of different sizes of the operands or memory allocation failure. See also `rvector::diff`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    const double b[] = {3., 5., 7., 9.};
    const rvector va (a, 4);
    rvector vb (4);
    vb.assign(b);

    std::cout << va - vb;
    std::cout << va - va;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-2.00e+000 -3.00e+000 -4.00e+000 -5.00e+000
0.00e+000 0.00e+000 0.00e+000 0.00e+000
```

2.3.20 sum

Function

```
rvector& rvector::sum (const rvector& v1, const rvector& v2)
throw (cvmexception);
```

assigns the result of addition of vectors v1 and v2 to a calling vector and returns a reference to the vector changed. It throws cvmexception in case of different sizes of the operands. See also rvector::operator + , rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    double b[] = {2., 3., 4., 5.};
    rvector va (a, 4);
    rvector vb (b, 4);
    rvector v (4);

    std::cout << v.sum(va, vb);
    std::cout << v.sum(v, va);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3.00e+00 5.00e+00 7.00e+00 9.00e+00
4.00e+00 7.00e+00 1.00e+01 1.30e+01
```

2.3.21 diff

Function

```
rvector& rvector::diff (const rvector& v1, const rvector& v2)
throw (cvmexception);
```

assigns the result of subtraction of vectors `v1` and `v2` to a calling vector and returns a reference to the vector changed. It throws `cvmexception` in case of different sizes of the operands. See also `rvector::operator -` , `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    double b[] = {2., 3., 4., 5.};
    rvector va (a, 4);
    rvector vb (b, 4);
    rvector v (4);

    std::cout << v.diff(va, vb);
    std::cout << v.diff(v, va);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-1.00e+00 -1.00e+00 -1.00e+00 -1.00e+00
-2.00e+00 -3.00e+00 -4.00e+00 -5.00e+00
```

2.3.22 operator +=

Operator

```
rvector& rvector::operator += (const rvector& v) throw (cvmexception);
```

adds vector v to a calling vector and returns a reference to the vector changed. It throws cvmexception in case of different sizes of the operands. See also rvector::operator + , rvector::sum, rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    rvector v1 (4);
    rvector v2 (4);
    v1.set(1.);
    v2.set(2.);

    v1 += v2;
    std::cout << v1;

    // well, you can do this too, but temporary object would be created
    v2 += v2;
    std::cout << v2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3.00e+00 3.00e+00 3.00e+00 3.00e+00
4.00e+00 4.00e+00 4.00e+00 4.00e+00
```

2.3.23 operator -=

Operator

```
rvector& rvector::operator -= (const rvector& v) throw (cvmexception);
```

subtracts vector v from calling vector and returns a reference to the vector changed. It throws cvmexception in case of different sizes of the operands. See also rvector::diff, rvector::operator - , rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    rvector v1 (4);
    rvector v2 (4);
    v1.set(1.);
    v2.set(2.);

    v1 -= v2;
    std::cout << v1;

    // well, you can do this too, but temporary object would be created
    v2 -= v2;
    std::cout << v2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-1.00e+00 -1.00e+00 -1.00e+00 -1.00e+00
0.00e+00 0.00e+00 0.00e+00 0.00e+00
```

2.3.24 operator - ()

Operator

```
rvector rvector::operator - () const throw (cvmexception);
```

creates an object of type `rvector` as a calling vector multiplied by -1. It can throw `cvmexception` in case of memory allocation failure. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
const rvector v (a, 4);

std::cout << - v;
```

prints

```
-1.00e+00 -2.00e+00 -3.00e+00 -4.00e+00
```

2.3.25 operator * (TR)

Operator

```
rvector rvector::operator * (TR d) const throw (cvmexception);
```

creates an object of type `rvector` as a product of a calling vector and number `d`. It throws `cvmexception` in case of memory allocation failure. See also `rvector::operator *=`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
const rvector v (a, 4);

std::cout << v * 5.;
```

prints

```
5.00e+00 1.00e+01 1.50e+01 2.00e+01
```

2.3.26 operator / (TR)

Operator

```
rvector rvector::operator / (TR d) const throw (cvmexception);
```

creates an object of type `rvector` as a quotient of a calling vector and number `d`. It throws `cvmexception` if `d` has absolute value equal or less than the smallest normalized positive number. It also throws exception in case of memory allocation failure. See also `rvector::operator /=`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    const rvector v (a, 4);

    std::cout << v / 2.;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
5.00e-01 1.00e+00 1.50e+00 2.00e+00
```

2.3.27 operator *=

Operator

```
rvector& rvector::operator *= (TR d);
```

multiplies calling vector by number `d` and returns a reference to the vector changed. See also `rvector::operator *` , `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v (4);
v.set(2.);
v *= 2.
std::cout << v;
```

prints

```
4.00e+00 4.00e+00 4.00e+00 4.00e+00
```

2.3.28 operator /=

Operator

```
rvector& rvector::operator /= (TR d) throw (cvmexception);
```

divides calling vector by number d and returns a reference to the vector changed. It throws cvmexception if d has absolute value equal or less than the smallest normalized positive number. See also rvector::operator / , rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    rvector v (4);
    v.set(3.);
    v /= 2.;
    std::cout << v;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.50e+00 1.50e+00 1.50e+00 1.50e+00
```

2.3.29 normalize

Function

```
rvector& rvector::normalize ();
```

normalizes calling vector so its Euclidean norm becomes equal to 1 if it was greater than the smallest normalized positive number before the call (otherwise it does nothing). See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v(4);
v(1) = 1.;
v(2) = 2.;
v(3) = 3.;
v(4) = 4.;
std::cout << v.normalize();
std::cout << v.norm() << std::endl;
```

prints

```
1.83e-01 3.65e-01 5.48e-01 7.30e-01
1.00e+00
```

2.3.30 operator * (const rvector&)

Operator

```
TR rvector::operator * (const rvector& v) const throw (cvmexception);
```

returns scalar product of a calling vector and v. It throws cvmexception if the operands have different sizes. See also rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4.};
    rvector v1(4);
    rvector v2(4);
    v1.assign(a);
    v2.assign(a);

    std::cout << v1 * v2 << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3.00e+01
```

2.3.31 operator * (const rmatrix&)

Operator

```
rvector rvector::operator * (const rmatrix& m) const
throw (cvmexception);
```

creates an object of type `rvector` as a product of a calling vector and a matrix `m`. Use `mult` function to avoid new object creation. Operator throws `cvmexception` if calling vector's size differs from number of rows in matrix `m`. See also `rvector`, `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    rvector v(2);
    rmatrix m(2, 3);
    v.set(2.);
    m.set(1.);

    std::cout << v * m;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
4.00e+00 4.00e+00 4.00e+00
```

2.3.32 mult (const rvector&, const rmatrix&)

Function

```
rvector& rvector::mult (const rvector& v, const rmatrix& m)
throw (cvmexception);
```

sets calling vector to be equal to product of vector v by matrix m and returns a reference to the object changed. See also rvector::mult (const rmatrix&, const rvector&), rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    rvector v2(2), v3(3);
    rmatrix m(2, 3);
    v2.set(2.);
    m.set(1.);

    std::cout << v3.mult(v2, m);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
4.00e+00 4.00e+00 4.00e+00
```

2.3.33 mult (const rmatrix&, const rvector&)

Function

```
rvector& rvector::mult (const rmatrix& m, const rvector& v)
throw (cvmexception);
```

sets calling vector to be equal to product of a matrix `m` by vector `v` and returns a reference to the vector changed. See also `rvector::mult (const rvector&, const rmatrix&)`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    rvector v2(2), v3(3);
    rmatrix m(2, 3);
    v3.set(2.);
    m.set(1.);

    std::cout << v2.mult(m, v3);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
6.00e+00 6.00e+00
```

2.3.34 rank1update

Function

```
rmatrix rvector::rank1update (const rvector& v) const;
```

creates an object of type rmatrix as rank-1 update of a calling vector and a vector v. The rank-1 update of vector-column x of size m and vector-row y of size n is defined as $m \times n$ matrix

$$\begin{pmatrix} x_1 y_1 & x_1 y_2 & \cdots & x_1 y_n \\ x_2 y_1 & x_2 y_2 & \cdots & x_2 y_n \\ \dots & \dots & \dots & \dots \\ x_m y_1 & x_m y_2 & \cdots & x_m y_n \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} \begin{pmatrix} y_1 & y_2 & \cdots & y_n \end{pmatrix}.$$

See also rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v1(3);
rvector v2(2);
v1.set(2.);
v2.set(2.);
std::cout << v1.rank1update (v2);
```

prints

```
4.00e+00 4.00e+00
4.00e+00 4.00e+00
4.00e+00 4.00e+00
```

2.3.35 solve

Functions

```
rvector& rvector::solve (const srmatrix& mA,
                         const rvector& vB, TR& dErr)
                         throw (cvmexception);
rvector& rvector::solve (const srmatrix& mA,
                         const rvector& vB)
                         throw (cvmexception);
```

set calling vector to be equal to solution x of linear equation $A * x = b$ where parameter `mA` is square matrix A and parameter `vB` is vector b. Every function returns a reference to the vector changed. The first version also sets output parameter `dErr` to be equal to the norm of computation error. These functions throw `cvmexception` in case of inappropriate sizes of the objects or when matrix A is close to singular. See also `operator % (srmatrix)`, `rvector::solve_tran`, `rvector`, `srmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (12);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -2., 1.};
    double b[] = {1., 2., 3.};
    srmatrix ma(m, 3);
    rvector  vb(b, 3);
    rvector  vx(3);
    double   dErr = 0.;

    std::cout << vx.solve (ma, vb, dErr);
    std::cout << dErr << std::endl;

    std::cout << ma * vx - vb;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+8.000000000000e+00 -8.000000000000e+00 +3.000000000000e+00
+6.661338147751e-15
+0.000000000000e+00 +0.000000000000e+00 +0.000000000000e+00
```

2.3.36 solve_tran

Functions

```
rvector& rvector::solve_tran (const srmatrix& mA,
                               const rvector& vB, TR& dErr)
                               throw (cvmexception);
rvector& rvector::solve_tran (const srmatrix& mA,
                               const rvector& vB)
                               throw (cvmexception);
```

set calling vector to be equal to solution x of linear equation $A^T * x = b$ (which is equivalent to $x * A = b$) where parameter mA is square matrix A and parameter vB is vector b. Every function returns a reference to the vector changed. The first version also sets output parameter dErr to be equal to the norm of computation error. These functions throw cvmexception in case of inappropriate sizes of the objects or when matrix A is close to singular. See also operator / (srmatrix), rvector::solve, rvector, srmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (12);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -2., 1.};
    double b[] = {1., 2., 3.};
    srmatrix ma(m, 3);
    rvector  vb(b, 3);
    rvector  vx(3);
    double   dErr = 0.;

    std::cout << vx.solve_tran (ma, vb, dErr);
    std::cout << dErr << std::endl;

    std::cout << vx * ma - vb;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+1.000000000000e+000 +0.000000000000e+000 +0.000000000000e+000
+7.105427357601e-015
+0.000000000000e+000 +0.000000000000e+000 +0.000000000000e+000
```

2.3.37 operator / (const srmatrix&)

Operator

```
rvector operator / (const srmatrix& mA) const throw (cvmexception);
```

returns solution x of linear equation $A^T * x = b$ (which is equivalent to $x * A = b$) where parameter mA is square matrix A and calling vector is b. This operator throws cvmexception in case of inappropriate sizes of the objects or when matrix A is close to singular. See also rvector::solve_tran, rvector, srmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (12);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -2., 1.};
    double b[] = {1., 2., 3.};
    srmatrix ma(m, 3);
    rvector  vb(b, 3);
    rvector  vx(3);

    vx = vb / ma;

    std::cout << vx * ma - vb;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+0.000000000000e+000 +0.000000000000e+000 +0.000000000000e+000
```

2.3.38 operator % (const srmatrix&)

Operator

```
rvector operator % (const srmatrix& mA) const throw (cvmexception);
```

returns solution x of linear equation $A * x = b$ where parameter mA is square matrix A and calling vector is b. This operator throws cvmexception in case of inappropriate sizes of the objects or when matrix A is close to singular. See also rvector::solve, rvector, srmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (12);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -2., 1.};
    double b[] = {1., 2., 3.};
    srmatrix ma(m, 3);
    rvector  vb(b, 3);
    rvector  vx(3);

    vx = vb % ma;

    std::cout << ma * vx - vb;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+0.000000000000e+000 +0.000000000000e+000 +0.000000000000e+000
```

2.3.39 solve_lu

Functions

```
rvector&
rvector::solve_lu (const srmatrix& mA, const srmatrix& mLU,
                   const int* pPivots, const rvector& vB, TR& dErr)
                   throw (cvmexception);

rvector&
rvector::solve_lu (const srmatrix& mA, const srmatrix& mLU,
                   const int* pPivots, const rvector& vB)
                   throw (cvmexception);
```

set calling vector to be equal to solution x of linear equation $A * x = b$ where parameter `mA` is square matrix A, parameter `mLU` is LU factorization of a matrix A, parameter `pPivots` is an array of pivot numbers created while factorizing matrix A and parameter `vB` is vector b. Every function returns a reference to the vector changed. The first version also sets output parameter `dErr` to be equal to the norm of computation error. These functions are useful when you need to solve few linear equations of kind $Ax = b$ with the same matrix A and different vectors b. In such case you save on matrix A factorization since it's needed to be performed just one time. These functions throw `cvmexception` in case of inappropriate sizes of the objects or when matrix A is close to singular. See also `rvector`, `srmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (12);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -2., 1.};
    double b1[] = {1., 2., 3.};
    double b2[] = {0., -1., -2.};
    srmatrix ma(m, 3);
    srmatrix mLU(3);
    rvector  vb1(b1, 3);
    rvector  vb2(b2, 3);
    rvector  vx1(3);
    rvector  vx2(3);
    iarray   nPivots(3);
    double   dErr = 0.;

    mLU.low_up(ma, nPivots);
    std::cout << vx1.solve_lu (ma, mLU, nPivots, vb1, dErr);
    std::cout << dErr << std::endl;
    std::cout << vx2.solve_lu (ma, mLU, nPivots, vb2);
```

```
    std::cout << ma * vx1 - vb1 << ma * vx2 - vb2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+8.000000000000e+000 -8.000000000000e+000 +3.000000000000e+000
+6.661338147751e-015
-5.000000000000e+000 +4.000000000000e+000 -1.000000000000e+000
+0.000000000000e+000 +0.000000000000e+000 +0.000000000000e+000
+0.000000000000e+000 +0.000000000000e+000 +0.000000000000e+000
```

2.3.40 gels

Function

```
rvector& rvector::gels (bool transpose,
                        const rmatrix& mA, const rvector& mB,
                        TR& dErr) throw (cvmexception);
```

solves overdetermined or underdetermined linear system

$$A * x = b$$

for $m \times n$ matrix A (or transposed one) where b is a vector of length k and $k = m$ in non-transposed case and $k = n$ otherwise. The algorithm uses QR or LQ factorization of A. It is assumed that A has full rank, infinity returned otherwise. Internally function uses ?GELS LAPACK routines. If $m > n$ and transpose=false or $m < n$ and transpose=true, then the system is overdetermined, thus the algorithm tries to find the least squares solution x of the problem

$$\|A * x - b\|_2 \to \min \quad \text{or} \quad \|A^H * x - b\|_2 \to \min$$

respectively. Real number dErr returns residual sum of squares. The system is underdetermined otherwise, and the algorithm finds its minimum norm solution. In this case bErr is set to zero. In both cases the solution computed satisfies $x = \mathrm{pinv}(A) * b$, but this algorithm is faster than pseudo inversion. Function sets calling object to be the solution and returns a reference to it. It throws cvmexception in case of inappropriate sizes of the operands. See also rvector, rmatrix::gels. Example:

```
using namespace cvm;
rmatrix a(7, 5);
rvector b(7), bt(5);
treal dErr;
a.randomize(-10., 10.);
b.randomize(-10., 10.);
bt.randomize(-10., 10.);

rvector x(5);
x.gels(false, a, b, dErr);
cvm::rvector xt(7);
xt.gels(true, a, bt, dErr);
std::cout << (a.pinv()*b - x).norm() << std::endl;
std::cout << (~a.pinv()*bt - xt).norm() << std::endl;
```

prints

```
+5.1347815e-016
+1.1752774e-015
```

2.3.41 gelsy

Function

```
rvector& rvector::gelsy (const rmatrix& mA, const rvector& mB, int& rank,
                         TR tol = cvmMachSp()) throw (cvmexception);
```

computes minimum-norm solution to linear least squares problem

$$\|A * x - b\|_2 \to \min$$

using complete orthogonal factorization of $m \times n$ matrix A. Here b is a vector of length m. Matrix A may be rank-deficient, function returns its effective rank in `rank` output parameter using `tol` tolerance. Internally function uses `?GELSY` LAPACK routines, see more details about the algorithm in those routines' documentation. Matrix A is passed as argument `mA`. Function sets calling object to be the solution and returns a reference to it. It throws `cvmexception` in case of inappropriate sizes of the operands. See also `rvector`, `rvector::gelss`, `rvector::gelsd`, `rmatrix::gelsy`. Example:

```
using namespace cvm;

rmatrix a(4, 5);
rvector b(4);
a.randomize(-10., 10.);
b.randomize(-10., 10.);
rvector x(5);
tint rank;

x.gelsy(a, b, rank);
std::cout << (a*x - b).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+2.1326688e-015
+4 +4
```

2.3.42 gelss

Function

```
rvector& rvector::gelss (const rmatrix& mA, const rvector& mB, int& rank,
                         rvector& sv, TR tol = cvmMachSp())
                         throw (cvmexception);
```

computes minimum-norm solution to linear least squares problem

$$\|A * x - b\|_2 \to \min$$

using singular value decomposition of $m \times n$ matrix A. Here b is a vector of length m. Matrix A may be rank-deficient, function returns its effective rank in `rank` output parameter using `tol` tolerance. This function also computes singular values of A in decreasing order and returns them in sv output parameter having $\min(m, n)$ size. Internally function uses `?GELSS` LAPACK routines, see more details about the algorithm in that routine's documentation. Matrix A is passed as argument `mA`. Function sets calling object to be the solution and returns a reference to it. It throws `cvmexception` in case of inappropriate sizes of the operands. See also `rvector`, `rvector::gelsy`, `rvector::gelsd`, `rmatrix::gelss`. Example:

```
using namespace cvm;

rmatrix a(4, 5);
rvector b(4);
a.randomize(-10., 10.);
b.randomize(-10., 10.);
rvector sv(4);
rvector x(5);
tint rank;

x.gelss(a, b, sv, rank);
std::cout << (a*x - b).norm() << std::endl;
std::cout << (sv - a.svd()).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+7.7937340e-015
+1.2338969e-014
+4 +4
```

2.3.43 gelsd

Function

```
rvector& rvector::gelsd (const rmatrix& mA, const rvector& mB, int& rank,
                         rvector& sv, TR tol = cvmMachSp())
                         throw (cvmexception);
```

computes minimum-norm solution to linear least squares problem

$$\|A * x - b\|_2 \to \min$$

using singular value decomposition of $m \times n$ matrix A and divide and conquer method. Here b is a vector of length m. Matrix A may be rank-deficient, function returns its effective rank in `rank` output parameter using `tol` tolerance. This function also computes singular values of A in decreasing order and returns them in `sv` output parameter having $\min(m, n)$ size. Internally function uses `?GELSD` LAPACK routines, see more details about the algorithm in that routine's documentation. Matrix A is passed as argument `mA`. Function sets calling object to be the solution and returns a reference to it. It throws `cvmexception` in case of inappropriate sizes of the operands. See also `rvector`, `rvector::gelsy`, `rvector::gelss`, `rmatrix::gelsd`. Example:

```
using namespace cvm;

rmatrix a(4, 5);
rvector b(4);
a.randomize(-10., 10.);
b.randomize(-10., 10.);
rvector sv(4);
rvector x(5);
tint rank;

x.gelsd(a, b, sv, rank);
std::cout << (a*x - b).norm() << std::endl;
std::cout << (sv - a.svd()).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+7.8441794e-015
+5.1021966e-015
+4 +4
```

2.3.44 svd

Functions

```
rvector&
rvector::svd (const rmatrix& mArg) throw (cvmexception);
rvector&
rvector::svd (const cmatrix& mArg) throw (cvmexception);

rvector&
rvector::svd (const rmatrix& mArg,
              srmatrix& mU, srmatrix& mVH) throw (cvmexception);
rvector&
rvector::svd (const cmatrix& mArg,
              scmatrix& mU, scmatrix& mVH) throw (cvmexception);
```

set calling vector to be equal to singular values

$$\sigma_1 \geqslant \sigma_2 \geqslant \cdots \geqslant \sigma_{\min(m,n)} \geqslant 0$$

of $m \times n$ matrix A (parameter `mArg`). These values are the main diagonal of a matrix Σ of the singular value decomposition

$$A = U\Sigma V^H$$

where U and V are orthogonal for real A and unitary for complex A. V^H is transposed V for real one and hermitian conjugated V for complex one. First $\min(m,n)$ columns of matrices U and V are left and right singular vectors of A respectively. Singular values and singular vectors satisfy

$$Av_i = \sigma_i u_i \quad \text{and} \quad A^H u_i = \sigma_i v_i$$

where u_i and v_i are i-th columns of U and V respectively. Third and fourth versions of functions set output parameter `mU` to be equal to a matrix U of size $m \times m$ and `mVH` to be equal to a matrix V^H of size $n \times n$. All functions return reference to the object they change and throw `cvmexception` in case of inappropriate calling object size (it must be equal to $\min(m,n)$), matrix `mU` size (must be $m \times m$), matrix `mVH` size (must be $n \times n$) or in case of convergence error. See also `rvector`, `rmatrix`, `cmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (10);

try {
    double m[] = {1., -1., 1., 2., -2., 1.,
                  3., -2., 1., 0., -2., 1.};
    rmatrix mA(m,4,3);
    rmatrix mSigma(4,3);
    rvector v(3);
    srmatrix mU(4), mVH(3);
```

```
    v.svd(mA, mU, mVH);
    mSigma.diag(0) = v;

    std::cout << mU << std::endl;
    std::cout << mVH << std::endl;
    std::cout << mSigma << std::endl;

    std::cout << (mA * ~mVH - mU * mSigma).norm() << std::endl;
    std::cout << (~mA * mU - ~(mSigma * mVH)).norm() << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-4.8425643615e-01 +1.9516809011e-01 +1.1506232201e-02 -8.5280286542e-01
+2.1685987119e-01 -3.4107922671e-01 -8.8948423927e-01 -2.1320071636e-01
+6.6237057295e-01 +7.1553688692e-01 -6.1787070600e-02 -2.1320071636e-01
-5.2889765022e-01 +5.7756501033e-01 -4.5262319054e-01 +4.2640143271e-01

-2.2124855498e-01 +8.5354150454e-01 -4.7171599183e-01
+9.5937301747e-01 +1.0365951763e-01 -2.6240830353e-01
-1.7507852602e-01 -5.1060905244e-01 -8.4179920723e-01

+4.9561500411e+00 +0.0000000000e+00 +0.0000000000e+00
+0.0000000000e+00 +2.5088408581e+00 +0.0000000000e+00
+0.0000000000e+00 +0.0000000000e+00 +3.7721919242e-01
+0.0000000000e+00 +0.0000000000e+00 +0.0000000000e+00

+1.3710111285e-15
+2.4829995848e-15
```

2.3.45 eig

Functions

```
rvector&
rvector::eig (const srsmatrix& mArg) throw (cvmexception);
rvector&
rvector::eig (const schmatrix& mArg) throw (cvmexception);

rvector&
rvector::eig (const srsmatrix& mArg,
              srmatrix& mEigVect) throw (cvmexception);
rvector&
rvector::eig (const schmatrix& mArg,
              scmatrix& mEigVect) throw (cvmexception);
```

solve symmetric eigenvalue problem and set calling vector to be equal to eigenvalues of symmetric or hermitian matrix `mArg`. The symmetric eigenvalue problem is defined as follows: given symmetric or hermitian matrix A, find the eigenvalues λ and the corresponding eigenvectors z that satisfy the equation

$$Az = \lambda z.$$

All n eigenvalues are real not only for real symmetric but also for complex Hermitian matrices A, and there exists an orthogonal system of n eigenvectors. If A is symmetric or Hermitian positive-definite matrix, all eigenvalues are positive. See [3] for further details. The third and fourth versions of functions set output parameter `mEigVect` to be equal to square matrix containing eigenvectors as columns. All functions return reference to the vector they change and throw `cvmexception` in case of inappropriate calling object sizes or in case of convergence error. See also `rvector`, `cvector::eig`, `srsmatrix`, `schmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (3);

try {
    srsmatrix m(3);
    srmatrix me(3);
    rvector v(3);
    m.randomize(1., 3.);

    v.eig (m, me);
    std::cout << v;

    std::cout << m * me(1) - me(1) * v(1);
    std::cout << m * me(2) - me(2) * v(2);
```

```
    std::cout << m * me(3) - me(3) * v(3);
    std::cout << me(1) * me(2) << std::endl; // orthogonality check

    schmatrix mc(3);
    scmatrix mce(3);
    mc.randomize_real(1., 3.);
    mc.randomize_imag(1., 3.);

    v.eig (mc, mce);
    std::cout << v;

    std::cout << mc * mce(1) - mce(1) * v(1);
    std::cout << mc * mce(2) - mce(2) * v(2);
    std::cout << mc * mce(3) - mce(3) * v(3);
    std::cout << mce(1) % mce(2) << std::endl; // orthogonality check
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-9.360e-01 +3.535e-01 +6.376e+00
-4.441e-16 -5.551e-16 -6.106e-16
+3.331e-16 +1.145e-16 +1.110e-16
-4.441e-16 +0.000e+00 -4.441e-16
+2.060e-17
-3.274e+00 +9.710e-01 +8.209e+00
(-4.441e-16,-1.221e-15) (-1.443e-15,-4.441e-16) (-8.882e-16,+4.683e-16)
(-5.551e-16,-2.776e-16) (+0.000e+00,-4.025e-16) (+6.661e-16,-2.461e-17)
(-5.551e-16,+0.000e+00) (+4.441e-16,-4.441e-16) (+0.000e+00,+3.896e-16)
(+1.608e-16,-2.261e-17)
```

2.3.46 gemv

Function

```
rvector& rvector::gemv (bool bLeft, const rmatrix& m, TR dAlpha,
                        const rvector& v, TR dBeta) throw (cvmexception);
```

calls one of ?GEMV routines of the BLAS library performing matrix-vector operation defined as

$$c = \alpha M \cdot v + \beta c \quad \text{or} \quad c = \alpha v \cdot M + \beta c,$$

where α and β are real numbers (parameters dAlpha and dBeta), M is matrix (parameter m) and v and c are vectors (parameter v and calling vector respectively). First operation is performed if bLeft passed is false and second one otherwise. Function returns a reference to the vector changed and throws cvmexception in case of inappropriate sizes of the operands. See also rvector, rmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (7);

try {
    double alpha = 1.3;
    double beta = -0.7;
    rmatrix m(4,3);
    rvector c(4);
    rvector v(3);
    m.randomize(-1., 2.); v.randomize(-1., 3.); c.randomize(0., 2.);

    std::cout << m * v * alpha + c * beta;
    std::cout << c.gemv(false, m, alpha, v, beta);
    std::cout << c * m * alpha + v * beta;
    std::cout << v.gemv(true, m, alpha, c, beta);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-3.5397829e-02 +3.1990410e-02 +3.2633344e-01 -5.4669713e-01
-3.5397829e-02 +3.1990410e-02 +3.2633344e-01 -5.4669713e-01
-4.7697026e-01 -2.2544922e-01 -5.5204984e-01
-4.7697026e-01 -2.2544922e-01 -5.5204984e-01
```

2.3.47 gbmv

Function

```
rvector& rvector::gbmv (bool bLeft, const srbmatrix& m, TR dAlpha,
                        const rvector& v, TR dBeta) throw (cvmexception);
```

calls one of ?GBMV routines of the BLAS library performing matrix-vector operation defined as

$$c = \alpha M \cdot v + \beta c \quad \text{or} \quad c = \alpha v \cdot M + \beta c,$$

where α and β are real numbers (parameters dAlpha and dBeta), M is band matrix (parameter m) and v and c are vectors (parameter v and calling vector respectively). First operation is performed if bLeft passed is false and second one otherwise. Function returns a reference to the vector changed and throws cvmexception in case of inappropriate sizes of the operands. See also rvector, srbmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (7);

try {
    double alpha = 1.3;
    double beta = -0.7;
    srbmatrix m(3, 1, 0);
    rvector c(3);
    rvector v(3);
    m.randomize(-1., 2.); v.randomize(-1., 3.); c.randomize(0., 2.);

    std::cout << m * v * alpha + c * beta;
    std::cout << c.gbmv(false, m, alpha, v, beta);
    std::cout << c * m * alpha + v * beta;
    std::cout << v.gbmv(true, m, alpha, c, beta);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+1.4551599e+00 -5.1882508e-01 -5.2088503e-02
+1.4551599e+00 -5.1882508e-01 -5.2088503e-02
+7.3471591e-01 -2.6952064e-01 -2.0478054e-01
+7.3471591e-01 -2.6952064e-01 -2.0478054e-01
```

2.3.48 randomize

Function

```
rvector& rvector::randomize (TR dFrom, TR dTo);
```

fills calling vector with pseudo-random numbers distributed between `dFrom` and `dTo`. It returns a reference to the vector changed. See also `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (7);

rvector v(4);
v.randomize(-2.,3.);
std::cout << v;
```

prints

```
-1.1160314e+000 2.5649586e+000 8.9345988e-001 -1.1631825e+000
```

2.4 cvector

This is end-user class encapsulating vector of complex numbers.

```
template <typename TR, typename TC>
class cvector : public Array<TR,TC> {
public:
    cvector ();
    explicit cvector (int nSize);
    cvector (std::initializer_list<TC> list);
    cvector (int nSize, TC c);
    cvector (TC* pD, int nSize, int nIncr = 1);
    cvector (const TC* pD, int nSize, int nIncr = 1);
    cvector (const cvector& v);
    cvector (const TR* pRe, const TR* pIm, int nSize,
             tint nIncrRe = 1, tint nIncrIm = 1);
    cvector (const rvector& vRe, const rvector& vIm);
    cvector (const TR* pA, int nSize,
             bool bRealPart = true, int nIncr = 1);
    explicit cvector (const rvector& v, bool bRealPart = true);
    rvector real ();
    rvector imag ();
    cvector& operator = (const cvector& v) throw (cvmexception);
    cvector& assign (const TC* p, int nIncr = 1);
    cvector& assign (int n, const TC* p, int nIncr = 1);
    cvector& assign (int n, const TC* p, int nSize, tint nIncr);
    cvector& assign (int n, const cvector& v) throw (cvmexception);
    cvector& set (TC x);
    cvector& assign_real (const rvector& vRe) throw (cvmexception);
    cvector& assign_imag (const rvector& vIm) throw (cvmexception);
    cvector& set_real (TR x);
    cvector& set_imag (TR x);
    cvector& resize (int nNewSize) throw (cvmexception);
    bool operator == (const cvector& v) const;
    bool operator != (const cvector& v) const;
    cvector& operator << (const cvector& v) throw (cvmexception);
    cvector operator + (const cvector& v) const throw (cvmexception);
    cvector operator - (const cvector& v) const throw (cvmexception);
    cvector& sum (const cvector& v1,
                  const cvector& v2) const throw (cvmexception);
    cvector& diff (const cvector& v1,
                   const cvector& v2) const throw (cvmexception);
    cvector& operator += (const cvector& v) throw (cvmexception);
    cvector& operator -= (const cvector& v) throw (cvmexception);
    cvector operator - () const throw (cvmexception);
```

```
cvector operator * (TR d) const;
cvector operator / (TR d) const throw (cvmexception);
cvector operator * (TC c) const;
cvector operator / (TC c) const throw (cvmexception);
cvector& operator *= (TR d);
cvector& operator /= (TR d) throw (cvmexception);
cvector& operator *= (TC c);
cvector& operator /= (TC c) throw (cvmexception);
cvector& normalize ();
cvector operator ~() const throw (cvmexception);
cvector& conj (const cvector& v) throw (cvmexception);
cvector& conj ();
TC operator * (const cvector& v) const throw (cvmexception);
TC operator % (const cvector& v) const throw (cvmexception);
cvector operator * (const cvector& v) const throw (cvmexception);
cvector& mult (const cvector& v, const cmatrix& m)
               throw (cvmexception);
cvector& mult (const cmatrix& m, const cvector& v)
               throw (cvmexception);
cmatrix rank1update_u (const cvector& v) const;
cmatrix rank1update_c (const cvector& v) const;
cvector& solve (const scmatrix& mA,
                const cvector& vB, TR& dErr) throw (cvmexception);
cvector& solve (const scmatrix& mA,
                const cvector& vB) throw (cvmexception);
rvector& solve_tran (const scmatrix& mA,
                     const cvector& vB, TR& dErr)
                     throw (cvmexception);
rvector& solve_tran (const scmatrix& mA,
                     const cvector& vB) throw (cvmexception);
rvector& solve_conj (const scmatrix& mA,
                     const cvector& vB, TR& dErr)
                     throw (cvmexception);
rvector& solve_conj (const scmatrix& mA,
                     const cvector& vB) throw (cvmexception);
rvector operator / (const scmatrix& mA) const throw (cvmexception);
rvector operator % (const scmatrix& mA) const throw (cvmexception);
cvector& solve_lu (const scmatrix& mA, const scmatrix& mLU,
                   const int* pPivots, const cvector& vB, TR& dErr)
                   throw (cvmexception);
cvector& solve_lu (const scmatrix& mA, const scmatrix& mLU,
                   const int* pPivots, const cvector& vB)
                   throw (cvmexception);
cvector& gels (bool conjugate, const cmatrix& mA, const cvector& mB,
               TC& cErr) throw (cvmexception);
```

```
    cvector& gelsy (const cmatrix& mA, const cvector& mB,
                    int& rank, TR tol = cvmMachSp())
                    throw (cvmexception);
    rvector& gelss (const cmatrix& mA, const cvector& mB,
                    rvector& sv, int& rank, TR tol = cvmMachSp())
                    throw (cvmexception);
    rvector& gelsd (const cmatrix& mA, const cvector& mB,
                    rvector& sv, int& rank, TR tol = cvmMachSp())
                    throw (cvmexception);
    cvector& eig (const srmatrix& mArg) throw (cvmexception);
    cvector& eig (const scmatrix& mArg) throw (cvmexception);
    cvector& eig (const srmatrix& mArg, scmatrix& mEigVect,
                  bool bRightVect = true) throw (cvmexception);
    cvector& eig (const scmatrix& mArg, scmatrix& mEigVect,
                  bool bRightVect = true) throw (cvmexception);
    cvector& geneig(const srmatrix& mA, const srmatrix& mB,
                     rvector& vBeta) throw (cvmexception);
    cvector& geneig(const srmatrix& mA, const srmatrix& mB,
                     rvector& vBeta, scmatrix& mEigVectLeft,
                     scmatrix& mEigVectRight) throw (cvmexception);
    cvector& geneig(const srmatrix& mA, const srmatrix& mB,
                     rvector& vBeta, scmatrix& mEigVect,
                     bool bRightVect = true) throw (cvmexception);
    cvector& geneig(const scmatrix& mA, const scmatrix& mB,
                     cvector& vBeta) throw (cvmexception);
    cvector& geneig(const scmatrix& mA, const scmatrix& mB,
                     cvector& vBeta, scmatrix& mEigVectLeft,
                     scmatrix& mEigVectRight) throw (cvmexception);
    cvector& geneig(const scmatrix& mA, const scmatrix& mB,
                     cvector& vBeta, scmatrix& mEigVect,
                     bool bRightVect = true) throw (cvmexception);
    cvector& gemv (bool bLeft, const cmatrix& m, TC cAlpha,
                   const cvector& v, TC dBeta) throw (cvmexception);
    cvector& gbmv (bool bLeft, const scbmatrix& m, TC dAlpha,
                   const cvector& v, TC dBeta) throw (cvmexception);
    cvector& randomize_real (TR dFrom, TR dTo);
    cvector& randomize_imag (TR dFrom, TR dTo);
};
```

2.4.1 cvector ()

Constructor

```
cvector::cvector ();
```

creates empty `cvector` object. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v;
std::cout << v.size() << std::endl;

v.resize (3);
v(1) = std::complex<double>(1.5, -1.);
std::cout << v;
```

prints

```
0
(1.50e+00,-1.00e+00) (0.00e+00,0.00e+00) (0.00e+00,0.00e+00)
```

2.4.2 cvector (int)

Constructor

```
explicit cvector::cvector (int nSize);
```

creates `cvector` object of size equal to `nSize`. It throws `cvmexception` in case of non-positive size passed or memory allocation failure. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v(3);
std::cout << v.size() << std::endl;
std::cout << v;
```

prints

```
3
(0.00e+00,0.00e+00) (0.00e+00,0.00e+00) (0.00e+00,0.00e+00)
```

2.4.3 cvector (std::initializer_list<TC>)

Constructor

```
cvector::cvector (std::initializer_list<TC> list);
```

creates cvector object and fills it with values provided in the initializer list. This constructor is available only if your compiler supports initializer lists (GCC 4.4 and higher, MS Visual Studio 2013 and higher, Apple LLVM 5.0 and higher). It throws cvmexception in case of memory allocation failure. See also cvector. Example:

```
using namespace cvm;
std::cout.setf(std::ios::scientific | std::ios::left);
std::cout.precision(3);

cvector v = { tcomplex(1.2, 3.4), tcomplex(3.4, 5.6), 99.99 };
std::cout << v;
```

prints

```
(1.200e+000,3.400e+000) (3.400e+000,5.600e+000) (9.999e+001,0.000e+000)
```

Also, if your compiler supports user-defined literals (GCC 4.7 and higher, Apple LLVM 5.0 and higher) you can write

```
cvector v = { 1.2+3.4_i, 3.4+5.6_i, 99.99 };
```

2.4.4 cvector (int, TC)

Constructor

```
cvector::cvector (int nSize, TC c);
```

creates `cvector` object of size equal to `nSize` and fills it with a value of `c`. It throws `cvmexception` in case of non-positive size passed or memory allocation failure. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v (3, std::complex<double>(1.5, -1.));
std::cout << v;
```

prints

```
(1.50e+00,-1.00e+00) (1.50e+00,-1.00e+00) (1.50e+00,-1.00e+00)
```

2.4.5 cvector (TC*,int,int)

Constructor

```
cvector::cvector (TC* pD, int nSize, int nIncr = 1);
```

creates `cvector` object of size equal to `nSize`. Unlike others, this constructor *does not allocate memory*. It just shares memory with an array pointed to by `pD` with distance between elements equal to `nIncr`. It throws `cvmexception` in case of non-positive size passed. See also `cvector`, `cvector (const TC*,int,int)`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
cvector v1 ((std::complex<double>*) a, 2, 2);

std::cout << v1;
v1(2) = std::complex<double> (9.99, 9.99);
std::cout << v1 << std::endl;

for (int i = 0; i < 6; i++) {
    std::cout << a[i] << " ";
}
std::cout << std::endl;

cvector v2 ((std::complex<double>*) a, 3);
std::cout << v2;
```

prints

```
(1.00e+000,2.00e+000) (5.00e+000,6.00e+000)
(1.00e+000,2.00e+000) (9.99e+000,9.99e+000)

1.00e+000 2.00e+000 3.00e+000 4.00e+000 9.99e+000 9.99e+000
(1.00e+000,2.00e+000) (3.00e+000,4.00e+000) (9.99e+000,9.99e+000)
```

2.4.6 cvector (const TC*,int,int)

Constructor

```
cvector::cvector (const TC* pD, int nSize, int nIncr = 1);
```

creates `cvector` object of size equal to `nSize` and copies `nSize` elements of array `pD` to it using increment `nIncr` (1 by default). The object created has an increment set to 1. It throws `cvmexception` in case of non-positive size passed. See also `cvector`, `cvector (TC*,int,int)`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

const double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
cvector v1 ((const std::complex<double>*) a, 2, 2);

std::cout << v1;
v1(2) = std::complex<double> (9.99, 9.99);
std::cout << v1 << std::endl;

for (int i = 0; i < 6; i++) {
    std::cout << a[i] << " ";
}
std::cout << std::endl;

cvector v2 ((std::complex<double>*) a, 3);
std::cout << v2;
```

prints

```
(1.00e+000,2.00e+000) (5.00e+000,6.00e+000)
(1.00e+000,2.00e+000) (9.99e+000,9.99e+000)

1.00e+000 2.00e+000 3.00e+000 4.00e+000 5.00e+000 6.00e+000
(1.00e+000,2.00e+000) (3.00e+000,4.00e+000) (5.00e+000,6.00e+000)
```

2.4.7 cvector (const cvector&)

Copy constructor

```
cvector::cvector (const cvector& v);
```

creates `cvector` object of size equal to size of vector `v` and sets every element of created vector to be equal to appropriate element of `v`. It throws `cvmexception` in case of memory allocation failure. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10.};
const cvector v ((std::complex<double>*) a, 3, 2);
cvector vc (v);

vc(1) = std::complex<double>(7.77,8.88);
std::cout << vc;
std::cout << v;
```

prints

```
(7.77e+00,8.88e+00) (5.00e+00,6.00e+00) (9.00e+00,1.00e+01)
(1.00e+00,2.00e+00) (5.00e+00,6.00e+00) (9.00e+00,1.00e+01)
```

2.4.8 cvector (const TR*,const TR*,int,int,int)

Constructor

```
cvector::cvector (const TR* pRe, const TR* pIm, int nSize,
                  tint nIncrRe = 1, tint nIncrIm = 1);
```

creates `cvector` object of size equal to `nSize` and copies every `nIncrRe`-th element of an array pointed to by `pRe` and every `nIncrIm`-th element of an array pointed to by `pIm` to real and imaginary part of the object created. It throws `cvmexception` in case of non-positive size passed or memory allocation failure. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double re[] = {1., 2., 3., 4., 5.};
double im[] = {5., 4., 3., 2., 1.};
cvector v (re, im, 3, 2);

std::cout << v;
re[0] = 7.77;
std::cout << v;

const double rec[] = {1., 2., 3.};
const double imc[] = {5., 4., 3.};
const cvector vc (rec, imc, 3);
std::cout << vc;
```

prints

```
(1.00e+00,5.00e+00) (3.00e+00,4.00e+00) (5.00e+00,3.00e+00)
(1.00e+00,5.00e+00) (3.00e+00,4.00e+00) (5.00e+00,3.00e+00)
(1.00e+00,5.00e+00) (2.00e+00,4.00e+00) (3.00e+00,3.00e+00)
```

2.4.9 cvector (const rvector&, const rvector&)

Constructor

```
cvector::cvector (const rvector& vRe, const rvector& vIm);
```

creates cvector object of size equal to vRe.size() and vIm.size() and copies vectors vRe and vIm to real and imaginary part of the object created. It throws cvmexception in case of non-equal sizes of the parameters passed or memory allocation failure. See also cvector and rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector vr(3), vi(3);
vr[1] = 1.;
vr[2] = 2.;
vr[3] = 3.;
vi[1] = 5.;
vi[2] = 4.;
vi[3] = 3.;

const cvector vc(vr, vi);
std::cout << vc;
```

prints

```
(1.00e+00,5.00e+00) (2.00e+00,4.00e+00) (3.00e+00,3.00e+00)
```

2.4.10 cvector (const TR*,int,bool,int)

Constructor

```
cvector::cvector (const TR* pA, int nSize,
                  bool bRealPart = true, int nIncr = 1);
```

creates `cvector` object of size equal to `nSize` and copies every `nIncr`-th element of an array pointed to by `pA` to real (if `bRealPart` is `true`) or imaginary (if `bRealPart` is `false`) part of the object created. Constructor throws `cvmexception` in case of non-positive size passed or memory allocation failure. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

const double a[] = {1., 2., 3., 4., 5.};
cvector v1 (a, 3, false, 2);
cvector v2 (a, 2);

std::cout << v1 << v2;
```

prints

```
(0.00e+00,1.00e+00) (0.00e+00,3.00e+00) (0.00e+00,5.00e+00)
(1.00e+00,0.00e+00) (2.00e+00,0.00e+00)
```

2.4.11 cvector (const rvector&,bool)

Constructor

```
explicit cvector::cvector (const rvector& v, bool bRealPart = true);
```

creates `cvector` object of size equal to `v.size()` and copies every element of a vector `v` to real (if `bRealPart` is `true`) or imaginary (if `bRealPart` is `false`) part of the object created. Constructor throws `cvmexception` in case of memory allocation failure. See also `cvector` and `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector vr (3);
vr(1) = 1.;
vr(2) = 2.;
vr(3) = 3.;

cvector v1 (vr);
cvector v2 (vr, false);
std::cout << v1 << v2;
```

prints

```
(1.00e+00,0.00e+00) (2.00e+00,0.00e+00) (3.00e+00,0.00e+00)
(0.00e+00,1.00e+00) (0.00e+00,2.00e+00) (0.00e+00,3.00e+00)
```

2.4.12 real

Function

```
rvector cvector::real ();
```

creates `rvector` object of size equal to size of a calling vector sharing memory with its real part. In other words, the vector returned is *l-value*. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector vc(3);
vc.set(std::complex<double>(1.,1.));
std::cout << vc << vc.real();
vc.real()(1) = 7.77;
std::cout << vc;
```

prints

```
(1.00e+00,1.00e+00) (1.00e+00,1.00e+00) (1.00e+00,1.00e+00)
1.00e+00 1.00e+00 1.00e+00
(7.77e+00,1.00e+00) (1.00e+00,1.00e+00) (1.00e+00,1.00e+00)
```

2.4.13 imag

function

```
rvector cvector::imag ();
```

creates `rvector` object of size equal to size of a calling vector sharing memory with its imaginary part. In other words, the vector returned is *l-value*. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector vc(3);
vc.set(std::complex<double>(1.,1.));
std::cout << vc << vc.imag();
vc.imag()(1) = 7.77;
std::cout << vc;
```

prints

```
(1.00e+00,1.00e+00) (1.00e+00,1.00e+00) (1.00e+00,1.00e+00)
1.00e+00 1.00e+00 1.00e+00
(1.00e+00,7.77e+00) (1.00e+00,1.00e+00) (1.00e+00,1.00e+00)
```

2.4.14 operator = (const cvector&)

Operator

```
cvector& cvector::operator = (const cvector& v) throw (cvmexception);
```

sets every element of a calling vector to be equal to appropriate element of a vector v and returns a reference to the vector changed. It throws cvmexception in case of different vector sizes. See also cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector v(3);
    cvector vc(3);
    v(1) = std::complex<double>(1.,2.);

    vc = v;
    std::cout << vc;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1.00e+00,2.00e+00) (0.00e+00,0.00e+00) (0.00e+00,0.00e+00)
```

2.4.15 assign(const TC*, int)

Function

```
cvector& cvector::assign (const TC* p, int nIncr = 1);
```

sets every element of a calling vector to be equal to every `nIncr`-th element of an array pointed to by parameter `p` and returns a reference to the vector changed. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

const double a[] = {1., 2., 3., 4., 5., 6., 7.};
cvector v1(3);
cvector v2(2);

v1.assign((const std::complex<double>*) a);
v2.assign((const std::complex<double>*) a, 2);
std::cout << v1;
std::cout << v2;
```

prints

```
(1.00e+00,2.00e+00) (3.00e+00,4.00e+00) (5.00e+00,6.00e+00)
(1.00e+00,2.00e+00) (5.00e+00,6.00e+00)
```

2.4.16 assign(int, const TC*, int)

Function

```
cvector& cvector::assign (int n, const TC* p, int nIncr = 1);
```

sets every element of a calling vector, starting from ℓ-based n-th one, to be equal to every nIncr-th element of an array pointed to by parameter p and returns a reference to the vector changed. See also cvector. Example:

```
using namespace cvm;

const double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10.};
cvector v1(4);
cvector v2(4);

v1.assign(3, (const std::complex<double>*) a);
v2.assign(2, (const std::complex<double>*) a, 2);
std::cout << v1;
std::cout << v2;
```

prints

```
(0,0) (0,0) (1,2) (3,4)
(0,0) (1,2) (5,6) (9,10)
```

2.4.17 assign(int, const TR*, int, int)

Function

```
cvector& cvector::assign (int n, const TC* p, int nSize, tint nIncr);
```

sets every element of a calling vector, starting from ℓ-based n-th one, up to nSize total, to be equal to every nIncr-th element of an array pointed to by parameter p and returns a reference to the vector changed. If n + nSize goes beyond vector boundaries assignment stops at the last element. See also cvector. Example:

```
using namespace cvm;
const double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10.};
cvector v1(4);

v1.assign(2, (const std::complex<double>*) a, 2, 3);
std::cout << v1;
```

prints

```
(0,0) (1,2) (7,8) (0,0)
```

2.4.18 assign (int, const cvector&)

Function

```
cvector& cvector::assign (int n, const cvector& v) throw (cvmexception);
```

sets every element of a calling vector's sub-vector beginning with 1-based index `n` to a vector `v` and returns reference to the vector changed. Function throws `cvmexception` if `n` is not positive or `v.size()+n-1` is greater than calling vector's size. See also `cvector`. Example:

```
using namespace cvm;

cvector v1(5);
cvector v2(2);
v1.set(std::complex<double>(1.,1.));
v2.set(std::complex<double>(2.,2.));
v1.assign(3, v2);
std::cout << v1;
```

prints

```
(1,1) (1,1) (2,2) (2,2) (1,1)
```

2.4.19 set(TC)

Function

```
cvector& cvector::set (TC x);
```

sets every element of a calling vector to be equal to parameter x and returns a reference to the vector changed. See also cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v(3);
v.set(std::complex<double>(3.,1.));
std::cout << v;
```

prints

```
(3.00e+00,1.00e+00) (3.00e+00,1.00e+00) (3.00e+00,1.00e+00)
```

2.4.20 assign_real

Function

```
cvector& cvector::assign_real (const rvector& vRe) throw (cvmexception);
```

sets real part of every element of a calling vector to be equal to appropriate element of a vector vRe and returns a reference to the vector changed. Function throws cvmexception in case of different sizes of the operands. See also cvector and rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v(3);
cvector vc(3);
v(1) = 1.; v(2) = 2.; v(3) = 3.;

vc.assign_real(v);
std::cout << vc;
```

prints

```
(1.00e+00,0.00e+00) (2.00e+00,0.00e+00) (3.00e+00,0.00e+00)
```

2.4.21 assign_imag

Function

```
cvector& cvector::assign_imag (const rvector& vIm) throw (cvmexception);
```

sets imaginary part of every element of a calling vector to be equal to appropriate element of a vector vIm and returns a reference to the vector changed. Function throws cvmexception in case of different sizes of the operands. See also cvector and rvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

rvector v(3);
cvector vc(3);
v(1) = 1.; v(2) = 2.; v(3) = 3.;

vc.assign_imag(v);
std::cout << vc;
```

prints

```
(0.00e+00,1.00e+00) (0.00e+00,2.00e+00) (0.00e+00,3.00e+00)
```

2.4.22 set_real

Function

```
cvector& cvector::set_real (TR x);
```

sets real part of every element of a calling vector to be equal to parameter x and returns a reference to the vector changed. See also cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v(3);
v.set_real(1.);
std::cout << v;
```

prints

```
(1.00e+00,0.00e+00) (1.00e+00,0.00e+00) (1.00e+00,0.00e+00)
```

2.4.23 set_imag

Function

```
cvector& cvector::set_imag (TR x);
```

sets imaginary part of every element of a calling vector to be equal to parameter `x` and returns a reference to the vector changed. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v(3);
v.set_imag(1.);
std::cout << v;
```

prints

```
(0.00e+00,1.00e+00) (0.00e+00,1.00e+00) (0.00e+00,1.00e+00)
```

2.4.24 resize

Function

```
cvector& cvector::resize (int nNewSize) throw (cvmexception);
```

changes size of a calling vector to be equal to `nNewSize` and returns a reference to the vector changed. In case of increasing of its size, the vector is filled up with zeroes. Function throws `cvmexception` in case of negative size passed or memory allocation failure. See also `basic_array::resize` and `cvector`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4.};
    rvector v (a, 3);
    std::cout << v;
    v.resize(2);
    std::cout << v;
    v.resize(4);
    std::cout << v;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (3,4) (5,6)
(1,2) (3,4)
(1,2) (3,4) (0,0) (0,0)
```

2.4.25 operator ==

Operator

```
bool cvector::operator == (const cvector& v) const;
```

compares calling vector with a vector `v` and returns `true` if they have the same sizes and their appropriate elements differ by not more than the smallest normalized positive number. Returns `false` otherwise. See also `cvector`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4.};
cvector v1 ((std::complex<double>*)a, 2);
cvector v2 (2);

v2(1) = std::complex<double>(1.,2.);
v2(2) = std::complex<double>(3.,4.);

std::cout << (v1 == v2) << std::endl;
```

prints

```
1
```

2.4.26 operator !=

Operator

```
bool cvector::operator != (const cvector& v) const;
```

compares calling vector with a vector `v` and returns `true` if they have different sizes or some of their appropriate elements differ by more than the smallest normalized positive number. Returns `false` otherwise. See also `cvector`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4.};
cvector v1 ((std::complex<double>*)a, 2);
cvector v2 (2);

std::cout << (v1 != v2) << std::endl;
```

prints

```
1
```

2.4.27 operator <<

Operator

```
cvector& cvector::operator << (const cvector& v) throw (cvmexception);
```

destroys calling vector, creates a new one as a copy of `v` and returns a reference to the vector changed. See also `cvector`. Operator throws `cvmexception` in case of memory allocation failure. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector v(2);
    cvector vc(3);
    v(1) = std::complex<double> (1.,2.);
    v(2) = std::complex<double> (3.,4.);
    std::cout << v << vc << std::endl;

    vc << v;
    std::cout << vc;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1.00e+000,2.00e+000) (3.00e+000,4.00e+000)
(0.00e+000,0.00e+000) (0.00e+000,0.00e+000) (0.00e+000,0.00e+000)

(1.00e+000,2.00e+000) (3.00e+000,4.00e+000)
```

2.4.28 operator +

Operator

```
cvector cvector::operator + (const cvector& v) const
throw (cvmexception);
```

creates an object of type `cvector` as a sum of a calling vector and a vector `v`. Operator throws `cvmexception` in case of different sizes of the operands or memory allocation failure. See also `cvector`, `cvector::sum`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector va(3);
    cvector vb(3);
    va.set(std::complex<double>(1.,1.));
    vb.set(std::complex<double>(2.,2.));

    std::cout << va + vb;
    std::cout << va + va;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(3.00e+000,3.00e+000) (3.00e+000,3.00e+000) (3.00e+000,3.00e+000)
(2.00e+000,2.00e+000) (2.00e+000,2.00e+000) (2.00e+000,2.00e+000)
```

2.4.29 operator -

Operator

```
cvector cvector::operator - (const cvector& v) const
throw (cvmexception);
```

creates an object of type `cvector` as a difference of a calling vector and a vector `v`. It throws `cvmexception` in case of different sizes of the operands or memory allocation failure. See also `cvector::diff`, `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector va(3);
    cvector vb(3);
    va.set(std::complex<double> (1.,1.));
    vb.set(std::complex<double> (2.,2.));

    std::cout << va - vb;
    std::cout << va - va;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-1.00e+000,-1.00e+000) (-1.00e+000,-1.00e+000) (-1.00e+000,-1.00e+000)
(0.00e+000,0.00e+000) (0.00e+000,0.00e+000) (0.00e+000,0.00e+000)
```

2.4.30 sum

Function

```
cvector& cvector::sum (const cvector& v1, const cvector& v2)
throw (cvmexception);
```

assigns the result of addition of vectors v1 and v2 to a calling vector and returns a reference to the vector changed. It throws cvmexception in case of different sizes of the operands. See also cvector::operator + , cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector va(3);
    cvector vb(3);
    cvector v(3);
    va.set(std::complex<double> (1.,1.));
    vb.set(std::complex<double> (2.,2.));

    std::cout << v.sum(va, vb);
    std::cout << v.sum(v, va);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(3.00e+000,3.00e+000) (3.00e+000,3.00e+000) (3.00e+000,3.00e+000)
(4.00e+000,4.00e+000) (4.00e+000,4.00e+000) (4.00e+000,4.00e+000)
```

2.4.31 diff

Function

```
cvector& cvector::diff (const cvector& v1, const cvector& v2)
throw (cvmexception);
```

assigns the result of subtraction of vectors `v1` and `v2` to a calling vector and returns a reference to the vector changed. It throws `cvmexception` in case of different sizes of the operands. See also `cvector::operator -` , `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector va(3);
    cvector vb(3);
    cvector v(3);
    va.set(std::complex<double> (1.,1.));
    vb.set(std::complex<double> (2.,2.));

    std::cout << v.diff(va, vb);
    std::cout << v.diff(v, va);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-1.00e+000,-1.00e+000) (-1.00e+000,-1.00e+000) (-1.00e+000,-1.00e+000)
(-2.00e+000,-2.00e+000) (-2.00e+000,-2.00e+000) (-2.00e+000,-2.00e+000)
```

2.4.32 operator +=

Operator

```
cvector& cvector::operator += (const cvector& v) throw (cvmexception);
```

adds vector v to a calling vector and returns a reference to the vector changed. It throws cvmexception in case of different sizes of the operands. See also cvector::operator + , cvector::sum, cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector v1(3);
    cvector v2(3);
    v1.set(std::complex<double> (1.,1.));
    v2.set(std::complex<double> (2.,2.));

    v1 += v2;
    std::cout << v1;

    // well, you can do this too, but temporary object would be created
    v2 += v2;
    std::cout << v2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(3.00e+000,3.00e+000) (3.00e+000,3.00e+000) (3.00e+000,3.00e+000)
(4.00e+000,4.00e+000) (4.00e+000,4.00e+000) (4.00e+000,4.00e+000)
```

2.4.33 operator -=

Operator

```
cvector& cvector::operator -= (const cvector& v) throw (cvmexception);
```

subtracts vector v from calling vector and returns a reference to the vector changed. It throws cvmexception in case of different sizes of the operands. See also cvector::diff, cvector::operator - , cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    cvector v1(3);
    cvector v2(3);
    v1.set(std::complex<double> (1.,1.));
    v2.set(std::complex<double> (2.,2.));

    v1 -= v2;
    std::cout << v1;

    // well, you can do this too, but temporary object would be created
    v2 -= v2;
    std::cout << v2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-1.00e+000,-1.00e+000) (-1.00e+000,-1.00e+000) (-1.00e+000,-1.00e+000)
(0.00e+000,0.00e+000) (0.00e+000,0.00e+000) (0.00e+000,0.00e+000)
```

2.4.34 operator - ()

Operator

```
cvector cvector::operator - () const throw (cvmexception);
```

creates an object of type `cvector` as a calling vector multiplied by -1. It can throw `cvmexception` in case of memory allocation failure. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
const cvector v ((std::complex<double>*) a, 2);

std::cout << - v;
```

prints

```
(-1.00e+000,-2.00e+000) (-3.00e+000,-4.00e+000)
```

2.4.35 operator * (TR)

Operator

```
cvector cvector::operator * (TR d) const;
```

creates an object of type cvector as a product of a calling vector and number d. See also cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
const cvector v ((std::complex<double>*) a, 2);

std::cout << v * 5.;
```

prints

```
(5.00e+000,1.00e+001) (1.50e+001,2.00e+001)
```

2.4.36 operator / (TR)

Operator

```
cvector cvector::operator / (TR d) const throw (cvmexception);
```

creates an object of type `cvector` as a quotient of a calling vector and number `d`. Operator throws `cvmexception` if `d` has absolute value equal or less than the smallest normalized positive number. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
const cvector v ((std::complex<double>*) a, 2);

std::cout << v / 4.;
```

prints

```
(2.50e-001,5.00e-001) (7.50e-001,1.00e+000)
```

2.4.37 operator * (TC)

Operator

```
cvector cvector::operator * (TC c) const;
```

creates an object of type `cvector` as a product of a calling vector and complex number `c`. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
const cvector v ((std::complex<double>*) a, 2);

std::cout << v * std::complex<double>(1.,1.);
```

prints

```
(-1.00e+000,3.00e+000) (-1.00e+000,7.00e+000)
```

2.4.38 operator / (TC)

Operator

```
cvector cvector::operator / (TC c) const
throw (cvmexception);
```

creates an object of type `cvector` as a quotient of a calling vector and complex number `c`. Operator throws `cvmexception` if `c` has absolute value equal or less than the smallest normalized positive number. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
const cvector v ((std::complex<double>*) a, 2);

std::cout << v / std::complex<double>(1.,1.);
```

prints

```
(1.50e+000,5.00e-001) (3.50e+000,5.00e-001)
```

2.4.39 operator *= (TR)

Operator

```
cvector& cvector::operator *= (TR d);
```

multiplies calling vector by real number d and returns a reference to the vector changed. See also cvector::operator * , cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
cvector v ((std::complex<double>*) a, 2);

std::cout << (v *= 2.);
```

prints

```
(2.00e+000,4.00e+000) (6.00e+000,8.00e+000)
```

2.4.40 operator /= (TR)

Operator

```
cvector& cvector::operator /= (TR d) throw (cvmexception);
```

divides calling vector by real number d and returns a reference to the vector changed. It throws cvmexception if d has absolute value equal or less than the smallest normalized positive number. See also cvector::operator / , cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
cvector v ((std::complex<double>*) a, 2);

std::cout << (v /= 2.);
```

prints

```
(5.00e-001,1.00e+000) (1.50e+000,2.00e+000)
```

2.4.41 operator *= (TC)

Operator

```
cvector& cvector::operator *= (TC c);
```

multiplies calling vector by complex number c and returns a reference to the vector changed. See also cvector::operator * , cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
cvector v ((std::complex<double>*) a, 2);

v *= std::complex<double>(1.,1.);
std::cout << v;
```

prints

```
(-1.00e+000,3.00e+000) (-1.00e+000,7.00e+000)
```

2.4.42 operator /= (TC)

Operator

```
cvector& cvector::operator /= (TC c) throw (cvmexception);
```

divides calling vector by complex number c and returns a reference to the vector changed. It throws cvmexception if c has absolute value equal or less than the smallest normalized positive number. See also cvector::operator / , cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
cvector v ((std::complex<double>*) a, 2);

v /= std::complex<double>(1.,1.);
std::cout << v;
```

prints

```
(1.50e+000,5.00e-001) (3.50e+000,5.00e-001)
```

2.4.43 normalize

Function

```
cvector& cvector::normalize ();
```

normalizes calling vector so its Euclidean norm becomes equal to 1 if it was greater than the smallest normalized positive number before the call (otherwise it does nothing). See also cvector. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
cvector v ((std::complex<double>*) a, 2);

std::cout << v.normalize();
std::cout << v.norm() << std::endl;
```

prints

```
(1.83e-01,3.65e-01) (5.48e-01,7.30e-01)
1.00e+00
```

2.4.44 conjugation

Operator and functions

```
cvector  cvector::operator ~ () const throw (cvmexception);
cvector& cvector::conj (const cvector& v) throw (cvmexception);
cvector& cvector::conj ();
```

implement complex conjugation. First operator creates an object of type `cvector` as complex conjugated calling vector (it throws `cvmexception` in case of memory allocation failure). Second function sets calling vector to be equal to vector v conjugated (it throws `cvmexception` in case of different sizes of the operands), third one makes it to be equal to conjugated itself. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 2., 3., 4.};
const cvector v ((std::complex<double>*) a, 2);
cvector vc(2);

std::cout << ~v;
std::cout << vc.conj(v);
std::cout << vc.conj();
```

prints

```
(1.00e+00,-2.00e+00) (3.00e+00,-4.00e+00)
(1.00e+00,-2.00e+00) (3.00e+00,-4.00e+00)
(1.00e+00,2.00e+00) (3.00e+00,4.00e+00)
```

2.4.45 operator * (const cvector&)

Operator

```
TC cvector::operator * (const cvector& v) const throw (cvmexception);
```

returns scalar product of a calling vector and a vector `v`. Operator throws `cvmexception` if the operands have different sizes. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
double b[] = {1., -1., 1., 2.};
const cvector v1((std::complex<double>*) a, 2);
const cvector v2((std::complex<double>*) b, 2);

std::cout << v1 * v2 << std::endl;
```

prints

```
(-2.00e+00,1.10e+01)
```

2.4.46 operator %

Operator

```
TC cvector::operator % (const cvector& v) const throw (cvmexception);
```

returns scalar product of complex conjugated a calling vector and a vector `v`. Operator throws `cvmexception` if the operands have different sizes. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4.};
double b[] = {1., -1., 1., 2.};
const cvector v1((std::complex<double>*) a, 2);
const cvector v2((std::complex<double>*) b, 2);

std::cout << v1 % v2 << std::endl;
std::cout << ~v1 * v2 << std::endl;
```

prints

```
(1.00e+01,-1.00e+00)
(1.00e+01,-1.00e+00)
```

2.4.47 operator * (const cmatrix&)

Operator

```
cvector cvector::operator * (const cmatrix& m) const
throw (cvmexception);
```

creates an object of type `cvector` as a product of a calling vector and a matrix `m`. Operator throws `cvmexception` if the calling vector's size is differ from number of rows of a matrix `m`'. See also `cvector::mult (const cvector&, const cmatrix&)`, `cvector`, `cmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 3., 2., 1.};
double b[] = {1., -1., 1., 2., -2., 1.,
              3., -2., 1., 2., -1., 3.};
const cvector v((std::complex<double>*) a, 3);
const cmatrix m((std::complex<double>*) b, 3, 2);

std::cout << v << m << std::endl << v * m;
```

prints

```
(1.00e+00,2.00e+00) (3.00e+00,3.00e+00) (2.00e+00,1.00e+00)
(1.00e+00,-1.00e+00) (3.00e+00,-2.00e+00)
(1.00e+00,2.00e+00) (1.00e+00,2.00e+00)
(-2.00e+00,1.00e+00) (-1.00e+00,3.00e+00)

(-5.00e+00,1.00e+01) (-1.00e+00,1.80e+01)
```

2.4.48 mult (const cvector&, const cmatrix&)

Function

```
cvector& cvector::mult (const cvector& v, const cmatrix& m)
throw (cvmexception);
```

sets calling vector to be equal to product of vector `v` and a matrix `m` and returns a reference to the object changed. Function throws `cvmexception` if case of inappropriate sizes of the operands. See also `cvector::mult (const cmatrix&, const cvector&)`, `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 1., 2., 3.};
    double b[] = {1., -1., 1., -1., 1., -1.,
                  2., -1., 2., -1., 2., -1.};
    const cvector v ((std::complex<double>*) a, 3);
    const cmatrix m ((std::complex<double>*) b, 3, 2);
    const scmatrix sm ((std::complex<double>*) b, 2);
    cvector vm (2);

    std::cout << vm.mult(v, m) << std::endl;
    std::cout << sm << std::endl;
    std::cout << vm.mult(vm, sm);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1.20e+01,0.00e+00) (1.80e+01,6.00e+00)

(1.00e+00,-1.00e+00) (1.00e+00,-1.00e+00)
(1.00e+00,-1.00e+00) (2.00e+00,-1.00e+00)

(3.60e+01,-2.40e+01) (5.40e+01,-1.80e+01)
```

2.4.49 mult (const cmatrix&, const cvector&)

Function

```
cvector& cvector::mult (const cmatrix& m, const cvector& v)
throw (cvmexception);
```

sets calling vector to be equal to product of matrix `m` and a vector `v` and returns a reference to the object changed. Function throws `cvmexception` if case of inappropriate sizes of the operands. See also `cvector::mult (const cvector&, const cmatrix&)`, `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 1., 2., 3.};
    double b[] = {1., -1., 1., -1., 1., -1.,
                  2., -1., 2., -1., 2., -1.};
    const cvector v ((std::complex<double>*) a, 3);
    const cmatrix m ((std::complex<double>*) b, 2, 3);
    const scmatrix sm ((std::complex<double>*) b, 2);
    cvector vm (2);

    std::cout << vm.mult(m, v) << std::endl;
    std::cout << sm << std::endl;;
    std::cout << vm.mult(vm, sm);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1.40e+01,3.00e+00) (1.70e+01,4.00e+00)

(1.00e+00,-1.00e+00) (1.00e+00,-1.00e+00)
(1.00e+00,-1.00e+00) (2.00e+00,-1.00e+00)

(3.80e+01,-2.40e+01) (5.50e+01,-2.00e+01)
```

2.4.50 rank1update_u

Function

```
cmatrix cvector::rank1update_u (const cvector& v) const;
```

creates an object of type `cmatrix` as rank-1 update (unconjugated) of a calling vector and a vector v. The rank-1 update (unconjugated) of vector-column x of size m and vector-row y of size n is defined as $m \times n$ matrix

$$\begin{pmatrix} x_1 y_1 & x_1 y_2 & \cdots & x_1 y_n \\ x_2 y_1 & x_2 y_2 & \cdots & x_2 y_n \\ \dots & \dots & \dots & \dots \\ x_m y_1 & x_m y_2 & \cdots & x_m y_n \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} \begin{pmatrix} y_1 & y_2 & \cdots & y_n \end{pmatrix}$$

See also `cvector`, `cmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 2., 3., -2., -1., 1.};
double b[] = {4., 5., 3., 2.};
cvector v1((std::complex<double>*) a, 3);
cvector v2((std::complex<double>*) b, 2);

std::cout << v1.rank1update_u (v2);
```

prints

```
(-6.00e+00,1.30e+01) (-1.00e+00,8.00e+00)
(2.20e+01,7.00e+00) (1.30e+01,0.00e+00)
(-9.00e+00,-1.00e+00) (-5.00e+00,1.00e+00)
```

2.4.51 rank1update_c

Function

```
cmatrix cvector::rank1update_c (const cvector& v) const;
```

creates an object of type cmatrix as rank-1 update (conjugated) of a calling vector and complex conjugated vector v. The rank-1 update (conjugated) operation of vector-column x of size m and complex conjugated vector-column y of size n is defined as $m \times n$ matrix

$$\begin{pmatrix} x_1y_1^* & x_1y_2^* & \cdots & x_1y_n^* \\ x_2y_1^* & x_2y_2^* & \cdots & x_2y_n^* \\ \multicolumn{4}{c}{\dotfill} \\ x_my_1^* & x_my_2^* & \cdots & x_my_n^* \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} \operatorname{conj} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} \begin{pmatrix} y_1^* & y_2^* & \ldots & y_n^* \end{pmatrix},$$

where y_i^* is i-th complex conjugated element of y. See also cvector, cmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 2., 3., -2., -1., 1.};
double b[] = {4., 5., 3., 2.};
cvector v1((std::complex<double>*) a, 3);
cvector v2((std::complex<double>*) b, 2);

std::cout << v1.rank1update_c (v2) << std::endl;
std::cout << v1.rank1update_u (~v2);
```

prints

```
(1.40e+01,3.00e+00) (7.00e+00,4.00e+00)
(2.00e+00,-2.30e+01) (5.00e+00,-1.20e+01)
(1.00e+00,9.00e+00) (-1.00e+00,5.00e+00)

(1.40e+01,3.00e+00) (7.00e+00,4.00e+00)
(2.00e+00,-2.30e+01) (5.00e+00,-1.20e+01)
(1.00e+00,9.00e+00) (-1.00e+00,5.00e+00)
```

2.4.52 solve

Functions

```
cvector&
cvector::solve (const scmatrix& mA,
                const cvector& vB, TR& dErr) throw (cvmexception);
cvector&
cvector::solve (const scmatrix& mA,
                const cvector& vB) throw (cvmexception);
```

set calling vector to be equal to solution x of linear equation $A * x = b$ where parameter `mA` is square complex matrix A and parameter `vB` is complex vector b. Every function returns a reference to the vector changed. The first version also sets output parameter `dErr` to be equal to norm of computation error. These functions throw `cvmexception` in case of inappropriate sizes of the objects or when the matrix A is close to singular. See also `operator % (scmatrix)`, `cvector`, `scmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (7);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -3.};
    double b[] = {1., 2., 5., -3.};
    scmatrix ma((std::complex<double>*) m, 2);
    cvector vb((std::complex<double>*) b, 2);
    cvector vx(2);
    double dErr = 0.;

    std::cout << vx.solve (ma, vb, dErr);
    std::cout << dErr << std::endl;
    std::cout << ma * vx - vb;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(3.5200000e+00,6.4000000e-01) (2.2400000e+00,-1.3200000e+00)
3.2788531e-15
(-7.7715612e-16,4.4408921e-16) (0.0000000e+00,0.0000000e+00)
```

2.4.53 solve_tran

Functions

```
cvector& cvector::solve_tran (const scmatrix& mA,
                              const cvector& vB, TR& dErr)
                              throw (cvmexception);
cvector& cvector::solve_tran (const scmatrix& mA,
                              const cvector& vB)
                              throw (cvmexception);
```

set calling vector to be equal to solution x of linear equation $A^T * b = b$ (which is equivalent to $x * A = b$) where parameter mA is square complex matrix A and parameter vB is complex vector b. Every function returns a reference to the vector changed. The first version also sets output parameter dErr to be equal to the norm of computation error. These functions throw cvmexception in case of inappropriate sizes of the objects or when the matrix A is close to singular. See also operator / (scmatrix), cvector::solve, cvector::solve_conj, cvector, scmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (7);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -3.};
    double b[] = {1., 2., 5., -3.};
    scmatrix ma((std::complex<double>*) m, 2);
    cvector vb((std::complex<double>*) b, 2);
    cvector vx(2);
    double dErr = 0.;

    std::cout << vx.solve_tran (ma, vb, dErr);
    std::cout << dErr << std::endl;
    std::cout << vx * ma - vb;
    std::cout << !ma * vx - vb;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(+1.6000000e-001,-8.8000000e-001) (+1.5600000e+000,-8.0000000e-002)
+3.7480513e-015
(+0.0000000e+000,+0.0000000e+000) (+0.0000000e+000,+4.4408921e-016)
(+0.0000000e+000,+0.0000000e+000) (+0.0000000e+000,+4.4408921e-016)
```

2.4.54 solve_conj

Functions

```
cvector& cvector::solve_conj (const scmatrix& mA,
                              const cvector& vB, TR& dErr)
                              throw (cvmexception);
cvector& cvector::solve_conj (const scmatrix& mA,
                              const cvector& vB)
                              throw (cvmexception);
```

set calling vector to be equal to solution x of linear equation $A^H * x = b$ (here A^H is conjugated A) where parameter **mA** is square complex matrix A and parameter **vB** is complex vector b. Every function returns a reference to the vector changed. The first version also sets output parameter dErr to be equal to the norm of computation error. These functions throw cvmexception in case of inappropriate sizes of the objects or when the matrix A is close to singular. See also cvector::solve, cvector::solve_tran, cvector, scmatrix, scmatrix::operator ~ . Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (7);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -3.};
    double b[] = {1., 2., 5., -3.};
    scmatrix ma((std::complex<double>*) m, 2);
    cvector vb((std::complex<double>*) b, 2);
    cvector vx(2);
    double dErr = 0.;

    std::cout << vx.solve_conj (ma, vb, dErr);
    std::cout << dErr << std::endl;
    std::cout << ~ma * vx - vb;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(+2.3200000e+000,+3.7600000e+000) (+2.1200000e+000,+1.6000000e-001)
+2.1932508e-015
(+0.0000000e+000,-4.4408921e-016) (-8.8817842e-016,-8.8817842e-016)
```

2.4.55 operator / (const scmatrix&)

Operator

```
cvector operator / (const scmatrix& mA) const throw (cvmexception);
```

returns solution x of linear equation $A^T * x = b$ (which is equivalent to $x * A = b$) where parameter `mA` is square complex matrix A and calling vector is b. This operator throws exception of type `cvmexception` in case of inappropriate sizes of the objects or when the matrix A is close to singular. See also `cvector::solve_tran`, `cvector`, `scmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (7);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -3.};
    double b[] = {1., 2., 5., -3.};
    scmatrix ma((std::complex<double>*) m, 2);
    cvector vb((std::complex<double>*) b, 2);
    cvector vx(2);

    vx = vb / ma;

    std::cout << vx * ma - vb;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(+0.0000000e+000,+0.0000000e+000) (+0.0000000e+000,+4.4408921e-016)
```

2.4.56 operator % (const scmatrix&)

Operator

```
cvector operator % (const scmatrix& mA) const throw (cvmexception);
```

returns solution x of linear equation $A * x = b$ where parameter `mA` is square complex matrix A and calling vector is b. This operator throws exception of type `cvmexception` in case of inappropriate sizes of the objects or when the matrix A is close to singular. See also `cvector::solve`, `cvector`, `scmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (7);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -3.};
    double b[] = {1., 2., 5., -3.};
    scmatrix ma((std::complex<double>*) m, 2);
    cvector vb((std::complex<double>*) b, 2);
    cvector vx(2);

    vx = vb % ma;

    std::cout << ma * vx - vb;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-6.6613381e-016,+4.4408921e-016) (+0.0000000e+000,+0.0000000e+000)
```

