Jupyter Notebook Moment Tensor Decomposition Tool

A program to decompose a full moment tensor, plot deviatoric, double-couple and CLVD tensors, and plot the solution in source-type space

Some Reading

Seismological Research Letters, Volume 60, No. 2, April - June, 1989

A Student's Guide to and Review of Moment Tensors

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ABSTRACT

A review of a moment tensor for describing a general seismic point source is presented to show a second order moment tensor can be related to simpler seismic source descriptions such as centers of expansion and double couples. A review of literature is followed by detailed algebraic expansions of the moment tensor into isotropic and deviatoric components. Specific numerical examples are provided in the appendices for use in testing algorithms for moment tensor decomposition.

Geophysical Journal International

Geophys. J. Int. (2012) **190,** 499–510 doi: 10.1111/j.1365-246X.2012.05490.x

A geometric comparison of source-type plots for moment tensors

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Accepted 2012 March 30. Received 2012 March 16; in original form 2011 December 5

SUMMARY

We describe a general geometric framework for thinking about source-type plots for moment tensors. We consider two fundamental examples, one where the source-type plot is on the unit sphere, and one where it is on the unit cube. The plot on the sphere is preferable to the plot on the cube: it is simpler, it embodies a more natural assumption about eigenvalue probabilities and it is more consistent with the conventional Euclidian definition of scalar seismic moment. We describe the source-type plots of Hudson, Pearce and Rogers in our geometric context, and we find that they are equivalent to a plot on the cube. We therefore suggest the plot on the sphere as an alternative.

Key words: Theoretical seismology.

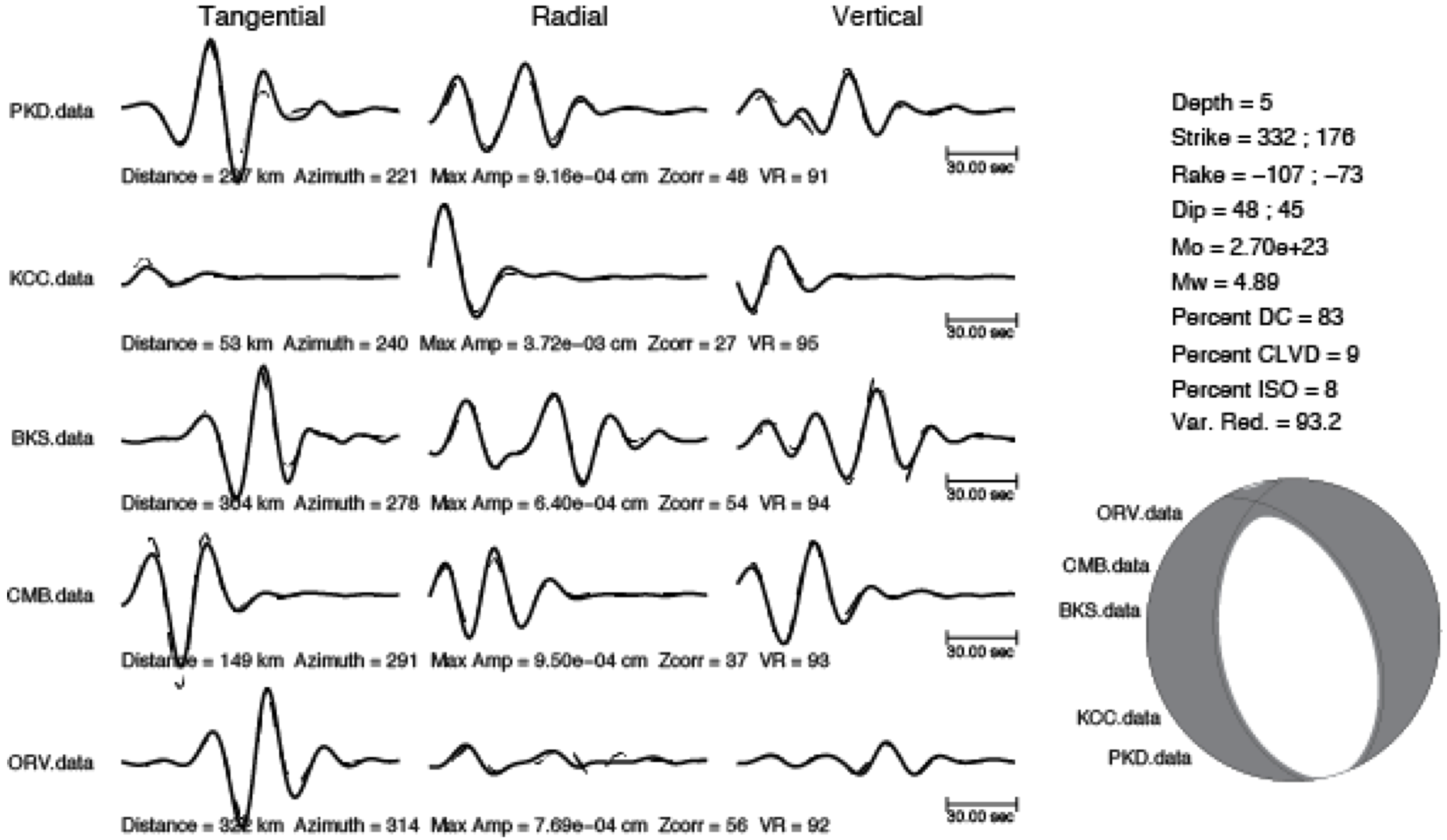
Bulletin of the Seismological Society of America, Vol. 105, No. 6, pp. 2987–3000, December 2015, doi: 10.1785/0120140334

Source-Type-Specific Inversion of Moment Tensors

by Avinash Nayak and Douglas S. Dreger

Abstract The mapping of the fit of seismic moment tensor (MT) solutions in source-type space helps to characterize uncertainty and solution uniqueness. Current practice relies on the forward testing of a distribution of randomly generated MTs in source-type space, which is slow and does not necessarily recover the true maximum fit surface. We design an iterative damped least-squares inversion scheme to invert waveforms and/or *P*-wave first motions for best-fitting MT solutions for specific source types. An event associated with the sinkhole at the Napoleonville salt dome, Louisiana, an industrial quarry explosion, and an earthquake at The Geysers geothermal field, northern California, are presented as examples. We find that the inversion method is more accurate and successful than the random-search approach in recovering the region of best-fitting MT solutions or source types and is substantially faster. The approach also enables the determination of the best-fitting MT for specified source types such as pure double couples, tensile cracks, or explosions, as well as compound mechanisms in a single numerical framework.

