Evolutionary Game Theory

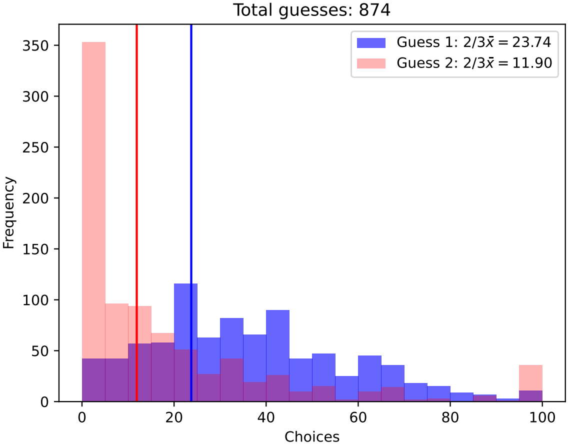
Vince Knight

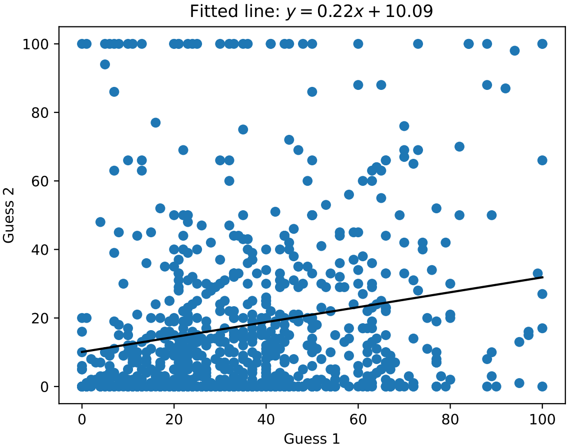
Keynesian Beauty Contest[1]

[1] Joseph A Schumpeter. *The General Theory of Employment, Interest and Money.* 1936.

Two Thirds of the Average

- Pick an integer between 0 and 100 (inclusive);
- Closest to two thirds of the average of all picked numbers wins.


Total guesses: 874
Guess 1: $2/3\bar{x} = 23.74$
Guess 2: $2/3\bar{x} = 11.90$
Frequency
Choices
0
50
100
150
200
250
300
350
0
20
40
60
80
100


Fitted line: $y = 0.22x + 10.09$
Guess 2
Guess 1
0
20
40
60
80
100
0
20
40
60
80
100

Definition

Considering an infinite population of individuals each of which represents an action from $\mathcal{A}$, we define the population profile as a vector $x \in [0,1]_{\mathbb{R}}^{|\mathcal{A}|}$. Note that:

$$\sum_{i\in\mathcal{A}} x_i = 1$$

Definition

The population dependent fitness of an individual of type i in a population x is denoted as $f_i : \mathbb{R}^{101}_{[0,1]} \to \mathbb{R}$.

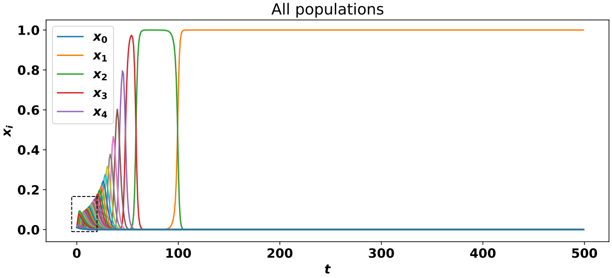
Definition

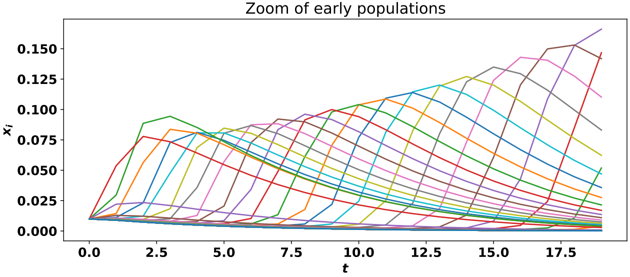
Replicator Dynamics Equation

$$\frac{dx_i}{dt} = x_i(f_i(x) - \phi) \text{ for all } i$$

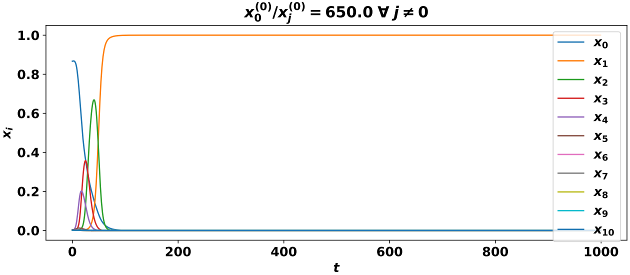
where:

