# Cooperation

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# Golden Balls

Watch the final stage of a game show.

## Definition

$$A = \begin{pmatrix} R & S \\ T & P \end{pmatrix} \qquad B = \begin{pmatrix} R & T \\ S & P \end{pmatrix}$$

with the following constraints:

$$T > R > P > S \qquad 2R > T + S$$

## Definition

Given a two player game $(A, B) \in \mathbb{R}^{m \times n^2}$, referred to as a stage game, a $T$-stage repeated game is a game in which players play that stage game for $T > 0$ repetitions. Players make decisions based on the full history of play over all the repetitions.

## Definition

A strategy for a player in a repeated game is a mapping from all possible histories of play to a probability distribution over the action set of the stage game.

$$\sum_{i=0}^{\infty} \delta^{i} u_i(s_1, s_2)$$

Probability of game continuing

**R**eward for mutual cooperation

$$U(s_C, s_C) = U(s_G, s_G) = U(s_G, s_C) = U(s_C, s_G) = \sum_{i=0}^{\infty} \delta^i R = \frac{R}{1-\delta}$$

$$U(s_D, s_D) = \sum_{i=0}^{\infty} \delta^i P = \frac{P}{1-\delta}$$

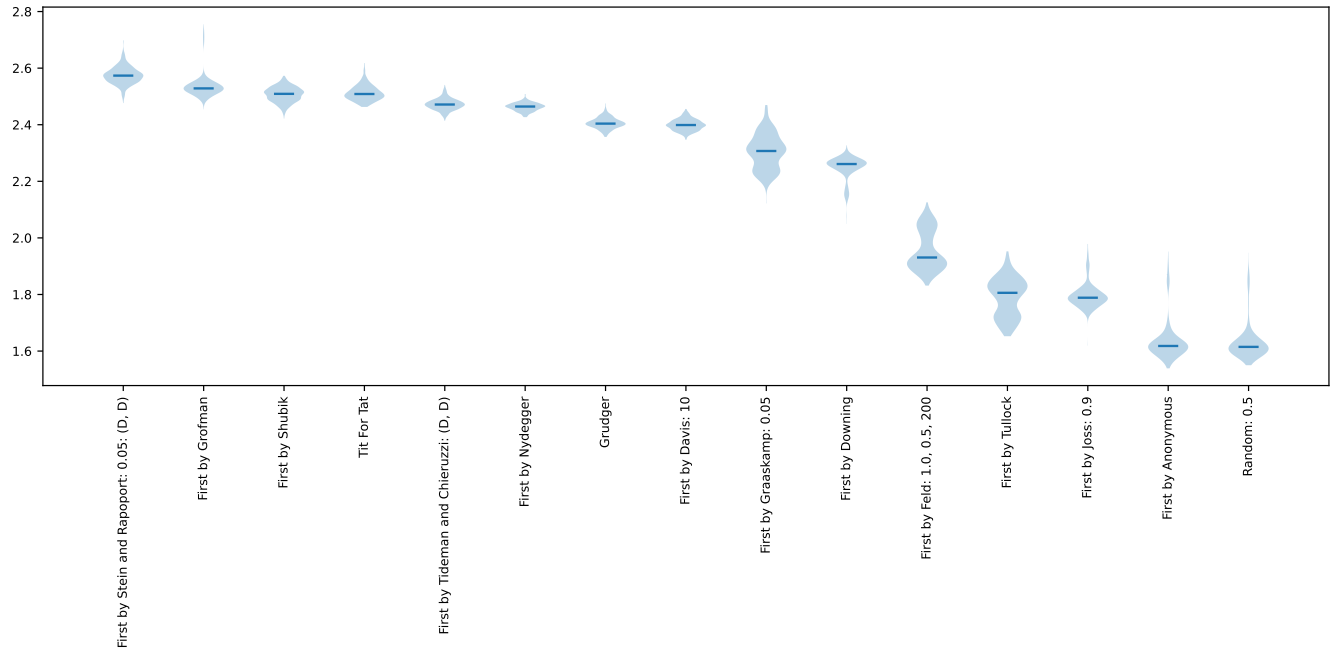
**P**unishment for mutual defection

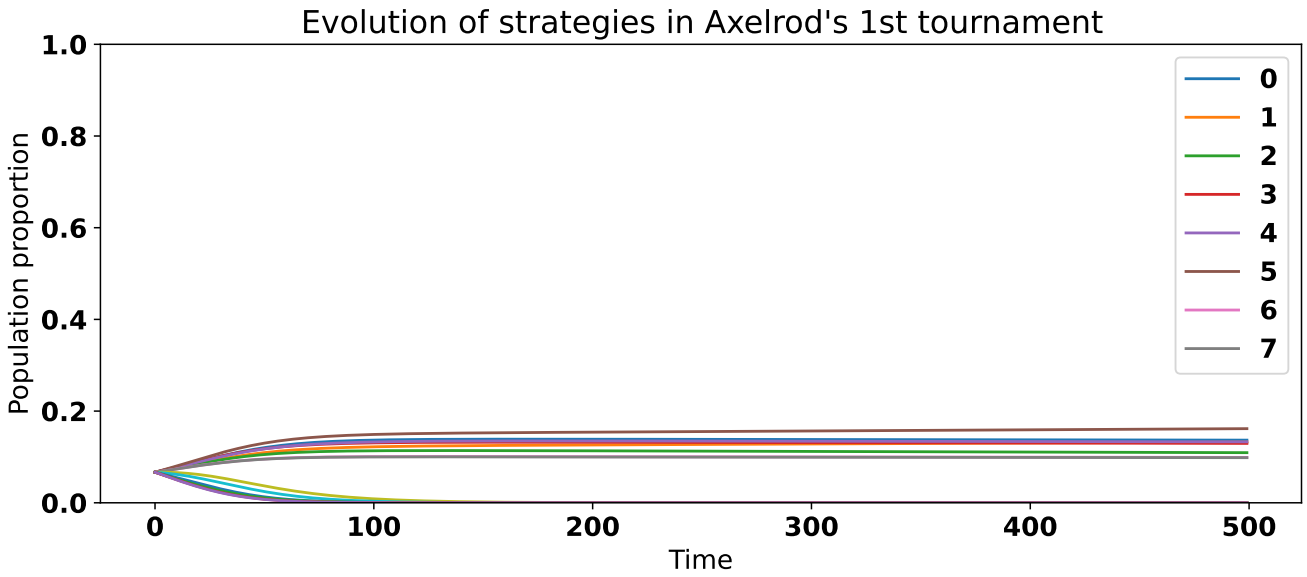
$$U(s_D, s_G) = T + \sum_{i=1}^{\infty} \delta^i P = T + \frac{P\delta}{1-\delta}$$