Example MT Inversion



Moment Tensor Method

$$U_n(\vec{x},t) = G_{ni,j}(\vec{x},t)M_{ij}$$

M=$(G^T G)^{-1}G^T$U

M_0^{iso}=$\frac{1}{3}(M_{11}+M_{22}+M_{33})$

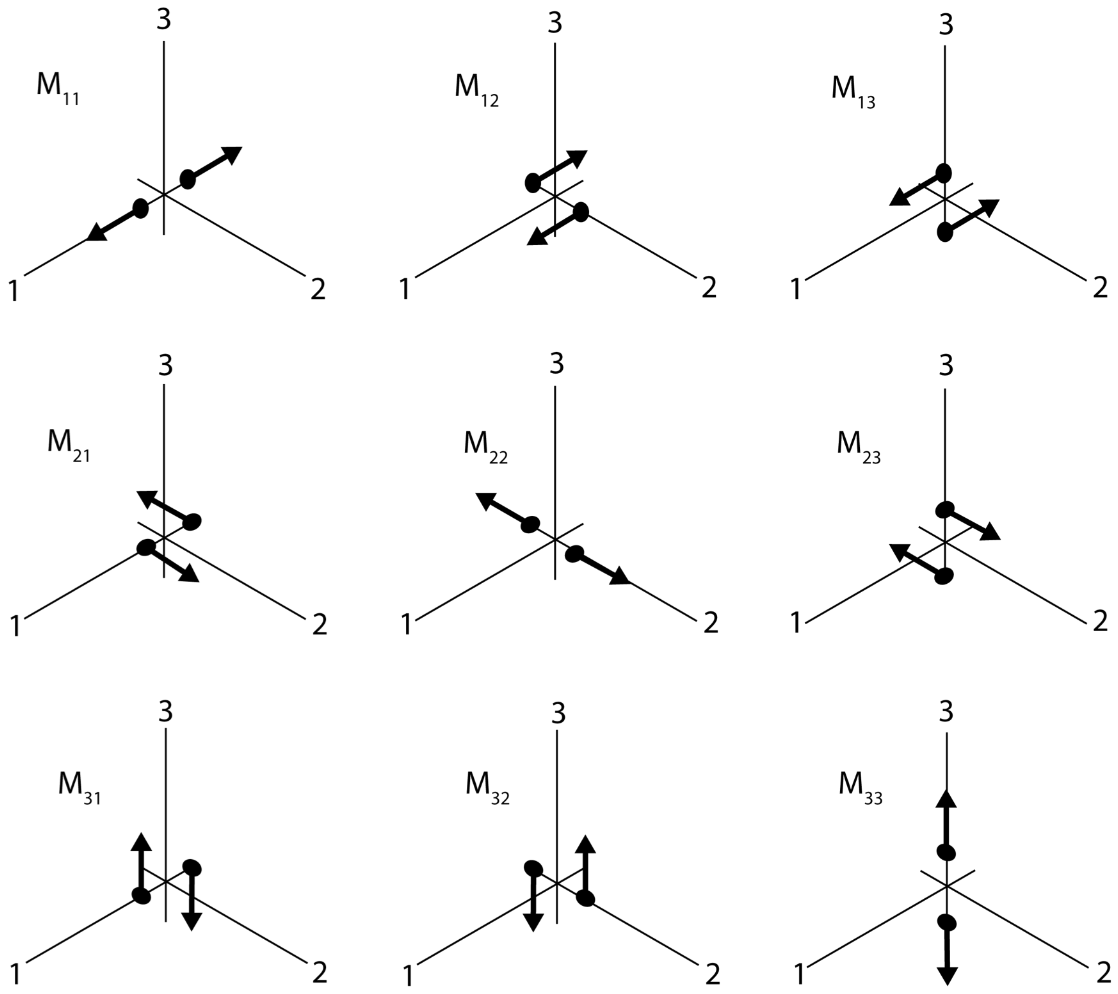
M^{dev}= M - M_0^{iso}I

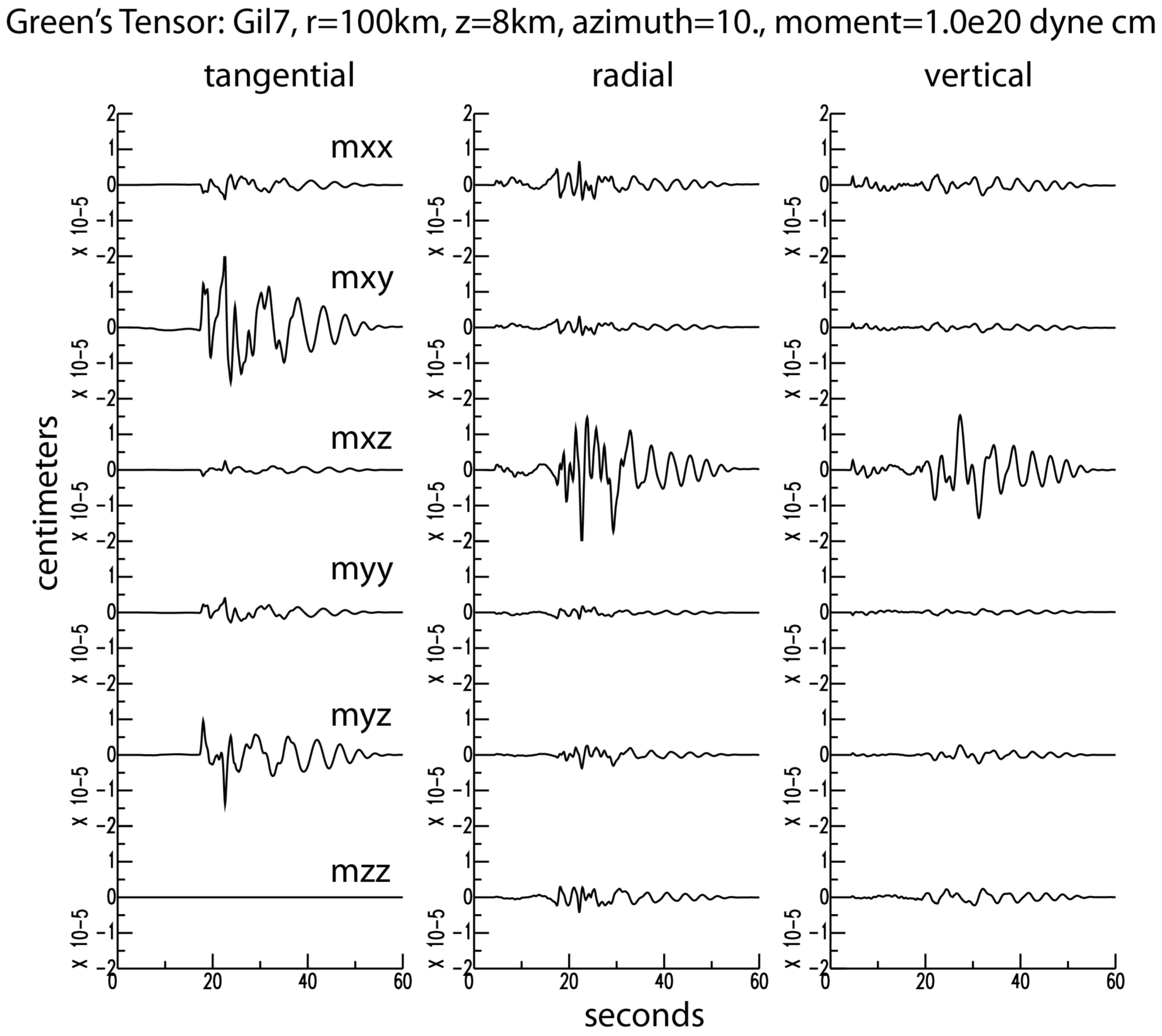
M=M_0^{iso}I + M^{dev}

M=M_0^{iso}I + $\begin{bmatrix} m_1^{dev} & & \\ & m_2^{dev} & \\ & & m_3^{dev} \end{bmatrix}$

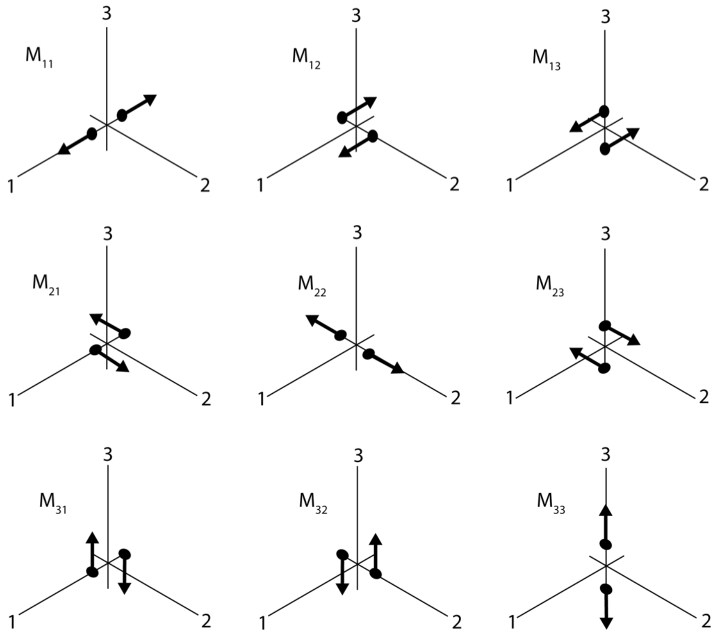
$$|m_1^{dev}| \geq |m_2^{dev}| \geq |m_3^{dev}|$$

$$M_0^{total} = M_0^{iso} + |m_1^{dev}|$$




Green's Tensor: Gil7, r=100km, z=8km, azimuth=10., moment=1.0e20 dyne cm
tangential
radial
vertical
mxx
mxy
mxz
myy
myz
mzz
centimeters
x 10-5
seconds

Moment Tensor Method



