**SSY080**

**Transformer, Signaler och System**

**Examiner: Silvia Muceli muceli@chalmers.se**

**Date: 05/01/21, Time: 4 h (14.00-18.00)**

**Grading system**

| | | | |
|---|---|---|---|
| **10 Quest A** | **1 point each** | **10 points in total** | **5/10 necessary to pass** |
| **3 Quest B** | **5 points each** | **15 points in total** | **7/15 necessary to pass** |

Note: only a **complete answer** results in the **full point** (A) / **points** (B).

| **Points** | **12-15** | **16-20** | **20-25** |
|---|---|---|---|
| **Final grade** | **3** | **4** | **5** |

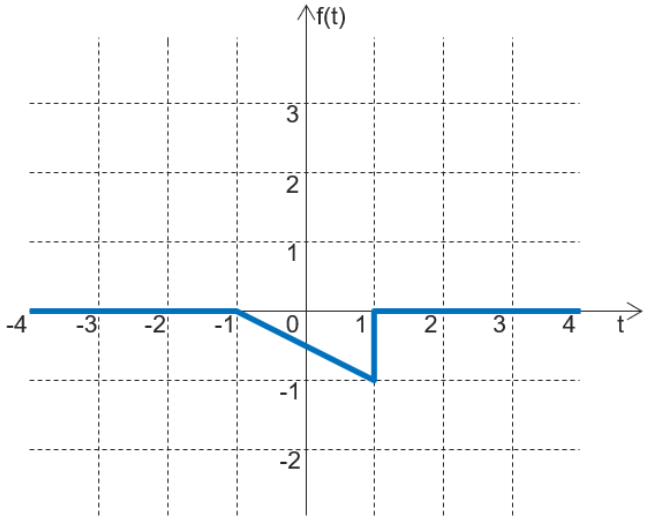
At the top of the first page, report **which questions you have answered** (e.g. A1, A3, A10, B2).

All answers must be written in **English**.

The solutions must be complete and easy to follow.

You can either write by hand or on a computer.

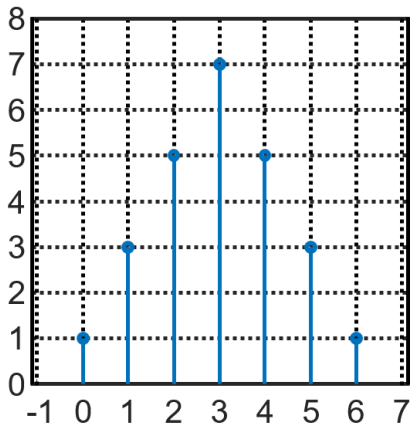
**A1.** Given the signal f(t) in the figure



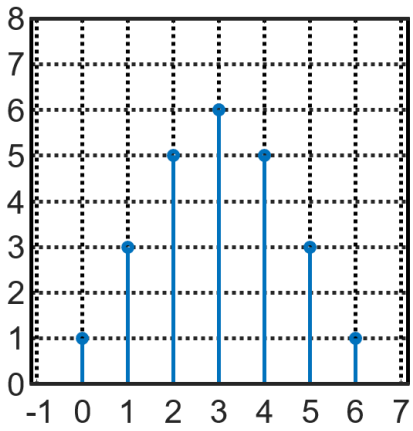

Plot the signal y(t) = 1 - f(t). Motivate your answer.

**A2.** Given the two sequences $x_1[n] = u[n] - u[n-4]$ and $x_2[n] = \delta[n] + 2\,\delta[n-1] + 2\delta[n-2] + \delta[n-3]$, determine the convolution x[n]= $x_1$[n] * $x_2$[n]. One of the 4 options (A, B, C, D) is correct. Motivate your answer.