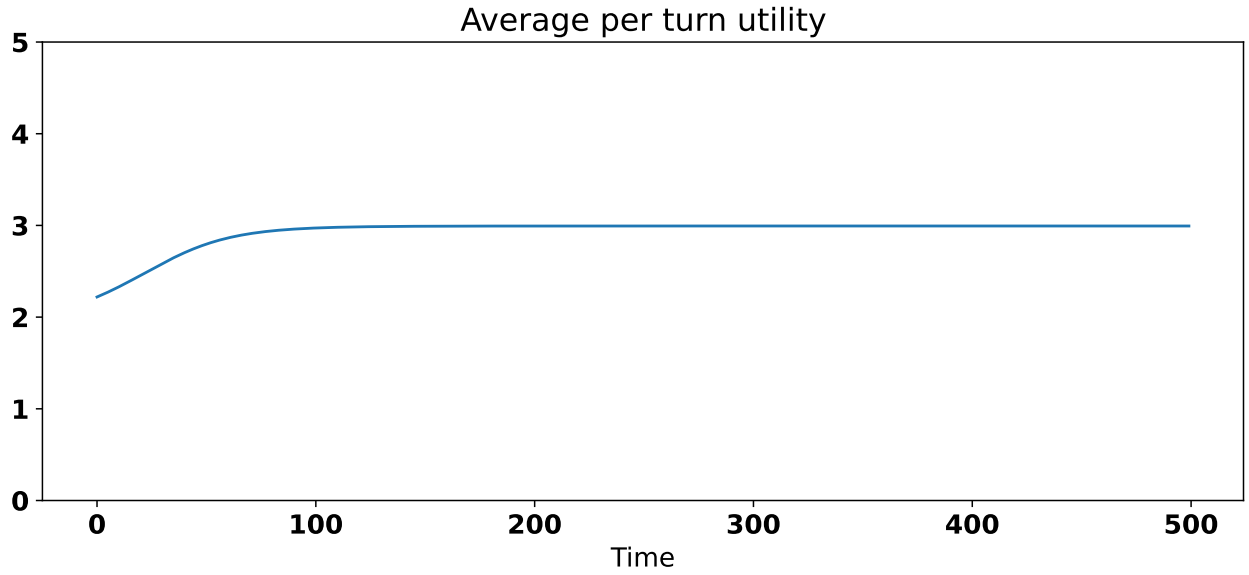
**T**emptation to defect

**Robert Axelrod**[1] (1943 - )
Courtesy of University of Michigan personal website,
https://commons.wikimedia.org/w/index.php?curid=20096037

[1] Robert Axelrod. "Effective Choice in the Prisoner's Dilemma". In: *The Journal of Conflict Resolution* 24.1 (1980), pp. 3–25. (Visited on 03/20/2024).


2.8
2.6
2.4
2.2
2.0
1.8
1.6
First by Stein and Rapoport: 0.05: (D, D)
First by Grofman
First by Shubik
Tit For Tat
First by Tideman and Chieruzzi: (D, D)
First by Nydegger
Grudger
First by Davis: 10
First by Graaskamp: 0.05
First by Downing
First by Feld: 1.0, 0.5, 200
First by Tullock
First by Joss: 0.9
First by Anonymous
Random: 0.5


Evolution of strategies in Axelrod's 1st tournament
Population proportion
Time
0
1
2
3
4
5
6
7
0.0
0.2
0.4
0.6
0.8
1.0
0
100
200
300
400
500


Average per turn utility
Time
0
1
2
3
4
5
0
100
200
300
400
500

Looking at the plot on the left, we see that there are only a few strategies that survive the evolutionary process. In fact those that do are ones that only cooperate against each other.

- **Be nice**
- **Be provocable**
- **Don't be envious**
- **Don't be too clever**

"The world of game theory is currently on fire."

MIT Technology Review

$P(C\,|\,CC)$ $P(C\,|\,CD)$

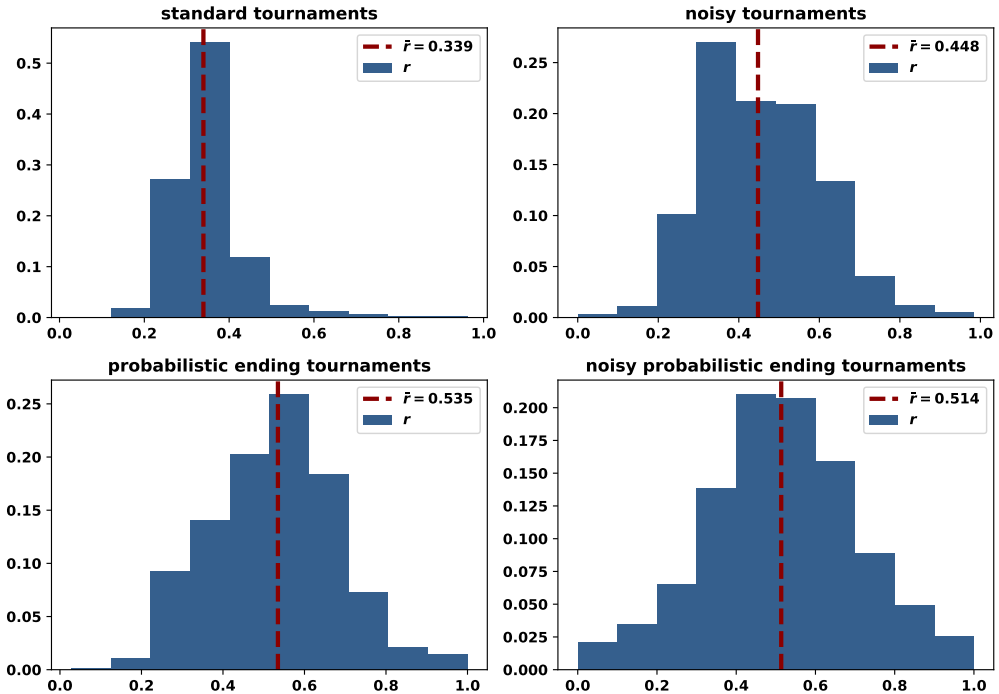
For $p = (\ p_1\ ,\ p_2\ ,\ p_3\ ,\ p_4\ )$ and $q = (q_1, q_2, q_3, q_4)$:

