String matching

Data Structures and Algorithms for Computational Linguistics III (ISCL-BA-07)

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University of Tübingen Seminar für Sprachwissenschaft

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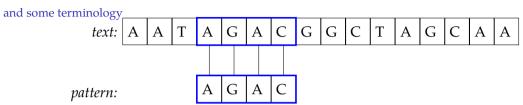
Finding patterns in a string

- Finding a pattern in a larger text is a common problem in many applications
- Typical example is searching in a text editor or word processor
- There are many more:
 - DNA sequencing / bioinformatics
 - Plagiarism detection
 - Search engines / information retrieval
 - Spell checking
 - ..

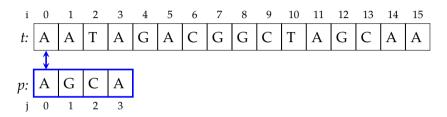
Types of problems

- The efficiency and usability of algorithms depend on some properties of the problem
- Typical applications are based on finding multiple occurrences of a single pattern in a text, where the pattern is much shorter than the text
- The efficiency of the algorithms may depend on the
 - relative size of the pattern
 - expected number of repetitions
 - size of the alphabet
 - whether the pattern is used once or many times
- Another related problem is searching for multiple patterns at once
- In some cases, fuzzy / approximate search may be required
- In some applications, preprocessing (indexing) the text to be searched may be beneficial

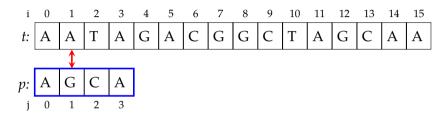
Problem definition



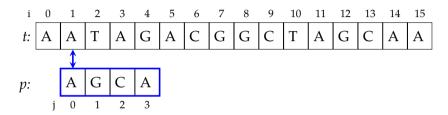
- We want to find all occurrences of pattern p (length m) in text t (length n)
- The characters in both t and p are from an alphabet Σ , in the example $\Sigma = \{A, C, G, T\}$
- The size of the alphabet (q) is often an important factor
- p occurs in t with shift s if p[0:m] == t[s:s+m], we have a match at s=3 in the example
- A string x is a prefix of string y, if y = xw for a possibly empty string w
- A string x is a suffix of string y, if y = wx for a possibly empty string w