2.4.57 solve_lu

Functions

```
cvector&
cvector::solve_lu (const scmatrix& mA, const scmatrix& mLU,
                   const int* pPivots, const cvector& vB, TR& dErr)
                   throw (cvmexception);
cvector&
cvector::solve_lu (const scmatrix& mA, const scmatrix& mLU,
                   const int* pPivots, const cvector& vB)
                   throw (cvmexception);
```

set calling vector to be equal to solution x of linear equation $A * x = b$ where parameter `mA` is square matrix A, parameter `mLU` is LU factorization of the matrix A, parameter `pPivots` is array of pivot numbers created while factorizing the matrix A and parameter `vB` is vector b. Every function returns a reference to the vector changed. The first version also sets output parameter `dErr` to be equal to norm of computation error. These functions are useful when you need to solve few linear equations of kind $Ax = b$ with the same matrix A and different vectors b. In such case you save on matrix A factorization since it's needed to be performed just one time. These functions throw `cvmexception` in case of inappropriate sizes of the objects or when the matrix A is close to singular. See also `cvector`, `scmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (7);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -3.};
    double b1[] = {1., 2., 5., -3.};
    double b2[] = {3., -1., 1., 7.};
    scmatrix ma((std::complex<double>*) m, 2);
    scmatrix mLU(2);
    cvector vb1((std::complex<double>*) b1, 2);
    cvector vb2((std::complex<double>*) b2, 2);
    cvector vx1(2);
    cvector vx2(2);
    iarray nPivots(2);
    double dErr = 0.;

    mLU.low_up(ma, nPivots);
    std::cout << vx1.solve_lu (ma, mLU, nPivots, vb1, dErr);
    std::cout << dErr << std::endl;
    std::cout << vx2.solve_lu (ma, mLU, nPivots, vb2);
    std::cout << ma * vx1 - vb1 << ma * vx2 - vb2;
```

```
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(3.5200000e+00,6.4000000e-01) (2.2400000e+00,-1.3200000e+00)
3.2788531e-15
(2.2800000e+00,1.9600000e+00) (3.6000000e-01,5.2000000e-01)
(-7.7715612e-16,4.4408921e-16) (0.0000000e+00,0.0000000e+00)
(-8.8817842e-16,0.0000000e+00) (-2.2204460e-16,0.0000000e+00)
```

2.4.58 gels

Function

```
cvector& cvector::gels (bool conjugate,
                        const cmatrix& mA, const cvector& mB,
                        TC& cErr) throw (cvmexception);
```

This function solves overdetermined or underdetermined linear system

$$A * x = b$$

for $m \times n$ matrix A (or its conjugate) where b is a vector of length k and $k = m$ in non-transposed case and $k = n$ otherwise. The algorithm uses QR or LQ factorization of A. It is assumed that A has full rank, infinity returned otherwise. Internally function uses `?GELS` LAPACK routines. If $m > n$ and `conjugate=false` or $m < n$ and `conjugate=true`, then the system is overdetermined, thus the algorithm tries to find the least squares solution x of the problem

$$\|A * x - b\|_2 \to \min \quad \text{or} \quad \|A^H * x - b\|_2 \to \min$$

respectively. Complex number `cErr` returns residuals. The system is underdetermined otherwise, and the algorithm finds its minimum norm solution. In this case `cErr` is set to zero. In both cases the solution computed satisfies $x = \mathrm{pinv}(A) * b$, but this algorithm is faster than pseudo inversion. Function sets calling object to be the solution and returns a reference to it. It throws `cvmexception` in case of inappropriate sizes of the operands. See also `cvector`, `cmatrix::gels`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (7);
cvm::cmatrix a(7, 5);
cvm::cvector b(7), bt(5);
tcomplex cErr, cErrc;
a.randomize_real(-10., 10.);
a.randomize_imag(-10., 10.);
b.randomize_real(-10., 10.);
b.randomize_imag(-10., 10.);
bt.randomize_real(-10., 10.);
bt.randomize_imag(-10., 10.);

cvm::cvector x(5);
x.gels(false, a, b, cErr);
cvm::cvector xt(7);
xt.gels(true, a, bt, cErrc);

std::cout << (a.pinv()*b - x).norm() << std::endl;
std::cout << cErr << std::endl;
std::cout << (~a.pinv()*bt - xt).norm() << std::endl;
std::cout << cErrc << std::endl;
```

prints

```
+1.0444134e-015
(-3.2469414e+001,+5.8375852e+000)
+1.9786436e-015
(+0.0000000e+000,+0.0000000e+000)
```

2.4.59 gelsy

Function

```
cvector& cvector::gelsy (const cmatrix& mA, const cvector& mB, int& rank,
                         TR tol = cvmMachSp()) throw (cvmexception);
```

These functions compute minimum-norm solution to linear least squares problem

$$\|A * x - b\|_2 \to \min$$

using complete orthogonal factorization of $m \times n$ matrix A. Here b is a vector of length m. Matrix A may be rank-deficient, function returns its effective rank in `rank` output parameter using `tol` tolerance. Internally function uses `?GELSY` LAPACK routines, see more details about the algorithm in those routines' documentation. Matrix A is passed as argument `mA`. Function sets calling object to be the solution and returns a reference to it. It throws `cvmexception` in case of inappropriate sizes of the operands. See also `cvector`, `cvector::gelss`, `cvector::gelsd`, `cmatrix::gelsy`. Example:

```
using namespace cvm;

cmatrix a(4, 5);
cvector b(4);
a.randomize_real(-10., 10.);
a.randomize_imag(-10., 10.);
b.randomize_real(-10., 10.);
b.randomize_imag(-10., 10.);
cvector x(5);
tint rank;

x.gelsy(a, b, rank);
std::cout << (a*x - b).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+6.0116296e-015
+4 +4
```

2.4.60 gelss

Function

```
cvector& cvector::gelss (const cmatrix& mA, const cvector& mB, int& rank,
                         rvector& sv, TR tol = cvmMachSp())
                         throw (cvmexception);
```

computes minimum-norm solution to linear least squares problem

$$\|A * x - b\|_2 \to \min$$

using the singular value decomposition of $m \times n$ matrix A. Here b is a vector of length m. Matrix A may be rank-deficient, function returns its effective rank in `rank` output parameter using `tol` tolerance. This function also computes singular values of A in decreasing order and returns them in sv output parameter having $\min(m, n)$ size. Internally function uses `?GELSS` LAPACK routines, see more details about the algorithm in that routine's documentation. Matrix A is passed as argument `mA`. Function sets calling object to be the solution and returns a reference to it. It throws `cvmexception` in case of inappropriate sizes of the operands. See also `cvector`, `cvector::gelsy`, `cvector::gelsd`, `cmatrix::gelss`. Example:

```
using namespace cvm;

cvm::cmatrix a(4, 5);
cvm::cvector b(4);
a.randomize_real(-10., 10.);
a.randomize_imag(-10., 10.);
b.randomize_real(-10., 10.);
b.randomize_imag(-10., 10.);
cvm::rvector sv(4);
cvm::cvector x(5);
tint rank;

x.gelss(a, b, sv, rank);
std::cout << (a*x - b).norm() << std::endl;
std::cout << (sv - a.svd()).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+9.4127010e-015
+4.4408921e-015
+4 +4
```

2.4.61 gelsd

Function

```
cvector& cvector::gelsd (const cmatrix& mA, const cvector& mB, int& rank,
                         rvector& sv, TR tol = cvmMachSp())
                         throw (cvmexception);
```

computes minimum-norm solution to linear least squares problem

$$\|A * x - b\|_2 \to \min$$

using the singular value decomposition of $m \times n$ matrix A and divide and conquer method. Here b is a vector of length m. Matrix A may be rank-deficient, function returns its effective rank in `rank` output parameter using `tol` tolerance. This function also computes singular values of A in decreasing order and returns them in `sv` output parameter having $\min(m, n)$ size. Internally function uses `?GELSD` LAPACK routines, see more details about the algorithm in that routine's documentation. Matrix A is passed as argument `mA`. Function sets calling object to be the solution and returns a reference to it. It throws `cvmexception` in case of inappropriate sizes of the operands. See also `cvector`, `cvector::gelsy`, `cvector::gelss`, `cmatrix::gelsd`. Example:

```
using namespace cvm;

cvm::cmatrix a(4, 5);
cvm::cvector b(4);
a.randomize_real(-10., 10.);
a.randomize_imag(-10., 10.);
b.randomize_real(-10., 10.);
b.randomize_imag(-10., 10.);
cvm::rvector sv(4);
cvm::cvector x(5);
tint rank;

x.gelsd(a, b, sv, rank);
std::cout << (a*x - b).norm() << std::endl;
std::cout << (sv - a.svd()).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+1.3491837e-014
+5.3290705e-015
+4 +4
```

2.4.62 eig

Functions

```
cvector&
cvector::eig (const srmatrix& mArg) throw (cvmexception);
cvector&
cvector::eig (const scmatrix& mArg) throw (cvmexception);

cvector&
cvector::eig (const srmatrix& mArg, scmatrix& mEigVect,
              bool bRightVect = true) throw (cvmexception);
cvector&
cvector::eig (const scmatrix& mArg, scmatrix& mEigVect,
              bool bRightVect = true) throw (cvmexception);
```

solve nonsymmetric eigenvalue problem and set calling vector to be equal to eigenvalues of a square matrix `mArg`. The nonsymmetric eigenvalue problem is defined as follows: given a nonsymmetric (or non-Hermitian) matrix A, find the eigenvalues λ and the corresponding eigenvectors z that satisfy the equation

$$Az = \lambda z.$$

Some eigenvalues may be complex even for real matrix A. Moreover, if real nonsymmetric matrix has a complex eigenvalue $a + bi$ corresponding to an eigenvector z, then $a - bi$ is also an eigenvalue. The eigenvalue $a - bi$ corresponds to the eigenvector whose elements are complex conjugate to the elements of z. The third and fourth versions of functions set output parameter `mEigVect` to be equal to square matrix containing eigenvectors as columns. They also compute "left" eigenvectors if parameter ḅRightVect is set to f̧alse. Left eigencectors satisfy

$$z^H A = \lambda z^H.$$

All functions return reference to the vector they change and throw `cvmexception` in case of inappropriate calling object sizes or in case of convergence error. See also `cvector`, `rvector::eig`, `scmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (2);

try {
    srmatrix m(3);
    scmatrix me(3);
    cvector vl(3);

    m(1,1) = 0.1;  m(1,2) = 0.2;  m(1,3) = 0.1;
    m(2,1) = 0.11; m(2,2) = 2.9;  m(2,3) = -8.4;
```

```
    m(3,1) = 0.;   m(3,2) = 2.91; m(3,3) = 8.2;
    scmatrix mc(m);

    std::cout << vl.eig (m, me) << std::endl;
    std::cout << mc * me(1) - me(1) * vl(1);
    std::cout << mc * me(2) - me(2) * vl(2);
    std::cout << mc * me(3) - me(3) * vl(3) << std::endl;

    std::cout << vl.eig (m, me, false) << std::endl;
    std::cout << ~(me(1)) * mc - ~(me(1)) * vl(1);
    std::cout << ~(me(2)) * mc - ~(me(2)) * vl(2);
    std::cout << ~(me(3)) * mc - ~(me(3)) * vl(3);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(+9.69e-002,+0.00e+000) (+5.55e+000,+4.17e+000) (+5.55e+000,-4.17e+000)

(+2.78e-017,+0.00e+000) (+7.24e-017,+0.00e+000) (+4.23e-017,+0.00e+000)
(-2.22e-016,-1.94e-016) (-7.11e-015,-4.88e-015) (+0.00e+000,+1.78e-015)
(-2.22e-016,+1.94e-016) (-7.11e-015,+4.88e-015) (+0.00e+000,-1.78e-015)

(+9.69e-002,+0.00e+000) (+5.55e+000,+4.17e+000) (+5.55e+000,-4.17e+000)

(+0.00e+000,+0.00e+000) (+4.19e-017,+0.00e+000) (+1.13e-017,+0.00e+000)
(+0.00e+000,-4.16e-017) (+2.22e-016,-1.78e-015) (-8.88e-016,+2.44e-015)
(+0.00e+000,+4.16e-017) (+2.22e-016,+1.78e-015) (-8.88e-016,-2.44e-015)
```

2.4.63 geneig

Functions

```
cvector&
cvector::geneig (const srmatrix& mA, const srmatrix& mB, rvector& vBeta)
                throw(cvmexception);
cvector&
cvector::geneig (const srmatrix& mA, const srmatrix& mB, rvector& vBeta,
                scmatrix& mEigVectLeft, scmatrix& mEigVectRight)
                throw(cvmexception);
cvector&
cvector::geneig (const srmatrix& mA, const srmatrix& mB, rvector& vBeta,
                scmatrix& mEigVect, bool bRightVect = true)
                throw(cvmexception);

cvector&
cvector::geneig (const scmatrix& mA, const scmatrix& mB, cvector& vBeta)
                throw(cvmexception);
cvector&
cvector::geneig (const scmatrix& mA, const scmatrix& mB, cvector& vBeta,
                scmatrix& mEigVectLeft, scmatrix& mEigVectRight)
                throw(cvmexception);
cvector&
cvector::geneig (const scmatrix& mA, const scmatrix& mB, cvector& vBeta,
                scmatrix& mEigVect, bool bRightVect = true)
                throw(cvmexception);
```

solve generalized eigenvalue problem and set calling vector to be equal to generalized eigenvalues of a square matrices `mA` and `mB`. A generalized eigenvalue for a pair of matrices (A, B) is a scalar λ or a ratio $\alpha/\beta = \lambda$, such that $A - \lambda B$ is singular. It is usually represented as the pair (α, β), as there is a reasonable interpretation for $\beta = 0$ and even for both being zero. The right generalized eigenvector v_j corresponding to the generalized eigenvalue λ_j of (A, B) satisfies

$$Av_j = \lambda_j Bv_j.$$

The left generalized eigenvector u_j corresponding to the generalized eigenvalue λ_j of (A, B) satisfies

$$u_j^H A = \lambda_j u_j^H B.$$

All functions return reference to the vector they change and throw `cvmexception` in case of inappropriate calling object sizes or in case of convergence error. See also `cvector`, `cvector::eig`, `scmatrix`. Example (real matrices):

```
using namespace cvm;
try {
    srmatrix a(5), b(5);
```

```
    a.randomize(-10., 10.);
    b.randomize(-10., 10.);
    cvector alpha(5);
    rvector beta(5);
    scmatrix eigVectLeft(5), eigVectRight(5);
    int i;

    alpha.geneig(a, b, beta, eigVectLeft, eigVectRight);
    for (i = 1; i <= 5; ++i) {
        std::cout << (scmatrix(a) -
            alpha[i] / beta[i] * scmatrix(b)).svd();
    }
    for (i = 1; i <= 5; ++i) {
        std::cout << (~(eigVectLeft(i)) * scmatrix(a) -
            (alpha[i] / beta[i]) * ~(eigVectLeft(i)) *
            scmatrix(b)).norm() << std::endl;
    }
    for (i = 1; i <= 5; ++i) {
        std::cout << (scmatrix(a) * eigVectRight(i) -
            (alpha[i] / beta[i]) * scmatrix(b) *
            eigVectRight(i)).norm() << std::endl;
    }

    alpha.geneig(a, b, beta, eigVectLeft, false);
    for (i = 1; i <= 5; ++i) {
        std::cout << (~(eigVectLeft(i)) * scmatrix(a) -
            (alpha[i] / beta[i]) * ~(eigVectLeft(i)) *
            scmatrix(b)).norm() << std::endl;
    }

    alpha.geneig(a, b, beta, eigVectRight);
    for (i = 1; i <= 5; ++i) {
        std::cout << (scmatrix(a) * eigVectRight(i) -
            (alpha[i] / beta[i]) * scmatrix(b) *
            eigVectRight(i)).norm() << std::endl;
    }
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
31.328 23.9648 18.6695 7.95943 1.3636e-015
31.328 23.9648 18.6695 7.95943 3.17647e-015
26.3359 20.9384 12.8015 8.20644 1.58238e-015
```

```
38.886 22.9187 21.6982 8.51899 4.17944e-015
26.3947 16.1351 13.8545 7.6506 2.99015e-015
2.32204e-014
2.16331e-014
1.53681e-014
1.23709e-014
5.32213e-015
1.25076e-014
1.33762e-014
2.10381e-014
1.99396e-014
1.22447e-014
2.32204e-014
2.16331e-014
1.53681e-014
1.23709e-014
5.32213e-015
1.25076e-014
1.33762e-014
2.10381e-014
1.99396e-014
1.22447e-014
```

Example (complex matrices):

```
using namespace cvm;
try {
    scmatrix a(5);
    scbmatrix b(5, 2, 1);
    a.randomize_real(-10., 10.);
    a.randomize_imag(-10., 10.);
    b.randomize_real(-10., 10.);
    b.randomize_imag(-10., 10.);
    cvector alpha(5);
    cvector beta(5);
    scmatrix eigVectLeft(5), eigVectRight(5);
    int i;

    alpha.geneig(a, b, beta, eigVectLeft, eigVectRight);
    for (i = 1; i <= 5; ++i) {
        std::cout << (a - alpha[i] / beta[i] * b).svd();
    }
    for (i = 1; i <= 5; ++i) {
        std::cout << (~(eigVectLeft(i)) * a -
            (alpha[i] / beta[i]) * ~(eigVectLeft(i)) * b).norm()
            << std::endl;
```

```
    }
    for (i = 1; i <= 5; ++i) {
        std::cout << (a * eigVectRight(i) -
            (alpha[i] / beta[i]) * b * eigVectRight(i)).norm()
            << std::endl;
    }

    alpha.geneig(a, b, beta, eigVectLeft, false);
    for (i = 1; i <= 5; ++i) {
        std::cout << (~(eigVectLeft(i)) * a -
            (alpha[i] / beta[i]) * ~(eigVectLeft(i)) * b).norm()
            << std::endl;
    }

    alpha.geneig(a, b, beta, eigVectRight);
    for (i = 1; i <= 5; ++i) {
        std::cout << (a * eigVectRight(i) -
            (alpha[i] / beta[i]) * b * eigVectRight(i)).norm()
            << std::endl;
    }
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
67.7195 51.2625 39.0367 13.472 1.34998e-015
46.2033 28.5441 20.5831 16.045 1.30494e-014
35.9992 22.556 16.2677 9.76767 8.3086e-016
37.362 22.8235 16.7452 6.18941 7.78931e-015
48.1894 33.972 19.4611 9.67851 8.82633e-015
3.15084e-014
3.18446e-014
1.79087e-014
1.32967e-014
1.55926e-014
2.81603e-014
1.82293e-014
2.18339e-014
2.17468e-014
2.38458e-014
3.15084e-014
3.18446e-014
1.79087e-014
1.32967e-014
```

```
1.55926e-014
2.81603e-014
1.82293e-014
2.18339e-014
2.17468e-014
2.38458e-014
```

2.4.64 gemv

Function

```
cvector& cvector::gemv (bool bLeft, const cmatrix& m, TC dAlpha,
                        const cvector& v, TC dBeta) throw (cvmexception);
```

calls one of ?GEMV routines of the BLAS library performing matrix-vector operation defined as

$$c = \alpha M \cdot v + \beta c \quad \text{or} \quad c = \alpha v \cdot M + \beta c,$$

where α and β are complex numbers (parameters dAlpha and dBeta), M is complex matrix (parameter m) and v and c are complex vectors (parameter v and calling vector respectively). First operation is performed if bLeft passed is false and second one otherwise. Function returns a reference to the vector changed and throws cvmexception in case of inappropriate calling object sizes. See also cvector, cmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    std::complex<double> alpha = std::complex<double>(1.3,-0.7);
    std::complex<double> beta  = std::complex<double>(0.15,-1.09);
    cmatrix m(3,2);
    cvector c(3);
    cvector v(2);
    m.randomize_real(-1., 2.); m.randomize_imag(0., 1.);
    v.randomize_real(-1., 3.); v.randomize_imag(2., 4.);
    c.randomize_real(0., 2.); c.randomize_imag(3., 7.);
    std::cout << m * v * alpha + c * beta;
    std::cout << c.gemv(false, m, alpha, v, beta);
    std::cout << c * m * alpha + v * beta;
    std::cout << v.gemv(true, m, alpha, c, beta);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(2.71e-01,2.44e+00) (2.20e+01,7.16e+00) (-7.89e-01,2.45e+00)
(2.71e-01,2.44e+00) (2.20e+01,7.16e+00) (-7.89e-01,2.45e+00)
(5.92e+01,-1.47e+01) (3.54e+01,-3.14e+00)
(5.92e+01,-1.47e+01) (3.54e+01,-3.14e+00)
```

2.4.65 gbmv

Function

```
cvector& cvector::gbmv (bool bLeft, const scbmatrix& m, TC dAlpha,
                        const cvector& v, TC dBeta) throw (cvmexception);
```

calls one of ?GBMV routines of the BLAS library performing matrix-vector operation defined as

$$c = \alpha M \cdot v + \beta c \quad \text{or} \quad c = \alpha v \cdot M + \beta c,$$

where α and β are complex numbers (parameters dAlpha and dBeta), M is complex band matrix (parameter m) and v and c are complex vectors (parameter v and calling vector respectively). First operation is performed if bLeft passed is false and second one otherwise. Function returns a reference to the vector changed and throws cvmexception in case of inappropriate calling object sizes. See also cvector, scbmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    std::complex<double> alpha = std::complex<double>(1.3,-0.7);
    std::complex<double> beta  = std::complex<double>(0.15,-1.09);
    scbmatrix m(3,1,0);
    cvector c(3);
    cvector v(3);
    m.randomize_real(-1., 2.); m.randomize_imag(0., 1.);
    v.randomize_real(-1., 3.); v.randomize_imag(2., 4.);
    c.randomize_real(0., 2.); c.randomize_imag(3., 7.);
    std::cout << m * v * alpha + c * beta;
    std::cout << c.gbmv(false, m, alpha, v, beta);
    std::cout << c * m * alpha + v * beta;
    std::cout << v.gbmv(true, m, alpha, c, beta);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(3.73e+00,7.96e-01) (6.89e+00,1.07e+01) (2.16e+00,3.28e+00)
(3.73e+00,7.96e-01) (6.89e+00,1.07e+01) (2.16e+00,3.28e+00)
(3.11e+01,2.51e+01) (-4.93e+00,1.34e+01) (1.70e+00,3.93e+00)
(3.11e+01,2.51e+01) (-4.93e+00,1.34e+01) (1.70e+00,3.93e+00)
```

2.4.66 randomize_real

Function

```
cvector& cvector::randomize_real (TR dFrom, TR dTo);
```

fills real part of a calling vector with pseudo-random numbers distributed between `dFrom` and `dTo`. Function returns a reference to the vector changed. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v(3);
v.randomize_real(-2.,3.);
std::cout << v;
```

prints

```
(-4.93e-01,0.00e+00) (1.37e+00,0.00e+00) (-1.49e-01,0.00e+00)
```

2.4.67 randomize_imag

Function

```
cvector& cvector::randomize_imag (TR dFrom, TR dTo);
```

fills imaginary part of a calling vector with pseudo-random numbers distributed between `dFrom` and `dTo`. Function returns a reference to the vector changed. See also `cvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

cvector v(3);
v.randomize_imag(-2.,3.);
std::cout << v;
```

prints

```
(0.00e+00,-4.37e-01) (0.00e+00,-1.59e+00) (0.00e+00,2.42e+00)
```

2.5 Matrix

This base class contains member functions common for all matrices. This class is not designed to be instantiated.

```
template <typename TR, typename TC>
class Matrix : public Array<TR,TC> {
public:
    int msize () const;
    int nsize () const;
    int ld () const;
    int rowofmax () const;
    int rowofmin () const;
    int colofmax () const;
    int colofmin () const;
    virtual TR norm1 () const;
    <typename TR, typename TC>
    friend std::ostream& operator >> <> (std::istream& is,
                                         const Array<TR,TC>& mIn);
    <typename TR, typename TC>
    friend std::ostream& operator << <> (std::ostream& os,
                                         const Array<TR,TC>& mOut);
};
```

2.5.1 msize

Function

```
int Matrix<TR,TC>::msize () const;
```

returns number of rows of a calling matrix. Function is *inherited* in all matrix classes of the library: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Matrix`. Example:

```
using namespace cvm;

rmatrix m (100, 200);
std::cout << m.msize() << std::endl;
```

prints

```
100
```

2.5.2 nsize

Function

```
int Matrix<TR,TC>::nsize () const;
```

returns number of columns of a calling matrix. Function is *inherited* in all matrix classes of the library: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Matrix`. Example:

```
using namespace cvm;

rmatrix m (100, 200);

std::cout << m.nsize() << std::endl;
```

prints

```
200
```

2.5.3 ld

Function

```
int Matrix<TR,TC>::ld () const;
```

returns leading dimension of a calling matrix. Leading dimension is equal to number of rows for every matrix except submatrices. For submatrices it's equal to number of rows of parent matrix. Function is *inherited* in all matrix classes of the library: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Matrix`. Example:

```
using namespace cvm;

rmatrix m (100, 200);
srmatrix ms (m, 30, 40, 5); // 5x5 submatrix
std::cout << ms.ld() << std::endl;
```

prints

```
100
```

2.5.4 rowofmax

Function

```
int Matrix<TR,TC>::rowofmax () const;
```

returns ℓ-based number of a calling matrix row where the element with the maximum absolute value is located. Function is *inherited*[5] in all matrix classes of the library: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Matrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., -3., 1., -1.};
rmatrix m (a, 2, 3);

std::cout << m << std::endl << m.rowofmax() << std::endl;
```

prints

```
1 2 1
0 -3 -1

2
```

[5]Calls virtual function inside

2.5.5 rowofmin

Function

```
int Matrix<TR,TC>::rowofmin () const;
```

returns ℓ-based number of a calling matrix row where the element with the minimum absolute value is located. Function is *inherited*[6] in all matrix classes of the library: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Matrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., -3., 1., -1.};
rmatrix m (a, 2, 3);

std::cout << m << std::endl << m.rowofmin() << std::endl;
```

prints

```
1 2 1
0 -3 -1

2
```

[6]Calls virtual function inside

2.5.6 colofmax

Function

```
int Matrix<TR,TC>::colofmax () const;
```

returns ℓ-based number of a calling matrix column where the element with the maximum absolute value is located. Function is *inherited*[7] in all matrix classes of the library: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Matrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., -3., 1., -1.};
rmatrix m (a, 2, 3);

std::cout << m << std::endl << m.colofmax() << std::endl;
```

prints

```
1 2 1
0 -3 -1

2
```

[7]Calls virtual function inside

2.5.7 colofmin

Function

```
int Matrix<TR,TC>::colofmin () const;
```

returns ℓ-based number of a calling matrix column where the element with the minimum absolute value is located. Function is *inherited*[8] in all matrix classes of the library: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srbmatrix`, `scbmatrix`, `srsmatrix` and `schmatrix`. See also `Matrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., -3., 1., -1.};
rmatrix m (a, 2, 3);

std::cout << m << std::endl << m.colofmin() << std::endl;
```

prints

```
1 2 1
0 -3 -1

1
```

[8]Calls virtual function inside

2.5.8 norm1

Virtual function

```
virtual TR Matrix<TR,TC>::norm1 () const;
```

returns 1-norm of a calling matrix that is defined as

$$\|A\|_1 = \max_{j=1,\dots,n} \sum_{i=1}^{m} |a_{ij}|,$$

where A is $m \times n$ matrix. Function is *inherited* in the following classes of the library: `rmatrix`, `cmatrix`, `srmatrix`, `scmatrix`, `srsmatrix` and `schmatrix`. It's *redefined* in `srbmatrix` and `scbmatrix`. See also `Array::norminf` and `Matrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., -3., 1., 0.};
rmatrix m (a, 2, 3);

std::cout << m << std::endl << m.norm1()
          << std::endl << m.norminf() << std::endl;
```

prints

```
1 2 1
0 -3 0

5
4
```

2.5.9 operator >> <> (std::istream& is, Matrix<TR,TC>& mIn)

Friend template operator

```
template <typename TR, typename TC>
friend std::istream& operator >> <> (std::istream& is,
                                      Matrix<TR,TC>& mIn);
```

fills calling Matrix (row by row) referenced by parameter `mIn` with numbers from `is` stream. See also `Array::operator >>` , `Matrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    std::ofstream os;
    os.open ("in.txt");
    os << 1.2 << " " << 2.3 << std::endl << 3.4;
    os.close ();

    std::ifstream is("in.txt");
    rmatrix m(3,2);
    is >> m;

    std::cout << m;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.20e+000 2.30e+000
3.40e+000 0.00e+000
0.00e+000 0.00e+000
```

2.5.10 `operator << <> (std::ostream& os, const Matrix<TR,TC>& mOut)`

Friend template operator

```
template <typename TR, typename TC>
friend std::ostream& operator << <> (std::ostream& os,
                                     const Matrix<TR,TC>& mOut);
```

writes matrix (row by row) referenced by parameter `mOut` into `os` stream. See also `Array::operator <<` , `Matrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

srmatrix m(3);
m(1,1) = 1.;
m(2,3) = 3.;

std::cout << m;
```

prints

```
1.00e+00 0.00e+00 0.00e+00
0.00e+00 0.00e+00 3.00e+00
0.00e+00 0.00e+00 0.00e+00
```

2.6 rmatrix

This is end-user class encapsulating matrix of real numbers.

```
template <typename TR>
class rmatrix : public Matrix <TR,TR> {
public:
    rmatrix ();
    rmatrix (int nM, int nN);
    rmatrix (TR* pD, int nM, int nN);
    rmatrix (const TR* pD, int nM, int nN);
    rmatrix (const rmatrix& m);
    explicit rmatrix (const rvector& v, bool bBeColumn = true);
    rmatrix (rmatrix& m, int nRow, int nCol, int nHeight, int nWidth);
    TR& operator () (int im, int in) throw (cvmexception);
    TR operator () (int im, int in) const throw (cvmexception);
    rvector operator () (int i) throw (cvmexception);
    const rvector operator () (int i) const throw (cvmexception);
    rvector operator [] (int i) throw (cvmexception);
    const rvector operator [] (int i) const throw (cvmexception);
    rvector diag (int i) throw (cvmexception);
    const rvector diag (int i) const throw (cvmexception);
    rmatrix& operator = (const rmatrix& m) throw (cvmexception);
    rmatrix& assign (const rvector& v) throw (cvmexception);
    rmatrix& assign (const TR* pD);
    rmatrix& assign (int nRow, int nCol, const rmatrix& m)
                     throw (cvmexception);
    rmatrix& set (TR x);
    rmatrix& resize (int nNewM, int nNewN) throw (cvmexception);
    bool operator == (const rmatrix& m) const;
    bool operator != (const rmatrix& m) const;
    rmatrix& operator << (const rmatrix& m) throw (cvmexception);
    rmatrix operator + (const rmatrix& m) const
                        throw (cvmexception);
    rmatrix operator - (const rmatrix& m) const
                        throw (cvmexception);
    rmatrix& sum (const rmatrix& m1,
                  const rmatrix& m2) throw (cvmexception);
    rmatrix& diff (const rmatrix& m1,
                   const rmatrix& m2) throw (cvmexception);
    rmatrix& operator += (const rmatrix& m) throw (cvmexception);
    rmatrix& operator -= (const rmatrix& m) throw (cvmexception);
    rmatrix operator - () const;
    rmatrix operator * (TR d) const;
    rmatrix operator / (TR d) const
```

```
                      throw (cvmexception);
    rmatrix& operator *= (TR d);
    rmatrix& operator /= (TR d) throw (cvmexception);
    rmatrix& normalize ();
    rmatrix operator ~ () const throw (cvmexception);
    rmatrix& transpose (const rmatrix& m) throw (cvmexception);
    rmatrix& transpose () throw (cvmexception);
    rvector operator * (const rvector& v) const
                      throw (cvmexception);
    rmatrix operator * (const rmatrix& m) const
                      throw (cvmexception);
    rmatrix& mult (const rmatrix& m1, const rmatrix& m2)
                 throw (cvmexception);
    rmatrix& rank1update (const rvector& vCol,
                          const rvector& vRow)
                          throw (cvmexception);
    rmatrix& swap_rows (int n1, int n2) throw (cvmexception);
    rmatrix& swap_cols (int n1, int n2) throw (cvmexception);
    rmatrix& solve (const srmatrix& mA,
                    const rmatrix& mB, TR& dErr)
                    throw (cvmexception);
    rmatrix& solve (const srmatrix& mA,
                    const rmatrix& mB) throw (cvmexception);
    rmatrix& solve_tran (const srmatrix& mA,
                         const rmatrix& mB, TR& dErr)
                         throw (cvmexception);
    rmatrix& solve_tran (const srmatrix& mA,
                         const rmatrix& mB) throw (cvmexception);
    rmatrix& solve_lu (const srmatrix& mA, const srmatrix& mLU,
                       const int* pPivots, const rmatrix& mB, TR& dErr)
                       throw (cvmexception);
    rmatrix& solve_lu (const srmatrix& mA, const srmatrix& mLU,
                       const int* pPivots, const rmatrix& mB)
                       throw (cvmexception);
    rvector svd () const throw (cvmexception);
    rvector svd (srmatrix& mU, srmatrix& mVH) const
                throw (cvmexception);
    rmatrix pinv (TR threshold = cvmMachSp()) const
                 throw (cvmexception);
    rmatrix& pinv (const rmatrix& mA, TR threshold = cvmMachSp())
                  throw (cvmexception);
    rmatrix gels (bool transpose, const rmatrix& mB,
                 rvector& vErr) const throw (cvmexception);
    rmatrix& gels (bool transpose, const rmatrix& mA, const rmatrix& mB,
                  rvector& vErr) throw (cvmexception);
```

```
rvector gels (bool transpose, const rvector& mB,
               TR& dErr) const throw (cvmexception);
rmatrix gelsy (const rmatrix& mB, int& rank,
               TR tol = cvmMachSp()) const throw (cvmexception);
rmatrix& gelsy (const rmatrix& mA, const rmatrix& mB, int& rank,
               TR tol = cvmMachSp()) throw (cvmexception);
rvector gelsy (const rvector& mB, int& rank,
               TR tol = cvmMachSp()) const throw (cvmexception);
rmatrix gelss (const rmatrix& mB, rvector& sv,
               int& rank, TR tol = cvmMachSp()) const
               throw (cvmexception);
rmatrix& gelss (const rmatrix& mA, const rmatrix& mB, rvector& sv,
               int& rank, TR tol = cvmMachSp())
               throw (cvmexception);
rvector gelss (const rvector& mB, rvector& sv,
               int& rank, TR tol = cvmMachSp()) const
               throw (cvmexception);
rmatrix gelsd (const rmatrix& mB, rvector& sv,
               int& rank, TR tol = cvmMachSp()) const
               throw (cvmexception);
rmatrix& gelsd (const rmatrix& mA, const rmatrix& mB, rvector& sv,
               int& rank, TR tol = cvmMachSp())
               throw (cvmexception);
rvector gelsd (const rvector& mB, rvector& sv,
               int& rank, TR tol = cvmMachSp()) const
               throw (cvmexception);
int rank (TR eps = cvmMachSp ()) const throw (cvmexception);
rmatrix& ger (TR dAlpha, const rvector& vCol,
              const rvector& vRow) throw (cvmexception);
rmatrix& gemm (const rmatrix& m1, bool bTrans1,
               const rmatrix& m2, bool bTrans2,
               TR dAlpha, TR dBeta) throw (cvmexception);
rmatrix& symm (bool bLeft, const srsmatrix& ms,
               const rmatrix& m, TR dAlpha, TR dBeta)
               throw (cvmexception);
void qr(rmatrix& mQ, srmatrix& mR) const throw (cvmexception);
void qr(srmatrix& mQ, rmatrix& mR) const throw (cvmexception);
void lq(srmatrix& mL, rmatrix& mQ) const throw (cvmexception);
void lq(rmatrix& mL, srmatrix& mQ) const throw (cvmexception);
void rq(srmatrix& mR, rmatrix& mQ) const throw (cvmexception);
void rq(rmatrix& mR, srmatrix& mQ) const throw (cvmexception);
void ql(rmatrix& mQ, srmatrix& mL) const throw (cvmexception);
void ql(srmatrix& mQ, rmatrix& mL) const throw (cvmexception);
rmatrix& vanish ();
rmatrix& randomize (TR dFrom, TR dTo);
```

```
};
```

2.6.1 rmatrix ()

Default constructor

```
rmatrix::rmatrix ();
```

creates empty `rmatrix` object. See also `rmatrix`. Example:

```
using namespace cvm;

rmatrix m;
std::cout << m.msize() << std::endl << m.nsize() << std::endl;
std::cout << m.size() << std::endl;

m.resize (2, 3);
std::cout << m;
```

prints

```
0
0
0
0 0 0
0 0 0
```

2.6.2 rmatrix (int,int)

Constructor

```
rmatrix::rmatrix (int nM, int nN);
```

creates $m \times n$ `rmatrix` object where m is passed in `nM` parameter (number of rows) and n is passed in `nN` (number of columns). Constructor sets all elements to zero. It throws `cvmexception` in case of non-positive sizes passed or memory allocation failure. See also `rmatrix`. Example:

```
using namespace cvm;

rmatrix m (3, 4);
std::cout << m.msize() << std::endl << m.nsize()
          << std::endl << m.size() << std::endl << m;
```

prints

```
3
4
12
0 0 0 0
0 0 0 0
0 0 0 0
```

2.6.3 rmatrix (TR*,int,int)

Constructor

```
rmatrix::rmatrix (TR* pD, int nM, int nN);
```

creates $m \times n$ `rmatrix` object where m is passed in `nM` parameter (number of rows) and n is passed in `nN` (number of columns). Unlike others, this constructor *does not allocate memory*. It just shares memory with an array pointed to by `pD`. It throws `cvmexception` in case of non-positive size passed. See also `rmatrix`, `rmatrix (const TR*,int,int)`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 2., 3., 4., 5., 6.};
rmatrix m (a, 2, 3);

m(1,1) = 7.77;
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << std::endl;
```

prints

```
7.77e+000 3.00e+000 5.00e+000
2.00e+000 4.00e+000 6.00e+000

7.77e+000 2.00e+000 3.00e+000
```

2.6.4 rmatrix (const TR*,int,int)

Constructor

```
rmatrix::rmatrix (const TR* pD, int nM, int nN);
```

creates $m \times n$ `rmatrix` object where m is passed in `nM` parameter (number of rows) and n is passed in `nN` (number of columns) and copies $m * n$ elements of an array `pD` to it `by colums`. It throws `cvmexception` in case of non-positive size passed. See also `rmatrix`, `rmatrix (TR*,int,int)`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
const double a[] = {1., 2., 3., 4., 5., 6.};
rmatrix m (a, 2, 3);

m(1,1) = 7.77;
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << std::endl;
```

prints

```
7.77e+000 3.00e+000 5.00e+000
2.00e+000 4.00e+000 6.00e+000

1.00e+000 2.00e+000 3.00e+000
```

2.6.5 rmatrix (const rmatrix&)

Copy constructor

```
rmatrix::rmatrix (const rmatrix& m);
```

creates `rmatrix` object as a copy of `m`. It throws `cvmexception` in case of memory allocation failure. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 2., 3., 4., 5., 6.};
rmatrix m (a, 2, 3);
rmatrix mc(m);

m(1,1) = 7.77;
std::cout << m << std::endl << mc;
```

prints

```
7.77e+000 3.00e+000 5.00e+000
2.00e+000 4.00e+000 6.00e+000

1.00e+000 3.00e+000 5.00e+000
2.00e+000 4.00e+000 6.00e+000
```

2.6.6 rmatrix (const rvector&,bool)

Constructor

```
explicit rmatrix::rmatrix (const rvector& v, bool bBeColumn = true);
```

creates `rmatrix` object containing `v.size()` rows and 1 column if `bBeColumn` is `true` or 1 row and `v.size()` columns otherwise. After that it copies vector `v`'s elements to the matrix created. Constructor throws `cvmexception` in case of memory allocation failure. See also `rmatrix`, `rvector`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
rvector v(3);
v(1) = 1.;
v(2) = 2.;
v(3) = 3.;

rmatrix mc (v);
rmatrix mr (v, false);

std::cout << mc << std::endl << mr;
```

prints

```
1.00e+000
2.00e+000
3.00e+000

1.00e+000 2.00e+000 3.00e+000
```

2.6.7 submatrix

Submatrix constructor

```
rmatrix::rmatrix (rmatrix& m, int nRow, int nCol,
                              int nHeight, int nWidth);
```

creates `rmatrix` object as *submatrix* of `m`. It means that the matrix object created shares memory with some part of `m`. This part is defined by its upper left corner (parameters `nRow` and `nCol`, both are ℓ-based) and its height and width (parameters `nHeight` and `nWidth`). See also `rmatrix`. Example:

```
using namespace cvm;
rmatrix m(4,5);
rmatrix subm(m, 2, 2, 2, 2);
subm.set(1.);

std::cout << m;
```

prints

```
0 0 0 0 0
0 1 1 0 0
0 1 1 0 0
0 0 0 0 0
```

2.6.8 operator (,)

Indexing operators

```
TR& rmatrix::operator () (int im, int in) throw (cvmexception);
TR rmatrix::operator () (int im, int in) const throw (cvmexception);
```

provide access to a particular element of a calling matrix. The first version is applicable to non-constant object. This version returns *l-value* in order to make possible write access to an element. Both operators are ℓ-based. Operators throw `cvmexception` if `im` is outside of `[1,msize()]` range or `in` is outside of `[1,nsize()]` range. Operators are *inherited* in the classes `srmatrix` and `srbmatrix`. Operators are *redefined* in the class `srsmatrix`. See also `rmatrix`, `Matrix::msize()`, `Matrix::nsize()`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix m (a, 2, 3);
    rmatrix ms(m);

    std::cout << m(1,1) << " " << m(2,3) << std::endl << std::endl;

    ms(2,2) = 7.77;
    std::cout << ms;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+00 6.00e+00

1.00e+00 3.00e+00 5.00e+00
2.00e+00 7.77e+00 6.00e+00
```

2.6.9 operator ()

Indexing operators

```
rvector rmatrix::operator () (int i) throw (cvmexception);
const rvector rmatrix::operator () (int i) const throw (cvmexception);
```

provide access to i-th column of a calling matrix. The first version is applicable to non-constant object and *returns l-value*, i.e. vector returned shares memory with i-th column of the matrix in order to make possible write access to it. The second version creates *copy* of a column and therefore is *not l-value*. Both operators are ℓ-based. Operators throw `cvmexception` if the parameter `i` is outside of `[1,nsize()]` range. Operators are *inherited* in the class `srmatrix`. Operators are *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `rmatrix`, `Matrix::nsize()`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix m (a, 2, 3);
    srmatrix ms(2);

    std::cout << m(2) << std::endl;

    ms(2) = m(3);
    std::cout << ms;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3.00e+00 4.00e+00

0.00e+00 5.00e+00
0.00e+00 6.00e+00
```

2.6.10 operator []

Indexing operators

```
rvector rmatrix::operator [] (int i) throw (cvmexception);
const rvector rmatrix::operator [] (int i) const throw (cvmexception);
```

provide access to i-th row of matrix. The first version is applicable to non-constant object and *returns l-value*, i.e. vector returned shares memory with i-th row of the matrix in order to make possible write access to it. The second version creates *copy* of a row and therefore is *not l-value*. Both operators are ℓ-based. Operators throw `cvmexception` if `i` is outside of `[1,msize()]` range. Operators are *inherited* in the class `srmatrix`. Operators are *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `rmatrix`, `Matrix::msize()`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix m (a, 2, 3);
    srmatrix ms(3);

    std::cout << m[1] << std::endl;

    ms[1] = m[2];
    std::cout << ms;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+00 3.00e+00 5.00e+00

2.00e+00 4.00e+00 6.00e+00
0.00e+00 0.00e+00 0.00e+00
0.00e+00 0.00e+00 0.00e+00
```

2.6.11 diag

Functions

```
rvector rmatrix::diag (int i) throw (cvmexception);
const rvector rmatrix::diag (int i) const throw (cvmexception);
```

provide access to i-th diagonal of a calling matrix, where $i = 0$ for main diagonal, $i < 0$ for lower diagonals and $i > 0$ for upper ones. The first version is applicable to non-constant object and *returns l-value*, i.e. vector returned shares memory with i-th diagonal of the matrix in order to make possible write access to it. The second version creates *copy* of a diagonal and therefore is *not l-value*. Functions throw `cvmexception` if parameter `i` is outside of `[-msize()+1,nsize()-1]` range. Functions are *inherited* in the classes `srmatrix` and `srbmatrix`. Functions are *redefined* in the class `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    rmatrix  m(2,3);
    const srmatrix ms(a,3);

    m.diag(-1).set(1.);
    m.diag(0).set(2.);
    m.diag(1).set(3.);
    m.diag(2).set(4.);
    std::cout << m << std::endl;

    std::cout << ms << std::endl;
    std::cout << ms.diag(0) << ms.diag(1);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
2 3 4
1 2 3

1 4 7
2 5 8
3 6 9

1 5 9
4 8
```

2.6.12 operator = (const rmatrix&)

Operator

```
rmatrix& rmatrix::operator = (const rmatrix& m) throw (cvmexception);
```

sets every element of a calling matrix to be equal to appropriate element of a matrix `m` and returns a reference to the matrix changed. Operator throws `cvmexception` in case of different matrix sizes. Operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix m1(a, 3, 2);
    rmatrix m2(3, 2);

    m2 = m1;
    std::cout << m2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+000 4.00e+000
2.00e+000 5.00e+000
3.00e+000 6.00e+000
```

2.6.13 assign (const TR*)

Function

```
rmatrix& rmatrix::assign (const rvector& v) throw (cvmexception);
rmatrix& rmatrix::assign (const TR* pD);
```

sets every element of a calling matrix to be equal to appropriate element of vector `v` or array pointed to by `pD` and returns a reference to the matrix changed. In first version it's assumed that vector passed is long enough to fill calling matrix. Function throws `cvmexception` otherwise. Function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
const double a[] = {1., 2., 3., 4., 5., 6.};
rmatrix m(2, 3);

m.assign(a);
std::cout << m;
```

prints

```
1.00e+000 3.00e+000 5.00e+000
2.00e+000 4.00e+000 6.00e+000
```

2.6.14 assign (int, int, const rmatrix&)

Function

```
rmatrix& rmatrix::assign (int nRow, int nCol, const rmatrix& m)
throw (cvmexception);
```

sets sub-matrix of a calling matrix beginning with ℓ-based row `nRow` and column `nCol` to a matrix `m` and returns a reference to the matrix changed. Function throws `cvmexception` if `nRow` or `nCol` are not positive or matrix `m` doesn't fit. Function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;

rmatrix m1(4,5);
rmatrix m2(2,2);
m1.set(1.);
m2.set(2.);
m1.assign(2,3,m2);
std::cout << m1;
```

prints

```
1 1 1 1 1
1 1 2 2 1
1 1 2 2 1
1 1 1 1 1
```

2.6.15 set (TR)

Function

```
rmatrix& rmatrix::set (TR x);
```

sets every element of a calling matrix to be equal to parameter `x` and returns a reference to the matrix changed. Use `vanish` to set every element of a calling matrix to be equal to zero. Function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
rmatrix m(2, 3);

m.set(3.);
std::cout << m;
```

prints

```
3.00e+000 3.00e+000 3.00e+000
3.00e+000 3.00e+000 3.00e+000
```

2.6.16 resize

Function

```
rmatrix& rmatrix::resize (int nNewM, int nNewN)
throw (cvmexception);
```

changes size of a calling matrix to `nNewM` by `nNewN` and returns a reference to the matrix changed. In case of increasing of its size, the matrix is filled up with zeroes. Function throws `cvmexception` in case of negative size passed or memory allocation failure. Function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    rmatrix m(a, 2, 3);
    std::cout << m << std::endl;
    m.resize (2, 2);
    std::cout << m << std::endl;
    m.resize (3, 3);
    std::cout << m;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+000 3.00e+000 5.00e+000
2.00e+000 4.00e+000 6.00e+000

1.00e+000 3.00e+000
2.00e+000 4.00e+000

1.00e+000 3.00e+000 0.00e+000
2.00e+000 4.00e+000 0.00e+000
0.00e+000 0.00e+000 0.00e+000
```

2.6.17 `operator ==`

Operator

```
bool rmatrix::operator == (const rmatrix& m) const;
```

compares calling matrix with a matrix `m` and returns `true` if they have the same sizes and their appropriate elements differ by not more than the smallest normalized positive number. Returns `false` otherwise. Operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
double a[] = {1., 2., 3., 4.};
rmatrix m1(a, 2, 2);
rmatrix m2(2, 2);

m2(1,1) = 1.; m2(1,2) = 3.;
m2(2,1) = 2.; m2(2,2) = 4.;

std::cout << (m1 == m2) << std::endl;
```

prints

```
1
```

2.6.18 operator !=

Operator

```
bool rmatrix::operator != (const rmatrix& m) const;
```

compares calling matrix with a matrix `m` and returns `true` if they have different sizes or at least one of their appropriate elements differs by more than the smallest normalized positive number. Returns `false` otherwise. Operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
double a[] = {1., 2., 3., 4.};
rmatrix m1(a, 2, 2);
rmatrix m2(2, 2);

m2(1,1) = 1.; m2(1,2) = 3.;
m2(2,1) = 2.; m2(2,2) = 4.;

std::cout << (m1 != m2) << std::endl;
```

prints

```
0
```

2.6.19 operator <<

Operator

```
rmatrix& rmatrix::operator << (const rmatrix& m)
throw (cvmexception);
```

destroys calling matrix, creates new one as a copy of `m` and returns a reference to the matrix changed. Operator throws `cvmexception` in case of memory allocation failure. Operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    rmatrix m(3,4);
    rmatrix mc(1,1);
    m(1,2) = 1.;
    m(3,4) = 2.;
    std::cout << m << mc << std::endl;

    mc << m;
    std::cout << mc;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
0.00e+000 1.00e+000 0.00e+000 0.00e+000
0.00e+000 0.00e+000 0.00e+000 0.00e+000
0.00e+000 0.00e+000 0.00e+000 2.00e+000
0.00e+000

0.00e+000 1.00e+000 0.00e+000 0.00e+000
0.00e+000 0.00e+000 0.00e+000 0.00e+000
0.00e+000 0.00e+000 0.00e+000 2.00e+000
```

2.6.20 operator +

Operator

```
rmatrix rmatrix::operator + (const rmatrix& m) const
throw (cvmexception);
```

creates an object of type `rmatrix` as a sum of a calling matrix and a matrix `m`. Operator throws `cvmexception` in case of different sizes of the operands. Operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::sum`, `rmatrix`. Example:

```
using namespace cvm;
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix ma(a,2,3);
    rmatrix mb(2,3);
    mb.set(1.);

    std::cout << ma + mb << std::endl;
    std::cout << ma + ma;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
2 4 6
3 5 7

2 6 10
4 8 12
```

2.6.21 operator -

Operator

```
rmatrix rmatrix::operator - (const rmatrix& m) const
throw (cvmexception);
```

creates an object of type `rmatrix` as a difference of a calling matrix and a matrix `m`. It throws `cvmexception` in case of different sizes of the operands. Operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::diff`, `rmatrix`. Example:

```
using namespace cvm;
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix ma(a,2,3);
    rmatrix mb(2,3);
    mb.set(1.);

    std::cout << ma - mb << std::endl;
    std::cout << ma - ma;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
0 2 4
1 3 5

0 0 0
0 0 0
```

2.6.22 sum

Function

```
rmatrix& rmatrix::sum (const rmatrix& m1, const rmatrix& m2)
throw (cvmexception);
```

assigns the result of addition of matrices `m1` and `m2` to a calling matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. Function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::operator +` , `rmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix m1(a, 2, 3);
    rmatrix m2(2, 3);
    rmatrix m(2, 3);
    m2.set(1.);

    std::cout << m.sum(m1, m2) << std::endl;
    std::cout << m.sum(m, m2);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
2 4 6
3 5 7

3 5 7
4 6 8
```

2.6.23 diff

Function

```
rmatrix& rmatrix::diff (const rmatrix& m1, const rmatrix& m2)
throw (cvmexception);
```

assigns the result of subtraction of matrices m1 and m2 to a calling matrix and returns a reference to the matrix changed. It throws cvmexception in case of different sizes of the operands. Function is *redefined* in the classes srmatrix, srbmatrix and srsmatrix. See also rmatrix::operator - , rmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const rmatrix m1(a, 2, 3);
    rmatrix m2(2, 3);
    rmatrix m(2, 3);
    m2.set(1.);

    std::cout << m.diff(m1, m2) << std::endl;
    std::cout << m.diff(m, m2);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
0 2 4
1 3 5

-1 1 3
0 2 4
```

2.6.24 operator +=

Operator

```
rmatrix& rmatrix::operator += (const rmatrix& m) throw (cvmexception);
```

adds matrix `m` to a calling matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. Operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::operator +` , `rmatrix::sum`, `rmatrix`. Example:

```
using namespace cvm;
try {
    rmatrix m1(2, 3);
    rmatrix m2(2, 3);
    m1.set(1.);
    m2.set(2.);

    m1 += m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 += m2;
    std::cout << m2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3 3
3 3 3

4 4 4
4 4 4
```

2.6.25 operator -=

Operator

`rmatrix& rmatrix::operator -= (const rmatrix& m) throw (cvmexception);`

subtracts matrix `m` from calling matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. Operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::operator -` , `rmatrix::diff`, `rmatrix`. Example:

```
using namespace cvm;
try {
    rmatrix m1(2, 3);
    rmatrix m2(2, 3);
    m1.set(1.);
    m2.set(2.);

    m1 -= m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 -= m2;
    std::cout << m2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-1 -1 -1
-1 -1 -1

0 0 0
0 0 0
```

2.6.26 operator - ()

Operator

`rmatrix rmatrix::operator - () const throw (cvmexception);`

creates an object of type `rmatrix` as a calling matrix multiplied by -1. Operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.};
const rmatrix ma(a, 2, 3);

std::cout << - ma;
```

prints

```
-1 -3 -5
-2 -4 -6
```

2.6.27 operator * (TR)

Operator

```
rmatrix rmatrix::operator * (TR d) const;
```

creates an object of type `rmatrix` as a product of a calling matrix and number `d`. Operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::operator *=` , `rmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.};
rmatrix m(a, 2, 3);

std::cout << m * 2.;
```

prints

```
2 6 10
4 8 12
```

2.6.28 operator / (TR)

Operator

```
rmatrix rmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `rmatrix` as a quotient of a calling matrix and number `d`. It throws `cvmexception` if `d` has absolute value equal or less than the smallest normalized positive number. Operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::operator /=` , `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    rmatrix m(a, 2, 3);

    std::cout << m / 2.;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
5.00e-01 1.50e+00 2.50e+00
1.00e+00 2.00e+00 3.00e+00
```

2.6.29 operator *= (TR)

Operator

```
rmatrix& rmatrix::operator *= (TR d);
```

multiplies calling matrix by number `d` and returns a reference to the matrix changed. Operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`, `rmatrix::operator *`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.};
rmatrix m(a, 2, 3);

m *= 2.;
std::cout << m;
```

prints

```
2 6 10
4 8 12
```

2.6.30 operator /= (TR)

Operator

`rmatrix& rmatrix::operator /= (TR d) throw (cvmexception);`

divides calling matrix by number `d` and returns a reference to the matrix changed. It throws `cvmexception` if `d` has absolute value equal or less than the smallest normalized positive number. Operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix::operator /` , `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    rmatrix m(a, 2, 3);

    m /= 2.;
    std::cout << m;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
5.00e-01 1.50e+00 2.50e+00
1.00e+00 2.00e+00 3.00e+00
```

2.6.31 normalize

Function

```
rmatrix& rmatrix::normalize ();
```

normalizes calling matrix so its Euclidean norm becomes equal to 1 if it was greater than the smallest normalized positive number before the call (otherwise the function does nothing). Function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 2., 3., 4., 5., 6.};
rmatrix m(a, 2, 3);

m.normalize();
std::cout << m;
```

prints

```
1.05e-01 3.14e-01 5.24e-01
2.10e-01 4.19e-01 6.29e-01
```

2.6.32 transposition

Operator and functions

```
rmatrix rmatrix::operator ~ () const throw (cvmexception);
rmatrix& rmatrix::transpose (const rmatrix& m) throw (cvmexception);
rmatrix& rmatrix::transpose () throw (cvmexception);
```

implement matrix transposition. First operator creates an object of type `rmatrix` as transposed calling matrix (it throws `cvmexception` in case of memory allocation failure). Second function sets calling matrix to be equal to matrix `m` transposed (it throws `cvmexception` in case of not appropriate sizes of the operands), third one makes it to be equal to transposed itself (it also throws `cvmexception` in case of memory allocation failure). Functions are *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    rmatrix m(a,2,3);
    rmatrix mt(3,2);
    std::cout << m << std::endl << ~m << std::endl ;
    mt.transpose(m);
    std::cout << mt << std::endl;
    mt.transpose();
    std::cout << mt;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 3 5
2 4 6

1 2
3 4
5 6

1 2
3 4
5 6

1 3 5
2 4 6
```

2.6.33 operator * (const rvector&)

Operator

```
rvector rmatrix::operator * (const rvector& v) const
throw (cvmexception);
```

creates an object of type `rvector` as a product of a calling matrix and a vector `v`. It throws `cvmexception` if the number of columns of the calling matrix differs from size of the vector `v`. Use `rvector::mult` to avoid new object creation. Function is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`, `rvector`. Example:

```
using namespace cvm;

try {
    rmatrix m(2, 3);
    rvector v(3);
    m.set(1.);
    v.set(1.);

    std::cout << m * v;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3
```

2.6.34 operator * (const rmatrix&)

Operator

```
rmatrix rmatrix::operator * (const rmatrix& m) const
throw (cvmexception);
```

creates an object of type `rmatrix` as a product of a calling matrix and a matrix `m`. It throws `cvmexception` if number of columns of a calling matrix differs from number of rows of the matrix `m`. Use `rmatrix::mult` to avoid new object creation. Operator is *redefined* in the classes `srmatrix`, `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;

try {
    rmatrix m1(2, 3);
    rmatrix m2(3, 2);
    m1.set(1.);
    m2.set(1.);

    std::cout << m1 * m2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3
3 3
```

2.6.35 mult

Function

```
rmatrix& rmatrix::mult (const rmatrix& m1, const rmatrix& m2)
throw (cvmexception);
```

sets calling matrix to be equal to product of matrix `m1` by matrix `m2` and returns a reference to the matrix changed. Function throws `cvmexception` in case of inappropriate sizes of the operands. Function is *inherited* in the class `srmatrix` and *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;

try {
    rmatrix m1(2, 3);
    rmatrix m2(3, 2);
    rmatrix m(2, 2);
    m1.set(1.);
    m2.set(1.);

    std::cout << m.mult(m1, m2) << std::endl;
    std::cout << m1.mult(m, m1);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3
3 3

6 6 6
6 6 6
```

2.6.36 rank1update

Function

```
rmatrix&
rmatrix::rank1update (const rvector& vCol, const rvector& vRow)
throw (cvmexception);
```

sets calling matrix to be equal to rank-1 update of vectors `vCol` and `vRow` and returns a reference to the matrix changed. Function throws `cvmexception` if number of rows of a calling matrix is not equal to `vCol.size()` or number of columns of a calling matrix is not equal to `vRow.size()`. Function is *inherited* in the class `srmatrix` and *not applicable* to objects of the classes `srbmatrix` and `srsmatrix` (i.e. exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also rvector::rank1update, `rmatrix`. Example:

```
using namespace cvm;

try {
    rvector vc(3), vr(2);
    rmatrix m(3, 2);
    vc(1) = 1.;
    vc(2) = 2.;
    vc(3) = 3.;
    vr(1) = 4.;
    vr(2) = 5.;

    std::cout << m.rank1update (vc, vr);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
4 5
8 10
12 15
```

2.6.37 swap_rows

Function

```
rmatrix& rmatrix::swap_rows (int n1, int n2) throw (cvmexception);
```

swaps two rows of a calling matrix and returns a reference to the matrix changed. `n1` and `n2` are numbers of rows to be swapped, both are ℓ-based). Function throws `cvmexception` if one of the parameters is outside of the range `[1,msize()]`. Function is *redefined* in the class `srmatrix` and *not applicable* to objects of the classes `srbmatrix` and `srsmatrix` (i.e. exception of type `cvmexception` would be thrown in case of using it for objects of those classes). See also `rmatrix`. Example:

```
using namespace cvm;
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    rmatrix m (a, 3, 2);

    std::cout << m << std::endl;
    std::cout << m.swap_rows(2,3);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 4
2 5
3 6

1 4
3 6
2 5
```

2.6.38 swap_cols

Function

```
rmatrix& rmatrix::swap_cols (int n1, int n2) throw (cvmexception);
```

swaps two columns of a calling matrix and returns a reference to the matrix changed. n1 and n2 are numbers of columns to be swapped, both are ℓ-based). Function throws cvmexception if one of the parameters is outside of the range [1,nsize()]. Function is *redefined* in the class srmatrix and *not applicable* to objects of the classes srbmatrix and srsmatrix (i.e. exception of type cvmexception would be thrown in case of using it for objects of those classes). See also rmatrix. Example:

```
using namespace cvm;
try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    rmatrix m (a, 2, 3);

    std::cout << m << std::endl;
    std::cout << m.swap_cols(2,3);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 3 5
2 4 6

1 5 3
2 6 4
```

2.6.39 solve

Functions

```
rmatrix&
rmatrix::solve (const srmatrix& mA,
                const rmatrix& mB, TR& dErr) throw (cvmexception);
rmatrix&
rmatrix::solve (const srmatrix& mA,
                const rmatrix& vB) throw (cvmexception);
```

set calling matrix to be equal to solution X of matrix linear equation $A * X = B$ where parameter `mA` is square matrix A and parameter `vB` is matrix B. Every function returns a reference to the matrix changed. The first version also sets output parameter `dErr` to be equal to the norm of computation error. Functions throw `cvmexception` in case of inappropriate sizes of the operands or when the matrix A is close to singular. Functions are *redefined* in the class `srmatrix` and *inherited* thereafter in the classes `srbmatrix` and `srsmatrix`. See also `rvector::solve`, `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (2);
try {
    srmatrix ma(3);
    rmatrix  mb(3,4);
    rmatrix  mx(3,4);
    double dErr;
    ma.randomize(-10., 10.);
    mb.randomize(-10., 10.);

    mx.solve (ma, mb, dErr);

    std::cout << ma * mx - mb << dErr << std::endl;
}
catch (cvmexception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-1.78e-015 +1.80e-016 -8.88e-016 +0.00e+000
+0.00e+000 -1.33e-015 +8.88e-016 +0.00e+000
+0.00e+000 +0.00e+000 -1.78e-015 +0.00e+000
+2.02e-015
```

2.6.40 solve_tran

Functions

```
rmatrix&
rmatrix::solve_tran (const srmatrix& mA,
                     const rmatrix& mB, TR& dErr) throw (cvmexception);
rmatrix&
rmatrix::solve_tran (const srmatrix& mA,
                     const rmatrix& vB) throw (cvmexception);
```

set calling matrix to be equal to solution X of matrix linear equation $A^T * X = B$ (which is equivalent to $X^T * A = B^T$) where parameter `mA` is square matrix A and parameter `vB` is matrix B. Every function returns a reference to the matrix changed. The first version also sets output parameter `dErr` to be equal to the norm of computation error. Functions throw `cvmexception` in case of inappropriate sizes of the operands or when the matrix A is close to singular. Functions are *redefined* in the class `srmatrix` and *inherited* thereafter in the classes `srbmatrix` and `srsmatrix`. See also `rmatrix::solve`, `rvector::solve_tran`, `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (3);

srmatrix ma(3);
rmatrix  mb(3,4);
rmatrix  mx(3,4);
double dErr;
ma.randomize(-10., 10.);
mb.randomize(-10., 10.);

mx.solve_tran (ma, mb, dErr);

std::cout << ~ma * mx - mb << dErr << std::endl;
std::cout << ~mx * ma - ~mb;
```

prints

```
-1.776e-015 -8.882e-016 -3.553e-015 +4.441e-016
+0.000e+000 +3.553e-015 +1.243e-014 -8.882e-016
+0.000e+000 +3.553e-015 -1.066e-014 +0.000e+000
+4.357e-015
-1.776e-015 +0.000e+000 +0.000e+000
-8.882e-016 +3.553e-015 +3.553e-015
-3.553e-015 +1.243e-014 -1.066e-014
+4.441e-016 -8.882e-016 +0.000e+000
```

2.6.41 solve_lu

Functions

```
rmatrix&
rmatrix::solve_lu (const srmatrix& mA, const srmatrix& mLU,
                   const int* pPivots, const rmatrix& mB, TR& dErr)
                   throw (cvmexception);
rmatrix&
rmatrix::solve_lu (const srmatrix& mA, const srmatrix& mLU,
                   const int* pPivots, const rmatrix& mB)
                   throw (cvmexception);
```

set calling matrix to be equal to solution X of matrix linear equation $A * X = B$ where parameter `mA` is square matrix A, parameter `mLU` is LU factorization of the matrix A, parameter `pPivots` is array of pivot numbers created while factorizing the matrix A and parameter `mB` is the matrix B. Every function returns a reference to the matrix changed. The first version also sets output parameter `dErr` to be equal to norm of computation error. These functions are useful when you need to solve few linear equations of kind $AX = B$ with the same matrix A and different matrices B. In such case you save on matrix A factorization since it's needed to be performed just one time. Functions throw `cvmexception` in case of inappropriate sizes of the operands or when the matrix A is close to singular. See also `rvector::solve`, `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., -1., 1., 2., -2., 1., 3., -2., 1.};
    srmatrix ma(a,3);
    srmatrix mLU(3);
    rmatrix  mb1(3,2);
    rmatrix  mb2(3,2);
    rmatrix  mx1(3,2);
    rmatrix  mx2(3,2);
    iarray   nPivots(3);
    double   dErr = 0.;
    mb1.randomize(-1.,3.);
    mb2.randomize(2.,5.);

    mLU.low_up(ma, nPivots);
    std::cout << mx1.solve_lu (ma, mLU, nPivots, mb1, dErr);
    std::cout << dErr << std::endl;
    std::cout << mx2.solve_lu (ma, mLU, nPivots, mb2) << std::endl;
    std::cout << ma * mx1 - mb1 << std::endl << ma * mx2 - mb2;
}
```

```
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3.85e+00 5.90e-01
-4.23e+00 -3.67e+00
2.10e+00 2.55e+00
7.04e-15
9.49e+00 8.93e+00
-1.00e+01 -1.42e+01
4.21e+00 7.55e+00

0.00e+00 0.00e+00
0.00e+00 0.00e+00
4.44e-16 -1.11e-16

4.44e-16 0.00e+00
-4.44e-16 0.00e+00
8.88e-16 0.00e+00
```

2.6.42 svd

Functions

```
rvector
rmatrix::svd () const throw (cvmexception);
rvector
rmatrix::svd (srmatrix& mU, srmatrix& mVH) const throw (cvmexception);
```

create object of type `rvector` as a vector of singular values of a calling matrix The second version sets output parameter `mU` to be equal to a matrix U of size $m \times m$ and `mVH` to be equal to a matrix V^H of size $n \times n$. Functions throw `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. Use `rvector::svd` in order to avoid new vector creation. Function is *redefined* in the classes `srmatrix`, `srbmatrix`, `srsmatrix`. See also `rvector`, `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double m[] = {1., -1., 1., 2., -2., 1.,
                  3., -2., 1., 0., -2., 1.};
    rmatrix mA(m,4,3);
    rmatrix mSigma(4,3);
    rvector v;
    srmatrix mU(4), mVH(3);

    v << mA.svd(mU, mVH);
    mSigma.diag(0) = v;

    std::cout << mU << std::endl;
    std::cout << mVH << std::endl;
    std::cout << mSigma << std::endl;

    std::cout << (mA * ~mVH - mU * mSigma).norm() << std::endl;
    std::cout << (~mA * mU - ~(mSigma * mVH)).norm() << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-4.84e-01 1.95e-01 1.15e-02 -8.53e-01
2.17e-01 -3.41e-01 -8.89e-01 -2.13e-01
6.62e-01 7.16e-01 -6.18e-02 -2.13e-01
-5.29e-01 5.78e-01 -4.53e-01 4.26e-01
```

```
-2.21e-01 8.54e-01 -4.72e-01
9.59e-01 1.04e-01 -2.62e-01
-1.75e-01 -5.11e-01 -8.42e-01

4.96e+00 0.00e+00 0.00e+00
0.00e+00 2.51e+00 0.00e+00
0.00e+00 0.00e+00 3.77e-01
0.00e+00 0.00e+00 0.00e+00

1.37e-15
2.48e-15
```

2.6.43 pinv

Functions

```
rmatrix  rmatrix::pinv (TR threshold = cvmMachSp()) const
                        throw (cvmexception);
rmatrix& rmatrix::pinv (const basic_rmatrix& mA,
                        TR threshold = cvmMachSp())
                        throw (cvmexception);
```

implement matrix pseudo inversion [6], p. 33 (or Moore-Penrose generalized inversion [5], p. 421). Strictly defined, $n \times m$ matrix A^+ is *pseudo inversion* of $m \times n$ matrix A if the following two equations are satisfied:

$$AA^+A = A,$$
$$A^+ = QA^H = A^H P$$

where Q and P are some matrices. To compute the pseudo inversion, we use Singular Value Decomposition (SVD)

$$A = U\Sigma V^H$$

of a matrix A, thus

$$A^+ = V\Sigma^{-1}U^H,$$

where Σ^{-1} is diagonal $n \times m$ matrix having inverted diagonal values of a matrix Σ if they are greater than some threshold, and zeros otherwise.