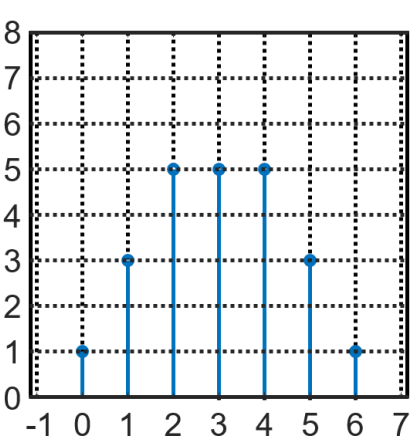
A



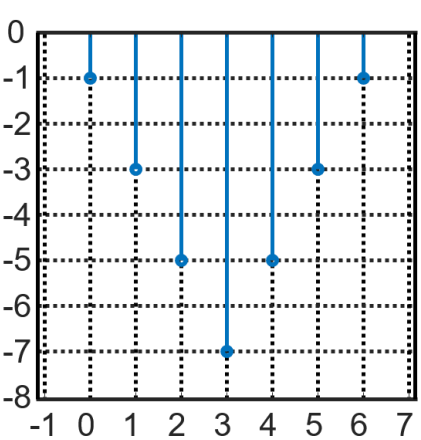

B



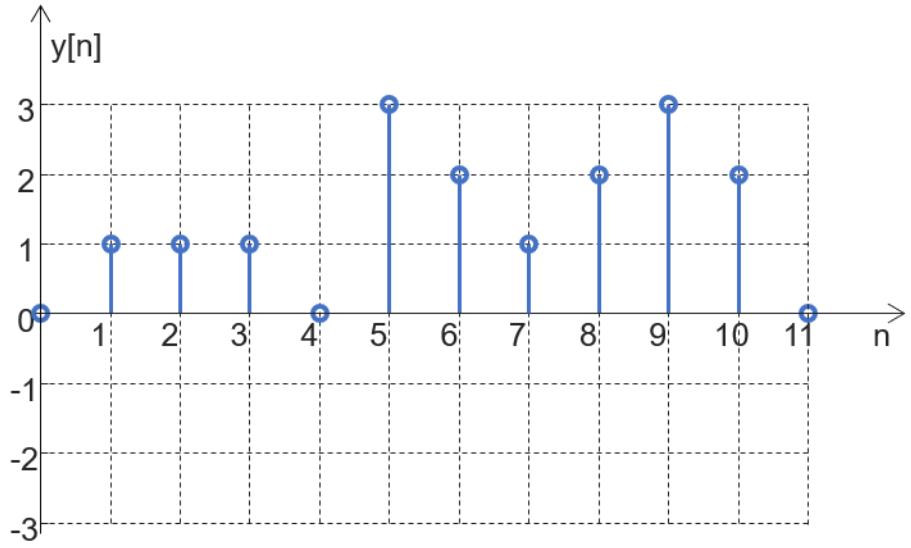

C



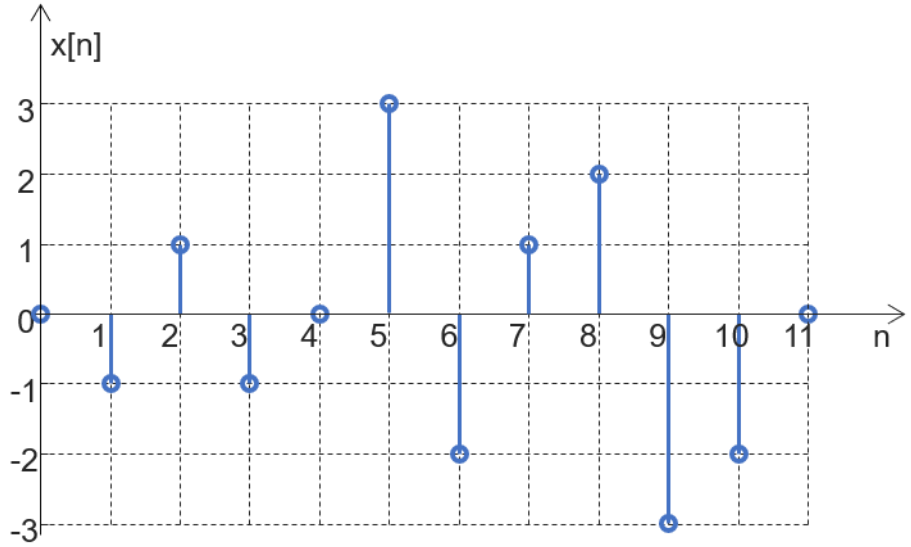

D



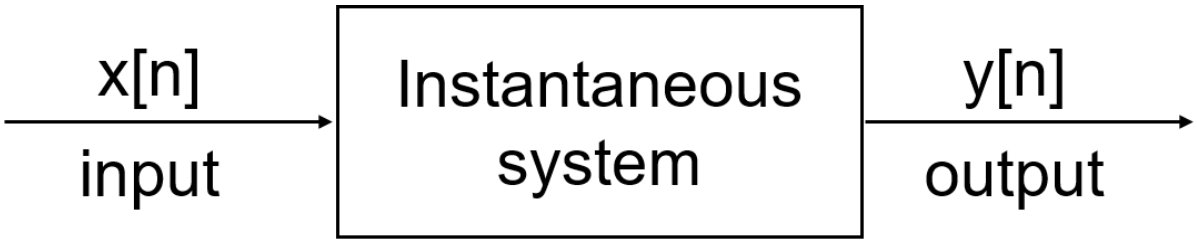
**A3.** Given an instantaneous system that provides as output the sequence y[n] represented below




when fed with the sequence x[n] as input,



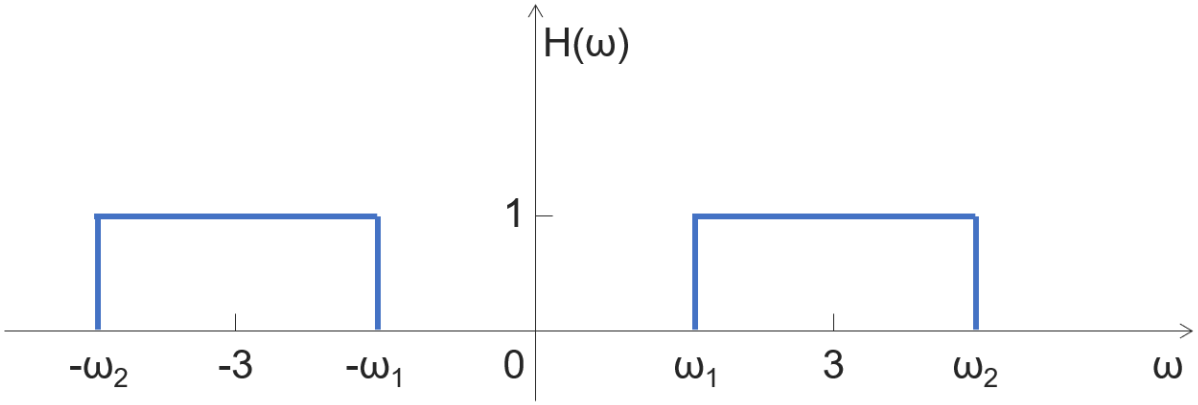

determine the mathematical function that relates the input x[n] to the output y[n] (i.e. y[n] = …). Motivate your answer.




Is the system invertible? Motivate your answer.

**A4.** Given a LTI system with

- input x(t)=sin(t)+cos(3t)
- and impulse response h(t) with Fourier transform $H(\omega)$ represented in the figure




Determine the values of $\omega_1$ and $\omega_2$ so that the resulting system output y(t) is equal to

$$y(t) = \alpha \cos(3t)$$

with α constant and ≠ 0. Motivate your answer.

**A5.** Given the periodic signal $x(t) = cos(3t) + 2\sin(9t)$, compute the coefficient $c_0$ of the complex Fourier series

$$x(t) = \sum_{k=-\infty}^{+\infty} c_k e^{jk\omega_0 t}.$$

Motivate your answer.

**A6.** Determine the inverse Laplace transform $f(t)$ of the function

$$F(s) = \frac{1}{s} - \frac{3}{2}\frac{1}{s+2}$$

Motivate your answer.

**A7.** Consider the LTI system described by the following differential equation

$$\frac{d^2y(t)}{dt^2} + 10\frac{dy(t)}{dt} + 16\,y(t) = x(t),$$

where x(t) is the system input and y(t) the system output. Determine the system transfer function in the Laplace domain. Motivate your answer.

**A8.** Given a system with transfer function

$$H(s) = \frac{1}{s^2 + 2s + 5}$$

determine if the system is stable. Motivate your answer.