$$M = M_0^{iso} I + \begin{bmatrix} m_1^{dev} & & \\ & m_2^{dev} & \\ & & m_3^{dev} \end{bmatrix}$$

isotropic

Double-couple

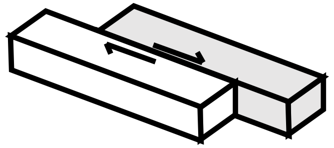
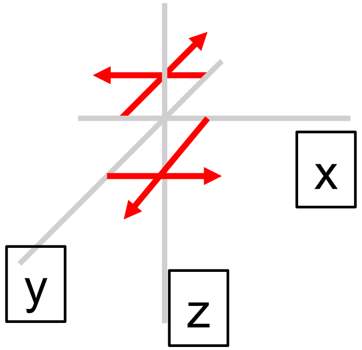
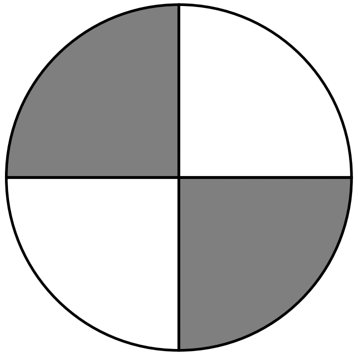
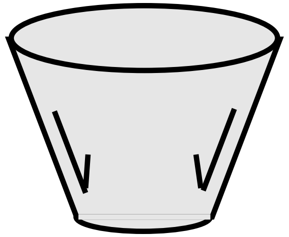
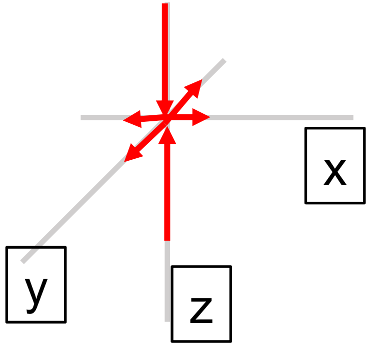
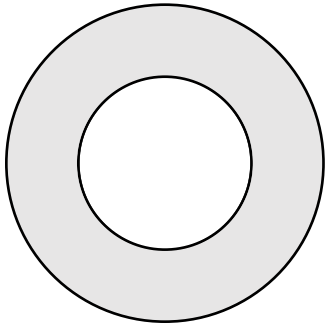
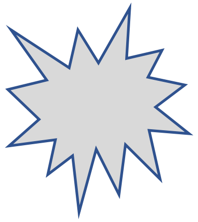
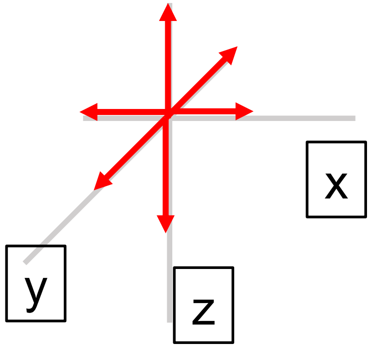
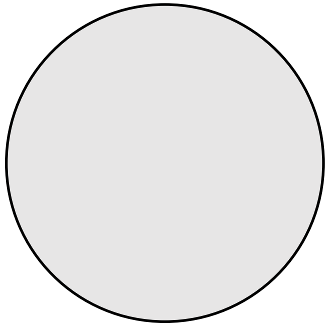
Compensated-Linear-Vector-Dipole (CLVD)

$$M = M_0^{iso} I \quad + . \quad m_1^{dev}\left(1 - 2\frac{-m_3^{dev}}{m_1^{dev}}\right)(a_1 a_1 - a_2 a_2). \quad + \quad m_1^{dev}\frac{-m_3^{dev}}{m_1^{dev}}(2a_1 a_1 - a_2 a_2 - a_3 a_3)$$