First version creates an object of type `rmatrix` as pseudo inverted calling matrix (it throws `cvmexception` in case of memory allocation failure). Second function sets calling matrix to be equal to matrix `mA` pseudo inverted (it throws `cvmexception` in case of not appropriate sizes of the operands). The threshold parameter sets the minimum distinguishable from zero singular value to be used to compute the pseudo inversion. All values equal or less than the threshold are treated as zeros. Functions are *inherited* in the classes `srmatrix` and `srsmatrix` and *redefined* in `srbmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;

try {
    rmatrix mA(3,4);
    mA(1,1) =  1.; mA(1,2) = -1.; mA(1,3) =  2.; mA(1,4) =  0.;
    mA(2,1) = -1.; mA(2,2) =  2.; mA(2,3) = -3.; mA(2,4) =  1.;
    mA(3,1) =  0.; mA(3,2) =  1.; mA(3,3) = -1.; mA(3,4) =  1.;
    rmatrix mX = mA.pinv(1.e-13);
    std::cout << mX << (mA * mX * mA - mA).norm2() << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+3.3333333e-001 -2.1510571e-016 +3.3333333e-001
+1.1111111e-001 +1.1111111e-001 +2.2222222e-001
+2.2222222e-001 -1.1111111e-001 +1.1111111e-001
+4.4444444e-001 +1.1111111e-001 +5.5555556e-001
+2.5460202e-015
```

Band matrix example:

```
using namespace cvm;

try {
    srbmatrix mA (40, 1, 0);
    mA.diag(0).randomize(-1.,1.);
    mA.diag(-1).randomize(5.,10.);

    srmatrix mX (40);
    mX.pinv(mA);
    std::cout << (mA * mX * mA - mA).norm2() << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+8.1956952e-14
```

2.6.44 gels

Functions

```
rmatrix  rmatrix::gels (bool transpose, const rmatrix& mB,
                        rvector& vErr) const throw (cvmexception);
rmatrix& rmatrix::gels (bool transpose,
                        const rmatrix& mA, const rmatrix& mB,
                        rvector& vErr) throw (cvmexception);
rvector  rmatrix::gels (bool transpose, const rvector& mB,
                        TR& dErr) const throw (cvmexception);
```

solve overdetermined or underdetermined linear systems

$$A * x = b$$

for $m \times n$ matrix A (or its transpose) where b is a vector of length k or systems

$$A * X = B$$

for multiple vectors b stored as columns of $k \times l$ matrix B where $k = m$ in non-transposed case and $k = n$ otherwise. The algorithm uses QR or LQ factorization of A. It is assumed that A has full rank, infinity returned otherwise. Internally functions use `?GELS` LAPACK routines. If $m > n$ and `transpose=false` or $m < n$ and `transpose=true`, then the system is overdetermined, thus the algorithm tries to find the least squares solution x of the problem

$$\|A * x - b\|_2 \to \min \quad \text{or} \quad \|A^H * x - b\|_2 \to \min$$

respectively. Output vector `vErr` of length l (or real number `dErr` for single vector b) returns residual sum of squares. The system is underdetermined otherwise, and the algorithm finds its minimum norm solution. In this case `vErr` (or `bErr`) is set to zero. In both cases the solution computed satisfies $x = \mathrm{pinv}(A) * b$, but this algorithm is faster than pseudo inversion. Matrix A is passed as argument `mA` or (in second case) it's a calling object. First and third versions return the solution as new matrix or vector object. Second version sets calling object to be the solution. Functions throw `cvmexception` in case of inappropriate sizes of the operands. Functions are *inherited* in all real matrix classes. See also `rmatrix`, `rvector::gels`, `rmatrix::pinv`. Example:

```
using namespace cvm;

rmatrix a(7, 5);
rmatrix b(7, 2);
rmatrix bt(5, 2);
rvector bv(5);
rvector vErr(2);
treal dErr;
a.randomize(-10., 10.);
b.randomize(-10., 10.);
```

```
bt.randomize(-10., 10.);
bv.randomize(-10., 10.);

rmatrix x = a.gels(false, b, vErr);
rvector vt = a.gels(true, bv, dErr);
rmatrix xt(7,2);
xt.gels(true, a, bt, vErr);

std::cout << (a.pinv()*b - x).norm() << std::endl;
std::cout << (~a.pinv()*bv - vt).norm() << std::endl;
std::cout << (~a.pinv()*bt - xt).norm() << std::endl;
```

prints

```
+2.6295387e-015
+5.8515615e-015
+6.6462869e-015
```

2.6.45 gelsy

Functions

```
rmatrix  rmatrix::gelsy (const rmatrix& mB, int& rank,
                         TR tol = cvmMachSp()) const throw (cvmexception);
rmatrix& rmatrix::gelsy (const rmatrix& mA, const rmatrix& mB, int& rank,
                         TR tol = cvmMachSp()) throw (cvmexception);
rvector  rmatrix::gelsy (const rvector& mB, int& rank,
                         TR tol = cvmMachSp()) const throw (cvmexception);
```

compute the minimum-norm solution to linear least squares problem

$$\|A * x - b\|_2 \rightarrow \min$$

using complete orthogonal factorization of $m \times n$ matrix A. Here b is a vector of length m. Multiple vectors b can be stored as columns of $m \times l$ matrix B. Matrix A may be rank-deficient, functions return its effective rank in `rank` output parameter using `tol` tolerance. Internally functions use `?GELSY` LAPACK routines, see more details about the algorithm in those routines' documentation. Matrix A is passed as argument `mA` or (in second case) it's a calling object. First and third versions return the solution as new matrix or vector object. Second version sets calling object to be the solution. Functions throw `cvmexception` in case of inappropriate sizes of the operands. Functions are *inherited* in all real matrix classes. See also `rmatrix`, `rvector::gelsy`, `rmatrix::gelss`, `rmatrix::gelsd`. Example:

```
using namespace cvm;

rmatrix a(4, 5);
rmatrix b(4, 2);
rvector bv(4);
a.randomize(-10., 10.);
b.randomize(-10., 10.);
bv.randomize(-10., 10.);
tint rank;

rmatrix x = a.gelsy(b, rank);
rvector xv = a.gelsy(bv, rank);
rmatrix x2(5,2);
x2.gelsy(a, b, rank);

std::cout << (a*x - b).norm() << std::endl;
std::cout << (a*xv - bv).norm() << std::endl;
std::cout << (a*x2 - b).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+3.3539762e-014
+5.2545344e-015
+3.3539762e-014
+4 +4
```

2.6.46 gelss

Functions

```
rmatrix  rmatrix::gelss (const rmatrix& mB, int& rank, rvector& sv,
                         TR tol = cvmMachSp()) const throw (cvmexception);
rmatrix& rmatrix::gelss (const rmatrix& mA, const rmatrix& mB,
                         rvector& sv, int& rank, TR tol = cvmMachSp())
                         throw (cvmexception);
rvector  rmatrix::gelss (const rvector& mB, rvector& sv, int& rank,
                         TR tol = cvmMachSp()) const throw (cvmexception);
```

compute minimum-norm solution to linear least squares problem

$$\|A * x - b\|_2 \to \min$$

using singular value decomposition of $m \times n$ matrix A. Here b is a vector of length m. Multiple vectors b can be stored as columns of $m \times l$ matrix B. Matrix A may be rank-deficient, functions return its effective rank in `rank` output parameter using `tol` tolerance. These functions also compute singular values of A in decreasing order and return them in `sv` output parameter having $\min(m, n)$ size. Internally functions use `?GELSS` LAPACK routines, see more details about the algorithm in those routines' documentation. Matrix A is passed as argument `mA` or (in second case) it's a calling object. First and third versions return the solution as new matrix or vector object. Second version sets calling object to be the solution. Functions throw `cvmexception` in case of inappropriate sizes of the operands. Functions are *inherited* in all real matrix classes. See also `rmatrix`, `rvector::gelss`, `rmatrix::gelsy`, `rmatrix::gelsd`. Example:

```
using namespace cvm;

rmatrix a(4, 5);
rmatrix b(4, 2);
rvector bv(4);
rvector sv(4);
tint rank;
a.randomize(-10., 10.);
b.randomize(-10., 10.);
bv.randomize(-10., 10.);

rmatrix x = a.gelss(b, sv, rank);
rvector xv = a.gelss(bv, sv, rank);
rmatrix xt(5,2);
xt.gelss(a, b, sv, rank);

std::cout << (a*x - b).norm() << std::endl;
std::cout << (a*xv - bv).norm() << std::endl;
std::cout << (a*xt - b).norm() << std::endl;
```

```
std::cout << (sv - a.svd()).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+1.9169279e-014
+8.9260826e-015
+1.9169279e-014
+8.1402897e-015
+4 +4
```

2.6.47 gelsd

Functions

```
rmatrix  rmatrix::gelsd (const rmatrix& mB, int& rank, rvector& sv,
                         TR tol = cvmMachSp()) const throw (cvmexception);
rmatrix& rmatrix::gelsd (const rmatrix& mA, const rmatrix& mB,
                         rvector& sv, int& rank, TR tol = cvmMachSp())
                         throw (cvmexception);
rvector  rmatrix::gelsd (const rvector& mB, rvector& sv, int& rank,
                         TR tol = cvmMachSp()) const throw (cvmexception);
```

compute minimum-norm solution to linear least squares problem

$$\|A * x - b\|_2 \to \min$$

using singular value decomposition of $m \times n$ matrix A and divide and conquer method. Here b is a vector of length m. Multiple vectors b can be stored as columns of $m \times l$ matrix B. Matrix A may be rank-deficient, functions return its effective rank in `rank` output parameter using `tol` tolerance. These functions also compute singular values of A in decreasing order and return them in `sv` output parameter having $\min(m, n)$ size. Internally functions use `?GELSD` LAPACK routines, see more details about the algorithm in those routines' documentation. Matrix A is passed as argument `mA` or (in second case) it's a calling object. First and third versions return the solution as new matrix or vector object. Second version sets calling object to be the solution. Functions throw `cvmexception` in case of inappropriate sizes of the operands. Functions are *inherited* in all real matrix classes. See also `rmatrix`, `rvector::gelsd`, `rmatrix::gelsy`, `rmatrix::gelss`. Example:

```
using namespace cvm;

rmatrix a(4, 5);
rmatrix b(4, 2);
rvector bv(4);
rvector sv(4);
tint rank;
a.randomize(-10., 10.);
b.randomize(-10., 10.);
bv.randomize(-10., 10.);

rmatrix x = a.gelsd(b, sv, rank);
rvector xv = a.gelsd(bv, sv, rank);
rmatrix xt(5,2);
xt.gelsd(a, b, sv, rank);

std::cout << (a*x - b).norm() << std::endl;
std::cout << (a*xv - bv).norm() << std::endl;
std::cout << (a*xt - b).norm() << std::endl;
```

```
std::cout << (sv - a.svd()).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+1.0341178e-014
+6.0443462e-015
+1.0341178e-014
+8.9260826e-015
+4 +4
```

2.6.48 rank

Function

`int rmatrix::rank (TR eps = cvmMachSp ()) const throw (cvmexception);`

returns rank of a calling matrix as number of singular values with normalized absolute value greater than or equal to parameter `eps` (this is the largest relative spacing by default). Function throws `cvmexception` in case of convergence error. Function is *inherited* in the classes `srmatrix`, `srbmatrix`, `srsmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;
try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    rmatrix m(a,3,4);

    std::cout << m << m.rank() << std::endl;
    m(3,4) = 13.;
    std::cout << m.rank() << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 4 7 10
2 5 8 11
3 6 9 12
2
3
```

2.6.49 ger

Function

```
rmatrix&
rmatrix::ger (TR dAlpha, const rvector& vCol, const rvector& vRow)
throw (cvmexception);
```

calls one of ?GER routines of the BLAS library performing rank-1 update matrix-vector operation defined as

$$M = \alpha \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} \begin{pmatrix} y_1 & y_2 & \cdots & y_n \end{pmatrix} + M,$$

where α is real number (parameter `dAlpha`), M is calling matrix and x and y are real vectors (parameters `vCol` and `vRow` respectively). Function returns a reference to the matrix changed and throws `cvmexception` in case of inappropriate sizes of the operands. Function is *inherited* in the class `srmatrix` and *not applicable* to objects of the classes `srbmatrix` and `srsmatrix` (i.e. `cvmexception` would be thrown in case of using it for objects of those classes). See also `rvector`, `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (4);
try {
    double alpha = 1.3;
    rmatrix m(3,4);
    rvector vc(3);
    rvector vr(4);
    m.randomize(-1., 2.); vc.randomize(-1., 3.); vr.randomize(0., 2.);

    std::cout << m + vc.rank1update (vr) * alpha << std::endl;
    std::cout << m.ger(alpha, vc, vr);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-1.7127e-01 2.9410e+00 1.3449e+00 3.6055e+00
1.9057e+00 2.6726e+00 1.7134e+00 2.2154e+00
1.7217e-01 1.3508e+00 8.8949e-01 2.2551e+00

-1.7127e-01 2.9410e+00 1.3449e+00 3.6055e+00
1.9057e+00 2.6726e+00 1.7134e+00 2.2154e+00
1.7217e-01 1.3508e+00 8.8949e-01 2.2551e+00
```

2.6.50 gemm

Function

```
rmatrix& rmatrix::gemm (const rmatrix& m1, bool bTrans1,
                        const rmatrix& m2, bool bTrans2,
                        TR dAlpha, TR dBeta) throw (cvmexception);
```

calls one of ?GEMM routines of the BLAS library performing matrix-matrix operation defined as

$$M = \alpha \mathcal{T}(M_1) \cdot \mathcal{T}(M_2) + \beta M,$$

where α and β are real numbers (parameters dAlpha and dBeta), M is calling matrix and M_1 and M_2 are matrices (parameters m1 and m2 respectively). Function $\mathcal{T}(M_i)$ transposes matrix M_i if appropriate boolean parameter bTrans* is equal to true and does nothing otherwise. Function returns a reference to the matrix changed and throws cvmexception in case of inappropriate sizes of the operands. Function is *inherited* in the class srmatrix and *not applicable* to objects of the classes srbmatrix and srsmatrix (i.e. exception of type cvmexception would be thrown in case of using it for objects of those classes). See also rmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (4);
try {
    double alpha = 1.3;
    double beta = -0.7;
    rmatrix m1(4,3); rmatrix m2(4,3);
    rmatrix m(3,3);
    m.randomize(-1., 2.); m1.randomize(-1., 3.); m2.randomize(0., 2.);
    std::cout << ~m1 * m2 * alpha + m * beta << std::endl;
    std::cout << m.gemm(m1, true, m2, false, alpha, beta);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
5.0504e+00 6.8736e+00 3.1171e+00
2.3915e+00 2.2544e+00 3.9205e+00
3.4607e+00 3.5351e+00 4.8622e+00

5.0504e+00 6.8736e+00 3.1171e+00
2.3915e+00 2.2544e+00 3.9205e+00
3.4607e+00 3.5351e+00 4.8622e+00
```

2.6.51 symm

Function

```
rmatrix& rmatrix::symm (bool bLeft, const srsmatrix& ms,
                        const rmatrix& m, TR dAlpha, TR dBeta)
                        throw (cvmexception);
```

calls one of ?SYMM routines of the BLAS library performing one of matrix-matrix operations defined as

$$M = \alpha M_s \cdot M_1 + \beta M \quad \text{or} \quad M = \alpha M_1 \cdot M_s + \beta M,$$

where α and β are real numbers (parameters dAlpha and dBeta), M is calling matrix, M_s is symmetric matrix and M_1 is real matrix (parameters ms and m respectively). First operation is performed if bLeft passed is true and second one otherwise. Function returns a reference to the matrix changed and throws cvmexception in case of inappropriate sizes of the operands. Function is *inherited* in the classes srmatrix and srsmatrix and *not applicable* to objects of the class srbmatrix (i.e. cvmexception would be thrown in case of using it for objects of that class). See also srsmatrix, rmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (4);
try {
    double alpha = 1.3;
    double beta = -0.7;
    rmatrix m1(3,4);
    rmatrix m2(4,3);
    srsmatrix ms(3);
    rmatrix m(3,4);
    m.randomize(-1., 2.); m1.randomize(-1., 3.); m2.randomize(0., 2.);
    ms.randomize(-3., 1.);

    std::cout << ms * m1 * alpha + m * beta << std::endl;
    std::cout << m.symm (true, ms, m1, alpha, beta) << std::endl;

    m.resize(4,3);
    std::cout << m2 * ms * alpha + m * beta << std::endl;
    std::cout << m.symm (false, ms, m2, alpha, beta);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-3.3733e+00 -5.0566e+00 -6.3018e+00 -5.4907e+00
-1.8629e+00 -1.5133e+00 -1.1372e+00 -2.5557e+00
```

```
-3.5695e+00 -1.0012e+01 -1.4239e+00 -6.1786e-01

-3.3733e+00 -5.0566e+00 -6.3018e+00 -5.4907e+00
-1.8629e+00 -1.5133e+00 -1.1372e+00 -2.5557e+00
-3.5695e+00 -1.0012e+01 -1.4239e+00 -6.1786e-01

-6.4072e+00 7.0534e-01 1.5349e+00
-4.8219e+00 -6.9891e+00 -5.1766e+00
6.8503e-01 3.5828e+00 -3.2174e+00
2.3469e-01 -9.3921e-01 -2.1961e+00

-6.4072e+00 7.0534e-01 1.5349e+00
-4.8219e+00 -6.9891e+00 -5.1766e+00
6.8503e-01 3.5828e+00 -3.2174e+00
2.3469e-01 -9.3921e-01 -2.1961e+00
```

2.6.52 qr

Functions

```
void rmatrix::qr (rmatrix& mQ, srmatrix& mR) const throw (cvmexception);
void rmatrix::qr (srmatrix& mQ, rmatrix& mR) const throw (cvmexception);
```

compute QR factorization as

$$M = QR$$

where M is calling matrix, orthogonal matrix Q and upper triangular (trapezoidal) matrix R are `mQ` and `mR` respectively. First version implements so-called "economy" algorithm which for given $m \times n$ matrix M computes $m \times n$ matrix Q and $n \times n$ matrix R. Second version is "full" mode one computing $m \times m$ matrix Q and $m \times n$ matrix R. Functions throw `cvmexception` in case if inappropriate sizes of the operands passed. Functions are *redefined* in the class `srmatrix`. See also `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;

treal a[] = {1., 2., 3., 4., 5., 6.};
const cvm::rmatrix mh(a, 2, 3);
const cvm::rmatrix mv(a, 3, 2);
cvm::srmatrix s2(2), s3(3);
cvm::rmatrix  h(2,3), v(3,2);

mh.qr(h,s3);
std::cout << (eye_real(2) - ~rmatrix(h,1,1,2,2)*rmatrix(h,1,1,2,2)).norm()
          << " " << (mh - h * s3).norm() << std::endl;
mh.qr(s2,h);
std::cout << (eye_real(2) - ~s2 * s2).norm()
          << " " << (mh - s2 * h).norm() << std::endl;
mv.qr(v,s2);
std::cout << (eye_real(2) - ~v * v).norm()
          << " " << (mv - v * s2).norm() << std::endl;
mv.qr(s3,v);
std::cout << (eye_real(3) - ~s3 * s3).norm()
          << " " << (mv - s3 * v).norm() << std::endl;
```

prints

```
+4.6933177e-16 +2.2342807e-15
+4.6933177e-16 +2.2342807e-15
+5.1302953e-16 +1.4432899e-15
+5.2889959e-16 +1.4432899e-15
```

2.6.53 lq

Functions

```
void rmatrix::lq (srmatrix& mL, rmatrix& mQ) const throw (cvmexception);
void rmatrix::lq (rmatrix& mL, srmatrix& mQ) const throw (cvmexception);
```

compute LQ factorization as

$$M = LQ$$

where M is calling matrix, lower triangular (trapezoidal) matrix L and orthogonal matrix Q are `mL` and `mQ` respectively. First version implements so-called "economy" algorithm which for given $m \times n$ matrix M computes $m \times m$ matrix L and $m \times n$ matrix Q. Second version is "full" mode one computing $m \times n$ matrix L and $n \times n$ matrix Q. Functions throw `cvmexception` in case of inappropriate sizes of the operands passed. Functions are *redefined* in the class `srmatrix`. See also `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;

treal a[] = {1., 2., 3., 4., 5., 6.};
const cvm::rmatrix mh(a, 2, 3);
const cvm::rmatrix mv(a, 3, 2);
cvm::srmatrix s2(2), s3(3);
cvm::rmatrix h(2,3), v(3,2);

mh.lq(s2,h);
std::cout << (eye_real(2) - h * ~h).norm() << " " << (mh - s2 * h)
             .norm() << std::endl;
mv.lq(s3,v);
std::cout << (eye_real(2) - ~rmatrix(v,1,1,2,2) * rmatrix(v,1,1,2,2))
             .norm() << " " << (mv - s3 * v).norm() << std::endl;
mh.lq(h,s3);
std::cout << (eye_real(3) - s3 * ~s3).norm() << " " << (mh - h * s3)
             .norm() << std::endl;
mv.lq(v,s2);
std::cout << (eye_real(2) - s2 * ~s2).norm() << " " << (mv - v * s2)
             .norm() << std::endl;
```

prints

```
+8.6355085e-016 +3.3893638e-015
+1.9229627e-016 +1.6011864e-015
+1.0030483e-015 +3.3893638e-015
+1.9229627e-016 +1.6011864e-015
```

2.6.54 rq

Functions

```
void rmatrix::rq (srmatrix& mR, rmatrix& mQ) const throw (cvmexception);
void rmatrix::rq (rmatrix& mR, srmatrix& mQ) const throw (cvmexception);
```

compute RQ factorization as

$$M = RQ$$

where M is calling matrix, upper triangular matrix R and orthogonal matrix Q are `mR` and `mQ` respectively. First version implements so-called "economy" algorithm which for given $m \times n$ matrix M computes $m \times m$ matrix R and $m \times n$ matrix Q. Second version is "full" mode one computing $m \times n$ matrix R and $n \times n$ matrix Q. Following this definition the implementation assumes that $m \leqslant n$ and throws `cvmexception` otherwise. Functions also throw `cvmexception` in case of inappropriate sizes of the operands passed. Functions are *redefined* in the class `srmatrix`. See also `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;

treal a[] = {1., 2., 3., 4., 5., 6.};
const cvm::rmatrix mh(a, 2, 3);
const cvm::rmatrix mv(a, 3, 2);
cvm::srmatrix s2(2), s3(3);
cvm::rmatrix  h(2,3), v(3,2);

mh.rq(h,s3);
std::cout << (eye_real(2) - rmatrix(s3,2,1,2,3) * ~rmatrix(s3,2,1,2,3))
             .norm() << " " << (mh - h * s3).norm() << std::endl;

mh.rq(s2,h);
std::cout << (eye_real(2) - h * ~h).norm()
          << " " << (mh - s2 * h).norm() << std::endl;
```

prints

```
+5.5511151e-016 +2.0471501e-015
+5.5511151e-016 +2.0471501e-015
```

2.6.55 ql

Functions

```
void rmatrix::ql (rmatrix& mQ, srmatrix& mL) const throw (cvmexception);
void rmatrix::ql (srmatrix& mQ, rmatrix& mL) const throw (cvmexception);
```

compute QL factorization as

$$M = QL$$

where M is calling matrix, orthogonal matrix Q and lower triangular matrix L are `mQ` and `mL` respectively. First version implements so-called "economy" algorithm which for given $m \times n$ matrix M computes $m \times n$ matrix Q and $n \times n$ matrix L. Second version is "full" mode one computing $m \times m$ matrix Q and $m \times n$ matrix L. Following this definition the implementation assumes that $m \geqslant n$ and throws `cvmexception` otherwise. Functions also throw `cvmexception` in case of inappropriate sizes of the operands passed. Functions are *redefined* in the class `srmatrix`. See also `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;

treal a[] = {1., 2., 3., 4., 5., 6.};
const cvm::rmatrix mv(a, 3, 2);
cvm::srmatrix s2(2), s3(3);
cvm::rmatrix v(3,2);

mv.ql(v,s2);
std::cout << (eye_real(2) - ~v * v).norm()
          << " " << (mv - v * s2).norm() << std::endl;

mv.ql(s3,v);
std::cout << (eye_real(2) - ~rmatrix(s3,1,2,3,2) * rmatrix(s3,1,2,3,2))
             .norm() << " " << (mv - s3 * v).norm() << std::endl;
```

prints

```
+0.0000000e+000 +2.0947646e-015
+0.0000000e+000 +2.0947646e-015
```

2.6.56 vanish

Function

```
rmatrix& rmatrix::vanish();
```

sets every element of a calling matrix to be equal to zero and returns a reference to the matrix changed. This function is faster than, for example, `rmatrix::set(TR)` with zero operand passed. Function is *redefined* in the classes `srmatrix`, `srsmatrix`, `srbmatrix`. See also `rmatrix`. Example:

```
using namespace cvm;

rmatrix m(3, 4);
m.randomize(0.,1.);

std::cout << m << std::endl;
std::cout << m.vanish ();
```

prints

```
0.856532 0.938261 0.275704 0.186834
0.651173 0.812159 0.100467 0.536912
0.0726646 0.695914 0.661824 0.554613

0 0 0 0
0 0 0 0
0 0 0 0
```

2.6.57 randomize

Function

```
rmatrix& rmatrix::randomize (TR dFrom, TR dTo);
```

fills calling matrix with pseudo-random numbers distributed between `dFrom` and `dTo`. Function returns a reference to the matrix changed. See also `rmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (7);

rmatrix m(3,4);
m.randomize(-2.,3.);
std::cout << m;
```

prints

```
9.6853542e-01 2.7761467e+00 2.3791009e+00 -3.4452345e-01
2.9029511e+00 -9.5519883e-01 -4.9131748e-01 -1.2561113e+00
1.5219886e+00 -1.4494461e+00 2.8193304e+00 4.8817408e-01
```

2.7 cmatrix

This is end-user class encapsulating matrix of complex numbers.

```
template <typename TR, typename TC>
class cmatrix : public Matrix <TR,TC> {
public:
    cmatrix ();
    cmatrix (int nM, int nN);
    cmatrix (TC* pD, int nM, int nN);
    cmatrix (const TC* pD, int nM, int nN);
    cmatrix (const cmatrix& m);
    explicit cmatrix (const cvector& v, bool bBeColumn = true);
    explicit cmatrix (const rmatrix& m, bool bRealPart = true);
    cmatrix (const TR* pRe, const TR* pIm, int nM, int nN);
    cmatrix (const rmatrix& mRe, const rmatrix& mIm);
    cmatrix (cmatrix& m, int nRow, int nCol, int nHeight, int nWidth);
    TC& operator () (int im, int in) throw (cvmexception);
    TC operator () (int im, int in) const throw (cvmexception);
    cvector operator () (int i) throw (cvmexception);
    const cvector operator () (int i) const throw (cvmexception);
    cvector operator [] (int i) throw (cvmexception);
    const cvector operator [] (int i) const throw (cvmexception);
    cvector diag (int i) throw (cvmexception);
    const cvector diag (int i) const throw (cvmexception);
    const rmatrix real () const;
    const rmatrix imag () const;
    cmatrix& operator = (const cmatrix& m) throw (cvmexception);
    cmatrix& assign (const cvector& v) throw (cvmexception);
    cmatrix& assign (const TC* pD);
    rmatrix& assign (int nRow, int nCol, const cmatrix& m)
                     throw (cvmexception);
    cmatrix& set (TC z);
    cmatrix& assign_real (const rmatrix& mRe) throw (cvmexception);
    cmatrix& assign_imag (const rmatrix& mIm) throw (cvmexception);
    cmatrix& set_real (TR d);
    cmatrix& set_imag (TR d);
    cmatrix& resize (int nNewM, int nNewN) throw (cvmexception);
    bool operator == (const cmatrix& v) const;
    bool operator != (const cmatrix& v) const;
    cmatrix& operator << (const cmatrix& m) throw (cvmexception);
    cmatrix operator + (const cmatrix& m) const throw (cvmexception);
    cmatrix operator - (const cmatrix& m) const throw (cvmexception);
    cmatrix& sum (const cmatrix& m1,
                  const cmatrix& m2) throw (cvmexception);
```

```
cmatrix& diff (const cmatrix& m1,
               const cmatrix& m2) throw (cvmexception);
cmatrix& operator += (const cmatrix& m) throw (cvmexception);
cmatrix& operator -= (const cmatrix& m) throw (cvmexception);
cmatrix operator - () const;
cmatrix operator * (TR d) const;
cmatrix operator / (TR d) const throw (cvmexception);
cmatrix operator * (TC z) const;
cmatrix operator / (TC z) const throw (cvmexception);
cmatrix& operator *= (TR d);
cmatrix& operator /= (TR d) throw (cvmexception);
cmatrix& operator *= (TC z);
cmatrix& operator /= (TC z) throw (cvmexception);
cmatrix& normalize ();
cmatrix operator ~() const throw (cvmexception);
cmatrix operator !() const throw (cvmexception);
cmatrix& conj (const cmatrix& m) throw (cvmexception);
cmatrix& conj () throw (cvmexception);
cmatrix& transpose (const cmatrix& m) throw (cvmexception);
cmatrix& transpose () throw (cvmexception);
cvector operator * (const cvector& v) const
                    throw (cvmexception);
cmatrix operator * (const cmatrix& m) const
                    throw (cvmexception);
cmatrix& mult (const cmatrix& m1, const cmatrix& m2)
               throw (cvmexception);
cmatrix& rank1update_u (const rvector& vCol,
                        const rvector& vRow) throw (cvmexception);
cmatrix& rank1update_c (const rvector& vCol,
                        const rvector& vRow) throw (cvmexception);
cmatrix& swap_rows (int n1, int n2) throw (cvmexception);
cmatrix& swap_cols (int n1, int n2) throw (cvmexception);
cmatrix& solve (const scmatrix& mA,
                const cmatrix& mB, TR& dErr)
                throw (cvmexception);
cmatrix& solve (const scmatrix& mA,
                const cmatrix& mB) throw (cvmexception);
cmatrix& solve_tran (const scmatrix& mA,
                     const cmatrix& mB, TR& dErr)
                     throw (cvmexception);
cmatrix& solve_tran (const scmatrix& mA,
                     const cmatrix& mB) throw (cvmexception);
cmatrix& solve_conj (const scmatrix& mA,
                     const cmatrix& mB, TR& dErr)
                     throw (cvmexception);
```

```
cmatrix& solve_conj (const scmatrix& mA,
                     const cmatrix& mB) throw (cvmexception);
cmatrix& solve_lu (const scmatrix& mA, const scmatrix& mLU,
                   const int* pPivots, const cmatrix& mB, TR& dErr)
                   throw (cvmexception);
cmatrix& solve_lu (const scmatrix& mA, const scmatrix& mLU,
                   const int* pPivots, const cmatrix& mB)
                   throw (cvmexception);
rvector svd () throw (cvmexception);
rvector svd (cmatrix& mLSingVect, cmatrix& mRSingVect)
             throw (cvmexception);
cmatrix pinv (TR threshold = cvmMachSp()) const
              throw (cvmexception);
cmatrix& pinv (const cmatrix& mA,
               TR threshold = cvmMachSp())
               throw (cvmexception);
int rank (TR eps = cvmMachSp()) const throw (cvmexception);
cmatrix gels (bool conjugate, const cmatrix& mB,
              cvector& vErr) const throw (cvmexception);
cmatrix& gels (bool conjugate, const cmatrix& mA, const cmatrix& mB,
               cvector& vErr) throw (cvmexception);
cvector gels (bool conjugate, const cvector& mB,
               TC& dErr) const throw (cvmexception);
cmatrix gelsy (const cmatrix& mB, int& rank,
               TR tol = cvmMachSp()) const throw (cvmexception);
cmatrix& gelsy (const cmatrix& mA, const cmatrix& mB, int& rank,
               TR tol = cvmMachSp()) throw (cvmexception);
cvector gelsy (const cvector& mB, int& rank,
               TR tol = cvmMachSp()) const throw (cvmexception);
rmatrix gelss (const cmatrix& mB, rvector& sv,
               int& rank, TR tol = cvmMachSp()) const
               throw (cvmexception);
rmatrix& gelss (const cmatrix& mA, const cmatrix& mB, rvector& sv,
               int& rank, TR tol = cvmMachSp())
               throw (cvmexception);
rvector gelss (const cvector& mB, rvector& sv,
               int& rank, TR tol = cvmMachSp()) const
               throw (cvmexception);
rmatrix gelsd (const cmatrix& mB, rvector& sv,
               int& rank, TR tol = cvmMachSp()) const
               throw (cvmexception);
rmatrix& gelsd (const cmatrix& mA, const cmatrix& mB, rvector& sv,
               int& rank, TR tol = cvmMachSp())
               throw (cvmexception);
rvector gelsd (const cvector& mB, rvector& sv,
```

```
                  int& rank, TR tol = cvmMachSp()) const
                  throw (cvmexception);
    void qr(cmatrix& mQ, scmatrix& mR) const throw (cvmexception);
    void qr(scmatrix& mQ, cmatrix& mR) const throw (cvmexception);
    void lq(scmatrix& mL, cmatrix& mQ) const throw (cvmexception);
    void lq(cmatrix& mL, scmatrix& mQ) const throw (cvmexception);
    void rq(scmatrix& mR, cmatrix& mQ) const throw (cvmexception);
    void rq(cmatrix& mR, scmatrix& mQ) const throw (cvmexception);
    void ql(cmatrix& mQ, scmatrix& mL) const throw (cvmexception);
    void ql(scmatrix& mQ, cmatrix& mL) const throw (cvmexception);
    cmatrix& vanish ();
    cmatrix& geru (TC alpha, const cvector& vCol,
                  const cvector& vRow) throw (cvmexception);
    cmatrix& gerc (TC alpha, const cvector& vCol,
                  const cvector& vRow) throw (cvmexception);
    cmatrix& gemm (const cmatrix& m1, bool bTrans1,
                  const cmatrix& m2, bool bTrans2,
                  TC dAlpha, TC dBeta) throw (cvmexception);
    cmatrix& hemm (bool bLeft, const schmatrix& ms, const cmatrix& m,
                  TC dAlpha, TC dBeta) throw (cvmexception);
    cmatrix& randomize_real (TR dFrom, TR dTo);
    cmatrix& randomize_imag (TR dFrom, TR dTo);
};
```

2.7.1 cmatrix ()

Constructor

```
cmatrix::cmatrix ();
```

creates empty `cmatrix` object. See also `cmatrix`. Example:

```
using namespace cvm;

cmatrix m;
std::cout << m.msize() << std::endl
          << m.nsize() << std::endl;

m << eye_complex(3);
std::cout << m;
```

prints

```
0
0
(1,0) (0,0) (0,0)
(0,0) (1,0) (0,0)
(0,0) (0,0) (1,0)
```

2.7.2 cmatrix (int,int)

Constructor

```
cmatrix::cmatrix (int nM, int nN);
```

creates $m \times n$ `cmatrix` object where m is passed in `nM` parameter (number of rows) and n is passed in `nN` (number of columns). Constructor throws `cvmexception` in case of non-positive sizes passed or memory allocation failure. See also `cmatrix`. Example:

```
using namespace cvm;

cmatrix m (3, 4);
std::cout << m.msize() << std::endl
          << m.nsize() << std::endl
          << m.size()  << std::endl << m;
```

prints

```
3
4
12
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
```

2.7.3 cmatrix (TC*,int,int)

Constructor

```
cmatrix::cmatrix (TC* pD, int nM, int nN);
```

creates $m \times n$ `cmatrix` object where m is passed in `nM` parameter (number of rows) and n is passed in `nN` (number of columns). Unlike others, this constructor *does not allocate memory*. It just shares memory with an array pointed to by `pD`. Constructor throws `cvmexception` in case of non-positive size passed. See also `cmatrix`, `cmatrix (const TC*,int,int)`. Example:

```
using namespace cvm;
double a[] = {5., 5., 5., 5., 5., 5.,
              5., 5., 5., 5., 5., 5.};
cmatrix m ((std::complex<double>*) a, 2, 3);

m(1,1) = std::complex<double>(1.,2.);
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " "
          << a[2] << " " << a[3] << std::endl;
```

prints

```
(1,2) (5,5) (5,5)
(5,5) (5,5) (5,5)

1 2 5 5
```

2.7.4 cmatrix (const TC*,int,int)

Constructor

```
cmatrix::cmatrix (const TC* pD, int nM, int nN);
```

creates $m \times n$ `cmatrix` object where m is passed in `nM` parameter (number of rows) and n is passed in `nN` (number of columns) and copies $m * n$ elements of an array `pD` to it `by colums`. Constructor throws `cvmexception` in case of non-positive size passed. See also `cmatrix`, `cmatrix (TC*,int,int)`. Example:

```
using namespace cvm;
const double a[] = {5., 5., 5., 5., 5., 5.,
                    5., 5., 5., 5., 5., 5.};
cmatrix m ((const std::complex<double>*) a, 2, 3);

m(1,1) = std::complex<double>(1.,2.);
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " "
          << a[2] << " " << a[3] << std::endl;
```

prints

```
(1,2) (5,5) (5,5)
(5,5) (5,5) (5,5)

5 5 5 5
```

2.7.5 cmatrix (const cmatrix&)

Copy constructor

```
cmatrix::cmatrix (const cmatrix& m);
```

creates `cmatrix` object as a copy of `m`. Constructor throws `cvmexception` in case of memory allocation failure. See also `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m ((std::complex<double>*) a, 2, 3);
cmatrix mc(m);

m(1,1) = std::complex<double>(7.77,7.77);
std::cout << m << std::endl << mc;
```

prints

```
(7.77,7.77) (5,6) (9,10)
(3,4) (7,8) (11,12)

(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)
```

2.7.6 cmatrix (const cvector&,bool)

Constructor

```
explicit cmatrix::cmatrix (const cvector& v, bool bBeColumn = true);
```

creates `cmatrix` object containing `v.size()` rows and 1 column if `bBeColumn` is `true` or 1 row and `v.size()` columns otherwise. After that it copies vector `v`'s elements to the matrix created. Constructor throws `cvmexception` in case of memory allocation failure. See also `cmatrix`, `cvector`. Example:

```
using namespace cvm;

cvector v(3);
v(1) = std::complex<double>(1.,2.);
v(2) = std::complex<double>(2.,3.);
v(3) = std::complex<double>(3.,4.);

cmatrix mc (v);
cmatrix mr (v, false);

std::cout << mc << std::endl << mr;
```

prints

```
(1,2)
(2,3)
(3,4)

(1,2) (2,3) (3,4)
```

2.7.7 cmatrix (const rmatrix&,bool)

Constructor

```
explicit cmatrix::cmatrix (const rmatrix& m, bool bRealPart = true);
```

creates `cmatrix` object containing `m.msize()` rows and `m.nsize()` columns and copies matrix `m` to its real part if `bRealPart` is `true` or to its imaginary part otherwise. Constructor throws `cvmexception` in case of memory allocation failure. See also `cmatrix`, `rmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.};
const rmatrix m (a, 2, 3);
cmatrix mr(m), mi(m, false);

std::cout << mr << std::endl << mi;
```

prints

```
(1,0) (3,0) (5,0)
(2,0) (4,0) (6,0)

(0,1) (0,3) (0,5)
(0,2) (0,4) (0,6)
```

2.7.8 cmatrix (const TR*,const TR*,int,int)

Constructor

```
cmatrix::cmatrix (const TR* pRe, const TR* pIm,
                  const int nM, const int nN);
```

creates `cmatrix` object of size `nM` by `nN` and copies every element of arrays pointed to by `pRe` and `pIm` to real and imaginary part of the matrix created respectively. Use `NULL` pointer to fill up appropriate part with zero values. Constructor throws `cvmexception` in case of memory allocation failure. See also `cmatrix`. Example:

```
using namespace cvm;

double re[] = {1., 2., 3., 4., 5., 6.};
double im[] = {6., 5., 4., 3., 2., 1.};
cmatrix m (re, im, 3, 2);
std::cout << m << std::endl;
re[1] = 7.77;
std::cout << m << std::endl;

const double re2[] = {1., 2., 3., 4.};
const cmatrix m2 (re2, NULL, 2, 2);
std::cout << m2;
```

prints

```
(1,6) (4,3)
(2,5) (5,2)
(3,4) (6,1)

(1,6) (4,3)
(2,5) (5,2)
(3,4) (6,1)

(1,0) (3,0)
(2,0) (4,0)
```

2.7.9 cmatrix (const rmatrix&, const rmatrix&)

Constructor

```
cmatrix::cmatrix (const rmatrix& mRe, const rmatrix& mIm);
```

creates `cmatrix` object of size `mRe.msize()` by `mRe.nsize()` (if one of these sizes differs from appropriate size of matrix `mIm` then constructor throws `cvmexception`) and copies matrices `mRe` and `mIm` to real and imaginary part of the matrix created respectively. Constructor throws `cvmexception` in case of memory allocation failure. See also `cmatrix`, `rmatrix`. Example:

```
using namespace cvm;

rmatrix mr(3,3), mi(3,3);
mr(1,1) = 1.;
mr(2,2) = 2.;
mr(3,3) = 3.;
mi(1,3) = 6.;
mi(2,2) = 5.;
mi(3,1) = 4.;

const cmatrix mc(mr, mi);
std::cout << mc;
```

prints

```
(1,0) (0,0) (0,6)
(0,0) (2,5) (0,0)
(0,4) (0,0) (3,0)
```

2.7.10 submatrix

Submatrix constructor

```
cmatrix::cmatrix (cmatrix& m, int nRow, int nCol,
                              int nHeight, int nWidth);
```

creates `cmatrix` object as *submatrix* of `m`. It means that the matrix object created shares memory with some part of `m`. This part is defined by its upper left corner (parameters `nRow` and `nCol`, both are ℓ-based) and its height and width (parameters `nHeight` and `nWidth`). See also `cmatrix`. Example:

```
using namespace cvm;

cmatrix m(4,5);
cmatrix subm(m, 2, 2, 2, 2);
subm.set(std::complex<double>(1.,1.));

std::cout << m;
```

prints

```
(0,0) (0,0) (0,0) (0,0) (0,0)
(0,0) (1,1) (1,1) (0,0) (0,0)
(0,0) (1,1) (1,1) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0) (0,0)
```

2.7.11 operator (,)

Indexing operators

```
TC& cmatrix::operator () (int im, int in) throw (cvmexception);
TC cmatrix::operator () (int im, int in) const throw (cvmexception);
```

provide access to a particular element of a calling matrix. The first version of operator is applicable to non-constant object. This version returns *l-value* in order to make possible write access to an element. Both operators are ℓ-based. Operators throw `cvmexception` if `im` is outside of `[1,msize()]` range or `in` is outside of `[1,nsize()]` range. Operators are *inherited* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix m ((std::complex<double>*) a, 2, 3);
    scmatrix ms(2);
    std::cout << m(1,1) << " "
              << m(2,3) << std::endl << std::endl;
    ms(2,2) = std::complex<double>(7.77,7.77);
    std::cout << ms;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (11,12)

(0,0) (0,0)
(0,0) (7.77,7.77)
```

2.7.12 operator ()

Indexing operators

```
cvector cmatrix::operator () (int i) throw (cvmexception);
const cvector cmatrix::operator () (int i) const throw (cvmexception);
```

provide access to i-th column of a calling matrix. The first version of operator is applicable to non-constant object and *returns l-value*, i.e. vector returned shares memory with i-th column of a calling matrix in order to make possible write access to it. The second version creates *copy* of a column and therefore is *not l-value*. Both operators are ℓ-based. The operators throw `cvmexception` if parameter `i` is outside of `[1,nsize()]` range. Operators are *inherited* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`, `Matrix::msize()`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix m ((std::complex<double>*) a, 2, 3);
    scmatrix ms(2);

    std::cout << m(2) << std::endl;

    ms(2) = m(3);
    std::cout << ms;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(5,6) (7,8)

(0,0) (9,10)
(0,0) (11,12)
```

2.7.13 operator []

Indexing operators

```
cvector cmatrix::operator [] (int i) throw (cvmexception);
const cvector cmatrix::operator [] (int i) const throw (cvmexception);
```

provide access to i-th row of a calling matrix. The first version of operator is applicable to non-constant object and *returns l-value*, i.e. the vector returned shares memory with i-th row of a calling matrix in order to make possible write access to it. The second version creates *copy* of a row and therefore is *not l-value*. Both operators are ℓ-based. The operators throw `cvmexception` if `i` is outside of `[1,msize()]` range. Operators are *inherited* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`, `Matrix::msize()`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix m ((std::complex<double>*) a, 2, 3);
    scmatrix ms(3);
    std::cout << m[1] << std::endl;
    ms[1] = m[2];
    std::cout << ms;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (5,6) (9,10)

(3,4) (7,8) (11,12)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.7.14 diag

Functions

```
cvector cmatrix::diag (int i) throw (cvmexception);
const cvector cmatrix::diag (int i) const throw (cvmexception);
```

provide access to i-th diagonal of a calling matrix, where $i = 0$ for main diagonal, $i < 0$ for lower diagonals and $i > 0$ for upper ones. The first version of function is applicable to non-constant object and *returns l-value*, i.e. the vector returned shares memory with i-th diagonal of a calling matrix in order to make possible write access to it. The second version creates *copy* of a diagonal and therefore is *not l-value*. Functions throw `cvmexception` if parameter `i` is outside of `[-msize()+1,nsize()-1]` range. Functions are *inherited* in the classes `scmatrix` and `scbmatrix`. Functions are *redefined* in the class `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                  10., 11., 12., 13., 14., 15., 16., 17., 18.};
    cmatrix m (2, 3);
    const scmatrix ms((std::complex<double>*)a, 3);
    m.diag(-1).set(std::complex<double>(1.,1.));
    m.diag(0).set(std::complex<double>(2.,2.));
    m.diag(1).set(std::complex<double>(3.,3.));
    m.diag(2).set(std::complex<double>(4.,4.));
    std::cout << m << std::endl;
    std::cout << ms << std::endl;
    std::cout << ms.diag(0) << ms.diag(1);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(2,2) (3,3) (4,4)
(1,1) (2,2) (3,3)

(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)

(1,2) (9,10) (17,18)
(7,8) (15,16)
```

2.7.15 real

Function

```
const rmatrix cmatrix::real () const;
```

creates an object of type `const rmatrix` as real part of a calling matrix. Please note that, unlike cvector::real, this function creates new object *not sharing* memory with real part of a calling matrix, i.e. the matrix returned is *not l-value*. Function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `rmatrix`, `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m((std::complex<double>*) a, 2, 3);

std::cout << m << std::endl << m.real();
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)

1 5 9
3 7 11
```

2.7.16 imag

Function

```
const rmatrix cmatrix::imag () const;
```

creates an object of type `const rmatrix` as imaginary part of a calling matrix. Please note that, unlike cvector::imag, this function creates new object *not sharing* memory with imaginary part of a calling matrix, i.e. the matrix returned is *not l-value*. Function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `rmatrix`, `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m((std::complex<double>*) a, 2, 3);

std::cout << m << std::endl << m.imag();
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)

2 6 10
4 8 12
```

2.7.17 operator = (const cmatrix&)

Operator

`cmatrix& cmatrix::operator = (const cmatrix& m) throw (cvmexception);`

sets every element of a calling rmatrix to be equal to appropriate element of a matrix `m` and returns a reference to the matrix changed. Operator throws `cvmexception` in case of different matrix sizes. Operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix m1((std::complex<double>*) a, 2, 3);
    cmatrix m2(2, 3);

    m2 = m1;
    std::cout << m2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)
```

2.7.18 assign (const TC*)

Function

```
cmatrix& cmatrix::assign (const cvector& v) throw (cvmexception);
cmatrix& cmatrix::assign (const TC* pD);
```

sets every element of a calling matrix to a value of appropriate element of a vector `v` or array pointed to by `pD` and returns a reference to the matrix changed. In first version it's assumed that vector passed is long enough to fill calling matrix. Function throws `cvmexception` otherwise. Function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

const double a[] = {1., 2., 3., 4., 5., 6.,
                    7., 8., 9., 10., 11., 12.};
cmatrix m(2, 3);

m.assign ((const std::complex<double>*) a);
std::cout << m;
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)
```

2.7.19 assign (int, int, const cmatrix&)

Function

```
cmatrix& cmatrix::assign (int nRow, int nCol, const cmatrix& m)
throw (cvmexception);
```

sets sub-matrix of a calling matrix beginning with ℓ-based row `nRow` and column `nCol` to a matrix `m` and returns a reference to the matrix changed. Function throws `cvmexception` if `nRow` or `nCol` are not positive or matrix `m` doesn't fit. Function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

rmatrix m1(4,5);
rmatrix m2(2,2);
m1.set(1.);
m2.set(2.);
m1.assign(2,3,m2);
std::cout << m1;
```

prints

```
(1,1) (1,1) (1,1) (1,1) (1,1)
(1,1) (1,1) (2,2) (2,2) (1,1)
(1,1) (1,1) (2,2) (2,2) (1,1)
(1,1) (1,1) (1,1) (1,1) (1,1)
```

2.7.20 set (TC)

Function

```
cmatrix& cmatrix::set (TC z);
```

sets every element of a calling matrix to a value of parameter z and returns a reference to the matrix changed. Use vanish to set every element of a calling matrix to be equal to zero. Function is *redefined* in the classes scmatrix, scbmatrix and schmatrix. See also cmatrix. Example:

```
using namespace cvm;

cmatrix m(3, 4);
m.set(std::complex<double>(3.,4.));
std::cout << m;
```

prints

```
(3,4) (3,4) (3,4) (3,4)
(3,4) (3,4) (3,4) (3,4)
(3,4) (3,4) (3,4) (3,4)
```

2.7.21 assign_real

Function

```
cmatrix& cmatrix::assign_real (const rmatrix& mRe) throw (cvmexception);
```

sets real part of every element of a calling matrix to a value of appropriate element of a matrix `mRe` and returns a reference to the matrix changed. Function throws `cvmexception` in case of different sizes of the operands. See also `cmatrix` and `rmatrix`. Function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. Example:

```
using namespace cvm;

rmatrix m (2,3);
cmatrix mc(2,3);
m.randomize (0., 1.);

mc.assign_real(m);
std::cout << mc;
```

prints

```
(0.126835,0) (0.57271,0) (0.28312,0)
(0.784417,0) (0.541673,0) (0.663869,0)
```

2.7.22 assign_imag

Function

```
cmatrix& cmatrix::assign_imag (const rmatrix& mIm) throw (cvmexception);
```

sets imaginary part of every element of a calling matrix to a value of appropriate element of a matrix `mIm` and returns a reference to the matrix changed. Function throws `cvmexception` in case of different sizes of the operands. Function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix` and `rmatrix`. Example:

```
using namespace cvm;

rmatrix m (2,3);
cmatrix mc(2,3);
m.randomize (0., 1.);

mc.assign_imag(m);
std::cout << mc;
```

prints

```
(0,0.13831) (0,0.267373) (0,0.482345)
(0,0.50618) (0,0.992401) (0,0.444777)
```

2.7.23 set_real

Function

```
cmatrix& cmatrix::set_real (TR d);
```

sets real part of every element of a calling matrix to a value of parameter d and returns a reference to the matrix changed. See also cmatrix. Function is *redefined* in the classes scmatrix, scbmatrix and schmatrix. Example:

```
using namespace cvm;

cmatrix m(2,3);
m.set_real(1.);
std::cout << m;
```

prints

```
(1,0) (1,0) (1,0)
(1,0) (1,0) (1,0)
```

2.7.24 set_imag

Function

```
cmatrix& cmatrix::set_imag (TR d);
```

sets imaginary part of every element of a calling matrix to a value of parameter d and returns a reference to the matrix changed. See also cmatrix. Function is *redefined* in the classes scmatrix, scbmatrix and schmatrix. Example:

```
using namespace cvm;

cmatrix m(2,3);
m.set_imag(1.);
std::cout << m;
```

prints

```
(0,1) (0,1) (0,1)
(0,1) (0,1) (0,1)
```

2.7.25 resize

Function

```
cmatrix& cmatrix::resize (int nNewM, int nNewN) throw (cvmexception);
```

changes size of a calling matrix to `nNewM` by `nNewN` and returns a reference to the matrix changed. In case of increasing of its size, the matrix is filled up with zeroes. Function throws `cvmexception` in case of negative size passed or memory allocation failure. Function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    cmatrix m((std::complex<double>*) a, 2, 3);
    std::cout << m << std::endl;
    m.resize (2, 2);
    std::cout << m << std::endl;
    m.resize (3, 3);
    std::cout << m;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)

(1,2) (5,6)
(3,4) (7,8)

(1,2) (5,6) (0,0)
(3,4) (7,8) (0,0)
(0,0) (0,0) (0,0)
```

2.7.26 operator ==

Operator

```
bool cmatrix::operator == (const cmatrix& m) const;
```

compares calling matrix with a matrix `m` and returns `true` if they have the same sizes and their appropriate elements differ by not more than the smallest normalized positive number. Returns `false` otherwise. Operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

cmatrix m1(2, 3);
cmatrix m2(2, 3);
m1.set_real(1.);
m2.set_real(1.);
std::cout << (m1 == m2) << std::endl;
```

prints

```
1
```

2.7.27 operator !=

Operator

```
bool cmatrix::operator != (const cmatrix& m) const;
```

compares calling matrix with a matrix `m` and returns `true` if they have different sizes or at least of their appropriate elements differs by more than the smallest normalized positive number. Returns `false` otherwise. Operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

cmatrix m1(2, 3);
cmatrix m2(2, 3);
m1.set_real(1.);
m2.set_real(1.);
std::cout << (m1 != m2) << std::endl;
```

prints

```
0
```

2.7.28 operator <<

Operator

```
cmatrix& cmatrix::operator << (const cmatrix& m) throw (cvmexception);
```

destroys calling matrix, creates new one as a copy of m and returns a reference to the matrix changed. Operator throws cvmexception in case of memory allocation failure. Oerator is *redefined* in the classes scmatrix, scbmatrix and schmatrix. See also cmatrix. Example:

```
using namespace cvm;

try {
    cmatrix m(2,3);
    cmatrix mc(1,1);
    m(1,2) = std::complex<double>(1.,2.);
    m(2,3) = std::complex<double>(2.,4.);
    std::cout << m << mc << std::endl;

    mc << m;
    std::cout << mc;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,0) (1,2) (0,0)
(0,0) (0,0) (2,4)
(0,0)

(0,0) (1,2) (0,0)
(0,0) (0,0) (2,4)
```

2.7.29 operator +

Operator

```
cmatrix cmatrix::operator + (const cmatrix& m) const
throw (cvmexception);
```

creates an object of type `cmatrix` as a sum of a calling matrix and a matrix `m`. Operator throws `cvmexception` in case of different sizes of the operands. Operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cvector::sum`, `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix ma ((std::complex<double>*) a, 2, 3);
    cmatrix mb (2, 3);
    mb.set (std::complex<double>(1.,1.));

    std::cout << ma + mb << std::endl;
    std::cout << ma + ma;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(2,3) (6,7) (10,11)
(4,5) (8,9) (12,13)

(2,4) (10,12) (18,20)
(6,8) (14,16) (22,24)
```

2.7.30 operator -

Operator

```
cmatrix cmatrix::operator - (const cmatrix& m) const
throw (cvmexception);
```

creates an object of type `cmatrix` as a difference of a calling matrix and a matrix `m`. It throws `cvmexception` in case of different sizes of the operands. Operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cvector::diff`, `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix ma ((std::complex<double>*) a, 2, 3);
    cmatrix mb (2, 3);
    mb.set (std::complex<double>(1.,1.));

    std::cout << ma - mb << std::endl;
    std::cout << ma - ma;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,1) (4,5) (8,9)
(2,3) (6,7) (10,11)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.7.31 sum

Function

```
cmatrix& cmatrix::sum (const cmatrix& m1, const cmatrix& m2)
throw (cvmexception);
```

assigns the result of addition of matrices `m1` and `m2` to a calling matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. Function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator +` , `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix ma ((std::complex<double>*) a, 2, 3);
    cmatrix mb (2, 3);
    cmatrix m (2, 3);
    mb.set (std::complex<double>(1.,1.));

    std::cout << m.sum(ma, mb) << std::endl;
    std::cout << m.sum(m, mb);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(2,3) (6,7) (10,11)
(4,5) (8,9) (12,13)

(3,4) (7,8) (11,12)
(5,6) (9,10) (13,14)
```

2.7.32 diff

Function

```
cmatrix& cmatrix::diff (const cmatrix& m1, const cmatrix& m2)
throw (cvmexception);
```

assigns the result of subtraction of matrices `m1` and `m2` to a calling matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. Function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator -` , `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix ma ((std::complex<double>*) a, 2, 3);
    cmatrix mb (2, 3);
    cmatrix m (2, 3);
    mb.set (std::complex<double>(1.,1.));

    std::cout << m.diff(ma, mb) << std::endl;
    std::cout << m.diff(m, mb);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,1) (4,5) (8,9)
(2,3) (6,7) (10,11)

(-1,0) (3,4) (7,8)
(1,2) (5,6) (9,10)
```

2.7.33 operator +=

Operator

```
cmatrix& cmatrix::operator += (const cmatrix& m) throw (cvmexception);
```

adds matrix `m` to a calling matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. Operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator +` , `cmatrix::sum`, `cmatrix`. Example:

```
using namespace cvm;

try {
    cmatrix m1(2, 3);
    cmatrix m2(2, 3);
    m1.set(std::complex<double>(1.,1.));
    m2.set(std::complex<double>(2.,2.));

    m1 += m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 += m2;
    std::cout << m2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(3,3) (3,3) (3,3)
(3,3) (3,3) (3,3)

(4,4) (4,4) (4,4)
(4,4) (4,4) (4,4)
```

2.7.34 operator -=

Operator

```
cmatrix& cmatrix::operator -= (const cmatrix& m) throw (cvmexception);
```

subtracts matrix `m` from calling matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. Operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator -` , `cmatrix::diff`, `cmatrix`. Example:

```
using namespace cvm;

try {
    cmatrix m1(2, 3);
    cmatrix m2(2, 3);
    m1.set(std::complex<double>(1.,1.));
    m2.set(std::complex<double>(2.,2.));

    m1 -= m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 -= m2;
    std::cout << m2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-1,-1) (-1,-1) (-1,-1)
(-1,-1) (-1,-1) (-1,-1)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.7.35 operator - ()

Operator

```
cmatrix cmatrix::operator - () const throw (cvmexception);
```

creates an object of type `cmatrix` as a calling matrix multiplied by -1. Operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
const cmatrix ma ((std::complex<double>*) a, 2, 3);

std::cout << - ma;
```

prints

```
(-1,-2) (-5,-6) (-9,-10)
(-3,-4) (-7,-8) (-11,-12)
```

2.7.36 operator * (TR)

Operator

```
cmatrix cmatrix::operator * (TR d) const;
```

creates an object of type cmatrix as a product of a calling matrix and real number d. Operator is *redefined* in the classes scmatrix, scbmatrix and schmatrix. See also cmatrix, cmatrix::operator *= . Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
const cmatrix ma ((std::complex<double>*) a, 2, 3);

std::cout << ma * 5.;
```

prints

```
(5,10) (25,30) (45,50)
(15,20) (35,40) (55,60)
```

2.7.37 operator / (TR)

Operator

```
cmatrix cmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `cmatrix` as a quotient of a calling matrix and real number `d`. It throws `cvmexception` if `d` has absolute value equal or less than the smallest normalized positive number. Operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator /=` , `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix ma ((std::complex<double>*) a, 2, 3);

    std::cout << ma / 4.;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0.25,0.5) (1.25,1.5) (2.25,2.5)
(0.75,1) (1.75,2) (2.75,3)
```

2.7.38 operator * (TC)

Operator

```
cmatrix cmatrix::operator * (TC z) const;
```

creates an object of type `cmatrix` as a product of a calling matrix and complex number `z`. Operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`, `cmatrix::operator *=` . Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
const cmatrix ma ((std::complex<double>*) a, 2, 3);

std::cout << ma * std::complex<double>(5.,2.);
```

prints

```
(1,12) (13,40) (25,68)
(7,26) (19,54) (31,82)
```

2.7.39 operator / (TC)

Operator

```
cmatrix cmatrix::operator / (TC z) const throw (cvmexception);
```

creates an object of type `cmatrix` as a quotient of a calling matrix and complex number `z`. It throws `cvmexception` if `z` has absolute value equal or less than the smallest normalized positive number. Operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator /=` , `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    const cmatrix ma ((std::complex<double>*) a, 2, 3);

    std::cout << ma / std::complex<double>(4.,2.);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0.4,0.3) (1.6,0.7) (2.8,1.1)
(1,0.5) (2.2,0.9) (3.4,1.3)
```

2.7.40 operator *= (TR)

Operator

```
cmatrix& cmatrix::operator *= (TR d);
```

multiplies calling matrix by real number d and returns a reference to the matrix changed. Operator is *redefined* in the classes scmatrix, scbmatrix and schmatrix. See also cmatrix, cmatrix::operator *. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix ma ((std::complex<double>*) a, 2, 3);

ma *= 2.;
std::cout << ma;
```

prints

```
(2,4) (10,12) (18,20)
(6,8) (14,16) (22,24)
```

2.7.41 operator /= (TR)

Operator

```
cmatrix& cmatrix::operator /= (TR d) throw (cvmexception);
```

divides calling matrix by real number `d` and returns a reference to the matrix changed. It throws `cvmexception` if `d` has absolute value equal or less than the smallest normalized positive number. Operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator /` , `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    cmatrix ma ((std::complex<double>*) a, 2, 3);

    ma /= 2.;
    std::cout << ma;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0.5,1) (2.5,3) (4.5,5)
(1.5,2) (3.5,4) (5.5,6)
```

2.7.42 operator *= (TC)

Operator

```
cmatrix& cmatrix::operator *= (TC z);
```

multiplies calling matrix by complex number z and returns a reference to the matrix changed. Operator is *redefined* in the classes scmatrix, scbmatrix and schmatrix. See also cmatrix::operator * , cmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix ma ((std::complex<double>*) a, 2, 3);

ma *= std::complex<double>(2.,1.);
std::cout << ma;
```

prints

```
(0,5) (4,17) (8,29)
(2,11) (6,23) (10,35)
```

2.7.43 operator /= (TC)

Operator

```
cmatrix& cmatrix::operator /= (TC z) throw (cvmexception);
```

divides calling matrix by complex number z and returns a reference to the matrix changed. It throws `cvmexception` if z has absolute value equal or less than the smallest normalized positive number. Operator is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::operator /` , `cmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.,
                  7., 8., 9., 10., 11., 12.};
    cmatrix ma ((std::complex<double>*) a, 2, 3);

    ma /= std::complex<double>(2.,1.);
    std::cout << ma;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0.8,0.6) (3.2,1.4) (5.6,2.2)
(2,1) (4.4,1.8) (6.8,2.6)
```

2.7.44 normalize

Function

```
cmatrix& cmatrix::normalize ();
```

normalizes calling matrix so its Euclidean norm becomes equal to 1 if it was greater than the smallest normalized positive number before the call (otherwise function does nothing). Function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix ma ((std::complex<double>*) a, 2, 3);

ma.normalize();
std::cout << ma << ma.norm() << std::endl;
```

prints

```
(0.0392232,0.0784465) (0.196116,0.235339) (0.353009,0.392232)
(0.11767,0.156893) (0.274563,0.313786) (0.431455,0.470679)
1
```

2.7.45 conjugation

Operator and functions

```
cmatrix cmatrix::operator ~ () const throw (cvmexception);
cmatrix& cmatrix::conj (const cmatrix& m) throw (cvmexception);
cmatrix& cmatrix::conj () throw (cvmexception);
```

implement complex matrix conjugation. First operator creates an object of type `cmatrix` as conjugated calling matrix (it throws `cvmexception` in case of memory allocation failure). Second function sets calling matrix to be equal to matrix `m` conjugated (it throws `cvmexception` in case of not appropriate sizes of the operands), third one makes it to be equal to conjugated itself (it also throws `cvmexception` in case of memory allocation failure). Functions are *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::tramspose`, `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m((std::complex<double>*) a, 2, 3);
cmatrix mc(3,2);
std::cout << m << std::endl << ~m << std::endl ;
mc.conj(m);
std::cout << mc << std::endl;
mc.conj();
std::cout << mc;
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)

(1,-2) (3,-4)
(5,-6) (7,-8)
(9,-10) (11,-12)

(1,-2) (3,-4)
(5,-6) (7,-8)
(9,-10) (11,-12)

(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)
```

2.7.46 transposition

Operator and functions

```
cmatrix cmatrix::operator ! () const throw (cvmexception);
cmatrix& cmatrix::transpose (const cmatrix& m) throw (cvmexception);
cmatrix& cmatrix::transpose () throw (cvmexception);
```

implement complex matrix transposition (*not* conjugation). First operator creates an object of type `cmatrix` as transposed calling matrix (it throws `cvmexception` in case of memory allocation failure). Second function sets calling matrix to be equal to matrix `m` transposed (it throws `cvmexception` in case of not appropriate sizes of the operands), third one makes it to be equal to transposed itself (it also throws `cvmexception` in case of memory allocation failure). Functions are *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix::conj`, `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m((std::complex<double>*) a, 2, 3);
cmatrix mc(3,2);
std::cout << m << std::endl << !m << std::endl ;
mc.transpose(m);
std::cout << mc << std::endl;
mc.transpose();
std::cout << mc;
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)

(1,2) (3,4)
(5,6) (7,8)
(9,10) (11,12)

(1,2) (3,4)
(5,6) (7,8)
(9,10) (11,12)

(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)
```

2.7.47 operator * (const cvector&)

Operator

```
cvector cmatrix::operator * (const cvector& v) const
throw (cvmexception);
```

creates an object of type cvector as a product of a calling matrix and a vector v. It throws cvmexception if number of columns of a calling matrix differs from size of a vector v. Use cvector::mult in order to avoid of new object creation. Function is *redefined* in the classes scmatrix, scbmatrix and schmatrix. See also cmatrix, cvector. Example:

```
using namespace cvm;

cmatrix m(2, 3);
cvector v(3);
m.set(std::complex<double>(1.,1.));
v.set(std::complex<double>(1.,1.));

std::cout << m * v;
```

prints

```
(0,6) (0,6)
```

2.7.48 operator * (const cmatrix&)

Operator

```
cmatrix cmatrix::operator * (const cmatrix& m) const
throw (cvmexception);
```

creates an object of type cmatrix as a product of a calling matrix and a matrix m. It throws cvmexception if number of columns of a calling matrix differs from number of rows of a matrix m. Use cmatrix::mult in order to avoid of new object creation. Operator is *redefined* in the classes scmatrix, scbmatrix and schmatrix. See also cmatrix. Example:

```
using namespace cvm;

cmatrix m1(2, 3);
cmatrix m2(3, 2);
m1.set(std::complex<double>(1.,1.));
m2.set(std::complex<double>(1.,1.));

std::cout << m1 * m2;
```

prints

```
(0,6) (0,6)
(0,6) (0,6)
```

2.7.49 mult

Function

```
cmatrix& cmatrix::mult (const cmatrix& m1, const cmatrix& m2)
throw (cvmexception);
```

sets calling matrix to be equal to product of matrix `m1` by matrix `m2` and returns a reference to the matrix changed. Function throws `cvmexception` in case of inappropriate sizes of the operands. Function is is *inherited* in the class `scmatrix` and *redefined* in the classes `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

cmatrix m1(2, 3);
cmatrix m2(3, 2);
scmatrix m(2);
m1.set(std::complex<double>(1.,1.));
m2.set(std::complex<double>(1.,1.));

m.mult(m1, m2);
std::cout << m;
```

prints

```
(0,6) (0,6)
(0,6) (0,6)
```

2.7.50 rank1update_u

Function

```
cmatrix&
cmatrix::rank1update_u (const cvector& vCol, const cvector& vRow)
throw (cvmexception);
```

sets calling matrix to be equal to rank-1 update (uncojugated) of vectors `vCol` and `vRow` and returns a reference to the matrix changed. Function throws `cvmexception` if number of rows of a calling matrix is not equal to `vCol.size()` or number of columns is not equal to `vRow.size()`. Function is *inherited* in the class `scmatrix` and *not applicable* to objects of the classes `scbmatrix` and `schmatrix` (i.e. `cvmexception` would be thrown in case of using it for objects of those classes). See also cvector::rank1update_u, `cmatrix`. Example:

```
using namespace cvm;

cvector vc(3), vr(2);
cmatrix m(3, 2);
vc.set(std::complex<double>(1.,1.));
vr.set(std::complex<double>(1.,1.));

std::cout << m.rank1update_u (vc, vr);
```

prints

```
(0,2) (0,2)
(0,2) (0,2)
(0,2) (0,2)
```

2.7.51 rank1update_c

Function

```
cmatrix&
cmatrix::rank1update_c (const cvector& vCol, const cvector& vRow)
throw (cvmexception);
```

sets calling matrix to be equal to rank-1 update (conjugated) of vectors `vCol` and `vRow` and returns a reference to the matrix changed. Function throws `cvmexception` if number of rows of a calling matrix is not equal to `vCol.size()` or number of columns is not equal to `vRow.size()`. Function is *inherited* in the class `scmatrix` and *not applicable* to objects of the classes `scbmatrix` and `schmatrix` (i.e. `cvmexception` would be thrown in case of using it for objects of those classes). See also cvector::rank1update_c, `cmatrix`. Example:

```
using namespace cvm;

cvector vc(3), vr(2);
cmatrix m(3, 2);
vc.set(std::complex<double>(1.,1.));
vr.set(std::complex<double>(1.,1.));

std::cout << m.rank1update_c (vc, vr);
```

prints

```
(2,0) (2,0)
(2,0) (2,0)
(2,0) (2,0)
```

2.7.52 swap_rows

Function

```
cmatrix& cmatrix::swap_rows (int n1, int n2) throw (cvmexception);
```

swaps two rows of a calling matrix and returns a reference to the matrix changed. n1 and n2 are numbers of rows to be swapped, both are ℓ-based). Function throws cvmexception if one of parameters is outside of the range [1,msize()]. Function is *redefined* in the class scmatrix and *not applicable* to objects of the classes scbmatrix and schmatrix (i.e. cvmexception would be thrown in case of using it for objects of those classes). See also cmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m ((std::complex<double>*)a, 3, 2);

std::cout << m << std::endl;
std::cout << m.swap_rows(2,3);
```

prints

```
(1,2) (7,8)
(3,4) (9,10)
(5,6) (11,12)

(1,2) (7,8)
(5,6) (11,12)
(3,4) (9,10)
```

2.7.53 swap_cols

Function

```
cmatrix& cmatrix::swap_cols (int n1, int n2) throw (cvmexception);
```

swaps two columns of a calling matrix and returns a reference to the matrix changed. n1 and n2 are numbers of columns to be swapped, both are ℓ-based). Function throws cvmexception if one of parameters is outside of the range [1,nsize()]. Function is *redefined* in the class scmatrix and *not applicable* to objects of the classes scbmatrix and schmatrix (i.e. cvmexception would be thrown in case of using it for objects of those classes). See also cmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m ((std::complex<double>*)a, 2, 3);

std::cout << m << std::endl;
std::cout << m.swap_cols(2,3);
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)

(1,2) (9,10) (5,6)
(3,4) (11,12) (7,8)
```

2.7.54 solve

Functions

```
cmatrix&
cmatrix::solve (const scmatrix& mA,
                const cmatrix& mB, TR& dErr) throw (cvmexception);
cmatrix&
cmatrix::solve (const scmatrix& mA,
                const cmatrix& vB) throw (cvmexception);
```

set calling matrix to be equal to solution X of the matrix linear equation $A * X = B$ where parameter `mA` is square matrix A and parameter `vB` is matrix B. Every function returns a reference to the matrix changed. The first version also sets output parameter `dErr` to be equal to the norm of computation error. Functions throw `cvmexception` in case of inappropriate sizes of operands or when matrix A is close to singular. Functions are *redefined* in the class `scmatrix` and *inherited* thereafter in the classes `scbmatrix` and `schmatrix`. See also `cvector::solve`, `cmatrix`, `scmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (4);

scmatrix ma(3);
cmatrix  mb(3,2);
cmatrix  mx(3,2);
double dErr;
ma.randomize_real(0.,10.); ma.randomize_imag(0.,10.);
mb.randomize_real(0.,10.); mb.randomize_imag(0.,10.);

mx.solve (ma, mb, dErr);
std::cout << mx << std::endl << ma * mx - mb
          << dErr << std::endl;
```

prints

```
(+3.2103e-001,+2.8598e-002) (+1.1907e+000,-1.1101e+000)
(+3.7451e-001,+4.2471e-001) (-2.3858e-001,+3.3028e-001)
(+3.0085e-001,-3.0427e-001) (-7.6874e-001,+4.1100e-001)

(+4.4409e-016,+0.0000e+000) (-8.8818e-016,+1.3323e-015)
(+7.7716e-016,-8.8818e-016) (+0.0000e+000,+1.7764e-015)
(+0.0000e+000,-8.8818e-016) (-1.7764e-015,+8.8818e-016)
+5.4774e-015
```

2.7.55 solve_tran

Functions

```
cmatrix&
cmatrix::solve_tran (const scmatrix& mA,
                     const cmatrix& mB, TR& dErr) throw (cvmexception);
cmatrix&
cmatrix::solve_tran (const scmatrix& mA,
                     const cmatrix& vB) throw (cvmexception);
```

set calling matrix to be equal to solution X of matrix linear equation $A^T * X = B$ (which is equivalent to $X^T * A = B^T$) where parameter `mA` is square matrix A and parameter `vB` is matrix B. Every function returns a reference to the matrix changed. The first version also sets output parameter `dErr` to be equal to the norm of computation error. Functions throw `cvmexception` in case of inappropriate sizes of the operands or when the matrix A is close to singular. Functions are *redefined* in the class `scmatrix` and *inherited* thereafter in the classes `scbmatrix` and `schmatrix`. See also `cmatrix::solve`, `cvector::solve_tran`, `cmatrix`, `scmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (3);

scmatrix ma(3);
cmatrix  mb(3,2);
cmatrix  mx(3,2);
double dErr;
ma.randomize_real(0.,10.); ma.randomize_imag(0.,10.);
mb.randomize_real(0.,10.); mb.randomize_imag(0.,10.);

mx.solve_tran (ma, mb, dErr);

std::cout << !ma * mx - mb << dErr << std::endl;
std::cout << !mx * ma - !mb << std::endl;
```

prints

```
(+6.661e-016,+1.776e-015) (+0.000e+000,+5.690e-016)
(-1.776e-015,-1.776e-015) (-8.882e-016,-8.882e-016)
(-2.665e-015,+1.776e-015) (+0.000e+000,-1.110e-015)
+7.079e-015
(+6.661e-016,+1.776e-015) (-1.776e-015,-1.776e-015) (-1.776e-015,+1.776e-015)
(+0.000e+000,+5.690e-016) (-8.882e-016,-8.882e-016) (+0.000e+000,-1.554e-015)
```

2.7.56 solve_conj

Functions

```
cmatrix&
cmatrix::solve_conj (const scmatrix& mA,
                     const cmatrix& mB, TR& dErr) throw (cvmexception);
cmatrix&
cmatrix::solve_conj (const scmatrix& mA,
                     const cmatrix& vB) throw (cvmexception);
```

set calling matrix to be equal to solution X of matrix linear equation $A^H * X = B$ (which is equivalent to $X^H * A = B^H$), where A^H is conjugated A. Here parameter mA is square matrix A and parameter vB is matrix B. Every function returns a reference to the matrix changed. The first version also sets output parameter dErr to be equal to the norm of computation error. Functions throw cvmexception in case of inappropriate sizes of the operands or when the matrix A is close to singular. Functions are *redefined* in the class scmatrix and *inherited* thereafter in the classes scbmatrix and schmatrix. See also cmatrix::solve, cvector::solve_conj, cmatrix, scmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (3);

scmatrix ma(3);
cmatrix  mb(3,2);
cmatrix  mx(3,2);
double dErr;
ma.randomize_real(0.,10.); ma.randomize_imag(0.,10.);
mb.randomize_real(0.,10.); mb.randomize_imag(0.,10.);

mx.solve_conj (ma, mb, dErr);

std::cout << ~ma * mx - mb << dErr << std::endl;
std::cout << ~mx * ma - ~mb << std::endl;
```

prints

```
(+7.105e-015,-1.776e-015) (+1.332e-015,+8.882e-016)
(+1.776e-015,-8.882e-016) (+8.882e-016,-8.882e-016)
(+0.000e+000,-6.661e-016) (-1.776e-015,-2.665e-015)
+1.169e-014
(+7.105e-015,+1.776e-015) (+1.776e-015,+8.882e-016) (-2.665e-015,+6.661e-016)
(+1.332e-015,-8.882e-016) (+8.882e-016,+8.882e-016) (-1.776e-015,+2.665e-015)
```

2.7.57 solve_lu

Functions

```
cmatrix&
cmatrix::solve_lu (const scmatrix& mA, const scmatrix& mLU,
                   const int* pPivots, const cmatrix& mB, TR& dErr)
                   throw (cvmexception);
cmatrix&
cmatrix::solve_lu (const scmatrix& mA, const scmatrix& mLU,
                   const int* pPivots, const cmatrix& vB)
                   throw (cvmexception);
```

set calling matrix to be equal to solution X of the matrix linear equation $A * X = B$ where parameter `mA` is square complex matrix A, parameter `mLU` is LU factorization of a matrix A, parameter `pPivots` is array of pivot numbers created while factorizing matrix A and parameter `vB` is matrix B. Every function returns a reference to the matrix changed. The first version also sets output parameter `dErr` to be equal to the norm of computation error. These functions are useful when you need to solve few linear equations of kind $AX = B$ with the same matrix A and different matrices B. In such case you save on matrix A factorization since it's needed to be performed just one time. Functions throw `cvmexception` in case of inappropriate sizes of operands or when matrix A is close to singular. See also `cvector::solve`, `cmatrix`, `scmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (4);
try {
    scmatrix ma(3);
    scmatrix mLU(3);
    cmatrix  mb1(3,2);
    cmatrix  mb2(3,2);
    cmatrix  mx1(3,2);
    cmatrix  mx2(3,2);
    iarray   nPivots(3);
    double   dErr = 0.;
    ma.randomize_real(0.,10.); ma.randomize_imag(0.,10.);
    mb1.randomize_real(0.,10.); mb1.randomize_imag(0.,10.);
    mb2.randomize_real(0.,10.); mb2.randomize_imag(0.,10.);

    mLU.low_up(ma, nPivots);
    std::cout << mx1.solve_lu (ma, mLU, nPivots, mb1, dErr);
    std::cout << dErr << std::endl;
    std::cout << mx2.solve_lu (ma, mLU, nPivots, mb2) << std::endl;
    std::cout << ma * mx1 - mb1 << std::endl << ma * mx2 - mb2;
```

```
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(4.2888e-01,8.2409e-02) (-1.1261e-01,-5.7778e-01)
(5.8052e-01,3.2179e-01) (2.5811e-01,-3.8609e-02)
(-3.1499e-02,-7.0014e-01) (1.2652e+00,4.5309e-01)
5.2931e-15
(3.0153e-01,-5.6606e-01) (-1.6308e-01,1.8217e-01)
(7.4971e-01,-1.1305e-01) (5.2187e-01,2.3441e-01)
(-1.9916e-01,1.4493e+00) (9.1046e-02,3.5242e-01)

(0.0000e+00,-8.8818e-16) (0.0000e+00,-8.8818e-16)
(-4.4409e-16,0.0000e+00) (0.0000e+00,-8.8818e-16)
(0.0000e+00,0.0000e+00) (0.0000e+00,0.0000e+00)

(-8.8818e-16,8.8818e-16) (-8.8818e-16,-1.7764e-15)
(0.0000e+00,0.0000e+00) (2.2204e-16,-8.8818e-16)
(4.4409e-16,-8.8818e-16) (1.3878e-17,-4.4409e-16)
```

2.7.58 svd

Functions

```
rvector
cmatrix::svd () throw (cvmexception);
rvector
cmatrix::svd (scmatrix& mU, scmatrix& mVH) throw (cvmexception);
```

create object of type `rvector` as vector of singular values of a calling matrix. The second version sets output parameter `mU` to be equal to matrix U of size $m \times m$ (and change size of the object if it's needed) and `mVH` to be equal to a matrix V^H of size $n \times n$. All functions throw `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. Use `rvector::svd` in order to avoid of new vector creation. Function is *redefined* in the classes `scmatrix`, `scbmatrix`, `schmatrix`. See also `rvector`, `cmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
try {
    double m[] = {1., -1., 1., 2., -2., 1.,
                  3., -2., 1., 0., -2., 1.};
    cmatrix mA((std::complex<double>*) m, 2, 3);
    cmatrix mSigma(2,3);
    rvector v(2);
    scmatrix mU(2), mVH(3);

    v = mA.svd(mU, mVH);
    mSigma.diag(0) = cvector(v);

    std::cout << mU << std::endl;
    std::cout << mVH << std::endl;
    std::cout << mSigma << std::endl;

    std::cout << (mA * ~mVH - mU * mSigma).norm() << std::endl;
    std::cout << (~mA * mU - ~(mSigma * mVH)).norm() << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-4.861e-01,0.000e+00) (8.739e-01,0.000e+00)
(7.956e-01,-3.616e-01) (4.425e-01,-2.012e-01)
```

```
(-7.590e-02,4.474e-01) (7.488e-01,-1.820e-01) (-4.474e-01,1.327e-02)
(8.084e-01,1.878e-01) (-1.576e-02,5.238e-01) (-1.878e-01,3.558e-02)
(1.065e-01,3.065e-01) (3.597e-01,4.669e-02) (8.727e-01,4.012e-02)

(5.452e+00,0.000e+00) (0.000e+00,0.000e+00) (0.000e+00,0.000e+00)
(0.000e+00,0.000e+00) (1.131e+00,0.000e+00) (0.000e+00,0.000e+00)

1.357e-15
1.267e-15
```

2.7.59 pinv

Functions

```
cmatrix  cmatrix::pinv (TR threshold = cvmMachSp()) const
                        throw (cvmexception);
cmatrix& cmatrix::pinv (const basic_rmatrix& mA,
                        TR threshold = cvmMachSp())
                        throw (cvmexception);
```

implement complex matrix pseudo inversion [6], p. 33 (or Moore-Penrose generalized inversion [5], p. 421). Strictly defined, $n \times m$ matrix A^+ is *pseudo inversion* of $m \times n$ matrix A if the following two equations are satisfied:

$$AA^+A = A,$$
$$A^+ = QA^H = A^H P$$

where Q and P are some matrices. To compute pseudo inversion, we use Singular Value Decomposition (SVD)

$$A = U\Sigma V^H$$

of a matrix A, thus

$$A^+ = V\Sigma^{-1}U^H,$$

where Σ^{-1} is diagonal $n \times m$ matrix having inverted diagonal values of a matrix Σ if they are greater than some threshold, and zeros otherwise.

