SSY080

Transformer, Signaler och System

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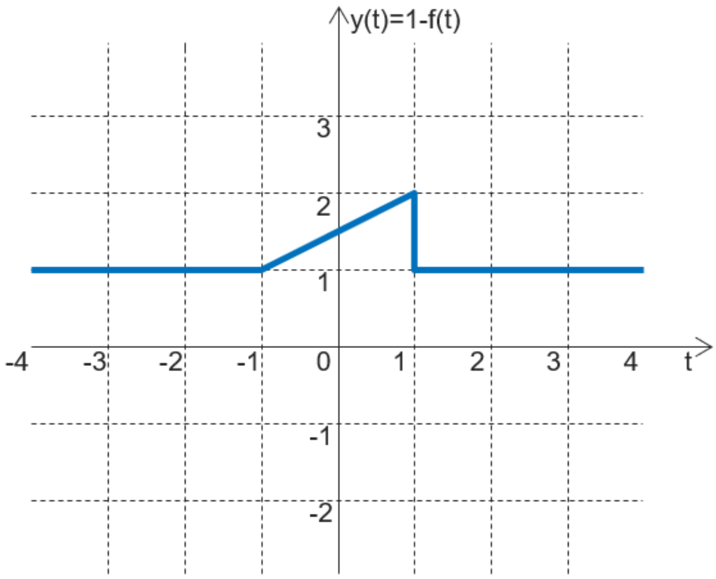
Date: 05/01/21, Time: 4 h (14.00-18.00)

Solution

A1.



- f(t): flip about the t axis

1: DC

A2. B is the correct solution.

D cannot be correct, because the convolution of two positive sequences yields a positive output.

A, B, and C differ for the sample x[3], which is obtained in case of maximal overlap between x1[n] and x2[n] ($\sum_{n=0}^{3} x_1[n]x_2[n] = 6$).

A3. The system is instantaneous. Therefore $y[n]$ depends only on $x[n]$. Comparing input and output, it can be observed that

$$y[n] = \begin{cases} x[n] & if\ x[n] \geq 0 \\ -x[n] & if\ x[n] < 0 \end{cases}$$

Therefore $y[n] = |x[n]|$

The system is not invertible. For instance, given $y[5] = 3$, we cannot conclude if $x[5] = 3$ or $x[5] = -3$.

A4. $Y(\omega) = H(\omega)X(\omega)$

$\sin(t)$ contributes to $X(\omega)$ with terms proportional to δ centered at $\omega = \pm 1$ radians/seconds, and we want to filter them out

$\cos(3t)$ contributes to $X(\omega)$ with terms proportional to δ centered at $\omega = \pm 3$ radians/seconds, and we want to maintain them

Therefore, $0 < \omega_1 < 3$, and $3 < \omega_2 < \infty$

A5. The terms cos and sin are oscillatory and do not contribute to DC. Therefore, $c_0 = 0$.

A6.

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{3}{2}\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} = u(t) - \frac{3}{2}e^{-2t}u(t) = \left(1 - \frac{3}{2}e^{-2t}\right)u(t)$$

A7. Transfer function: ratio of the system output to input in the Laplace domain, assuming zero initial conditions

$$s^2\, Y(s) + 10\, s\, Y(s) + 16\, Y(s) = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 + 10\, s + 16}$$

A8. The system has poles in $p = \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm j2$, i.e. in the LHP. Therefore, the system is stable.

A9.

$$\sum_{n=0}^{+\infty} 0.4^n = \sum_{n=0}^{+\infty} 0.4^n\, z^{-n}\Big|_{z=1} = \mathcal{Z}\{0.4^n\, u[n]\}\Big|_{z=1} = \frac{z}{z-0.4}\Big|_{z=1} = \frac{1}{0.6} = \frac{10}{6} = \frac{5}{3}$$

A10.

$$f[k] = \mathcal{Z}^{-1}\{F[z]\} = 4\,\mathcal{Z}^{-1}\left\{\frac{1}{z-3}\right\} + 5\,\mathcal{Z}^{-1}\left\{\frac{1}{z-2}\right\} = [4\,(3)^{k-1} + 5\,(2)^{k-1}]\,u[k-1]$$

B1. a. Fundamental frequency

$$\omega_1 = 2 \quad \left[\frac{\text{r}}{\text{s}}\right], T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{2} = \pi \quad [\text{s}]$$
$$\omega_2 = 4 \quad \left[\frac{\text{r}}{\text{s}}\right], T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{4} = \frac{\pi}{2} \quad [\text{s}]$$