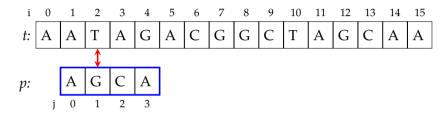
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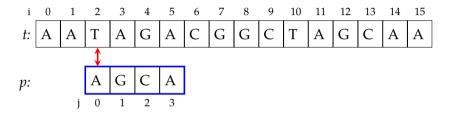
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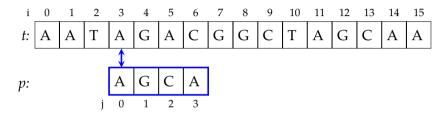
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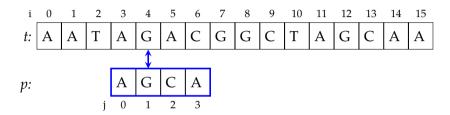
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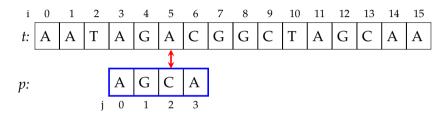
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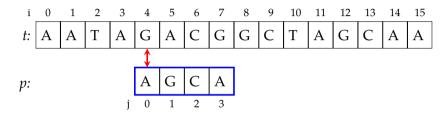
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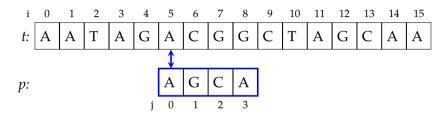
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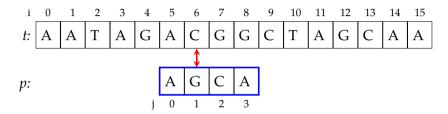
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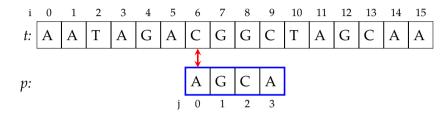
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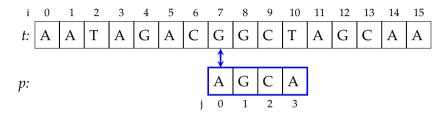
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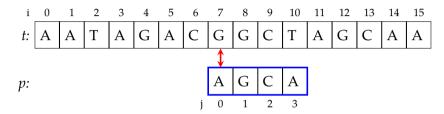
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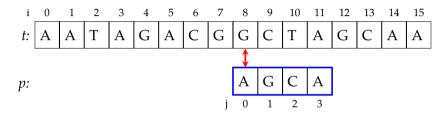
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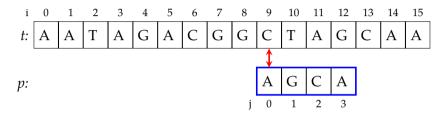
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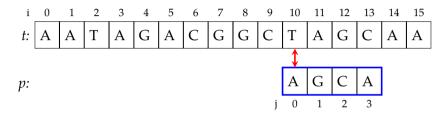
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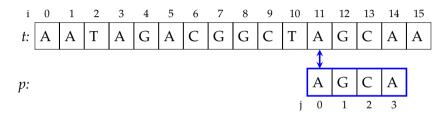
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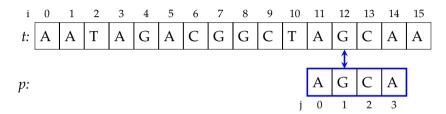
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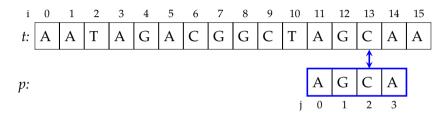
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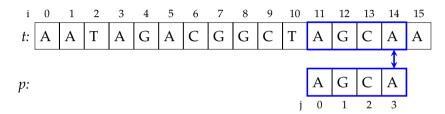
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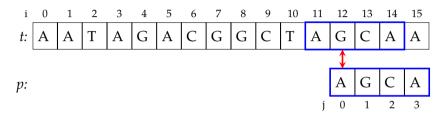
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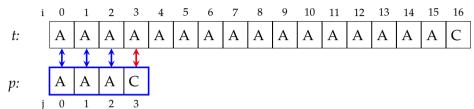
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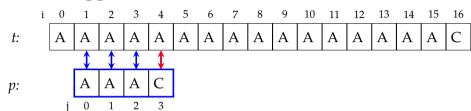


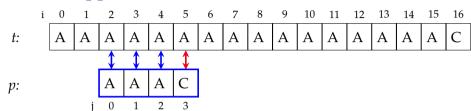
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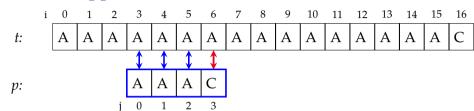


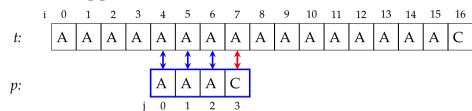
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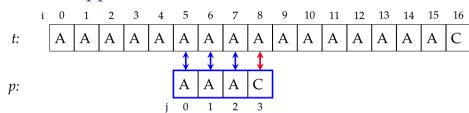