a^1 are eigenvectors and $a_1 a_1$ are eigenvector dyadics

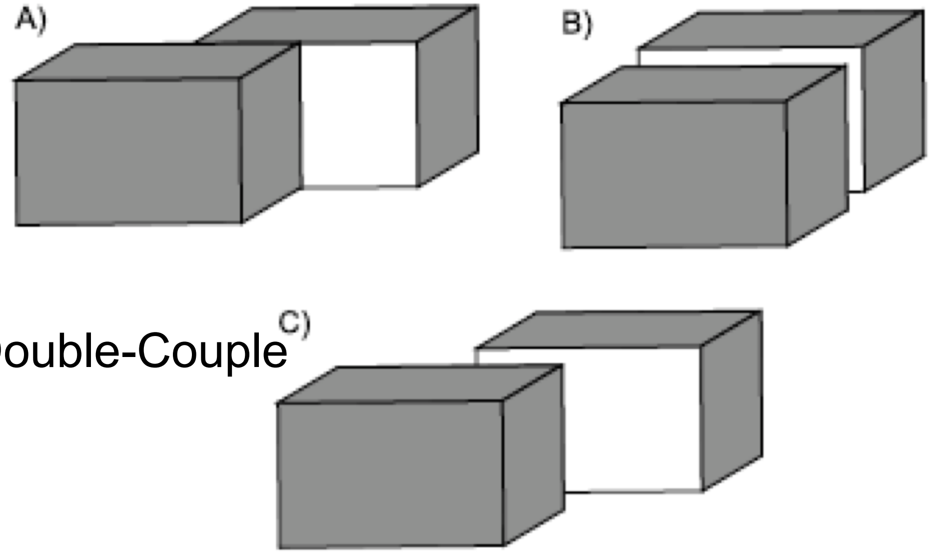
$$a_1 a_1 = \begin{bmatrix} a_x^1 a_x^1 & a_x^1 a_y^1 & a_x^1 a_z^1 \\ a_y^1 a_x^1 & a_y^1 a_y^1 & a_y^1 a_z^1 \\ a_z^1 a_x^1 & a_z^1 a_y^1 & a_z^1 a_z^1 \end{bmatrix}$$

Seismic moment tensors

Model Example	Source Type	M	Couples	Focal Mechanism
 Fault-slip	Double-couple (DC)	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$		
 Ring Fault	Compensated linear vector dipole (CLVD)	$\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$		
 Explosion	Isotropic	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		

