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On the Ludwig Integration Algorithm for Triangular Subregions

M. L. X. DOS SANTOS AND NILSON R. RABELO

By using area coordinates one obtains algebraic expressions for the computation of two-dimensional integrals with oscillatory integrands over triangular subregions that are simpler than those published recently [1]. All the advantages of previous methods [1], [2] are retained. Moreover, the new equation is evaluated only once for each triangular subregion.

INTRODUCTION

Pogorzelski's method [1] for the application of the Ludwig integration algorithm for triangular subregions can be simplified by using area (also called natural or simplex) coordinates instead of rectangular Cartesian ones. The resulting expressions have been applied to the computation of radiation patterns of reflector antennas modeled by plane triangular elements as shown in Fig. 1. Results obtained for parabolic reflectors confirm Pogorzelski's claims on the advantages of triangular subdomains in comparison with rectangular ones for evaluating two-dimensional integrals with oscillatory integrands.

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M. L. X. dos Santos is with the Instituto Tecnológico de Aeronáutica (ITA), 12.200—São José dos Campos, SP, Brazil.

N. R. Rabelo is with the Instituto de Pesquisas Espaciais (INPE), 12.200—São José dos Campos, SP, Brazil.

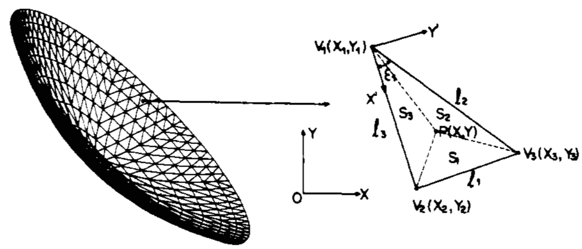

Fig. 1. Triangle-modeled parabolic reflector antenna and triangular subregion geometry.

The area coordinate expression for the radiation integral is simpler in form and as suitable as Pogorzelski's expressions for programming into a numerical integration subroutine. The amplitude and phase of the oscillatory integrand are also approximated by linear functions. Consequently, all the advantages of the Ludwig integration scheme are retained. Unlike the Pogorzelski method, which requires computing an algebraic expression for each triangle side, the area coordinate formula is computed only once for any given subregion.

FORMULATION

By adopting Pogorzelski's notation [1] one writes the radiation integral in the form

$$I = \iint_S F(x, y) e^{-jf(x,y)}\, dx\, dy \tag{1}$$

where $F(x, y)$ and $f(x, y)$ are, respectively, the amplitude and the phase functions; S stands for the area of the triangular subregion shown in Fig. 1.

According to the Ludwig algorithm, the functions $F(x, y)$ and $f(x, y)$ are approximated by the linear functions

$$F(x, y) \equiv Ax + By + C \tag{2}$$

$$f(x, y) \equiv \alpha x + \beta y + \gamma \tag{3}$$

where the coefficients A, B, C, α, β, and γ are determined by point-matching of $F(x, y)$ and $f(x, y)$ at the triangle vertices.

In order to determine an algebraic expression for the integral (1) with the approximations (2) and (3) one can use area, also called natural or simplex, coordinates [3]–[6]. Consider a generic triangular subregion as shown in Fig. 1. In this figure Oxy is a global rectangular Cartesian coordinate system, $P(x, y)$ is any point within the triangle, $\ell_i (i = 1, 2, 3)$ is the length of the side opposite to the vertex $V_i(x_i, y_i)$, S_i is the area of the internal triangle with vertex P and base ℓ_i, and ε_1 is the angle $V_2 \hat{V}_1 V_3$.

The area coordinates $s_i (i = 1, 2, 3)$ of any point $P(s_1, s_2, s_3)$ within the triangle are defined by

$$s_i = S_i / S \tag{4}$$

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so that

$$\sum_i s_i = 1 \tag{5}$$

where S is the area of the triangle $V_1 V_2 V_3$.

Let $V_1x'y'$ be a local rectangular Cartesian coordinate system whose origin is at V_1 and whose x' axis is along ℓ_3 (Fig. 1). Coordinates x' and y' of point P can be shown to be

$$x' = \ell_3 s_2 + (\ell_2 \cos \varepsilon_1) s_3 \tag{6}$$

$$y' = (2S/\ell_3) s_3. \tag{7}$$

Using (5)–(7) and the appropriate coordinate transformation, (1)–(3) can be written in the form

$$I = 2S\int_0^1 \int_0^{(1-s_1)} F_a(s_1, s_2) e^{-jf_a(s_1, s_2)} ds_2\, ds_1 \tag{8}$$

with

$$F_a(s_1, s_2) = A's_1 + B's_2 + C' \tag{9}$$

$$f_a(s_1, s_2) = \alpha' s_1 + \beta' s_2 + \gamma' \tag{10}$$

where

$$A' + C' = F_a(V_1) \quad B' + C' = F_a(V_2) \quad C' = F_a(V_3)$$
$$\alpha' + \gamma' = f_a(V_1) \quad \beta' + \gamma' = f_a(V_2) \quad \gamma' = f_a(V_3).$$

The notation $F_a(V_i)[f_a(V_i)](i = 1,2,3)$ stands for the value of $F_a(s_1, s_2)[f_a(s_1, s_2)]$ at the vertex V_i.