**A9.** Compute the following summation using the z-transform $\sum_{n=0}^{+\infty}(0.4)^n$.

**A10.** Determine the inverse z transform $f[k]$ of the function

$$F[z] = \frac{4}{z-3} + \frac{5}{z-2}$$

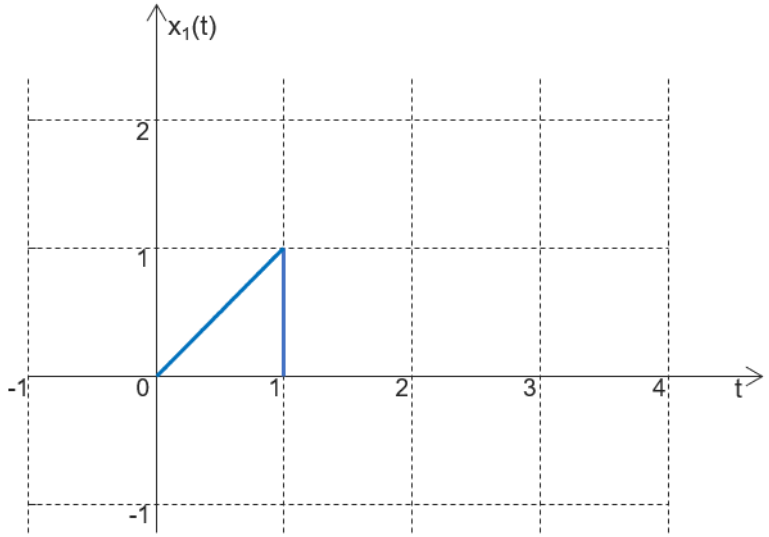
Motivate your answer.

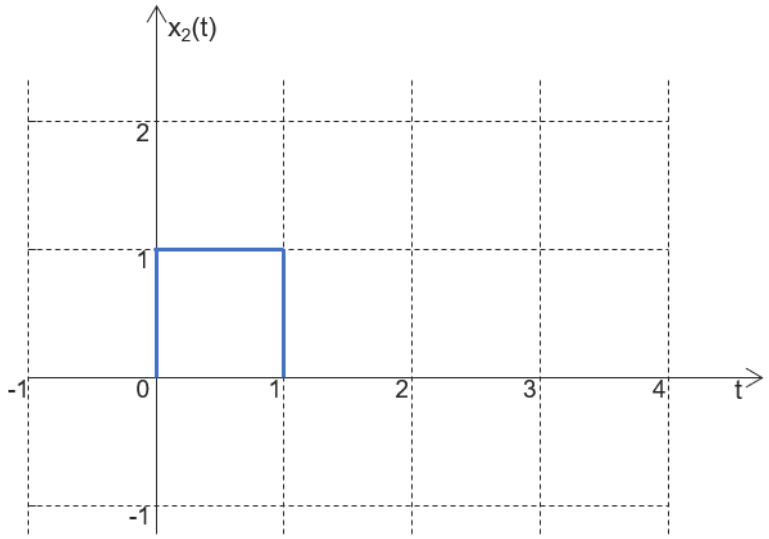
**B1.** Given the signal $x(t) = 7 + sin(2t) + 5\cos\left(4t + \frac{\pi}{3}\right)$,

a. Determine the fundamental frequency $\omega_0$
b. Determine the coefficients $c_k$ of the complex Fourier series