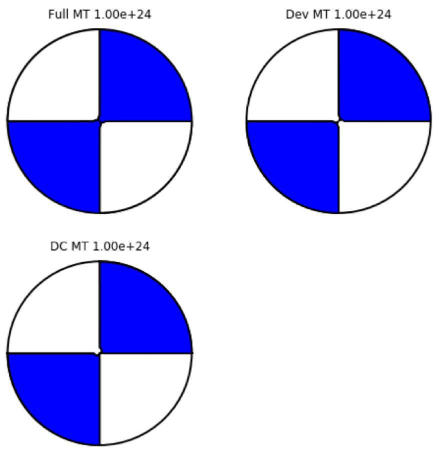
Shear Fault

Tensile Crack

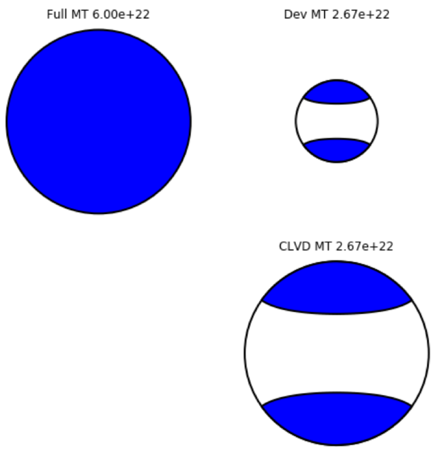


Crack-Double-Couple

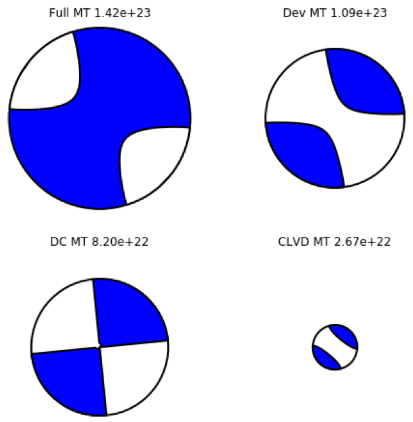
Double-Couple



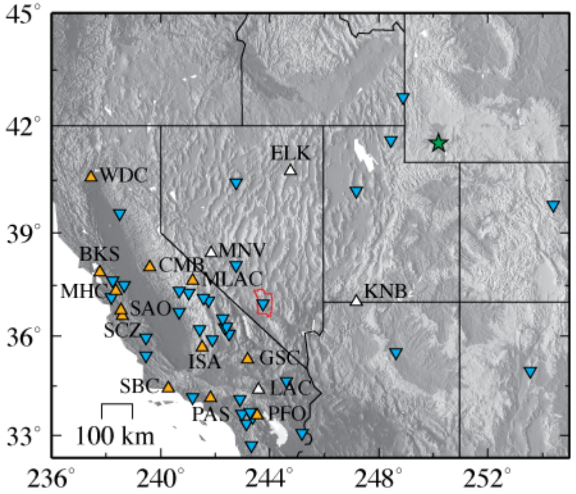
Tensile Crack

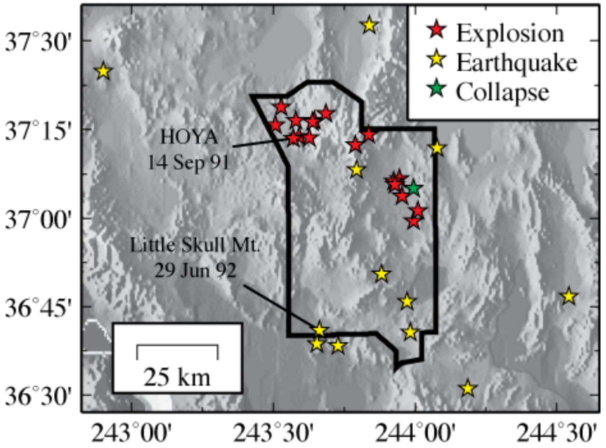


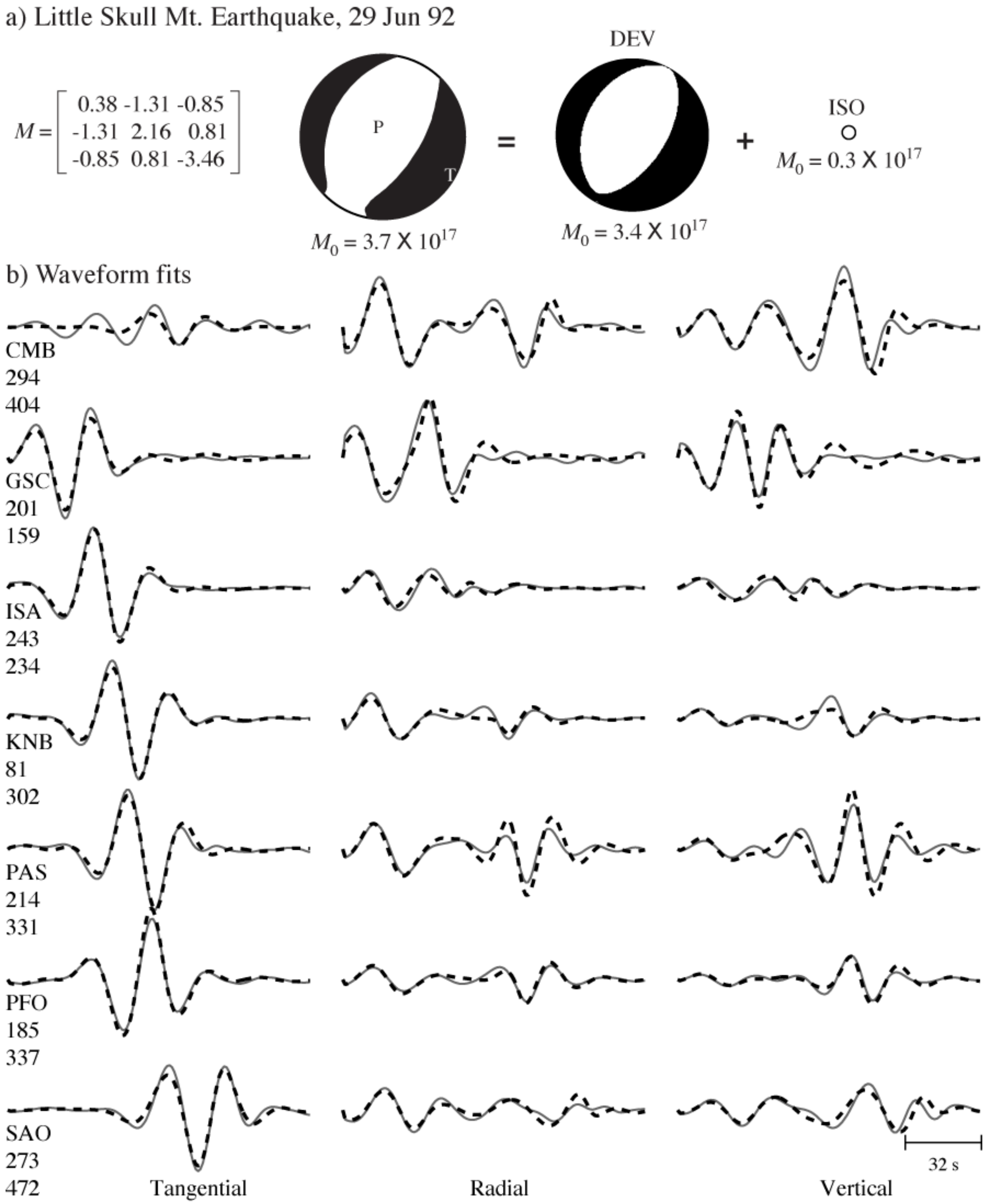