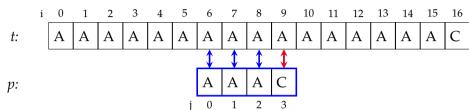


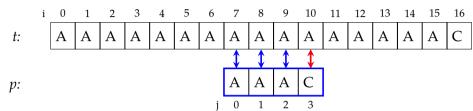


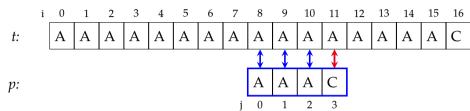


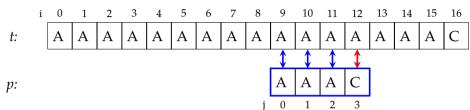


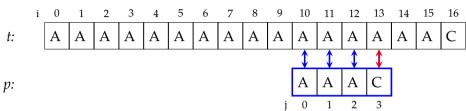


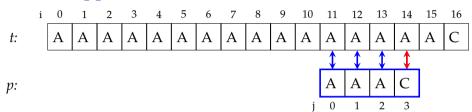


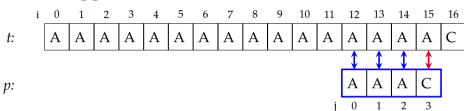


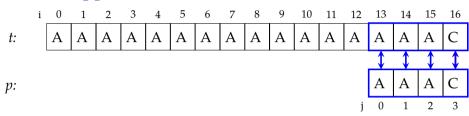




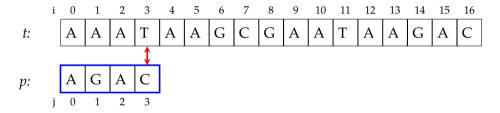




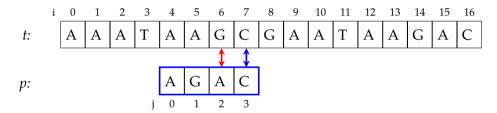




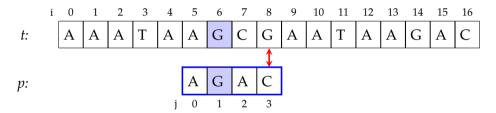
- Worst-case complexity of the method is O(nm)
- Crucially, most of the comparisons are redundant
 - for i>0 and any comparison with j=0,1,2, we already inspected corresponding i values
- The main idea for more advanced algorithms is to avoid this unnecessary comparisons



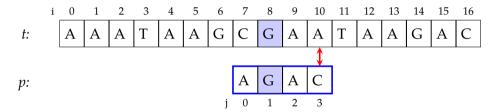
- The main idea is to start comparing from the end of p
- If t[i] does not occur in p, shift m steps
- Otherwise, align the last occurrence of t[i] in p with t[i]



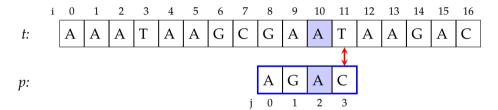
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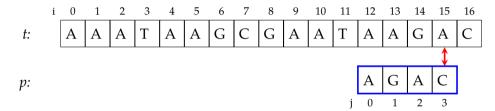
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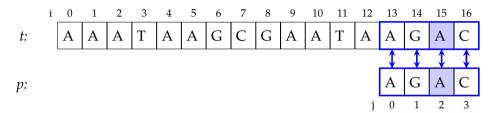
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implementation and analysis

- On average, the algorithm performs better than brute-force
- In worst case the complexity of the algorithm is O(nm), example:

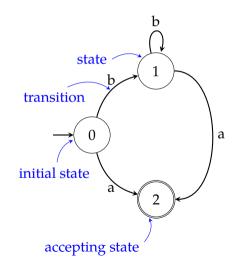
```
t=aaa\dots a,p=baa\dots a
```

• Faster versions exist (O(n + m + q))

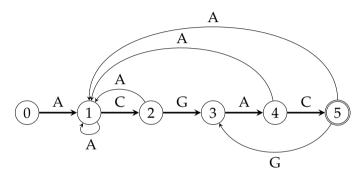
```
last = {}
for j in range(m):
 last[P[j]] = j
i, j = m-1, m-1
while i < n:
  if T[i] == P[i]:
    if j == 0:
      return i
    else:
      i -= 1
      i -= 1
  else:
    k = last.get(T[i], -1)
    i += m + min(j, k+1)
    i = m - 1
return None
```

