Problem Decomposition

(Also available for Pyret)

Students take a closer look at how functions can work together by investigating the relationship between revenue, cost, and profit.

Prerequisites	Restating the Problem
Relevant Standards CC-Math	*Select one or more standards from the menu on the left (⌘-click on Mac, Ctrl-click elsewhere).*
Lesson Goals	Students will be able to: • Write a ***function*** that explicitly uses another function. • Explain the benefits and drawbacks of functions that depend on each other. • Explain the difference between bottom-up and top-down strategies.
Student-Facing Lesson Goals	• I can explain the benefits and drawbacks of functions that use other functions. • I can write a function that uses another function.
Materials	• Lesson slides (Google Slides) • Design Recipe: revenue (PDF (Page 38)) • Design Recipe: cost (PDF (Page 39)) • Design Recipe: profit (PDF (Page 40)) Bootstrap Formative Assessments • Bootstrap: Algebra - Coordinates, Circles of Evaluation, & Code (Quizizz) • Bootstrap:Algebra - Data Types & Circles of Evaluation (Desmos Activity) • Bootstrap:Algebra - Circles of Evaluation Review(Blank Template) (Desmos Activity) • Bootstrap:Algebra - Contracts, Domain/Range, Data Types, & Functions (Quizizz) • Bootstrap:Algebra - Data Types, Circles of Evaluation, and Contracts (Desmos Activity)
Preparation	• Make sure all materials have been gathered • Decide how students will be grouped in pairs
Supplemental Resources	• Function Composition Dynamic Illustrator I (Geogebra) • Composition of Functions (Geogebra Quiz) • Composition of Functions 2 (Quizizz)
Key Points for the Facilitator	• There are several ways to write the `profit` function - use this opportunity for discussion and to promote higher-order critical thinking. • If students are struggling with understanding the basics of the problem, start by coming up with examples of `cost` and `revenue` . If Sally sells one glass, what is her total revenue? How much does it cost her to produce that one glass? • Ensure students understand the difference between "revenue" and "profit", and that "cost" refers to the cost of *making* the lemonade, not the amount Sally is charging.

Glossary

function :: a mathematical object that consumes inputs and produces an output

Warmup

Students should have their workbook, pencil, and be logged into WeScheme and have their workbooks with a pen or pencil.

Problem Decomposition

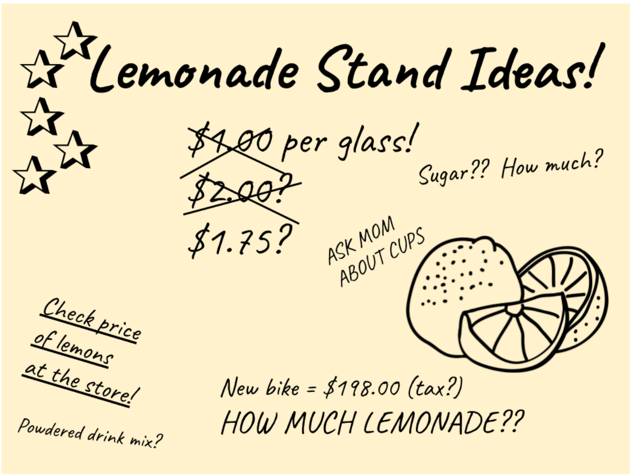
30 minutes

Overview

Students are introduced to word problems that can be broken down into *multiple* problems, the solutions to which can be composed to solve other problems. They adapt the Design Recipe to handle this situation.

Launch

Display the following image:



Notice and Wonder

What do you notice? What do you wonder?

One example of a *relationship* we can find in this situation is that Sally takes in $1.75 for every glass she sells: $revenue = \$1.75 \times glasses$

What other relationships can you find here?

(Give students a chance to discuss and brainstorm)

- Every glass sold brings in $1.75 in **revenue**
- Every glass sold costs $0.30 in **costs**, such as lemonds, sugar and water
- Every glass sold brings in some amount of **profit**: it costs a certain amount to make, but it brings in another amount in revenue

Investigate

Students form groups and brainstorm their ideas for functions. Students can use any strategies they've learned so far.

> **Strategies for English Language Learners**
>
> MLR 7 - Compare and Connect There are several correct ways to write the functions needed for Sally's Lemonade. Have students compare methods and develop understanding and language related to mathematical representation and methods. What are the advantages of the different solutions? What are some drawbacks?

- **What is the difference between *revenue* and *profit*?** *Revenue is the total amount of money that comes in, profit is the remaining money after cost has been subtracted.*
- **How could Sally *increase* her profits?** *By decreasing her costs, raising her prices (which increases revenue), by selling more lemonade.*
- **What is the *relationship* between profit, cost, and revenue?** *Profit = Revenue - Cost*

Students work with their partners to develop their function models for revenue (Page 38), cost (Page 39), and profit (Page 40), using the Design Recipe.

While students are working, walk the room and gauge student understanding. There is more than one correct way to write the `profit` function! Encourage discussion between students and push students to develop their thinking on the advantages and disadvantages of each correct solution.

Synthesis

This activity started with a situation, and students modeled that situation with functions. One part of the model was *profit*, which can be written several ways, for example:

```
(define (profit g) (- (* 1.75 g) (* 0.30 g)))
(define (profit g) (* (- 1.75 0.30) g))
(define (profit g) (* 1.45 g))
(define (profit g) (- (revenue g) (cost g)))
```

- Which way is "best", and why?
- If lemons gets more expensive, which way requires the least amount of change?
- If sugar gets less expensive, which way requires the least amount of change?

Big Ideas

1. `profit` can be *decomposed* into a simple function that uses the `cost` and `revenue` functions.
2. Decomposing a problem allows us to solve it in smaller pieces, which are also easier to test!
3. These pieces can also be re-used, resulting in writing less code, and less *duplicate* code.
4. Duplicate code means more places to make mistakes, especially when that code needs to be changed.

Top-Down vs. Bottom-Up 20 minutes

Overview

Students explore problem decomposition as an explicit strategy, and learn about two ways of decomposing.

Launch

Top-Down and *Bottom-Up* design are two different strategies for problem decomposition.

When thinking Bottom-Up, we start with the small, easier relationships first and then build our way to the larger relationships. In the Lemond Stand example, we had you write the lower-level functions - `cost` and `revenue` - *first*, and then gave you the chance to use them in the higher-level `profit` function. This is called **Bottom-Up** design. When thinking Top-Down, we start with the "big picture" and then worry about the details later. For example, we could have *started* with `profit`, and kept track of all the lower-level functions we would need to write. This is called **Top-Down** design.

Investigate

Consider the following situation:

Jamal's trip to Thailand requires him to drive 20 miles to the airport, take a plane 9,000 miles to Thailand, and then a bus 6 miles to his hotel. The average speed when driving to the airport is 40mph, the average speed of an airplane is 575mph, and the average speed of his shuttle bus is 15mph

Setting aside time spent waiting at the airport or for the bus, how long is Jamal in transit?

This problem can be decomposed in Top-Down or Bottom-Up fashion. Describe what your steps would be in each solution (for extra credit, you can actually compute the answer!).

Synthesize

Make sure that students see *both* strategies, and have them discuss which they prefer and why.