Common period $T = m_1 T_1 = m_2 T_2,$ with $m_1, m_2 \in \mathbb{Z}$

$$T = \pi \ [s], m_1 = 1, m_2 = 2$$
$$\omega_0 = \frac{2\pi}{\pi} = 2 \quad \left[\frac{\text{r}}{\text{s}}\right]$$

b. Coefficients

$$x(t) = 7 + sin(2t) + 5\cos\left(4t + \frac{\pi}{3}\right) = 7 + \frac{e^{j2t} - e^{-j2t}}{2j} + 5\frac{e^{j4t}e^{\frac{j\pi}{3}} + e^{-j4t}e^{-\frac{j\pi}{3}}}{2} =$$
$$= 7e^{jk\omega_0 t}\Big|_{k=0} + \frac{1}{2j}e^{jk\omega_0 t}\Big|_{k=1} - \frac{1}{2j}e^{jk\omega_0 t}\Big|_{k=-1} + \frac{5}{2}e^{\frac{j\pi}{3}}\,e^{jk\omega_0 t}\Big|_{k=2} + \frac{5}{2}\,e^{-\frac{j\pi}{3}}\,e^{jk\omega_0 t}\Big|_{k=-2}$$

$$c_k = \begin{cases} 7 & 0 \\ \pm\frac{1}{2j} & k = \pm 1 \\ \frac{5}{2j}e^{\pm\frac{j\pi}{3}} & k = \pm 2 \\ 0 & otherwise \end{cases}$$

B2. a. $y_a(t) = x_1(t) * x_2(t) = \int_{-\infty}^{+\infty} x_1(\tau)\, x_2(t-\tau)\ d\tau$

$t \leq 0$

$y_a(t) = 0$

$0 \leq t \leq 1$

$$y_a(t) = \int_0^t \tau\, d\tau = \frac{1}{2}\tau^2\Big|_0^t = \frac{1}{2}t^2$$

$1 \leq t \leq 2$

$$y_a(t) = \int_{-1+t}^{1} \tau\, d\tau = \frac{1}{2}\tau^2\Big|_{t-1}^{1} = \frac{1}{2} - \frac{1}{2}(t-1)^2 = t - \frac{1}{2}\, t^2$$

$t \geq 2$

$y_a(t) = 0$

b.

$$y_b(t) = x_2(t) * x_1(t) = \int_{-\infty}^{+\infty} x_2(\tau)\, x_1(t-\tau)\ d\tau$$

$t \leq 0$

$y_b(t) = 0$

$0 \leq t \leq 1$

$$y_b(t) = \int_0^t (t-\tau)\, d\tau = t\,\tau|_0^t - \frac{1}{2}\tau^2\Big|_0^t = t^2 - \frac{1}{2}t^2 = \frac{1}{2}t^2$$

$1 \leq t \leq 2$

$$y_b(t) = \int_{-1+t}^{1} (t-\tau)\, d\tau = t\,\tau|_{t-1}^{1} - \frac{1}{2}\tau^2\Big|_{t-1}^{1} = t\,(1-t+1) - \frac{1}{2} + \frac{1}{2}(t-1)^2 = t - \frac{1}{2}\, t^2$$

