1) $$\frac{P(\text{alice} \mid \text{data})}{P(\text{bob} \mid \text{data})} = \frac{P(\text{alice})}{P(\text{bob})} \times \frac{P(\text{data} \mid \text{alice})}{P(\text{data} \mid \text{bob})}$$

$$\frac{P(\text{data} \mid \text{alice})}{P(\text{data} \mid \text{bob})} = \frac{\left(e^{-10} \times \frac{10^{12}}{12!}\right) \times \left(e^{-10} \times \frac{10^{10}}{10!}\right) \times \cdots}{\left(e^{-15} \times \frac{15^{12}}{12!}\right) \times \left(e^{-15} \times \frac{15^{10}}{10!}\right) \times \cdots}$$

$$= \left(\frac{2}{3}\right)^{12} \times \left(\frac{2}{3}\right)^{10} \times \left(\frac{2}{3}\right)^{11} \cdots\cdots \times e^{5 \times 5} = 254.08$$

$$\frac{P(\text{alice} \mid \text{data})}{P(\text{bob} \mid \text{data})} = 10 \times 254.08$$

& $P(\text{alice} \mid \text{data}) + P(\text{bob} \mid \text{data}) = 1$

$$\therefore P(\text{alice} \mid \text{data}) = \frac{1}{1 + \frac{1}{2541}} \approx 1$$

2a) $\theta \sim N(5, 9) \qquad X \sim N(\theta, 4)$

$P(X=6) \times P(\theta = x \mid x=6) = P(X=6 \mid \theta = x) \times P(\theta = x)$

difficult to calculate
∴ will ignore by normalization
as it won't be fxn of x

$\rightarrow$ constant $\times\ P(\theta = x \mid x=6) = \frac{1}{2\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{6-x}{2}\right)^2} \times \frac{1}{3\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-5}{3}\right)^2}$

$$\rightarrow P(\theta = x \mid x=6) = \text{constant} \times \exp\left(-\frac{1}{2}\left[\frac{1}{36}(13x^2 - 148x + 424)\right]\right)$$

$$\rightarrow P(\theta = x \mid X=6) = \text{constant} \times \exp\left(-\frac{1}{2}\left[\frac{x - \frac{74}{13}}{\sqrt{\frac{36}{13}}}\right]^2\right)$$

$$\therefore P(\theta = x \mid X=6) \sim N\left(\frac{74}{13}, \frac{36}{13}\right)$$

b) $\bar{X} = \dfrac{X_1 + X_2 + X_3 + X_4}{4} \sim N\left(\dfrac{x_1 + x_2 + x_3 + x_4}{4}, \dfrac{4 \times 4}{4^2}\right) = N(6, 1)$

Again: $P(\bar{X} = 6) \times P(\theta = x \mid \bar{X} = 6) = P(\bar{X} = 6 \mid \theta = x) \times P(\theta = x)$

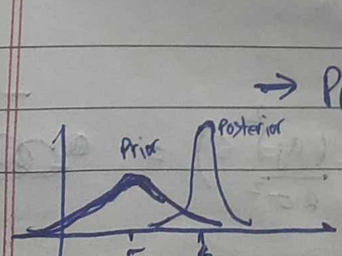
$\rightarrow$ constant $\times\ P(\theta = x \mid \bar{x} = 6) = \frac{1}{\sqrt{2\pi}} \times \exp\left(-\frac{1}{2}(6-x)^2\right) \times \frac{1}{3\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-5}{3}\right)^2\right)$

$\times$ constant

$\cdot\ (\ldots)$

$$P(\theta = x \mid \bar{x} = 6) = \text{constant} \times \exp\left(-\frac{1}{2}\left[\frac{x - \frac{59}{10}}{\sqrt{\frac{9}{10}}}\right]^2\right)$$

$$\rightarrow P(\theta = x \mid \bar{x} = 6) \sim N(5.9, 0.9)$$



True value is probably closer to 6

c) Variance ↓ as uncertainty of predictions ↓ ~~Mean incr~~

Mean increases and goes closer to 6 and variance keeps decreasing as the same signal is repeated

d) i) Prior : $N(100, 15^2)$

$$\mu_{post} = \frac{80 \times \frac{1}{10^2} + 100 \times \frac{1}{15^2}}{\frac{1}{10^2} + \frac{1}{15^2}} = 88.73$$

ii) Prior : $N(100, 15^2)$

$$\mu_{post} = \frac{150 \times \frac{1}{10^2} + 100 \times \frac{1}{15^2}}{\frac{1}{10^2} + \frac{1}{15^2}} = 129.92$$

5) (Read along with code), parameters: $a_1, a_2, a_3 \ldots$ (coefficients)

$$\ell(\theta) = \Sigma \begin{cases} \ln\left(\text{sigmoid}(a_1 x_1 + a_2 x_2 \ldots)\right) & \text{if blue (1)} \\ \ln\left(1 - \text{sigmoid}(a_1 x_1 + a_2 x_2 \ldots)\right) & \text{if orange (0)} \end{cases}$$

$$\cancel{\frac{d}{da_1}(\ell(\theta)) = \Sigma \left\{ \frac{1}{\text{Sigmoid}(\Sigma a_i x_i)} \times \right.}$$

