Somato-dendritic prediction error learning

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In the following we describe an extended version of the somato-dendritic prediction error learning scheme described in [1]. We consider a two-compartment neuron where the dendrite receives synaptic contacts and the soma is stimulated externally. Somatic (U) and dendritic (V) potentials evolve according to:

$$\begin{aligned}\dot{U} &= -g_L(U - E_L) - g_D(U - V) + I_{NaK}(U) + I_{ext} \\ \dot{V} &= -g_L(V - E_L) - g_S(V - U) - g_E(V - E_E) - g_I(V - E_I)\end{aligned} \tag{1}$$

Outside of refractory periods, the neuron generates spikes with a sigmoid rate

$$\phi(U) = \frac{r_{max}}{1 + \exp(-\beta(U - \alpha))}. \tag{2}$$

To these formal spikes, we attach actual somatic action potentials by defining $I_{NaK}(U, t)$. For X the set of formal spike times:

$$\begin{aligned}I_{NaK}(U, t) &= \sum_{s \in X} i_{NaK}(U, t - s) \\ i_{NaK}(U, \tau) &= -g_{Na}(U - E_{Na})\Theta(1 - \tau) - g_K(U - E_K)\Theta(\tau - 1)\Theta(3 - \tau),\end{aligned} \tag{3}$$

with Θ the Heaviside step function.

We take the value of U obtained from exclusive control of the dendrite to be the dendritic prediction V_w^* of the somatic potential U:

$$\dot{V_w^*} = -g_L(V_w^* - E_L) - g_D(V_W^* - V) \tag{4}$$

For learning, we want to know how this prediction depends on the weight parameters. Focusing on w_E and taking the partial derivative we get:

$$\begin{aligned}\frac{\partial \dot{V}_w^*}{\partial w_E} &= -(g_L + g_D)\frac{\partial V_w^*}{\partial w_E} + g_D \frac{\partial V}{\partial w_E} \\ \frac{\partial \dot{V}}{\partial w_E} &= -(g_L + g_S + g_E(w_E))\frac{\partial V}{\partial w_E} + g_S \frac{\partial V_w^*}{\partial w_E} + (E_E - V)\frac{\partial}{\partial w_E} g_E(w_E)\end{aligned} \tag{5}$$

Synaptic conductances decay exponentially, with a presynaptic spike leading to an instantaneous increase in g_E or g_I given by the current respective weight. Hence, $\frac{\partial}{\partial w_E} g_E(w_E)$ in the above equation is simply the convolution of the input synaptic spike train with the response kernel - in our case a decaying exponential.

Learning is driven by a somato-dendritic prediction error. We define a plasticity-inducing variable:

$$\mathrm{PI}(t) = (S(t) - \phi(V_w^*))\, h(V_w^*) \frac{\partial V_w^*}{\partial w_E}, \tag{6}$$

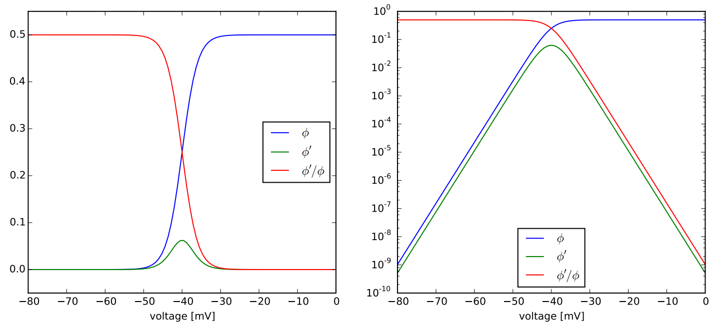

Figure 1: $\phi(V)$ for $\alpha = -40$, $\beta = 0.5$, $r_{max} = 0.5$ on normal and logarithmic scales

a low-pass filtered version of which is consolidated into persistent synaptic weight change:

$$\begin{aligned} \tau_\Delta \dot{\Delta} &= \mathrm{PI}(t) - \Delta \\ \dot{w} &= \eta \Delta, \end{aligned} \tag{7}$$

with η the learning rate. The process $S(t)$ represents the observation of somatic spikes at the dendrite, via backpropagation. When assuming instant and perfect backpropagation, we can equate $S(t)$ with the somatic spiking. More realistically, and progressing towards a local learning rule, $S(t)$ can also be computed by the dendrite itself by observing its voltage. We envision this process as a thresholding mechanism which enters a refractory period that depends on the instantaneous dendritic voltage where higher voltages enforce a faster sampling process. Thus we define the process $S(t)$ to exhibit a spike if and only if:

$$V(t) > \theta \text{ and } t - t_{ls} > t_{ref}(V(t)), \tag{8}$$

where t_{ls} is the time of the last spike and the function $t_{ref}(V)$ is defined as:

$$t_{ref}(V) = \begin{cases} 0 & V < \theta \\ t^0_{ref} \exp(-(V-\theta)/\theta_0) & V \geq \theta \end{cases}$$

As a weighting function we chose

$$h(V) = \frac{\phi'(V)}{\phi(V)}, \tag{9}$$

although the learning theory requires h only to be positive.

References

[1] Robert Urbanczik and Walter Senn. Learning by the dendritic prediction of somatic spiking. *Neuron*, 81(3):521–528, 2014.