$t \geq 2$

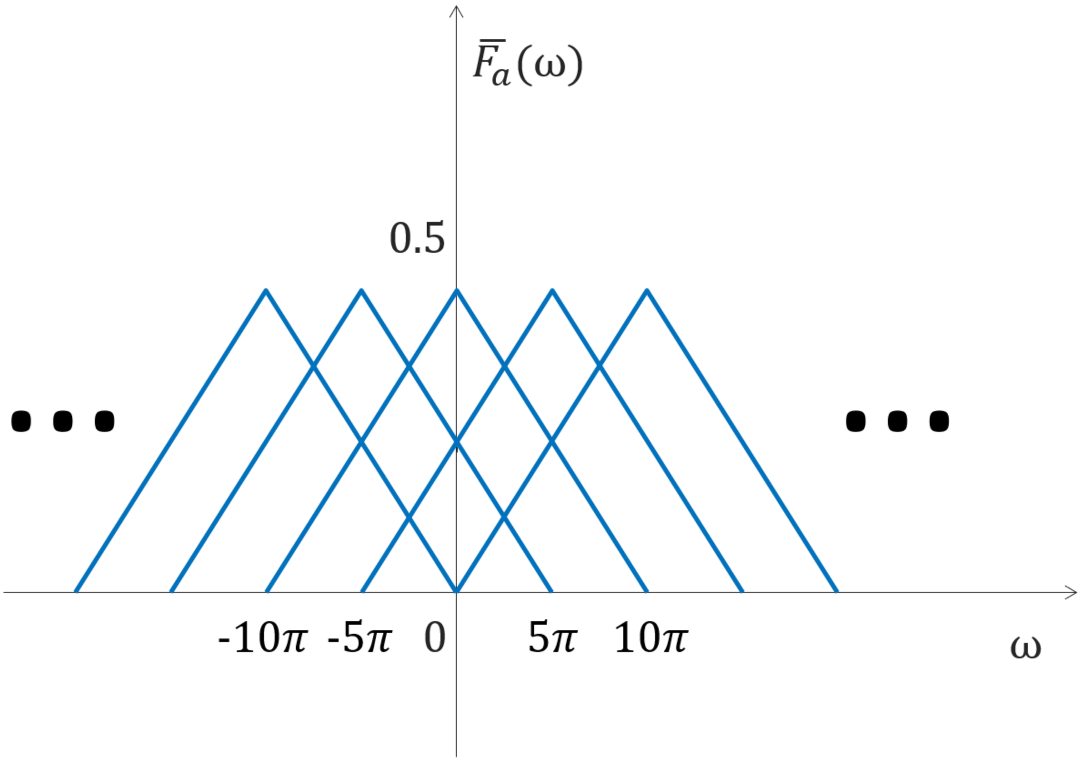
$y_b(t) = 0$

$y_a(t) = y_b(t)$, as expected since the convolution is commutative

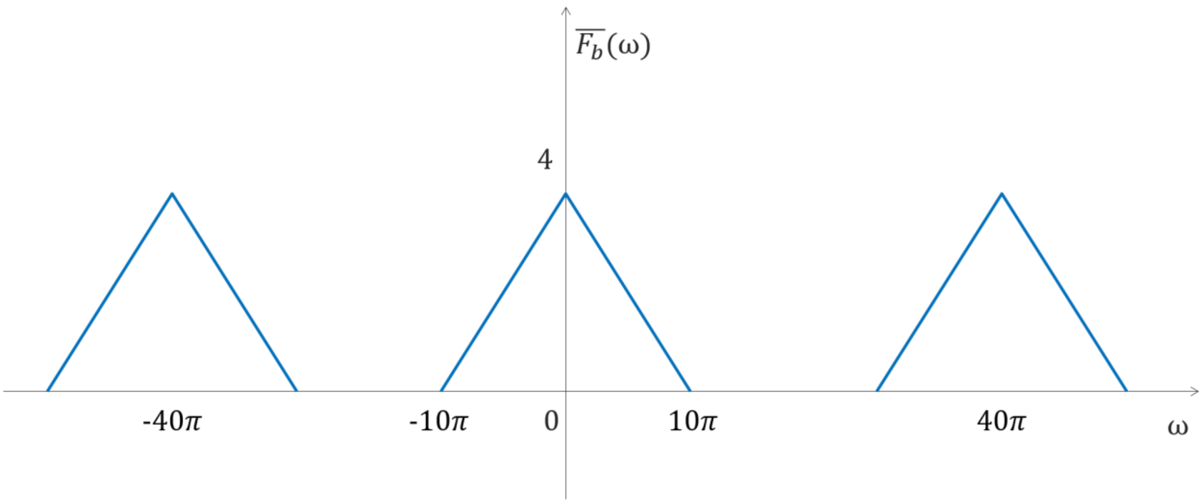
B3.

$$\bar{F}(\omega) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} F(\omega - n\,\omega_s)$$

a. $\omega_{sA} = \frac{2\pi}{T_A} = \frac{2\pi}{0.4} = 5\,\pi \quad [\frac{radian}{second}]$



b. $\omega_{sB} = \frac{2\pi}{T_B} = \frac{2\pi}{0.05} = 40\,\pi \quad [\frac{radian}{second}]$



Note: this figure is not in scale.

Case a: aliasing

Case b: sampling higher than Nyquist rate, we can recover the original signal (even using a practical filter).