A quick introduction to FSA

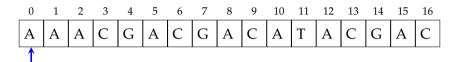
- Another efficient way to search a string is building a finite state automaton for the pattern
- An FSA is a directed graph where edges have labels
- One of the states is the *initial state*
- Some states are accepting states
- We will study FSA more in-depth soon

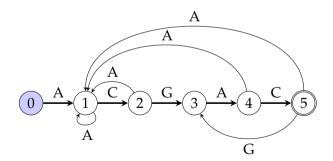


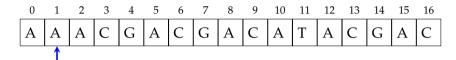
An FSA for the pattern ACGAC

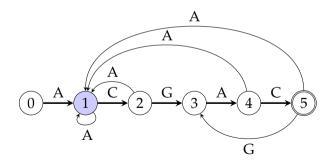


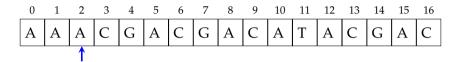
- Start at state 0, switch states based on the input
- All unspecified transitions go to state 0
- When at the accepting state, announce success

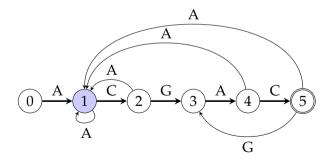


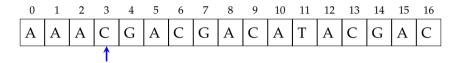


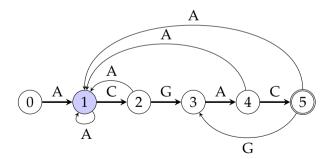


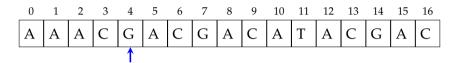


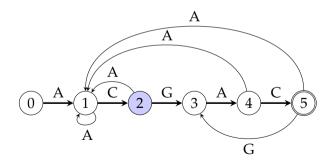


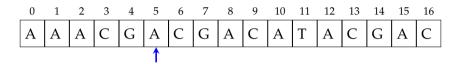


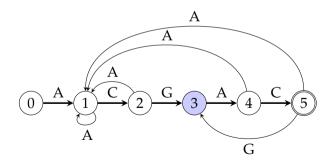


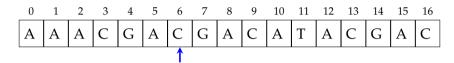


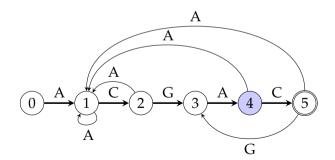


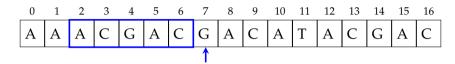


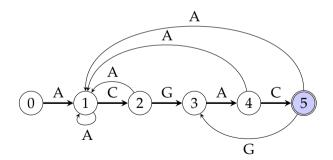


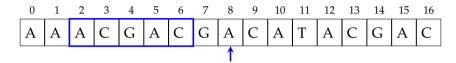


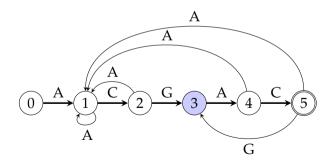


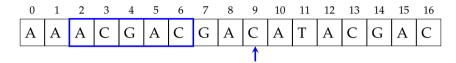


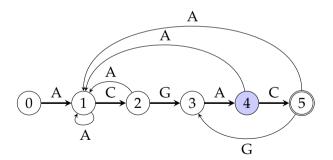


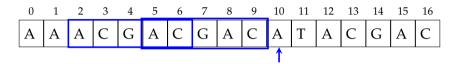


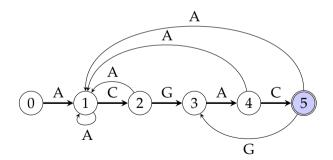


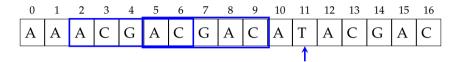


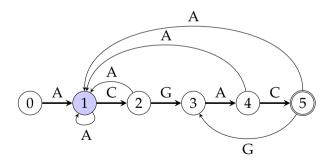


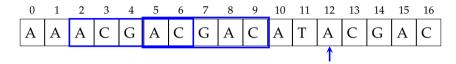


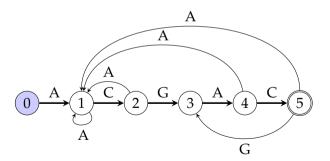


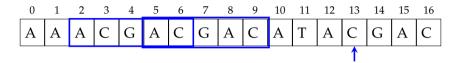


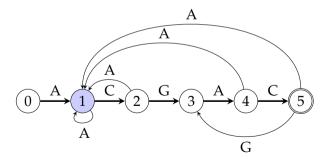


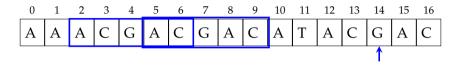