Prior: $a_j \sim N(0, \underset{\text{fixed/given}}{\sigma})$

For MAP $\frac{\partial}{\partial \theta}\left(\ell(\theta) + \ln(\text{prior})\right) = 0$

$$\frac{\partial}{\partial \theta}\left(\ln(\text{prior})\right) = \frac{\partial}{\partial a_j}\left(\text{constant} - \frac{(a_j)^2}{2\sigma^2}\right) = \frac{-a_j}{\sigma^2}$$

$\cancel{\frac{\partial}{\partial a_1}(\ell(\theta)) = \sum \frac{1}{\text{sigmoid}(x)} \times} \ \text{sigmoid}(x)\ (1-\text{sigm.}$

$$\frac{\partial}{\partial a_1}(\ell(\theta)) = \sum \begin{cases} \frac{1}{\text{sigmoid}(\sum a_i x_i)} \times [\text{sigmoid}(\sum a_i x_i)](1-\text{sigmoid}(\sum a_i x_i)) \times x_1 & (\text{if blue}) \\ \frac{1}{1-\text{sigmoid}(\sum a_i x_i)} \times -\text{sigmoid}(\sum a_i x_i) \times (1-\text{sigmoid}(\sum a_i x_i)) \times x_1 & (\text{if orange}) \end{cases}$$

$$= \sum \begin{cases} (1-\text{sigmoid}(\sum a_i x_i))\, x_1 & (\text{if blue}) \\ -\,\text{sigmoid}(\sum a_i x_i) \times x_1 & (\text{if orange}) \end{cases}$$

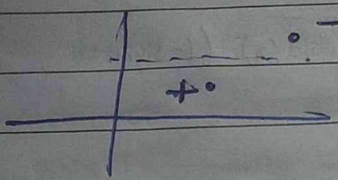
~~we need to solve~~

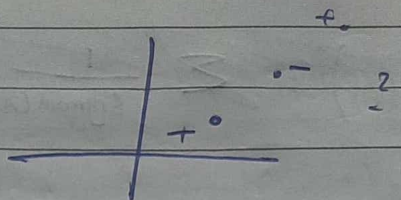
$$\Rightarrow \sum_{x \in \text{blue}} x_1 - \sum \text{sigmoid}(\sum a_i x_i) \times x_1 - \frac{a_j}{\sigma^2} = 0$$

$\therefore$ we need to solve: $\sum_{x \in \text{blue}} x_j = \left[\sum \text{sigmoid}(\sum a_i x_i) \times x_j\right] + \frac{a_j}{\sigma^2}$

for $a_1, a_2, a_3 \dots$ for $j \in \{1, 2 \dots n$ dims$\}$

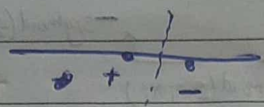
a)



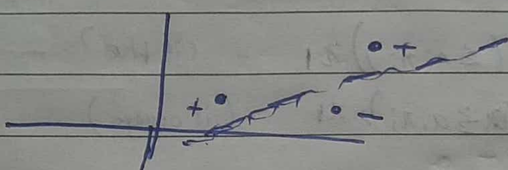


VC-dim = 2

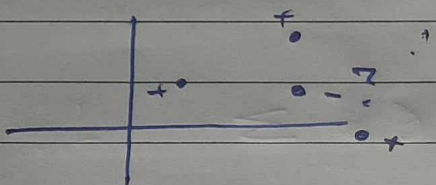
b) for 1D: VC-dim = 2



for 2D:



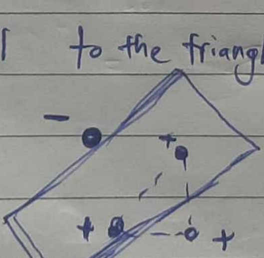
VC-dim = 3 (for non colinear)



~~for nD: $\text{sign}(a_1 x_1 + a_2 x_2 \dots + a_n x_n + b) = 1$ or -1~~

~~$\rightarrow \text{sign}(a^T x + b) = 1$ or -1~~

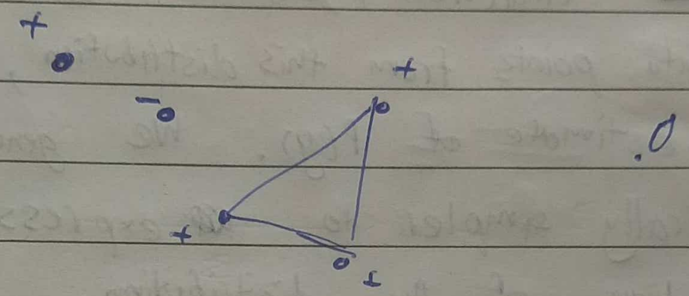
for 3D: any 4 points can be found that are shatterable
ex. vertices of tetrahedron ⇒
(points should be non coplanar) ① if all are + then trivial,
② if 3 are + rest −, the plane exists parallel to the triangle plane and between the point and plane



③ if 2+, 2− then case is also simple

to find a counter example of vc_dim = 5
take the tetrahedron case and place the 5th point

as "+". beyond the "−" as so:



$\therefore$ vc-dim $= 4$

! For ND vc-dim $= n+1$ (?)