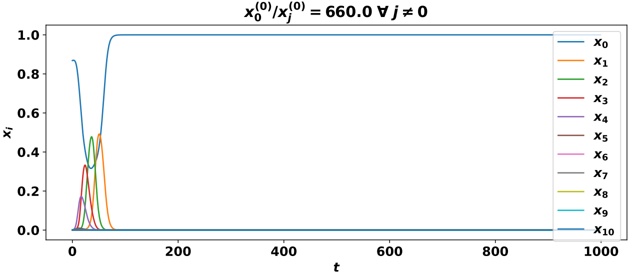
$$\phi = \sum_{i=0}^{N} x_i f_i(x)$$

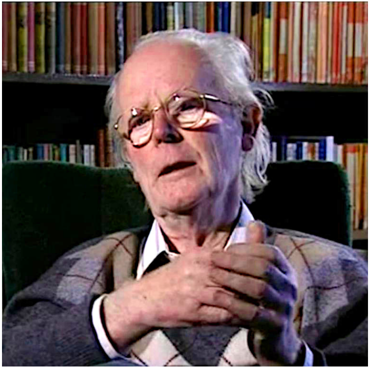

All populations
1.0
0.8
0.6
0.4
0.2
0.0
x_i
x_0
x_1
x_2
x_3
x_4
0
100
200
300
400
500
t


Zoom of early populations
0.150
0.125
0.100
0.075
0.050
0.025
0.000
x_i
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
t

We see that over time, the population emerges to all guessing 1.
So everyone wins.
Note that everyone guessing 0 also is stable.


$x_0^{(0)}/x_j^{(0)} = 650.0\ \forall\ j \neq 0$
1.0
0.8
0.6
0.4
0.2
0.0
x_i
0
200
400
600
800
1000
t
x_0
x_1
x_2
x_3
x_4
x_5
x_6
x_7
x_8
x_9
x_{10}


$x_0^{(0)}/x_j^{(0)} = 660.0\ \forall\ j \neq 0$
1.0
0.8
0.6
0.4
0.2
0.0
x_i
0
200
400
600
800
1000
t
x_0
x_1
x_2
x_3
x_4
x_5
x_6
x_7
x_8
x_9
x_{10}



John Maynard-Smith[2] (1920 - 2004)

[2] J.M. Smith. *The Theory of Evolution*. A Pelican original. Penguin, 1977. ISBN: 9780140204339.

Definition

In a population game when considering a pairwise contest game we assume that individuals are randomly matched and play some game with utility matrices A, A^T. For a population profile x this gives a compact expression for the fitness:

$$f = Ax$$

Definition

In a pairwise interaction game the fitness of a strategy σ in a population x is given by:

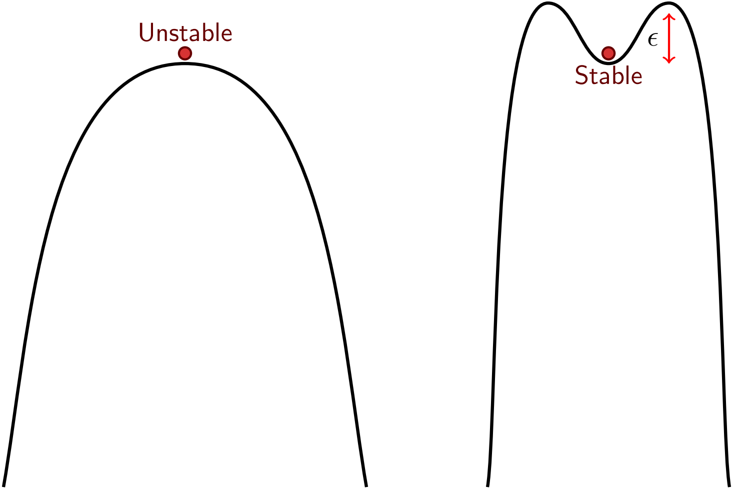
$$u(\sigma, x) = \sum_{i=1}^{|\mathcal{A}|} \sigma_i f_i(x)$$

Definition

A strategy σ^* is called an **Evolutionary Stable Strategy** if there exists an $0 < \bar{\epsilon} < 1$ such that for every $0 < \epsilon < \bar{\epsilon}$ and every $\sigma \neq \sigma^*$ σ^* is:

$$u(\sigma^*, x_\epsilon) > u(\sigma, x_\epsilon)$$

Where x_ϵ is the post entry population where a proportion ϵ of the population are σ.


Unstable
Stable
ϵ

Theorem

If σ^* is an ESS in a pairwise contest population game then for all $\sigma \neq \sigma^*$:
1. $u(\sigma^*, \sigma^*) > u(\sigma, \sigma^*)$ OR 2. $u(\sigma^*, \sigma^*) = u(\sigma, \sigma^*)$ and $u(\sigma^*, \sigma) > u(\sigma, \sigma)$
Conversely, if either (1) or (2) holds for all $\sigma \neq \sigma^*$ in a two player normal form game then σ^* is an ESS.

An evolutionary game theoretic model of rhino horn devaluation[a]

[a]Nikoleta E. Glynatsi, Vincent Knight, and Tamsin E. Lee. "An evolutionary game theoretic model of rhino horn devaluation". In: *Ecological Modelling* 389 (2018), pp. 33–40. ISSN: 0304-3800.

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