First version creates an object of type `cmatrix` as pseudo inverted calling matrix (it throws `cvmexception` in case of memory allocation failure). Second function sets calling matrix to be equal to matrix `mA` pseudo inverted (it throws `cvmexception` in case of not appropriate sizes of the operands). The threshold parameter sets the minimum distinguishable from zero singular value to be used to compute the pseudo inversion. All values equal or less than the threshold are treated as zeros. Functions are *inherited* in the classes `scmatrix` and `schmatrix` and *redefined* in `scbmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;
try {
    cmatrix mA(2,3);
    mA(1,1) = tcomplex(1.,2.);  mA(1,2) = tcomplex(-1.,3.);
    mA(1,3) = tcomplex(0.,-1.);
    mA(2,1) = tcomplex(1.,-2.); mA(2,2) = tcomplex(1.,-4.);
    mA(2,3) = tcomplex(1.,0.);

    cmatrix mX = mA.pinv();
    std::cout << mX << (mA * mX * mA - mA).norm2() << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(+3.2407407e-01,-1.1111111e-01) (+2.5925926e-01,-1.8518519e-02)
(-1.3888889e-01,-2.7777778e-02) (-5.5555556e-02,+1.6666667e-01)
(+1.4814815e-01,+2.3148148e-01) (+1.6666667e-01,+1.4814815e-01)
+3.3217718e-15
```

Band matrix example:

```
using namespace cvm;

try {
    scbmatrix mA (40, 1, 0);
    mA.diag(0).randomize_real(-1.,1.);
    mA.diag(0).randomize_imag(-3.,2.);
    mA.diag(-1).randomize_real(5.,10.);
    mA.diag(-1).randomize_imag(-3.,7.);

    scmatrix mX (40);
    mX.pinv(mA);
    std::cout << (mA * mX * mA - mA).norm2() << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+6.3113999e-14
```

2.7.60 gels

Functions

```
cmatrix  cmatrix::gels (bool conjugate, const cmatrix& mB,
                        cvector& vErr) const throw (cvmexception);
cmatrix& cmatrix::gels (bool conjugate,
                        const cmatrix& mA, const cmatrix& mB,
                        cvector& vErr) throw (cvmexception);
cvector  cmatrix::gels (bool conjugate, const cvector& mB,
                        TC& cErr) const throw (cvmexception);
```

solve overdetermined or underdetermined linear systems

$$A * x = b$$

for $m \times n$ matrix A (or its conjugated) where b is a vector of length k or systems

$$A * X = B$$

for multiple vectors b stored as columns of $k \times l$ matrix B where $k = m$ in non-transposed case and $k = n$ otherwise. The algorithm uses QR or LQ factorization of A. It is assumed that A has full rank, infinity returned otherwise. Internally functions use ?GELS LAPACK routines. If $m > n$ and `conjugate=false` or $m < n$ and `conjugate=true`, then the system is overdetermined, thus the algorithm tries to find the least squares solution x of the problem

$$\|A * x - b\|_2 \to \min \quad \text{or} \quad \|A^H * x - b\|_2 \to \min$$

respectively. Output vector `vErr` of length l (or complex number `cErr` for single vector b) returns residual sum of squares. The system is underdetermined otherwise, and the algorithm finds its minimum norm solution. In this case `vErr` (or `cErr`) is set to zero. In both cases the solution computed satisfies $x = \mathrm{pinv}(A) * b$, but this algorithm is faster than pseudo inversion. Matrix A is passed as argument `mA` or (in second case) it's calling object. First and third versions return the solution as new matrix or vector object. Second version sets calling object to be the solution. Functions throw `cvmexception` in case of inappropriate sizes of the operands. Functions are *inherited* in all complex matrix classes. See also `cmatrix`, `cvector::gels`, `cmatrix::pinv`. Example:

```
using namespace cvm;

cmatrix a(7, 5);
cmatrix b(7, 2);
cmatrix bt(5, 2);
cvector bv(5);
cvector vErr(2);
tcomplex dErr;
a.randomize_real(-10., 10.);
a.randomize_imag(-10., 10.);
```

```
b.randomize_real(-10., 10.);
b.randomize_imag(-10., 10.);
bt.randomize_real(-10., 10.);
bt.randomize_imag(-10., 10.);
bv.randomize_real(-10., 10.);
bv.randomize_imag(-10., 10.);

cmatrix x = a.gels(false, b, vErr);
cvector vt = a.gels(true, bv, dErr);
cmatrix xt(7,2);
xt.gels(true, a, bt, vErr);

std::cout << (a.pinv()*b - x).norm() << std::endl;
std::cout << (~a.pinv()*bv - vt).norm() << std::endl;
std::cout << (~a.pinv()*bt - xt).norm() << std::endl;
```

prints

```
+1.3258216e-015
+1.0696705e-015
+1.1685394e-015
```

2.7.61 gelsy

Functions

```
cmatrix  cmatrix::gelsy (const cmatrix& mB, int& rank,
                         TR tol = cvmMachSp()) const throw (cvmexception);
cmatrix& cmatrix::gelsy (const cmatrix& mA, const cmatrix& mB, int& rank,
                         TR tol = cvmMachSp()) throw (cvmexception);
cvector  cmatrix::gelsy (const cvector& mB, int& rank,
                         TR tol = cvmMachSp()) const throw (cvmexception);
```

compute the minimum-norm solution to the linear least squares problem

$$\|A * x - b\|_2 \to \min$$

using complete orthogonal factorization of $m \times n$ matrix A. Here b is a vector of length m. Multiple vectors b can be stored as columns of $m \times l$ matrix B. Matrix A may be rank-deficient, functions return its effective rank in `rank` output parameter using `tol` tolerance. Internally functions use `?GELSY` LAPACK routines, see more details about the algorithm in those routines' documentation. Matrix A is passed as argument `mA` or (in second case) it's calling object. First and third versions return the solution as new matrix or vector object. Second version sets calling object to be the solution. Functions throw `cvmexception` in case of inappropriate sizes of the operands. Functions are *inherited* in all complex matrix classes. See also `cmatrix`, `cvector::gelsy`, `cmatrix::gelss`, `cmatrix::gelsd`. Example:

```
using namespace cvm;

cmatrix a(4, 5);
cmatrix b(4, 2);
cvector bv(4);
a.randomize_real(-10., 10.);
a.randomize_imag(-10., 10.);
b.randomize_real(-10., 10.);
b.randomize_imag(-10., 10.);
bv.randomize_real(-10., 10.);
bv.randomize_imag(-10., 10.);
tint rank;

cmatrix x = a.gelsy(b, rank);
cvector xv = a.gelsy(bv, rank);
cmatrix x2(5,2);
x2.gelsy(a, b, rank);

std::cout << (a*x - b).norm() << std::endl;
std::cout << (a*xv - bv).norm() << std::endl;
std::cout << (a*x2 - b).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+8.9701623e-015
+6.9342294e-015
+8.9701623e-015
+4 +4
```

2.7.62 gelss

Functions

```
cmatrix  cmatrix::gelss (const cmatrix& mB, int& rank, rvector& sv,
                         TR tol = cvmMachSp()) const throw (cvmexception);
cmatrix& cmatrix::gelss (const cmatrix& mA, const cmatrix& mB,
                         rvector& sv, int& rank, TR tol = cvmMachSp())
                         throw (cvmexception);
cvector  cmatrix::gelss (const cvector& mB, rvector& sv, int& rank,
                         TR tol = cvmMachSp()) const throw (cvmexception);
```

compute the minimum-norm solution to the linear least squares problem

$$\|A * x - b\|_2 \to \min$$

using singular value decomposition of $m \times n$ matrix A. Here b is a vector of length m. Multiple vectors b can be stored as columns of $m \times l$ matrix B. Matrix A may be rank-deficient, functions return its effective rank in `rank` output parameter using `tol` tolerance. These functions also compute singular values of A in decreasing order and return them in `sv` output parameter having $\min(m, n)$ size. Internally functions use `?GELSS` LAPACK routines, see more details about the algorithm in those routines' documentation. Matrix A is passed as argument `mA` or (in second case) it's calling object. First and third versions return the solution as new matrix or vector object. Second version sets calling object to be the solution. Functions throw `cvmexception` in case of inappropriate sizes of the operands. Functions are *inherited* in all complex matrix classes. See also `cmatrix`, `cvector::gelss`, `cmatrix::gelsy`, `cmatrix::gelsd`. Example:

```
using namespace cvm;

cmatrix a(4, 5);
cmatrix b(4, 2);
cvector bv(4);
rvector sv(4);
tint rank;
a.randomize_real(-10., 10.);
a.randomize_imag(-10., 10.);
b.randomize_real(-10., 10.);
b.randomize_imag(-10., 10.);
bv.randomize_real(-10., 10.);
bv.randomize_imag(-10., 10.);

cmatrix x = a.gelss(b, sv, rank);
cvector xv = a.gelss(bv, sv, rank);
cmatrix xt(5,2);
xt.gelss(a, b, sv, rank);
```

```
std::cout << (a*x - b).norm() << std::endl;
std::cout << (a*xv - bv).norm() << std::endl;
std::cout << (a*xt - b).norm() << std::endl;
std::cout << (sv - a.svd()).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+2.3444861e-013
+4.6335022e-014
+2.3444861e-013
+1.1234667e-014
+4 +4
```

2.7.63 gelsd

Functions

```
cmatrix  cmatrix::gelsd (const cmatrix& mB, int& rank, rvector& sv,
                         TR tol = cvmMachSp()) const throw (cvmexception);
cmatrix& cmatrix::gelsd (const cmatrix& mA, const cmatrix& mB,
                         rvector& sv, int& rank, TR tol = cvmMachSp())
                         throw (cvmexception);
cvector  cmatrix::gelsd (const cvector& mB, rvector& sv, int& rank,
                         TR tol = cvmMachSp()) const throw (cvmexception);
```

compute the minimum-norm solution to the linear least squares problem

$$\|A * x - b\|_2 \to \min$$

using singular value decomposition of $m \times n$ matrix A and divide and conquer method. Here b is a vector of length m. Multiple vectors b can be stored as columns of $m \times l$ matrix B. Matrix A may be rank-deficient, functions return its effective rank in `rank` output parameter using `tol` tolerance. These functions also compute singular values of A in decreasing order and return them in `sv` output parameter having $\min(m, n)$ size. Internally functions use `?GELSD` LAPACK routines, see more details about the algorithm in those routines' documentation. Matrix A is passed as argument `mA` or (in second case) it's calling object. First and third versions return the solution as new matrix or vector object. Second version sets calling object to be the solution. Functions throw `cvmexception` in case of inappropriate sizes of the operands. Functions are *inherited* in all complex matrix classes. See also `cmatrix`, `cvector::gelsd`, `cmatrix::gelsy`, `cmatrix::gelss`. Example:

```
using namespace cvm;

cmatrix a(4, 5);
cmatrix b(4, 2);
cvector bv(4);
rvector sv(4);
tint rank;
a.randomize_real(-10., 10.);
a.randomize_imag(-10., 10.);
b.randomize_real(-10., 10.);
b.randomize_imag(-10., 10.);
bv.randomize_real(-10., 10.);
bv.randomize_imag(-10., 10.);

cmatrix x = a.gelsd(b, sv, rank);
cvector xv = a.gelsd(bv, sv, rank);
cmatrix xt(5,2);
xt.gelsd(a, b, sv, rank);
```

```
std::cout << (a*x - b).norm() << std::endl;
std::cout << (a*xv - bv).norm() << std::endl;
std::cout << (a*xt - b).norm() << std::endl;
std::cout << (sv - a.svd()).norm() << std::endl;
std::cout << rank << " " << a.rank() << std::endl;
```

prints

```
+1.2865301e-014
+1.2824878e-014
+1.2865301e-014
+1.2809491e-014
+4 +4
```

2.7.64 rank

Function

```
int cmatrix::rank (TR eps = cvmMachSp()) const throw (cvmexception);
```

returns rank of a calling matrix as number of singular values with normalized absolute value greater than or equal to parameter eps (this is the largest relative spacing by default). Function throws `cvmexception` in case of convergence error. Function is *inherited* in the classes `scmatrix`, `scbmatrix`, `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m(a, NULL, 3, 4);

std::cout << m << m.rank() << std::endl;
m(3,4) = std::complex<double>(0.,1.);
std::cout << m.rank() << std::endl;
```

prints

```
(1,0) (4,0) (7,0) (10,0)
(2,0) (5,0) (8,0) (11,0)
(3,0) (6,0) (9,0) (12,0)
2
3
```

2.7.65 qr

Functions

```
void cmatrix::qr (cmatrix& mQ, scmatrix& mR) const throw (cvmexception);
void cmatrix::qr (scmatrix& mQ, cmatrix& mR) const throw (cvmexception);
```

compute QR factorization as

$$M = QR$$

where M is calling matrix, unitary matrix Q and upper triangular matrix R are `mQ` and `mR` respectively. First version implements so-called "economy" algorithm which for given $m \times n$ matrix M computes $m \times n$ matrix Q and $n \times n$ matrix R. Second version is "full" mode one computing $m \times m$ matrix Q and $m \times n$ matrix R. Functions throw `cvmexception` in case of inappropriate sizes of the operands. Functions are *redefined* in the class `scmatrix`. See also `cmatrix`, `scmatrix`. Example:

```
using namespace cvm;

treal ar[] = {1., 2., 3., 4., 5., 6.};
treal ai[] = {1., -1., 2., -2., 3., -3.};
const cvm::cmatrix mh(ar, ai, 2, 3);
const cvm::cmatrix mv(ar, ai, 3, 2);
cvm::scmatrix s2(2), s3(3);
cvm::cmatrix  h(2,3), v(3,2);

mh.qr(h,s3);
std::cout << (eye_complex(2)-~cmatrix(h,1,1,2,2)*cmatrix(h,1,1,2,2)).norm()
          << " " << (mh - h * s3).norm() << std::endl;
mh.qr(s2,h);
std::cout << (eye_complex(2) - ~s2 * s2).norm()
          << " " << (mh - s2 * h).norm() << std::endl;
mv.qr(v,s2);
std::cout << (eye_complex(2) - ~v * v).norm()
          << " " << (mv - v * s2).norm() << std::endl;
mv.qr(s3,v);
std::cout << (eye_complex(3) - ~s3 * s3).norm()
          << " " << (mv - s3 * v).norm() << std::endl;
```

prints

```
+2.5145832e-16 +1.3506446e-15
+2.5145832e-16 +1.3506446e-15
+2.5367068e-16 +1.4432899e-15
+3.8435519e-16 +1.4432899e-15
```

2.7.66 lq

Functions

```
void cmatrix::lq (scmatrix& mL, cmatrix& mQ) const throw (cvmexception);
void cmatrix::lq (cmatrix& mL, scmatrix& mQ) const throw (cvmexception);
```

compute LQ factorization as

$$M = LQ$$

where M is calling matrix, upper triangular (trapezoidal) matrix L and unitary matrix Q are `mL` and `mQ` respectively. First version implements so-called "economy" algorithm which for given $m \times n$ matrix M computes $m \times m$ matrix L and $m \times n$ matrix Q. Second version is "full" mode one computing $m \times n$ matrix L and $n \times n$ matrix Q. Functions throw `cvmexception` in case of inappropriate sizes of the operands. Functions are *redefined* in the class `scmatrix`. See also `cmatrix`, `scmatrix`. Example:

```
using namespace cvm;

treal ar[] = {1., 2., 3., 4., 5., 6.};
treal ai[] = {1., -1., 2., -2., 3., -3.};
const cvm::cmatrix mh(ar, ai, 2, 3);
const cvm::cmatrix mv(ar, ai, 3, 2);
cvm::scmatrix s2(2), s3(3);
cvm::cmatrix  h(2,3), v(3,2);

mh.lq(s2,h);
std::cout << (eye_complex(2) - h * ~h).norm() << " " << (mh - s2 * h)
             .norm() << std::endl;
mv.lq(s3,v);
std::cout << (eye_complex(2) - ~cmatrix(v,1,1,2,2) * cmatrix(v,1,1,2,2))
             .norm() << " " << (mv - s3 * v).norm() << std::endl;
mh.lq(h,s3);
std::cout << (eye_complex(3) - s3 * ~s3).norm() << " " << (mh - h * s3)
             .norm() << std::endl;
mv.lq(v,s2);
std::cout << (eye_complex(2) - s2 * ~s2).norm() << " " << (mv - v * s2)
             .norm() << std::endl;
```

prints

```
+1.3374617e-015 +7.4806297e-015
+6.6728929e-016 +2.9205408e-015
+1.4349634e-015 +7.4806297e-015
+6.7217793e-016 +2.9205408e-015
```

2.7.67 rq

Functions

```
void cmatrix::rq (scmatrix& mR, cmatrix& mQ) const throw (cvmexception);
void cmatrix::rq (cmatrix& mR, scmatrix& mQ) const throw (cvmexception);
```

compute RQ factorization as

$$M = RQ$$

where M is calling matrix, upper triangular matrix R and unitary matrix Q are `mR` and `mQ` respectively. First version implements so-called "economy" algorithm which for given $m \times n$ matrix M computes $m \times m$ matrix R and $m \times n$ matrix Q. Second version is "full" mode one computing $m \times n$ matrix R and $n \times n$ matrix Q. Following this definition the implementation assumes that $m \leqslant n$ and throws `cvmexception` otherwise. Functions also throw `cvmexception` in case of inappropriate sizes of the operands passed. Functions are *redefined* in the class `srmatrix`. See also `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;

treal ar[] = {1., 2., 3., 4., 5., 6.};
treal ai[] = {1., -1., 2., -2., 3., -3.};
const cvm::cmatrix mh(ar, ai, 2, 3);
const cvm::cmatrix mv(ar, ai, 3, 2);
cvm::scmatrix s2(2), s3(3);
cvm::cmatrix  h(2,3), v(3,2);

mh.rq(h,s3);
std::cout << (eye_complex(2) - cmatrix(s3,2,1,2,3) * ~cmatrix(s3,2,1,2,3))
             .norm() << " " << (mh - h * s3).norm() << std::endl;

mh.rq(s2,h);
std::cout << (eye_complex(2) - h * ~h).norm()
          << " " << (mh - s2 * h).norm() << std::endl;
```

prints

```
+3.1898416e-016 +2.2342807e-015
+3.1898416e-016 +2.2342807e-015
```

2.7.68 ql

Functions

```
void cmatrix::ql (cmatrix& mQ, scmatrix& mL) const throw (cvmexception);
void cmatrix::ql (scmatrix& mQ, cmatrix& mL) const throw (cvmexception);
```

compute QL factorization as

$$M = QL$$

where M is calling matrix, unitary matrix Q and lower triangular matrix L are `mQ` and `mL` respectively. First version implements so-called "economy" algorithm which for given $m \times n$ matrix M computes $m \times n$ matrix Q and $n \times n$ matrix L. Second version is "full" mode one computing $m \times m$ matrix Q and $m \times n$ matrix L. Following this definition the implementation assumes that $m \geqslant n$ and throws `cvmexception` otherwise. Functions also throw `cvmexception` in case of inappropriate sizes of the operands passed. Functions are *redefined* in the class `scmatrix`. See also `cmatrix`, `scmatrix`. Example:

```
using namespace cvm;

treal ar[] = {1., 2., 3., 4., 5., 6.};
treal ai[] = {1., -1., 2., -2., 3., -3.};
const cvm::cmatrix mv(ar, ai, 3, 2);
cvm::scmatrix s2(2), s3(3);
cvm::cmatrix v(3,2);

mv.ql(v,s2);
std::cout << (eye_complex(2) - ~v * v).norm()
          << " " << (mv - v * s2).norm() << std::endl;

mv.ql(s3,v);
std::cout << (eye_complex(2) - ~cmatrix(s3,1,2,3,2) * cmatrix(s3,1,2,3,2))
             .norm() << " " << (mv - s3 * v).norm() << std::endl;
```

prints

```
+1.1857187e-016 +9.4857497e-016
+1.1857187e-016 +9.4857497e-016
```

2.7.69 vanish

Function

```
cmatrix& cmatrix::vanish();
```

sets every element of a calling matrix to be equal to zero and returns a reference to the matrix changed. This function is faster than, for example, `cmatrix::set(TC)` with zero operand passed. Function is *redefined* in the classes `scmatrix`, `scbmatrix`, `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;

cmatrix m(4, 3);
m.randomize_real(0.,1.);
m.randomize_imag(1.,2.);

std::cout << m << std::endl;
std::cout << m.vanish ();
```

prints

```
(0.851527,1.16376) (0.557512,1.90188) (0.0343638,1.52068)
(0.478042,1.29106) (0.561724,1.19764) (0.320994,1.35804)
(0.264534,1.40986) (0.113468,1.75137) (0.37727,1.54994)
(0.521409,1.83035) (0.559465,1.35072) (0.809198,1.12537)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.7.70 geru

Function

```
cmatrix&
cmatrix::geru (TC dAlpha, const cvector& vCol, const cvector& vRow)
throw (cvmexception);
```

calls one of ?GERU routines of the BLAS library performing rank-1 update (unconjugated) matrix-vector operation defined as

$$M = \alpha \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} \begin{pmatrix} y_1 & y_2 & \cdots & y_n \end{pmatrix} + M,$$

where α is complex number (parameter dAlpha), M is calling matrix and x and y are complex vectors (parameters vCol and vRow respectively). Function returns a reference to the matrix changed and throws cvmexception in case of inappropriate sizes of the operands. Function is *inherited* in the class scmatrix and *not applicable* to objects of the classes scbmatrix and schmatrix (i.e. cvmexception would be thrown in case of using it for objects of those classes). See also cvector, cmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (5);

std::complex<double> alpha = std::complex<double>(1.2,4.11);
cmatrix m(3,2);
cvector vc(3);
cvector vr(2);
m.randomize_real(-1., 2.); m.randomize_imag(-3., 2.);
vc.randomize_real(-1., 3.); vc.randomize_imag(1., 3.);
vr.randomize_real(0., 2.); vr.randomize_imag(-1., 2.);

std::cout << m + vc.rank1update_u (vr) * alpha << std::endl;
std::cout << m.geru(alpha, vc, vr);
```

prints

```
(2.88144e+00,3.54299e+00) (-8.14760e+00,-1.03789e+00)
(6.33361e-01,3.35209e+00) (-4.81787e+00,-8.53964e+00)
(5.44811e-01,1.37156e+00) (-5.97006e+00,-5.00794e+00)

(2.88144e+00,3.54299e+00) (-8.14760e+00,-1.03789e+00)
(6.33361e-01,3.35209e+00) (-4.81787e+00,-8.53964e+00)
(5.44811e-01,1.37156e+00) (-5.97006e+00,-5.00794e+00)
```

2.7.71 gerc

Function

```
cmatrix&
cmatrix::gerc (TC dAlpha, const cvector& vCol, const cvector& vRow)
throw (cvmexception);
```

calls one of ?GERC routines of the BLAS library performing rank-1 update (conjugated) matrix-vector operation defined as

$$M = \alpha \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} \begin{pmatrix} y_1^* & y_2^* & \dots & y_n^* \end{pmatrix} + M,$$

where α is complex number (parameter `dAlpha`), M is calling matrix and x and y are complex vectors (parameters `vCol` and `vRow` respectively). Function returns a reference to the matrix changed and throws `cvmexception` in case of inappropriate sizes of the operands. Function is *inherited* in the class `scmatrix` and *not applicable* to objects of the classes `scbmatrix` and `schmatrix` (i.e. `cvmexception` would be thrown in case of using it for objects of those classes). See also `cvector`, `cmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (5);

std::complex<double> alpha = std::complex<double>(1.2,4.11);
cmatrix m(3,2);
cvector vc(3);
cvector vr(2);
m.randomize_real(-1., 2.); m.randomize_imag(-3., 2.);
vc.randomize_real(-1., 3.); vc.randomize_imag(1., 3.);
vr.randomize_real(0., 2.); vr.randomize_imag(-1., 2.);

std::cout << m + vc.rank1update_c (vr) * alpha << std::endl;
std::cout << m.gerc(alpha, vc, vr);
```

prints

```
(1.27138e+01,1.58049e+01) (1.00616e+01,2.21197e+01)
(1.93326e+01,1.41763e+01) (1.74769e+01,2.49013e+01)
(8.09961e+00,1.36259e+01) (5.86738e+00,1.97800e+01)

(1.27138e+01,1.58049e+01) (1.00616e+01,2.21197e+01)
(1.93326e+01,1.41763e+01) (1.74769e+01,2.49013e+01)
(8.09961e+00,1.36259e+01) (5.86738e+00,1.97800e+01)
```

2.7.72 gemm

Function

```
cmatrix& cmatrix::gemm (const cmatrix& m1, bool bConj1,
                        const cmatrix& m2, bool bConj2,
                        TC dAlpha, TC dBeta) throw (cvmexception);
```

calls one of ?GEMM routines of the BLAS library performing matrix-matrix operation defined as

$$M = \alpha\,\mathcal{C}(M_1)\cdot\mathcal{C}(M_2) + \beta M,$$

where α and β are complex numbers (parameters dAlpha and dBeta), M is calling matrix and M_1 and M_2 are matrices (parameters m1 and m2 respectively). Function $\mathcal{C}(M_i)$ congugates matrix M_i if appropriate boolean parameter bConj* is equal to true and does nothing otherwise. Function returns a reference to the matrix changed and throws cvmexception in case of inappropriate sizes of the operands. Function is *inherited* in the class scmatrix and *not applicable* to objects of the classes scbmatrix and schmatrix (i.e. cvmexception would be thrown in case of using it for objects of those classes). See also cmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);

try {
    std::complex<double> alpha = std::complex<double>(1.1,2.1);
    std::complex<double> beta = std::complex<double>(0.71,0.12);
    cmatrix m1(4,3); cmatrix m2(4,3);
    cmatrix m(3,3);
    m.randomize_real(-1., 2.); m.randomize_imag(1., 3.);
    m1.randomize_real(-1., 3.); m1.randomize_imag(-2., 4.);
    m2.randomize_real(0., 2.); m2.randomize_imag(-3., 2.);

    std::cout << ~m1 * m2 * alpha + m * beta << std::endl;
    std::cout << m.gemm(m1, true, m2, false, alpha, beta);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(3.225e+01,3.611e+01) (2.042e+01,1.206e+01) (5.065e+01,-2.261e+01)
(3.009e+01,3.665e+00) (2.167e+01,-3.327e+00) (4.305e+01,-1.960e+01)
(1.156e+01,-4.966e+00) (4.067e+00,-1.181e+01) (1.121e+01,-2.684e+01)

(3.225e+01,3.611e+01) (2.042e+01,1.206e+01) (5.065e+01,-2.261e+01)
```

```
(3.009e+01,3.665e+00) (2.167e+01,-3.327e+00) (4.305e+01,-1.960e+01)
(1.156e+01,-4.966e+00) (4.067e+00,-1.181e+01) (1.121e+01,-2.684e+01)
```

2.7.73 hemm

Function

```
cmatrix& cmatrix::hemm (bool bLeft, const schmatrix& ms,
                        const cmatrix& m, TC dAlpha, TC dBeta)
                        throw (cvmexception);
```

calls one of ?HEMM routines of the BLAS library performing one of matrix-matrix operations defined as

$$M = \alpha M_h \cdot M_1 + \beta M \quad \text{or} \quad M = \alpha M_1 \cdot M_h + \beta M,$$

where α and β are complex numbers (parameters dAlpha and dBeta), M is calling matrix, M_h is hermitian matrix and M_1 is complex matrix (parameters ms and m respectively). First operation is performed if bLeft passed is true and second one otherwise. Function returns a reference to the matrix changed and throws cvmexception in case of inappropriate sizes of the operands. Function is *inherited* in the classes scmatrix and schmatrix and *not applicable* to objects of the class scbmatrix (i.e. cvmexception would be thrown in case of using it for objects of that class). See also schmatrix, cmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);

try {
    std::complex<double> alpha = std::complex<double>(1.3,0.21);
    std::complex<double> beta = std::complex<double>(0.5,-0.1);
    cmatrix m1(2,3);
    cmatrix m2(3,2);
    schmatrix ms(2);
    cmatrix m(2,3);
    m.randomize_real(-1., 2.); m.randomize_imag(1., 3.);
    m1.randomize_real(-1., 3.); m1.randomize_imag(1., 2.);
    m2.randomize_real(0., 2.); m2.randomize_imag(-3., -1.);
    ms.randomize_real(-3., 1.); ms.randomize_imag(-1.3, 4.);

    std::cout << ms * m1 * alpha + m * beta << std::endl;
    std::cout << m.hemm (true, ms, m1, alpha, beta) << std::endl;

    m.resize(3,2);
    m.randomize_real(-1.4, 1.3); m.randomize_imag(1.1, 3.);
    std::cout << m2 * ms * alpha + m * beta << std::endl;
    std::cout << m.hemm (false, ms, m2, alpha, beta);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1.096e+00,-7.692e+00) (-7.923e+00,-3.909e+00) (-1.324e+01,-5.264e+00)
(2.415e+00,1.240e+00) (4.384e-01,-1.771e+00) (7.495e-01,-2.740e+00)

(1.096e+00,-7.692e+00) (-7.923e+00,-3.909e+00) (-1.324e+01,-5.264e+00)
(2.415e+00,1.240e+00) (4.384e-01,-1.771e+00) (7.495e-01,-2.740e+00)

(-5.007e+00,1.010e+01) (2.341e+00,3.248e+00)
(-8.753e+00,7.854e+00) (3.152e+00,4.491e+00)
(-9.162e+00,6.401e+00) (-1.168e+00,3.973e+00)

(-5.007e+00,1.010e+01) (2.341e+00,3.248e+00)
(-8.753e+00,7.854e+00) (3.152e+00,4.491e+00)
(-9.162e+00,6.401e+00) (-1.168e+00,3.973e+00)
```

2.7.74 randomize_real

Function

`cmatrix& cmatrix::randomize_real (TR dFrom, TR dTo);`

fills real part of a calling matrix with pseudo-random numbers distributed between `dFrom` and `dTo`. Function returns a reference to the matrix changed. Function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);

cmatrix m(2,3);
m.randomize_real(-1., 2.);
std::cout << m;
```

prints

```
(1.090e+00,0.000e+00) (-6.375e-01,0.000e+00) (1.248e+00,0.000e+00)
(-1.272e-01,0.000e+00) (-8.557e-01,0.000e+00) (4.848e-01,0.000e+00)
```

2.7.75 randomize_imag

Function

```
cmatrix& cmatrix::randomize_imag (TR dFrom, TR dTo);
```

fills imaginary part of a calling matrix with pseudo-random numbers distributed between `dFrom` and `dTo`. Function returns a reference to the matrix changed. Function is *redefined* in the classes `scmatrix`, `scbmatrix` and `schmatrix`. See also `cmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);

cmatrix m(2,3);
m.randomize_imag(-1., 2.);
std::cout << m;
```

prints

```
(0.000e+00,1.113e+00) (0.000e+00,6.615e-01) (0.000e+00,1.017e+00)
(0.000e+00,-3.397e-01) (0.000e+00,1.577e+00) (0.000e+00,8.071e-01)
```

2.8 srmatrix

This is end-user class encapsulating square matrix of real numbers.

```
template <typename TR>
class srmatrix : public rmatrix <TR>, public SqMatrix <TR,TR> {
public:
    srmatrix ();
    explicit srmatrix (int nMN);
    srmatrix (TR* pD, int nMN);
    srmatrix (const TR* pD, int nMN);
    srmatrix (const srmatrix& m);
    srmatrix (const rmatrix& m);
    explicit srmatrix (const rvector& v);
    srmatrix (rmatrix& m, int nRow, int nCol, int nSize);
    TR& operator () (int im, int in) throw (cvmexception);
    TR operator () (int im, int in) const throw (cvmexception);
    rvector operator () (int i) throw (cvmexception);
    const rvector operator () (int i) const throw (cvmexception);
    rvector operator [] (int i) throw (cvmexception);
    const rvector operator [] (int i) const throw (cvmexception);
    srmatrix& operator = (const srmatrix& m) throw (cvmexception);
    srmatrix& assign (const rvector& v) throw (cvmexception);
    srmatrix& assign (const TR* pD);
    srmatrix& assign (int nRow, int nCol, const rmatrix& m)
                      throw (cvmexception);
    srmatrix& set (TR x);
    srmatrix& resize (int nNewMN) throw (cvmexception);
    srmatrix& operator << (const srmatrix& m) throw (cvmexception);
    srmatrix operator + (const srmatrix& m) const
                         throw (cvmexception);
    srmatrix operator - (const srmatrix& m) const
                         throw (cvmexception);
    srmatrix& sum (const srmatrix& m1,
                   const srmatrix& m2) throw (cvmexception);
    srmatrix& diff (const srmatrix& m1,
                    const srmatrix& m2) throw (cvmexception);
    srmatrix& operator += (const srmatrix& m) throw (cvmexception);
    srmatrix& operator -= (const srmatrix& m) throw (cvmexception);
    srmatrix operator - () const;
    srmatrix& operator ++ ();
    srmatrix& operator ++ (int);
    srmatrix& operator -- ();
    srmatrix& operator -- (int);
    srmatrix operator * (TR d) const;
```

```
srmatrix operator / (TR d) const throw (cvmexception);
srmatrix& operator *= (TR d);
srmatrix& operator /= (TR d) throw (cvmexception);
srmatrix& normalize ();
srmatrix operator ~ () const throw (cvmexception);
srmatrix& transpose (const srmatrix& m) throw (cvmexception);
srmatrix& transpose ();
rvector operator * (const rvector& v) const throw (cvmexception);
rmatrix operator * (const rmatrix& m) const throw (cvmexception);
srmatrix operator * (const srmatrix& m) const throw (cvmexception);
srmatrix& operator *= (const srmatrix& m) throw (cvmexception);
srmatrix& swap_rows (int n1, int n2) throw (cvmexception);
srmatrix& swap_cols (int n1, int n2) throw (cvmexception);
rvector solve (const rvector& vB) const throw (cvmexception);
rmatrix solve (const rmatrix& mB) const throw (cvmexception);
rvector solve (const rvector& vB, TR& dErr) const
              throw (cvmexception);
rmatrix solve (const rmatrix& mB, TR& dErr) const
              throw (cvmexception);
rvector solve_tran (const rvector& vB) const throw (cvmexception);
rmatrix solve_tran (const rmatrix& mB) const throw (cvmexception);
rvector solve_tran (const rvector& vB, TR& dErr) const
                   throw (cvmexception);
rmatrix solve_tran (const rmatrix& mB, TR& dErr) const
                   throw (cvmexception);
rvector operator % (const rvector& vB) const throw (cvmexception);
rvector operator / (const rvector& vB) const throw (cvmexception);
rvector solve_lu (const srmatrix& mLU, const int* pPivots,
                  const rvector& vB, TR& dErr) throw (cvmexception);
rvector solve_lu (const srmatrix& mLU, const int* pPivots,
                  const rvector& vB) throw (cvmexception);
rmatrix solve_lu (const srmatrix& mLU, const int* pPivots,
                  const rmatrix& mB, TR& dErr) throw (cvmexception);
rmatrix solve_lu (const srmatrix& mLU, const int* pPivots,
                  const rmatrix& mB) throw (cvmexception);
TR det () const throw (cvmexception);
srmatrix& low_up (const srmatrix& m,
                  int* nPivots) throw (cvmexception);
srmatrix low_up (int* nPivots) const throw (cvmexception);
TR cond () const throw (cvmexception);
srmatrix& inv (const srmatrix& mArg) throw (cvmexception);
srmatrix inv () const throw (cvmexception);
srmatrix& exp (const srmatrix& m,
              TR tol = cvmMachSp ()) throw (cvmexception);
srmatrix exp (TR tol = cvmMachSp ()) const throw (cvmexception);
```

```
    srmatrix& polynom (const srmatrix& m, const rvector& v)
                       throw (cvmexception);
    srmatrix polynom (const rvector& v) const throw (cvmexception);
    cvector eig (scmatrix& mEigVect,
                 bool bRightVect = true) const throw (cvmexception);
    cvector eig () const throw (cvmexception);
    srmatrix& cholesky (const srsmatrix& m) throw (cvmexception);
    srmatrix& bunch_kaufman (const srsmatrix& m,
                             int* pivots) throw (cvmexception);
    void qr(srmatrix& mQ, srmatrix& mR) const throw (cvmexception);
    void lq(srmatrix& mL, srmatrix& mQ) const throw (cvmexception);
    void rq(srmatrix& mR, srmatrix& mQ) const throw (cvmexception);
    void ql(srmatrix& mQ, srmatrix& mL) const throw (cvmexception);
    srmatrix& identity ();
    srmatrix& vanish ();
    srmatrix& randomize (TR dFrom, TR dTo);
};
```

2.8.1 srmatrix ()

Constructor

`srmatrix::srmatrix ();`

creates empty `srmatrix` object. See also `srmatrix`. Example:

```
using namespace cvm;

srmatrix m;
std::cout << m.msize() << std::endl << m.nsize() << std::endl;
std::cout << m.size() << std::endl;

m.resize (3);
std::cout << m;
```

prints

```
0
0
0
0 0 0
0 0 0
0 0 0
```

2.8.2 srmatrix (int)

Constructor

```
explicit srmatrix::srmatrix (int nMN);
```

creates $n \times n$ `srmatrix` object where n is passed in `nMN` parameter. Constructor sets all elements to zero. Constructor throws `cvmexception` in case of non-positive size passed or memory allocation failure. See also `srmatrix`. Example:

```
using namespace cvm;

srmatrix m (4);
std::cout << m.msize() << std::endl << m.nsize()
          << std::endl << m.size() << std::endl << m;
```

prints

```
4
4
16
0 0 0 0
0 0 0 0
0 0 0 0
0 0 0 0
```

2.8.3 srmatrix (TR*,int)

Constructor

```
srmatrix::srmatrix (TR* pD, int nMN);
```

creates $n \times n$ `srmatrix` object where n is passed in `nMN` parameter. Unlike others, this constructor *does not allocate memory*. It just shares memory with an array pointed to by `pD`. Constructor throws `cvmexception` in case of non-positive size passed. See also `srmatrix`, `srmatrix (const TR*,int)`. Example:

```
using namespace cvm;
double a[] = {1., 1., 1., 1., 1., 1., 1., 1., 1.};
srmatrix m (a, 3);

m(1,1) = 5.;
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << std::endl;
```

prints

```
5 1 1
1 1 1
1 1 1

5 1 1
```

2.8.4 srmatrix (const TR*,int)

Constructor

```
srmatrix::srmatrix (const TR* pD, int nMN);
```

creates $n \times n$ srmatrix object where n is passed in nMN parameter and copies $n * n$ elements of an array pD to it by colums. Constructor throws cvmexception in case of non-positive size passed. See also srmatrix, srmatrix (TR*,int). Example:

```
using namespace cvm;
const double a[] = {1., 1., 1., 1., 1., 1., 1., 1., 1.};
srmatrix m (a, 3);

m(1,1) = 5.;
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << std::endl;
```

prints

```
5 1 1
1 1 1
1 1 1

1 1 1
```

2.8.5 srmatrix (const srmatrix&)

Copy constructor

```
srmatrix::srmatrix (const srmatrix& m);
```

creates `srmatrix` object as a copy of `m`. Constructor throws `cvmexception` in case of memory allocation failure. See also `srmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
srmatrix m(a, 3);
srmatrix mc(m);

m(1,1) = 7.77;
std::cout << m << std::endl << mc;
```

prints

```
7.77 4 7
2 5 8
3 6 9

1 4 7
2 5 8
3 6 9
```

2.8.6 srmatrix (const rmatrix&)

Constructor

```
srmatrix::srmatrix (const rmatrix& m);
```

creates `srmatrix` object as a copy of a matrix `m`. It's assumed that $m \times n$ matrix `m` must have equal sizes, i.e. $m = n$ is satisfied. Constructor throws `cvmexception` if this is not true or in case of memory allocation failure. Please note that this constructor is *not explicit* anymore. See also `srmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.};
rmatrix m(a, 2, 3);
std::cout << m << std::endl;

m.resize(3, 3);
srmatrix ms (m);
std::cout << ms;
```

prints

```
1 3 5
2 4 6

1 3 5
2 4 6
0 0 0
```

2.8.7 srmatrix (const rvector&)

Constructor

```
explicit srmatrix::srmatrix (const rvector& v);
```

creates `srmatrix` object of size `v.size()` by `v.size()` and assigns vector `v` to its main diagonal. Constructor throws `cvmexception` in case of memory allocation failure. See also `srmatrix`, `rvector`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5.};
rvector v(a, 5);
srmatrix m(v);
std::cout << m;
```

prints

```
1 0 0 0 0
0 2 0 0 0
0 0 3 0 0
0 0 0 4 0
0 0 0 0 5
```

2.8.8 submatrix

Submatrix constructor

```
srmatrix::srmatrix (rmatrix& m, int nRow, int nCol, int nSize);
```

creates `srmatrix` object as *submatrix* of `m`. It means that the matrix object created shares memory with some part of `m`. This part is defined by its upper left corner (parameters `nRow` and `nCol`, both are ℓ-based) and its size (parameter `nSize`). See also `srmatrix`. Example:

```
using namespace cvm;

rmatrix m(4,5);
srmatrix subm(m, 2, 2, 2);
subm.set(1.);

std::cout << m;
```

prints

```
0 0 0 0 0
0 1 1 0 0
0 1 1 0 0
0 0 0 0 0
```

2.8.9 operator (,)

Indexing operators

```
TR& srmatrix::operator () (int im, int in) throw (cvmexception);
TR srmatrix::operator () (int im, int in) const throw (cvmexception);
```

provide access to a particular element of a calling matrix. The first version is applicable to non-constant object. This version returns *l-value* in order to make possible write access to a particular element. Both operators are ℓ-based. Operators throw `cvmexception` if some of parameters passed is outside of `[1,msize()]` range. Operators are *inherited* in the class `srbmatrix` and *redefined* in the class `srsmatrix`. See also `srmatrix`, `Matrix::msize()`, `Matrix::nsize()`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    const srmatrix m (a, 3);
    srmatrix ms(m);

    std::cout << m(1,1) << " " << m(2,3) << std::endl << std::endl;

    ms(2,2) = 7.77;
    std::cout << ms;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 8

1 4 7
2 7.77 8
3 6 9
```

2.8.10 operator ()

Indexing operators

```
rvector srmatrix::operator () (int i) throw (cvmexception);
const rvector srmatrix::operator () (int i) const throw (cvmexception);
```

provide access to i-th column of a calling matrix. The first version is applicable to non-constant object and *returns l-value*, i.e. the vector returned shares memory with i-th column of a calling matrix in order to make possible write access to it. The second version creates *copy* of a column and therefore it's *not l-value*. Both operators are ℓ-based. Operators throw `cvmexception` if parameter `i` is outside of `[1,nsize()]` range. Operators are *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    const srmatrix m (a, 3);
    srmatrix ms(3);

    std::cout << m(2) << std::endl;

    ms(2) = m(3);
    std::cout << ms;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
4.00e+00 5.00e+00 6.00e+00

0.00e+00 7.00e+00 0.00e+00
0.00e+00 8.00e+00 0.00e+00
0.00e+00 9.00e+00 0.00e+00
```

2.8.11 operator []

Indexing operators

```
rvector srmatrix::operator [] (int i) throw (cvmexception);
const rvector srmatrix::operator [] (int i) const throw (cvmexception);
```

provide access to i-th row of a calling matrix. The first version is applicable to non-constant object and *returns l-value*, i.e. the vector returned shares memory with i-th row of a calling matrix in order to make possible write access to it. The second version creates *copy* of a row and therefore it's *not l-value*. Both operators are ℓ-based. Operators throw `cvmexception` if parameter `i` is outside of `[1,msize()]` range. Operators are *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`, `Matrix::msize()`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    const srmatrix m (a, 3);
    srmatrix ms(3);

    std::cout << m[2] << std::endl;

    ms[2] = m[3];
    std::cout << ms;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
2.00e+00 5.00e+00 8.00e+00

0.00e+00 0.00e+00 0.00e+00
3.00e+00 6.00e+00 9.00e+00
0.00e+00 0.00e+00 0.00e+00
```

2.8.12 operator = (const srmatrix&)

Operator

```
srmatrix& srmatrix::operator = (const srmatrix& m)
throw (cvmexception);
```

sets every element of a calling matrix to a value of appropriate element of a matrix `m` and returns a reference to the matrix changed. Operator throws `cvmexception` in case of different sizes of the operands. Operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    const srmatrix m1(a, 3);
    srmatrix m2(3);

    m2 = m1;
    std::cout << m2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.00e+00 4.00e+00 7.00e+00
2.00e+00 5.00e+00 8.00e+00
3.00e+00 6.00e+00 9.00e+00
```

2.8.13 assign (const TR*)

Function

```
srmatrix& srmatrix::assign (const rvector& v) throw (cvmexception);
srmatrix& srmatrix::assign (const TR* pD);
```

sets every element of a calling matrix to a value of appropriate element of vector `v` or array pointed to by `pD` and returns a reference to the matrix changed. In first version it's assumed that vector passed is long enough to fill calling matrix. Function throws `cvmexception` otherwise. Function is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

const double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
srmatrix m(3);

m.assign(a);
std::cout << m;
```

prints

```
1.00e+00 4.00e+00 7.00e+00
2.00e+00 5.00e+00 8.00e+00
3.00e+00 6.00e+00 9.00e+00
```

2.8.14 assign (int, int, const rmatrix&)

Function

```
srmatrix& srmatrix::assign (int nRow, int nCol, const rmatrix& m)
throw (cvmexception);
```

sets sub-matrix of a calling matrix beginning with ℓ-based row `nRow` and column `nCol` to a matrix `m` and returns a reference to the matrix changed. Function throws `cvmexception` if `nRow` or `nCol` are not positive or matrix `m` doesn't fit. Function is *redefined* in the class `srsmatrix`. See also `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;

srmatrix m1(5);
rmatrix m2(2,3);
m1.set(1.);
m2.set(2.);
m1.assign(2,3,m2);
std::cout << m1;
```

prints

```
1 1 1 1 1
1 1 2 2 2
1 1 2 2 2
1 1 1 1 1
1 1 1 1 1
```

2.8.15 set (TR)

Function

```
srmatrix& srmatrix::set (TR x);
```

sets every element of a calling matrix to a value of parameter `x` and returns a reference to the matrix changed. Use `vanish` to set every element of a calling matrix to be equal to zero. Function is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
srmatrix m(3);

m.set(3.);
std::cout << m;
```

prints

```
3.00e+00 3.00e+00 3.00e+00
3.00e+00 3.00e+00 3.00e+00
3.00e+00 3.00e+00 3.00e+00
```

2.8.16 resize

Function

`srmatrix& srmatrix::resize (int nNewMN) throw (cvmexception);`

changes size of a calling matrix to `nNewMN` by `nNewMN` and returns a reference to the matrix changed. In case of increasing of its size, calling matrix is filled up with zeroes. Function throws `cvmexception` in case of negative size passed or memory allocation failure. Function is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4.};
    srmatrix m(a, 2);

    std::cout << m << std::endl;

    m.resize (3);
    std::cout << m;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 3
2 4

1 3 0
2 4 0
0 0 0
```

2.8.17 operator <<

Operator

```
srmatrix& srmatrix::operator << (const srmatrix& m)
throw (cvmexception);
```

destroys calling matrix, creates new one as a copy of m and returns a reference to the matrix changed. Operator throws cvmexception in case of memory allocation failure. Operator is *redefined* in the classes srbmatrix and srsmatrix. See also srmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

try {
    srmatrix m(3);
    srmatrix mc(1);
    m(1,2) = 1.;
    m(2,3) = 2.;
    std::cout << m << std::endl << mc << std::endl;

    mc << m;
    std::cout << mc;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
0.00e+00 1.00e+00 0.00e+00
0.00e+00 0.00e+00 2.00e+00
0.00e+00 0.00e+00 0.00e+00

0.00e+00

0.00e+00 1.00e+00 0.00e+00
0.00e+00 0.00e+00 2.00e+00
0.00e+00 0.00e+00 0.00e+00
```

2.8.18 operator +

Operator

```
srmatrix srmatrix::operator + (const srmatrix& m) const
throw (cvmexception);
```

creates an object of type `srmatrix` as a sum of a calling matrix and a matrix `m`. Operator throws `cvmexception` in case of different sizes of the operands. Operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::sum`, `srmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double b[] = {10., 20., 30., 40., 50., 60., 70., 80., 90.};
    srmatrix m1(a, 3);
    srmatrix m2(b, 3);

    std::cout << m1 + m2 << std::endl << m1 + m1;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
11 44 77
22 55 88
33 66 99

2 8 14
4 10 16
6 12 18
```

2.8.19 operator -

Operator

```
srmatrix srmatrix::operator - (const srmatrix& m) const
throw (cvmexception);
```

creates an object of type `srmatrix` as a difference of a calling matrix and a matrix `m`. It throws `cvmexception` in case of different sizes of the operands. Operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::diff`, `srmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double b[] = {10., 20., 30., 40., 50., 60., 70., 80., 90.};
    srmatrix m1(a, 3);
    srmatrix m2(b, 3);

    std::cout << m2 - m1 << std::endl << m1 - m1;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
9 36 63
18 45 72
27 54 81

0 0 0
0 0 0
0 0 0
```

2.8.20 sum

Function

```
srmatrix& srmatrix::sum (const srmatrix& m1, const srmatrix& m2)
throw (cvmexception);
```

assigns the result of addition of matrices `m1` and `m2` to a calling matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. Function is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::operator +` , `srmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    const srmatrix m1(a, 3);
    srmatrix m2(3);
    srmatrix m(3);
    m2.set(1.);

    std::cout << m.sum(m1, m2) << std::endl;
    std::cout << m.sum(m, m2);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
2 5 8
3 6 9
4 7 10

3 6 9
4 7 10
5 8 11
```

2.8.21 diff

Function

```
srmatrix& srmatrix::diff (const srmatrix& m1, const srmatrix& m2)
throw (cvmexception);
```

assigns the result of subtraction of matrices `m1` and `m2` to a calling matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. Function is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::operator -` , `srmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    const srmatrix m1(a, 3);
    srmatrix m2(3);
    srmatrix m(3);
    m2.set(1.);

    std::cout << m.diff(m1, m2) << std::endl;
    std::cout << m.diff(m, m2);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
0 3 6
1 4 7
2 5 8

-1 2 5
0 3 6
1 4 7
```

2.8.22 operator +=

Operator

```
srmatrix& srmatrix::operator += (const srmatrix& m) throw (cvmexception);
```

adds matrix `m` to a calling matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. Operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::operator +` , `srmatrix::sum`, `srmatrix`. Example:

```
using namespace cvm;
try {
    srmatrix m1(3);
    srmatrix m2(3);
    m1.set(1.);
    m2.set(2.);

    m1 += m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 += m2;
    std::cout << m2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3 3
3 3 3
3 3 3

4 4 4
4 4 4
4 4 4
```

2.8.23 operator -=

Operator

```
srmatrix& srmatrix::operator -= (const srmatrix& m) throw (cvmexception);
```

subtracts matrix m from calling matrix and returns a reference to the matrix changed. It throws cvmexception in case of different sizes of the operands. Operator is *redefined* in the classes srbmatrix and srsmatrix. See also srmatrix::operator - , srmatrix::diff, srmatrix. Example:

```
using namespace cvm;

try {
    srmatrix m1(3);
    srmatrix m2(3);
    m1.set(1.);
    m2.set(2.);

    m1 -= m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 -= m2;
    std::cout << m2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-1 -1 -1
-1 -1 -1
-1 -1 -1

0 0 0
0 0 0
0 0 0
```

2.8.24 operator - ()

Operator

```
srmatrix srmatrix::operator - () const throw (cvmexception);
```

creates an object of type `srmatrix` as a calling matrix multiplied by -1. Operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
srmatrix m(a, 3);

std::cout << -m;
```

prints

```
-1 -4 -7
-2 -5 -8
-3 -6 -9
```

2.8.25 operator ++

Operator

```
srmatrix& srmatrix::operator ++ ();
srmatrix& srmatrix::operator ++ (int);
```

adds identity matrix to a calling matrix and returns a reference to the matrix changed. Operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
srmatrix m (a, 3);

m++;
std::cout << m << std::endl;
std::cout << ++m;
```

prints

```
2 4 7
2 6 8
3 6 10

3 4 7
2 7 8
3 6 11
```

2.8.26 operator --

Operator

```
srmatrix& srmatrix::operator -- ();
srmatrix& srmatrix::operator -- (int);
```

subtracts identity matrix from calling matrix and returns a reference to the matrix changed. Operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
srmatrix m (a, 3);

m--;
std::cout << m << std::endl;
std::cout << --m;
```

prints

```
0 4 7
2 4 8
3 6 8

-1 4 7
2 3 8
3 6 7
```

2.8.27 operator * (TR)

Operator

```
srmatrix srmatrix::operator * (TR d) const;
```

creates an object of type `srmatrix` as a product of a calling matrix and number `d`. Operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::operator *=` , `srmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
srmatrix m(a, 3);

std::cout << m * 5.;
```

prints

```
5 20 35
10 25 40
15 30 45
```

2.8.28 operator / (TR)

Operator

```
srmatrix srmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `srmatrix` as a quotient of a calling matrix and number `d`. It throws `cvmexception` if `d` has absolute value equal or less than the smallest normalized positive number.)perator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::operator /=` , `srmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    srmatrix m(a, 3);

    std::cout << m / 4.;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
0.25 1 1.75
0.5 1.25 2
0.75 1.5 2.25
```

2.8.29 operator *= (TR)

Operator

```
srmatrix& srmatrix::operator *= (TR d);
```

multiplies calling matrix by number `d` and returns a reference to the matrix changed. Operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::operator *` , `srmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
srmatrix m(a, 3);

m *= 2.;
std::cout << m;
```

prints

```
2 8 14
4 10 16
6 12 18
```

2.8.30 operator /= (TR)

Operator

`srmatrix& srmatrix::operator /= (TR d) throw (cvmexception);`

divides calling matrix by number `d` and returns a reference to the matrix changed. It throws `cvmexception` if `d` has absolute value equal or less than the smallest normalized positive number. Operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::operator /` , `srmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    srmatrix m(a, 3);

    m /= 2.;
    std::cout << m;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
0.5 2 3.5
1 2.5 4
1.5 3 4.5
```

2.8.31 normalize

Function

```
srmatrix& srmatrix::normalize ();
```

normalizes calling matrix so its Euclidean norm becomes equal to 1 if it was greater than the smallest normalized positive number before the call (otherwise function does nothing). Function is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
srmatrix m(a, 3);

m.normalize();
std::cout << m << m.norm() << std::endl;
```

prints

```
5.923e-02 2.369e-01 4.146e-01
1.185e-01 2.962e-01 4.739e-01
1.777e-01 3.554e-01 5.331e-01
1.000e+00
```

2.8.32 transposition

Operator and functions

```
srmatrix srmatrix::operator ~ () const throw (cvmexception);
srmatrix& srmatrix::transpose (const srmatrix& m) throw (cvmexception);
srmatrix& srmatrix::transpose ();
```

implement matrix transposition. First operator creates an object of type `srmatrix` as transposed calling matrix (it throws `cvmexception` in case of memory allocation failure). Second function sets calling matrix to be equal to matrix `m` transposed (it throws `cvmexception` in case of not appropriate sizes of the operands), third one makes it to be equal to transposed itself. Functions are *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    srmatrix m(a,3);
    srmatrix mt(3);
    std::cout << ~m << std::endl ;
    mt.transpose(m);
    std::cout << mt << std::endl;
    mt.transpose();
    std::cout << mt;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 2 3
4 5 6
7 8 9

1 2 3
4 5 6
7 8 9

1 4 7
2 5 8
3 6 9
```

2.8.33 operator * (const rvector&)

Operator

```
rvector srmatrix::operator * (const rvector& v) const
throw (cvmexception);
```

creates an object of type `rvector` as a product of a calling matrix and a vector `v`. It throws `cvmexception` if number of columns of a calling matrix differs from size of a vector `v`. Use `rvector::mult` in order to avoid new object creation. Function is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`, `rvector`. Example:

```
using namespace cvm;

try {
    srmatrix m(3);
    rvector v(3);
    m.set(1.);
    v.set(1.);

    std::cout << m * v;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3 3
```

2.8.34 operator * (const rmatrix&)

Operator

```
rmatrix srmatrix::operator * (const rmatrix& m) const
throw (cvmexception);
```

creates an object of type `rmatrix` as a product of a calling matrix and a matrix `m`. It throws `cvmexception` if number of columns of a calling matrix differs from number of rows of a matrix `m`. Use `rmatrix::mult` in order to avoid new object creation. Operator is *redefined* in the classes `srbmatrix` and `srsmatrix`. See also `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;

try {
    srmatrix ms(3);
    rmatrix m(3,2);
    ms.set(1.);
    m.set(1.);

    std::cout << ms * m;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3
3 3
3 3
```

2.8.35 operator * (const srmatrix&)

Operator

```
srmatrix srmatrix::operator * (const srmatrix& m) const
throw (cvmexception);
```

creates an object of type `srmatrix` as a product of a calling matrix and a matrix `m`. It throws `cvmexception` if the operands have different sizes. Use `rmatrix::mult` in order to avoid new object creation. Operator is *inherited* in the class `srbmatrix` and *redefined* in `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

try {
    srmatrix m1(3);
    srmatrix m2(3);
    m1.set(1.);
    m2.set(1.);

    std::cout << m1 * m2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3 3
3 3 3
3 3 3
```

2.8.36 operator *= (const srmatrix&)

Operator

```
srmatrix& srmatrix::operator *= (const srmatrix& m)
throw (cvmexception);
```

sets calling matrix to be equal to product of itself and square matrix m and returns a reference to the object it changes. Operator throws cvmexception in case of different sizes of the operands. Operator is *inherited* in the class srbmatrix and *redefined* in srsmatrix. See also srmatrix. Example:

```
using namespace cvm;

try {
    srmatrix m1(3);
    srmatrix m2(3);
    m1.set(1.);
    m2.set(1.);

    m1 *= m2;
    std::cout << m1 << std::endl;
    m1 *= m1;
    std::cout << m1;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
3 3 3
3 3 3
3 3 3

27 27 27
27 27 27
27 27 27
```

2.8.37 swap_rows

Function

```
srmatrix& srmatrix::swap_rows (int n1, int n2) throw (cvmexception);
```

swaps two rows of a calling matrix and returns a reference to the matrix changed. `n1` and `n2` are numbers of rows to be swapped, both are ℓ-based). Function throws `cvmexception` if one of the parameters is outside of the range `[1,msize()]`. Function is *not applicable* to objects of the classes `srbmatrix` and `srsmatrix` (i.e. `cvmexception` would be thrown in case of using it for objects of those classes). See also `srmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    srmatrix m (a, 3);

    std::cout << m << std::endl;
    std::cout << m.swap_rows(2,3);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 4 7
2 5 8
3 6 9

1 4 7
3 6 9
2 5 8
```

2.8.38 swap_cols

Function

`srmatrix& srmatrix::swap_cols (int n1, int n2) throw (cvmexception);`

swaps two columns of a calling matrix and returns a reference to the matrix changed. `n1` and `n2` are numbers of columns to be swapped, both are ℓ-based). Function throws `cvmexception` if one of the parameters is outside of the range `[1,nsize()]`. Function is *not applicable* to objects of the classes `srbmatrix` and `srsmatrix` (i.e. `cvmexception` would be thrown in case of using it for objects of those classes). See also `srmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    srmatrix m (a, 3);

    std::cout << m << std::endl;
    std::cout << m.swap_cols(2,3);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 4 7
2 5 8
3 6 9

1 7 4
2 8 5
3 9 6
```

2.8.39 solve

Functions

```
rvector srmatrix::solve (const rvector& vB) const throw (cvmexception);
rmatrix srmatrix::solve (const rmatrix& mB) const throw (cvmexception);
rvector srmatrix::solve (const rvector& vB, TR& dErr) const
throw (cvmexception);
rmatrix srmatrix::solve (const rmatrix& mB, TR& dErr) const
throw (cvmexception);
```

return solution of linear equation of kind $A * x = b$ or $A * X = B$ where A is calling matrix. The first and third versions solve equation $A * x = b$ where vector b is passed in parameter `vB` and the second and fourth versions solve equation $A * X = B$ where matrix B is passed in parameter `mB`. The last two versions also set output parameter `dErr` to be equal to the norm of computation error. Functions throw `cvmexception` in case of inappropriate sizes of the operands. Function is *inherited* in the classes `srbmatrix` and `srsmatrix`. See also `rvector::solve`, `rmatrix::solve`, `rmatrix::solve_tran`, `srmatrix::solve_tran`, `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    srmatrix ma(a, 3);
    rmatrix  mb(3,4);
    rmatrix  mx(3,4);
    double dErr;

    mb(1).set(1.);
    mb(2).set(2.);
    mb(3).set(3.);
    mb(1,4) = 1.; mb(2,4) = 2.; mb(3,4) = 3.;

    mx = ma.solve (mb, dErr);
    std::cout << mx << dErr
              << std::endl << ma * mx - mb << std::endl;

    rvector vb(3), vx(3);
    vb = mb(2);
    vx = ma.solve (vb, dErr);
    std::cout << vx << dErr << std::endl << ma * vx - vb;
}
catch (std::exception& e) {
```

```
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-3.333e-01 -6.667e-01 -1.000e+00 1.000e+00
3.333e-01 6.667e-01 1.000e+00 0.000e+00
6.661e-16 1.332e-15 0.000e+00 0.000e+00
3.301e-14
0.000e+00 0.000e+00 0.000e+00 0.000e+00
-1.110e-16 -2.220e-16 0.000e+00 0.000e+00
2.220e-16 4.441e-16 0.000e+00 0.000e+00

-6.667e-01 6.667e-01 1.332e-15
3.301e-14
0.000e+00 -2.220e-16 4.441e-16
```

2.8.40 solve_tran

Functions

```
rvector srmatrix::solve_tran (const rvector& vB)
const throw (cvmexception);
rmatrix srmatrix::solve_tran (const rmatrix& mB)
const throw (cvmexception);
rvector srmatrix::solve_tran (const rvector& vB, TR& dErr) const
throw (cvmexception);
rmatrix srmatrix::solve_tran (const rmatrix& mB, TR& dErr) const
throw (cvmexception);
```

return solution of linear equation of kind $A^T * x = b$ or $A^T * X = B$ where A is calling matrix. The first and third versions solve equation $A^T * x = b$ where vector b is passed in parameter `vB` and the second and fourth versions solve equation $A^T * X = B$ where matrix B is passed in parameter `mB`. The last two versions also set output parameter `dErr` to be equal to the norm of computation error. Functions throw `cvmexception` in case of inappropriate sizes of the operands. Function is *inherited* in the classes `srbmatrix` and `srsmatrix`. See also `rvector::solve`, `rmatrix::solve`, `srmatrix::solve`, `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (3);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    srmatrix ma(a, 3);
    rmatrix  mb(3,4);
    rmatrix  mx(3,4);
    double dErr;

    mb(1).set(1.);
    mb(2).set(2.);
    mb(3).set(3.);
    mb(1,4) = 1.; mb(2,4) = 2.; mb(3,4) = 3.;

    mx = ma.solve_tran (mb, dErr);
    std::cout << mx << dErr
              << std::endl << ~ma * mx - mb << std::endl;

    rvector vb(3), vx(3);
    vb = mb(2);
    vx = ma.solve_tran (vb, dErr);
    std::cout << vx << dErr << std::endl << vx * ma - vb;
}
```

```
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-1.000e+000 -2.000e+000 -3.000e+000 -3.333e-001
+1.000e+000 +2.000e+000 +3.000e+000 +6.667e-001
+0.000e+000 +0.000e+000 -1.332e-015 +0.000e+000
+3.513e-014
+0.000e+000 +0.000e+000 +0.000e+000 +0.000e+000
+0.000e+000 +0.000e+000 +8.882e-016 -2.220e-016
+0.000e+000 +0.000e+000 -2.665e-015 +0.000e+000

-2.000e+000 +2.000e+000 +0.000e+000
+3.168e-014
+0.000e+000 +0.000e+000 +0.000e+000
```

2.8.41 operator % (const rvector&)

Operator

```
rvector operator % (const rvector& vB) const throw (cvmexception);
```

returns solution x of linear equation $A^T * x = b$ (which is equivalent to $x * A = b$) where calling matrix is square matrix A and a vector b is passed in parameter `vB`. This operator throws exception of type `cvmexception` in case of inappropriate sizes of the objects or when matrix A is close to singular. See also `rvector::solve_tran`, `srmatrix::solve_tran`, `rvector.operator /`, `rvector`, `srmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (12);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -2., 1.};
    double b[] = {1., 2., 3.};
    srmatrix ma(m, 3);
    rvector  vb(b, 3);
    rvector  vx(3);

    vx = ma % vb;

    std::cout << vx * ma - vb;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+0.000000000000e+000 +0.000000000000e+000 +0.000000000000e+000
```

2.8.42 operator / (const rvector&)

Operator

```
rvector operator / (const rvector& vB) const throw (cvmexception);
```

returns solution x of linear equation $A * x = b$ where calling matrix is square matrix A and a vector b is passed in parameter `vB`. This operator throws exception of type `cvmexception` in case of inappropriate sizes of the objects or when matrix A is close to singular. It's *redefined* in classes `srbmatrix` and `srsmatrix`. See also `rvector::solve`, `srmatrix::solve`, `rvector.operator %`, `rvector`, `srmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (12);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -2., 1.};
    double b[] = {1., 2., 3.};
    srmatrix ma(m, 3);
    rvector  vb(b, 3);
    rvector  vx(3);

    vx = ma / vb;

    std::cout << ma * vx - vb;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+0.000000000000e+000 +0.000000000000e+000 +0.000000000000e+000
```

2.8.43 solve_lu

Functions

```
rvector
srmatrix::solve_lu (const srmatrix& mLU, const int* pPivots,
                    const rvector& vB, TR& dErr) throw (cvmexception);
rvector
srmatrix::solve_lu (const srmatrix& mLU, const int* pPivots,
                    const rvector& vB) throw (cvmexception);

rmatrix
srmatrix::solve_lu (const srmatrix& mLU, const int* pPivots,
                    const rmatrix& mB, TR& dErr) throw (cvmexception);
rmatrix
srmatrix::solve_lu (const srmatrix& mLU, const int* pPivots,
                    const rmatrix& mB) throw (cvmexception);
```

create object of type `rvector` or `rmatrix` as solution x or X of a matrix linear equation $A * x = b$ or $A * X = B$ respectively. Here A is calling matrix, parameter `mLU` is LU factorization of a matrix A, parameter `pPivots` is an array of pivot numbers created while factorizing the matrix A and parameters `vB` and `mB` are vector b and matrix B respectively. The first and third version also set output parameter `dErr` to be equal to the norm of computation error. These functions are useful when you need to solve few linear equations of kind $A * x = b$ or $A * X = B$ with the same matrix A and different vectors b or matrices B. In such case you save on matrix A factorization since it's needed to be performed just one time. Functions throw `cvmexception` in case of inappropriate sizes of the operands or when the matrix A is close to singular. Function is *inherited* in the classes `srbmatrix` and `srsmatrix`. See also `rvector::solve`, `rmatrix`, `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
try {
    double a[] = {1., -1., 1., 2., -2., 1., 3., -2., 1.};
    srmatrix ma(a,3);
    srmatrix mLU(3);
    rmatrix  mb1(3,2); rvector vb1(3);
    rmatrix  mb2(3,2); rvector vb2(3);
    rmatrix  mx1(3,2); rvector vx1(3);
    rmatrix  mx2(3,2); rvector vx2(3);
    iarray   nPivots(3);
    double   dErr = 0.;
    mb1.randomize(-1.,3.); vb1.randomize(-2.,4.);
    mb2.randomize(-2.,5.); vb2.randomize(-3.,1.);
```

```
    mLU.low_up(ma, nPivots);
    mx1 = ma.solve_lu (mLU, nPivots, mb1, dErr);
    std::cout << mx1 << dErr << std::endl;
    mx2 = ma.solve_lu (mLU, nPivots, mb2);
    std::cout << mx2 << std::endl;;
    std::cout << ma * mx1 - mb1 << std::endl << ma * mx2 - mb2;

    vx1 = ma.solve_lu (mLU, nPivots, vb1, dErr);
    std::cout << vx1 << dErr << std::endl;
    vx2 = ma.solve_lu (mLU, nPivots, vb2);
    std::cout << vx2 << std::endl;;
    std::cout << ma * vx1 - vb1 << std::endl << ma * vx2 - vb2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
2.807e+00 1.107e+00
-3.651e-01 -4.843e+00
-5.412e-01 3.095e+00
6.438e-15
-7.639e-01 1.082e+01
-2.869e-01 -1.110e+01
4.890e-01 3.443e+00

0.000e+00 -4.441e-16
1.110e-16 -4.441e-16
-4.441e-16 4.441e-16

0.000e+00 -4.441e-16
0.000e+00 8.882e-16
0.000e+00 -4.441e-16
-1.651e+00 2.361e-01 -6.384e-02
3.828e-15
-5.886e+00 7.038e+00 -3.125e+00

0.000e+00 0.000e+00 0.000e+00

0.000e+00 0.000e+00 2.220e-16
```

2.8.44 det

Function

```
TR srmatrix::det () const throw (cvmexception);
```

returns determinant of a calling matrix. It uses the LU factorization internally and may throw the same exceptions as the factorizer. Function is *inherited* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    srmatrix m(a, 3);

    std::cout << m << std::endl << m.det() << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.000e+00 4.000e+00 7.000e+00
2.000e+00 5.000e+00 8.000e+00
3.000e+00 6.000e+00 1.000e+01

-3.000e+00
```

2.8.45 low_up

Functions

```
srmatrix&
srmatrix::low_up (const srmatrix& m, int* nPivots) throw (cvmexception);
srmatrix
srmatrix::low_up (int* nPivots) const throw (cvmexception);
```

compute the LU factorization of square matrix A as

$$A = PLU$$

where P is permutation matrix, L is lower triangular matrix with unit diagonal elements and U is upper triangular matrix. All functions store result as the matrix L without main diagonal combined with U. All functions return pivot indices as array of integers (it should support at least `msize()` elements) pointed to by `nPivots` so i-th row was interchanged with `nPivots[i]`-th row. The first version sets calling matrix to be equal to matrix `m`'s LU factorization and the second one creates an object of type `srmatrix` as calling matrix LU factorization. Functions throw `cvmexception` in case of inappropriate sizes of the operands or when the matrix to be factorized is close to singular. It is recommended to use `iarray` for pivot values. Function is *redefined* in the class `srbmatrix` and *inherited* in `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    srmatrix m(a, 3);
    srmatrix mLU(3), mLo(3), mUp(3);
    iarray naPivots(3);

    mLU.low_up (m, naPivots);

    mLo.identity ();

    mLo.diag(-2) = mLU.diag(-2);
    mLo.diag(-1) = mLU.diag(-1);
    mUp.diag(0) = mLU.diag(0);
    mUp.diag(1) = mLU.diag(1);
    mUp.diag(2) = mLU.diag(2);

    std::cout << mLo << std::endl << mUp
              << std::endl << naPivots << std::endl;
```

```
    mLU = mLo * mUp;
    for (int i = 3; i >= 1; i--) {
        mLU.swap_rows (i, naPivots[i]);
    }
    std::cout << mLU;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.000e+00 0.000e+00 0.000e+00
3.333e-01 1.000e+00 0.000e+00
6.667e-01 5.000e-01 1.000e+00

3.000e+00 6.000e+00 1.000e+01
0.000e+00 2.000e+00 3.667e+00
0.000e+00 0.000e+00 -5.000e-01

3 3 3

1.000e+00 4.000e+00 7.000e+00
2.000e+00 5.000e+00 8.000e+00
3.000e+00 6.000e+00 1.000e+01
```

2.8.46 cond

Function

```
TR srmatrix::cond () const throw (cvmexception);
```

returns condition number reciprocal of a calling matrix A in the infinity-norm:

$$\kappa_\infty = \|A\|_\infty \|A^{-1}\|_\infty .$$

Less value returned means that matrix A is closer to singular. Zero value returned means estimation underflow or that matrix A is singular. The condition number is used for error analysis of systems of linear equations. Function throws `cvmexception` in case of LAPACK subroutines failure. Function is *inherited* in the classes `srbmatrix` and `srsmatrix`. See also `srmatrix::solve`, `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    srmatrix m(a, 3);
    std::cout << m.cond() << std::endl
              << m.det() << std::endl << std::endl;
    m(3,3) = 10.;
    std::cout << m.cond() << std::endl << m.det() << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
0.000e+00
0.000e+00

7.519e-03
-3.000e+00
```

2.8.47 inv

Functions

```
srmatrix& srmatrix::inv (const srmatrix& m) throw (cvmexception);
srmatrix srmatrix::inv () const throw (cvmexception);
```

implement matrix inversion. The first version sets calling matrix to be equal to `m` inverted and the second one creates an object of type `srmatrix` as inverted calling matrix. Functions throw `cvmexception` in case of inappropriate sizes of the operands or when the matrix to be inverted is close to singular. Function is *redefined* in the class `srsmatrix` and *inherited* in `srbmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (10);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    srmatrix m(a, 3);
    srmatrix mi(3);
    mi.inv (m);
    std::cout << mi << std::endl << mi * m - eye_real(3);
    std::cout << std::endl << mi.inv() * mi - eye_real(3);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-6.6666666667e-01 -6.6666666667e-01 1.0000000000e+00
-1.3333333333e+00 3.6666666667e+00 -2.0000000000e+00
1.0000000000e+00 -2.0000000000e+00 1.0000000000e+00

0.0000000000e+00 0.0000000000e+00 1.7763568394e-15
1.7763568394e-15 3.5527136788e-15 0.0000000000e+00
0.0000000000e+00 0.0000000000e+00 1.7763568394e-15

0.0000000000e+00 1.7763568394e-15 -1.7763568394e-15
-8.8817841970e-16 3.5527136788e-15 -3.5527136788e-15
0.0000000000e+00 0.0000000000e+00 -1.7763568394e-15
```

2.8.48 exp

Functions

```
srmatrix& srmatrix::exp (const srmatrix& m, TR tol = cvmMachSp ())
throw (cvmexception);
```

```
srmatrix srmatrix::exp (TR tol = cvmMachSp ()) const
throw (cvmexception);
```

compute exponent of square matrix using Padé approximation defined as

$$R_{pq}(z) = D_{pq}(z)^{-1} N_{pq}(z) = 1 + z + \cdots + z^p/p!\,,$$

where

$$N_{pq}(z) = \sum_{k=0}^{p} \frac{(p+q-k)!p!}{(p+q)!k!(q-k)!} z^k,$$
$$D_{pq}(z) = \sum_{k=0}^{q} \frac{(p+q-k)!p!}{(p+q)!k!(q-k)!} (-z)^k$$

along with the matrix normalizing as described in [2], p. 572. Functions use `DMEXP` (or `SMEXP` for `float` version) FORTRAN subroutine implementing the algorithm. The first version sets calling matrix to be equal to exponent of `m` and returns a reference to the matrix changed. The second version creates an object of type `srmatrix` as exponent of a calling matrix. The algorithm uses parameter `tol` as $\varepsilon(p,q)$ in order to choose constants p and q so that

$$\varepsilon(p,q) \geqslant 2^{3-(p+q)} \frac{p!q!}{(p+q)!(p+q+1)!}.$$

This parameter is equal to the largest relative spacing by default. Functions throw `cvmexception` in case of inappropriate sizes of the operands or when LAPACK subroutine fails. Functions are *inherited* in the classes `srbmatrix` and `srsmatrix`. The second version is *redefined* in `srbmatrix`. See also `srmatrix`. Example (see [2], p. 567, example 11.2.2):

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (15);
try {
    srmatrix m(2);
    m(1,1) = -49.;
    m(1,2) = 24.;
    m(2,1) = -64.;
    m(2,2) = 31.;
```

```
    std::cout << m << std::endl << m.exp();
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-4.900000000000000e+01 2.400000000000000e+01
-6.400000000000000e+01 3.100000000000000e+01

-7.357587581448284e-01 5.518190996581556e-01
-1.471517599088415e+00 1.103638240715692e+00
```

Matlab output:

```
-7.357587581446907e-001    5.518190996580505e-001
-1.471517599088136e+000    1.103638240715478e+000
```

2.8.49 polynomial

Functions

```
srmatrix& srmatrix::polynom (const srmatrix& m, const rvector& v)
throw (cvmexception);

srmatrix srmatrix::polynom (const rvector& v) const
throw (cvmexception);
```

compute matrix polynomial defined as

$$p(A) = b_0 I + b_1 A + \cdots + b_q A^q$$

using the Horner's rule:

$$p(A) = \sum_{k=0}^{r} B_k (A^s)^k, \quad s = \text{floor}(\sqrt{q}\,),\ r = \text{floor}(q/s)$$

where

$$B_k = \begin{cases} \sum\limits_{i=0}^{s-1} b_{sk+i} A^i, & k = 0, 1, \ldots, r-1 \\ \sum\limits_{i=0}^{q-sr} b_{sr+i} A^i, & k = r. \end{cases}$$

See also [2], p. 568. The coefficients $b_0, b_1, \ldots, b_q$ are passed in the parameter v, where q is equal to `v.size()-1`, so the functions compute matrix polynomial equal to

$$\mathrm{v}[1] * \mathrm{I} + \mathrm{v}[2] * \mathrm{m} + \cdots + \mathrm{v}[\mathrm{v.size}()] * \mathrm{m}^{\mathrm{v.size}()-1}$$

The first version sets calling matrix to be equal to the polynomial of `m` and the second one creates an object of type `srmatrix` as the polynomial of a calling matrix. Functions use `DPOLY` (or `SPOLY` for `float` version) FORTRAN subroutine implementing the Horner's algorithm. Functions throw `cvmexception` in case of inappropriate sizes of the operands. Functions are *inherited* in the class `srbmatrix` and *redefined* in `srsmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (15);
try {
    double a[] = {2.2, 1.3, 1.1, -0.9, 0.2,
                  -0.45, 45, -30, 10, 3, 3.2};
    const rvector v(a, 11);
    srmatrix m(2), mp(2);
    m(1,1) = 1.;
    m(1,2) = 0.5;
```

```
    m(2,1) = -1.;
    m(2,2) = 0.3;

    mp.polynom (m, v);
    std::cout << mp;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-7.963641665999998e+00 -7.551532476200001e+00
1.510306495240000e+01 2.608503800680002e+00
```