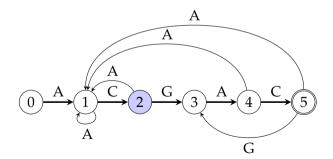


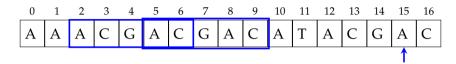


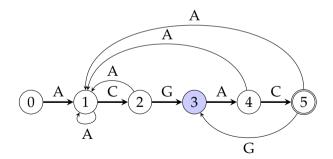


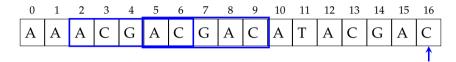


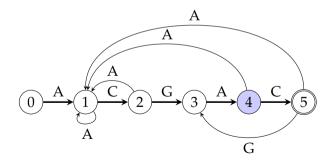




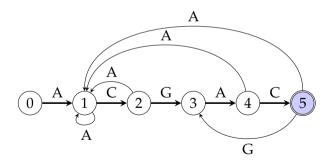








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FSA for string matching

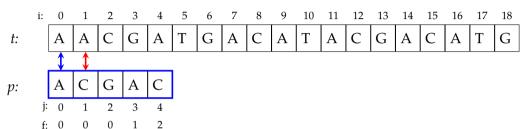
how to build the automaton

- An FSA results in O(n) time matching, however, we need to first build the automaton
- At any state of the automaton, we want to know which state to go for the failing matches
- Given substring s recognized by a state and a non-matching input symbol a, we want to find the longest prefix of s such that it is also a suffix of sa
- A naive attempt results in $O(qm^3)$ time for building the automaton (where q is the size of the alphabet m is the length of the pattern)
- If stored in a matrix, the space requirement is $O(m^2)$
- Better (faster) algorithms exist for construction these automaton (we will cover some later in this course)

Knuth-Morris-Pratt (KMP) algorithm

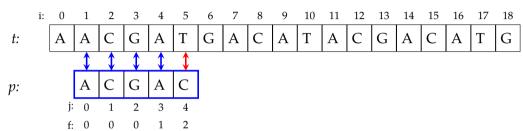
- The KMP algorithm is probably the most popular algorithm for string matching
- The idea is similar to the FSA approach: on failure, continue comparing from the longest matched prefix so far
- However, we rely on a simpler data structure (a function/table that tells us where to back up)
- Construction of the table is also faster

KMP algorithm

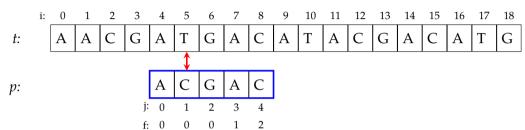


- In case of a match, increment both i and j
- On failure, or at the end of the pattern, decide which new p[j] compare with t[i] based on a function f
- f[j-1] tells which j value to resume the comparisons from