$P(C\,|\,DC)$ $P(C\,|\,DD)$

$$u(p,q) = \begin{vmatrix} -1 + p_1 q_1 & -1 + p_1 & -1 + q_1 & f_1 \\ p_2 q_3 & -1 + p_2 & q_3 & f_2 \\ p_3 q_2 & p_3 & -1 + q_2 & f_3 \\ p_4 q_4 & p_4 & q_4 & f_4 \end{vmatrix}$$

## Properties of Winning Iterated Prisoner's Dilemma Strategies[a]

[a] Nikoleta E. Glynatsi, Vincent Knight, and Marc Harper. *Properties of Winning Iterated Prisoner's Dilemma Strategies*. 2024. arXiv: 2001.05911 [cs.GT].



- Be nice in non-noisy environments or when game lengths are longer
- Be provocable in tournaments with short matches, and generous in tournaments with noise
- Be a little bit envious
- Be clever
- Adapt to the environment (including the population of strategies).

Evolution reinforces cooperation with the emergence of self-recognition mechanisms: An empirical study of strategies in the Moran process for the iterated prisoner's dilemma[a]

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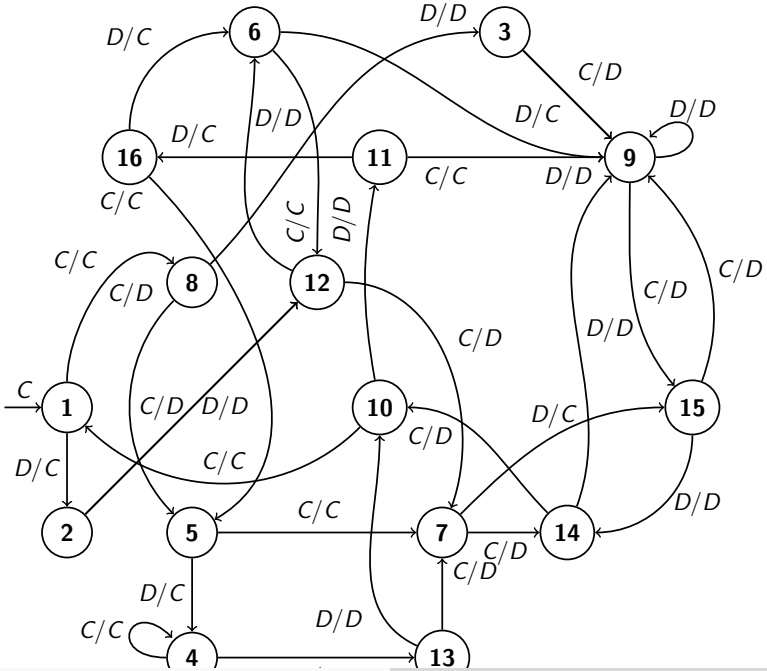
[a] Vincent Knight et al. "Evolution reinforces cooperation with the emergence of self-recognition mechanisms: An empirical study of strategies in the Moran process for the iterated prisoner's dilemma". In: *PloS one* 13.10 (2018), e0204981.



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