Matlab output:

```
-7.963641665999999e+000   -7.551532476200002e+000
 1.510306495240000e+001    2.608503800680002e+000
```

2.8.50 eig

Functions

```
cvector srmatrix::eig (scmatrix& mEigVect, bool bRightVect = true) const
throw (cvmexception);

cvector srmatrix::eig () const throw (cvmexception);
```

solve the nonsymmetric eigenvalue problem and return complex vector with eigenvalues of a calling matrix. The first version sets output parameter `mEigVect` to be equal to square matrix containing right (if parameter `bRightVect` is true, which is default value) or left (if parameter `bRightVect` is false) eigenvectors as columns. All functions throw `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. Functions are *inherited* in the class `srbmatrix` and *redefined* in `srsmatrix`. See also `cvector`, `scmatrix` and `srmatrix`. Example:

```
using namespace cvm;
try {
    scmatrix m(3), me(3);
    cvector vl(3);

    m(1,1) = 0.1;  m(1,2) = 0.2;  m(1,3) = 0.1;
    m(2,1) = 0.11; m(2,2) = -2.9; m(2,3) = -8.4;
    m(3,1) = 0.;   m(3,2) = 2.91; m(3,3) = 8.2;

    vl = m.eig (me);
    std::cout << vl;

    m(2,2) = 2.9;
    vl = m.eig (me);
    std::cout << vl << std::endl;

    std::cout.setf (std::ios::scientific | std::ios::showpos);
    std::cout.precision (1);

    std::cout << m * me(1) - me(1) * vl(1);
    std::cout << m * me(2) - me(2) * vl(2);
    std::cout << m * me(3) - me(3) * vl(3);
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(-0.0555784,0) (0.285327,0) (5.17025,0)
(0.0968985,-1.38778e-017) (5.55155,4.1733) (5.55155,-4.1733)
```

```
(+1.4e-017,+2.8e-017) (-1.0e-016,-5.4e-017) (+3.9e-017,+4.3e-017)
(-5.0e-016,+1.7e-016) (-7.1e-015,+2.9e-015) (+3.6e-015,+2.7e-015)
(-3.1e-016,+2.8e-017) (-2.7e-015,-8.9e-016) (+8.9e-016,+3.6e-015)
```

2.8.51 Cholesky

Function

```
srmatrix& srmatrix::cholesky (const srsmatrix& m)
throw (cvmexception);
```

forms the Cholesky factorization of symmetric positive-definite matrix A defined as

$$A = U^T U,$$

where U is upper triangular matrix. It utilizes one of ?POTRF routines of the LAPACK library. Function sets calling matrix to be equal to the factorization of symmetric positive-definite matrix m. Function throws cvmexception in case of inappropriate sizes of the operands or in case of convergence error. See also srmatrix and srsmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 1., 2., 5., -1., 1., -1., 20.};
    const srsmatrix m(a, 3);
    srmatrix h(3);

    h.cholesky(m);
    std::cout << h << std::endl;
    std::cout << ~h * h - m;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 2 1
0 1 -3
0 0 3.16228

0 0 0
0 0 0
0 0 0
```

2.8.52 Bunch-Kaufman

Function

```
srmatrix& srmatrix::bunch_kaufman (const srsmatrix& m, int* pivots)
throw (cvmexception);
```

forms the Bunch-Kaufman factorization of symmetric matrix (cited from the MKL library documentation):

$$A = PUDU^T P^T,$$

where A is the input matrix passed in parameter `m`, P is a permutation matrix, U and L are upper and lower triangular matrices with unit diagonal, and D is a symmetric block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. U and L have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of D. It utilizes one of `?SYTRF` routines of the LAPACK library. Function sets calling matrix to be equal to the factorization of symmetric positive-definite matrix `m`. Function throws `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. See also `srmatrix` and `srsmatrix`. Function is mostly designed to be used for subsequent calls of `?SYTRS`, `?SYCON` and `?SYTRI` routines of the LAPACK library. Currently it's used internally in `srmatrix::det` flow when argument is symmetric but not positive-definite.

2.8.53 qr

Function

```
void srmatrix::qr (srmatrix& mQ, srmatrix& mR) const throw (cvmexception);
```

computes QR factorization as

$$M = QR$$

where M is calling square matrix, orthogonal matrix Q and upper triangular matrix R are `mQ` and `mR` respectively. Function throws `cvmexception` in case if inappropriate sizes of the operands passed. See also `rmatrix`, `rmatrix::qr`, `srmatrix`. Example:

```
using namespace cvm;

treal a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
const cvm::srmatrix m(a, 3);
cvm::srmatrix q(3), r(3);

m.qr(q,r);
std::cout << (eye_real(3) - ~q * q).norm()
          << " " << (m - q * r).norm() << std::endl;
```

prints

```
+5.2889959e-16 +7.0854500e-15
```

2.8.54 lq

Function

`void srmatrix::lq (srmatrix& mL, srmatrix& mQ) const throw (cvmexception);`

computes LQ factorization as

$$M = LQ$$

where M is calling square matrix, lower triangular matrix L and orthogonal matrix Q are `mL` and `mQ` respectively. Function throws `cvmexception` in case if inappropriate sizes of the operands passed. See also `rmatrix`, `rmatrix::lq`, `srmatrix`. Example:

```
using namespace cvm;

treal a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
const cvm::srmatrix m(a, 3);
cvm::srmatrix l(3), q(3);

m.lq(l,q);
std::cout << (eye_real(3) - ~q * q).norm()
          << " " << (m - l * q).norm() << std::endl;
```

prints

```
+7.3329369e-016 +8.1523942e-015
```

2.8.55 ql

Function

```
void srmatrix::ql (srmatrix& mQ, srmatrix& mL) const throw (cvmexception);
```

computes QL factorization as

$$M = QL$$

where M is calling square matrix, orthogonal matrix Q and lower triangular matrix L are `mQ` and `mL` respectively. Function throws `cvmexception` in case if inappropriate sizes of the operands passed. See also `rmatrix`, `rmatrix::ql`, `srmatrix`. Example:

```
using namespace cvm;

treal a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
const cvm::srmatrix m(a, 3);
cvm::srmatrix q(3), l(3);

m.ql(q,l);
std::cout << (eye_real(3) - ~q * q).norm()
          << " " << (m - q * l).norm() << std::endl;
```

prints

```
+1.6146017e-015 +4.3341378e-015
```

2.8.56 rq

Function

```
void srmatrix::rq (srmatrix& mR, srmatrix& mQ) const throw (cvmexception);
```

computes RQ factorization as

$$M = RQ$$

where M is calling square matrix, upper triangular matrix R and orthogonal matrix Q are mR and mQ respectively. Function throws cvmexception in case if inappropriate sizes of the operands passed. See also rmatrix, rmatrix::rq, srmatrix. Example:

```
using namespace cvm;

treal a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
const cvm::srmatrix m(a, 3);
cvm::srmatrix r(3), q(3);

m.rq(r,q);
std::cout << (eye_real(3) - ~q * q).norm()
          << " " << (m - r * q).norm() << std::endl;
```

prints

```
+3.7030557e-016 +2.2752801e-015
```

2.8.57 identity

Function

```
srmatrix& srmatrix::identity();
```

sets calling matrix to be equal to identity matrix and returns a reference to the matrix changed. Function is *redefined* in the classes srbmatrix and srsmatrix. See also srmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
srmatrix m(3);
m.randomize(0.,1.);

std::cout << m << std::endl;
std::cout << m.identity();
```

prints

```
9.423e-01 2.950e-01 8.429e-01
2.013e-01 3.250e-01 2.904e-01
7.920e-01 2.405e-02 7.801e-01

1.000e+00 0.000e+00 0.000e+00
0.000e+00 1.000e+00 0.000e+00
0.000e+00 0.000e+00 1.000e+00
```

2.8.58 vanish

Function

```
srmatrix& srmatrix::vanish();
```

sets every element of a calling matrix to be equal to zero and returns a reference to the matrix changed. This function is faster than `srmatrix::set(TR)` with zero operand passed. Function is *redefined* in the classes `srsmatrix` and `srbmatrix`. See also `srmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
srmatrix m(3);
m.randomize(0.,1.);

std::cout << m << std::endl;
std::cout << m.vanish ();
```

prints

```
1.747e-01 7.563e-01 5.163e-01
9.657e-01 6.619e-01 8.036e-01
6.392e-01 6.658e-01 6.495e-01

0.000e+00 0.000e+00 0.000e+00
0.000e+00 0.000e+00 0.000e+00
0.000e+00 0.000e+00 0.000e+00
```

2.8.59 randomize

Function

`srmatrix& srmatrix::randomize (TR dFrom, TR dTo);`

fills calling matrix with pseudo-random numbers distributed between `dFrom` and `dTo`. Function returns a reference to the matrix changed. See also `srmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (7);

srmatrix m(3);
m.randomize(-2.,3.);
std::cout << m;
```

prints

```
-1.6790979e+00 5.0233467e-02 -1.9559008e+00
-1.7987609e-01 -5.2092044e-01 -1.8211615e+00
6.8242439e-01 9.0688803e-01 -1.7891171e+00
```

2.9 scmatrix

This is end-user class encapsulating square matrix of complex numbers.

```
template <typename TR, typename TC>
class scmatrix : public cmatrix <TR,TC>, public SqMatrix <TR,TC> {
public:
    scmatrix ();
    explicit scmatrix (int nMN);
    scmatrix (TC* pD, int nMN);
    scmatrix (const TC* pD, int nMN);
    scmatrix (const scmatrix& m);
    explicit scmatrix (const cmatrix& m);
    explicit scmatrix (const cvector& v);
    explicit scmatrix (const srmatrix& m, bool bRealPart = true);
    scmatrix (const TR* pRe, const TR* pIm, int nMN);
    scmatrix (const srmatrix& mRe, const srmatrix& mIm);
    scmatrix (cmatrix& m, int nRow, int nCol, int nSize);
    TC& operator () (int im, int in) throw (cvmexception);
    TC operator () (int im, int in) const throw (cvmexception);
    cvector operator () (int i) throw (cvmexception);
    const cvector operator () (int i) const throw (cvmexception);
    cvector operator [] (int i) throw (cvmexception);
    const cvector operator [] (int i) const throw (cvmexception);
    const srmatrix real () const;
    const srmatrix imag () const;
    scmatrix& operator = (const scmatrix& m) throw (cvmexception);
    scmatrix& assign (const cvector& v) throw (cvmexception);
    scmatrix& assign (const TC* pD);
    scmatrix& assign (int nRow, int nCol, const cmatrix& m)
                      throw (cvmexception);
    scmatrix& set (TC x);
    scmatrix& assign_real (const srmatrix& mRe) throw (cvmexception);
    scmatrix& assign_imag (const srmatrix& mIm) throw (cvmexception);
    scmatrix& set_real (TR d);
    scmatrix& set_imag (TR d);
    scmatrix& resize (int nNewMN) throw (cvmexception);
    scmatrix& operator << (const scmatrix& m) throw (cvmexception);
    scmatrix operator + (const scmatrix& m) const
                         throw (cvmexception);
    scmatrix operator - (const scmatrix& m) const
                         throw (cvmexception);
    scmatrix& sum (const scmatrix& m1,
                   const scmatrix& m2) throw (cvmexception);
    scmatrix& diff (const scmatrix& m1,
```

```
                const scmatrix& m2) throw (cvmexception);
scmatrix& operator += (const scmatrix& m) throw (cvmexception);
scmatrix& operator -= (const scmatrix& m) throw (cvmexception);
scmatrix operator - () const;
scmatrix& operator ++ ();
scmatrix& operator ++ (int);
scmatrix& operator -- ();
scmatrix& operator -- (int);
scmatrix operator * (TR d) const;
scmatrix operator / (TR d) const throw (cvmexception);
scmatrix operator * (TC z) const;
scmatrix operator / (TC z) const throw (cvmexception);
scmatrix& operator *= (TR d);
scmatrix& operator /= (TR d) throw (cvmexception);
scmatrix& operator *= (TC z);
scmatrix& operator /= (TC z) throw (cvmexception);
scmatrix& normalize ();
scmatrix operator ~ () const;
scmatrix operator ! () const;
scmatrix& conj (const scmatrix& m) throw (cvmexception);
scmatrix& conj ();
scmatrix& transpose (const scmatrix& m) throw (cvmexception);
scmatrix& transpose ();
cvector operator * (const cvector& v) const
                    throw (cvmexception);
cmatrix operator * (const cmatrix& m) const
                    throw (cvmexception);
scmatrix operator * (const scmatrix& m) const
                     throw (cvmexception);
scmatrix& operator *= (const scmatrix& m)
                       throw (cvmexception);
scmatrix& swap_rows (int n1, int n2) throw (cvmexception);
scmatrix& swap_cols (int n1, int n2) throw (cvmexception);
cvector solve (const cvector& vB) const throw (cvmexception);
cmatrix solve (const cmatrix& mB) const throw (cvmexception);
cvector solve (const cvector& vB, TR& dErr) const
               throw (cvmexception);
cmatrix solve (const cmatrix& mB, TR& dErr) const
               throw (cvmexception);
cvector solve_tran (const cvector& vB) const throw (cvmexception);
cmatrix solve_tran (const cmatrix& mB) const throw (cvmexception);
cvector solve_tran (const cvector& vB, TR& dErr) const
                    throw (cvmexception);
cmatrix solve_tran (const cmatrix& mB, TR& dErr) const
                    throw (cvmexception);
```

```
    cvector solve_conj (const cvector& vB) const throw (cvmexception);
    cmatrix solve_conj (const cmatrix& mB) const throw (cvmexception);
    cvector solve_conj (const cvector& vB, TR& dErr) const
                        throw (cvmexception);
    cmatrix solve_conj (const cmatrix& mB, TR& dErr) const
                        throw (cvmexception);
    cvector operator % (const cvector& vB) const throw (cvmexception);
    cvector operator / (const cvector& vB) const throw (cvmexception);
    cvector solve_lu (const scmatrix& mLU, const int* pPivots,
                      const cvector& vB, TR& dErr) throw (cvmexception);
    cvector solve_lu (const scmatrix& mLU, const int* pPivots,
                      const cvector& vB) throw (cvmexception);
    cmatrix solve_lu (const scmatrix& mLU, const int* pPivots,
                      const cmatrix& mB, TR& dErr) throw (cvmexception);
    cmatrix solve_lu (const scmatrix& mLU, const int* pPivots,
                      const cmatrix& mB) throw (cvmexception);
    TC det () const throw (cvmexception);
    scmatrix& low_up (const scmatrix& m,
                      int* nPivots) throw (cvmexception);
    scmatrix low_up (int* nPivots) const throw (cvmexception);
    TR cond () const throw (cvmexception);
    scmatrix& inv (const scmatrix& mArg) throw (cvmexception);
    scmatrix inv () const throw (cvmexception);
    scmatrix& exp (const scmatrix& mArg, TR tol = cvmMachSp ())
                   throw (cvmexception);
    scmatrix exp (TR tol = cvmMachSp ()) const throw (cvmexception);
    scmatrix& polynom (const scmatrix& m, const cvector& v)
                       throw (cvmexception);
    scmatrix polynom (const cvector& v) const
                      throw (cvmexception);
    cvector eig (scmatrix& mEigVect, bool bRightVect = true) const
                 throw (cvmexception);
    cvector eig () const throw (cvmexception);
    scmatrix& cholesky (const schmatrix& m) throw (cvmexception);
    scmatrix& bunch_kaufman (const schmatrix& m,
                             int* pivots) throw (cvmexception);
    void qr(scmatrix& mQ, scmatrix& mR) const throw (cvmexception);
    void lq(scmatrix& mL, scmatrix& mQ) const throw (cvmexception);
    void rq(scmatrix& mR, scmatrix& mQ) const throw (cvmexception);
    void ql(scmatrix& mQ, scmatrix& mL) const throw (cvmexception);
    scmatrix& identity ();
    scmatrix& vanish ();
    scmatrix& randomize_real (TR dFrom, TR dTo);
    scmatrix& randomize_imag (TR dFrom, TR dTo);
};
```

2.9.1 scmatrix ()

Constructor

```
scmatrix::scmatrix ();
```

creates empty scmatrix object. See also scmatrix. Example:

```
using namespace cvm;

scmatrix m;
std::cout << m.msize() << std::endl
          << m.nsize() << std::endl
          << m.size() << std::endl;
m.resize(3);
std::cout << m;
```

prints

```
0
0
0
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.9.2 scmatrix (int)

Constructor

```
explicit scmatrix::scmatrix (int nMN);
```

creates $n \times n$ `scmatrix` object where n is passed in `nMN` parameter. Constructor throws `cvmexception` in case of non-positive size passed or memory allocation failure. See also `scmatrix`. Example:

```
using namespace cvm;

scmatrix m (4);
std::cout << m.msize() << std::endl
          << m.nsize() << std::endl
          << m.size() << std::endl << m;
```

prints

```
4
4
16
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
```

2.9.3 scmatrix (TC*,int)

Constructor

```
scmatrix::scmatrix (TC* pD, int nMN);
```

creates $n \times n$ `scmatrix` object where n is passed in `nMN` parameter. Unlike others, this constructor *does not allocate memory*. It just shares memory with an array pointed to by `pD`. Constructor throws `cvmexception` in case of non-positive size passed. See also `scmatrix`, `scmatrix (const TC*,int)`. Example:

```
using namespace cvm;
double a[] = {1., 1., 1., 1., 1., 1., 1., 1., 1.,
              1., 1., 1., 1., 1., 1., 1., 1., 1.};
scmatrix m ((std::complex<double>*) a, 3);

m(1,1) = std::complex<double>(2.,3.);
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << " "
          << a[3] << " " << a[4] << " " << a[5] << " " << std::endl;
```

prints

```
(2,3) (1,1) (1,1)
(1,1) (1,1) (1,1)
(1,1) (1,1) (1,1)

2 3 1 1 1 1
```

2.9.4 scmatrix (const TC*,int)

Constructor

```
scmatrix::scmatrix (const TC* pD, int nMN);
```

creates $n \times n$ `scmatrix` object where n is passed in `nMN` parameter and copies $n*n$ elements of an array `pD` to it `by colums`. Constructor throws `cvmexception` in case of non-positive size passed. See also `scmatrix`, `scmatrix (TC*,int)`. Example:

```
using namespace cvm;
const double a[] = {1., 1., 1., 1., 1., 1., 1., 1., 1.,
                    1., 1., 1., 1., 1., 1., 1., 1., 1.};
scmatrix m ((const std::complex<double>*) a, 3);

m(1,1) = std::complex<double>(2.,3.);
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << " "
          << a[3] << " " << a[4] << " " << a[5] << " " << std::endl;
```

prints

```
(2,3) (1,1) (1,1)
(1,1) (1,1) (1,1)
(1,1) (1,1) (1,1)

1 1 1 1 1 1
```

2.9.5 scmatrix (const scmatrix&)

Copy constructor

```
scmatrix::scmatrix (const scmatrix& m)
```

creates `scmatrix` object as a copy of `m`. Constructor throws `cvmexception` in case of memory allocation failure. See also `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
scmatrix m ((std::complex<double>*) a, 2);
scmatrix mc(m);

m(1,1) = std::complex<double>(7.77,7.77);
std::cout << m << std::endl << mc;
```

prints

```
(7.77,7.77) (5,6)
(3,4) (7,8)

(1,2) (5,6)
(3,4) (7,8)
```

2.9.6 scmatrix (const cmatrix&)

Constructor

`explicit scmatrix::scmatrix (const cmatrix& m)`

creates `scmatrix` object as a copy of a matrix `m`. It's assumed that $m \times n$ matrix `m` must have equal sizes, i.e. $m = n$ is satisfied. Cnstructor throws `cvmexception` if this is not true or in case of memory allocation failure. Please note that this constructor is *not explicit* anymore. See also `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.,
              7., 8., 9., 10., 11., 12.};
cmatrix m((std::complex<double>*) a, 2, 3);
std::cout << m << std::endl;

m.resize(3, 3);
scmatrix ms (m);
std::cout << ms;
```

prints

```
(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)

(1,2) (5,6) (9,10)
(3,4) (7,8) (11,12)
(0,0) (0,0) (0,0)
```

2.9.7 scmatrix (const cvector&)

Constructor

```
explicit scmatrix::scmatrix (const cvector& v);
```

creates `scmatrix` object of size `v.size()` by `v.size()` and assigns vector `v` to its main diagonal. Constructor throws `cvmexception` in case of memory allocation failure. See also `scmatrix`, `cvector`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
cvector v((std::complex<double>*) a, 4);
scmatrix m(v);
std::cout << m;
```

prints

```
(1,2) (0,0) (0,0) (0,0)
(0,0) (3,4) (0,0) (0,0)
(0,0) (0,0) (5,6) (0,0)
(0,0) (0,0) (0,0) (7,8)
```

2.9.8 scmatrix (const srmatrix&,bool)

Constructor

```
explicit scmatrix::scmatrix (const srmatrix& m, bool bRealPart = true);
```

creates `scmatrix` object having the same dimension as real matrix `m` and copies matrix `m` to its real part if `bRealPart` is `true` or to its imaginary part otherwise. See also `scmatrix`, `srmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
const srmatrix m(a, 3);
scmatrix mr(m), mi(m, false);
std::cout << mr << std::endl << mi;
```

prints

```
(1,0) (4,0) (7,0)
(2,0) (5,0) (8,0)
(3,0) (6,0) (9,0)

(0,1) (0,4) (0,7)
(0,2) (0,5) (0,8)
(0,3) (0,6) (0,9)
```

2.9.9 scmatrix (const TR*,const TR*,int)

Constructor

```
scmatrix::scmatrix (const TR* pRe, const TR1* pIm, int nMN);
```

creates `scmatrix` object of size `nMN` by `nMN` and copies every element of arrays pointed to by `pRe` and `pIm` to real and imaginary part of the matrix created respectively. Use `NULL` pointer to fill up appropriate part with zero values. Constructor throws `cvmexception` in case of memory allocation failure. See also `scmatrix`. Example:

```
using namespace cvm;

double re[] = {1., 2., 3., 4.};
double im[] = {4., 3., 2., 1.};
scmatrix m(re, im, 2);
std::cout << m << std::endl;
re[0] = 7.777;
std::cout << m << std::endl;

const double rec[] = {1., 2., 3., 4.};
const scmatrix mc (rec, NULL, 2);
std::cout << mc;
```

prints

```
(1,4) (3,2)
(2,3) (4,1)

(1,4) (3,2)
(2,3) (4,1)

(1,0) (3,0)
(2,0) (4,0)
```

2.9.10 scmatrix (const srmatrix&, const srmatrix&)

Constructor

```
scmatrix::scmatrix (const srmatrix& mRe, const srmatrix& mIm);
```

creates `scmatrix` object of the same size as `mRe` and `mIm` has (it throws `cvmexception` if `mRe` and `mIm` have different sizes) and copies matrices `mRe` and `mIm` to real and imaginary part of the matrix created. Constructor throws `cvmexception` in case of memory allocation failure. See also `scmatrix`, `srmatrix`. Example:

```
using namespace cvm;

srmatrix mr(3), mi(3);
mr.set(1.);
mi.set(2.);
const scmatrix mc(mr, mi);
std::cout << mc;
```

prints

```
(1,2) (1,2) (1,2)
(1,2) (1,2) (1,2)
(1,2) (1,2) (1,2)
```

2.9.11 submatrix

Submatrix constructor

```
scmatrix::scmatrix (cmatrix& m, int nRow, int nCol, int nSize);
```

creates `scmatrix` object as *submatrix* of `m`. It means that the matrix object created shares memory with some part of `m`. This part is defined by its upper left corner (parameters `nRow` and `nCol`, both are ℓ-based) and its size (parameter `nSize`). See also `scmatrix`. Example:

```
using namespace cvm;

cmatrix m(4,5);
scmatrix subm(m,2,2,2);
subm.set(std::complex<double>(1.,2.));
std::cout << m;
```

prints

```
(0,0) (0,0) (0,0) (0,0) (0,0)
(0,0) (1,2) (1,2) (0,0) (0,0)
(0,0) (1,2) (1,2) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0) (0,0)
```

2.9.12 operator (,)

Indexing operators

```
TC& scmatrix::operator () (int im, int in) throw (cvmexception);
TC scmatrix::operator () (int im, int in) const throw (cvmexception);
```

provide access to a particular element of a calling matrix. The first version of operator is applicable to non-constant object. This version returns *l-value* in order to make possible write access to an element. Both operators are ℓ-based. Operators throw `cvmexception` if some of parameters passed is outside of `[1,msize()]` range. Operators are *inherited* in the class `scbmatrix` and *redefined* in the class `schmatrix`. See also `scmatrix`, `Matrix::msize()`, `Matrix::nsize()`. Example:

```
using namespace cvm;

scmatrix m (3);
m.set(std::complex<double>(1.,2.));
std::cout << m(1,1) << std::endl;

m(2,2) = std::complex<double>(7.77,7.77);
std::cout << m;
```

prints

```
(1,2)
(1,2) (1,2) (1,2)
(1,2) (7.77,7.77) (1,2)
(1,2) (1,2) (1,2)
```

2.9.13 operator ()

Indexing operators

```
cvector scmatrix::operator () (int i) throw (cvmexception);
const rcector scmatrix::operator () (int i) const throw (cvmexception);
```

provide access to i-th column of a calling matrix. The first version of operator is applicable to non-constant object and *returns l-value*, i.e. vector returned shares memory with i-th column of a calling matrix in order to make possible write access to it. The second version creates *copy* of a column and therefore is *not l-value*. Both operators are ℓ-based. Operators throw `cvmexception` if parameter `i` is outside of `[1,nsize()]` range. Operators are *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                  10., 11., 12., 13., 14., 15., 16., 17., 18.};
    const scmatrix m ((std::complex<double>*)a, 3);
    scmatrix ms(3);
    std::cout << m(2) << std::endl;

    ms(2) = m(3);
    std::cout << ms;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(7,8) (9,10) (11,12)

(0,0) (13,14) (0,0)
(0,0) (15,16) (0,0)
(0,0) (17,18) (0,0)
```

2.9.14 operator []

Indexing operators

```
cvector scmatrix::operator [] (int i) throw (cvmexception);
const cvector scmatrix::operator [] (int i) const throw (cvmexception);
```

provide access to i-th row of a calling matrix. The first version is applicable to non-constant object and *returns l-value*, i.e. vector returned shares memory with i-th row of a calling matrix in order to make possible write access to it. The second version creates *copy* of a row and therefore is *not l-value*. Both operators are ℓ-based. Operators throw `cvmexception` if parameter `i` is outside of `[1,msize()]` range. Operators are *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`, `Matrix::msize()`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                  10., 11., 12., 13., 14., 15., 16., 17., 18.};
    const scmatrix m ((std::complex<double>*)a, 3);
    scmatrix ms(3);
    std::cout << m[2] << std::endl;

    ms[2] = m[3];
    std::cout << ms;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(3,4) (9,10) (15,16)

(0,0) (0,0) (0,0)
(5,6) (11,12) (17,18)
(0,0) (0,0) (0,0)
```

2.9.15 real

Function

```
const srmatrix scmatrix::real () const;
```

creates an object of type `const srmatrix` as real part of a calling matrix. Please note that, unlike cvector::real, this function creates new object *not sharing* memory with real part of a calling matrix, i.e. matrix returned is *not l-value*. Function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `srmatrix`, `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
scmatrix m((std::complex<double>*) a, 2);
std::cout << m << std::endl << m.real();
```

prints

```
(1,2) (5,6)
(3,4) (7,8)

1 5
3 7
```

2.9.16 imag

Function

```
const srmatrix scmatrix::imag () const;
```

creates an object of type `const srmatrix` as imaginary part of a calling matrix. Please note that, unlike cvector::imag, this function creates new object *not sharing* memory with imaginary part of a calling matrix, i.e. matrix returned is *not l-value*. Function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `srmatrix`, `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
scmatrix m((std::complex<double>*) a, 2);
std::cout << m << std::endl << m.imag();
```

prints

```
(1,2) (5,6)
(3,4) (7,8)

2 6
4 8
```

2.9.17 operator = (const scmatrix&)

Operator

```
scmatrix& scmatrix::operator = (const scmatrix& m)
throw (cvmexception);
```

sets every element of a calling matrix to a value of appropriate element of a matrix m and returns a reference to the matrix changed. Operator throws cvmexception in case of different matrix sizes. Operator is *redefined* in the classes scbmatrix and schmatrix. See also scmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    const scmatrix m1((std::complex<double>*) a, 2);
    scmatrix m2(2);

    m2 = m1;
    std::cout << m2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (5,6)
(3,4) (7,8)
```

2.9.18 assign (const TC*)

Function

```
scmatrix& scmatrix::assign (const cvector& v) throw (cvmexception);
scmatrix& scmatrix::assign (const TC* pD);
```

sets every element of a calling matrix to a value of appropriate element of vector `v` or array pointed to by `pD` and returns a reference to the matrix changed. In first version it's assumed that vector passed is long enough to fill calling matrix. Function throws `cvmexception` otherwise. Function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m(3);
m.assign((std::complex<double>*) a);
std::cout << m;
```

prints

```
(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)
```

2.9.19 assign (int, int, const cmatrix&)

Function

```
scmatrix& scmatrix::assign (int nRow, int nCol, const cmatrix& m)
throw (cvmexception);
```

sets sub-matrix of a calling matrix beginning with ℓ-based row `nRow` and column `nCol` to a matrix `m` and returns a reference to the matrix changed. Function throws `cvmexception` if `nRow` or `nCol` are not positive or matrix `m` doesn't fit. Function is *redefined* in the class `schmatrix`. See also `cmatrix`, `scmatrix`. Example:

```
using namespace cvm;

scmatrix m1(5);
cmatrix m2(2,3);
m1.set(std::complex<double>(1.,1.));
m2.set(std::complex<double>(2.,2.));
m1.assign(2,3,m2);
std::cout << m1;
```

prints

```
(1,1) (1,1) (1,1) (1,1) (1,1)
(1,1) (1,1) (2,2) (2,2) (2,2)
(1,1) (1,1) (2,2) (2,2) (2,2)
(1,1) (1,1) (1,1) (1,1) (1,1)
(1,1) (1,1) (1,1) (1,1) (1,1)
```

2.9.20 set (TC)

Function

```
scmatrix& scmatrix::set (TC x);
```

sets every element of a calling matrix to a value of parameter `x` and returns a reference to the matrix changed. Use `vanish` to set every element of a calling matrix to be equal to zero. Function is *redefined* in the classes `scbmatrix` and *not applicable* to objects of the class `srsmatrix` (i.e. `cvmexception` would be thrown in case of using it for objects of this class). See also `scmatrix`. Example:

```
using namespace cvm;

scmatrix m(3);
m.set(std::complex<double>(1.,2.));
std::cout << m;
```

prints

```
(1,2) (1,2) (1,2)
(1,2) (1,2) (1,2)
(1,2) (1,2) (1,2)
```

2.9.21 assign_real

Function

```
scmatrix& scmatrix::assign_real (const srmatrix& mRe)
throw (cvmexception);
```

sets real part of every element of a calling matrix to a value of appropriate element of a matrix `mRe` and returns a reference to the matrix changed. Function throws `cvmexception` in case of different sizes of the operands. Function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix` and `srmatrix`. Example:

```
using namespace cvm;
scmatrix mc(3);
srmatrix m(3);
m.set(1.);
mc.assign_real(m);
std::cout << mc;
```

prints

```
(1,0) (1,0) (1,0)
(1,0) (1,0) (1,0)
(1,0) (1,0) (1,0)
```

2.9.22 assign_imag

Function

```
scmatrix& scmatrix::assign_imag (const srmatrix& mIm)
throw (cvmexception);
```

sets imaginary part of every element of a calling matrix to a value of appropriate element of a matrix `mIm` and returns a reference to the matrix changed. Function throws `cvmexception` in case of different sizes of the operands. Function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix` and `srmatrix`. Example:

```
using namespace cvm;
scmatrix mc(3);
srmatrix m(3);
m.set(1.);
mc.assign_imag(m);
std::cout << mc;
```

prints

```
(0,1) (0,1) (0,1)
(0,1) (0,1) (0,1)
(0,1) (0,1) (0,1)
```

2.9.23 set_real

Function

```
scmatrix& scmatrix::set_real (TR d);
```

sets real part of every element of a calling matrix to a value of parameter d and returns a reference to the matrix changed. Function is *redefined* in the classes scbmatrix and schmatrix. See also scmatrix. Example:

```
using namespace cvm;

scmatrix m(3);
m.set_real(1.);
std::cout << m;
```

prints

```
(1,0) (1,0) (1,0)
(1,0) (1,0) (1,0)
(1,0) (1,0) (1,0)
```

2.9.24 set_imag

Function

```
scmatrix& scmatrix::set_imag (TR d);
```

sets imaginary part of every element of a calling matrix to a value of parameter `d` and returns a reference to the matrix changed. Function is *redefined* in the class `scbmatrix` and *not allowed* in `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

scmatrix m(3);
m.set_imag(1.);
std::cout << m;
```

prints

```
(0,1) (0,1) (0,1)
(0,1) (0,1) (0,1)
(0,1) (0,1) (0,1)
```

2.9.25 resize

Function

```
scmatrix& scmatrix::resize (int nNewMN);
throw (cvmexception);
```

changes size of a calling matrix to nNewMN by nNewMN and returns a reference to the matrix changed. In case of increasing of its size, the matrix is filled up with zeroes. Function throws cvmexception in case of negative size passed or memory allocation failure. Function is *redefined* in the classes scbmatrix and schmatrix. See also scmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    scmatrix m((std::complex<double>*) a, 2);

    std::cout << m << std::endl;
    m.resize (3);
    std::cout << m;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (5,6)
(3,4) (7,8)

(1,2) (5,6) (0,0)
(3,4) (7,8) (0,0)
(0,0) (0,0) (0,0)
```

2.9.26 operator <<

Operator

```
scmatrix& scmatrix::operator << (const scmatrix& m)
throw (cvmexception);
```

destroys calling matrix, creates new one as a copy of `m` and returns a reference to the matrix changed. Operator throws `cvmexception` in case of memory allocation failure. Operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

try {
    scmatrix m(3);
    scmatrix mc(1);
    m(1,2) = 1.;
    m(2,3) = 2.;
    std::cout << m << std::endl << mc << std::endl;

    mc << m;
    std::cout << mc;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,0) (1,0) (0,0)
(0,0) (0,0) (2,0)
(0,0) (0,0) (0,0)

(0,0)

(0,0) (1,0) (0,0)
(0,0) (0,0) (2,0)
(0,0) (0,0) (0,0)
```

2.9.27 operator +

Operator

```
scmatrix scmatrix::operator + (const scmatrix& m) const
throw (cvmexception);
```

creates an object of type `scmatrix` as a sum of a calling matrix and a matrix `m`. Operator throws `cvmexception` in case of different sizes of the operands. Operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::sum`, `scmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    double b[] = {10., 20., 30., 40., 50., 60., 70., 80.};
    scmatrix m1((std::complex<double>*) a, 2);
    scmatrix m2((std::complex<double>*) b, 2);

    std::cout << m1 + m2 << std::endl << m1 + m1;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(11,22) (55,66)
(33,44) (77,88)

(2,4) (10,12)
(6,8) (14,16)
```

2.9.28 operator -

Operator

```
scmatrix scmatrix::operator - (const scmatrix& m) const
throw (cvmexception);
```

creates an object of type `scmatrix` as a difference of a calling matrix and a matrix `m`. It throws `cvmexception` in case of different sizes of the operands. Operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::diff`, `scmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    double b[] = {10., 20., 30., 40., 50., 60., 70., 80.};
    scmatrix m1((std::complex<double>*) a, 2);
    scmatrix m2((std::complex<double>*) b, 2);

    std::cout << m2 - m1 << std::endl << m1 - m1;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(9,18) (45,54)
(27,36) (63,72)

(0,0) (0,0)
(0,0) (0,0)
```

2.9.29 sum

Function

```
scmatrix& scmatrix::sum (const scmatrix& m1, const scmatrix& m2)
throw (cvmexception);
```

assigns the result of addition of matrices `m1` and `m2` to a calling matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. Function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator +` , `scmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                  10., 11., 12., 13., 14., 15., 16., 17., 18.};
    const scmatrix m1((std::complex<double>*)a, 3);
    scmatrix m2(3);
    scmatrix m(3);
    m2.set(std::complex<double>(1.,1.));

    std::cout << m.sum(m1, m2) << std::endl;
    std::cout << m.sum(m, m2);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(2,3) (8,9) (14,15)
(4,5) (10,11) (16,17)
(6,7) (12,13) (18,19)

(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)
(7,8) (13,14) (19,20)
```

2.9.30 diff

Function

```
scmatrix& scmatrix::diff (const scmatrix& m1, const scmatrix& m2)
throw (cvmexception);
```

assigns the result of subtraction of matrices `m1` and `m2` to a calling matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. Function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator -` , `scmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                  10., 11., 12., 13., 14., 15., 16., 17., 18.};
    const scmatrix m1((std::complex<double>*)a, 3);
    scmatrix m2(3);
    scmatrix m(3);
    m2.set(std::complex<double>(1.,1.));

    std::cout << m.diff(m1, m2) << std::endl;
    std::cout << m.diff(m, m2);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,1) (6,7) (12,13)
(2,3) (8,9) (14,15)
(4,5) (10,11) (16,17)

(-1,0) (5,6) (11,12)
(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
```

2.9.31 operator +=

Operator

```
scmatrix& scmatrix::operator += (const scmatrix& m) throw (cvmexception);
```

adds matrix `m` to a calling matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. Operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator +` , `scmatrix::sum`, `scmatrix`. Example:

```
using namespace cvm;

try {
    scmatrix m1(3);
    scmatrix m2(3);
    m1.set(std::complex<double>(1.,2.));
    m2.set(std::complex<double>(3.,4.));

    m1 += m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 += m2;
    std::cout << m2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(4,6) (4,6) (4,6)
(4,6) (4,6) (4,6)
(4,6) (4,6) (4,6)

(6,8) (6,8) (6,8)
(6,8) (6,8) (6,8)
(6,8) (6,8) (6,8)
```

2.9.32 operator -=

Operator

`scmatrix& scmatrix::operator -= (const scmatrix& m) throw (cvmexception);`

subtracts matrix `m` from calling matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. Operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator -` , `scmatrix::diff`, `scmatrix`. Example:

```
using namespace cvm;

try {
    scmatrix m1(3);
    scmatrix m2(3);
    m1.set(std::complex<double>(1.,2.));
    m2.set(std::complex<double>(3.,4.));

    m1 -= m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 -= m2;
    std::cout << m2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-2,-2) (-2,-2) (-2,-2)
(-2,-2) (-2,-2) (-2,-2)
(-2,-2) (-2,-2) (-2,-2)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.9.33 operator - ()

Operator

```
scmatrix scmatrix::operator - () const throw (cvmexception);
```

creates an object of type `scmatrix` as a calling matrix multiplied by -1. Operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
scmatrix m((std::complex<double>*) a, 2);
std::cout << -m;
```

prints

```
(-1,-2) (-5,-6)
(-3,-4) (-7,-8)
```

2.9.34 operator ++

Operator

```
scmatrix& scmatrix::operator ++ ();
scmatrix& scmatrix::operator ++ (int);
```

adds identity matrix to a calling matrix and returns a reference to the matrix changed. Operator is *redefined* in the classes scbmatrix and schmatrix. See also scmatrix. Example:

```
using namespace cvm;

scmatrix m(3);
m.set(std::complex<double>(1.,1.));
m++;
std::cout << m << std::endl;
std::cout << ++m;
```

prints

```
(2,1) (1,1) (1,1)
(1,1) (2,1) (1,1)
(1,1) (1,1) (2,1)

(3,1) (1,1) (1,1)
(1,1) (3,1) (1,1)
(1,1) (1,1) (3,1)
```

2.9.35 operator --

Operator

```
scmatrix& scmatrix::operator -- ();
scmatrix& scmatrix::operator -- (int);
```

subtracts identity matrix from calling matrix and returns a reference to the matrix changed. Operator is *redefined* in the classes scbmatrix and schmatrix. See also scmatrix. Example:

```
using namespace cvm;

scmatrix m(3);
m.set(std::complex<double>(1.,1.));
m--;
std::cout << m << std::endl;
std::cout << --m;
```

prints

```
(0,1) (1,1) (1,1)
(1,1) (0,1) (1,1)
(1,1) (1,1) (0,1)

(-1,1) (1,1) (1,1)
(1,1) (-1,1) (1,1)
(1,1) (1,1) (-1,1)
```

2.9.36 operator * (TR)

Operator

```
scmatrix scmatrix::operator * (TR d) const;
```

creates an object of type `scmatrix` as a product of a calling matrix and real number `d`. Operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator *=` , `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*) a, 3);
std::cout << m * 5.;
```

prints

```
(5,10) (35,40) (65,70)
(15,20) (45,50) (75,80)
(25,30) (55,60) (85,90)
```

2.9.37 operator / (TR)

Operator

```
scmatrix scmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `scmatrix` as a quotient of a calling matrix and real number `d`. It throws `cvmexception` if `d` has absolute value equal or less than the smallest normalized positive number. Operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator /=` , `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*) a, 3);
std::cout << m / 2.;
```

prints

```
(0.5,1) (3.5,4) (6.5,7)
(1.5,2) (4.5,5) (7.5,8)
(2.5,3) (5.5,6) (8.5,9)
```

2.9.38 operator * (TC)

Operator

```
scmatrix scmatrix::operator * (TC z) const;
```

creates an object of type scmatrix as a product of a calling matrix and complex number z. Operator is *redefined* in the classes scbmatrix and schmatrix. See also scmatrix, scmatrix::operator *= . Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*) a, 3);
std::cout << m * std::complex<double>(1.,1.);
```

prints

```
(-1,3) (-1,15) (-1,27)
(-1,7) (-1,19) (-1,31)
(-1,11) (-1,23) (-1,35)
```

2.9.39 operator / (TC)

Operator

```
scmatrix scmatrix::operator / (TC z) const throw (cvmexception);
```

creates an object of type `scmatrix` as a quotient of a calling matrix and complex number `z`. It throws `cvmexception` if `z` has absolute value equal or less than the smallest normalized positive number. Operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator /=` , `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*) a, 3);
std::cout << m / std::complex<double>(1.,1.);
```

prints

```
(1.5,0.5) (7.5,0.5) (13.5,0.5)
(3.5,0.5) (9.5,0.5) (15.5,0.5)
(5.5,0.5) (11.5,0.5) (17.5,0.5)
```

2.9.40 operator *= (TR)

Operator

```
scmatrix& scmatrix::operator *= (TR d);
```

multiplies calling matrix by real number d and returns a reference to the matrix changed. It's *redefined* in the classes scbmatrix and schmatrix. See also scmatrix::operator * , scmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*) a, 3);
m *= 5.;
std::cout << m;
```

prints

```
(5,10) (35,40) (65,70)
(15,20) (45,50) (75,80)
(25,30) (55,60) (85,90)
```

2.9.41 operator /= (TR)

Operator

```
scmatrix& scmatrix::operator /= (TR d) throw (cvmexception);
```

divides calling matrix by real number d and returns a reference to the matrix changed. It throws cvmexception if d has absolute value equal or less than the smallest normalized positive number. Operator is *redefined* in the classes scbmatrix and schmatrix. See also scmatrix::operator / , scmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*) a, 3);
m /= 2.;
std::cout << m;
```

prints

```
(0.5,1) (3.5,4) (6.5,7)
(1.5,2) (4.5,5) (7.5,8)
(2.5,3) (5.5,6) (8.5,9)
```

2.9.42 operator *= (TC)

Operator

```
scmatrix& scmatrix::operator *= (TC z);
```

multiplies calling matrix by complex number `z` and returns a reference to the matrix changed. It's *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`, `scmatrix::operator *` . Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*) a, 3);
m *= std::complex<double>(2.,1.);
std::cout << m;
```

prints

```
(0,5) (6,23) (12,41)
(2,11) (8,29) (14,47)
(4,17) (10,35) (16,53)
```

2.9.43 operator /= (TC)

Operator

```
scmatrix& scmatrix::operator /= (TC z) throw (cvmexception);
```

divides calling matrix by complex number z and returns a reference to the matrix changed. It throws `cvmexception` if z has absolute value equal or less than the smallest normalized positive number. Operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::operator /` , `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*) a, 3);
m /= std::complex<double>(2.,1.);
std::cout << m;
```

prints

```
(0.8,0.6) (4.4,1.8) (8,3)
(2,1) (5.6,2.2) (9.2,3.4)
(3.2,1.4) (6.8,2.6) (10.4,3.8)
```

2.9.44 normalize

Function

```
scmatrix& scmatrix::normalize ();
```

normalizes calling matrix so its Euclidean norm becomes equal to 1 if it was greater than the smallest normalized positive number before the call (otherwise function does nothing). Function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (5);

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
scmatrix m((std::complex<double>*) a, 2);
m.normalize();
std::cout << m << m.norm() << std::endl;
```

prints

```
(7.00140e-002,1.40028e-001) (3.50070e-001,4.20084e-001)
(2.10042e-001,2.80056e-001) (4.90098e-001,5.60112e-001)
1.00000e+000
```

2.9.45 conjugation

Operator and functions

```
scmatrix scmatrix::operator ~ () const throw (cvmexception);
scmatrix& scmatrix::conj (const scmatrix& m) throw (cvmexception);
scmatrix& scmatrix::conj () throw (cvmexception);
```

implement complex matrix conjugation. First operator creates an object of type scmatrix as conjugated calling matrix (it throws cvmexception in case of memory allocation failure). Second function sets calling matrix to be equal to matrix m conjugated (it throws cvmexception in case of not appropriate sizes of the operands), third one makes it to be equal to conjugated itself (it also throws cvmexception in case of memory allocation failure). Functions are *redefined* in the classes scbmatrix and schmatrix. See also scmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*) a, 3);
scmatrix mc(3);
std::cout << m << std::endl << ~m << std::endl ;
mc.conj(m);
std::cout << mc << std::endl;
mc.conj();
std::cout << mc;
```

prints

```
(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)

(1,-2) (3,-4) (5,-6)
(7,-8) (9,-10) (11,-12)
(13,-14) (15,-16) (17,-18)

(1,-2) (3,-4) (5,-6)
(7,-8) (9,-10) (11,-12)
(13,-14) (15,-16) (17,-18)

(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)
```

2.9.46 transposition

Operator and functions

```
scmatrix scmatrix::operator ! () const throw (cvmexception);
scmatrix& scmatrix::transpose (const scmatrix& m) throw (cvmexception);
scmatrix& scmatrix::transpose () throw (cvmexception);
```

implement complex matrix transposition (*not* conjugation). First operator creates an object of type `scmatrix` as transposed calling matrix (it throws `cvmexception` in case of memory allocation failure). Second function sets calling matrix to be equal to matrix `m` transposed (it throws `cvmexception` in case of not appropriate sizes of the operands), third one makes it to be equal to transposed itself (it also throws `cvmexception` in case of memory allocation failure). Functions are *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*) a, 3);
scmatrix mc(3);
std::cout << m << std::endl << !m << std::endl ;
mc.transpose(m);
std::cout << mc << std::endl;
mc.transpose();
std::cout << mc;
```

prints

```
(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)

(1,2) (3,4) (5,6)
(7,8) (9,10) (11,12)
(13,14) (15,16) (17,18)

(1,2) (3,4) (5,6)
(7,8) (9,10) (11,12)
(13,14) (15,16) (17,18)

(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)
```

2.9.47 operator * (const cvector&)

Operator

```
cvector scmatrix::operator * (const cvector& v) const
throw (cvmexception);
```

creates an object of type `cvector` as a product of a calling matrix and a vector `v`. It throws `cvmexception` if number of columns of a calling matrix differs from size of a vector `v`. Use `cvector::mult` in order to avoid new object creation. Function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`, `cvector`. Example:

```
using namespace cvm;

try {
    scmatrix m(3);
    cvector v(3);
    m.set(std::complex<double>(1.,1.));
    v.set(std::complex<double>(1.,1.));
    std::cout << m * v;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,6) (0,6) (0,6)
```

2.9.48 operator * (const cmatrix&)

Operator

```
cmatrix scmatrix::operator * (const cmatrix& m) const
throw (cvmexception);
```

creates an object of type `cmatrix` as a product of a calling matrix and a matrix `m`. It throws `cvmexception` if number of columns of a calling matrix differs from number of rows of a matrix `m`. Use `cmatrix::mult` in order to avoid new object creation. Operator is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `cmatrix`, `scmatrix`. Example:

```
using namespace cvm;

try {
    scmatrix ms(3);
    cmatrix m(3,2);
    ms.set(std::complex<double>(1.,1.));
    m.set(std::complex<double>(1.,1.));
    std::cout << ms * m;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,6) (0,6)
(0,6) (0,6)
(0,6) (0,6)
```

2.9.49 operator * (const scmatrix&)

Operator

```
scmatrix scmatrix::operator * (const scmatrix& m) const
throw (cvmexception);
```

creates an object of type scmatrix as a product of a calling matrix and a matrix m. It throws cvmexception if the operands have different sizes. Use cmatrix::mult in order to avoid new object creation. Operator is *inherited* in the class scbmatrix and *redefined* in schmatrix. See also scmatrix. Example:

```
using namespace cvm;

try {
    scmatrix m1(3), m2(3);
    m1.set(std::complex<double>(1.,1.));
    m2.set(std::complex<double>(1.,1.));
    std::cout << m1 * m2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,6) (0,6) (0,6)
(0,6) (0,6) (0,6)
(0,6) (0,6) (0,6)
```

2.9.50 operator *= (const scmatrix&)

Operator

```
scmatrix& scmatrix::operator *= (const scmatrix& m)
throw (cvmexception);
```

sets calling matrix to be equal to product of itself by matrix m and returns a reference to the object it changes. Operator throws cvmexception in case of different sizes of the operands. Operator is *inherited* in the class scbmatrix and *redefined* in schmatrix. See also scmatrix. Example:

```
using namespace cvm;

try {
    scmatrix m1(3), m2(3);
    m1.set(std::complex<double>(1.,2.));
    m2.set(std::complex<double>(2.,1.));
    m1 *= m2;
    m2 *= m2;
    std::cout << m1 << std::endl << m2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(0,15) (0,15) (0,15)
(0,15) (0,15) (0,15)
(0,15) (0,15) (0,15)

(9,12) (9,12) (9,12)
(9,12) (9,12) (9,12)
(9,12) (9,12) (9,12)
```

2.9.51 swap_rows

Function

```
scmatrix& scmatrix::swap_rows (int n1, int n2) throw (cvmexception);
```

swaps two rows of a calling matrix and returns a reference to the matrix changed. `n1` and `n2` are numbers of rows to be swapped, both are ℓ-based). Function throws `cvmexception` if one of parameters is outside of the range `[1,msize()]`. Function is *not applicable* to objects of the classes `scbmatrix` and `schmatrix` (i.e. `cvmexception` would be thrown in case of using it for objects of those classes). See also `scmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                  10., 11., 12., 13., 14., 15., 16., 17., 18.};
    scmatrix m ((std::complex<double>*)a, 3);

    std::cout << m << std::endl;
    std::cout << m.swap_rows(2,3);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)

(1,2) (7,8) (13,14)
(5,6) (11,12) (17,18)
(3,4) (9,10) (15,16)
```

2.9.52 swap_cols

Function

```
scmatrix& scmatrix::swap_cols (int n1, int n2) throw (cvmexception);
```

swaps two columns of a calling matrix and returns a reference to the matrix changed. `n1` and `n2` are numbers of columns to be swapped, both are ℓ-based). Function throws `cvmexception` if one of parameters is outside of the range `[1,nsize()]`. Function is *not applicable* to objects of the classes `scbmatrix` and `schmatrix` (i.e. `cvmexception` would be thrown in case of using it for objects of those classes). See also `scmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                  10., 11., 12., 13., 14., 15., 16., 17., 18.};
    scmatrix m ((std::complex<double>*)a, 3);

    std::cout << m << std::endl;
    std::cout << m.swap_cols(2,3);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)

(1,2) (13,14) (7,8)
(3,4) (15,16) (9,10)
(5,6) (17,18) (11,12)
```

2.9.53 solve

Functions

```
cvector scmatrix::solve (const cvector& vB) const throw (cvmexception);
cmatrix scmatrix::solve (const cmatrix& mB) const throw (cvmexception);
cvector scmatrix::solve (const cvector& vB, TR& dErr) const
throw (cvmexception);
cmatrix scmatrix::solve (const cmatrix& mB, TR& dErr) const
throw (cvmexception);
```

return solution of linear equation of kind $A * x = b$ or $A * X = B$ where A is calling matrix. The first and third versions solve the equation $A * x = b$ where vector b is passed in parameter `vB` and the second and fourth versions solve the equation $A * X = B$ where matrix B is passed in parameter `mB`. The last two versions also set output parameter `dErr` to be equal to the norm of computation error. Functions throw `cvmexception` in case of inappropriate sizes of the operands. Functions are *inherited* in the classes `scbmatrix` and `schmatrix`. See also `cvector::solve`, `cmatrix::solve`, `scmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (2);
try {
    double re[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double im[] = {-1., 2., -3., -4., 5., -6., 7., -8., 9.};
    scmatrix ma(re, im, 3);
    cmatrix  mb(3,2);
    cmatrix  mx(3,2);
    double dErr;

    mb(1).set(std::complex<double>(1.,1.));
    mb(1,2) = std::complex<double>(1.,1.);
    mb(2,2) = std::complex<double>(2.,2.);
    mb(3,2) = std::complex<double>(3.,3.);

    mx.solve (ma, mb, dErr);
    std::cout << mx << dErr
              << std::endl << ma * mx - mb << std::endl;

    cvector vb(3), vx(3);
    vb = mb(2);
    vx = ma.solve (vb, dErr);
    std::cout << vx << dErr << std::endl << ma * vx - vb;
}
catch (std::exception& e) {
```

```
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-6.25e-002,-2.71e-001) (-1.25e-001,+1.13e+000)
(+1.25e-001,+2.08e-001) (+2.50e-001,-2.50e-001)
(+6.25e-002,+6.25e-002) (+1.25e-001,+1.25e-001)
+1.45e-014
(+0.00e+000,+0.00e+000) (+0.00e+000,+0.00e+000)
(-2.22e-016,+4.44e-016) (+0.00e+000,+0.00e+000)
(+2.22e-016,+0.00e+000) (+0.00e+000,+0.00e+000)

(-1.25e-001,+1.13e+000) (+2.50e-001,-2.50e-001) (+1.25e-001,+1.25e-001)
+7.41e-015
(+0.00e+000,+0.00e+000) (+0.00e+000,+0.00e+000) (+0.00e+000,+0.00e+000)
```

2.9.54 solve_tran

Functions

```
cvector scmatrix::solve_tran (const cvector& vB)
const throw (cvmexception);
cmatrix scmatrix::solve_tran (const cmatrix& mB)
const throw (cvmexception);
cvector scmatrix::solve_tran (const cvector& vB, TR& dErr) const
throw (cvmexception);
cmatrix scmatrix::solve_tran (const cmatrix& mB, TR& dErr) const
throw (cvmexception);
```

return solution of linear equation of kind $A^T * x = b$ or $A^T * X = B$ (equivalent to $x * A = b$ and $X * A = B$) where A is calling matrix. The first and third versions solve equation $A^T * x = b$ where vector b is passed in parameter `vB` and the second and fourth versions solve equation $A^T * X = B$ where matrix B is passed in parameter `mB`. The last two versions also set output parameter `dErr` to be equal to the norm of computation error. Functions throw `cvmexception` in case of inappropriate sizes of the operands. Function is *inherited* in the classes `scbmatrix` and `schmatrix`. See also `cvector::solve`, `cmatrix::solve`, `scmatrix::solve`, `scmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (2);
try {
    double re[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double im[] = {-1., 2., -3., -4., 5., -6., 7., -8., 9.};
    scmatrix ma(re, im, 3);
    cmatrix  mb(3,2);
    cmatrix  mx(3,2);
    double dErr;

    mb(1).set(std::complex<double>(1.,1.));
    mb(1,2) = std::complex<double>(1.,1.);
    mb(2,2) = std::complex<double>(2.,2.);
    mb(3,2) = std::complex<double>(3.,3.);

    mx.solve_tran (ma, mb, dErr);
    std::cout << mx << dErr
              << std::endl << !ma * mx - mb
              << std::endl << !mx * ma - !mb << std::endl;

    cvector vb(3), vx(3);
    vb = mb(2);
```

```
    vx = ma.solve_tran (vb, dErr);
    std::cout << vx << dErr << std::endl << vx * ma - vb;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(+3.13e-002,-5.31e-001) (+9.38e-002,-9.38e-002)
(+6.25e-002,+6.25e-002) (+1.88e-001,+1.88e-001)
(+3.13e-002,+4.69e-001) (+9.38e-002,+2.40e-001)
+1.02e-014
(-2.22e-016,-2.22e-016) (+0.00e+000,+0.00e+000)
(-4.44e-016,-4.44e-016) (+0.00e+000,+0.00e+000)
(+8.88e-016,-8.88e-016) (+0.00e+000,+0.00e+000)

(-2.22e-016,-2.22e-016) (-4.44e-016,-4.44e-016) (+8.88e-016,-8.88e-016)
(+0.00e+000,+0.00e+000) (+0.00e+000,+0.00e+000) (+0.00e+000,+0.00e+000)

(+9.38e-002,-9.38e-002) (+1.88e-001,+1.88e-001) (+9.38e-002,+2.40e-001)
+1.02e-014
(+0.00e+000,+0.00e+000) (+0.00e+000,+0.00e+000) (+0.00e+000,+0.00e+000)
```

2.9.55 solve_conj

Functions

```
cvector scmatrix::solve_conj (const cvector& vB)
const throw (cvmexception);
cmatrix scmatrix::solve_conj (const cmatrix& mB)
const throw (cvmexception);
cvector scmatrix::solve_conj (const cvector& vB, TR& dErr) const
throw (cvmexception);
cmatrix scmatrix::solve_conj (const cmatrix& mB, TR& dErr) const
throw (cvmexception);
```

return solution of linear equation of kind $A^H * x = b$ or $A^H * X = B$ (which is equivalent to $X^H * A = B^H$), where A^H is conjugated A. Here A is calling matrix. The first and third versions solve equation $A^H * x = b$ where vector b is passed in parameter vB and the second and fourth versions solve equation $A^H * X = B$ where matrix B is passed in parameter mB. The last two versions also set output parameter dErr to be equal to the norm of computation error. Functions throw cvmexception in case of inappropriate sizes of the operands. Function is *inherited* in the classes scbmatrix and schmatrix. See also cvector::solve, cmatrix::solve, scmatrix::solve, scmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (2);
try {
    double re[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double im[] = {-1., 2., -3., -4., 5., -6., 7., -8., 9.};
    scmatrix ma(re, im, 3);
    cmatrix  mb(3,2);
    cmatrix  mx(3,2);
    double dErr;

    mb(1).set(std::complex<double>(1.,1.));
    mb(1,2) = std::complex<double>(1.,1.);
    mb(2,2) = std::complex<double>(2.,2.);
    mb(3,2) = std::complex<double>(3.,3.);

    mx.solve_conj (ma, mb, dErr);
    std::cout << mx << dErr
              << std::endl << ~ma * mx - mb
              << std::endl << ~mx * ma - ~mb << std::endl;

    cvector vb(3), vx(3);
    vb = mb(2);
```

```
    vx = ma.solve_conj (vb, dErr);
    std::cout << vx << dErr << std::endl << ~ma * vx - vb;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-5.31e-001,+3.13e-002) (-9.38e-002,+9.38e-002)
(+6.25e-002,+6.25e-002) (+1.88e-001,+1.88e-001)
(+4.69e-001,+3.13e-002) (+2.40e-001,+9.38e-002)
+1.02e-014
(-2.22e-016,-2.22e-016) (+0.00e+000,+0.00e+000)
(-4.44e-016,-4.44e-016) (+0.00e+000,+0.00e+000)
(-8.88e-016,+8.88e-016) (+0.00e+000,+0.00e+000)

(-2.22e-016,+2.22e-016) (-4.44e-016,+4.44e-016) (-8.88e-016,-8.88e-016)
(+0.00e+000,+0.00e+000) (+0.00e+000,+0.00e+000) (+0.00e+000,+0.00e+000)

(-9.38e-002,+9.38e-002) (+1.88e-001,+1.88e-001) (+2.40e-001,+9.38e-002)
+1.02e-014
(+0.00e+000,+0.00e+000) (+0.00e+000,+0.00e+000) (+0.00e+000,+0.00e+000)
```

2.9.56 operator % (const cvector&)

Operator

```
cvector operator % (const cvector& vB) const throw (cvmexception);
```

returns solution x of linear equation $A^T * x = b$ (which is equivalent to $x * A = b$) where calling matrix is square matrix A and a vector b is passed in parameter vB. This operator throws exception of type cvmexception in case of inappropriate sizes of the objects or when matrix A is close to singular. See also cvector::solve_tran, scmatrix::solve_tran, cvector.operator /, cvector, scmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (7);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -3.};
    double b[] = {1., 2., 5., -3.};
    scmatrix ma((std::complex<double>*) m, 2);
    cvector vb((std::complex<double>*) b, 2);
    cvector vx(2);

    vx = ma % vb;

    std::cout << vx * ma - vb;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(+0.0000000e+000,+0.0000000e+000) (+0.0000000e+000,+4.4408921e-016)
```

2.9.57 operator / (const cvector&)

Operator

```
cvector operator / (const cvector& vB) const throw (cvmexception);
```

returns solution x of linear equation $A * x = b$ where calling matrix is square matrix A and a vector b is passed in parameter `vB`. This operator throws exception of type `cvmexception` in case of inappropriate sizes of the objects or when matrix A is close to singular. It's *redefined* in class `scbmatrix`. See also `cvector::solve`, `scmatrix::solve`, `cvector.operator %`, `cvector`, `scmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (7);

try {
    double m[] = {1., -1., 1., 2., -2., 1., 3., -3.};
    double b[] = {1., 2., 5., -3.};
    scmatrix ma((std::complex<double>*) m, 2);
    cvector vb((std::complex<double>*) b, 2);
    cvector vx(2);

    vx = ma / vb;

    std::cout << ma * vx - vb;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-6.6613381e-016,+4.4408921e-016) (+0.0000000e+000,+0.0000000e+000)
```

2.9.58 solve_lu

Functions

```
cvector
scmatrix::solve_lu (const scmatrix& mLU, const int* pPivots,
                    const cvector& vB, TR& dErr) throw (cvmexception);
cvector
scmatrix::solve_lu (const scmatrix& mLU, const int* pPivots,
                    const cvector& vB) throw (cvmexception);
cmatrix
scmatrix::solve_lu (const scmatrix& mLU, const int* pPivots,
                    const cmatrix& mB, TR& dErr) throw (cvmexception);
cmatrix
scmatrix::solve_lu (const scmatrix& mLU, const int* pPivots,
                    const cmatrix& mB) throw (cvmexception);
```

create object of type `cvector` or `cmatrix` as solution x or X of a matrix linear equation $A * x = b$ or $A * X = B$ respectively. Here A is calling matrix, parameter `mLU` is LU factorization of the matrix A, parameter `pPivots` is an array of pivot numbers created while factorizing the matrix A and parameters `vB` and `mB` are the vector b and matrix B respectively. The first and third version also set output parameter `dErr` to be equal to the norm of computation error. These functions are useful when you need to solve few linear equations of kind $A * x = b$ or $A * X = B$ with the same matrix A and different vectors b or matrices B. In such case you save on matrix A factorization since it's needed to be performed just one time. Functions throw `cvmexception` in case of inappropriate sizes of the operands or when the matrix A is close to singular. Function is *inherited* in the classes `scbmatrix` and `schmatrix`. See also `cvector::solve`, `cmatrix`, `scmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    scmatrix ma(3);
    scmatrix mLU(3);
    cmatrix  mb1(3,2); cvector vb1(3);
    cmatrix  mb2(3,2); cvector vb2(3);
    cmatrix  mx1(3,2); cvector vx1(3);
    cmatrix  mx2(3,2); cvector vx2(3);
    iarray   nPivots(3);
    double   dErr = 0.;
    ma.randomize_real(-1.1,3.); ma.randomize_imag(-3.7,3.);
    mb1.randomize_real(-1.,3.); mb1.randomize_imag(-1.,3.);
    vb1.randomize_real(-2.,3.); vb1.randomize_imag(-3.,1.);
    mb2.randomize_real(-5.,1.); mb2.randomize_imag(-4.,1.);
    vb2.randomize_real(-1.,6.); vb1.randomize_imag(-4.,4.);
```

```
    mLU.low_up(ma, nPivots);
    mx1 = ma.solve_lu (mLU, nPivots, mb1, dErr);
    std::cout << mx1 << dErr << std::endl;
    mx2 = ma.solve_lu (mLU, nPivots, mb2);
    std::cout << mx2 << std::endl;;
    std::cout << ma * mx1 - mb1 << std::endl << ma * mx2 - mb2;
    vx1 = ma.solve_lu (mLU, nPivots, vb1, dErr);
    std::cout << vx1 << dErr << std::endl;
    vx2 = ma.solve_lu (mLU, nPivots, vb2);
    std::cout << vx2 << std::endl;;
    std::cout << ma * vx1 - vb1 << std::endl << ma * vx2 - vb2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-3.91e-001,-1.62e-001) (-3.17e-001,2.39e-001)
(-3.00e-001,-7.91e-001) (4.71e-001,-9.13e-001)
(-2.34e-001,1.09e+000) (1.10e-001,2.72e-001)
2.78e-015
(-2.60e-001,-5.48e-001) (-3.09e-002,-9.62e-001)
(8.77e-001,8.41e-001) (-6.02e-001,1.87e+000)
(4.20e-003,-9.72e-001) (6.18e-001,-5.64e-001)

(0.00e+000,-2.64e-016) (-1.11e-016,1.11e-016)
(0.00e+000,-2.22e-016) (2.22e-016,-4.44e-016)
(0.00e+000,0.00e+000) (-5.55e-017,0.00e+000)

(6.66e-016,-2.22e-016) (6.18e-016,0.00e+000)
(0.00e+000,1.11e-016) (0.00e+000,0.00e+000)
(0.00e+000,0.00e+000) (-4.44e-016,0.00e+000)
(2.61e-001,2.97e-001) (1.95e+000,-1.07e-001) (-5.51e-001,-1.03e-001)
1.96e-015
(1.26e-001,4.07e-001) (-4.82e-001,-1.14e-002) (2.59e-001,1.60e-001)

(1.11e-016,0.00e+000) (5.55e-017,0.00e+000) (1.11e-016,-2.22e-016)

(-1.11e-016,-7.61e-017) (-2.22e-016,-8.94e-017) (0.00e+000,4.07e-017)
```

2.9.59 det

Function

```
TC scmatrix::det () const throw (cvmexception);
```

returns determinant of a calling matrix. It uses the LU factorization inside and may throw the same exceptions as the factorizer. Function is *inherited* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

try {
    double re[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double im[] = {-1., 2., -3., -4., 5., -6., 7., -8., 9.};
    const scmatrix m(re, im, 3);

    std::cout << m << std::endl << m.det() << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,-1) (4,-4) (7,7)
(2,2) (5,5) (8,-8)
(3,-3) (6,-6) (9,9)

(-192,-192)
```

2.9.60 low_up

Functions

```
scmatrix& scmatrix::low_up (const scmatrix& m, int* nPivots)
throw (cvmexception);

scmatrix scmatrix::low_up (int* nPivots) const
throw (cvmexception);
```

compute LU factorization of a calling matrix as

$$A = PLU$$

where P is permutation matrix, L is lower triangular matrix with unit diagonal elements and U is upper triangular matrix. All functions store result as the matrix L without main diagonal combined with U. All functions return pivot indices as an array of integers (it should support at least `msize()` elements) pointed to by `nPivots` so i-th row was interchanged with `nPivots[i]`-th row. The first version sets calling matrix to be equal to the `m`'s LU factorization and the second one creates an object of type `scmatrix` as calling matrix LU factorization. Functions throw `cvmexception` in case of inappropriate sizes of the operands or when the matrix to be factorized is close to singular. It is recommended to use `iarray` for pivot values. Function is *redefined* in the class `scbmatrix` and *inherited* in `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

try {
    double re[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double im[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.1};
    scmatrix m(re, im, 3);
    scmatrix mLU(3), mLo(3), mUp(3);
    iarray naPivots(3);

    mLU.low_up (m, naPivots);

    mLo.identity ();
    mLo.diag(-2) = mLU.diag(-2);
    mLo.diag(-1) = mLU.diag(-1);
    mUp.diag(0) = mLU.diag(0);
    mUp.diag(1) = mLU.diag(1);
    mUp.diag(2) = mLU.diag(2);

    std::cout << mLo << std::endl << mUp
              << std::endl << naPivots << std::endl;

    mLU = mLo * mUp;
```

```
    for (int i = 3; i >= 1; i--) {
        mLU.swap_rows (i, naPivots[i]);
    }
    std::cout << mLU << std::endl << m - mLU;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,0) (0,0) (0,0)
(0.333333,0) (1,0) (0,0)
(0.666667,0) (0.5,0) (1,0)

(3,3) (6,6) (9,9.1)
(0,0) (2,2) (4,3.96667)
(0,0) (0,0) (-1.11022e-016,-0.05)

3 3 3

(1,1) (4,4) (7,7)
(2,2) (5,5) (8,8)
(3,3) (6,6) (9,9.1)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.9.61 cond

Function

```
TR scmatrix::cond () const throw (cvmexception);
```

returns condition number reciprocal of a calling matrix A in the infinity-norm:

$$\kappa_\infty = \|A\|_\infty \|A^{-1}\|_\infty .$$

Less value returned means that matrix A is closer to singular. Zero value returned means estimation underflow or that matrix A is singular. The condition number is used for error analysis of systems of linear equations. Function throws `cvmexception` in case of LAPACK subroutines failure. Function is *inherited* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix::solve`, `scmatrix`. Example:

```
using namespace cvm;

try {
    double re[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double im[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};

    scmatrix m(re, im, 3);
    std::cout << m.cond() << std::endl
              << m.det() << std::endl << std::endl;

    m(3,3) = std::complex<double>(9.,10.);
    std::cout << m.cond() << std::endl << m.det() << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1.54198e-018
(1.33227e-015,-1.33227e-015)

0.0050679
(6,-1.33227e-015)
```

2.9.62 inv

Functions

```
scmatrix& scmatrix::inv (const scmatrix& m) throw (cvmexception);
scmatrix scmatrix::inv () const throw (cvmexception);
```

implement matrix inversion. The first version sets calling matrix to be equal to `m` inverted and the second one creates an object of type `scmatrix` as inverted calling matrix. Functions throw `cvmexception` in case of inappropriate sizes of the operands or when the matrix to be inverted is close to singular. Function is *redefined* in the class `schmatrix` and *inherited* in `scbmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double re[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double im[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    scmatrix m(re, im, 3);
    scmatrix mi(3);

    mi.inv (m);
    std::cout << mi << std::endl << mi * m - eye_complex(3);
    std::cout << std::endl << mi.inv() * mi - eye_complex(3);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-8.33e-001,-1.67e-001) (6.67e-001,1.33e+000) (4.81e-016,-1.00e+000)
(3.33e-001,1.67e+000) (-1.67e-001,-3.83e+000) (-5.37e-016,2.00e+000)
(2.22e-016,-1.00e+000) (-4.44e-016,2.00e+000) (2.22e-016,-1.00e+000)

(-1.11e-016,1.11e-016) (0.00e+000,2.22e-016) (8.33e-017,1.26e-015)
(4.44e-016,-5.00e-016) (0.00e+000,7.77e-016) (7.22e-016,-1.15e-015)
(-1.11e-016,1.11e-016) (2.22e-016,-6.66e-016) (0.00e+000,1.11e-016)

(4.44e-016,4.44e-016) (-1.18e-015,1.33e-015) (6.66e-016,-7.77e-016)
(-6.85e-016,8.88e-016) (1.33e-015,3.16e-030) (-8.33e-016,-1.77e-030)
(-5.09e-016,6.66e-016) (7.96e-016,-1.78e-015) (-4.44e-016,1.11e-016)
```

2.9.63 exp

Functions

```
scmatrix& scmatrix::exp (const scmatrix& m, TR tol = cvmMachSp ())
throw (cvmexception);
scmatrix scmatrix::exp (TR tol = cvmMachSp ()) const
throw (cvmexception);
```

compute exponent of a calling matrix using Padé approximation defined as

$$R_{pq}(z) = D_{pq}(z)^{-1} N_{pq}(z) = 1 + z + \cdots + z^p/p!\,,$$

where

$$N_{pq}(z) = \sum_{k=0}^{p} \frac{(p+q-k)!p!}{(p+q)!k!(q-k)!} z^k,$$
$$D_{pq}(z) = \sum_{k=0}^{q} \frac{(p+q-k)!p!}{(p+q)!k!(q-k)!} (-z)^k$$

along with the matrix normalizing as described in [2], p. 572. Functions use ZMEXP (or CMEXP for float version) FORTRAN subroutine implementing the algorithm. The first version sets calling matrix to be equal to exponent of m and returns a reference to the matrix changed. The second version creates an object of type scmatrix as exponent of a calling matrix. The algorithm uses parameter tol as $\varepsilon(p, q)$ in order to choose constants p and q so that

$$\varepsilon(p, q) \geqslant 2^{3-(p+q)} \frac{p!q!}{(p+q)!(p+q+1)!}.$$

This parameter is equal to the largest relative spacing by default. Functions throw cvmexception in case of inappropriate sizes of the operands or when LAPACK subroutine fails. Functions are *inherited* in the classes scbmatrix and schmatrix. The second version is *redefined* in scbmatrix. See also scmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (15);
try {
    scmatrix m(2);
    m(1,1) = std::complex<double>(-49.,1.);
    m(1,2) = std::complex<double>(24.,1.);
    m(2,1) = std::complex<double>(-64.,1.);
    m(2,2) = std::complex<double>(31.,1.);

    m = m.exp();
    std::cout << m(1,1) << std::endl << "    "
```

```
              << m(1,2) << std::endl;
    std::cout << m(2,1) << std::endl << "    "
              << m(2,2) << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(-4.508497580070061e-001,7.900659666739228e-001)
   (3.199576050798058e-001,-6.081804753524478e-001)
(-7.584316151932173e-001,1.666747485117903e+000)
   (5.295040786048336e-001,-1.278050361026397e+000)
```

MATLAB output:

```
  Column 1

    -4.508497580070262e-001 +7.900659666739607e-001i
    -7.584316151932523e-001 +1.666747485117982e+000i

  Column 2

     3.199576050798204e-001 -6.081804753524764e-001i
     5.295040786048589e-001 -1.278050361026457e+000i
```

2.9.64 polynomial

Functions

```
scmatrix& scmatrix::polynom (const scmatrix& m, const cvector& v)
throw (cvmexception);

scmatrix scmatrix::polynom (const cvector& v) const
throw (cvmexception);
```

compute matrix polynomial defined as

$$p(A) = b_0 I + b_1 A + \cdots + b_q A^q$$

using the Horner's rule:

$$p(A) = \sum_{k=0}^{r} B_k (A^s)^k, \quad s = \text{floor}(\sqrt{q}\,),\; r = \text{floor}(q/s)$$

where

$$B_k = \begin{cases} \sum\limits_{i=0}^{s-1} b_{sk+i} A^i, & k = 0, 1, \ldots, r-1 \\ \sum\limits_{i=0}^{q-sr} b_{sr+i} A^i, & k = r. \end{cases}$$

See also [2], p. 568. The coefficients $b_0, b_1, \ldots, b_q$ are passed in parameter v, where q is equal to `v.size()-1`, so functions compute matrix polynomial equal to

$$v[1] * I + v[2] * m + \cdots + v[v.size()] * m^{v.size()-1}$$

The first version sets calling matrix to be equal to the polynomial of `m` and the second one creates an object of type `scmatrix` as the polynomial of a calling matrix. Functions use `ZPOLY` (or `CPOLY` for `float` version) FORTRAN subroutine implementing the Horner's algorithm. Functions throw `cvmexception` in case of inappropriate sizes of the operands. Functions are *inherited* in the class `scbmatrix` and *redefined* in `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (15);
try {
    const double re[] = {2.2, 1.3, 1.1, -0.9, 0.2,
                         -0.45, 45., -30., 10., 3., 0.};
    const double im[] = {0.5, -2, 0, 1, 3,
                         -3., 30., 0., -9., 0., 1.};
    const cvector v(re, im, 11);
    scmatrix m(2), mp(2);
```

```
    m(1,1) = std::complex<double>(0.1, -0.2);
    m(1,2) = std::complex<double>(0.1, -0.2);
    m(2,1) = std::complex<double>(0.5, -0.6);
    m(2,2) = std::complex<double>(0.3, -0.4);

    mp.polynom (m, v);
    std::cout << mp(1,1) << std::endl << "   "
              << mp(1,2) << std::endl;
    std::cout << mp(2,1) << std::endl << "   "
              << mp(2,2) << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(2.485652665600000e+000,3.791263308800001e+000)
   (2.817786176000004e-001,2.301942860800001e+000)
(-8.835069888000001e-001,8.052028620800002e+000)
   (1.903009862400001e+000,6.666306188800002e+000)
```

MATLAB output:

```
  Column 1

     2.485652665600000e+000 +3.791263308800001e+000i
    -8.835069887999991e-001 +8.052028620800002e+000i

  Column 2

     2.817786176000000e-001 +2.301942860800001e+000i
     1.903009862399999e+000 +6.666306188800003e+000i
```

2.9.65 eig

Functions

```
cvector scmatrix::eig (scmatrix& mEigVect, bool bRightVect = true) const
throw (cvmexception);

cvector scmatrix::eig () const throw (cvmexception);
```

solve the nonsymmetric eigenvalue problem and return complex vector with eigenvalues of a calling matrix. The first version sets output parameter `mEigVect` to be equal to square matrix containing right (if parameter `bRightVect` is true, which is default value) or left (if parameter `bRightVect` is false) eigenvectors as columns. All functions throw `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. Functions are *inherited* in the class `scbmatrix` and *redefined* in `schmatrix`. See also `cvector`, `scmatrix`. Example:

```
using namespace cvm;
try {
    double re[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
    double im[] = {1., 2., 3., 4., 5., 6., 7., 8., 10.};
    scmatrix m(re, im, 3);
    scmatrix me(3);
    cvector vl(3);

    vl = m.eig (me);
    std::cout << vl << std::endl;
    std::cout.setf (std::ios::scientific | std::ios::left);
    std::cout.precision (2);
    std::cout << m * me(1) - me(1) * vl(1);
    std::cout << m * me(2) - me(2) * vl(2);
    std::cout << m * me(3) - me(3) * vl(3);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(16.1096,16.7004) (-1.09351,-0.88358) (-0.0161248,0.183218)

(-9.44e-016,-3.55e-015) (-1.11e-016,-1.07e-014) (2.66e-015,-1.42e-014)
(-5.55e-016,-4.44e-016) (-1.80e-015,9.44e-016) (-2.00e-015,7.22e-016)
(9.92e-016,1.24e-015) (1.05e-015,2.78e-017) (1.64e-015,9.30e-016)
```

2.9.66 Cholesky

Function

```
scmatrix& scmatrix::cholesky (const schmatrix& m)
throw (cvmexception);
```

forms the Cholesky factorization of hermitian positive-definite matrix A defined as

$$A = U^T U,$$

where U is upper triangular matrix. It utilizes one of ?POTRF routines of the LAPACK library. Function sets calling matrix to be equal to the factorization of hermitian positive-definite matrix m. Function throws cvmexception in case of inappropriate sizes of the operands or in case of convergence error. See also scmatrix and schmatrix. Example:

```
using namespace cvm;

try {
    double r[] = {1., 2., 1., 2., 15., -1., 1., -1., 20.};
    double i[] = {0., -1., 2., 1., 0., 3., -2., -3., 0.};
    const schmatrix m(r, i, 3);
    scmatrix c(3);

    c.cholesky(m);

    std::cout << c << std::endl;
    std::cout << ~c * c - m;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(1,0) (2,1) (1,-2)
(0,-0) (3.16228,0) (-0.316228,0.632456)
(0,0) (-0,0) (3.80789,0)

(0,0) (0,0) (0,0)
(0,0) (1.77636e-015,0) (0,0)
(0,0) (0,0) (0,0)
```

2.9.67 Bunch-Kaufman

Function

```
scmatrix& scmatrix::bunch_kaufman (const schmatrix& m, int* pivots)
throw (cvmexception);
```

forms the Bunch-Kaufman factorization of hermitian matrix (cited from the MKL library documentation):

$$A = PUDU^T P^T,$$

where A is the input matrix passed in parameter `m`, P is permutation matrix, U and L are upper and lower triangular matrices with unit diagonal, and D is symmetric block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. U and L have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of D. It utilizes one of `?SYTRF` routines of the LAPACK library. Function sets calling matrix to be equal to the factorization of hermitian positive-definite matrix `m`. Function throws `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. See also `scmatrix` and `schmatrix`. Function is mostly designed to be used for subsequent calls of `?SYTRS`, `?SYCON` and `?SYTRI` routines of the LAPACK library. Currently it's used internally in `scmatrix::det` flow when argument is hermitian but not positive-definite matrix.