$$x(t) = \sum_{k=-\infty}^{+\infty} c_k e^{jk\omega_0 t}.$$

**B2. a.** Compute the convolution between $x_1(t)$ and $x_2(t)$.



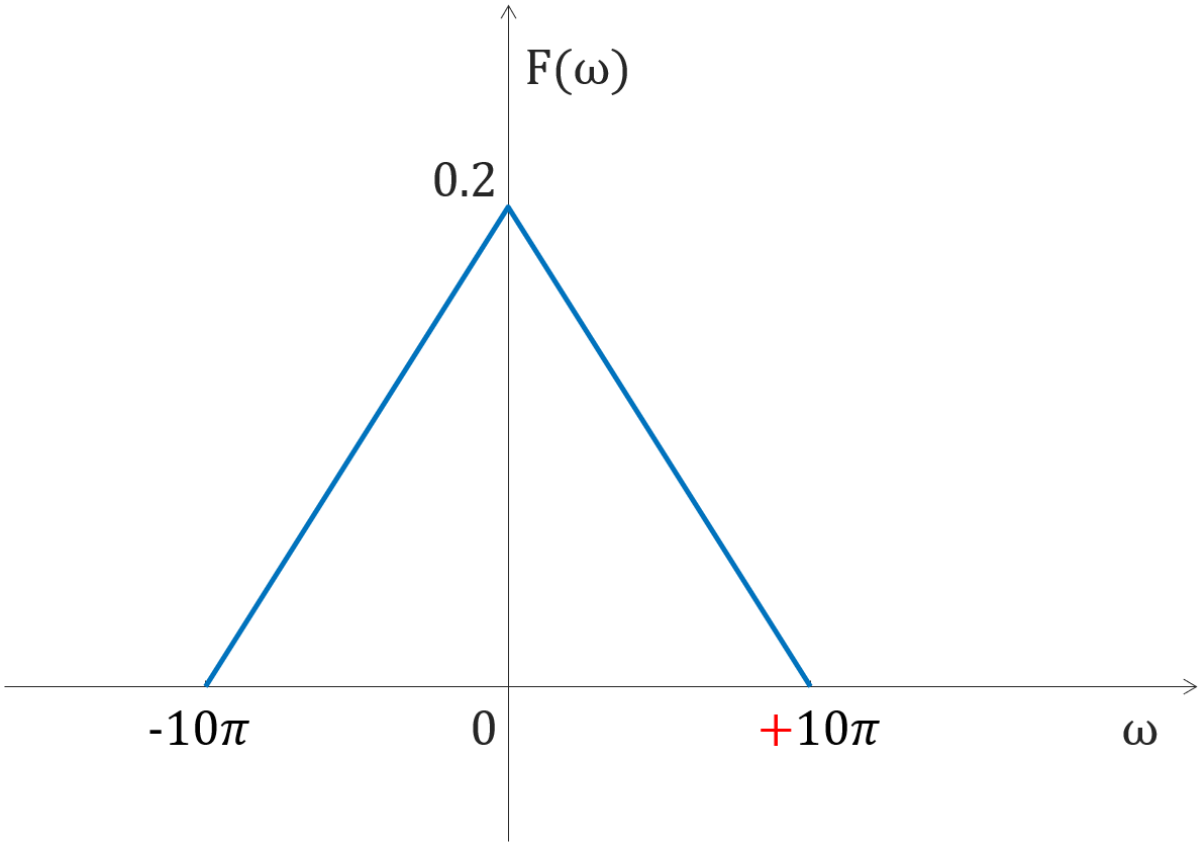

**b.** Show that the commutative property of the convolution holds true by computing the convolution between $x_2(t)$ and $x_1(t)$.

NOTE: If you use a software to calculate the integrals, you need to indicate which software you used. Even if you solved the integrals with a software, you need to indicate the mathematical expression of the integrals.

I report a random example to clarify what I mean:

- $\int_a^b x(\tau)d\tau = \frac{b^2-a^2}{2}$ will NOT be considered correct, because the mathematical expression of $x(\tau)$ is not specified
- $\int_a^b x(\tau)d\tau = \int_a^b \tau\, d\tau = \frac{\tau^2}{2}\Big|_a^b = \frac{b^2-a^2}{2}$ will be considered correct
- In case you use a software to calculate the integrals, $\int_a^b x(\tau)d\tau = \int_a^b \tau\, d\tau = \frac{b^2-a^2}{2}$ will also be considered correct because the expression of $x(\tau)$ has been explicitly reported (provided that you indicate which software you used to calculate the integral).

**B3.** Given a signal $f(t)$ with Fourier transform $F(\omega)$ sketched in the following figure




with $\omega$ expressed in radians / second, plot the Fourier spectrum $\bar{F}(\omega)$ of the signal $\bar{f}(t)$ obtained by sampling the signal $f(t)$ at a rate of $\mathcal{F}_s = \frac{1}{T}$ in the following two cases

a. $T = 0.4\ seconds$
b. $T = 0.05\ seconds$

Explain the procedure you followed.

Based on the two plots (a and b), discuss which of the two sampling intervals $T$ (a or b) is suitable in order to be able to recover $f(t)$ from its samples. Motivate your answer.