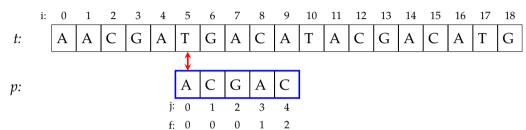
KMP algorithm



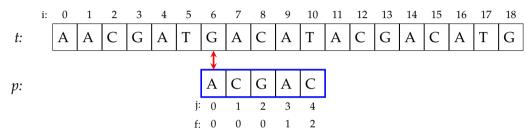
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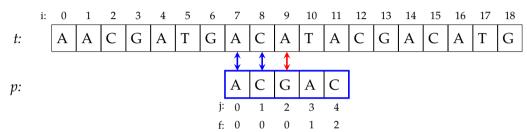
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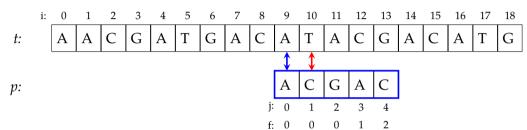
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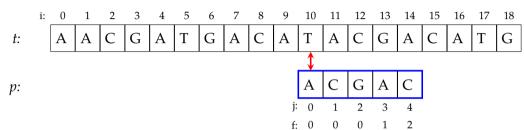
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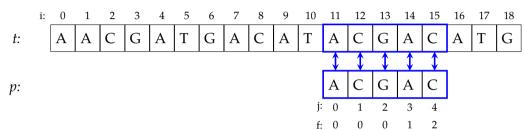
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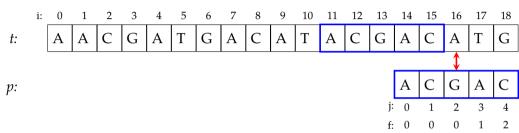
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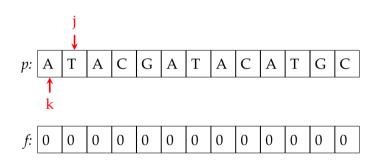
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Complexity of the KMP algorithm

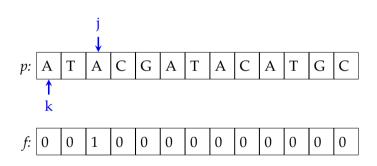
- In the while loop, we either increase i, or shift the comparison
- As a result, the loop runs at most 2n times, complexity is O(n)

```
i, j = 0, 0
while i < n:
  if T[i] == P[j]:
    if j == m - 1:
      return i - m + 1
    else:
      i += 1
      i += 1
  elif j > 0:
    j = fail[k - 1]
  else:
    i += 1
return None
```