2.9.68 qr

Function

```
void scmatrix::qr (scmatrix& mQ, scmatrix& mR) const throw (cvmexception);
```

computes QR factorization as

$$M = QR$$

where M is calling square matrix, unitary matrix Q and upper triangular matrix R are `mQ` and `mR` respectively. Function throws `cvmexception` in case of inappropriate sizes of the operands. See also `cmatrix`, `cmatrix::qr`, `scmatrix`. Example:

```
using namespace cvm;

treal ar[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
treal ai[] = {1., -1., 2., -2., 3., -3., 4., -4., 5.};
const cvm::scmatrix m(ar, ai, 3);
cvm::scmatrix q(3), r(3);

m.qr(q,r);
std::cout << (eye_complex(3) - ~q * q).norm()
          << " " << (m - q * r).norm() << std::endl;
```

prints

```
+3.7957075e-16 +1.9641850e-15
```

2.9.69 lq

Function

```
void scmatrix::lq (scmatrix& mL, scmatrix& mQ) const throw (cvmexception);
```

computes LQ factorization as

$$M = LQ$$

where M is calling square matrix, lower triangular matrix L and unitary matrix Q are `mL` and `mQ` respectively. Function throws `cvmexception` in case of inappropriate sizes of the operands. See also `cmatrix`, `cmatrix::lq`, `scmatrix`. Example:

```
using namespace cvm;

treal ar[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
treal ai[] = {1., -1., 2., -2., 3., -3., 4., -4., 5.};
const cvm::scmatrix m(ar, ai, 3);
cvm::scmatrix l(3), q(3);

m.lq(l,q);
std::cout << (eye_complex(3) - ~q * q).norm()
          << " " << (m - l * q).norm() << std::endl;
```

prints

```
+3.5331153e-016 +3.0526086e-015
```

2.9.70 rq

Function

```
void scmatrix::rq (scmatrix& mR, scmatrix& mQ) const throw (cvmexception);
```

computes RQ factorization as

$$M = RQ$$

where M is calling square matrix, upper triangular matrix R and unitary matrix Q are `mR` and `mQ` respectively. Function throws `cvmexception` in case if inappropriate sizes of the operands passed. See also `cmatrix`, `cmatrix::rq`, `srmatrix`. Example:

```
using namespace cvm;

treal ar[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
treal ai[] = {1., -1., 2., -2., 3., -3., 4., -4., 5.};
const cvm::scmatrix m(ar, ai, 3);
cvm::scmatrix r(3), q(3);

m.rq(r,q);
std::cout << (eye_complex(3) - ~q * q).norm()
          << " " << (m - r * q).norm() << std::endl;
```

prints

```
+3.3357248e-016 +3.4488467e-015
```

2.9.71 ql

Function

```
void scmatrix::ql (scmatrix& mQ, scmatrix& mL) const throw (cvmexception);
```

computes QL factorization as

$$M = QL$$

where M is calling square matrix, unitary matrix Q and lower triangular matrix L are `mQ` and `mL` respectively. Function throws `cvmexception` in case if inappropriate sizes of the operands passed. See also `cmatrix`, `cmatrix::ql`, `scmatrix`. Example:

```
using namespace cvm;

treal ar[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.};
treal ai[] = {1., -1., 2., -2., 3., -3., 4., -4., 5.};
const cvm::scmatrix m(ar, ai, 3);
cvm::scmatrix q(3), l(3);

m.ql(q,l);
std::cout << (eye_complex(3) - ~q * q).norm()
          << " " << (m - q * l).norm() << std::endl;
```

prints

```
+8.8904536e-016 +4.5789129e-015
```

2.9.72 identity

Function

```
scmatrix& scmatrix::identity();
```

sets calling matrix to be equal to identity matrix and returns a reference to the matrix changed. Function is *redefined* in the classes scbmatrix and schmatrix. See also scmatrix. Example:

```
using namespace cvm;

scmatrix m(3);
m.randomize_real(0.,3.);
m.randomize_imag(-1.,2.);

std::cout << m << std::endl;
std::cout << m.identity();
```

prints

```
(1.31162,-0.52501) (2.8612,-0.531144) (1.31849,0.547838)
(1.19929,1.48253) (0.535417,0.41316) (0.459883,1.7019)
(0.415937,-0.491134) (2.0969,-0.218024) (0.545305,1.17866)

(1,0) (0,0) (0,0)
(0,0) (1,0) (0,0)
(0,0) (0,0) (1,0)
```

2.9.73 vanish

Function

```
scmatrix& scmatrix::vanish();
```

sets every element of a calling matrix to be equal to zero and returns a reference to the matrix changed. This function is faster than `scmatrix::set(TC)` with zero operand passed. Function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

scmatrix m(3);
m.randomize_real(0.,3.);
m.randomize_imag(-1.,2.);

std::cout << m << std::endl;
std::cout << m.vanish();
```

prints

```
(1.34834,-0.758385) (0.837825,-0.225532) (0.367687,0.791833)
(2.23698,-0.183142) (2.6878,0.741111) (0.495865,0.698904)
(0.584124,0.00491348) (1.31574,0.687643) (0.482131,1.66482)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.9.74 randomize_real

Function

```
scmatrix& scmatrix::randomize_real (TR dFrom, TR dTo);
```

fills real part of a calling matrix with pseudo-random numbers distributed between `dFrom` and `dTo`. Function returns a reference to the matrix changed. Function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
scmatrix m(3);
m.randomize_real(0.,3.);
std::cout << m;
```

prints

```
(1.56e+000,0.00e+000) (2.39e+000,0.00e+000) (2.41e+000,0.00e+000)
(3.73e-002,0.00e+000) (2.61e+000,0.00e+000) (1.36e+000,0.00e+000)
(2.71e+000,0.00e+000) (1.69e+000,0.00e+000) (2.68e+000,0.00e+000)
```

2.9.75 randomize_imag

Function

```
scmatrix& scmatrix::randomize_imag (TR dFrom, TR dTo);
```

fills imaginary part of a calling matrix with pseudo-random numbers distributed between `dFrom` and `dTo`. Function returns a reference to the matrix changed. Function is *redefined* in the classes `scbmatrix` and `schmatrix`. See also `scmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
scmatrix m(3);
m.randomize_imag(0.,3.);
std::cout << m;
```

prints

```
(0.00e+000,1.58e+000) (0.00e+000,2.38e+000) (0.00e+000,2.64e+000)
(0.00e+000,1.62e-002) (0.00e+000,4.26e-002) (0.00e+000,2.21e+000)
(0.00e+000,2.39e+000) (0.00e+000,6.95e-001) (0.00e+000,4.30e-001)
```

2.10 BandMatrix

This base class contains member functions common for band matrices. This class is not designed to be instantiated.

```
template <typename TR, typename TC>
class BandMatrix {
public:
    int lsize () const;
    int usize () const;
};
```

2.10.1 lsize

Function

```
int BandMatrix<TR,TC>::lsize () const;
```

returns number of lower sub-diagonals of a calling matrix. Function is *inherited* in the classes `srbmatrix` and `scbmatrix`. See also `BandMatrix`. Example:

```
using namespace cvm;

srbmatrix m(4,2,1);
m.set(1.);
std::cout << m;
std::cout << m.msize() << " " << m.nsize() << " " << m.size();
std::cout << " " << m.lsize() << " " << m.usize() << std::endl;
```

prints

```
1 1 0 0
1 1 1 0
1 1 1 1
0 1 1 1
4 4 16 2 1
```

2.10.2 usize

Function

```
int BandMatrix<TR,TC>::usize () const;
```

returns number of upper sub-diagonals of a calling matrix. Function is *inherited* in the classes `srbmatrix` and `scbmatrix`. See also `BandMatrix`. Example:

```
using namespace cvm;

srbmatrix m(4,2,1);
m.set(1.);
std::cout << m;
std::cout << m.msize() << " " << m.nsize() << " " << m.size() ;
std::cout << " " << m.lsize() << " " << m.usize() << std::endl;
```

prints

```
1 1 0 0
1 1 1 0
1 1 1 1
0 1 1 1
4 4 16 2 1
```

2.11 srbmatrix

This is end-user class encapsulating square band matrix of real numbers. This class utilizes band storage for its elements.

```
template <typename TR>
class srbmatrix : public srmatrix <TR>, public BandMatrix <TR,TR> {
public:
    srbmatrix ();
    explicit srbmatrix (int nMN);
    srbmatrix (int nMN);
    srbmatrix (TR* pD, int nMN, int nKL, int nKU);
    srbmatrix (const TR* pD, int nMN, int nKL, int nKU);
    srbmatrix (const srbmatrix& m);
    srbmatrix (const rmatrix& m, int nKL, int nKU);
    explicit srbmatrix (const rvector& v);
    TR& operator () (int im, int in) throw (cvmexception);
    TR operator () (int im, int in) const throw (cvmexception);
    const rvector operator () (int i) const throw (cvmexception);
    const rvector operator [] (int i) const throw (cvmexception);
    srbmatrix& operator = (const srbmatrix& m) throw (cvmexception);
    srbmatrix& assign (const rvector& v) throw (cvmexception);
    srbmatrix& assign (const TR* pD);
    srbmatrix& set (TR x);
    srbmatrix& resize (int nNewMN) throw (cvmexception);
    srbmatrix& resize_lu (int nNewKL, int nNewKU) throw (cvmexception);
    bool operator == (const srbmatrix& v) const;
    bool operator != (const srbmatrix& v) const;
    srbmatrix& operator << (const srbmatrix& m) throw (cvmexception);
    srbmatrix operator + (const srbmatrix& m) const
                          throw (cvmexception);
    srbmatrix operator - (const srbmatrix& m) const
                          throw (cvmexception);
    srbmatrix& sum (const srbmatrix& m1,
                    const srbmatrix& m2) throw (cvmexception);
    srbmatrix& diff (const srbmatrix& m1,
                     const srbmatrix& m2) throw (cvmexception);
    srbmatrix& operator += (const srbmatrix& m) throw (cvmexception);
    srbmatrix& operator -= (const srbmatrix& m) throw (cvmexception);
    srbmatrix operator - () const;
    srbmatrix& operator ++ ();
    srbmatrix& operator ++ (int);
    srbmatrix& operator -- ();
    srbmatrix& operator -- (int);
    srbmatrix operator * (TR d) const;
```

```
    srbmatrix operator / (TR d) const throw (cvmexception);
    srbmatrix& operator *= (TR d);
    srbmatrix& operator /= (TR d) throw (cvmexception);
    srbmatrix& normalize ();
    srbmatrix operator ~ () const throw (cvmexception);
    srbmatrix& transpose (const srbmatrix& m) throw (cvmexception);
    srbmatrix& transpose () throw (cvmexception);
    rvector operator * (const rvector& v) const throw (cvmexception);
    rmatrix operator * (const rmatrix& m) const throw (cvmexception);
    srmatrix operator * (const srmatrix& m) const throw (cvmexception);
    srbmatrix operator * (const srbmatrix& m) const throw (cvmexception);
    srbmatrix& low_up (const srbmatrix& m,
                       int* nPivots) throw (cvmexception);
    srbmatrix low_up (int* nPivots) const throw (cvmexception);
    rvector operator / (const rvector& vB) const throw (cvmexception);
    srbmatrix& identity ();
    srbmatrix& vanish ();
    srbmatrix& randomize (TR dFrom, TR dTo);
};
```

2.11.1 srbmatrix ()

Constructor

`srbmatrix::srbmatrix ();`

creates empty `srbmatrix` object. See also `srbmatrix`. Example:

```
using namespace cvm;

srbmatrix m;
std::cout << m.msize() << " " << m.nsize() << " " << m.size();
std::cout << " " << m.lsize() << " " << m.usize() << std::endl;

m.resize (3);
m.resize_lu(1,0);
m.set(1.);
std::cout << m;
```

prints

```
0 0 0 0 0
1 0 0
1 1 0
0 1 1
```

2.11.2 srbmatrix (int)

Constructor

```
explicit srbmatrix::srbmatrix (int nMN);
```

creates $n \times n$ `srbmatrix` object where n is passed in `nMN` parameter. The matrix created is diagonal, i.e. $k_l = k_u = 0$. Constructor throws `cvmexception` in case of non-positive size passed or memory allocation failure. See also `srbmatrix`. Example:

```
using namespace cvm;

srbmatrix m(4);
std::cout << m.msize() << " " << m.nsize() << " " << m.size();
std::cout << " " << m.lsize() << " " << m.usize() << std::endl;

m.set(1.);
std::cout << m;
```

prints

```
4 4 4 0 0
1 0 0 0
0 1 0 0
0 0 1 0
0 0 0 1
```

2.11.3 srbmatrix (int,int,int)

Constructor

```
srbmatrix::srbmatrix (int nMN, int nKL, int nKU);
```

creates $n \times n$ `srbmatrix` object where n is passed in `nMN` parameter. The matrix created has `nKL` sub-diagonals and `nKU` super-diagonals. Constructor throws `cvmexception` in case of non-positive size or negative number of sub-diagonals or super-diagonals passed or in case of memory allocation failure. See also `srbmatrix`. Example:

```
using namespace cvm;

srbmatrix m(5,1,1);
m.set(1.);
std::cout << m << std::endl
          << m.msize() << " " << m.nsize() << " " << m.size()
          << " " << m.lsize() << " " << m.usize() << std::endl;
```

prints

```
1 1 0 0 0
1 1 1 0 0
0 1 1 1 0
0 0 1 1 1
0 0 0 1 1

5 5 15 1 1
```

2.11.4 srbmatrix (TR*,int,int,int)

Constructor

```
srbmatrix::srbmatrix (TR* pD, int nMN, int nKL, int nKU);
```

creates $n \times n$ `srbmatrix` object where n is passed in `nMN` parameter. The matrix created has k_l=`nKL` sub-diagonals and k_u=`nKU` super-diagonals. Unlike others, this constructor *does not allocate memory*. It just shares memory with an array pointed to by `pD`. Please note that this array must contain at least $(k_l + k_u + 1) * n$ elements. Constructor throws `cvmexception` in case of non-positive size or negative number of sub- or super-diagonals passed. See also `srbmatrix`, `srbmatrix (const TR*,int,int,int)`. Example:

```
using namespace cvm;
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix ml(a,4,1,0);
srbmatrix mu(a,4,0,1);
ml(2,1) = 9.;
std::cout << ml << std::endl << mu << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << " "
          << a[3] << " " << a[4] << " " << a[5] << " " << std::endl;
```

prints

```
1 0 0 0
9 3 0 0
0 4 5 0
0 0 6 7

9 3 0 0
0 4 5 0
0 0 6 7
0 0 0 8

1 9 3 4 5 6
```

2.11.5 srbmatrix (const TR*,int,int,int)

Constructor

```
srbmatrix::srbmatrix (const TR* pD, int nMN, int nKL, int nKU);
```

creates $n \times n$ `srbmatrix` object where n is passed in `nMN` parameter. The matrix created has k_l=`nKL` sub-diagonals and k_u=`nKU` super-diagonals. Then constructor copies $(k_l + k_u + 1) * n$ elements of an array `pD` to the matrix according to `band storage`. Constructor throws `cvmexception` in case of non-positive size or negative number of sub- or super-diagonals passed. See also `srbmatrix`, `srbmatrix (TR*,int,int,int)`. Example:

```
using namespace cvm;
const double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix ml(a,4,1,0);
srbmatrix mu(a,4,0,1);
ml(2,1) = 9.;
std::cout << ml << std::endl << mu << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << " "
          << a[3] << " " << a[4] << " " << a[5] << " " << std::endl;
```

prints

```
1 0 0 0
9 3 0 0
0 4 5 0
0 0 6 7

2 3 0 0
0 4 5 0
0 0 6 7
0 0 0 8

1 2 3 4 5 6
```

2.11.6 srbmatrix (const srbmatrix&)

Copy constructor

```
srbmatrix::srbmatrix (const srbmatrix& m);
```

creates `srbmatrix` object as a copy of `m`. Constructor throws `cvmexception` in case of memory allocation failure. See also `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m(a,4,1,0);
srbmatrix mc(m);
m(1,1) = 7.77;
std::cout << m << std::endl << mc;
```

prints

```
7.77 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7

1 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7
```

2.11.7 srbmatrix (const rmatrix&,int,int)

Constructor

```
srbmatrix::srbmatrix (const rmatrix& m, int nKL, int nKU);
```

creates `srbmatrix` object as a copy of "sliced" matrix `m`, i.e. it copies main diagonal, `nKL` sub-diagonals and `nKU` super-diagonals of a matrix `m`. It's assumed that $m \times n$ matrix `m` must have equal sizes, i.e. $m = n$ is satisfied. Constructor throws `cvmexception` if this is not true or in case of memory allocation failure. See also `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16.};
srmatrix m(a,4);
srbmatrix mb(m,1,2);
std::cout << m << std::endl << mb;
```

prints

```
1 5 9 13
2 6 10 14
3 7 11 15
4 8 12 16

1 5 9 0
2 6 10 14
0 7 11 15
0 0 12 16
```

2.11.8 srbmatrix (const rvector&)

Constructor

```
explicit srbmatrix::srbmatrix (const rvector& v);
```

creates `srbmatrix` object of size `v.size()` by `v.size()` and assigns vector `v` to its main diagonal. Constructor throws `cvmexception` in case of memory allocation failure. See also `srbmatrix`, `rvector`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5.};
rvector v(a, 5);
srbmatrix m(v);
std::cout << m;
```

prints

```
1 0 0 0 0
0 2 0 0 0
0 0 3 0 0
0 0 0 4 0
0 0 0 0 5
```

2.11.9 operator (,)

Indexing operators

```
TR& srbmatrix::operator () (int im, int in) throw (cvmexception);
TR srbmatrix::operator () (int im, int in) const throw (cvmexception);
```

provide access to a particular element of a calling band matrix. The first version is applicable to non-constant objects. This version returns *l-value* in order to make possible write access to an element. Only elements located on main diagonal or on non-zero sub- or super-diagonals are l-values. All other values located outside this area are not writable. Both operators are ℓ-based. Operators throw `cvmexception` if some of parameters passed is outside of `[1,msize()]` range or in case of attempt to write to a non-writable element[9]. See also `srbmatrix`, `BandMatrix::lsize()` and `BandMatrix::usize()`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    srbmatrix m (a,4,1,0);

    m(2,2) = 7.77;
    std::cout << m << std::endl;
    std::cout << m(3,2) << " " << m(1,4) << std::endl;

    m(1,3) = 7.77;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1.00e+000 0.00e+000 0.00e+000 0.00e+000
2.00e+000 7.77e+000 0.00e+000 0.00e+000
0.00e+000 4.00e+000 5.00e+000 0.00e+000
0.00e+000 0.00e+000 6.00e+000 7.00e+000

4.00e+000 0.00e+000
Exception: Attempt to change a read-only element
```

[9] Here I use `type_proxy<T>` class originally described in [4], p. 217.

2.11.10 operator ()

Indexing operator

```
const rvector srbmatrix::operator () (int i) const throw (cvmexception);
```

provides access to i-th column of a calling band matrix. Unlike srmatrix::operator (), this operator creates only *copy* of a column and therefore it returns *not l-value*. Operator is ℓ-based. It throws `cvmexception` if parameter `i` is outside of `[1,nsize()]` range. See also `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);

std::cout << m << std::endl;
std::cout << m(2);
```

prints

```
1 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7

0 3 4 0
```

2.11.11 operator []

Indexing operator

```
const rvector srbmatrix::operator [] (int i) const throw (cvmexception);
```

provides access to i-th row of a calling band matrix. Unlike srmatrix::operator [], this operator creates only *copy* of a column and therefore it returns *not l-value*. Operator is ℓ-based. It throws `cvmexception` if parameter `i` is outside of `[1,nsize()]` range. See also `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);

std::cout << m << std::endl;
std::cout << m[2];
```

prints

```
1 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7

2 3 0 0
```

2.11.12 operator = (const srbmatrix&)

Operator

```
srbmatrix& srbmatrix::operator = (const srbmatrix& m)
throw (cvmexception);
```

sets every element of a calling band matrix to a value of appropriate element of band matrix `m` and returns a reference to the matrix changed. Operator throws `cvmexception` in case of different matrix sizes or in case of different numbers of sub- or super-diagonals. See also `srbmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    const srbmatrix m1(a,4,1,0);
    srbmatrix m2(4,1,0);

    m2 = m1;
    std::cout << m2;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7
```

2.11.13 assign (const TR*)

Function

```
srbmatrix& srbmatrix::assign (const rvector& v) throw (cvmexception);
srbmatrix& srbmatrix::assign (const TR* pD);
```

sets every element of a calling band matrix to a value of appropriate element of vector `v` or array pointed to by `pD` and returns a reference to the matrix changed. In first version it's assumed that vector passed is long enough to fill calling matrix. Function throws `cvmexception` otherwise. In other words this array must contain at least $(k_l + k_u + 1)n$ elements. See also `srbmatrix`. Example:

```
using namespace cvm;

const double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m(4,0,1);

m.assign(a);
std::cout << m;
```

prints

```
2 3 0 0
0 4 5 0
0 0 6 7
0 0 0 8
```

2.11.14 set (TR)

Function

```
srbmatrix& srbmatrix::set (TR x);
```

sets every element of a calling band matrix to a value of parameter x and returns a reference to the matrix changed. Use vanish to set every element of a calling matrix to be equal to zero. See also srbmatrix. Example:

```
using namespace cvm;

srbmatrix m(8,2,1);
m.set(3.);
std::cout << m;
```

prints

```
3 3 0 0 0 0 0 0
3 3 3 0 0 0 0 0
3 3 3 3 0 0 0 0
0 3 3 3 3 0 0 0
0 0 3 3 3 3 0 0
0 0 0 3 3 3 3 0
0 0 0 0 3 3 3 3
0 0 0 0 0 3 3 3
```

2.11.15 resize

Function

```
srbmatrix& srbmatrix::resize (int nNewMN) throw (cvmexception);
```

changes size of a calling band matrix to `nNewMN` by `nNewMN` and returns a reference to the matrix changed. In case of increasing of its size, calling matrix is filled up with zeroes. This function doesn't change number of sub- ore super-diagonals. Like any band matrix class member function, this function doesn't change non-referred elements. See number 8 appearing after resize in example below. Function throws `cvmexception` in case of non-positive size passed or memory allocation failure. See also `srbmatrix.resize_lu`, `srbmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    srbmatrix m(a,4,1,0);
    std::cout << m << std::endl;
    m.resize (5);
    std::cout << m;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7

1 0 0 0 0
2 3 0 0 0
0 4 5 0 0
0 0 6 7 0
0 0 0 8 0
```

2.11.16 resize_lu

Function

```
srbmatrix& srbmatrix::resize_lu (int nNewKL, int nNewKU)
throw (cvmexception);
```

changes number of sub- and super-diagonals of a calling band matrix to `nNewKL` and `nNewKU` respectively and returns a reference to the matrix changed. In case of increasing of the numbers, calling matrix is filled up with zeroes. Function throws `cvmexception` in case of negative number passed or memory allocation failure. See also `srbmatrix::resize`, `srbmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    srbmatrix m(a,4,1,0);
    std::cout << m << std::endl;
    m.resize_lu (0,1);
    m.diag(1).set(9.);
    std::cout << m;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7

1 9 0 0
0 3 9 0
0 0 5 9
0 0 0 7
```

2.11.17 operator ==

Operator

```
bool srbmatrix::operator == (const srbmatrix& m) const;
```

compares calling band matrix with band matrix `m` and returns `true` if they have the same sizes, tha same numbers of sub- and super-diagonals and their appropriate elements differ by not more than the smallest normalized positive number. Returns `false` otherwise. See also `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4.};
srbmatrix m1(a,2,1,0);
srbmatrix m2(2,1,0);

std::cout << m1 << std::endl;

m2(1,1) = 1.;
m2(2,1) = 2.; m2(2,2) = 3.;

std::cout << (m1 == m2) << std::endl;
```

prints

```
1 0
2 3

1
```

2.11.18 operator !=

Operator

```
bool srbmatrix::operator != (const srbmatrix& m) const;
```

compares calling band matrix with band matrix `m` and returns `true` if they have different sizes, different numbers of sub- or super-diagonals or at least one of their appropriate elements differs by more than the smallest normalized positive number. Returns `false` otherwise. See also `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4.};
srbmatrix m1(a,2,1,0);
srbmatrix m2(2,1,0);

std::cout << m1 << std::endl;

m2(1,1) = 1.;
m2(2,1) = 2.; m2(2,2) = 3.;

std::cout << (m1 != m2) << std::endl;
```

prints

```
1 0
2 3

0
```

2.11.19 operator <<

Operator

```
srbmatrix& srbmatrix::operator << (const srbmatrix& m)
throw (cvmexception);
```

destroys calling band matrix, creates new one as a copy of `m` and returns a reference to the matrix changed. Operator throws `cvmexception` in case of memory allocation failure. See also `srbmatrix`. Example:

```
using namespace cvm;

try {
    srbmatrix m(3,1,0);
    srmatrix mc(1);
    m(2,1) = 1.;
    m(2,2) = 2.;
    std::cout << m << std::endl << mc << std::endl;

    mc << m;
    std::cout << mc;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
0 0 0
1 2 0
0 0 0

0

0 0 0
1 2 0
0 0 0
```

2.11.20 operator +

Operator

```
srbmatrix srbmatrix::operator + (const srbmatrix& m) const
throw (cvmexception);
```

creates an object of type `srbmatrix` as a sum of a calling band matrix and band matrix `m`. It throws `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `srbmatrix::sum`, `srbmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (1);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    double b[] = {10., 20., 30., 40., 50., 60., 70., 80.};
    srbmatrix m1(a,4,1,0);
    srbmatrix m2(b,4,1,0);

    std::cout << m1 + m2 << std::endl << m1 + m1;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1.1e+01 0.0e+00 0.0e+00 0.0e+00
2.2e+01 3.3e+01 0.0e+00 0.0e+00
0.0e+00 4.4e+01 5.5e+01 0.0e+00
0.0e+00 0.0e+00 6.6e+01 7.7e+01

2.0e+00 0.0e+00 0.0e+00 0.0e+00
4.0e+00 6.0e+00 0.0e+00 0.0e+00
0.0e+00 8.0e+00 1.0e+01 0.0e+00
0.0e+00 0.0e+00 1.2e+01 1.4e+01
```

2.11.21 operator -

Operator

```
srbmatrix srbmatrix::operator - (const srbmatrix& m) const
throw (cvmexception);
```

creates an object of type `srbmatrix` as a difference of a calling band matrix and band matrix `m`. It throws `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `srbmatrix::diff`, `srbmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (1);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    double b[] = {10., 20., 30., 40., 50., 60., 70., 80.};
    srbmatrix m1(a,4,1,0);
    srbmatrix m2(b,4,1,0);

    std::cout << m2 - m1 << std::endl << m1 - m1;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
9.0e+00 0.0e+00 0.0e+00 0.0e+00
1.8e+01 2.7e+01 0.0e+00 0.0e+00
0.0e+00 3.6e+01 4.5e+01 0.0e+00
0.0e+00 0.0e+00 5.4e+01 6.3e+01

0.0e+00 0.0e+00 0.0e+00 0.0e+00
0.0e+00 0.0e+00 0.0e+00 0.0e+00
0.0e+00 0.0e+00 0.0e+00 0.0e+00
0.0e+00 0.0e+00 0.0e+00 0.0e+00
```

2.11.22 sum

Function

```
srbmatrix& srbmatrix::sum (const srbmatrix& m1, const srbmatrix& m2)
throw (cvmexception);
```

assigns the result of addition of band matrices `m1` and `m2` to a calling band matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `srbmatrix::operator +` , `srbmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const srbmatrix m1(a,3,1,0);
    srbmatrix m2(3,1,0);
    srbmatrix m(3,1,0);
    m2.set(1.);
    std::cout << m1 << std::endl << m2 << std::endl;
    std::cout << m.sum(m1, m2) << std::endl;
    std::cout << m.sum(m, m2);
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 0 0
2 3 0
0 4 5

1 0 0
1 1 0
0 1 1

2 0 0
3 4 0
0 5 6

3 0 0
4 5 0
0 6 7
```

2.11.23 diff

Function

```
srbmatrix& srbmatrix::diff (const srbmatrix& m1, const srbmatrix& m2)
throw (cvmexception);
```

assigns the result of subtraction of band matrices m1 and m2 to a calling band matrix and returns a reference to the matrix changed. It throws cvmexception in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also srbmatrix::operator - , srbmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6.};
    const srbmatrix m1(a,3,1,0);
    srbmatrix m2(3,1,0);
    srbmatrix m(3,1,0);
    m2.set(1.);
    std::cout << m1 << std::endl << m2 << std::endl;
    std::cout << m.diff(m1, m2) << std::endl;
    std::cout << m.diff(m, m2);
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 0 0
2 3 0
0 4 5

1 0 0
1 1 0
0 1 1

0 0 0
1 2 0
0 3 4

-1 0 0
0 1 0
0 2 3
```

2.11.24 operator +=

Operator

```
srbmatrix& srbmatrix::operator += (const srbmatrix& m)
throw (cvmexception);
```

adds band matrix `m` to a calling band matrix and returns a reference to the matrix changed. Operator throws `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `srbmatrix::operator +` , `srbmatrix::sum`, `srbmatrix`. Example:

```
using namespace cvm;

try {
    srbmatrix m1(4,0,1);
    srbmatrix m2(4,0,1);
    m1.set(1.);
    m2.set(2.);

    m1 += m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 += m2;
    std::cout << m2;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
3 3 0 0
0 3 3 0
0 0 3 3
0 0 0 3

4 4 0 0
0 4 4 0
0 0 4 4
0 0 0 4
```

2.11.25 operator -=

Operator

```
srbmatrix& srbmatrix::operator -= (const srbmatrix& m)
throw (cvmexception);
```

subtracts band matrix `m` from calling band matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `srbmatrix::operator -` , `srbmatrix::diff`, `srbmatrix`. Example:

```
using namespace cvm;

try {
    srbmatrix m1(4,0,1);
    srbmatrix m2(4,0,1);
    m1.set(1.);
    m2.set(4.);

    m2 -= m1;
    std::cout << m2 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 -= m2;
    std::cout << m2;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
3 3 0 0
0 3 3 0
0 0 3 3
0 0 0 3

0 0 0 0
0 0 0 0
0 0 0 0
0 0 0 0
```

2.11.26 operator - ()

Operator

```
srbmatrix srbmatrix::operator - () const throw (cvmexception);
```

creates an object of type `srbmatrix` as a calling band matrix multiplied by -1. See also `srbmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific |
                std::ios::left |
                std::ios::showpos);
std::cout.precision (1);
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m(a,4,1,0);

std::cout << -m;
```

prints

```
-1.0e+00 +0.0e+00 +0.0e+00 +0.0e+00
-2.0e+00 -3.0e+00 +0.0e+00 +0.0e+00
+0.0e+00 -4.0e+00 -5.0e+00 +0.0e+00
+0.0e+00 +0.0e+00 -6.0e+00 -7.0e+00
```

2.11.27 operator ++

Operator

```
srbmatrix& srbmatrix::operator ++ ();
srbmatrix& srbmatrix::operator ++ (int);
```

adds identity matrix to a calling band matrix and returns a reference to the matrix changed. See also srbmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);

m++;
std::cout << m << std::endl;
std::cout << ++m;
```

prints

```
2 0 0 0
2 4 0 0
0 4 6 0
0 0 6 8

3 0 0 0
2 5 0 0
0 4 7 0
0 0 6 9
```

2.11.28 operator --

Operator

```
srbmatrix& srbmatrix::operator -- ();
srbmatrix& srbmatrix::operator -- (int);
```

subtracts identity matrix from calling band matrix and returns a reference to the matrix changed. See also srbmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);

m--;
std::cout << m << std::endl;
std::cout << --m;
```

prints

```
0 0 0 0
2 2 0 0
0 4 4 0
0 0 6 6

-1 0 0 0
2 1 0 0
0 4 3 0
0 0 6 5
```

2.11.29 operator * (TR)

Operator

```
srbmatrix srbmatrix::operator * (TR d) const;
```

creates an object of type `srbmatrix` as a product of a calling band matrix and number `d`. See also `srbmatrix::operator *=` , `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);
std::cout << m * 5. << std::endl;
```

prints

```
5 0 0 0
10 15 0 0
0 20 25 0
0 0 30 35
```

2.11.30 operator / (TR)

Operator

```
srbmatrix srbmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `srbmatrix` as a quotient of a calling band matrix and number `d`. It throws `cvmexception` if `d` has absolute value equal or less than the smallest normalized positive number. See also `srbmatrix::operator /=` , `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);
std::cout << m / 2. << std::endl;
```

prints

```
0.5 0 0 0
1 1.5 0 0
0 2 2.5 0
0 0 3 3.5
```

2.11.31 operator *= (TR)

Operator

```
srbmatrix& srbmatrix::operator *= (TR d);
```

multiplies calling band matrix by number d and returns a reference to the matrix changed. See also srbmatrix::operator * , srbmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);
m *= 2.;
std::cout << m;
```

prints

```
2 0 0 0
4 6 0 0
0 8 10 0
0 0 12 14
```

2.11.32 operator /= (TR)

Operator

```
srbmatrix& srbmatrix::operator /= (TR d) throw (cvmexception);
```

divides calling matrix by number `d` and returns a reference to the matrix changed. It throws `cvmexception` if `d` has absolute value equal or less than the smallest normalized positive number. See also `srbmatrix::operator /` , `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);
m /= 2.;
std::cout << m;
```

prints

```
0.5 0 0 0
1 1.5 0 0
0 2 2.5 0
0 0 3 3.5
```

2.11.33 normalize

Function

```
srbmatrix& srbmatrix::normalize ();
```

normalizes calling band matrix so its Euclidean norm becomes equal to 1 if it was greater than the smallest normalized positive number before the call (otherwise function does nothing). See also `srbmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);

m.normalize();
std::cout << m << m.norm() << std::endl;
```

prints

```
8.452e-02 0.000e+00 0.000e+00 0.000e+00
1.690e-01 2.535e-01 0.000e+00 0.000e+00
0.000e+00 3.381e-01 4.226e-01 0.000e+00
0.000e+00 0.000e+00 5.071e-01 5.916e-01
1.000e+00
```

2.11.34 transposition

Operator and functions

```
srbmatrix srbmatrix::operator ~ () const throw (cvmexception);
srbmatrix& srbmatrix::transpose (const srbmatrix& m) throw (cvmexception);
srbmatrix& srbmatrix::transpose () throw (cvmexception);
```

implement band matrix transposition. First operator creates an object of type `srbmatrix` as transposed calling band matrix (it throws `cvmexception` in case of memory allocation failure). Second function sets calling band matrix to be equal to band matrix `m` transposed (it throws `cvmexception` in case of not appropriate sizes of the operands or in case of memory allocation failure), third one makes it to be equal to transposed itself. See also `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
srbmatrix m (a,4,1,0);
srbmatrix mt(4,0,1);
std::cout << ~m << std::endl ;
mt.transpose(m);
std::cout << mt << std::endl;
mt.transpose();
std::cout << mt;
```

prints

```
1 2 0 0
0 3 4 0
0 0 5 6
0 0 0 7

1 2 0 0
0 3 4 0
0 0 5 6
0 0 0 7

1 0 0 0
2 3 0 0
0 4 5 0
0 0 6 7
```

2.11.35 operator * (const rvector&)

Operator

```
rvector srbmatrix::operator * (const rvector& v) const
throw (cvmexception);
```

creates an object of type `rvector` as a product of a calling band matrix and a vector `v`. It throws `cvmexception` if number of columns of a calling matrix differs from size of a vector `v`. Use `rvector::mult` in order to avoid new object creation. See also `srbmatrix` and `rvector`. Example:

```
using namespace cvm;

try {
    srbmatrix m (4,1,0);
    rvector v(4);
    m.set(1.);
    v.set(1.);

    std::cout << m * v;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 2 2 2
```

2.11.36 operator * (const rmatrix&)

Operator

```
rmatrix srbmatrix::operator * (const rmatrix& m) const
throw (cvmexception);
```

creates an object of type `rmatrix` as a product of a calling band matrix and a matrix `m`. It throws `cvmexception` if number of columns of a calling matrix differs from number of rows of a matrix `m`. Use `rmatrix::mult` in order to avoid new object creation. See also `rmatrix` and `srbmatrix`. Example:

```
using namespace cvm;

try {
    srbmatrix mb (4,1,0);
    rmatrix m(4,2);
    mb.set(1.);
    m.set(1.);

    std::cout << mb * m;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 1
2 2
2 2
2 2
```

2.11.37 operator * (const srmatrix&)

Operator

```
srmatrix srbmatrix::operator * (const srmatrix& m) const
throw (cvmexception);
```

creates an object of type `srmatrix` as a product of a calling band matrix and a matrix `m`. It throws `cvmexception` if the operands have different sizes. Use `rmatrix::mult` in order to avoid new object creation. See also `srmatrix` and `srbmatrix`. Example:

```
using namespace cvm;

try {
    srbmatrix mb(4,1,0);
    srmatrix ms(4);
    mb.set(1.);
    ms.set(1.);

    std::cout << mb * ms;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
1 1 1 1
2 2 2 2
2 2 2 2
2 2 2 2
```

2.11.38 operator * (const srbmatrix&)

Operator

```
srbmatrix srbmatrix::operator * (const srbmatrix& m) const
throw (cvmexception);
```

creates an object of type `srbmatrix` as a product of a calling band matrix and band matrix `m`. It throws `cvmexception` if the operands have different sizes. Use `rmatrix::mult` in order to avoid new object creation. See also `srbmatrix`. Example:

```
using namespace cvm;

try {
    srbmatrix m1(7,1,0);
    srbmatrix m2(7,1,1);
    m1.set(1.);
    m2.set(1.);

    std::cout << m1 * m2;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 1 0 0 0 0 0
2 2 1 0 0 0 0
1 2 2 1 0 0 0
0 1 2 2 1 0 0
0 0 1 2 2 1 0
0 0 0 1 2 2 1
0 0 0 0 1 2 2
```

2.11.39 low_up

Functions

```
srbmatrix&
srbmatrix::low_up (const srbmatrix& m, int* nPivots) throw (cvmexception);
srbmatrix
srbmatrix::low_up (int* nPivots) const throw (cvmexception);
```

compute LU factorization of a calling band matrix as

$$A = PLU$$

where P is permutation matrix, L is lower triangular matrix with unit diagonal elements and U is upper triangular matrix. All functions store result as matrix L without main diagonal combined with U. All functions return pivot indices as array of integers (it should support at least `msize()` elements) pointed to by `nPivots` so i-th row was interchanged with `nPivots[i]`-th row. The first version sets calling matrix to be equal to the `m`'s LU factorization and the second one creates an object of type `srbmatrix` as calling band matrix's LU factorization. Functions throw `cvmexception` in case of inappropriate sizes of the operands or when matrix to be factorized is close to singular. The first version also changes number of super-diagonals to be equal to $k_l + k_u$ in order to keep the result of factorization. It is recommended to use `iarray` for pivot values. This function is provided mostly for solving multiple systems of linear equations using `srmatrix::solve_lu` function, See also `srbmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (4);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
    srbmatrix ma(a,4,1,0);
    srbmatrix mLU(4,1,0);
    rmatrix  mb1(4,2); rvector vb1(4);
    rmatrix  mb2(4,2); rvector vb2(4);
    rmatrix  mx1(4,2); rvector vx1(4);
    rmatrix  mx2(4,2); rvector vx2(4);
    iarray   nPivots(4);
    double   dErr = 0.;
    mb1.randomize(-1.,3.); vb1.randomize(-2.,4.);
    mb2.randomize(-2.,5.); vb2.randomize(-3.,1.);

    mLU.low_up(ma, nPivots);
    mx1 = ma.solve_lu (mLU, nPivots, mb1, dErr);
    std::cout << mx1 << dErr << std::endl << std::endl;
    mx2 = ma.solve_lu (mLU, nPivots, mb2);
```

```
    std::cout << mx2 << std::endl;;
    std::cout << ma * mx1 - mb1 << std::endl << ma * mx2 - mb2;

    vx1 = ma.solve_lu (mLU, nPivots, vb1, dErr);
    std::cout << vx1 << dErr << std::endl;
    vx2 = ma.solve_lu (mLU, nPivots, vb2);
    std::cout << vx2 << std::endl;;
    std::cout << ma * vx1 - vb1 << std::endl << ma * vx2 - vb2;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
7.6327e-02 -4.7386e-01
-2.9523e-01 9.7577e-01
1.7288e-01 -3.5093e-01
1.0595e-01 4.7363e-01
1.1832e-15

3.1963e+00 4.8622e+00
-4.9904e-01 -2.6575e+00
8.2183e-01 2.3294e+00
-6.1693e-01 -1.8015e+00

0.0000e+00 0.0000e+00
0.0000e+00 -2.2204e-16
0.0000e+00 -4.4409e-16
0.0000e+00 0.0000e+00

0.0000e+00 0.0000e+00
8.8818e-16 0.0000e+00
0.0000e+00 -4.4409e-16
-4.4409e-16 4.4409e-16
7.8933e-01 7.0543e-01 -1.6338e-02 -2.6206e-01
1.4832e-15
-1.5505e+00 5.8987e-01 -8.4977e-01 7.3059e-01

0.0000e+00 0.0000e+00 0.0000e+00 0.0000e+00

-2.2204e-16 0.0000e+00 0.0000e+00 4.4409e-16
```

2.11.40 operator / (const rvector&)

Operator

```
rvector operator / (const rvector& vB) const throw (cvmexception);
```

returns solution x of linear equation $A * x = b$ where calling matrix is square band matrix A and a vector b is passed in parameter vB. This operator throws exception of type cvmexception in case of inappropriate sizes of the objects or when matrix A is close to singular. See also rvector::solve, srmatrix::solve, rvector.operator %, rvector, srmatrix, srbmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (5);

try {
    srbmatrix ma(4,2,1);
    rvector  vb(4);
    rvector  vx(4);
    ma.randomize(-1.,1.);
    vb.randomize(-2.,2.);

    vx = ma / vb;

    std::cout << ma * vx - vb;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+0.00000e+000 +2.22045e-016 +2.22045e-016 +0.00000e+000
```

2.11.41 identity

Function

```
srbmatrix& srbmatrix::identity();
```

sets calling band matrix to be equal to identity matrix and returns a reference to the matrix changed. Function doesn't change numbers of sub- and super-diagonals. See also srbmatrix. Example:

```
using namespace cvm;

srbmatrix m(4);
m.randomize(0.,1.);
std::cout << m << std::endl;
std::cout << m.identity();
```

prints

```
0.327372 0 0 0
0 0.955718 0 0
0 0 0.0960723 0
0 0 0 0.291818

1 0 0 0
0 1 0 0
0 0 1 0
0 0 0 1
```

2.11.42 vanish

Function

```
srbmatrix& srbmatrix::vanish();
```

sets every element of a calling band matrix to be equal to zero and returns a reference to the matrix changed. This function is faster than `srbmatrix::set(TR)` with zero operand passed. See also `srbmatrix`. Example:

```
using namespace cvm;

srbmatrix m(4,1,0);
m.randomize(0.,1.);
std::cout << m << std::endl;
std::cout << m.vanish();
```

prints

```
0.337138 0 0 0
0.101199 0.522843 0 0
0 0.258522 0.123447 0
0 0 0.591723 0.661489

0 0 0 0
0 0 0 0
0 0 0 0
0 0 0 0
```

2.11.43 randomize

Function

```
srbmatrix& srbmatrix::randomize (TR dFrom, TR dTo);
```

fills calling band matrix with pseudo-random numbers distributed between `dFrom` and `dTo`. Function returns a reference to the matrix changed. See also `srbmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (7);
srbmatrix m(4,1,0);
m.randomize(0.,1.);
std::cout << m << std::endl;
```

prints

```
+3.4281442e-01 +0.0000000e+00 +0.0000000e+00 +0.0000000e+00
+7.9808954e-01 +5.9761345e-01 +0.0000000e+00 +0.0000000e+00
+0.0000000e+00 +1.1670278e-01 +6.5645314e-02 +0.0000000e+00
+0.0000000e+00 +0.0000000e+00 +4.2225410e-01 +7.5563829e-02
```

2.12 scbmatrix

This is end-user class encapsulating square band matrix of complex numbers. This class utilizes band storage for its elements.

```
template <typename TR, typename TC>
class scbmatrix : public scmatrix <TR,TC>, public BandMatrix <TR,TC> {
public:
    scbmatrix ();
    explicit scbmatrix (int nMN);
    scbmatrix (int nMN);
    scbmatrix (TC* pD, int nMN, int nKL, int nKU);
    scbmatrix (const TC* pD, int nMN, int nKL, int nKU);
    scbmatrix (const scbmatrix& m);
    scbmatrix (const cmatrix& m, int nKL, int nKU);
    explicit scbmatrix (const cvector& v);
    explicit scbmatrix (const srbmatrix& m, bool bRealPart = true);
    scbmatrix (const srbmatrix& mRe, const srbmatrix& mIm);
    TC& operator () (int im, int in) throw (cvmexception);
    TC operator () (int im, int in) const throw (cvmexception);
    const cvector operator () (int i) const throw (cvmexception);
    const cvector operator [] (int i) const throw (cvmexception);
    const srbmatrix real () const;
    const srbmatrix imag () const;
    scbmatrix& operator = (const scbmatrix& m) throw (cvmexception);
    scbmatrix& assign (const cvector& v) throw (cvmexception);
    scbmatrix& assign (const TC* pD);
    scbmatrix& set (TC z);
    scbmatrix& assign_real (const srbmatrix& mRe) throw (cvmexception);
    scbmatrix& assign_imag (const srbmatrix& mIm) throw (cvmexception);
    scbmatrix& set_real (TR d);
    scbmatrix& set_imag (TR d);
    scbmatrix& resize (int nNewMN) throw (cvmexception);
    scbmatrix& resize_lu (int nNewKL, int nNewKU) throw (cvmexception);
    bool operator == (const scbmatrix& v) const;
    bool operator != (const scbmatrix& v) const;
    scbmatrix& operator << (const scbmatrix& m) throw (cvmexception);
    scbmatrix operator + (const scbmatrix& m) const
                          throw (cvmexception);
    scbmatrix operator - (const scbmatrix& m) const
                          throw (cvmexception);
    scbmatrix& sum (const scbmatrix& m1,
                    const scbmatrix& m2) throw (cvmexception);
    scbmatrix& diff (const scbmatrix& m1,
                     const scbmatrix& m2) throw (cvmexception);
```

```
    scbmatrix& operator += (const scbmatrix& m) throw (cvmexception);
    scbmatrix& operator -= (const scbmatrix& m) throw (cvmexception);
    scbmatrix operator - () const;
    scbmatrix& operator ++ ();
    scbmatrix& operator ++ (int);
    scbmatrix& operator -- ();
    scbmatrix& operator -- (int);
    scbmatrix operator * (TR d) const;
    scbmatrix operator / (TR d) const throw (cvmexception);
    scbmatrix operator * (TC z) const;
    scbmatrix operator / (TC z) const throw (cvmexception);
    scbmatrix& operator *= (TR d);
    scbmatrix& operator /= (TR d) throw (cvmexception);
    scbmatrix& operator *= (TC z);
    scbmatrix& operator /= (TC z) throw (cvmexception);
    scbmatrix& normalize ();
    scbmatrix operator ~ () const;
    scbmatrix operator ! () const;
    scbmatrix& conj (const scbmatrix& m) throw (cvmexception);
    scbmatrix& conj ();
    scbmatrix& transpose (const scbmatrix& m) throw (cvmexception);
    scbmatrix& transpose ();
    cvector operator * (const cvector& v) const throw (cvmexception);
    cmatrix operator * (const cmatrix& m) const throw (cvmexception);
    scmatrix operator * (const scmatrix& m) const throw (cvmexception);
    scbmatrix operator * (const scbmatrix& m) const throw (cvmexception);
    scbmatrix& low_up (const scbmatrix& m,
                       int* nPivots) throw (cvmexception);
    scbmatrix low_up (int* nPivots) const throw (cvmexception);
    cvector operator / (const cvector& vB) const throw (cvmexception);
    scbmatrix& identity ();
    scbmatrix& vanish ();
    scbmatrix& randomize_real (TR dFrom, TR dTo);
    scbmatrix& randomize_imag (TR dFrom, TR dTo);
};
```

2.12.1 scbmatrix ()

Constructor

```
scbmatrix::scbmatrix ();
```

creates empty `scbmatrix` object. See also `scbmatrix`. Example:

```
using namespace cvm;

scbmatrix m;
std::cout << m.msize() << " " << m.nsize() << " " << m.size() ;
std::cout << " " << m.lsize() << " " << m.usize() << std::endl;
m.resize(3);
m.resize_lu(1,0);
m.set(std::complex<double>(1.,2.));
std::cout << m;
```

prints

```
0 0 0 0 0
(1,2) (0,0) (0,0)
(1,2) (1,2) (0,0)
(0,0) (1,2) (1,2)
```

2.12.2 scbmatrix (int)

Constructor

```
explicit scbmatrix::scbmatrix (int nMN);
```

creates $n \times n$ `scbmatrix` object where n is passed in `nMN` parameter. The matrix created is diagonal, i.e. $k_l = k_u = 0$. Constructor throws `cvmexception` in case of non-positive size passed or memory allocation failure. See also `scbmatrix`. Example:

```
using namespace cvm;

scbmatrix m(4);
std::cout << m.msize() << " " << m.nsize() << " " << m.size() ;
std::cout << " " << m.lsize() << " " << m.usize() << std::endl;
m.set(std::complex<double>(1.,2.));
std::cout << m;
```

prints

```
4 4 4 0 0
(1,2) (0,0) (0,0) (0,0)
(0,0) (1,2) (0,0) (0,0)
(0,0) (0,0) (1,2) (0,0)
(0,0) (0,0) (0,0) (1,2)
```

2.12.3 scbmatrix (int,int,int)

Constructor

```
scbmatrix::scbmatrix (int nMN, int nKL, int nKU);
```

creates $n \times n$ `scbmatrix` object where n is passed in `nMN` parameter. The matrix created has `nKL` sub-diagonals and `nKU` super-diagonals. Constructor throws `cvmexception` in case of non-positive size or negative number of sub-diagonals or super-diagonals passed or in case of memory allocation failure. See also `scbmatrix`. Example:

```
using namespace cvm;

scbmatrix m(4,1,1);
m.set(std::complex<double>(1.,2.));
std::cout << m << std::endl
          << m.msize() << " " << m.nsize() << " " << m.size()
          << " " << m.lsize() << " " << m.usize() << std::endl;
```

prints

```
(1,2) (1,2) (0,0) (0,0)
(1,2) (1,2) (1,2) (0,0)
(0,0) (1,2) (1,2) (1,2)
(0,0) (0,0) (1,2) (1,2)

4 4 12 1 1
```

2.12.4 scbmatrix (TC*,int,int,int)

Constructor

```
scbmatrix::scbmatrix (TC* pD, int nMN, int nKL, int nKU);
```

creates $n \times n$ `scbmatrix` object where n is passed in `nMN` parameter. The matrix created has k_l=`nKL` sub-diagonals and k_u=`nKU` super-diagonals. Unlike others, this constructor *does not allocate memory*. It just shares memory with an array pointed to by `pD`. Please note that this array must contain at least $(k_l + k_u + 1) * n$ elements. Constructor throws `cvmexception` in case of non-positive size or negative number of sub- or super-diagonals passed. See also `scbmatrix`, `scbmatrix (const TC*,int,int,int)`. Example:

```
using namespace cvm;
double a[] = {1., 1., 1., 1., 1., 1., 1., 1.,
              1., 1., 1., 1., 1., 1., 1., 1.};
scbmatrix ml ((std::complex<double>*)a,4,1,0);
scbmatrix mu ((std::complex<double>*)a,4,0,1);
ml(2,1) = std::complex<double>(5.,5.);
std::cout << ml << std::endl << mu << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << " "
          << a[3] << " " << a[4] << " " << a[5] << " " << std::endl;
```

prints

```
(1,1) (0,0) (0,0) (0,0)
(5,5) (1,1) (0,0) (0,0)
(0,0) (1,1) (1,1) (0,0)
(0,0) (0,0) (1,1) (1,1)

(5,5) (1,1) (0,0) (0,0)
(0,0) (1,1) (1,1) (0,0)
(0,0) (0,0) (1,1) (1,1)
(0,0) (0,0) (0,0) (1,1)

1 1 5 5 1 1
```

2.12.5 scbmatrix (const TC*,int,int,int)

Constructor

```
scbmatrix::scbmatrix (const TC* pD, int nMN, int nKL, int nKU);
```

creates $n \times n$ `scbmatrix` object where n is passed in `nMN` parameter. The matrix created has k_l=`nKL` sub-diagonals and k_u=`nKU` super-diagonals. Then constructor copies $(k_l + k_u + 1) * n$ elements of an array `pD` to the matrix according to `band storage`. Constructor throws `cvmexception` in case of non-positive size or negative number of sub- or super-diagonals passed. See also `scbmatrix`, `scbmatrix (TC*,int,int,int)`. Example:

```
using namespace cvm;
const double a[] = {1., 1., 1., 1., 1., 1., 1., 1.,
                    1., 1., 1., 1., 1., 1., 1., 1.};
scbmatrix ml ((const std::complex<double>*)a,4,1,0);
scbmatrix mu ((const std::complex<double>*)a,4,0,1);
ml(2,1) = std::complex<double>(5.,5.);
std::cout << ml << std::endl << mu << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << " "
          << a[3] << " " << a[4] << " " << a[5] << " " << std::endl;
```

prints

```
(1,1) (0,0) (0,0) (0,0)
(5,5) (1,1) (0,0) (0,0)
(0,0) (1,1) (1,1) (0,0)
(0,0) (0,0) (1,1) (1,1)

(1,1) (1,1) (0,0) (0,0)
(0,0) (1,1) (1,1) (0,0)
(0,0) (0,0) (1,1) (1,1)
(0,0) (0,0) (0,0) (1,1)

1 1 1 1 1 1
```

2.12.6 scbmatrix (const scbmatrix&)

Copy constructor

```
scbmatrix::scbmatrix (const scbmatrix& m);
```

creates `scbmatrix` object as a copy of `m`. Constructor throws `cvmexception` in case of memory allocation failure. See also `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16.};
scbmatrix m ((std::complex<double>*)a,4,1,0);
scbmatrix mc(m);
m(1,1) = 7.77;
std::cout << m << std::endl << mc;
```

prints

```
(7.77,0) (0,0) (0,0) (0,0)
(3,4) (5,6) (0,0) (0,0)
(0,0) (7,8) (9,10) (0,0)
(0,0) (0,0) (11,12) (13,14)

(1,2) (0,0) (0,0) (0,0)
(3,4) (5,6) (0,0) (0,0)
(0,0) (7,8) (9,10) (0,0)
(0,0) (0,0) (11,12) (13,14)
```

2.12.7 scbmatrix (const cmatrix&,int,int)

Constructor

```
scbmatrix::scbmatrix (const cmatrix& m, int nKL, int nKU);
```

creates `scbmatrix` object as a copy of "sliced" matrix `m`, i.e. it copies main diagonal, `nKL` sub-diagonals and `nKU` super-diagonals of a matrix `m`. It's assumed that $m \times n$ matrix `m` must have equal sizes, i.e. $m = n$ is satisfied. Constructor throws `cvmexception` if this is not true or in case of memory allocation failure. See also `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16., 17., 18.};
scmatrix m((std::complex<double>*)a,3);
scbmatrix mb(m,1,0);
std::cout << m << std::endl << mb;
```

prints

```
(1,2) (7,8) (13,14)
(3,4) (9,10) (15,16)
(5,6) (11,12) (17,18)

(1,2) (0,0) (0,0)
(3,4) (9,10) (0,0)
(0,0) (11,12) (17,18)
```

2.12.8 scbmatrix (const cvector&)

Constructor

```
explicit scbmatrix::scbmatrix (const cvector& v);
```

creates `scbmatrix` object of size `v.size()` by `v.size()` and assigns vector `v` to its main diagonal. Constructor throws `cvmexception` in case of memory allocation failure. See also `scbmatrix`, `cvector`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6.};
cvector v((std::complex<double>*)a,3);
scbmatrix m(v);
std::cout << m;
```

prints

```
(1,2) (0,0) (0,0)
(0,0) (3,4) (0,0)
(0,0) (0,0) (5,6)
```

2.12.9 scbmatrix (const srbmatrix&,bool)

Constructor

```
explicit scbmatrix::scbmatrix (const srbmatrix& m, bool bRealPart = true);
```

creates `scbmatrix` object having the same dimension and the same numbers of sub- and super-diagonals as real matrix `m` and copies the matrix `m` to its real part if `bRealPart` is `true` or to its imaginary part otherwise. See also `scbmatrix`, `srbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
const srbmatrix m(a,4,1,0);
scbmatrix mr(m), mi(m, false);
std::cout << mr << std::endl << mi;
```

prints

```
(1,0) (0,0) (0,0) (0,0)
(2,0) (3,0) (0,0) (0,0)
(0,0) (4,0) (5,0) (0,0)
(0,0) (0,0) (6,0) (7,0)

(0,1) (0,0) (0,0) (0,0)
(0,2) (0,3) (0,0) (0,0)
(0,0) (0,4) (0,5) (0,0)
(0,0) (0,0) (0,6) (0,7)
```

2.12.10 scbmatrix (const srbmatrix&, const srbmatrix&)

Constructor

```
scbmatrix::scbmatrix (const srbmatrix& mRe, const srbmatrix& mIm);
```

creates `scbmatrix` object of the same size as `mRe` and `mIm` has (it throws `cvmexception` if `mRe` and `mIm` have different sizes or different numbers of sub- or super-diagonals) and copies matrices `mRe` and `mIm` to real and imaginary part of the matrix created respectively. Constructor throws `cvmexception` in case of memory allocation failure. See also `scbmatrix`, `srbmatrix`. Example:

```
using namespace cvm;

srbmatrix mr(4,1,0), mi(4,1,0);
mr.set(1.);
mi.set(2.);
const scbmatrix m(mr,mi);
std::cout << m;
```

prints

```
(1,2) (0,0) (0,0) (0,0)
(1,2) (1,2) (0,0) (0,0)
(0,0) (1,2) (1,2) (0,0)
(0,0) (0,0) (1,2) (1,2)
```

2.12.11 operator (,)

Indexing operators

```
TC& scbmatrix::operator () (int im, int in) throw (cvmexception);
TC scbmatrix::operator () (int im, int in) const throw (cvmexception);
```

provide access to a particular element of a calling band matrix. The first version of operator is applicable to non-constant object. This version returns *l-value* in order to make possible write access to an element. Only elements located on main diagonal or on non-zero sub- or super-diagonals are l-values. All other values located outside this area are not writable. Both operators are ℓ-based. Operators throw `cvmexception` if some of parameters passed is outside of `[1,msize()]` range or in case of attempt to write to non-writable element[10]. See also `scbmatrix`, `BandMatrix::lsize()` and `BandMatrix::usize()`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
    srbmatrix m (a,3,1,0);

    m(2,1) = 7.77;
    std::cout << m << std::endl;
    std::cout << m(3,2) << " " << m(1,3) << std::endl;

    m(1,3) = 7.77;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1.00e+00 0.00e+00 0.00e+00
7.77e+00 3.00e+00 0.00e+00
0.00e+00 4.00e+00 5.00e+00

4.00e+00 0.00e+00
Exception: Attempt to change a read-only element
```

[10] Here I use `type_proxy<T>` class originally described in [4], p. 217.

2.12.12 operator ()

Indexing operator

```
const cvector scbmatrix::operator () (int i) const throw (cvmexception);
```

provides access to i-th column of a calling band matrix. Unlike scmatrix::operator (), this operator creates only *copy* of a column and therefore it returns *not l-value*. Operator is ℓ-based. It throws `cvmexception` if parameter `i` is outside of `[1,nsize()]` range. See also `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
scbmatrix m ((std::complex<double>*)a,3,1,0);
std::cout << m << std::endl;
std::cout << m(2);
```

prints

```
(1,2) (0,0) (0,0)
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)

(0,0) (5,6) (7,8)
```

2.12.13 operator []

Indexing operator

```
const cvector scbmatrix::operator [] (int i) const throw (cvmexception);
```

provides access to i-th row of a calling band matrix. Unlike scmatrix::operator [], this operator creates only *copy* of a column and therefore it returns *not l-value*. Operator is ℓ-based. It throws `cvmexception` if parameter `i` is outside of `[1,nsize()]` range. See also `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
scbmatrix m ((std::complex<double>*)a,3,1,0);
std::cout << m << std::endl;
std::cout << m[3];
```

prints

```
(1,2) (0,0) (0,0)
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)

(0,0) (7,8) (9,10)
```

2.12.14 real

Function

`const srbmatrix scbmatrix::real () const;`

creates an object of type `const srbmatrix` as real part of a calling band matrix. Please note that, unlike cvector::real, this function creates new object *not sharing* memory with real part of a calling matrix, i.e. the matrix returned is *not l-value*. See also `srbmatrix`, `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
scbmatrix m ((std::complex<double>*)a,3,1,0);
std::cout << m << std::endl;
std::cout << m.real();
```

prints

```
(1,2) (0,0) (0,0)
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)

1 0 0
3 5 0
0 7 9
```

2.12.15 imag

Function

```
const srbmatrix scbmatrix::imag () const;
```

creates an object of type `const srbmatrix` as imaginary part of a calling band matrix. Please note that, unlike cvector::imag, this function creates new object *not sharing* memory with imaginary part of a calling matrix, i.e. the matrix returned is *not l-value*. See also `srbmatrix`, `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
scbmatrix m ((std::complex<double>*)a,3,1,0);
std::cout << m << std::endl;
std::cout << m.imag();
```

prints

```
(1,2) (0,0) (0,0)
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)

2 0 0
4 6 0
0 8 10
```

2.12.16 operator = (const scbmatrix&)

Operator

```
scbmatrix& scbmatrix::operator = (const scbmatrix& m)
throw (cvmexception);
```

sets every element of a calling band matrix to a value of appropriate element of band matrix m and returns a reference to the matrix changed. Operator throws cvmexception in case of different matrix sizes or in case of different numbers of sub- or super-diagonals. See also scbmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
    scbmatrix m1((std::complex<double>*)a,3,1,0);
    scbmatrix m2(3,1,0);

    m2 = m1;
    std::cout << m2;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(1.00e+00,2.00e+00) (0.00e+00,0.00e+00) (0.00e+00,0.00e+00)
(3.00e+00,4.00e+00) (5.00e+00,6.00e+00) (0.00e+00,0.00e+00)
(0.00e+00,0.00e+00) (7.00e+00,8.00e+00) (9.00e+00,1.00e+01)
```

2.12.17 assign (const TC*)

Function

```
scbmatrix& scbmatrix::assign (const cvector& v) throw (cvmexception);
scbmatrix& scbmatrix::assign (const TC* pD);
```

sets every element of a calling band matrix to a value of appropriate element of vector v or array pointed to by pD and returns a reference to the matrix changed. In first version it's assumed that vector passed is long enough to fill calling matrix. Function throws cvmexception otherwise. In other words this array must contain at least $(k_l + k_u + 1)n$ elements. See also scbmatrix. Example:

```
using namespace cvm;

const double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
scbmatrix m(3,0,1);
m.assign((const std::complex<double>*)a);
std::cout << m;
```

prints

```
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)
(0,0) (0,0) (11,12)
```

2.12.18 set (TC)

Function

```
scbmatrix& scbmatrix::set (TC z);
```

sets every element of a calling band matrix to a value of parameter z and returns a reference to the matrix changed. Use vanish to set every element of a calling matrix to be equal to zero. See also scbmatrix. Example:

```
using namespace cvm;

scbmatrix m(4,1,0);
m.set(std::complex<double>(1.,2.));
std::cout << m;
```

prints

```
(1,2) (0,0) (0,0) (0,0)
(1,2) (1,2) (0,0) (0,0)
(0,0) (1,2) (1,2) (0,0)
(0,0) (0,0) (1,2) (1,2)
```

2.12.19 assign_real

Function

```
scbmatrix& scbmatrix::assign_real (const srbmatrix& mRe)
throw (cvmexception);
```

sets real part of every element of a calling band matrix to a value of appropriate element of band matrix `mRe` and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. See also `scbmatrix` and `srbmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
srbmatrix m (3,0,1);
scbmatrix mc(3,0,1);
m.randomize (0., 1.);

mc.assign_real(m);
std::cout << mc;
```

prints

```
(5.44e-01,0.00e+00) (5.48e-02,0.00e+00) (0.00e+00,0.00e+00)
(0.00e+00,0.00e+00) (3.66e-01,0.00e+00) (3.49e-01,0.00e+00)
(0.00e+00,0.00e+00) (0.00e+00,0.00e+00) (8.00e-01,0.00e+00)
```

2.12.20 assign_imag

Function

```
scbmatrix& scbmatrix::assign_imag (const srbmatrix& mIm)
throw (cvmexception);
```

sets imaginary part of every element of a calling band matrix to a value of appropriate element of bandmatrix `mIm` and returns a reference to the matrix changed. Function throws `cvmexception` in case of different sizes of the operands. See also `scbmatrix` and `srbmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
srbmatrix m (3,0,1);
scbmatrix mc(3,0,1);
m.randomize (0., 1.);

mc.assign_imag(m);
std::cout << mc;
```

prints

```
(0.00e+00,5.53e-01) (0.00e+00,2.16e-01) (0.00e+00,0.00e+00)
(0.00e+00,0.00e+00) (0.00e+00,1.57e-01) (0.00e+00,1.12e-01)
(0.00e+00,0.00e+00) (0.00e+00,0.00e+00) (0.00e+00,7.03e-01)
```

2.12.21 set_real

Function

```
scbmatrix& scbmatrix::set_real (TR d);
```

sets real part of every element of a calling band matrix to a value of parameter `d` and returns a reference to the matrix changed. See also `scbmatrix`. Example:

```
using namespace cvm;

scbmatrix m(4,0,1);
m.set_real(1.);
std::cout << m;
```

prints

```
(1,0) (1,0) (0,0) (0,0)
(0,0) (1,0) (1,0) (0,0)
(0,0) (0,0) (1,0) (1,0)
(0,0) (0,0) (0,0) (1,0)
```

2.12.22 set_imag

Function

`scbmatrix& scbmatrix::set_imag (TR d);`

sets imaginary part of every element of a calling band matrix to a value of parameter `d` and returns a reference to the matrix changed. See also `scbmatrix`. Example:

```
using namespace cvm;

scbmatrix m(4,0,1);
m.set_imag(1.);
std::cout << m;
```

prints

```
(0,1) (0,1) (0,0) (0,0)
(0,0) (0,1) (0,1) (0,0)
(0,0) (0,0) (0,1) (0,1)
(0,0) (0,0) (0,0) (0,1)
```

2.12.23 resize

Function

```
scbmatrix& scbmatrix::resize (int nNewMN) throw (cvmexception);
```

changes size of a calling band matrix to `nNewMN` by `nNewMN` and returns a reference to the matrix changed. In case of increasing of its size, calling matrix is filled up with zeroes. This function doesn't change number of sub- ore super-diagonals. Like any band matrix class member function, this function doesn't change non-referred elements. See number `(11,12)` appearing after resize in example below. Function throws `cvmexception` in case of non-positive size passed or memory allocation failure. See also `scbmatrix.resize_lu`, `scbmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
    scbmatrix m((std::complex<double>*)a,3,1,0);
    std::cout << m << std::endl;
    m.resize (4);
    std::cout << m;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(1,2) (0,0) (0,0)
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)

(1,2) (0,0) (0,0) (0,0)
(3,4) (5,6) (0,0) (0,0)
(0,0) (7,8) (9,10) (0,0)
(0,0) (0,0) (11,12) (0,0)
```

2.12.24 resize_lu

Function

```
scbmatrix& scbmatrix::resize_lu (int nNewKL, int nNewKU)
throw (cvmexception);
```

changes number of sub- and super-diagonals of a calling band matrix to `nNewKL` by `nNewKU` respectively and returns a reference to the matrix changed. In case of increasing of the numbers, calling matrix is filled up with zeroes. Function throws `cvmexception` in case of negative number passed or memory allocation failure. See also `scbmatrix::resize`, `scbmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9., 10., 11., 12.};
    scbmatrix m((std::complex<double>*)a,3,1,0);
    std::cout << m << std::endl;
    m.resize_lu (0,1);
    m.diag(1).set(std::complex<double>(9.,9.));
    std::cout << m;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(1,2) (0,0) (0,0)
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)

(1,2) (9,9) (0,0)
(0,0) (5,6) (9,9)
(0,0) (0,0) (9,10)
```

2.12.25 operator ==

Operator

```
bool scbmatrix::operator == (const scbmatrix& m) const;
```

compares calling band matrix with band matrix `m` and returns `true` if they have the same sizes, the same numbers of sub- and super-diagonals and their appropriate elements differ by not more than the smallest normalized positive number. Returns `false` otherwise. See also `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
scbmatrix m1((std::complex<double>*)a,2,1,0);
scbmatrix m2(2,1,0);
std::cout << m1 << std::endl;

m2(1,1) = std::complex<double>(1.,2.);
m2(2,1) = std::complex<double>(3.,4.);
m2(2,2) = std::complex<double>(5.,6.);

std::cout << (m1 == m2) << std::endl;
```

prints

```
(1,2) (0,0)
(3,4) (5,6)

1
```

2.12.26 operator !=

Operator

`bool scbmatrix::operator != (const scbmatrix& m) const;`

compares calling band matrix with band matrix `m` and returns `true` if they have different sizes, different numbers of sub- or super-diagonals or at least one of their appropriate elements differs by more than the smallest normalized positive number. Returns `false` otherwise. See also `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.};
scbmatrix m1((std::complex<double>*)a,2,1,0);
scbmatrix m2(2,1,0);
std::cout << m1 << std::endl;

m2(1,1) = std::complex<double>(1.,2.);
m2(2,1) = std::complex<double>(3.,4.);
m2(2,2) = std::complex<double>(5.,6.00001);

std::cout << (m1 != m2) << std::endl;
```

prints

```
(1,2) (0,0)
(3,4) (5,6)

1
```

2.12.27 operator <<

Operator

```
scbmatrix& scbmatrix::operator << (const scbmatrix& m)
throw (cvmexception);
```

destroys calling band matrix, creates new one as a copy of `m` and returns a reference to the matrix changed. Operator throws `cvmexception` in case of memory allocation failure. See also `scbmatrix`. Example:

```
using namespace cvm;

try {
    scbmatrix m(3,1,0);
    scbmatrix mc(1);
    m(2,1) = std::complex<double>(1.,2.);
    m(2,2) = std::complex<double>(3.,4.);
    std::cout << m << std::endl << mc << std::endl;

    mc << m;
    std::cout << mc;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(0,0) (0,0) (0,0)
(1,2) (3,4) (0,0)
(0,0) (0,0) (0,0)

(0,0)

(0,0) (0,0) (0,0)
(1,2) (3,4) (0,0)
(0,0) (0,0) (0,0)
```

2.12.28 operator +

Operator

```
scbmatrix scbmatrix::operator + (const scbmatrix& m) const
throw (cvmexception);
```

creates an object of type `scbmatrix` as a sum of a calling band matrix and band matrix `m`. It throws `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `scbmatrix::sum`, `scbmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.,
                  9., 10., 11., 12.};
    double b[] = {10., 20., 30., 40., 50., 60.,
                  70., 80., 90., 100., 110., 120.};
    scbmatrix m1((std::complex<double>*)a,3,0,1);
    scbmatrix m2((std::complex<double>*)b,3,0,1);

    std::cout << m1 << std::endl << m2 << std::endl;
    std::cout << m1 + m2 << std::endl << m1 + m1;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)
(0,0) (0,0) (11,12)