Crack-Double-Couple



Western US

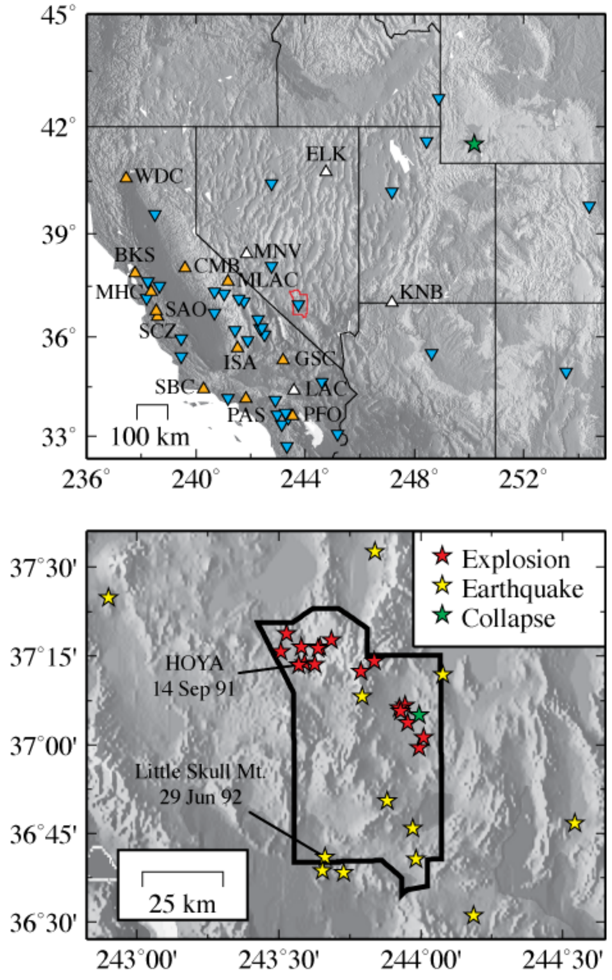


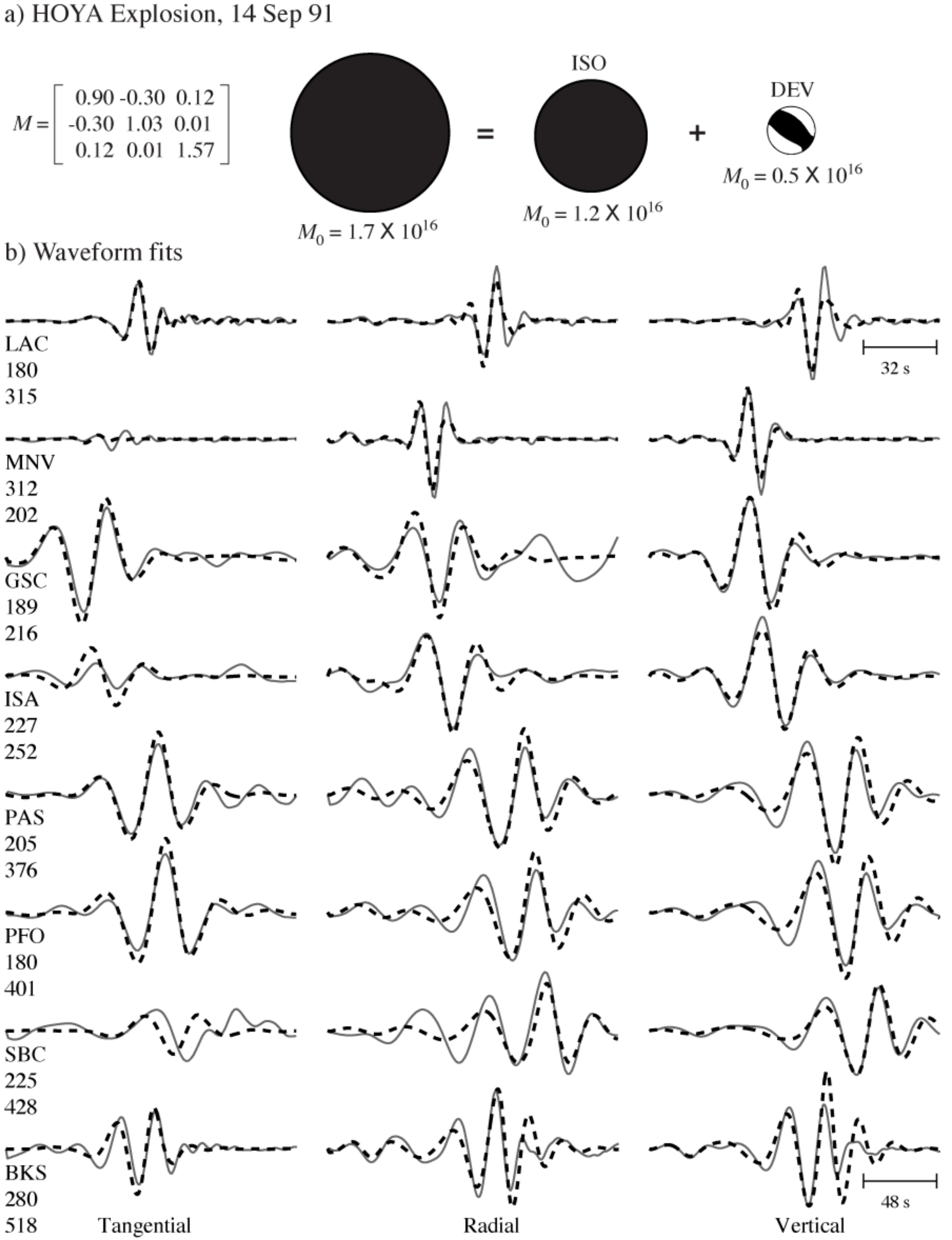




Ford et al., 2009

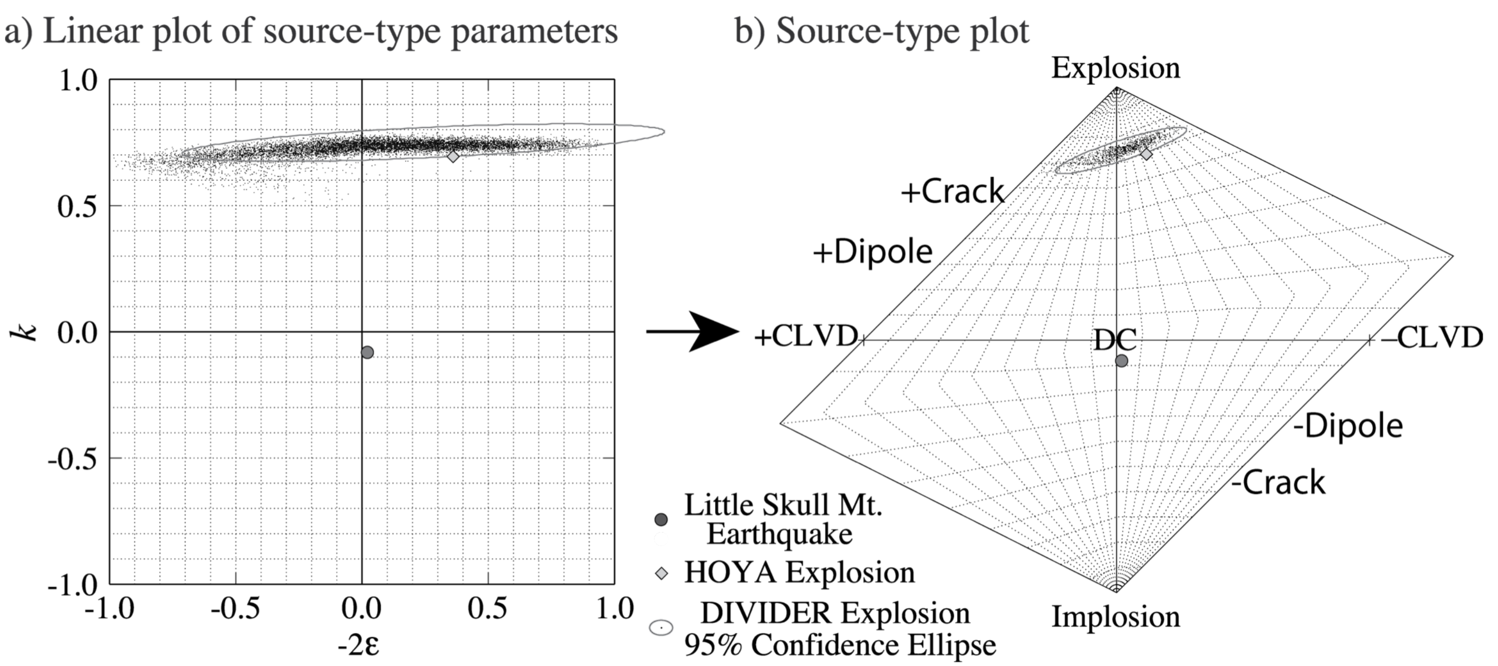
Western US



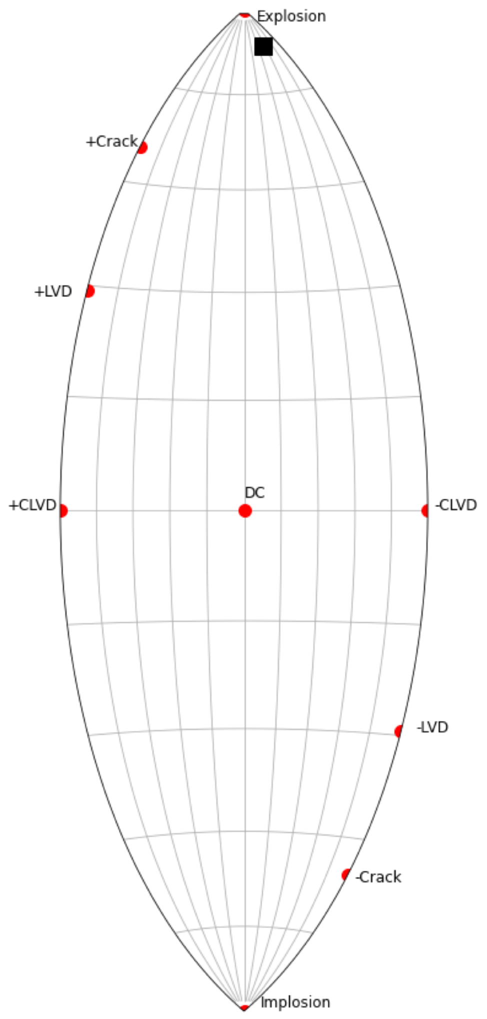


Ford et al., 2009

Hudson Source-Type Plot (Hudson et al., 1989)