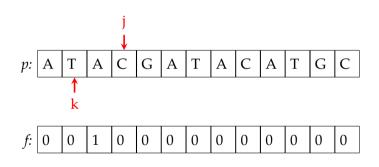
```
f = [0] * m
j, k = 1, 0
while j < m:
    if P[j] == P[k]:
        f[j] = k + 1
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        k += 1
elif k > 0:
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```



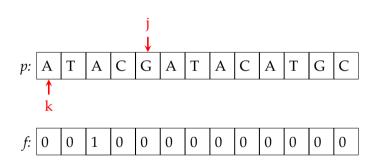
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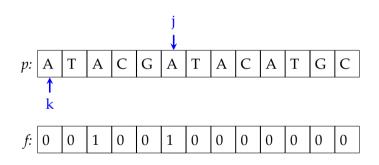
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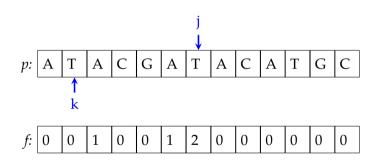
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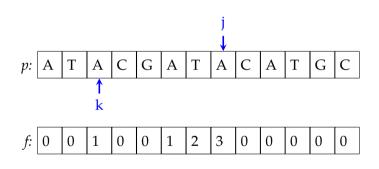
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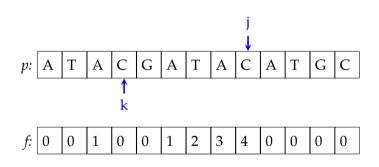
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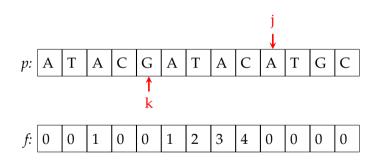
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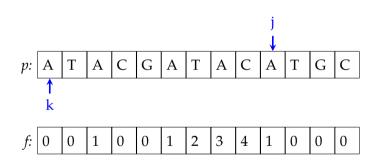
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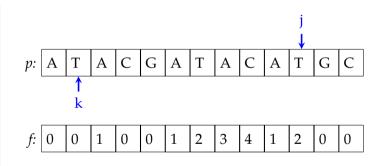
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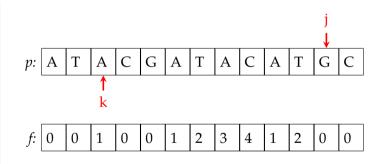
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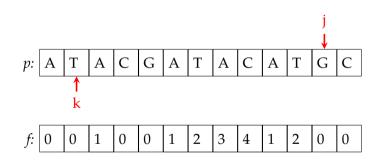
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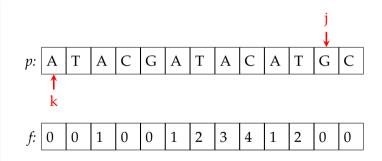
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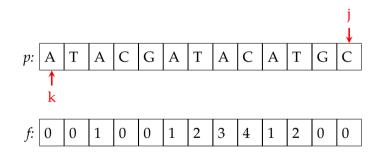
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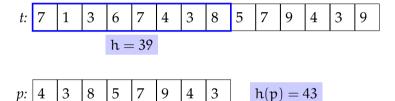


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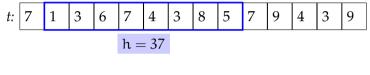


Rabin-Karp algorithm

- Rabin-Karp string matching algorithm is another interesting algorithm
- The idea is instead of matching the string itself, matching the hash of it (based on a hash function)
- If a match found, we need to verify the match may be because of a hash collision
- Otherwise, the algorithm makes a single comparison for each position in the text
- However, a hash should be computed for each position (with size m)
- Rolling hash functions avoid this complication

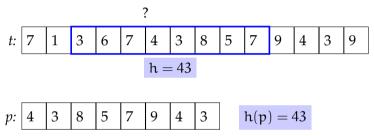


- A rolling hash function changes the hash value only based on the item coming in and going out of the window
- To reduce collisions, better rolling-hash functions (e.g., polynomial hash functions) can also be used

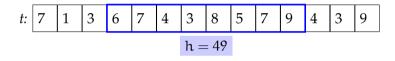


p:	4	3	8	5	7	9	4	3		h(p) = 43
•							l .		I	

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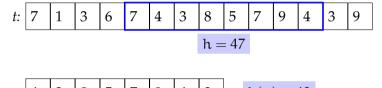


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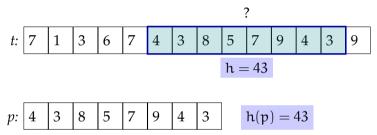
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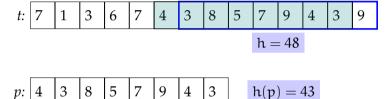


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Summary

- String matching is an important problem with wide range of applications
- The choice of algorithm largely depends on the problem
- We will revisit the problem on regular expressions and finite-state automata
- Reading: Goodrich, Tamassia, and Goldwasser (2013, chapter 13)

Next:

- Algorithms on strings: edit distance / alignment
- Reading: Goodrich, Tamassia, and Goldwasser (2013, chapter 13), Jurafsky and Martin (2009, section 3.11, or 2.5 in online draft)

Acknowledgments, credits, references

- Goodrich, Michael T., Roberto Tamassia, and Michael H. Goldwasser (2013). Data Structures and Algorithms in Python. John Wiley & Sons, Incorporated. ISBN: 9781118476734.
- Jurafsky, Daniel and James H. Martin (2009). Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition. second edition. Pearson Prentice Hall. ISBN: 978-0-13-504196-3.

A.4