(30,40) (50,60) (0,0)
(0,0) (70,80) (90,100)
(0,0) (0,0) (110,120)

(33,44) (55,66) (0,0)
(0,0) (77,88) (99,110)
(0,0) (0,0) (121,132)

(6,8) (10,12) (0,0)
(0,0) (14,16) (18,20)
(0,0) (0,0) (22,24)
```

2.12.29 operator -

Operator

```
scbmatrix scbmatrix::operator - (const scbmatrix& m) const
throw (cvmexception);
```

creates an object of type `scbmatrix` as a difference of a calling band matrix and band matrix `m`. It throws `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `scbmatrix::diff`, `scbmatrix`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8.,
                  9., 10., 11., 12.};
    double b[] = {10., 20., 30., 40., 50., 60.,
                  70., 80., 90., 100., 110., 120.};
    scbmatrix m1((std::complex<double>*)a,3,0,1);
    scbmatrix m2((std::complex<double>*)b,3,0,1);

    std::cout << m1 << std::endl << m2 << std::endl;
    std::cout << m1 - m2 << std::endl << m1 - m1;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)
(0,0) (0,0) (11,12)

(30,40) (50,60) (0,0)
(0,0) (70,80) (90,100)
(0,0) (0,0) (110,120)

(-27,-36) (-45,-54) (0,0)
(0,0) (-63,-72) (-81,-90)
(0,0) (0,0) (-99,-108)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.12.30 sum

Function

```
scbmatrix& scbmatrix::sum (const scbmatrix& m1, const scbmatrix& m2)
throw (cvmexception);
```

assigns the result of addition of band matrices `m1` and `m2` to a calling band matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also `scbmatrix::operator +` , `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.,
              9., 10., 11., 12.};
const scbmatrix m1((std::complex<double>*)a,3,1,0);
scbmatrix m2(3,1,0);
scbmatrix m(3,1,0);
m2.set(std::complex<double>(1.,1.));
std::cout << m1 << std::endl << m2 << std::endl;
std::cout << m.sum(m1, m2) << std::endl;
std::cout << m.sum(m, m2);
```

prints

```
(1,2) (0,0) (0,0)
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)

(1,1) (0,0) (0,0)
(1,1) (1,1) (0,0)
(0,0) (1,1) (1,1)

(2,3) (0,0) (0,0)
(4,5) (6,7) (0,0)
(0,0) (8,9) (10,11)

(3,4) (0,0) (0,0)
(5,6) (7,8) (0,0)
(0,0) (9,10) (11,12)
```

2.12.31 diff

Function

```
scbmatrix& scbmatrix::diff (const scbmatrix& m1, const scbmatrix& m2)
throw (cvmexception);
```

assigns the result of subtraction of band matrices m1 and m2 to a calling band matrix and returns a reference to the matrix changed. It throws cvmexception in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also scbmatrix::operator - , scbmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.,
              9., 10., 11., 12.};
const scbmatrix m1((std::complex<double>*)a,3,1,0);
scbmatrix m2(3,1,0);
scbmatrix m(3,1,0);
m2.set(std::complex<double>(1.,1.));
std::cout << m1 << std::endl << m2 << std::endl;
std::cout << m.diff(m1, m2) << std::endl;
std::cout << m.diff(m, m2);
```

prints

```
(1,2) (0,0) (0,0)
(3,4) (5,6) (0,0)
(0,0) (7,8) (9,10)

(1,1) (0,0) (0,0)
(1,1) (1,1) (0,0)
(0,0) (1,1) (1,1)

(0,1) (0,0) (0,0)
(2,3) (4,5) (0,0)
(0,0) (6,7) (8,9)

(-1,0) (0,0) (0,0)
(1,2) (3,4) (0,0)
(0,0) (5,6) (7,8)
```

2.12.32 operator +=

Operator

```
scbmatrix& scbmatrix::operator += (const scbmatrix& m)
throw (cvmexception);
```

adds band matrix m to a calling band matrix and returns a reference to the matrix changed. Operator throws cvmexception in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also scbmatrix::operator + , scbmatrix::sum, scbmatrix. Example:

```
using namespace cvm;

try {
    scbmatrix m1(4,0,1);
    scbmatrix m2(4,0,1);
    m1.set(std::complex<double>(1.,2.));
    m2.set(std::complex<double>(3.,4.));

    m1 += m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 += m2;
    std::cout << m2;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(4,6) (4,6) (0,0) (0,0)
(0,0) (4,6) (4,6) (0,0)
(0,0) (0,0) (4,6) (4,6)
(0,0) (0,0) (0,0) (4,6)

(6,8) (6,8) (0,0) (0,0)
(0,0) (6,8) (6,8) (0,0)
(0,0) (0,0) (6,8) (6,8)
(0,0) (0,0) (0,0) (6,8)
```

2.12.33 operator -=

Operator

```
scbmatrix& scbmatrix::operator -= (const scbmatrix& m)
throw (cvmexception);
```

subtracts band matrix m from calling band matrix and returns a reference to the matrix changed. It throws cvmexception in case of different sizes or different numbers of sub- or super-diagonals of the operands. See also scbmatrix::operator - , scbmatrix::diff, scbmatrix. Example:

```
using namespace cvm;

try {
    scbmatrix m1(4,0,1);
    scbmatrix m2(4,0,1);
    m1.set(std::complex<double>(1.,2.));
    m2.set(std::complex<double>(3.,4.));

    m1 -= m2;
    std::cout << m1 << std::endl;

    // well, you can do this too, but temporary object would be created
    m2 -= m2;
    std::cout << m2;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(-2,-2) (-2,-2) (0,0) (0,0)
(0,0) (-2,-2) (-2,-2) (0,0)
(0,0) (0,0) (-2,-2) (-2,-2)
(0,0) (0,0) (0,0) (-2,-2)

(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
```

2.12.34 operator - ()

Operator

```
scbmatrix scbmatrix::operator - () const throw (cvmexception);
```

creates an object of type `scbmatrix` as a calling band matrix multiplied by -1. See also `scbmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific |
                std::ios::left |
                std::ios::showpos);
std::cout.precision (2);

double a[] = {1., 2., 3., 4., 5., 6., 7., 8.,
              9., 10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,1,0);

std::cout << -m;
```

prints

```
(-1.00e+000,-2.00e+000) (+0.00e+000,+0.00e+000) (+0.00e+000,+0.00e+000)
(-3.00e+000,-4.00e+000) (-5.00e+000,-6.00e+000) (+0.00e+000,+0.00e+000)
(+0.00e+000,+0.00e+000) (-7.00e+000,-8.00e+000) (-9.00e+000,-1.00e+001)
```

2.12.35 operator ++

Operator

```
scbmatrix& scbmatrix::operator ++ ();
scbmatrix& scbmatrix::operator ++ (int);
```

adds identity matrix to a calling band matrix and returns a reference to the matrix changed. See also scbmatrix. Example:

```
using namespace cvm;

scbmatrix m(4,1,0);
m.set(std::complex<double>(1.,1.));

m++;
std::cout << m << std::endl;
std::cout << ++m;
```

prints

```
(2,1) (0,0) (0,0) (0,0)
(1,1) (2,1) (0,0) (0,0)
(0,0) (1,1) (2,1) (0,0)
(0,0) (0,0) (1,1) (2,1)

(3,1) (0,0) (0,0) (0,0)
(1,1) (3,1) (0,0) (0,0)
(0,0) (1,1) (3,1) (0,0)
(0,0) (0,0) (1,1) (3,1)
```

2.12.36 operator --

Operator

```
scbmatrix& scbmatrix::operator -- ();
scbmatrix& scbmatrix::operator -- (int);
```

subtracts identity matrix from calling band matrix and returns a reference to the matrix changed. See also `scbmatrix`. Example:

```
using namespace cvm;

scbmatrix m(4,1,0);
m.set(std::complex<double>(1.,1.));

m--;
std::cout << m << std::endl;
std::cout << --m;
```

prints

```
(0,1) (0,0) (0,0) (0,0)
(1,1) (0,1) (0,0) (0,0)
(0,0) (1,1) (0,1) (0,0)
(0,0) (0,0) (1,1) (0,1)

(-1,1) (0,0) (0,0) (0,0)
(1,1) (-1,1) (0,0) (0,0)
(0,0) (1,1) (-1,1) (0,0)
(0,0) (0,0) (1,1) (-1,1)
```

2.12.37 operator * (TR)

Operator

```
scbmatrix scbmatrix::operator * (TR d) const;
```

creates an object of type `scbmatrix` as a product of a calling band matrix and real number `d`. See also `scbmatrix::operator *=` , `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,0,1);
std::cout << m * 5.;
```

prints

```
(15,20) (25,30) (0,0)
(0,0) (35,40) (45,50)
(0,0) (0,0) (55,60)
```

2.12.38 operator / (TR)

Operator

```
scbmatrix scbmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `scbmatrix` as a quotient of a calling band matrix and real number `d`. It throws `cvmexception` if `d` has absolute value equal or less than the smallest normalized positive number. See also `scbmatrix::operator /=` , `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,0,1);
std::cout << m / 2.;
```

prints

```
(1.5,2) (2.5,3) (0,0)
(0,0) (3.5,4) (4.5,5)
(0,0) (0,0) (5.5,6)
```

2.12.39 operator * (TC)

Operator

```
scbmatrix scbmatrix::operator * (TC z) const;
```

creates an object of type scbmatrix as a product of a calling band matrix and complex number z. See also scbmatrix::operator *= , scbmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,0,1);
std::cout << m * std::complex<double>(1.,1.);
```

prints

```
(-1,7) (-1,11) (0,0)
(0,0) (-1,15) (-1,19)
(0,0) (0,0) (-1,23)
```

2.12.40 operator / (TC)

Operator

```
scbmatrix scbmatrix::operator / (TC z) const throw (cvmexception);
```

creates an object of type `scbmatrix` as a quotient of a calling band matrix and complex number `z`. It throws `cvmexception` if `z` has absolute value equal or less than the smallest normalized positive number. See also `scbmatrix::operator /=` , `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,0,1);
std::cout << m / std::complex<double>(1.,1.);
```

prints

```
(3.5,0.5) (5.5,0.5) (0,0)
(0,0) (7.5,0.5) (9.5,0.5)
(0,0) (0,0) (11.5,0.5)
```

2.12.41 operator *= (TR)

Operator

`scbmatrix& scbmatrix::operator *= (TR d);`

multiplies calling band matrix by real number `d` and returns a reference to the matrix changed. See also `scbmatrix::operator *` , `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,0,1);
m *= 5.;
std::cout << m;
```

prints

```
(15,20) (25,30) (0,0)
(0,0) (35,40) (45,50)
(0,0) (0,0) (55,60)
```

2.12.42 operator /= (TR)

Operator

```
scbmatrix& scbmatrix::operator /= (TR d) throw (cvmexception);
```

divides calling band matrix by real number d and returns a reference to the matrix changed. It throws cvmexception if d has absolute value equal or less than the smallest normalized positive number. See also scbmatrix::operator / , scbmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,0,1);
m /= 2.;
std::cout << m;
```

prints

```
(1.5,2) (2.5,3) (0,0)
(0,0) (3.5,4) (4.5,5)
(0,0) (0,0) (5.5,6)
```

2.12.43 operator *= (TC)

Operator

```
scbmatrix& scbmatrix::operator *= (TC z);
```

multiplies calling band matrix by complex number z and returns a reference to the matrix changed. See also `scbmatrix::operator *` , `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,0,1);
m *= std::complex<double>(1.,1.);
std::cout << m;
```

prints

```
(-1,7) (-1,11) (0,0)
(0,0) (-1,15) (-1,19)
(0,0) (0,0) (-1,23)
```

2.12.44 operator /= (TC)

Operator

`scbmatrix& scbmatrix::operator /= (TC z) throw (cvmexception);`

divides calling band matrix by complex number z and returns a reference to the matrix changed. It throws cvmexception if z has absolute value equal or less than the smallest normalized positive number. See also scbmatrix::operator / , scbmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,0,1);
m /= std::complex<double>(1.,1.);
std::cout << m;
```

prints

```
(3.5,0.5) (5.5,0.5) (0,0)
(0,0) (7.5,0.5) (9.5,0.5)
(0,0) (0,0) (11.5,0.5)
```

2.12.45 normalize

Function

```
scbmatrix& scbmatrix::normalize ();
```

normalizes calling band matrix so its Euclidean norm becomes equal to 1 if it was greater than the smallest normalized positive number before the call (otherwise function does nothing). See also scbmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12.};
scbmatrix m((std::complex<double>*)a,3,0,1);

m.normalize();
std::cout << m << m.norm() << std::endl;
```

prints

```
(1.18e-001,1.57e-001) (1.97e-001,2.36e-001) (0.00e+000,0.00e+000)
(0.00e+000,0.00e+000) (2.76e-001,3.15e-001) (3.54e-001,3.94e-001)
(0.00e+000,0.00e+000) (0.00e+000,0.00e+000) (4.33e-001,4.72e-001)
1.00e+000
```

2.12.46 conjugation

Operator and functions

```
scbmatrix scbmatrix::operator ~ () const throw(cvmexception);
scbmatrix& scbmatrix::conj (const scbmatrix& m) throw(cvmexception);
scbmatrix& scbmatrix::conj () throw(cvmexception);
```

implement complex band matrix conjugation. First operator creates `scbmatrix` object as conjugated calling band matrix (it throws `cvmexception` in case of memory allocation failure). Second function sets calling matrix to be equal to matrix `m` conjugated (it throws `cvmexception` in case of not appropriate sizes or numbers of sub- or super-diagonals of the operands), third one makes it to be equal to conjugated itself (it also throws `cvmexception` in case of memory allocation failure). See also `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16.};
scbmatrix m((std::complex<double>*)a,4,1,0);
scbmatrix mc(4,0,1);
std::cout << m << std::endl << ~m << std::endl ;
mc.conj(m);
std::cout << mc << std::endl;
mc.conj();
std::cout << mc;
```

prints

```
(1,2) (0,0) (0,0) (0,0)
(3,4) (5,6) (0,0) (0,0)
(0,0) (7,8) (9,10) (0,0)
(0,0) (0,0) (11,12) (13,14)

(1,-2) (3,-4) (0,0) (0,0)
(0,0) (5,-6) (7,-8) (0,0)
(0,0) (0,0) (9,-10) (11,-12)
(0,0) (0,0) (0,0) (13,-14)

(1,-2) (3,-4) (0,0) (0,0)
(0,0) (5,-6) (7,-8) (0,0)
(0,0) (0,0) (9,-10) (11,-12)
(0,0) (0,0) (0,0) (13,-14)

(1,2) (0,0) (0,0) (0,0)
(3,4) (5,6) (0,0) (0,0)
(0,0) (7,8) (9,10) (0,0)
(0,0) (0,0) (11,12) (13,14)
```

2.12.47 transposition

Operator and functions

```
scbmatrix scbmatrix::operator ! () const throw (cvmexception);
scbmatrix& scbmatrix::transpose (const scbmatrix& m) throw (cvmexception);
scbmatrix& scbmatrix::transpose () throw (cvmexception);
```

implement complex band matrix transposition (*not* conjugation). First operator creates an object of type `scbmatrix` as transposed calling matrix (it throws `cvmexception` in case of memory allocation failure). Second function sets calling matrix to be equal to matrix `m` transposed (it throws `cvmexception` in case of not appropriate sizes of the operands), third one makes it to be equal to transposed itself (it also throws `cvmexception` in case of memory allocation failure). See also `scbmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
              10., 11., 12., 13., 14., 15., 16.};
scbmatrix m((std::complex<double>*)a,4,1,0);
scbmatrix mc(4,0,1);
std::cout << m << std::endl << !m << std::endl ;
mc.transpose(m);
std::cout << mc << std::endl;
mc.transpose();
std::cout << mc;
```

prints

```
(1,2) (0,0) (0,0) (0,0)
(3,4) (5,6) (0,0) (0,0)
(0,0) (7,8) (9,10) (0,0)
(0,0) (0,0) (11,12) (13,14)

(1,2) (3,4) (0,0) (0,0)
(0,0) (5,6) (7,8) (0,0)
(0,0) (0,0) (9,10) (11,12)
(0,0) (0,0) (0,0) (13,14)

(1,2) (3,4) (0,0) (0,0)
(0,0) (5,6) (7,8) (0,0)
(0,0) (0,0) (9,10) (11,12)
(0,0) (0,0) (0,0) (13,14)

(1,2) (0,0) (0,0) (0,0)
(3,4) (5,6) (0,0) (0,0)
(0,0) (7,8) (9,10) (0,0)
(0,0) (0,0) (11,12) (13,14)
```

2.12.48 operator * (const cvector&)

Operator

```
cvector scbmatrix::operator * (const cvector& v) const
throw (cvmexception);
```

creates an object of type `cvector` as a product of a calling band matrix and a vector v. It throws `cvmexception` if number of columns of a calling matrix differs from size of a vector v. Use `cvector::mult` in order to avoid new object creation. See also `scbmatrix` and `cvector`. Example:

```
using namespace cvm;

try {
    scbmatrix m (4,1,0);
    cvector v(4);
    m.set(std::complex<double>(1.,1.));
    v.set(std::complex<double>(1.,1.));

    std::cout << m * v;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(0,2) (0,4) (0,4) (0,4)
```

2.12.49 operator * (const cmatrix&)

Operator

```
cmatrix scbmatrix::operator * (const cmatrix& m) const
throw (cvmexception);
```

creates an object of type `cmatrix` as a product of a calling band matrix and a matrix `m`. It throws `cvmexception` if number of columns of a calling matrix differs from number of rows of a matrix `m`. Use `cmatrix::mult` in order to avoid new object creation. See also `cmatrix` and `scbmatrix`. Example:

```
using namespace cvm;

try {
    scbmatrix mb(4,1,0);
    cmatrix m(4,2);
    mb.set(std::complex<double>(1.,1.));
    m.set(std::complex<double>(1.,1.));

    std::cout << mb * m;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(0,2) (0,2)
(0,4) (0,4)
(0,4) (0,4)
(0,4) (0,4)
```

2.12.50 operator * (const scmatrix&)

Operator

```
scmatrix scbmatrix::operator * (const scmatrix& m) const
throw (cvmexception);
```

creates an object of type `scmatrix` as a product of a calling band matrix and a matrix `m`. It throws `cvmexception` if the operands have different sizes. Use `cmatrix::mult` in order to avoid new object creation. See also `scmatrix` and `scbmatrix`. Example:

```
using namespace cvm;

try {
    scbmatrix mb(4,1,0);
    scmatrix m(4);
    mb.set(std::complex<double>(1.,1.));
    m.set(std::complex<double>(1.,1.));

    std::cout << mb * m;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(0,2) (0,2) (0,2) (0,2)
(0,4) (0,4) (0,4) (0,4)
(0,4) (0,4) (0,4) (0,4)
(0,4) (0,4) (0,4) (0,4)
```

2.12.51 operator * (const scbmatrix&)

Operator

```
scbmatrix scbmatrix::operator * (const scbmatrix& m) const
throw (cvmexception);
```

creates an object of type `scbmatrix` as a product of a calling band matrix and band matrix `m`. It throws `cvmexception` if the operands have different sizes. Use `cmatrix::mult` in order to avoid new object creation. See also `scbmatrix`. Example:

```
using namespace cvm;

try {
    scbmatrix m1(5,1,0);
    scbmatrix m2(5,1,1);
    m1.set(std::complex<double>(1.,1.));
    m2.set(std::complex<double>(1.,1.));

    std::cout << m1 * m2;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(0,2) (0,2) (0,0) (0,0) (0,0)
(0,4) (0,4) (0,2) (0,0) (0,0)
(0,2) (0,4) (0,4) (0,2) (0,0)
(0,0) (0,2) (0,4) (0,4) (0,2)
(0,0) (0,0) (0,2) (0,4) (0,4)
```

2.12.52 low_up

Functions

```
scbmatrix&
scbmatrix::low_up (const scbmatrix& m, int* nPivots) throw (cvmexception);
scbmatrix
scbmatrix::low_up (int* nPivots) const throw (cvmexception);
```

compute LU factorization of a calling band matrix as

$$A = PLU$$

where P is permutation matrix, L is lower triangular matrix with unit diagonal elements and U is upper triangular matrix. All functions store the result as matrix L without main diagonal combined with U. All functions return pivot indices as array of integers (it should support at least `msize()` elements) pointed to by `nPivots` so i-th row was interchanged with `nPivots[i]`-th row. The first version sets calling matrix to be equal to matrix `m`'s LU factorization and the second one creates an object of type `scbmatrix` as calling matrix's LU factorization. Functions throw `cvmexception` in case of inappropriate sizes of the operands or when matrix to be factorized is close to singular. The first version also changes numbers of super-diagonals to be equal to $k_l + k_u$ in order to keep result of factorization. It is recommended to use `iarray` for pivot values. This function is provided mostly for solving multiple systems of linear equations using `scmatrix::solve_lu` function, See also `scbmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
try {
    double a[] = {1., 2., 3., 4., 5., 6., 7., 8., 9.,
                  10., 11., 12.};
    scbmatrix ma((std::complex<double>*)a,3,1,0);
    scbmatrix mLU(3,1,0);
    cmatrix  mb1(3,2); cvector vb1(3);
    cmatrix  mb2(3,2); cvector vb2(3);
    cmatrix  mx1(3,2); cvector vx1(3);
    cmatrix  mx2(3,2); cvector vx2(3);
    iarray   nPivots(3);
    double   dErr = 0.;
    mb1.randomize_real(-1.,3.); mb1.randomize_imag(1.,5.);
    mb2.randomize_real(-2.,5.); mb2.randomize_imag(-3.,0.);
    vb1.randomize_real(-2.,4.); vb1.randomize_imag(-4.,1.);
    vb2.randomize_real(-3.,1.); vb2.randomize_imag(4.,5.);

    mLU.low_up(ma, nPivots);
```

```
    mx1 = ma.solve_lu (mLU, nPivots, mb1, dErr);
    std::cout << mx1 << dErr << std::endl << std::endl;
    mx2 = ma.solve_lu (mLU, nPivots, mb2);
    std::cout << mx2 << std::endl;;
    std::cout << ma * mx1 - mb1 << std::endl << ma * mx2 - mb2;

    vx1 = ma.solve_lu (mLU, nPivots, vb1, dErr);
    std::cout << vx1 << dErr << std::endl;
    vx2 = ma.solve_lu (mLU, nPivots, vb2);
    std::cout << vx2 << std::endl;;
    std::cout << ma * vx1 - vb1 << std::endl << ma * vx2 - vb2;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(1.20e+000,4.02e-002) (1.82e+000,1.23e+000)
(-6.55e-001,1.37e-001) (-6.41e-001,-8.72e-001)
(7.75e-001,4.70e-002) (5.35e-001,8.11e-001)
1.45e-015

(-4.52e-001,-2.68e-002) (-1.09e+000,2.01e-001)
(6.08e-001,-4.76e-001) (5.48e-001,-1.95e-001)
(-3.46e-001,1.57e-001) (-3.38e-001,-7.54e-002)

(0.00e+000,4.44e-016) (-2.22e-016,8.88e-016)
(-2.22e-016,2.22e-016) (0.00e+000,0.00e+000)
(-1.11e-016,0.00e+000) (-3.33e-016,-6.66e-016)

(0.00e+000,0.00e+000) (2.22e-016,2.22e-016)
(0.00e+000,0.00e+000) (4.44e-016,-2.22e-016)
(8.88e-016,5.55e-016) (0.00e+000,0.00e+000)
(-1.28e+000,-5.12e-001) (8.22e-001,1.59e-001) (-6.45e-001,-3.74e-001)
1.31e-015
(1.26e+000,1.50e+000) (-5.13e-001,-4.66e-001) (5.97e-001,7.01e-001)

(0.00e+000,8.88e-016) (-4.44e-016,4.44e-016) (-8.88e-016,0.00e+000)

(2.22e-016,-8.88e-016) (4.44e-016,-8.88e-016) (-2.22e-016,8.88e-016)
```

2.12.53 operator / (const cvector&)

Operator

```
cvector operator / (const cvector& vB) const throw (cvmexception);
```

returns solution x of linear equation $A * x = b$ where calling matrix is square band matrix A and a vector b is passed in parameter vB. This operator throws exception of type cvmexception in case of inappropriate sizes of the objects or when matrix A is close to singular. See also cvector::solve, scmatrix::solve, cvector.operator %, cvector, scmatrix, scbmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (12);

try {
    scbmatrix ma(4,2,1);
    cvector  vb(4);
    cvector  vx(4);
    ma.randomize_real(-1.,1.);
    ma.randomize_imag(-1.,1.);
    vb.randomize_real(-2.,2.);
    vb.randomize_imag(-2.,2.);

    vx = ma / vb;

    std::cout << (ma * vx - vb).norm() << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
+8.082545620881e-016
```

2.12.54 identity

Function

```
scbmatrix& scbmatrix::identity();
```

sets calling band matrix to be equal to identity matrix and returns a reference to the matrix changed. Function doesn't change numbers of sub- and super-diagonals. See also scbmatrix. Example:

```
using namespace cvm;

srbmatrix m(4);
m.randomize(0.,1.);
std::cout << m << std::endl;
std::cout << m.identity();
```

prints

```
(0.576128,1.42595) (0,0) (0,0) (0,0)
(0.956359,-0.919523) (0.869716,-0.704093) (0,0) (0,0)
(0,0) (0.0959807,0.0616779) (0.632618,1.1793) (0,0)
(0,0) (0,0) (0.532182,-0.870724) (0.338023,1.22892)

(1,0) (0,0) (0,0) (0,0)
(0,0) (1,0) (0,0) (0,0)
(0,0) (0,0) (1,0) (0,0)
(0,0) (0,0) (0,0) (1,0)
```

2.12.55 vanish

Function

```
scbmatrix& scbmatrix::vanish();
```

sets every element of a calling band matrix to be equal to zero and returns a reference to the matrix changed. This function is faster than `scbmatrix::set(TR)` with zero operand passed. See also `scbmatrix`. Example:

```
using namespace cvm;

scbmatrix m(4,1,0);
m.randomize_real(0.,1.);
m.randomize_imag(-1.,2.);
std::cout << m << std::endl;
std::cout << m.vanish();
```

prints

```
(0.584094,0.985931) (0,0) (0,0) (0,0)
(0.197546,0.0150761) (0.483413,-0.733848) (0,0) (0,0)
(0,0) (0.844356,1.97848) (0.814692,1.50194) (0,0)
(0,0) (0,0) (0.118931,-0.720756) (0.936796,-0.582232)

(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
```

2.12.56 randomize_real

Function

```
scbmatrix& scbmatrix::randomize_real (TR dFrom, TR dTo);
```

fills real part of a calling band matrix with pseudo-random numbers distributed between dFrom and dTo. Function returns a reference to the matrix changed. See also scbmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
scbmatrix m(3,0,1);
m.randomize_real(0.,3.);
std::cout << m;
```

prints

```
(1.78e+000,0.00e+000) (1.17e+000,0.00e+000) (0.00e+000,0.00e+000)
(0.00e+000,0.00e+000) (1.09e-002,0.00e+000) (6.05e-001,0.00e+000)
(0.00e+000,0.00e+000) (0.00e+000,0.00e+000) (2.49e+000,0.00e+000)
```

2.12.57 randomize_imag

Function

```
scbmatrix& scbmatrix::randomize_imag (TR dFrom, TR dTo);
```

fills imaginary part of a calling band matrix with pseudo-random numbers distributed between dFrom and dTo. Function returns a reference to the matrix changed. See also scbmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
scbmatrix m(3,0,1);
m.randomize_imag(0.,3.);
std::cout << m;
```

prints

```
(0.00e+000,1.80e+000) (0.00e+000,1.68e-001) (0.00e+000,0.00e+000)
(0.00e+000,0.00e+000) (0.00e+000,1.05e+000) (0.00e+000,1.40e+000)
(0.00e+000,0.00e+000) (0.00e+000,0.00e+000) (0.00e+000,1.98e+000)
```

2.13 srsmatrix

This is end-user class encapsulating symmetric matrix of real numbers.

```
template <typename TR>
class srsmatrix : public srmatrix <TR> {
public:
    srsmatrix ();
    explicit srsmatrix (int nMN);
    srsmatrix (TR* pD, int nMN, TR tol = cvmMachSp());
    srsmatrix (const TR* pD, int nMN, TR tol = cvmMachSp());
    srsmatrix (const srsmatrix& m);
    explicit srsmatrix (const rmatrix& m, TR tol = cvmMachSp());
    explicit srsmatrix (const rvector& v);
    srsmatrix (srsmatrix& m, int nRowCol, int nSize);
    TR operator () (int im, int in) const throw (cvmexception);
    const rvector operator () (int i) const throw (cvmexception);
    const rvector operator [] (int i) const throw (cvmexception);
    const rvector diag (int i) const throw (cvmexception);
    srsmatrix& operator = (const srsmatrix& m) throw (cvmexception);
    srsmatrix& assign (const rvector& v, TR tol = cvmMachSp())
                       throw (cvmexception);
    srsmatrix& assign (const TR* pD, TR tol = cvmMachSp())
                       throw (cvmexception);
    srsmatrix& assign (int nRowCol, const srsmatrix& m)
                       throw (cvmexception);
    srsmatrix& set (TR x);
    srsmatrix& set (int nRow, int nCol, TR x);
    srsmatrix& set_diag (int i, const rvector& v) throw (cvmexception);
    srsmatrix& resize (int nNewMN) throw (cvmexception);
    bool operator == (const srsmatrix& m) const;
    bool operator != (const srsmatrix& m) const;
    srsmatrix& operator << (const srsmatrix& m) throw (cvmexception);
    srsmatrix operator + (const srsmatrix& m) const
                          throw (cvmexception);
    srsmatrix operator - (const srsmatrix& m) const
                          throw (cvmexception);
    srsmatrix& sum (const srsmatrix& m1,
                    const srsmatrix& m2) throw (cvmexception);
    srsmatrix& diff (const srsmatrix& m1,
                     const srsmatrix& m2) throw (cvmexception);
    srsmatrix& operator += (const srsmatrix& m) throw (cvmexception);
    srsmatrix& operator -= (const srsmatrix& m) throw (cvmexception);
    srsmatrix operator - () const;
    srsmatrix& operator ++ ();
```

```
    srsmatrix& operator ++ (int);
    srsmatrix& operator -- ();
    srsmatrix& operator -- (int);
    srsmatrix operator * (TR d) const;
    srsmatrix operator / (TR d) const throw (cvmexception);
    srsmatrix& operator *= (TR d);
    srsmatrix& operator /= (TR d) throw (cvmexception);
    srsmatrix& normalize ();
    srsmatrix operator ~ () const throw (cvmexception);
    srsmatrix& transpose (const srsmatrix& m) throw (cvmexception);
    srsmatrix& transpose ();
    rvector operator * (const rvector& v) const throw (cvmexception);
    rmatrix operator * (const rmatrix& m) const throw (cvmexception);
    srmatrix operator * (const srsmatrix& m) const throw (cvmexception);
    rvector operator / (const rvector& vB) const throw (cvmexception);
    srsmatrix& syrk (TR alpha,
                     const rvector& v, TR beta) throw (cvmexception);
    srsmatrix& syrk (bool bTransp, TR alpha,
                     const rmatrix& m, TR beta) throw (cvmexception);
    srsmatrix& syr2k (TR alpha,
                     const rvector& v1, const rvector& v2, TR beta)
                     throw (cvmexception);
    srsmatrix& syr2k (bool bTransp, TR alpha,
                     const rmatrix& m1, const rmatrix& m2, TR beta)
                     throw (cvmexception);
    srsmatrix& inv (const srsmatrix& mArg) throw (cvmexception);
    srsmatrix inv () const throw (cvmexception);
    srsmatrix& exp (const srsmatrix& m,
                   TR tol = cvmMachSp()) throw (cvmexception);
    srsmatrix exp (TR tol = cvmMachSp()) const throw (cvmexception);
    srsmatrix& polynom (const srsmatrix& m, const rvector& v)
                        throw (cvmexception);
    srsmatrix polynom (const rvector& v) const throw (cvmexception);
    rvector eig (srmatrix& mEigVect) const throw (cvmexception);
    rvector eig () const throw (cvmexception);
    srmatrix cholesky () const throw (cvmexception);
    srmatrix bunch_kaufman () const throw (cvmexception);
    srsmatrix& identity ();
    srsmatrix& vanish ();
    srsmatrix& randomize (TR dFrom, TR dTo);
};
```

2.13.1 srsmatrix ()

Constructor

```
srsmatrix::srsmatrix ();
```

creates empty `srsmatrix` object. See also `srsmatrix`. Example:

```
using namespace cvm;

srsmatrix m;
std::cout << m.msize() << " " << m.nsize() << " " << m.size()
          << std::endl << std::endl;
m.resize (3);
m.set(1.);
std::cout << m;
```

prints

```
0 0 0

1 1 1
1 1 1
1 1 1
```

2.13.2 srsmatrix (int)

Constructor

```
explicit srsmatrix::srsmatrix (int nMN);
```

creates $n \times n$ `srsmatrix` object where n is passed in `nMN` parameter. Constructor throws `cvmexception` in case of non-positive size passed or memory allocation failure. See also `srsmatrix`. Example:

```
using namespace cvm;

srsmatrix m(4);
std::cout << m.msize() << " " << m.nsize() << " " << m.size()
          << std::endl << std::endl;
m.set(1.);
std::cout << m;
```

prints

```
4 4 16

1 1 1 1
1 1 1 1
1 1 1 1
1 1 1 1
```

2.13.3 srsmatrix (TR*,int)

Constructor

```
srsmatrix::srsmatrix (TR* pD, int nMN, TR tol = cvmMachSp());
```

creates $n \times n$ `srsmatrix` object where n is passed in `nMN` parameter. Unlike others, this constructor *does not allocate memory*. It just shares memory with an array pointed to by `pD`. Constructor throws `cvmexception` if the matrix created doesn't appear to be symmetric or in case of non-positive size passed. Symmetry tolerance is set by parameter `tol`. If subsequent application flow would change the array passed so it becomes not symmetric matrix anymore then results are not predictable. See also `srsmatrix`, `srsmatrix (const TR*,int)`. Example:

```
using namespace cvm;
double a[] = {1., 1., 1., 1., 1., 1., 1., 1., 1.};
srsmatrix m (a, 3);
m.set(2,1,5.);
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << " "
          << a[3] << " " << a[4] << " " << a[5] << " " << std::endl;
```

prints

```
1 5 1
5 1 1
1 1 1

1 5 1 5 1 1
```

2.13.4 srsmatrix (const TR*,int)

Constructor

```
srsmatrix::srsmatrix (const TR* pD, int nMN, TR tol = cvmMachSp());
```

creates $n \times n$ `srsmatrix` object where n is passed in `nMN` parameter and copies $n * n$ elements of an array `pD` to it `by colums`. Constructor throws `cvmexception` if the matrix created doesn't appear to be symmetric or in case of non-positive size passed. Symmetry tolerance is set by parameter `tol`. See also `srsmatrix`, `srsmatrix (TR*,int)`. Example:

```
using namespace cvm;
const double a[] = {1., 1., 1., 1., 1., 1., 1., 1., 1.};
srsmatrix m (a, 3);
m.set(2,1,5.);
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << " "
          << a[3] << " " << a[4] << " " << a[5] << " " << std::endl;
```

prints

```
1 5 1
5 1 1
1 1 1

1 1 1 1 1 1
```

2.13.5 srsmatrix (const srsmatrix&)

Copy constructor

```
srsmatrix::srsmatrix (const srsmatrix& m);
```

creates `srsmatrix` object as a copy of `m`. Constructor throws `cvmexception` in case of memory allocation failure. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m (a, 3);
srmatrix mc(m);
m.set(2,3,7.77);
std::cout << m << std::endl << mc;
```

prints

```
1 2 3
2 5 7.77
3 7.77 9

1 2 3
2 5 6
3 6 9
```

2.13.6 srsmatrix (const rmatrix&)

Constructor

```
explicit srsmatrix::srsmatrix (const rmatrix& m, TR tol = cvmMachSp());
```

creates `srsmatrix` object as a copy of a matrix `m`. It's assumed that $m \times n$ matrix `m` must have equal sizes, i.e. $m = n$ is satisfied, and must be symmetric. Symmetry tolerance is set by parameter `tol`. Constructor throws `cvmexception` if this is not true or in case of memory allocation failure. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
rmatrix m(a, 3, 3);
srsmatrix ms(m);
std::cout << ms;
```

prints

```
1 2 3
2 5 6
3 6 9
```

2.13.7 srsmatrix (const rvector&)

Constructor

```
explicit srsmatrix::srsmatrix (const rvector& v);
```

creates `srsmatrix` object of size `v.size()` by `v.size()` and assigns vector `v` to its main diagonal. Constructor throws `cvmexception` in case of memory allocation failure. See also `srsmatrix`, `rvector`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5.};
const rvector v(a, 5);
srsmatrix m(v);
std::cout << m;
```

prints

```
1 0 0 0 0
0 2 0 0 0
0 0 3 0 0
0 0 0 4 0
0 0 0 0 5
```

2.13.8 submatrix

Submatrix constructor

```
srsmatrix::srsmatrix (srsmatrix& m, int nRowCol, int nSize);
```

creates `srmatrix` object as *submatrix* of symmetric matrix `m`. It means that the matrix created shares memory with some part of `m`. This part is defined by its upper left corner (parameter `nRowCol`, ℓ-based) and its size (parameter `nSize`). See also `srsmatrix`. Example:

```
using namespace cvm;

srsmatrix m(5);
srsmatrix subm(m, 2, 2);
subm.set(1.);
std::cout << m;
```

prints

```
0 0 0 0 0
0 1 1 0 0
0 1 1 0 0
0 0 0 0 0
```

2.13.9 operator (,)

Indexing operator

```
TR srsmatrix::operator () (int im, int in) const throw (cvmexception);
```

provides access to a particular element of a calling matrix. Unlike indexing operators in other classes, this operator doesn't return *l-value* because this would make the matrix non-symmetric. Operator is ℓ-based. It throws `cvmexception` if some of parameters passed is outside of `[1,msize()]` range. See also `srsmatrix`, `Matrix::msize()`, `Matrix::nsize()`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a, 3);
std::cout << m(1,1) << " " << m(2,3) << std::endl;
```

prints

```
1 6
```

2.13.10 operator ()

Indexing operator

```
const rvector srsmatrix::operator () (int i) const throw (cvmexception);
```

provides access to `i`-th column of a calling matrix. Unlike indexing operators in other classes, this operator doesn't return *l-value* because this would make the matrix non-symmetric. Operator creates an object of class `rvector` as *copy* of a column and therefore it's *not l-value*. Operator is ℓ-based. It throws `cvmexception` if parameter `i` is outside of `[1,nsize()]` range. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a, 3);
std::cout << m(1) << m(2) << m(3);
```

prints

```
1 2 3
2 5 6
3 6 9
```

2.13.11 operator []

Indexing operator

```
const rvector srsmatrix::operator [] (int i) const throw (cvmexception);
```

provides access to i-th row of a calling matrix. Unlike indexing operators in other classes, this operator doesn't return *l-value* because this would make the matrix non-symmetric. Operator creates an object of class rvector as *copy* of a row and therefore it's *not l-value*. Operator is ℓ-based. It throws cvmexception if parameter i is outside of [1,msize()] range. See also srsmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a, 3);
std::cout << m[1] << m[2] << m[3];
```

prints

```
1 2 3
2 5 6
3 6 9
```

2.13.12 diag

Function

```
const rvector srsmatrix::diag (int i) const throw (cvmexception);
```

provides access to i-th diagonal of a calling matrix, where $i = 0$ for main diagonal, $i < 0$ for lower diagonals and $i > 0$ for upper ones. Unlike `diag` function in other classes, this one doesn't return *l-value* because this would make the matrix non-symmetric. Function creates an object of class `rvector` as *copy* of a diagonal and therefore it's *not l-value*. Function is ℓ-based. It throws `cvmexception` if parameter `i` is outside of `[-msize()+1,nsize()-1]` range. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a, 3);
std::cout << m << std::endl
          << m.diag(-2) << m.diag(-1) << m.diag(0)
          << m.diag(1) << m.diag(2);
```

prints

```
1 2 3
2 5 6
3 6 9

3
2 6
1 5 9
2 6
3
```

2.13.13 `operator = (const srsmatrix&)`

Operator

```
srsmatrix& srsmatrix::operator = (const srsmatrix& m)
throw (cvmexception);
```

sets every element of a calling symmetric matrix to a value of appropriate element of symmetric matrix `m` and returns a reference to the matrix changed. Operator throws `cvmexception` in case of different sizes of the operands. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
const srsmatrix m1(a, 3);
srsmatrix m2(3);

m2 = m1;
std::cout << m2;
```

prints

```
1 2 3
2 5 6
3 6 9
```

2.13.14 assign (const TR*)

Function

```
srsmatrix& srsmatrix::assign (const rvector& v, TR tol = cvmMachSp())
throw (cvmexception);

srsmatrix& srsmatrix::assign (const TR* pD, TR tol = cvmMachSp())
throw (cvmexception);
```

sets every element of a calling matrix to a value of appropriate element of vector `v` or array pointed to by `pD` and returns a reference to the matrix changed. In first version it's assumed that vector passed is long enough to fill calling matrix. Function throws `cvmexception` otherwise. Function throws `cvmexception` if the matrix changed doesn't appear to be symmetric. Symmetry tolerance is set by parameter `tol`. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(3);
m.assign(a);
std::cout << m;
```

prints

```
1 2 3
2 5 6
3 6 9
```

2.13.15 assign (int, const srsmatrix&)

Function

```
srsmatrix& srsmatrix::assign (int nRowCol, const srsmatrix& m)
throw (cvmexception);
```

sets main sub-matrix of a calling symmetric matrix beginning with ℓ-based row `nRowCol` to symmetric matrix `m` and returns a reference to the matrix changed. Function throws `cvmexception` if `nRowCol` is not positive or matrix `m` doesn't fit. See also `srsmatrix`. Example:

```
using namespace cvm;

srsmatrix m1(5);
srsmatrix m2(2);
m1.set(1.);
m2.set(2.);
m1.assign(2,m2);
std::cout << m1;
```

prints

```
1 1 1 1 1
1 2 2 1 1
1 2 2 1 1
1 1 1 1 1
1 1 1 1 1
```

2.13.16 set (TR)

Function

```
srsmatrix& srsmatrix::set (TR x);
```

sets every element of a calling matrix to a value of parameter `x` and returns a reference to the matrix changed. See also `srsmatrix`. Example:

```
using namespace cvm;

srsmatrix m(3);
m.set(3.);
std::cout << m;
```

prints

```
3 3 3
3 3 3
3 3 3
```

2.13.17 set (int,int,TR)

Function

```
srsmatrix& srsmatrix::set (int nRow, int nCol, TR x);
```

sets both elements located on `nRow`'s row and `nCol`'s column and on `nCol`'s row and `nRow`'s column to a value of parameter `x` and returns a reference to the matrix changed (thus the matrix remains symmetric). Parameters passed are ℓ-based. Function throws `cvmexception` if any of the parameters passed is outside of `[1,msize()]` range. See also `srsmatrix`. Example:

```
using namespace cvm;

srsmatrix m(3);
m.set(3.);
m.set(1,3,7.);
std::cout << m;
```

prints

```
3 3 7
3 3 3
7 3 3
```

2.13.18 set_diag (int,rvector)

Function

```
srsmatrix& srsmatrix::set_diag (int i, const rvector& v)
throw (cvmexception);
```

assigns vector v to i-th diagonal of a calling matrix, where $i = 0$ for main diagonal, $i < 0$ for lower diagonals and $i > 0$ for upper ones. If $i \neq 0$, then function assigns the vector to both i-th and $-i$-th diagonals (thus calling matrix remains symmetric). Function returns a reference to the matrix changed. Parameter i is *not* ℓ-based. Function throws cvmexception if parameter i is outside of [-msize()+1,nsize()-1] range or if vector v passed has size not equal to msize()-abs(i). See also srsmatrix. Example:

```
using namespace cvm;

srsmatrix m(3);
rvector v(2);
m.set(3.);
v.set(1.);
m.set_diag(1,v);
std::cout << m;
```

prints

```
3 1 3
1 3 1
3 1 3
```

2.13.19 resize

Function

```
srsmatrix& srsmatrix::resize (int nNewMN) throw (cvmexception);
```

changes size of a calling matrix to `nNewMN` by `nNewMN` and returns a reference to the matrix changed. In case of increasing of its size, calling matrix is filled up with zeroes. Function throws `cvmexception` in case of negative size passed or memory allocation failure. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a,3);
std::cout << m << std::endl;
m.resize(4);
std::cout << m;
```

prints

```
1 2 3
2 5 6
3 6 9

1 2 3 0
2 5 6 0
3 6 9 0
0 0 0 0
```

2.13.20 operator ==

Operator

```
bool srsmatrix::operator == (const srsmatrix& m) const;
```

compares calling matrix with symmetric matrix `m` and returns `true` if they have the same sizes and their appropriate elements differ by not more than the smallest normalized positive number. Returns `false` otherwise. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 2., 3.};
srsmatrix m1(a, 2);
srsmatrix m2(2);
m2.set(1,1,1.);
m2.set(1,2,2.);
m2.set(2,2,3.);

std::cout << (m1 == m2) << std::endl;
```

prints

```
1
```

2.13.21 operator !=

Operator

```
bool srsmatrix::operator != (const srsmatrix& m) const;
```

compares calling matrix with symmetric matrix `m` and returns `true` if they have different sizes or at least one of their appropriate elements differs by more than the smallest normalized positive number. Returns `false` otherwise. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 2., 3.};
srsmatrix m1(a, 2);
srsmatrix m2(2);
m2.set(1,1,1.0001);
m2.set(1,2,2.);
m2.set(2,2,3.);

std::cout << (m1 != m2) << std::endl;
```

prints

```
1
```

2.13.22 operator <<

Operator

```
srsmatrix& srsmatrix::operator << (const srsmatrix& m)
throw (cvmexception);
```

destroys calling matrix, creates new one as a copy of `m` and returns a reference to the matrix changed. Operator throws `cvmexception` in case of memory allocation failure. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a, 3);
srsmatrix mc(1);
std::cout << m << std::endl << mc << std::endl;
mc << m;
std::cout << mc;
```

prints

```
1 2 3
2 5 6
3 6 9

0

1 2 3
2 5 6
3 6 9
```

2.13.23 operator +

Operator

```
srsmatrix srsmatrix::operator + (const srsmatrix& m) const
throw (cvmexception);
```

creates an object of type `srsmatrix` as a sum of a calling symmetric matrix and symmetric matrix `m`. It throws `cvmexception` in case of different sizes of the operands. See also `srsmatrix::sum`, `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m1(a, 3);
srsmatrix m2(3);
m2.set(1.);
std::cout << m1 + m2 << std::endl << m1 + m1;
```

prints

```
2 3 4
3 6 7
4 7 10

2 4 6
4 10 12
6 12 18
```

2.13.24 operator -

Operator

```
srsmatrix srsmatrix::operator - (const srsmatrix& m) const
throw (cvmexception);
```

creates an object of type `srsmatrix` as a difference of a calling symmetric matrix and symmetric matrix `m`. It throws `cvmexception` in case of different sizes of the operands. See also `srsmatrix::diff`, `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m1(a, 3);
srsmatrix m2(3);
m2.set(1.);
std::cout << m1 - m2 << std::endl << m1 - m1;
```

prints

```
0 1 2
1 4 5
2 5 8

0 0 0
0 0 0
0 0 0
```

2.13.25 sum

Function

```
srsmatrix& srsmatrix::sum (const srsmatrix& m1, const srsmatrix& m2)
throw (cvmexception);
```

assigns the result of addition of symmetric matrices `m1` and `m2` to a calling symmetric matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. See also `srsmatrix::operator +` , `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
const srsmatrix m1(a, 3);
srsmatrix m2(3);
srsmatrix m(3);
m2.set(1.);

std::cout << m.sum(m1, m2) << std::endl;
std::cout << m.sum(m, m2);
```

prints

```
2 3 4
3 6 7
4 7 10

3 4 5
4 7 8
5 8 11
```

2.13.26 diff

Function

```
srsmatrix& srsmatrix::diff (const srsmatrix& m1, const srsmatrix& m2)
throw (cvmexception);
```

assigns the result of subtraction of symmetric matrices `m1` and `m2` to a calling symmetric matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. See also `srsmatrix::operator -` , `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
const srsmatrix m1(a, 3);
srsmatrix m2(3);
srsmatrix m(3);
m2.set(1.);

std::cout << m.diff(m1, m2) << std::endl;
std::cout << m.diff(m, m2);
```

prints

```
0 1 2
1 4 5
2 5 8

-1 0 1
0 3 4
1 4 7
```

2.13.27 operator +=

Operator

```
srsmatrix& srsmatrix::operator += (const srsmatrix& m)
throw (cvmexception);
```

adds symmetric matrix `m` to a calling symmetric matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. See also `srsmatrix::operator +` , `srsmatrix::sum`, `srsmatrix`. Example:

```
using namespace cvm;

srsmatrix m1(3);
srsmatrix m2(3);
m1.set(1.);
m2.set(2.);

m1 += m2;
std::cout << m1 << std::endl;

// well, you can do this too, but temporary object would be created
m2 += m2;
std::cout << m2;
```

prints

```
3 3 3
3 3 3
3 3 3

4 4 4
4 4 4
4 4 4
```

2.13.28 operator -=

Operator

```
srsmatrix& srsmatrix::operator -= (const srsmatrix& m)
throw (cvmexception);
```

subtracts symmetric matrix `m` from calling symmetric matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. See also `srsmatrix::operator -` , `srsmatrix::diff`, `srsmatrix`. Example:

```
using namespace cvm;

srsmatrix m1(3);
srsmatrix m2(3);
m1.set(1.);
m2.set(2.);

m1 -= m2;
std::cout << m1 << std::endl;

// well, you can do this too, but temporary object would be created
m2 -= m2;
std::cout << m2;
```

prints

```
-1 -1 -1
-1 -1 -1
-1 -1 -1

0 0 0
0 0 0
0 0 0
```

2.13.29 operator - ()

Operator

```
srsmatrix srsmatrix::operator - () const throw (cvmexception);
```

creates an object of type `srsmatrix` as a calling symmetric matrix multiplied by -1. See also `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a, 3);
std::cout << -m;
```

prints

```
-1 -2 -3
-2 -5 -6
-3 -6 -9
```

2.13.30 operator ++

Operator

```
srsmatrix& srsmatrix::operator ++ ();
srsmatrix& srsmatrix::operator ++ (int);
```

adds identity matrix to a calling symmetric matrix and returns a reference to the matrix changed. See also srsmatrix. Example:

```
using namespace cvm;

srsmatrix m(4);
m.set(4.);
m++;
std::cout << m << std::endl;
std::cout << ++m;
```

prints

```
5 4 4 4
4 5 4 4
4 4 5 4
4 4 4 5

6 4 4 4
4 6 4 4
4 4 6 4
4 4 4 6
```

2.13.31 operator --

Operator

```
srsmatrix& srsmatrix::operator -- ();
srsmatrix& srsmatrix::operator -- (int);
```

subtracts identity matrix from calling symmetric matrix and returns a reference to the matrix changed. See also srsmatrix. Example:

```
using namespace cvm;

srsmatrix m(4);
m.set(4.);
m--;
std::cout << m << std::endl;
std::cout << --m;
```

prints

```
3 4 4 4
4 3 4 4
4 4 3 4
4 4 4 3

2 4 4 4
4 2 4 4
4 4 2 4
4 4 4 2
```

2.13.32 operator * (TR)

Operator

```
srsmatrix srsmatrix::operator * (TR d) const;
```

creates an object of type srsmatrix as a product of a calling symmetric matrix and number d. See also srsmatrix::operator *= , srsmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a,3);
std::cout << m * 5.;
```

prints

```
5 10 15
10 25 30
15 30 45
```

2.13.33 operator / (TR)

Operator

```
srsmatrix srsmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `srsmatrix` as a quotient of a calling symmetric matrix and number `d`. It throws `cvmexception` if `d` has absolute value equal or less than the smallest normalized positive number. See also `srsmatrix::operator /=` , `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a,3);
std::cout << m / 4.;
```

prints

```
0.25 0.5 0.75
0.5 1.25 1.5
0.75 1.5 2.25
```

2.13.34 operator *= (TR)

Operator

```
srsmatrix& srsmatrix::operator *= (TR d);
```

multiplies calling symmetric matrix by number `d` and returns a reference to the matrix changed. See also `srsmatrix::operator *` , `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a,3);
m *= 2.;
std::cout << m;
```

prints

```
2 4 6
4 10 12
6 12 18
```

2.13.35 operator /= (TR)

Operator

```
srsmatrix& srsmatrix::operator /= (TR d) throw (cvmexception);
```

divides calling symmetric matrix by number `d` and returns a reference to the matrix changed. It throws `cvmexception` if `d` has absolute value equal or less than the smallest normalized positive number. See also `srsmatrix::operator /` , `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a,3);
m /= 2.;
std::cout << m;
```

prints

```
0.5 1 1.5
1 2.5 3
1.5 3 4.5
```

2.13.36 normalize

Function

```
srsmatrix& srsmatrix::normalize ();
```

normalizes calling symmetric matrix so its Euclidean norm becomes equal to 1 if it was greater than the smallest normalized positive number before the call (otherwise function does nothing). See also `srsmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a,3);
m.normalize();
std::cout << m << m.norm() << std::endl;
```

prints

```
6.984e-002 1.397e-001 2.095e-001
1.397e-001 3.492e-001 4.191e-001
2.095e-001 4.191e-001 6.286e-001
1.000e+000
```

2.13.37 transposition

Operator and functions

```
srsmatrix srsmatrix::operator ~ () const throw (cvmexception);
srsmatrix& srsmatrix::transpose (const srsmatrix& m) throw (cvmexception);
srsmatrix& srsmatrix::transpose ();
```

do nothing since calling matrix is symmetric. They are provided to reimplement similar member functions and operator of the class srmatrix. See also srsmatrix, srmatrix. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a,3);
std::cout << m - ~m;
```

prints

```
0 0 0
0 0 0
0 0 0
```

2.13.38 operator * (const rvector&)

Operator

```
rvector srsmatrix::operator * (const rvector& v) const
throw (cvmexception);
```

creates an object of type `rvector` as a product of a calling symmetric matrix and a vector v. It throws `cvmexception` if number of columns of a calling matrix differs from size of a vector v. Use `rvector::mult` in order to avoid new object creation. See also `srsmatrix` and `rvector`. Example:

```
using namespace cvm;

try {
    srsmatrix m (4);
    rvector v(4);
    m.set(1.);
    v.set(1.);

    std::cout << m * v;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
4 4 4 4
```

2.13.39 operator * (const rmatrix&)

Operator

```
rmatrix srsmatrix::operator * (const rmatrix& m) const
throw (cvmexception);
```

creates an object of type `rmatrix` as a product of a calling symmetric matrix and a matrix `m`. It throws `cvmexception` if number of columns of a calling matrix differs from number of rows of a matrix `m`. Use `rmatrix::mult` in order to avoid new object creation. See also `rmatrix` and `srsmatrix`. Example:

```
using namespace cvm;

try {
    srsmatrix ms(4);
    rmatrix m(4,2);
    ms.set(1.);
    m.set(2.);
    std::cout << ms * m;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
8 8
8 8
8 8
8 8
```

2.13.40 operator * (const srmatrix&)

Operator

```
srmatrix srsmatrix::operator * (const srmatrix& m) const
throw (cvmexception);
```

creates an object of type `srmatrix` as a product of a calling symmetric matrix and a matrix `m`. It throws `cvmexception` if operands have different sizes. Use `rmatrix::mult` in order to avoid new object creation. See also `srmatrix` and `srsmatrix`. Example:

```
using namespace cvm;

try {
    srsmatrix ms(3);
    srmatrix m(3);
    ms.set(1.);
    m.set(2.);
    std::cout << ms * m << std::endl;

    double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
    const srsmatrix ms2(a, 3);
    std::cout << ms2 * ms;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
6 6 6
6 6 6
6 6 6

6 6 6
13 13 13
18 18 18
```

2.13.41 operator / (const rvector&)

Operator

```
rvector operator / (const rvector& vB) const throw (cvmexception);
```

returns solution x of linear equation $A * x = b$ where calling matrix is square symmetric matrix A and a vector b is passed in parameter vB. This operator throws exception of type cvmexception in case of inappropriate sizes of the objects or when matrix A is close to singular. See also rvector::solve, srmatrix::solve, rvector.operator %, rvector, srsmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (12);

try {
    double m[] = {3., 2., 3., 2., 5., 6., 3., 6., 9.};
    double b[] = {1., 2., 3.};
    srsmatrix ma(m, 3);
    rvector  vb(b, 3);
    rvector  vx(3);

    vx = ma / vb;

    std::cout << ma * vx - vb;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
-1.110223024625e-016 -2.220446049250e-016 +4.440892098501e-016
```

2.13.42 syrk

Functions

```
srsmatrix&
srsmatrix::syrk (TR alpha, const rvector& v, TR beta)
throw (cvmexception);

srsmatrix&
srsmatrix::syrk (bool bTransp, TR alpha, const rmatrix& m, TR beta)
throw (cvmexception);
```

call one of ?SYRK routines of the BLAS library performing matrix-vector operation defined for the first version as rank-1 update operation

$$A = \alpha \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \begin{pmatrix} v_1 & v_2 & \cdots & x_n \end{pmatrix} + \beta A,$$

and for the second version as

$$A = \alpha M M^T + \beta A \quad \text{or} \quad A = \alpha M^T M + \beta A.$$

Here α and β are real numbers (parameters `alpha` and `beta`), M is real matrix (parameter `m`), A is calling symmetric matrix and v is real vector (parameter `v`). First operation for the second version is performed if `bTransp` passed is `false` and second one otherwise. Function returns a reference to the matrix changed and throws `cvmexception` in case of inappropriate sizes of the operands. See also `rvector`, `rmatrix` and `srsmatrix`.

Example:

```
using namespace cvm;
const treal alpha = 2.12;
const treal beta = -3.07;
rvector v(3);
srsmatrix ms(3), ms2(3);
v.randomize(-3.,2.);
ms.randomize(-1.,2.);
ms2 = ms;
ms.syrk (alpha, v, beta);
ms2 = alpha * srsmatrix(v.rank1update(v)) + beta * ms2;
std::cout << ms - ms2;
```

prints

```
0 0 0
0 0 0
0 0 0
```

Example:

```
using namespace cvm;
const treal alpha = 2.12;
const treal beta = -3.07;
rmatrix m(3,3);
srsmatrix ms(3), ms2(3);
m.randomize(-1.,2.);
ms.randomize(-1.,2.);
ms2 = ms;
ms.syrk (false, alpha, m, beta);
ms2 = alpha * srsmatrix (m * ~m) + beta * ms2;
std::cout << ms - ms2;
```

prints

```
0 0 0
0 0 0
0 0 0
```

Example:

```
using namespace cvm;
const treal alpha = 2.12;
const treal beta = -3.07;
rmatrix m(3,3);
srsmatrix ms(3), ms2(3);
m.randomize(-1.,2.);
ms.randomize(-1.,2.);
ms2 = ms;
ms.syrk (true, alpha, m, beta);
ms2 = alpha * srsmatrix (~m * m) + beta * ms2;
std::cout << ms - ms2;
```

prints

```
0 0 0
0 0 0
0 0 0
```

2.13.43 syr2k

Functions

```
srsmatrix&
srsmatrix::syr2k (TR alpha, const rvector& v1,
                  const rvector& v2, TR beta) throw (cvmexception);
srsmatrix&
srsmatrix::syr2k (bool bTransp, TR alpha, const rmatrix& m1,
                  const rmatrix& m2, TR beta) throw (cvmexception);
```

call one of ?SYR2K routines of the BLAS library performing matrix-vector operation defined for the first version as rank-1 update operation

$$A = \alpha v_1 v_2' + \alpha v_2 v_1' + \beta A,$$

and for the second version as

$$A = \alpha M_1 M_2^T + \alpha M_2 M_1^T + \beta A \quad \text{or} \quad A = \alpha M_1^T M_2 + \alpha M_2^T M_1 + \beta A.$$

Here α and β are real numbers (parameters `alpha` and `beta`), M_1 and M_2 are real matrices (parameters `m1` and `m2`), A is calling symmetric matrix and v_1 and v_2 are real vectors (parameters `v1` and `v2`). First operation for the second version is performed if `bTransp` passed is `false` and second one otherwise. Function returns a reference to the matrix changed and throws `cvmexception` in case of inappropriate sizes of the operands. See also `rvector`, `rmatrix` and `srsmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (2);
const treal alpha = 2.12;
const treal beta = -3.07;
rvector v1(4);
rvector v2(4);
srsmatrix ms(4), ms2(4);
v1.randomize(-1.,3.);
v2.randomize(-1.,3.);
ms.randomize(-1.,3.);
ms2 = ms;
ms.syr2k (alpha, v1, v2, beta);
ms2 = alpha * srsmatrix(v1.rank1update(v2) + v2.rank1update(v1)) + beta * ms2;
std::cout << ms - ms2;
```

prints

```
+0.00e+000 -1.33e-015 +0.00e+000 +0.00e+000
-1.33e-015 +0.00e+000 +4.44e-016 +0.00e+000
+0.00e+000 +4.44e-016 -5.55e-017 +0.00e+000
+0.00e+000 +0.00e+000 +0.00e+000 -8.88e-016
```

Example:

```
const treal alpha = 2.12;
const treal beta = -3.07;
rmatrix m1(3,3), m2(3,3);
srsmatrix ms(3), ms2(3);
m1.randomize(-2.,2.);
m2.randomize(-2.,2.);
ms.randomize(-1.,2.);
ms2 = ms;
ms.syr2k (false, alpha, m1, m2, beta);
ms2 = alpha * srsmatrix (m1 * ~m2 + m2 * ~m1) + beta * ms2;
std::cout << ms - ms2;
```

prints

```
+1.11e-015 +0.00e+000 +0.00e+000
+0.00e+000 +8.88e-016 +1.78e-015
+0.00e+000 +1.78e-015 -1.78e-015
```

Example:

```
const treal alpha = 2.12;
const treal beta = -3.07;
rmatrix m1(3,3), m2(3,3);
srsmatrix ms(3), ms2(3);
m1.randomize(-2.,2.);
m2.randomize(-2.,2.);
ms.randomize(-1.,2.);
ms2 = ms;
ms.syr2k (true, alpha, m1, m2, beta);
ms2 = alpha * srsmatrix (~m1 * m2 + ~m2 * m1) + beta * ms2;
std::cout << ms - ms2;
```

prints

```
+0.00e+000 +1.78e-015 +0.00e+000
+1.78e-015 +0.00e+000 +1.33e-015
+0.00e+000 +1.33e-015 +0.00e+000
```

2.13.44 inv

Functions

```
srsmatrix& srsmatrix::inv (const srsmatrix& m) throw (cvmexception);
srsmatrix srsmatrix::inv () const throw (cvmexception);
```

implement symmetric matrix inversion. The first version sets calling symmetric matrix to be equal to symmetric matrix `m` inverted and the second one creates an object of type `srsmatrix` as inverted calling matrix. Functions throw `cvmexception` in case of inappropriate sizes of the operands or when matrix to be inverted is close to singular. See also `srsmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (5);

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.05};
const srsmatrix m(a, 3);
const srsmatrix mi = m.inv();

std::cout << mi << std::endl;
std::cout << mi * m - eye_real(3);
```

prints

```
1.85000e+002 -2.00000e+000 -6.00000e+001
-2.00000e+000 1.00000e+000 0.00000e+000
-6.00000e+001 0.00000e+000 2.00000e+001

0.00000e+000 0.00000e+000 0.00000e+000
0.00000e+000 0.00000e+000 0.00000e+000
0.00000e+000 0.00000e+000 0.00000e+000
```

2.13.45 exp

Functions

```
srsmatrix& srsmatrix::exp (const srsmatrix& m, TR tol = cvmMachSp())
throw (cvmexception);

srsmatrix srsmatrix::exp (TR tol = cvmMachSp()) const
throw (cvmexception);
```

compute exponent of a calling symmetric matrix using Padé approximation defined as

$$R_{pq}(z) = D_{pq}(z)^{-1} N_{pq}(z) = 1 + z + \cdots + z^p/p!\,,$$

where

$$N_{pq}(z) = \sum_{k=0}^{p} \frac{(p+q-k)!p!}{(p+q)!k!(q-k)!} z^k,$$
$$D_{pq}(z) = \sum_{k=0}^{q} \frac{(p+q-k)!p!}{(p+q)!k!(q-k)!} (-z)^k$$

along with a matrix normalizing as described in [2], p. 572. Functions use `DMEXP` (or `SMEXP` for `float` version) FORTRAN subroutine implementing the algorithm. The first version sets calling symmetric matrix to be equal to exponent of symmetric matrix `m` and returns reference to the matrix changed. The second version creates an object of type `srsmatrix` as exponent of a calling matrix. The algorithm uses parameter `tol` as $\varepsilon(p, q)$ in order to choose constants p and q so that

$$\varepsilon(p, q) \geqslant 2^{3-(p+q)} \frac{p!q!}{(p+q)!(p+q+1)!}.$$

This parameter is equal to the largest relative spacing by default. Functions throw `cvmexception` in case of inappropriate sizes of the operands or when LAPACK subroutine fails. See also `srsmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (15);

double a[] = {1., 2., 1., 2., 0., -1., 1., -1., 2.};
const srsmatrix m(a, 3);
std::cout << m.exp();
```

prints

```
9.198262499129184e+000 5.558586002658855e+000 3.852443363622591e+000
5.558586002658857e+000 5.345819135506593e+000 -1.706142639036265e+000
3.852443363622590e+000 -1.706142639036266e+000 1.090440513816545e+001
```

Matlab output:

```
Columns 1 through 2

  9.198262499129212e+000    5.558586002658862e+000
  5.558586002658865e+000    5.345819135506588e+000
  3.852443363622600e+000   -1.706142639036258e+000

Column 3

  3.852443363622601e+000
 -1.706142639036260e+000
  1.090440513816545e+001
```

2.13.46 polynomial

Functions

```
srsmatrix& srsmatrix::polynom (const srsmatrix& m, const rvector& v)
throw (cvmexception);

srsmatrix srsmatrix::polynom (const rvector& v) const
throw (cvmexception);
```

compute symmetric matrix polynomial defined as

$$p(A) = b_0 I + b_1 A + \cdots + b_q A^q$$

using the Horner's rule:

$$p(A) = \sum_{k=0}^{r} B_k (A^s)^k, \quad s = \text{floor}(\sqrt{q}\,),\ r = \text{floor}(q/s)$$

where

$$B_k = \begin{cases} \sum_{i=0}^{s-1} b_{sk+i} A^i, & k = 0, 1, \ldots, r-1 \\ \sum_{i=0}^{q-sr} b_{sr+i} A^i, & k = r. \end{cases}$$

See also [2], p. 568. Coefficients $b_0, b_1, \ldots, b_q$ are passed in parameter v, where q is equal to `v.size()-1`, so functions compute matrix polynomial equal to

$$\mathrm{v}[1] * \mathrm{I} + \mathrm{v}[2] * \mathrm{m} + \cdots + \mathrm{v}[\mathrm{v.size}()] * \mathrm{m}^{\mathrm{v.size}()-1}$$

The first version sets calling symmetric matrix to be equal to the polynomial of symmetric matrix `m` and the second one creates an object of type `srsmatrix` as the polynomial of a calling symmetric matrix. Functions use `DPOLY` (or `SPOLY` for `float` version) FORTRAN subroutine implementing the Horner's algorithm. Functions throw `cvmexception` in case of inappropriate sizes of the operands. See also `srsmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (7);
double a[] = {1., 2., 1., 2., 0., -1., 1., -1., 2.};
double av[] = {2.2, 1.3, 1.1, -0.9, 0.2,
               -0.45, 45, -30, 10, 3, 3.2};
const rvector v(av, 11);
const srsmatrix m(a, 3);

std::cout << m.polynom (v);
```

prints

```
6.2127400e+004 2.3998000e+004 3.4100550e+004
2.3998000e+004 2.8026850e+004 1.0102550e+004
3.4100550e+004 1.0102550e+004 5.2024850e+004
```

Matlab output:

```
  Columns 1 through 2

    6.212740000000001e+004    2.399800000000000e+004
    2.399800000000000e+004    2.802685000000000e+004
    3.410055000000000e+004    1.010255000000000e+004

  Column 3

    3.410055000000000e+004
    1.010255000000000e+004
    5.202485000000000e+004
```

2.13.47 eig

Functions

```
rvector srsmatrix::eig (srmatrix& mEigVect) const throw (cvmexception);
rvector srsmatrix::eig () const throw (cvmexception);
```

solve the symmetric eigenvalue problem and return real vector with eigenvalues of a calling symmetric matrix. The first version sets output parameter `mEigVect` to be equal to square matrix containing orthogonal eigenvectors as columns. All functions throw `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. See also `rvector`, `srmatrix` and `srsmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (10);
double a[] = {1., 2., 1., 2., 0., -1., 1., -1., 2.};
const srsmatrix m(a, 3);
srmatrix me(3);
rvector v(3);

v = m.eig(me);
std::cout << v << std::endl;

std::cout << m * me(1) - me(1) * v(1);
std::cout << m * me(2) - me(2) * v(2);
std::cout << m * me(3) - me(3) * v(3);
```

prints

```
-2.0489173395e+000 2.3568958679e+000 2.6920214716e+000

4.4408920985e-016 0.0000000000e+000 5.5511151231e-016
-1.1102230246e-016 2.2204460493e-016 2.2204460493e-016
0.0000000000e+000 -1.1102230246e-016 -4.4408920985e-016
```

2.13.48 Cholesky

Function

```
srmatrix srsmatrix::cholesky () const throw (cvmexception);
```

forms the Cholesky factorization of symmetric positive-definite matrix A defined as

$$A = U^T U,$$

where U is upper triangular matrix. It utilizes one of ?POTRF routines of the LAPACK library. Function creates an object of type srmatrix as factorization of a calling matrix. Function throws cvmexception in case of convergence error. See also srmatrix and srsmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 1., 2., 5., -1., 1., -1., 20.};
    const srsmatrix m(a, 3);

    srmatrix h = m.cholesky();
    std::cout << h << std::endl;
    std::cout << ~h * h - m;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
1 2 1
0 1 -3
0 0 3.16228

0 0 0
0 0 0
0 0 0
```

2.13.49 Bunch-Kaufman

Function

```
srmatrix srsmatrix::bunch_kaufman () throw (cvmexception);
```

forms the Bunch-Kaufman factorization of a calling symmetric matrix (cited from the MKL library documentation):

$$A = PUDU^T P^T,$$

where A is calling symmetric matrix, P is permutation matrix, U and L are upper and lower triangular matrices with unit diagonal, and D is symmetric block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. U and L have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of D. It utilizes one of ?SYTRF routines of the LAPACK library. Function creates an object of type `srmatrix` as factorization of a calling matrix. Function throws `cvmexception` in case of convergence error. See also `srmatrix` and `srsmatrix`. Function is mostly designed to be used for subsequent calls of ?SYTRS, ?SYCON and ?SYTRI routines of the LAPACK library. Currently it's used internally in `srmatrix::det` flow when argument is symmetric but not positive-definite.

2.13.50 identity

Function

```
srsmatrix& srsmatrix::identity();
```

sets calling symmetric matrix to be equal to identity matrix and returns a reference to the matrix changed. See also `srsmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
srsmatrix m(3);
m.randomize(0.,1.);

std::cout << m << std::endl;
std::cout << m.identity();
```

prints

```
1.329e-001 8.527e-001 3.110e-001
8.527e-001 6.152e-001 3.247e-001
3.110e-001 3.247e-001 9.145e-001

1.000e+000 0.000e+000 0.000e+000
0.000e+000 1.000e+000 0.000e+000
0.000e+000 0.000e+000 1.000e+000
```

2.13.51 vanish

Function

```
srsmatrix& srsmatrix::vanish();
```

sets every element of a calling symmetric matrix to be equal to zero and returns a reference to the matrix changed. See also `srsmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (3);
srsmatrix m(3);
m.randomize(0.,1.);

std::cout << m << std::endl;
std::cout << m.vanish();
```

prints

```
1.422e-001 1.477e-001 1.445e-001
1.477e-001 8.893e-001 1.669e-002
1.445e-001 1.669e-002 7.766e-001

0.000e+000 0.000e+000 0.000e+000
0.000e+000 0.000e+000 0.000e+000
0.000e+000 0.000e+000 0.000e+000
```

2.13.52 randomize

Function

```
srsmatrix& srsmatrix::randomize (TR dFrom, TR dTo);
```

fills calling symmetric matrix with pseudo-random numbers distributed between `dFrom` and `dTo` keeping it to be symmetric. Function returns a reference to the matrix changed. See also `srsmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (7);

srsmatrix m(3);
m.randomize(-2.,3.);
std::cout << m;
```

prints

```
-1.2277291e+000 3.6610004e-001 2.1380047e+000
3.6610004e-001 1.0336924e+000 -1.8565630e+000
2.1380047e+000 -1.8565630e+000 1.7774285e+000
```

2.14 schmatrix

This is end-user class encapsulating hermitian matrix of complex numbers.