c) Tape and Tape (2012) Lune



$$\varepsilon = \frac{-m_3^{dev}}{\left|m_1^{dev}\right|}, \qquad \left|m_1^{dev}\right| \geq \left|m_2^{dev}\right| \geq \left|m_3^{dev}\right| \quad \& \quad m_1^{dev} + m_2^{dev} + m_3^{dev} = 0$$

m are eigenvalues

$$\kappa = \frac{M_0^{iso}}{\left|M_0^{iso}\right| + \left|m_1^{dev}\right|}$$

From Ford et al. (2010)

$$\tan\gamma = \frac{-\lambda_1 + 2\lambda_2 + \lambda_3}{\sqrt{3}(\lambda_1 - \lambda_3)}$$

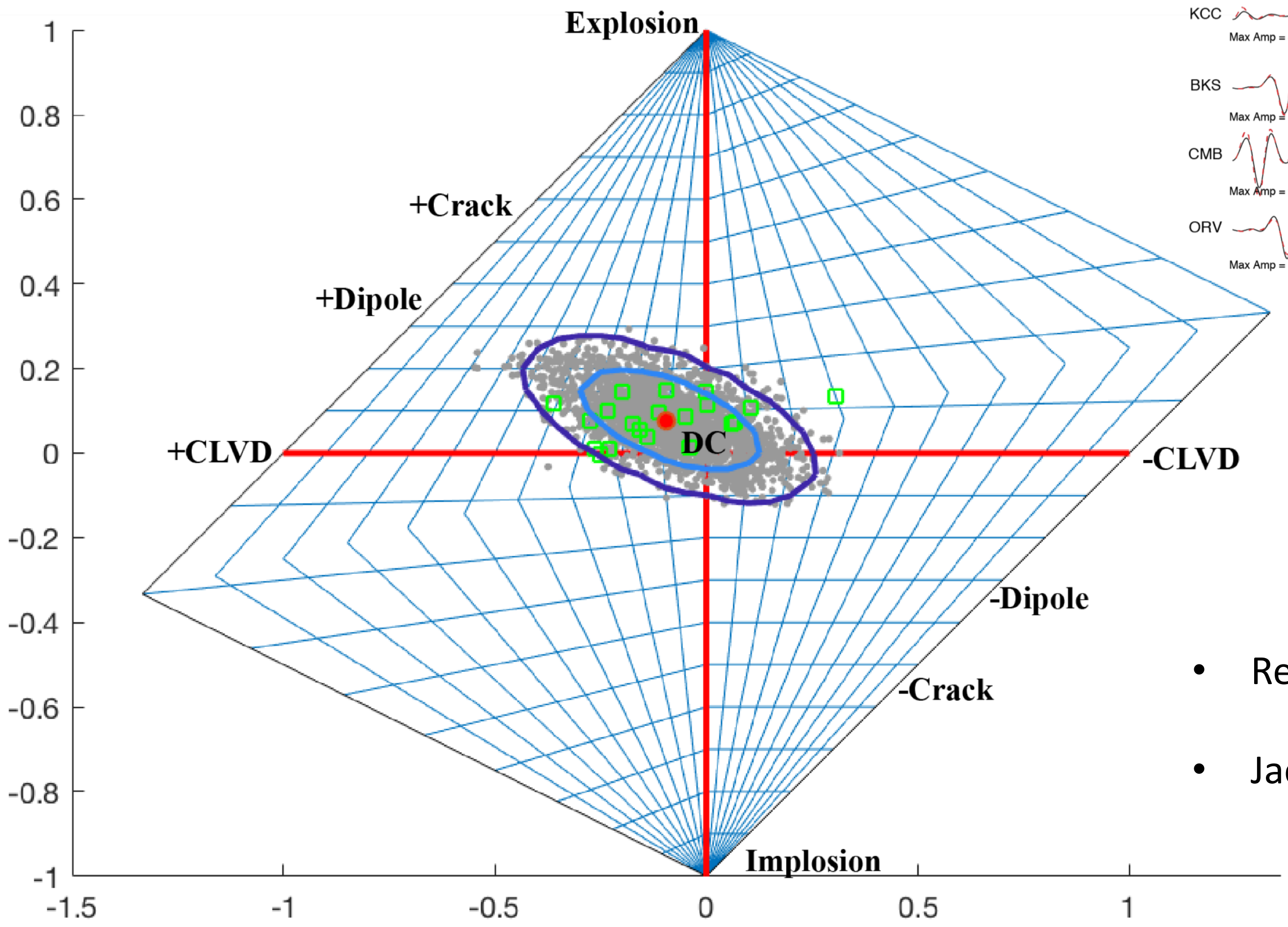
$$\cos\beta = \frac{\lambda_1 + \lambda_2 + \lambda_3}{\sqrt{3(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)}}$$