The analytical evaluation of the integral (8) gives

$$\begin{aligned} I = {} & 2S \exp(-j\gamma')\{[-C' + j(A'\beta' + B'\alpha')/(\alpha'\beta')]/(\alpha'\beta') \\ & +[A' + C' - j((B' - 2A')\alpha' + A'\beta')/(\alpha'(\beta' - \alpha'))]/ \\ & [\alpha'(\beta' - \alpha')] \cdot \exp(-j\alpha') \\ & +[B' + C' - j((A' - 2B')\beta' + B'\alpha')/(\beta'(\alpha' - \beta'))]/ \\ & [\beta'(\alpha' - \beta')] \cdot \exp(-j\beta')\} \\ & \qquad (\text{if } \alpha' \neq 0, \beta' \neq 0, \alpha' \neq \beta') \end{aligned} \tag{11}$$

$$\begin{aligned} I = {} & (2S/\beta'^2) \exp(-j\gamma')\{[(A' - B' + C') \\ & -j((A'/2 + C')\beta' - (A' - 2B')/\beta')] \\ & -[(B' + C') + j(A' - 2B')/\beta']\exp(-j\beta')\} \\ & \qquad (\text{if } \alpha' \equiv 0) \end{aligned} \tag{12}$$

$$\begin{aligned} I = {} & (2S/\alpha'^2) \exp(-j\gamma')\{[(B' - A' + C') \\ & -j((B'/2 + C')\alpha' - (B' - 2A')/\alpha')] \\ & -[(A' + C') + j(B' - 2A')/\alpha']\exp(-j\alpha')\} \\ & \qquad (\text{if } \beta' \equiv 0) \end{aligned} \tag{13}$$

$$\begin{aligned} I = {} & (2S/\delta'^2) \exp(-j\gamma')\{[-C' + j(A' + B')/\delta'] \\ & +[(A' + B' + C') - j((A' + B')/\delta' \\ & - ((A' + B')/2 + C')\delta')] \cdot \exp(-j\delta')\} \\ & \qquad (\text{if } \alpha' \equiv \beta' = \delta' \neq 0) \end{aligned} \tag{14}$$

$$\begin{aligned} I = {} & S[(A' + B')/3 + C']\exp(-j\gamma') \\ & \qquad (\text{if } \alpha' \equiv \beta' \equiv 0). \end{aligned} \tag{15}$$

The evaluation of the integral (1) over a right triangle has been performed elsewhere [7]. The equations developed therein correspond to a particular case of the general equations (11)–(15).

The computation of (11)–(15) is simpler than that of Pogorzelski's alternative expressions, for it is not necessary either to determine the angular and the linear coefficients of the triangle sides (therefore, it is meaningless whether they are vertical or not), or to substitute for the integration limits. Moreover, Pogorzelski's expressions must be evaluated three times since $I = I_1 + I_2 + I_3$, while (11)–(15) give the integral value in a single computation.

Equations (11)–(15) have been implemented into a numerical integration subroutine as part of a program for the numerical calculation of radiation patterns of reflector antennas modeled by plane triangle facets using the Physical Optics approximation. Preliminary results are anticipated to be stimulating [4].

Conclusion

By using area (or natural) coordinates one can obtain closed-form algebraic expressions for integrals with oscillatory integrands over triangular subregions, which are simpler than those published recently [1]. Since no change is made either in the Physical Optics or the form of the amplitude and phase functions approximations, all the advantages of the Ludwig integration algorithm are retained.

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Propagation Characteristics of Eccentric Core Fibers Using Point-Matching Method

A. ALPHONES AND G. S. SANYAL

The propagation characteristics of homogeneous single-mode optical fibers with eccentric core are analyzed by a numerical method based on the Improved Point-Matching Method (IPMM). The variation of fundamental mode propagation constant has been computed for various eccentricities with different fiber parameters.

I. Introduction

In the fabrication of single-mode glass fiber waveguides, precise control of diameter is necessary. During the drawing process, the core and cladding may not be concentric throughout the length of the fiber. Yet, no papers have been published discussing the problem of eccentric core fibers except those dealing with the effect of eccentricity on splicing loss. Since core and cladding are not concentric to each other, a numerical approach can be used to analyze the propagation characteristics of this fiber. Goell [1] described a circular-harmonic computer analysis of a rectangular homogeneous dielectric waveguide. He expanded electromagnetic fields in terms of circular harmonics and employed the point-matching method to approximately satisfy boundary conditions. James and Gallett [2] also pointed out the usefullness of this approach. This approach may be used to treat homogeneous noncircular fibers with core/cladding index difference. But the convergence of the results is poor for structures which deviate significantly from the circular shape [3]–[5]. When the geometry of the boundaries is as shown in Fig. 1, it is difficult to obtain closed-form analytical expressions to satisfy the boundary condition. This letter describes the results of computer analyses on the propagation characteristics of eccentric core single-mode optical fiber.

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The authors are with the Centre for Research and Training in Radar and Communication, Indian Institute of Technology, Kharagpur, India 721 302.

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