```
template <typename TR, typename TC>
class schmatrix : public scmatrix <TR,TC> {
public:
    schmatrix ();
    explicit schmatrix (int nMN);
    schmatrix (TC* pD, int nMN, TR tol = cvmMachSp());
    schmatrix (const TC* pD, int nMN, TR tol = cvmMachSp());
    schmatrix (const schmatrix& m);
    explicit schmatrix (const cmatrix& m, TR tol = cvmMachSp());
    explicit schmatrix (const rvector& v);
    explicit schmatrix (const srsmatrix& m);
    schmatrix (const TR* pRe, const TR* pIm, int nMN,
               TR tol = cvmMachSp());
    schmatrix (const srmatrix& mRe, const srmatrix& mIm,
               TR tol = cvmMachSp());
    schmatrix (schmatrix& m, int nRowCol, int nSize);
    TC operator () (int im, int in) const throw (cvmexception);
    const cvector operator () (int i) const throw (cvmexception);
    const cvector operator [] (int i) const throw (cvmexception);
    const cvector diag (int i) const throw (cvmexception);
    const srsmatrix real () const;
    const srmatrix imag () const;
    schmatrix& operator = (const schmatrix& m) throw (cvmexception);
    schmatrix& assign (const cvector& v, TR tol = cvmMachSp())
                       throw (cvmexception);
    schmatrix& assign (const TC* pD, TR tol = cvmMachSp())
                       throw (cvmexception);
    schmatrix& assign (int nRowCol, const schmatrix& m)
                       throw (cvmexception);
    schmatrix& set (int nRow, int nCol, TC z);
    schmatrix& set_diag (int i, const cvector& v) throw (cvmexception);
    schmatrix& set_main_diag (const rvector& v) throw (cvmexception);
    schmatrix& assign_real (const srsmatrix& m) throw (cvmexception);
    schmatrix& set_real (TR d) throw (cvmexception);
    schmatrix& resize (int nNewMN) throw (cvmexception);
    bool operator == (const schmatrix& v) const;
    bool operator != (const schmatrix& v) const;
    schmatrix& operator << (const schmatrix& m) throw (cvmexception);
    schmatrix operator + (const schmatrix& m) const
                          throw (cvmexception);
    schmatrix operator - (const schmatrix& m) const
```

```
                     throw (cvmexception);
schmatrix& sum (const schmatrix& m1,
                const schmatrix& m2) throw (cvmexception);
schmatrix& diff (const schmatrix& m1,
                 const schmatrix& m2) throw (cvmexception);
schmatrix& operator += (const schmatrix& m) throw (cvmexception);
schmatrix& operator -= (const schmatrix& m) throw (cvmexception);
schmatrix operator - () const;
schmatrix& operator ++ ();
schmatrix& operator ++ (int);
schmatrix& operator -- ();
schmatrix& operator -- (int);
schmatrix operator * (TR d) const;
schmatrix operator / (TR d) const throw (cvmexception);
scmatrix operator * (TC z) const;
scmatrix operator / (TC z) const throw (cvmexception);
schmatrix& operator *= (TR d);
schmatrix& operator /= (TR d) throw (cvmexception);
schmatrix& normalize ();
schmatrix operator ~ () const;
schmatrix operator ! () const;
schmatrix& conj (const schmatrix& m) throw (cvmexception);
schmatrix& conj ();
schmatrix& transpose (const schmatrix& m) throw (cvmexception);
schmatrix& transpose ();
cvector operator * (const cvector& v) const throw (cvmexception);
cmatrix operator * (const cmatrix& m) const throw (cvmexception);
scmatrix operator * (const scmatrix& m) const throw (cvmexception);
cvector operator / (const cvector& vB) const throw (cvmexception);
schmatrix& herk (TR alpha,
                 const cvector& v, TR beta) throw (cvmexception);
schmatrix& herk (bool bTransp, TR alpha,
                 const cmatrix& m, TR beta) throw (cvmexception);
schmatrix& her2k (TC alpha,
                 const cvector& v1, const cvector& v2, TR beta)
                 throw (cvmexception);
schmatrix& her2k (bool bTransp, TC alpha,
                 const cmatrix& m1, const cmatrix& m2, TR beta)
                 throw (cvmexception);
schmatrix& inv (const schmatrix& mArg) throw (cvmexception);
schmatrix inv () const throw (cvmexception);
schmatrix& exp (const schmatrix& m,
                TR tol = cvmMachSp ()) throw (cvmexception);
schmatrix exp (TR tol = cvmMachSp ()) const throw (cvmexception);
schmatrix& polynom (const schmatrix& m, const rvector& v)
```

```
                        throw (cvmexception);
    schmatrix polynom (const rvector& v) const throw (cvmexception);
    rvector eig (scmatrix& mEigVect) const throw (cvmexception);
    rvector eig () const throw (cvmexception);
    scmatrix cholesky () const throw (cvmexception);
    scmatrix bunch_kaufman () const throw (cvmexception);
    schmatrix& identity ();
    schmatrix& vanish ();
    schmatrix& randomize_real (TR dFrom, TR dTo);
    schmatrix& randomize_imag (TR dFrom, TR dTo);
};
```

2.14.1 schmatrix ()

Constructor

```
schmatrix::schmatrix ();
```

creates empty `schmatrix` object. See also `schmatrix`. Example:

```
using namespace cvm;

schmatrix m;
std::cout << m.msize() << " " << m.nsize() << " "
          << m.size() << std::endl;

m.resize (3);
std::cout << m;
```

prints

```
0 0 0
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.14.2 schmatrix (int)

Constructor

```
explicit schmatrix::schmatrix (int nMN);
```

creates $n \times n$ `schmatrix` object where n is passed in `nMN` parameter. Constructor throws `cvmexception` in case of non-positive size passed or memory allocation failure. See also `schmatrix`. Example:

```
using namespace cvm;

schmatrix m (4);
std::cout << m.msize() << std::endl
          << m.nsize() << std::endl
          << m.size() << std::endl << m;
```

prints

```
4
4
16
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
```

2.14.3 schmatrix (TC*,int)

Constructor

```
schmatrix::schmatrix (TC* pD, int nMN, TR tol = cvmMachSp());
```

creates $n \times n$ `scmatrix` object where n is passed in `nMN` parameter. Unlike others, this constructor *does not allocate memory*. It just shares memory with an array pointed to by `pD`. Constructor throws `cvmexception` if the matrix created doesn't appear to be hermitian (tolerance is set by parameter `tol`) or in case of non-positive size passed. If subsequent application flow would change the array passed so it becomes not hermitian matrix anymore then results are not predictable. See also `schmatrix`, `schmatrix (const TC*,int)`. Example:

```
using namespace cvm;
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
m.set(2,1,std::complex<double>(8.,8.));
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << " "
          << a[3] << " " << a[4] << " " << a[5] << " " << std::endl;
```

prints

```
(1,0) (8,-8) (-1,-2)
(8,8) (2,0) (0,-3)
(-1,2) (0,3) (3,0)

1 0 8 8 -1 2
```

2.14.4 schmatrix (const TC*,int)

Constructor

```
schmatrix::schmatrix (const TC* pD, int nMN, TR tol = cvmMachSp());
```

creates $n \times n$ scmatrix object where n is passed in nMN parameter and copies $n*n$ elements of an array pD to it by colums. Constructor throws cvmexception if the matrix created doesn't appear to be hermitian (tolerance is set by parameter tol) or in case of non-positive size passed. See also schmatrix, schmatrix (TC*,int). Example:

```
using namespace cvm;
const double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
                    0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((const std::complex<double>*)a, 3);
m.set(2,1,std::complex<double>(8.,8.));
std::cout << m << std::endl;
std::cout << a[0] << " " << a[1] << " " << a[2] << " "
          << a[3] << " " << a[4] << " " << a[5] << " " << std::endl;
```

prints

```
(1,0) (8,-8) (-1,-2)
(8,8) (2,0) (0,-3)
(-1,2) (0,3) (3,0)

1 0 2 1 -1 2
```

2.14.5 schmatrix (const schmatrix&)

Copy constructor

`schmatrix::schmatrix (const schmatrix& m)`

creates `schmatrix` object as a copy of `m`. Constructor throws `cvmexception` in case of memory allocation failure. See also `schmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left |
                std::ios::showpos);
std::cout.precision (1);
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
scmatrix mc(m);

m.set(1,2, std::complex<double>(7.7,7.7));
std::cout << m << std::endl << mc;
```

prints

```
(+1.0e+000,+0.0e+000) (+7.7e+000,+7.7e+000) (-1.0e+000,-2.0e+000)
(+7.7e+000,-7.7e+000) (+2.0e+000,+0.0e+000) (+0.0e+000,-3.0e+000)
(-1.0e+000,+2.0e+000) (+0.0e+000,+3.0e+000) (+3.0e+000,+0.0e+000)

(+1.0e+000,+0.0e+000) (+2.0e+000,-1.0e+000) (-1.0e+000,-2.0e+000)
(+2.0e+000,+1.0e+000) (+2.0e+000,+0.0e+000) (+0.0e+000,-3.0e+000)
(-1.0e+000,+2.0e+000) (+0.0e+000,+3.0e+000) (+3.0e+000,+0.0e+000)
```

2.14.6 schmatrix (const cmatrix&)

Constructor

`explicit schmatrix::schmatrix (const cmatrix& m, TR tol = cvmMachSp())`

creates `schmatrix` object as a copy of a matrix `m`. It's assumed that $m \times n$ matrix `m` must have equal sizes, i.e. $m = n$ is satisfied and it has to be hermitian one (tolerance is set by parameter `tol`). Constructor throws `cvmexception` if this is not true or in case of memory allocation failure. See also `schmatrix` and `cmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
cmatrix m ((std::complex<double>*)a, 3, 3);
scmatrix mch(m);
std::cout << mch;
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

2.14.7 schmatrix (const rvector&)

Constructor

```
explicit schmatrix::schmatrix (const rvector& v);
```

creates `schmatrix` object of size `v.size()` by `v.size()` and assigns vector `v` to its main diagonal. Constructor throws `cvmexception` in case of memory allocation failure. See also `schmatrix` and `rvector`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 4., 5.};
const rvector v (a, 5);
schmatrix m(v);
std::cout << m.msize() << " " << m.nsize() << " "
          << m.size() << std::endl << m;
```

prints

```
5 5 25
(1,0) (0,0) (0,0) (0,0) (0,0)
(0,0) (2,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (3,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (4,0) (0,0)
(0,0) (0,0) (0,0) (0,0) (5,0)
```

2.14.8 schmatrix (const srsmatrix&)

Constructor

```
explicit schmatrix::schmatrix (const srsmatrix& m);
```

creates `schmatrix` object having the same dimension as real symmetric matrix `m` and copies matrix `m` to its real part. See also `schmatrix` and `srsmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 2., 3., 2., 5., 6., 3., 6., 9.};
srsmatrix m(a, 3);
schmatrix mch(m);
std::cout << mch;
```

prints

```
(1,0) (2,0) (3,0)
(2,0) (5,0) (6,0)
(3,0) (6,0) (9,0)
```

2.14.9 schmatrix (const TR*,const TR*,int)

Constructor

```
schmatrix::schmatrix (const TR* pRe, const TR1* pIm, int nMN,
                      TR tol = cvmMachSp());
```

creates `schmatrix` object of size `nMN` by `nMN` and copies every element of arrays pointed to by `pRe` and `pIm` to real and imaginary part of the matrix created respectively. Use `NULL` pointer to fill up appropriate part with zero values. Constructor throws `cvmexception` if the matrix created doesn't appear to be hermitian (tolerance is et by parameter `tol`) or in case of memory allocation failure. See also `schmatrix`. Example:

```
using namespace cvm;

double re[] = {1., 2., -1., 2., 2., 0., -1., 0., 3.};
double im[] = {0., 1., 2., -1., 0., 3., -2., -3., 0.};
schmatrix m(re, im, 3);
std::cout << m;
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

2.14.10 schmatrix (const srmatrix&, const srmatrix&)

Constructor

```
schmatrix::schmatrix (const srmatrix& mRe, const srmatrix& mIm,
                      TR tol = cvmMachSp());
```

creates `schmatrix` object of the same size as `mRe` and `mIm` has (it throws `cvmexception` if `mRe` and `mIm` have different sizes) and copies matrices `mRe` and `mIm` to real and imaginary part of the matrix created respectively. Constructor throws `cvmexception` if the matrix created doesn't appear to be hermitian (tolerance is et by parameter `tol`) or in case of memory allocation failure. See also `schmatrix`, `srmatrix`. Example:

```
using namespace cvm;

double re[] = {1., 2., -1., 2., 2., 0., -1., 0., 3.};
double im[] = {0., 1., 2., -1., 0., 3., -2., -3., 0.};
srmatrix mr(re, 3);
srmatrix mi(im, 3);
schmatrix m(mr, mi);
std::cout << m;
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

2.14.11 submatrix

Submatrix constructor

```
schmatrix::schmatrix (schmatrix& m, int nRowCol, int nSize);
```

creates `scmatrix` object as *submatrix* of `m`. It means that the matrix object created shares memory with some part of `m`. This part is defined by its upper left corner (parameter `nRowCol`, 1-based) and its size (parameter `nSize`). See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
schmatrix m2 (m, 1, 2);

std::cout << m2 << std::endl;
m2.set_real(7.7);
std::cout << m;
```

prints

```
(1,0) (2,-1)
(2,1) (2,0)

(7.7,0) (7.7,-1) (-1,-2)
(7.7,1) (7.7,0) (0,-3)
(-1,2) (0,3) (3,0)
```

2.14.12 operator (,)

Indexing operator

```
TC schmatrix::operator () (int im, int in) const throw (cvmexception);
```

returns value of element of a calling hermitian matrix located on `im`-row and `in`-th column. Operator is ℓ-based. It throws `cvmexception` if some of parameters passed is outside of `[1,msize()]` range. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
std::cout << m << std::endl;
std::cout << m(3,2) << " " << m(1,3) << std::endl;
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)

(0,3) (-1,-2)
```

2.14.13 operator ()

Indexing operator

```
const cvector schmatrix::operator () (int i) const throw (cvmexception);
```

provides access to i-th column of a calling hermitian matrix. Unlike scmatrix::operator (), this operator creates only *copy* of a column and therefore it returns *not l-value*. Operator is ℓ-based. It throws `cvmexception` if parameter `i` is outside of `[1,nsize()]` range. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
std::cout << m << std::endl;
std::cout << m(1) << std::endl << m(3);
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)

(1,0) (2,1) (-1,2)

(-1,-2) (0,-3) (3,0)
```

2.14.14 operator []

Indexing operator

```
const cvector schmatrix::operator [] (int i) const throw (cvmexception);
```

provides access to i-th row of a calling hermitian matrix. Unlike scmatrix::operator [], this operator creates only *copy* of a column and therefore it returns *not l-value*. Operator is ℓ-based. It throws `cvmexception` if parameter `i` is outside of `[1,nsize()]` range. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
std::cout << m << std::endl;
std::cout << m[1] << std::endl << m[3];
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)

(1,0) (2,-1) (-1,-2)

(-1,2) (0,3) (3,0)
```

2.14.15 diag

Functions

```
const cvector schmatrix::diag (int i) const throw (cvmexception);
```

provide access to i-th diagonal of a calling matrix, where $i = 0$ for main diagonal, $i < 0$ for lower diagonals and $i > 0$ for upper ones. Unlike cmatrix::diag, this operator creates only *copy* of a diagonal and therefore it returns *not l-value*. Function throws `cvmexception` if parameter `i` is outside of `[-msize()+1,nsize()-1]` range. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
std::cout << m << std::endl;
std::cout << m.diag(-2)
          << m.diag(-1)
          << m.diag(0)
          << m.diag(1)
          << m.diag(2);
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)

(-1,2)
(2,1) (0,3)
(1,0) (2,0) (3,0)
(2,-1) (0,-3)
(-1,-2)
```

2.14.16 real

Function

```
const srsmatrix schmatrix::real () const;
```

creates an object of type `const srsmatrix` as real part of a calling hermitian matrix. Please note that, unlike cvector::real, this function creates new object *not sharing* memory with real part of a calling matrix, i.e. the matrix returned is *not l-value*. See also `srsmatrix`, `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
std::cout << m << std::endl;
std::cout << m.real();
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)

1 2 -1
2 2 0
-1 0 3
```

2.14.17 imag

Function

```
const srmatrix schmatrix::imag () const;
```

creates an object of type `const srmatrix` as imaginary part of a calling matrix. Please note that, unlike cvector::imag, this function creates new object *not sharing* memory with imaginary part of a calling matrix, i.e. the matrix returned is *not l-value*. See also `srmatrix`, `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m ((std::complex<double>*)a, 3);
std::cout << m << std::endl;
std::cout << m.imag();
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)

0 -1 -2
1 0 -3
2 3 0
```

2.14.18 operator = (const schmatrix&)

Operator

```
schmatrix& schmatrix::operator = (const schmatrix& m)
throw (cvmexception);
```

sets every element of a calling hermitian matrix to a value of appropriate element of hermitian matrix m and returns a reference to the matrix changed. Operator throws cvmexception in case of different matrix sizes. See also schmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
                  0., 3., -1., -2., 0., -3., 3., 0.};
    schmatrix m1((std::complex<double>*)a, 3);
    schmatrix m2(3);

    m2 = m1;
    std::cout << m2;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

2.14.19 assign (const TC*)

Function

```
schmatrix& schmatrix::assign (const cvector& v, TR tol = cvmMachSp())
throw (cvmexception);

schmatrix& schmatrix::assign (const TC* pD, TR tol = cvmMachSp())
throw (cvmexception);
```

sets every element of a calling hermitian matrix to a value of appropriate element of vector v or array pointed to by pD and returns a reference to the matrix changed. In first version it's assumed that vector passed is long enough to fill calling matrix. Function throws cvmexception otherwise. Function throws cvmexception if the matrix changed doesn't appear to be hermitian (tolerance is set by parameter tol). See also schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m(3);
m.assign((std::complex<double>*)a);
std::cout << m;
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

2.14.20 assign (int, int, const schmatrix&)

Function

```
schmatrix& schmatrix::assign (int nRowCol, const schmatrix& m)
throw (cvmexception);
```

sets main sub-matrix of a calling hermitian matrix beginning with ℓ-based row `nRowCol` to hermitian matrix `m` and returns a reference to the matrix changed. Function throws `cvmexception` if `nRowCol` is not positive or matrix `m` doesn't fit. See also `schmatrix`. Example:

```
using namespace cvm;

schmatrix m1(5);
schmatrix m2(2);
m2.set_main_diag(rvector(2,2.));
m2.set(1,2,std::complex<double>(2.,2.));
m1.assign(2,m2);
std::cout << m1;
```

prints

```
(0,0) (0,0) (0,0) (0,0) (0,0)
(0,0) (2,0) (2,2) (0,0) (0,0)
(0,0) (2,-2) (2,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0) (0,0)
(0,0) (0,0) (0,0) (0,0) (0,0)
```

2.14.21 set (int,int,TC)

Function

```
schmatrix& schmatrix::set (int nRow, int nCol, TC z) throw (cvmexception);
```

sets an element located on nRow's row (ℓ-based) and nCol's column (ℓ-based) to a value of parameter z and an element located on nCol's row and nRow's column to a value of parameter z conjugated (keeping calling matrix to be hermitian). Function returns a reference to the matrix changed. It throws cvmexception if any of parameters passed is outside of [1,msize()] range. It also throws in case of assigning complex number with non-zero imaginary part to any element on the main diagonal. See also schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);

m.set(1,2,std::complex<double>(7.7,7.7));
m.set(3,3,11.11);
std::cout << m;
```

prints

```
(1,0) (7.7,7.7) (-1,-2)
(7.7,-7.7) (2,0) (0,-3)
(-1,2) (0,3) (11.11,0)
```

2.14.22 set_diag

Function

```
schmatrix& schmatrix::set_diag (int i, const cvector& v)
throw (cvmexception);
```

sets i-th diagonal of a calling hermitian matrix, $i < 0$ for lower diagonals and $i > 0$ for upper ones, to be equal to complex vector passed in parameter v. Function also sets $-i$-th diagonal to be equal to a vector v conjugated, thus keeping calling matrix to be hermitian. Parameter i is *not* 1-based. Function returns a reference to the matrix changed and throws cvmexception if parameter i passed is outside of [-msize()+1,nsize()-1] range or equal to zero. It also throws if vector v passed has size not equal to msize()-abs(i). Use schmatrix::set_main_diag to set main diagonal. See also schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
cvector v(2);
v.set(std::complex<double>(7.7,7.7));
m.set_diag(1, v);
std::cout << m;
```

prints

```
(1,0) (7.7,7.7) (-1,-2)
(7.7,-7.7) (2,0) (7.7,7.7)
(-1,2) (7.7,-7.7) (3,0)
```

2.14.23 set_main_diag

Function

```
schmatrix& schmatrix::set_main_diag (const rvector& v)
throw (cvmexception);
```

sets main diagonal of a calling hermitian matrix to be equal to real vector passed in parameter v. Function returns a reference to the matrix changed. It throws `cvmexception` if vector v passed has size not equal to `msize()`. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
rvector v(3);
v.set(7.7);
m.set_main_diag(v);
std::cout << m;
```

prints

```
(7.7,0) (2,-1) (-1,-2)
(2,1) (7.7,0) (0,-3)
(-1,2) (0,3) (7.7,0)
```

2.14.24 assign_real

Function

```
schmatrix& schmatrix::assign_real (const srsmatrix& m)
throw (cvmexception);
```

sets real part of a calling hermitian matrix to be equal to real symmetric matrix m. Function returns a reference to the matrix changed. It throws cvmexception in case of different sizes of the operands. See also schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
srsmatrix ms(3);
ms.set(7.);
m.assign_real(ms);
std::cout << m;
```

prints

```
(7,0) (7,-1) (7,-2)
(7,1) (7,0) (7,-3)
(7,2) (7,3) (7,0)
```

2.14.25 set_real

Function

```
schmatrix& schmatrix::set_real (TR d);
```

sets every element of real part of a calling hermitian matrix to be equal to real number d. Function returns a reference to the matrix changed. See also schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
m.set_real(7.);
std::cout << m;
```

prints

```
(7,0) (7,-1) (7,-2)
(7,1) (7,0) (7,-3)
(7,2) (7,3) (7,0)
```

2.14.26 resize

Function

```
schmatrix& schmatrix::resize (int nNewMN) throw (cvmexception);
```

changes size of a calling hermitian matrix to `nNewMN` by `nNewMN` and returns a reference to the matrix changed. In case of increasing of its size, calling matrix is filled up with zeroes. Function throws `cvmexception` in case of non-positive size passed or memory allocation failure. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
std::cout << m << std::endl;
m.resize(4);
std::cout << m;
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)

(1,0) (2,-1) (-1,-2) (0,0)
(2,1) (2,0) (0,-3) (0,0)
(-1,2) (0,3) (3,0) (0,0)
(0,0) (0,0) (0,0) (0,0)
```

2.14.27 operator ==

Operator

```
bool schmatrix::operator == (const schmatrix& m) const;
```

compares calling hermitian matrix with hermitian matrix `m` and returns `true` if they have the same sizes and their appropriate elements differ by not more than the smallest normalized positive number. Returns `false` otherwise. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m1((std::complex<double>*)a,3);
schmatrix m2(3);
m2.assign((std::complex<double>*)a);
std::cout << (m1 == m2) << std::endl;
```

prints

```
1
```

2.14.28 operator !=

Operator

`bool schmatrix::operator != (const schmatrix& m) const;`

compares calling hermitian matrix with hermitian matrix `m` and returns `true` if they have different sizes or at least one of their appropriate elements differs by more than the smallest normalized positive number. Returns `false` otherwise. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m1((std::complex<double>*)a,3);
schmatrix m2(3);
m2.assign((std::complex<double>*)a);
m2.set(2,1,std::complex<double>(2.,1.000001));
std::cout << (m1 != m2) << std::endl;
```

prints

```
1
```

2.14.29 operator <<

Operator

```
schmatrix& schmatrix::operator << (const schmatrix& m)
throw (cvmexception);
```

destroys calling hermitian matrix, creates new one as a copy of `m` and returns a reference to the matrix changed. Operator throws `cvmexception` in case of memory allocation failure. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
schmatrix mc(1);
std::cout << m << std::endl;
std::cout << mc << std::endl;
mc << m;
std::cout << mc;
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)

(0,0)

(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

2.14.30 operator +

Operator

```
schmatrix schmatrix::operator + (const schmatrix& m) const
throw (cvmexception);
```

creates an object of type `schmatrix` as a sum of a calling hermitian matrix and hermitian matrix `m`. It throws `cvmexception` in case of different sizes of the operands. See also `schmatrix::sum`, `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
double b[] = {1., 0., 1., 1., 1., 1., 1., -1., 1., 0.,
              1., 1., 1., -1., 1., -1., 1., 0.};
schmatrix m1((std::complex<double>*)a,3);
schmatrix m2((std::complex<double>*)b,3);
std::cout << m1 + m2;
```

prints

```
(2,0) (3,-2) (0,-3)
(3,2) (3,0) (1,-4)
(0,3) (1,4) (4,0)
```

2.14.31 operator -

Operator

```
schmatrix schmatrix::operator - (const schmatrix& m) const
throw (cvmexception);
```

creates an object of type `schmatrix` as a difference of a calling hermitian matrix and hermitian matrix `m`. It throws `cvmexception` in case of different sizes of the operands. See also `schmatrix::diff`, `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
double b[] = {1., 0., 1., 1., 1., 1., 1., -1., 1., 0.,
              1., 1., 1., -1., 1., -1., 1., 0.};
schmatrix m1((std::complex<double>*)a,3);
schmatrix m2((std::complex<double>*)b,3);
std::cout << m1 - m2;
```

prints

```
(0,0) (1,0) (-2,-1)
(1,0) (1,0) (-1,-2)
(-2,1) (-1,2) (2,0)
```

2.14.32 sum

Function

```
schmatrix& schmatrix::sum (const schmatrix& m1, const schmatrix& m2)
throw (cvmexception);
```

assigns the result of addition of hermitian matrices `m1` and `m2` to a calling hermitian matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. See also `schmatrix::operator +` , `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
double b[] = {1., 0., 1., 1., 1., 1., 1., -1., 1., 0.,
              1., 1., 1., -1., 1., -1., 1., 0.};
schmatrix m1((std::complex<double>*)a,3);
schmatrix m2((std::complex<double>*)b,3);
schmatrix m(3);
std::cout << m.sum(m1, m2);
```

prints

```
(2,0) (3,-2) (0,-3)
(3,2) (3,0) (1,-4)
(0,3) (1,4) (4,0)
```

2.14.33 diff

Function

```
schmatrix& schmatrix::diff (const schmatrix& m1, const schmatrix& m2)
throw (cvmexception);
```

assigns the result of subtraction of hermitian matrices m1 and m2 to a calling hermitian matrix and returns a reference to the matrix changed. It throws cvmexception in case of different sizes of the operands. See also schmatrix::operator - , schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
double b[] = {1., 0., 1., 1., 1., 1., 1., -1., 1., 0.,
              1., 1., 1., -1., 1., -1., 1., 0.};
schmatrix m1((std::complex<double>*)a,3);
schmatrix m2((std::complex<double>*)b,3);
schmatrix m(3);
std::cout << m.diff(m1, m2);
```

prints

```
(0,0) (1,0) (-2,-1)
(1,0) (1,0) (-1,-2)
(-2,1) (-1,2) (2,0)
```

2.14.34 operator +=

Operator

```
schmatrix& schmatrix::operator += (const schmatrix& m)
throw (cvmexception);
```

adds hermitian matrix `m` to a calling hermitian matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. See also `schmatrix::operator +`, `schmatrix::sum`, `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
double b[] = {1., 0., 1., 1., 1., 1., 1., -1., 1., 0.,
              1., 1., 1., -1., 1., -1., 1., 0.};
schmatrix m1((std::complex<double>*)a,3);
schmatrix m2((std::complex<double>*)b,3);
m1 += m2;
m2 += m2;
std::cout << m1 << std::endl;
std::cout << m2;
```

prints

```
(2,0) (3,-2) (0,-3)
(3,2) (3,0) (1,-4)
(0,3) (1,4) (4,0)

(2,0) (2,-2) (2,-2)
(2,2) (2,0) (2,-2)
(2,2) (2,2) (2,0)
```

2.14.35 operator -=

Operator

```
schmatrix& schmatrix::operator -= (const schmatrix& m)
throw (cvmexception);
```

subtracts hermitian matrix `m` from calling hermitian matrix and returns a reference to the matrix changed. It throws `cvmexception` in case of different sizes of the operands. See also `schmatrix::operator -` , `schmatrix::diff`, `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
double b[] = {1., 0., 1., 1., 1., 1., 1., -1., 1., 0.,
              1., 1., 1., -1., 1., -1., 1., 0.};
schmatrix m1((std::complex<double>*)a,3);
schmatrix m2((std::complex<double>*)b,3);
m1 -= m2;
m2 -= m2;
std::cout << m1 << std::endl;
std::cout << m2;
```

prints

```
(0,0) (1,0) (-2,-1)
(1,0) (1,0) (-1,-2)
(-2,1) (-1,2) (2,0)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.14.36 operator - ()

Operator

```
schmatrix schmatrix::operator - () const throw (cvmexception);
```

creates an object of type `schmatrix` as a calling hermitian matrix multiplied by -1. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
std::cout << -m;
```

prints

```
(-1,0) (-2,1) (1,2)
(-2,-1) (-2,0) (0,3)
(1,-2) (0,-3) (-3,0)
```

2.14.37 operator ++

Operator

```
schmatrix& schmatrix::operator ++ ();
schmatrix& schmatrix::operator ++ (int);
```

adds identity matrix to a calling hermitian matrix and returns a reference to the matrix changed. See also schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
m++;
std::cout << m << std::endl;
std::cout << ++m;
```

prints

```
(2,0) (2,-1) (-1,-2)
(2,1) (3,0) (0,-3)
(-1,2) (0,3) (4,0)

(3,0) (2,-1) (-1,-2)
(2,1) (4,0) (0,-3)
(-1,2) (0,3) (5,0)
```

2.14.38 operator --

Operator

```
schmatrix& schmatrix::operator -- ();
schmatrix& schmatrix::operator -- (int);
```

subtracts identity matrix from calling hermitian matrix and returns a reference to the matrix changed. See also schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
m--;
std::cout << m << std::endl;
std::cout << --m;
```

prints

```
(0,0) (2,-1) (-1,-2)
(2,1) (1,0) (0,-3)
(-1,2) (0,3) (2,0)

(-1,0) (2,-1) (-1,-2)
(2,1) (0,0) (0,-3)
(-1,2) (0,3) (1,0)
```

2.14.39 operator * (TR)

Operator

```
schmatrix schmatrix::operator * (TR d) const;
```

creates an object of type `schmatrix` as a product of a calling hermitian matrix and real number d. See also schmatrix::operator *= , schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
std::cout << m * 5.;
```

prints

```
(5,0) (10,-5) (-5,-10)
(10,5) (10,0) (0,-15)
(-5,10) (0,15) (15,0)
```

2.14.40 operator / (TR)

Operator

```
schmatrix schmatrix::operator / (TR d) const throw (cvmexception);
```

creates an object of type `schmatrix` as a quotient of a calling hermitian matrix and real number `d`. It throws `cvmexception` if `d` has absolute value equal or less than the smallest normalized positive number. See also `schmatrix::operator /=` , `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
std::cout << m / 2.;
```

prints

```
(0.5,0) (1,-0.5) (-0.5,-1)
(1,0.5) (1,0) (0,-1.5)
(-0.5,1) (0,1.5) (1.5,0)
```

2.14.41 operator * (TC)

Operator

```
scmatrix schmatrix::operator * (TC z) const;
```

creates an object of type `scmatrix` as a product of a calling hermitian matrix and complex number `z`. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
std::cout << m * std::complex<double>(1.,1.);
```

prints

```
(1,1) (3,1) (1,-3)
(1,3) (2,2) (3,-3)
(-3,1) (-3,3) (3,3)
```

2.14.42 operator / (TC)

Operator

```
scmatrix schmatrix::operator / (TC z) const throw (cvmexception);
```

creates an object of type `scmatrix` as a quotient of a calling hermitian matrix and complex number `z`. It throws `cvmexception` if `z` has absolute value equal or less than the smallest normalized positive number. See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
std::cout << m / std::complex<double>(1.,1.);
```

prints

```
(0.5,-0.5) (0.5,-1.5) (-1.5,-0.5)
(1.5,-0.5) (1,-1) (-1.5,-1.5)
(0.5,1.5) (1.5,1.5) (1.5,-1.5)
```

2.14.43 operator *= (TR)

Operator

```
schmatrix& schmatrix::operator *= (TR d);
```

multiplies calling hermitian matrix by real number d and returns a reference to the matrix changed. See also schmatrix::operator * , schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
m *= 5.;
std::cout << m;
```

prints

```
(5,0) (10,-5) (-5,-10)
(10,5) (10,0) (0,-15)
(-5,10) (0,15) (15,0)
```

2.14.44 operator /= (TR)

Operator

```
schmatrix& schmatrix::operator /= (TR d) throw (cvmexception);
```

divides calling hermitian matrix by real number `d` and returns a reference to the matrix changed. It throws `cvmexception` if `d` has absolute value equal or less than the smallest normalized positive number. See also `schmatrix::operator /` , `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
m /= 2.;
std::cout << m;
```

prints

```
(0.5,0) (1,-0.5) (-0.5,-1)
(1,0.5) (1,0) (0,-1.5)
(-0.5,1) (0,1.5) (1.5,0)
```

2.14.45 normalize

Function

```
schmatrix& schmatrix::normalize ();
```

normalizes calling hermitian matrix so its Euclidean norm becomes equal to 1 if it was greater than the smallest normalized positive number before the call (otherwise function does nothing). See also schmatrix. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);

m.normalize();
std::cout << m << m.norm() << std::endl;
```

prints

```
(1.39e-001,0.00e+000) (2.77e-001,-1.39e-001) (-1.39e-001,-2.77e-001)
(2.77e-001,1.39e-001) (2.77e-001,0.00e+000) (0.00e+000,-4.16e-001)
(-1.39e-001,2.77e-001) (0.00e+000,4.16e-001) (4.16e-001,0.00e+000)
1.00e+000
```

2.14.46 conjugation

Operator and functions

```
schmatrix schmatrix::operator ~ () const throw (cvmexception);
schmatrix& schmatrix::conj (const schmatrix& m) throw (cvmexception);
schmatrix& schmatrix::conj () throw (cvmexception);
```

do nothing since matrix calling is hermitian. They are provided to reimplement similar member functions and operator of the class scmatrix. See also schmatrix. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);

std::cout << m - ~m;
```

prints

```
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.14.47 transposition

Operator and functions

```
schmatrix schmatrix::operator ! () const throw (cvmexception);
schmatrix& schmatrix::transpose (const schmatrix& m) throw (cvmexception);
schmatrix& schmatrix::transpose () throw (cvmexception);
```

implement complex hermitian matrix transposition (*not* conjugation). First operator creates an object of type `schmatrix` as transposed calling matrix (it throws `cvmexception` in case of memory allocation failure). Second function sets calling matrix to be equal to matrix `m` transposed (it throws `cvmexception` in case of not appropriate sizes of the operands), third one makes it to be equal to transposed itself (it also throws `cvmexception` in case of memory allocation failure). See also `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
schmatrix mt(3);
std::cout << m << std::endl << !m << std::endl ;
mt.transpose(m);
std::cout << mt << std::endl;
mt.transpose();
std::cout << mt;
```

prints

```
(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)

(1,0) (2,1) (-1,2)
(2,-1) (2,0) (0,3)
(-1,-2) (0,-3) (3,0)

(1,0) (2,1) (-1,2)
(2,-1) (2,0) (0,3)
(-1,-2) (0,-3) (3,0)

(1,0) (2,-1) (-1,-2)
(2,1) (2,0) (0,-3)
(-1,2) (0,3) (3,0)
```

2.14.48 operator * (const cvector&)

Operator

```
cvector schmatrix::operator * (const cvector& v) const
throw (cvmexception);
```

creates an object of type `cvector` as a product of a calling hermitian matrix and a vector v. It throws `cvmexception` if number of columns of a calling matrix differs from size of a vector v. Use `cvector::mult` in order to avoid new object creation. See also `schmatrix` and `cvector`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
try {
    schmatrix m((std::complex<double>*)a,3);
    cvector v(3);
    v.set(std::complex<double>(1.,1.));

    std::cout << m * v;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(5,-1) (6,2) (-3,7)
```

2.14.49 operator * (const cmatrix&)

Operator

```
cmatrix schmatrix::operator * (const cmatrix& m) const
throw (cvmexception);
```

creates an object of type `cmatrix` as a product of a calling hermitian matrix and a matrix `m`. It throws `cvmexception` if number of columns of a calling matrix differs from number of rows of a matrix `m`. Use `cmatrix::mult` in order to avoid new object creation. See also `cmatrix` and `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
try {
    schmatrix ms((std::complex<double>*)a,3);
    cmatrix m(3,2);
    m.set(std::complex<double>(1.,1.));

    std::cout << ms * m;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(5,-1) (5,-1)
(6,2) (6,2)
(-3,7) (-3,7)
```

2.14.50 operator * (const scmatrix&)

Operator

```
scmatrix schmatrix::operator * (const scmatrix& m) const
throw (cvmexception);
```

creates an object of type `scmatrix` as a product of a calling hermitian matrix and a matrix `m`. It throws `cvmexception` if the operands have different sizes. Use `cmatrix::mult` in order to avoid new object creation. See also `scmatrix` and `schmatrix`. Example:

```
using namespace cvm;

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
try {
    schmatrix ms((std::complex<double>*)a,3);
    scmatrix m(3);
    m.set(std::complex<double>(1.,1.));

    std::cout << ms * m;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(5,-1) (5,-1) (5,-1)
(6,2) (6,2) (6,2)
(-3,7) (-3,7) (-3,7)
```

2.14.51 operator / (const cvector&)

Operator

```
cvector operator / (const cvector& vB) const throw (cvmexception);
```

returns solution x of linear equation $A * x = b$ where calling matrix is square hermitian matrix A and a vector b is passed in parameter vB. This operator throws exception of type cvmexception in case of inappropriate sizes of the objects or when matrix A is close to singular. See also cvector::solve, scmatrix::solve, cvector.operator %, cvector, scmatrix, schmatrix. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (2);

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
try {
    schmatrix ma((std::complex<double>*)a,3);
    cvector vb(3);
    cvector vx(3);
    vb.set(std::complex<double>(1.,2.));

    vx = ma / vb;

    std::cout << ma * vx - vb;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(+0.00e+000,+1.78e-015) (-8.88e-016,+3.55e-015) (-3.55e-015,-1.78e-015)
```

2.14.52 herk

Functions

```
schmatrix&
schmatrix::herk (TR alpha, const cvector& v, TR beta)
                 throw (cvmexception);
schmatrix&
schmatrix::herk (bool bTransp, TR alpha, const cmatrix& m, TR beta)
                 throw (cvmexception);
```

call ZHERK routine of the BLAS library performing matrix-vector operation defined for the first version as rank-1 update operation

$$A = \alpha \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \begin{pmatrix} v_1^* & v_2^* & \cdots & v_n^* \end{pmatrix} + \beta A,$$

and for the second version as

$$A = \alpha M M^H + \beta A \quad \text{or} \quad A = \alpha M^H M + \beta A.$$

Here α and β are real numbers (parameters alpha and beta), M is complex matrix (parameter m), A is calling hermitian matrix and v is complex vector (parameter v). First operation for the second version is performed if bTransp passed is false and second one otherwise. Function returns a reference to the matrix changed and throws cvmexception in case of inappropriate sizes of the operands. See also cvector, cmatrix and schmatrix Example:

```
using namespace cvm;
const treal a[] = {1., -1., 2., 2., 3., -3.};
const cvector v((std::complex<double>*)a, 3);
const treal alpha = 2.12;
const treal beta = -3.07;
schmatrix mh(3), mh2(3);
mh.randomize_real(-1.,2.);
mh.randomize_imag(-2.,3.);
mh2 = mh;
mh.herk (alpha, v, beta);
mh2 = alpha * schmatrix(v.rank1update_c(v)) + beta * mh2;
std::cout << mh - mh2;
```

prints

```
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

Example:

```
cmatrix m(3,3);
const treal alpha = 2.12;
const treal beta = -3.07;
schmatrix mh(3), mh2(3);
m.randomize_real(-1.,2.);
m.randomize_imag(-2.,3.);
mh.randomize_real(-1.,2.);
mh.randomize_imag(-2.,3.);
mh2 = mh;
mh.herk (false, alpha, m, beta);
mh2 = alpha * schmatrix (m * ~m) + beta * mh2;
std::cout << mh - mh2;
```

prints

```
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

Example:

```
cmatrix m(3,3);
const treal alpha = 2.12;
const treal beta = -3.07;
schmatrix mh(3), mh2(3);
m.randomize_real(-1.,2.);
m.randomize_imag(-2.,3.);
mh.randomize_real(-1.,2.);
mh.randomize_imag(-2.,3.);
mh2 = mh;
mh.herk (true, alpha, m, beta);
mh2 = alpha * schmatrix (~m * m) + beta * mh2;
std::cout << mh - mh2;
```

prints

```
(3.55271e-015,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (1.77636e-015,0)
```

2.14.53 her2k

Functions

```
schmatrix&
schmatrix::her2k (TC alpha, const cvector& v1,
                  const cvector& v2, TR beta) throw (cvmexception);
schmatrix&
schmatrix::her2k (bool bTransp, TC alpha, const cmatrix& m1,
                  const cmatrix& m2, TR beta) throw (cvmexception);
```

call one of ?HER2K routines of the BLAS library performing matrix-vector operation defined for the first version as rank-1 update operation

$$A = \alpha v_1 v_2^* + \alpha^* v_2 v_1^* + \beta A,$$

and for the second version as

$$A = \alpha M_1 M_2^H + \alpha^* M_2 M_1^H + \beta A \quad \text{or} \quad A = \alpha M_1^H M_2 + \alpha^* M_2^H M_1 + \beta A.$$

Here α is complex and β is real number (parameters `alpha` and `beta`), M_1 and M_2 are complex matrices (parameters `m1` and `m2`), A is calling hermitian matrix and v_1 and v_2 are complex vectors (parameters `v1` and `v2`). First operation for the second version is performed if `bTransp` passed is `false` and second one otherwise. Function returns a reference to the matrix changed and throws `cvmexception` in case of inappropriate sizes of the operands. See also `cvector`, `cmatrix` and `schmatrix` Example:

```
using namespace cvm;
std::cout.setf(std::ios::scientific | std::ios::showpos);
std::cout.precision(2);
const tcomplex alpha(2.12,-0.14);
const tcomplex alphac(2.12,0.14);
const treal beta = -3.07;
cvector v1(3), v2(3);
schmatrix mh(3), mh2(3);
v1.randomize_real(-1.,2.);
v1.randomize_imag(-2.,3.);
v2.randomize_real(-1.,2.);
v2.randomize_imag(-2.,3.);
mh.randomize_real(-1.,2.);
mh.randomize_imag(-2.,3.);
mh2 = mh;
mh.her2k (alpha, v1, v2, beta);
mh2 = schmatrix(alpha * v1.rank1update_c(v2) +
                alphac * v2.rank1update_c(v1)) + beta * mh2;
std::cout << mh - mh2;
```

prints

```
(+1.78e-015,+0.00e+000) (+0.00e+000,+0.00e+000) (+0.00e+000,+0.00e+000)
(+0.00e+000,+0.00e+000) (-1.78e-015,+0.00e+000) (+0.00e+000,+1.78e-015)
(+0.00e+000,+0.00e+000) (+0.00e+000,-1.78e-015) (+4.44e-016,+0.00e+000)
```

Example:

```
using namespace cvm;
std::cout.setf(std::ios::scientific | std::ios::showpos);
std::cout.precision(2);
const tcomplex alpha(2.12,-0.14);
const tcomplex alphac(2.12,0.14);
const treal beta = -3.07;
cmatrix m1(3,3), m2(3,3);
schmatrix mh(3), mh2(3);
m1.randomize_real(-1.,2.);
m1.randomize_imag(-2.,3.);
m2.randomize_real(-1.,2.);
m2.randomize_imag(-2.,3.);
mh.randomize_real(-1.,2.);
mh.randomize_imag(-2.,3.);
mh2 = mh;
mh.her2k (false, alpha, m1, m2, beta);
mh2 = schmatrix(alpha * m1 * ~m2 +
                alphac * m2 * ~m1, 1.e-14) + beta * mh2;
std::cout << mh - mh2;
```

prints

```
(+0.00e+000,+4.44e-016) (+0.00e+000,+0.00e+000) (+0.00e+000,+1.78e-015)
(+3.55e-015,+0.00e+000) (+4.44e-016,-8.88e-016) (+0.00e+000,+0.00e+000)
(+8.88e-016,+8.88e-016) (+0.00e+000,+0.00e+000) (+8.88e-016,+1.78e-015)
```

Example:

```
using namespace cvm;
std::cout.setf(std::ios::scientific | std::ios::showpos);
std::cout.precision(2);
const tcomplex alpha(2.12,-0.14);
const tcomplex alphac(2.12,0.14);
const treal beta = -3.07;
cmatrix m1(3,3), m2(3,3);
schmatrix mh(3), mh2(3);
m1.randomize_real(-1.,2.);
m1.randomize_imag(-2.,3.);
m2.randomize_real(-1.,2.);
m2.randomize_imag(-2.,3.);
mh.randomize_real(-1.,2.);
```

```
mh.randomize_imag(-2.,3.);
mh2 = mh;
mh.her2k (true, alpha, m1, m2, beta);
mh2 = schmatrix(alpha * ~m1 * m2 +
                alphac * ~m2 * m1, 1.e-14) + beta * mh2;
std::cout << mh - mh2;
```

prints

```
(+3.55e-015,-8.88e-016) (+0.00e+000,+0.00e+000) (+0.00e+000,-1.78e-015)
(-8.88e-016,+1.78e-015) (+0.00e+000,-4.44e-016) (+8.88e-016,+0.00e+000)
(+0.00e+000,+3.55e-015) (-1.33e-015,-1.78e-015) (+0.00e+000,+0.00e+000)
```

2.14.54 inv

Functions

```
schmatrix& schmatrix::inv (const schmatrix& m) throw (cvmexception);
schmatrix schmatrix::inv () const throw (cvmexception);
```

implement hermitian matrix inversion. The first version sets calling hermitian matrix to be equal to hermitian matrix `m` inverted and the second one creates an object of type `schmatrix` as inverted calling matrix. Functions throw `cvmexception` in case of inappropriate sizes of the operands or when matrix to be inverted is close to singular. See also `schmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);

double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
const schmatrix mi = m.inv();

std::cout << mi << std::endl;
std::cout << mi * m - eye_complex(3);
```

prints

```
(-1.50e+000,0.00e+000) (-1.67e-016,8.33e-017) (-5.00e-001,-1.00e+000)
(-1.67e-016,-8.33e-017) (-1.00e+000,0.00e+000) (0.00e+000,-1.00e+000)
(-5.00e-001,1.00e+000) (0.00e+000,1.00e+000) (-1.50e+000,0.00e+000)

(2.22e-016,0.00e+000) (1.11e-016,0.00e+000) (2.78e-017,0.00e+000)
(2.78e-016,2.22e-016) (4.44e-016,0.00e+000) (0.00e+000,-4.44e-016)
(2.22e-016,-4.44e-016) (0.00e+000,-8.88e-016) (-2.22e-016,0.00e+000)
```

2.14.55 exp

Functions

```
schmatrix& schmatrix::exp (const schmatrix& m, TR tol = cvmMachSp ())
throw (cvmexception);

schmatrix schmatrix::exp (TR tol = cvmMachSp ()) const
throw (cvmexception);
```

compute exponent of a calling hermitian matrix using Padé approximation defined as

$$R_{pq}(z) = D_{pq}(z)^{-1} N_{pq}(z) = 1 + z + \cdots + z^p/p!\,,$$

where

$$N_{pq}(z) = \sum_{k=0}^{p} \frac{(p+q-k)!p!}{(p+q)!k!(q-k)!} z^k,$$
$$D_{pq}(z) = \sum_{k=0}^{q} \frac{(p+q-k)!p!}{(p+q)!k!(q-k)!} (-z)^k$$

along with a matrix normalizing as described in [2], p. 572. Functions use `ZMEXP` (or `CMEXP` for `float` version) FORTRAN subroutine implementing the algorithm. The first version sets calling hermitian matrix to be equal to exponent of hermitian matrix `m` and returns reference to the matrix changed. The second version creates an object of type `schmatrix` as exponent of a calling matrix. The algorithm uses parameter `tol` as $\varepsilon(p,q)$ in order to choose constants p and q so that

$$\varepsilon(p,q) \geqslant 2^{3-(p+q)} \frac{p!q!}{(p+q)!(p+q+1)!}.$$

This parameter is equal to the largest relative spacing by default. Functions throw `cvmexception` in case of inappropriate sizes of the operands or when LAPACK subroutine fails. See also `schmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left |
                std::ios::showpos);
std::cout.precision (15);
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
schmatrix me(3);
me.exp(m);
std::cout << "Column 1" << std::endl
<< me(1,1) << std::endl << me(2,1) << std::endl << me(3,1) << std::endl
```

```
          << "Column 2" << std::endl
<< me(1,2) << std::endl << me(2,2) << std::endl << me(3,2) << std::endl
          << "Column 3" << std::endl
<< me(1,3) << std::endl << me(2,3) << std::endl << me(3,3) << std::endl;
```

prints

```
Column 1
(+2.673228708372002e+002,+1.091141066389412e-014)
(+3.071187567026803e+002,+1.535593783513402e+002)
(-1.749365628720766e+002,+3.498731257441531e+002)
Column 2
(+3.071187567026803e+002,-1.535593783513401e+002)
(+4.422594337092766e+002,+1.919736460939478e-015)
(-9.600094996571151e-015,+5.034325040954932e+002)
Column 3
(-1.749365628720765e+002,-3.498731257441531e+002)
(+6.184072406183948e-015,-5.034325040954932e+002)
(+5.744416275398805e+002,+1.540673883337074e-014)
```

Matlab output:

```
  Column 1

     2.673228708371998e+002 -7.105427357601002e-015i
     3.071187567026802e+002 +1.535593783513401e+002i
    -1.749365628720764e+002 +3.498731257441527e+002i

  Column 2

     3.071187567026802e+002 -1.535593783513401e+002i
     4.422594337092769e+002 -5.489286670342458e-016i
     3.549798266275454e-015 +5.034325040954932e+002i

  Column 3

    -1.749365628720763e+002 -3.498731257441526e+002i
    -1.776065298147746e-014 -5.034325040954931e+002i
     5.744416275398801e+002 -2.096383162906490e-014i
```

2.14.56 polynomial

Functions

```
schmatrix& schmatrix::polynom (const schmatrix& m, const rvector& v)
throw (cvmexception);

schmatrix schmatrix::polynom (const rvector& v) const
throw (cvmexception);
```

compute hermitian matrix polynomial defined as

$$p(A) = b_0 I + b_1 A + \cdots + b_q A^q$$

using the Horner's rule:

$$p(A) = \sum_{k=0}^{r} B_k (A^s)^k, \quad s = \mathrm{floor}(\sqrt{q}\,),\ r = \mathrm{floor}(q/s)$$

where

$$B_k = \begin{cases} \sum\limits_{i=0}^{s-1} b_{sk+i} A^i, & k = 0, 1, \ldots, r-1 \\ \sum\limits_{i=0}^{q-sr} b_{sr+i} A^i, & k = r. \end{cases}$$

See also [2], p. 568. *Real* coefficients $b_0, b_1, \ldots, b_q$ are passed in parameter v, where q is equal to `v.size()-1`, so functions compute matrix polynomial equal to

$$\mathrm{v}[1] * \mathrm{I} + \mathrm{v}[2] * \mathrm{m} + \cdots + \mathrm{v}[\mathrm{v.size()}] * \mathrm{m}^{\mathrm{v.size()}-1}$$

The first version sets calling hermitian matrix to be equal to the polynomial of hermitian matrix `m` and the second one creates an object of type `schmatrix` as the polynomial of a calling symmetric matrix. Functions use `ZPOLY` (or `CPOLY` for `float` version) FORTRAN subroutine implementing the Horner's algorithm. Functions throw `cvmexception` in case of inappropriate sizes of the operands. See also `schmatrix`. Example:

```
using namespace cvm;
std::cout.setf (std::ios::scientific | std::ios::left |
                std::ios::showpos);
std::cout.precision (10);
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
double re[]={2.2,1.3,1.1,-0.9,0.2,-0.45,45.,-30.,10.,3.,1.13};
const rvector vr(re, 11);
schmatrix mp(3);

mp.polynom (m, vr);
```

```
std::cout << "Column 1" << std::endl
<< mp(1,1) << std::endl << mp(2,1) << std::endl << mp(3,1) << std::endl
          << "Column 2" << std::endl
<< mp(1,2) << std::endl << mp(2,2) << std::endl << mp(3,2) << std::endl
          << "Column 3" << std::endl
<< mp(1,3) << std::endl << mp(2,3) << std::endl << mp(3,3) << std::endl;
```

prints

```
Column 1
(+1.2319548758e+008,+0.0000000000e+000)
(+1.4179323916e+008,+7.0896619580e+007)
(-8.0802738460e+007,+1.6160547692e+008)
Column 2
(+1.4179323916e+008,-7.0896619580e+007)
(+2.0399822604e+008,+0.0000000000e+000)
(+0.0000000000e+000,+2.3250209650e+008)
Column 3
(-8.0802738460e+007,-1.6160547692e+008)
(+0.0000000000e+000,-2.3250209650e+008)
(+2.6498872674e+008,+0.0000000000e+000)
```

Matlab output:

```
  Column 1

     1.231954875800000e+008
     1.417932391600000e+008 +7.089661958000000e+007i
    -8.080273845999999e+007 +1.616054769200000e+008i

  Column 2

     1.417932391600000e+008 -7.089661958000000e+007i
     2.039982260400000e+008
                          0 +2.325020965000000e+008i

  Column 3

    -8.080273845999999e+007 -1.616054769200000e+008i
                          0 -2.325020965000000e+008i
     2.649887267400000e+008
```

2.14.57 eig

Functions

```
rvector schmatrix::eig (scmatrix& mEigVect) const throw (cvmexception);
rvector schmatrix::eig () const throw (cvmexception);
```

solve the symmetric eigenvalue problem and return real vector with eigenvalues of a calling hermitian matrix. The first version sets output parameter `mEigVect` to be equal to square matrix containing orthogonal eigenvectors as columns. All functions throw `cvmexception` in case of inappropriate sizes of the operands or in case of convergence error. See also `rvector`, `scmatrix` and `schmatrix`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::left);
std::cout.precision (2);
double a[] = {1., 0., 2., 1., -1., 2., 2., -1., 2., 0.,
              0., 3., -1., -2., 0., -3., 3., 0.};
schmatrix m((std::complex<double>*)a,3);
scmatrix me(3);
rvector v(3);

v = m.eig(me);
std::cout << v << std::endl;
cvector vc(v);

std::cout << m * me(1) - me(1) * vc(1);
std::cout << m * me(2) - me(2) * vc(2);
std::cout << m * me(3) - me(3) * vc(3);
// orthogonality check:
std::cout << me(1) % me(2) << std::endl;
std::cout << me(2) % me(3) << std::endl;
std::cout << me(1) % me(3) << std::endl;
```

prints

```
-8.13e-001 -3.44e-001 7.16e+000

(1.39e-016,2.22e-016) (5.25e-017,-1.11e-016) (1.94e-016,1.67e-016)
(4.86e-016,4.44e-016) (7.63e-017,0.00e+000) (3.33e-016,2.22e-016)
(-2.22e-016,-8.88e-016) (-5.55e-017,-8.88e-016) (8.88e-016,-5.55e-017)
(-5.17e-017,-9.74e-017)
(-5.81e-017,-5.40e-017)
(2.37e-018,-3.56e-017)
```

2.14.58 Cholesky

Function

```
scmatrix schmatrix::cholesky () const throw (cvmexception);
```

forms the Cholesky factorization of hermitian positive-definite matrix A defined as

$$A = U^H U,$$

where U is upper triangular matrix. It utilizes one of ?POTRF routines of the LAPACK library. Function creates an object of type scmatrix as factorization of a calling matrix. It throws cvmexception in case of convergence error. See also scmatrix and schmatrix. Example:

```
using namespace cvm;

try {
    double a[] = {3., 0., 2., 1., -1., 2., 2., -1., 3., 0.,
                  0., 3., -1., -2., 0., -3., 5., 0.};
    const schmatrix m((std::complex<double>*)a,3);

    scmatrix h = m.cholesky();
    std::cout << h << std::endl;
    std::cout << ~h * h - m;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
(1.73205,0) (1.1547,-0.57735) (-0.57735,-1.1547)
(0,0) (1.1547,0) (0,-1.1547)
(0,0) (0,0) (1.41421,0)

(-4.44089e-016,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.14.59 Bunch-Kaufman

Function

```
scmatrix schmatrix::bunch_kaufman () throw (cvmexception);
```

forms the Bunch-Kaufman factorization of a calling hermitian matrix (cited from the MKL library documentation):

$$A = PUDU^H P^H,$$

where A is calling matrix, P is permutation matrix, U and L are upper and lower triangular matrices with unit diagonal, and D is symmetric block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. U and L have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of D. It utilizes one of `?HETRF` routines of the LAPACK library. Function creates an object of type `scmatrix` as factorization of a calling matrix. It throws `cvmexception` in case of convergence error. See also `scmatrix` and `schmatrix`. Function is mostly designed to be used for subsequent calls of `?HETRS`, `?HECON` and `?HETRI` routines of the LAPACK library. Currently it's used internally in `scmatrix::det` flow when argument is hermitian but not positive-definite matrix.

2.14.60 identity

Function

```
schmatrix& schmatrix::identity();
```

sets calling hermitian matrix to be equal to identity matrix and returns a reference to the matrix changed. See also schmatrix. Example:

```
using namespace cvm;

schmatrix m(3);
m.randomize_real(0.,3.);
m.randomize_imag(-1.,2.);

std::cout << m << std::endl;
std::cout << m.identity();
```

prints

```
(1.93548,0) (1.84027,1.08353) (0.429579,-0.614093)
(1.84027,-1.08353) (1.76922,0) (1.71364,1.82788)
(0.429579,0.614093) (1.71364,-1.82788) (0.824915,0)

(1,0) (0,0) (0,0)
(0,0) (1,0) (0,0)
(0,0) (0,0) (1,0)
```

2.14.61 vanish

Function

```
schmatrix& schmatrix::vanish();
```

sets every element of a calling hermitian matrix to be equal to zero and returns a reference to the matrix changed. See also schmatrix. Example:

```
using namespace cvm;

schmatrix m(3);
m.randomize_real(0.,3.);
m.randomize_imag(-1.,2.);

std::cout << m << std::endl;
std::cout << m.vanish();
```

prints

```
(1.95499,0) (1.03925,0.830378) (0.951628,0.563677)
(1.03925,-0.830378) (0.150426,0) (2.29365,-0.580218)
(0.951628,-0.563677) (2.29365,0.580218) (0.0841395,0)

(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
(0,0) (0,0) (0,0)
```

2.14.62 randomize_real

Function

```
schmatrix& schmatrix::randomize_real (TR dFrom, TR dTo);
```

fills real part of a calling hermitian matrix with pseudo-random numbers distributed between `dFrom` and `dTo` keeping it to be hermitian. Function returns a reference to the matrix changed. See also `schmatrix`. Example:

```
using namespace cvm;

schmatrix m(3);
m.randomize_real(0.,3.);
std::cout << m << std::endl;
```

prints

```
(1.98245,0) (2.72103,0) (0.167272,0)
(2.72103,0) (0.0285653,0) (1.63765,0)
(0.167272,0) (1.63765,0) (1.15882,0)
```

2.14.63 randomize_imag

Function

```
schmatrix& schmatrix::randomize_imag (TR dFrom, TR dTo);
```

fills imaginary part of a calling hermitian matrix with pseudo-random numbers distributed between dFrom and dTo keeping it to be hermitian. Function returns a reference to the matrix changed. See also schmatrix. Example:

```
using namespace cvm;

schmatrix m(3);
m.randomize_imag(0.,3.);
std::cout << m << std::endl;
```

prints

```
(0,0) (0,0.13834) (0,2.39903)
(0,-0.13834) (0,0) (0,0.609577)
(0,-2.39903) (0,-0.609577) (0,0)
```

2.15 cvmexception

The CVM library exceptions class.

```
class cvmexception : public std::exception
{
public:
    explicit cvmexception (int nCause, ...);
    int cause () const;
    virtual const char* what () const;
    static int getNextCause ();
    static bool add (int nNewCause, const char* szNewMessage);
};
```

2.15.1 cause

Function

```
int cvmexception::cause () const;
```

returns a numeric code of an exception thrown. Possible codes can be found in `cvm.h` file. See also `cvmexception`. Example:

```
using namespace cvm;

try {
    rvector v(10);
    v[11] = 1.;
}
catch (cvmexception& e) {
    std::cout << "Exception " << e.cause () << ": "
              << e.what () << std::endl;
}
```

prints

```
Exception 2: Out of range
```

2.15.2 what

Function

```
virtual const char* cvmexception::what () const throw();
```

returns a string describing an exception happened. This function overrides `std::exception::what()`. This allows you to catch just one type of exception in your application. See also `cvmexception`. Example:

```
using namespace cvm;

try {
    double a[] = {1., 2., 1., 2.};
    const srsmatrix m(a, 2);
    std::cout << m;
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
Exception: The matrix passed doesn't appear to be symmetric
```

2.15.3 Customization

Constructor and functions

```
explicit cvmexception (int nCause, ...);
static bool cvmexception::add (int nNewCause, const char* szNewMessage);
static int cvmexception::getNextCause ();
```

allow to add and use customized exception codes and messages. See also `cvmexception`. Example:

```
using namespace cvm;

const int nNextCause = cvmexception::getNextCause();
cvmexception::add (nNextCause,
                   "My first exception with %d parameter");
cvmexception::add (nNextCause + 1,
                   "My second exception with %s parameter");

try {
    throw cvmexception (nNextCause, 1234);
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}

try {
    throw cvmexception (nNextCause + 1, "Hi!");
}
catch (std::exception& e) {
    std::cout << "Exception: " << e.what () << std::endl;
}
```

prints

```
Exception: My first exception with 1234 parameter
Exception: My second exception with Hi! parameter
```

2.16 Utilities

These functions have `cvm` namespace scope and can be used for different purposes.

```
template <typename T>
T* cvmMalloc (size_t nEls) throw (cvmexception);
template <typename T>
T* cvmAddRef (const T* pD);
template <typename T>
int cvmFree (T*& pD);
void cvmExit ();
treal cvmMachMin ();
treal cvmMachSp ();
srmatrix eye_real (int nM);
scmatrix eye_complex (int nM);
operator * (,);
```

2.16.1 cvmMalloc

Since version 6.0 if `CVM_USE_POOL_MANAGER` macro is not defined (this is the default) this function simply calls `::new T[nEls]` operator.

```
template <typename T>
T* cvmMalloc (size_t nEls) throw (cvmexception);
```

allocates `nEls` units of type `T` from the CVM library's memory pool and returns a pointer to the memory allocated. See also `cvmAddRef` and `cvmFree`. Example:

```
using namespace cvm;

double* p = cvmMalloc<double> (10);
p[0] = 1.;
p[1] = 2.;
p[2] = 3.;

rvector v(3);
v.assign(p);
std::cout << v;

cvmFree (p);
```

prints

```
1 2 3
```

2.16.2 cvmAddRef

Since version 6.0 it's available only if CVM_USE_POOL_MANAGER is defined.

```
template <typename T>
void cvmAddRef (const T* pD);
```

increments a reference counter for a memory block pointed to by pD if this block was allocated from the CVM library's memory pool (using cvmMalloc function). If pD points to a foreign memory block then the function does nothing. See also cvmAddRef. Example:

```
using namespace cvm;

double* p = cvmMalloc<double> (10);
p[0] = 1.;
p[1] = 2.;
p[2] = 3.;

cvmAddRef (p);
cvmFree (p);    // this call doesn't deallocate memory
cvmFree (p);    // this one does
```

2.16.3 cvmFree

Since version 6.0 if CVM_USE_POOL_MANAGER macro is not defined (this is the default) then it simply calls ::delete[] pD operator if pD!=NULL and returns zero.

```
template <typename T>
int cvmFree (T*& pD);
```

decrements a reference counter for a memory block pointed to by pD if this block was allocated from the CVM library's memory pool (using cvmMalloc function) and returns the reference counter it changed. If the function returns zero then it sets the pointer pD to be equal to NULL and "frees" the memory, i.e. returns the memory block to a list of free ones (see CVM memory management for details). If pD points to a foreign memory block then the function does nothing and returns -1. See also cvmAddRef. Example:

```
using namespace cvm;

double* pf = new double[10];
double* p  = cvmMalloc<double> (10);

cvmAddRef (p);

std::cout << cvmFree (p) << " ";
std::cout << p << std::endl;

std::cout << cvmFree (p) << " ";
std::cout << p << std::endl;

std::cout << cvmFree (pf) << " ";
std::cout << pf << std::endl;

delete[] pf;
```

prints

```
1 003C66B0
0 00000000
-1 003C7A40
```

2.16.4 cvmExit

Since version 6.0 it's available only if CVM_USE_POOL_MANAGER is defined.

```
void cvmExit ();
```

destroys the CVM library's memory pool if CVM_USE_POOL_MANAGER is defined. Otherwise does nothing. All objects created using this pool are not accessible after calling of this function. Call this function in the last expression only if you have problems with memory deallocation while finishing execution of your program (I was not able to experience such problems, this function is provided just in case). See also cvmMalloc. Example:

```
using namespace cvm;

int main(int argc, char* argv[])
    try {
        rvector v(3);
        v[1] = v[2] = v[3] = 1.;
        std::cout << v.norm2() << std::endl;
    }
    catch (cvmexception& e) {
        std::cout << "Exception " << e.cause ()
                  << ": " << e.what () << std::endl;
    }

    cvmExit ();
    return 1;
}
```

prints

```
1.73205
```

2.16.5 cvmMachMin

Function

```
treal cvmMachMin ();
```

returns the smallest normalized positive number, i.e. `numeric_limits<treal>::min()` where `treal` is `typedef`'ed as `double` by default or as `float` for float version of the library. See also `cvmMachSp`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (15);
std::cout << cvmMachMin() << std::endl;
```

on Intel Pentium® III machine prints

```
+2.225073858507201e-308
```

2.16.6 cvmMachSp

Function

`treal cvmMachSp ();`

returns the largest relative spacing or, in other words, the difference between 1 and the least value greater than 1 that is representable, i.e. `numeric_limits<treal>::epsilon()` where `treal` is `typedef`'ed as `double` by default or as `float` for float version of the library. See also `cvmMachMin`. Example:

```
using namespace cvm;

std::cout.setf (std::ios::scientific | std::ios::showpos);
std::cout.precision (15);
std::cout << cvmMachSp() << std::endl;
```

on Intel Pentium® III machine prints

```
+2.220446049250313e-016
```

2.16.7 eye_real

Function

```
srmatrix eye_real (int nM);
```

creates a nM by nM object of type srmatrix equal to identity matrix. See also srmatrix.
Example:

```
using namespace cvm;

std::cout << eye_real (4);
```

prints

```
1 0 0 0
0 1 0 0
0 0 1 0
0 0 0 1
```

2.16.8 eye_complex

Function

```
scmatrix eye_complex (int nM);
```

creates a nM by nM object of type scmatrix equal to identity matrix. See also scmatrix.
Example:

```
using namespace cvm;

std::cout << eye_complex (4);
```

prints

```
(1,0) (0,0) (0,0) (0,0)
(0,0) (1,0) (0,0) (0,0)
(0,0) (0,0) (1,0) (0,0)
(0,0) (0,0) (0,0) (1,0)
```

2.16.9 operator *

Operators

```
inline rvector operator * (TR d, const rvector& v);
inline rmatrix operator * (TR d, const rmatrix& m);
inline srmatrix operator * (TR d, const srmatrix& m);
inline srbmatrix operator * (TR d, const srbmatrix& m);
inline srsmatrix operator * (TR d, const srsmatrix& m);
inline cvector operator * (TR d, const cvector& v);
inline cmatrix operator * (TR d, const cmatrix& m);
inline scmatrix operator * (TR d, const scmatrix& m);
inline scbmatrix operator * (TR d, const scbmatrix& m);
inline schmatrix operator * (TR d, const schmatrix& m);
inline cvector operator * (std::complex<TR> c, const cvector& v);
inline cmatrix operator * (std::complex<TR> c, const cmatrix& m);
inline scmatrix operator * (std::complex<TR> c, const scmatrix& m);
inline scbmatrix operator * (std::complex<TR> c, const scbmatrix& m);
inline schmatrix operator * (std::complex<TR> c, const schmatrix& m);
inline rvector operator * (CVM_LONGEST_INT d, const rvector& v);
inline rmatrix operator * (CVM_LONGEST_INT d, const rmatrix& m);
inline srmatrix operator * (CVM_LONGEST_INT d, const srmatrix& m);
inline srbmatrix operator * (CVM_LONGEST_INT d, const srbmatrix& m);
inline srsmatrix operator * (CVM_LONGEST_INT d, const srsmatrix& m);
inline cvector operator * (CVM_LONGEST_INT d, const cvector& v);
inline cmatrix operator * (CVM_LONGEST_INT d, const cmatrix& m);
inline scmatrix operator * (CVM_LONGEST_INT d, const scmatrix& m);
inline scbmatrix operator * (CVM_LONGEST_INT d, const scbmatrix& m);
inline schmatrix operator * (CVM_LONGEST_INT d, const schmatrix& m);
```

provide an ability to make left-sided multiplication of numbers by different CVM objects. Example:

```
using namespace cvm;

const schmatrix scm = eye_complex (4);
std::cout << std::complex<double>(2.,1.) * scm << std::endl;

rvector v(3);
v(1) = 1.;
v(2) = 2.;
v(3) = 3.;

std::cout << 3. * v;
```

prints

```
(2,1) (0,0) (0,0) (0,0)
(0,0) (2,1) (0,0) (0,0)
(0,0) (0,0) (2,1) (0,0)
(0,0) (0,0) (0,0) (2,1)

3 6 9
```

2.17 Pool Manager

The memory management mechanism described below is no longer supported by default. It was a good solution more than 20 years ago when memory allocation operator was relatively expensive, but now standard allocator does this job much faster. However, the algorithm described below may be useful as an error detection helper. You will need to rebuild the library in debug mode with `CVM_USE_POOL_MANAGER` *defined in order to use it.*

Since its third release the CVM library implements nontrivial memory management which should be described in detail. Earlier memory was being allocated using operator `new` in every constructor, and freed with help of `delete` in every destructor. Let us consider an operation of multiplying of a vector `a` by matrix `A` and assignment of result to a vector `b`:

```
b = a * A;
```

This harmless code calls two constructors (a constructor allocating memory for output and a copy constructor, returning output to a calling function), as well as two destructors deleting those temporary objects[11]. If sizes of the objects are relatively small, your processor will be longer allocating memory than multiplying[12]. The idea of nontrivial memory management came from Jeff Alger's book [1]. Author suggests some approaches to memory allocation (overloading of operators `new` and `delete` or implementation of a class controlling a pool), and also some ways of a pool compaction (Baker's algorithm and in-place compaction) and references counting (using master or "genius" pointers). I decided to implement a class controlling a pool in the CVM library.

Pool is controlled by `MemoryPool` class. It allocates memory by blocks (so-called "outer blocks"). The memory block concept is implemented in a class `MemoryBlocks`. Size of an outer block equals to the nearest degree of 2 of a requested number of bytes multiplied by two. This can be illustrated on the following example. Let us suppose that it's required to allocate memory for storage of a vector consisting of 1000 units:

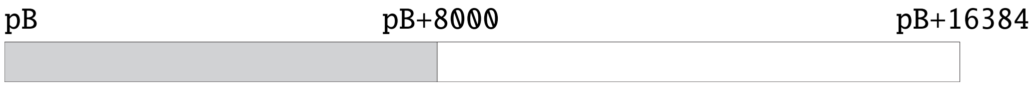
```
rvector v(1000);
```

If at the moment of execution of this statement there is no free block of size 8000 or greater in a list of free memory blocks, then the following (simplified) code will be executed:

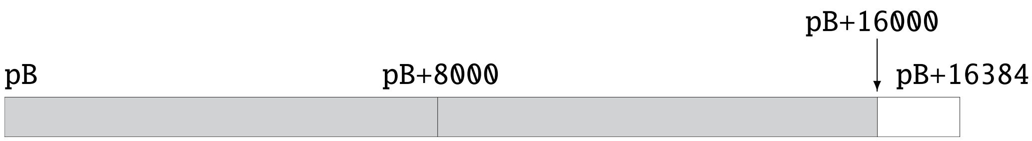
```
const int nUpBytes = up_value (nBytes);
const int nRest    = nUpBytes - nBytes;
try {
    pB = ::new tbyte[nUpBytes];
}
catch (const std::bad_alloc&) {
    throw (cvmexception (CVM_OUTOFMEMORY));
}
m_lOutBlocks.push_back (pB);
m_blocks.AddPair (pB, nBytes, nRest);
```

[11] In order to avoid those memory allocations you can use `b.mult(a,A);`
[12] At least under Win32

where variable `nBytes` has value of 8000 (`1000*sizeof(treal)`), and function `up_value` returns the nearest degree of two multiplied by two, i.e. 16384. Thus, the allocated outer block can be represented as follows:

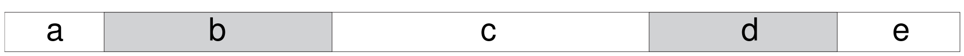


The block used for storage 1000 units of a vector `v` (its start address here is stored in the pointer `pB`) is filled. The remaining block (named as "free block") consists of `nRest=8384` bytes. Further, if application needs one more block of the same size (it happens in most cases while execution of a copy constructor), the memory will be allocated from this free block without calling of operator `new`. The result will be the following:

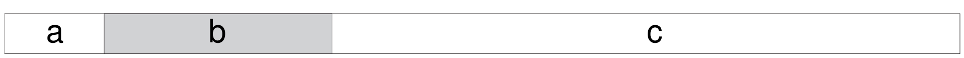


Remaining free block of 384 bytes will be utilized for memory allocation of small objects. In case of creation of an object of size greater than 384 bytes one more outer block will be created, etc. While using of this scheme sooner or later memory begins to be like a sieve of free and occupied blocks. To avoid this chaos, I have applied the logic of blocks releasing, which acts like an algorithm of free space compaction. Difference of this logic from the algorithms of compaction described in [1] is that occupied blocks are never being moved.

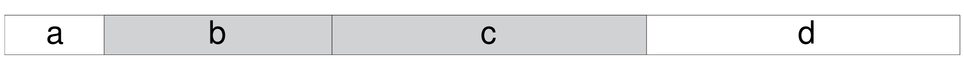
Let's say one of outer blocks looks like



And the block `d` is going to be released. In this case neighbor blocks `c` and `e` will be checked and if one of them (or both, as in this case) will appear as free, then it will be joined with the block being released. The result of release of the block `d` will be the following:



And if the following object to be created will not find a room in the block `a`, but find a room in the block `c`, the result of memory allocation will be the following:



So, there is no any chaos.

3 Functional Classes

3.1 Functions

Classes `rfunction` and `cfunction` encapsulate elementary functions of real or complex numbers. Typically they are used to convert strings entered by a user to computable expressions without need to parse same expression over and over again. Also might be used to simplify computable expressions and to compute derivatives analytically, i.e. without using numerical methods. Functions can have zero, one or more than one variables and can be parameterized. See online documentation for details.

Operators and functions supported in expressions:

$+ - * /$ ^	add, subtract, multiply, divide, power
`exp(x)`	e^x
`sqrt(x)`	$\sqrt{x}$
`log(x)`	natural logarithm of x
`log10(x)`	common (base 10) logarithm of x
`sin(x)`	sine of x
`cos(x)`	cosine of x
`tan(x)`	tangent of x
`asin(x)`	arc sine of x
`acos(x)`	arc cosine of x
`atan(x)`	arc tangent of x
`sinh(x)`	hyperbolic sine of x
`cosh(x)`	hyperbolic cosine of x
`tanh(x)`	hyperbolic tangent of x
`sinint(x)`	sine integral of x, $\int_0^x \frac{\sin t}{t} dt$
`cosint(x)`	cosine integral of x, $-\int_x^\infty \frac{\cos t}{t} dt$
`sign(x)`	sign of x, $\begin{cases} -1 \text{ if } x < 0 \\ 0 \text{ if } x = 0 \\ 1 \text{ if } x > 0 \end{cases}$
`abs(x)`	$\lvert x \rvert$
`iif(x,expr1,expr2)`	immediate if $\begin{cases} \text{expr1 if } x < 0 \\ \text{expr2 if } x \geqslant 0 \end{cases}$
`sat(x,y)`	satellite function of x and y, $\begin{cases} 1 \text{ if } x > \lvert y \rvert \\ 0 \text{ if } -\lvert y \rvert \leqslant x \leqslant \lvert y \rvert \\ -1 \text{ if } x < -\lvert y \rvert \end{cases}$

`i` 1 (one)
`0` (empty field means zero)

Example:

```
using namespace cvm;
try {
    rfunction rf("{x,y} cos(x)*cos(y) + x + x + x");
    std::cout << rf.simp() << std::endl;
    std::cout << rf.drv(0) << std::endl;
    std::cout << rf.drv(1) << std::endl;
    double vars[] = {0., 0.};
    std::cout << rf(vars) << std::endl;
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
{x,y} x*3+cos(x)*cos(y)
{x,y} 3-sin(x)*cos(y)
{x,y} -sin(y)*cos(x)
1
```

3.2 Function vectors and matrices

Classes `rfvector`, `cfvector`, `rfmatrix` and `cfmatrix` encapsulate vectors and matrices of elementary functions of real or complex numbers. See online documentation for details. Example:

```
using namespace cvm;
try {
    rfunction f1("{x} x+sin(x/2)");
    rfunction f2(2.);
    // note: (f2^2) in parenthesis to follow priority:
    std::cout << (f1*f2 + (f2^2)).simp() << std::endl;

    string_array sa;
    sa.push_back ("{x,y} x+y");
    sa.push_back ("{x,y} x-y");
    rfvector fv1(sa), fv2(sa);

    rfunction f = fv1 * fv2;
    std::cout << f << std::endl;
    std::cout << f.drv(0) << std::endl; // derivative by x
```

```
    std::cout << f.drv(1) << std::endl; // derivative by y


    double vars[] = {1., 2.};
    std::cout << f(vars) << std::endl;

    string_array sa1;
    sa1.push_back ("{x,y} x");
    sa1.push_back ("{x,y} y");
    sa1.push_back ("{x,y} 1");
    sa1.push_back ("{x,y} 2");
    sa1.push_back ("{x,y} 3");
    sa1.push_back ("{x,y} 4");
    rfmatrix fm(2,3,sa1);

    rfvector fvm = fv1 * fm;
    std::cout << fvm << std::endl;

    std::cout << std::endl;
    string_array sa2;
    sa2.push_back ("{x,y} 2*x^2+3*y^3");
    sa2.push_back ("{x,y} 4*x^3-5*y^2");
    sa2.push_back ("{x,y} -x^4+6*y^2");
    rfvector fv(sa2);

    rfmatrix fmj = fv.jacobian();
    std::cout << fv << std::endl << fmj << std::endl;
    std::cout << fmj(vars);
}
catch (std::exception& e) {
    std::cout << "Exception " << e.what () << std::endl;
}
```

prints

```
{x} (x+sin(x/2))*2+4
{x,y} (x+y)^2+(x-y)^2
{x,y} 4*x
{x,y} 2*(x+y)+(-2)*(x-y)
10
{x,y} (x+y)*x+(x-y)*y {x,y} x+y+(x-y)*2 {x,y} (x+y)*3+(x-y)*4


{x,y} 2*x^2+3*y^3 {x,y} 4*x^3-5*y^2 {x,y} -x^4+6*y^2

{x,y} 4*x {x,y} 9*y^2
```

```
{x,y} 12*x^2 {x,y} (-10)*y
{x,y} (-4)*x^3 {x,y} 12*y

4 36
12 -20
-4 24
```

References

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