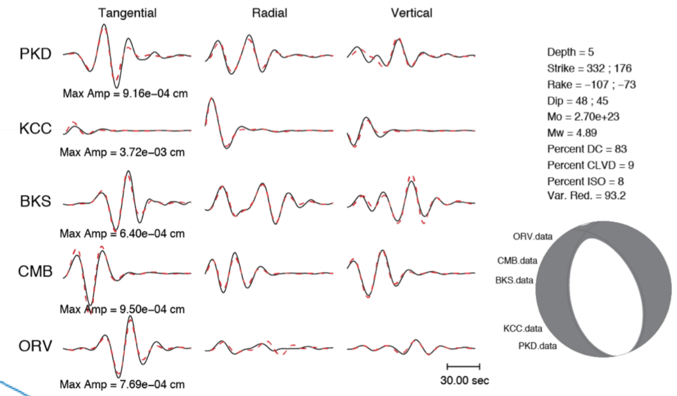
$$\delta = \frac{\pi}{2} - \beta,$$

λ are eigenvalues

γ is longitude, δ is latitude and β is colatitude on a unit sphere

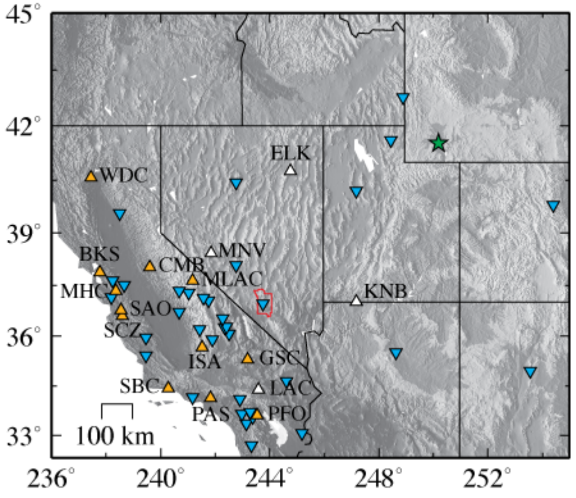
Source-Type Projection
&
Uncertainties

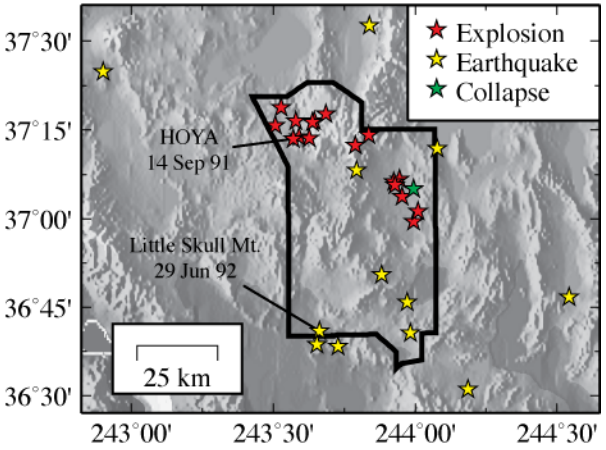


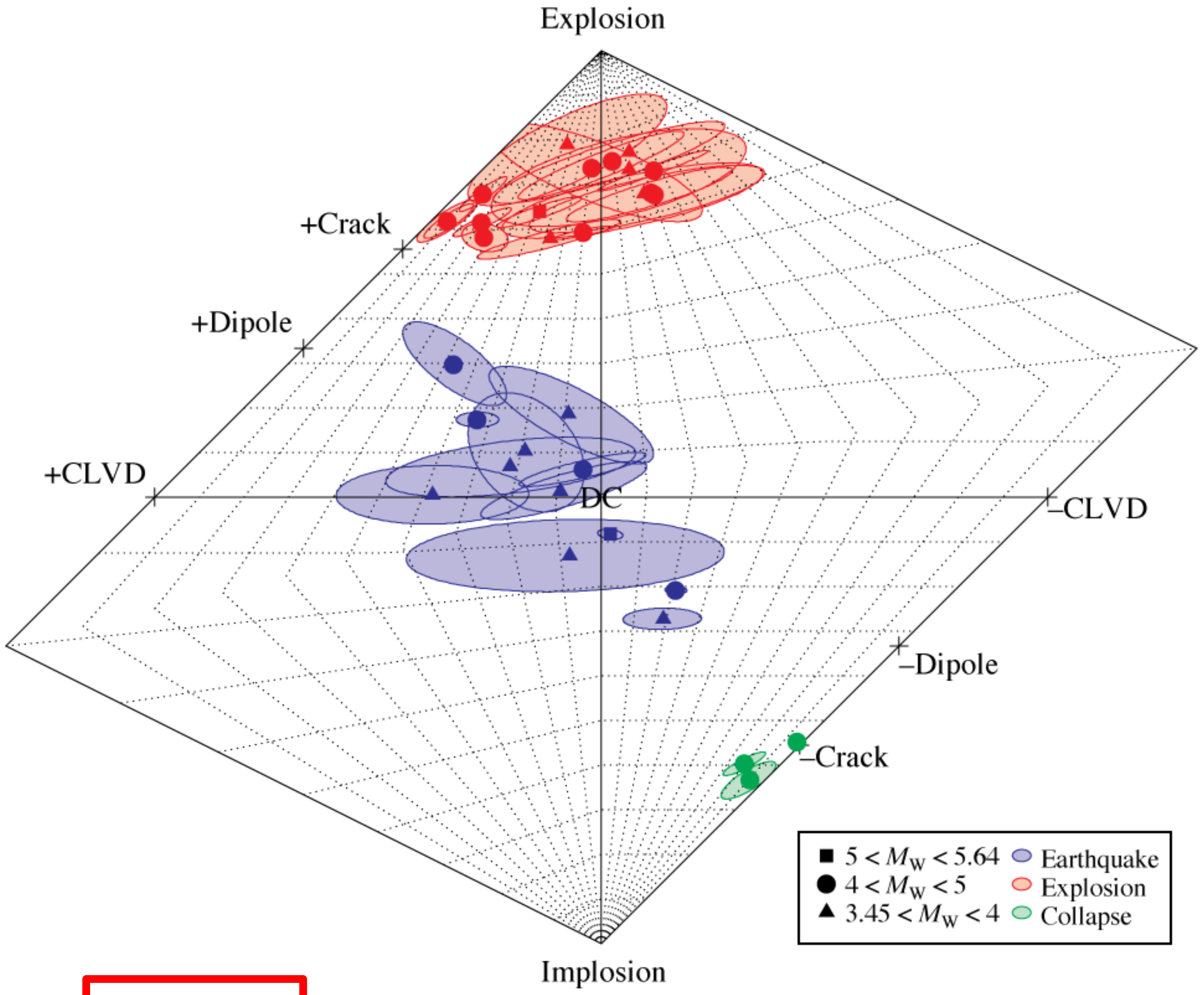


- Residual Bootstrap
- Jackknife 3-Component Stations

Western US







$$\boxed{\varepsilon = \frac{-m_3^{dev}}{\left|m_1^{dev}\right|},}\qquad \left|m_1^{dev}\right| \geq \left|m_2^{dev}\right| \geq \left|m_3^{dev}\right| \quad \& \quad m_1^{dev} + m_2^{dev} + m_3^{dev} = 0$$

$$\boxed{\kappa = \frac{M_0^{iso}}{\left|M_0^{iso}\right| + \left|m_1^{dev}\right|}}$$

Ford et al. (2009)

Jupyter Notebook MT Decomposition Tool

- Enter a moment tensor
- Optionally enter a filename containing a source-type fitting parameter

- Decompose the moment tensor and provide information on the scalar moment, moment magnitude and the isotropic, deviatoric, double-couple and CLVD components
- Graphically plot the moment tensor in source-type space

- Optionally plot a source-type fitting parameter distribution