Course name:
Empirical software engineering DAT246/DIT278 (formerly DAT245)

Date:
January 13, 2020

Time:
14:00 –18:00

Responsible teachers:
Richard Torkar and David Mattos.

Teacher will visit the exam room.
Twice (at 15:00 and at 16:15)

Telephone number:
0723526147 (David Mattos)

Examiner:
Richard Torkar

Aids:
Approved calculator by Chalmers.

Grades:
Maximum points: 56
Chalmers:
Grade 3: 28 – 36 [50% , 65%[
Grade 4: 37 – 47 [65% , 85%[
Grade 5: 48 – 56 [85% , 100%]
GU:
Grade G: 28 – 36 [50% , 85%[
Grade VG: 37 – 56 [85% , 100%]

Appeal of Grading:
February 11th , 2020 at 14.00 – 15:00 (Jupiter, floor 4, room 473)

Time and place for presentation of solutions:
February 11th , 2020 at 14.00 – 15:00 (Jupiter, floor 4, room 473)

“In science, progress is possible. In fact, if one believes in Bayes' theorem, scientific progress is inevitable as predictions are made and as beliefs are tested and refined.”

— Nate Silver

Text for questions 1 and 2

Screen time might be physically changing kids' brains
Source and adapted from: https://www.technologyreview.com/f/614672/screen-time-might-be-physically-changing-kids-brains/
Date: Nov 4, 2019

A study published today [1] in JAMA Pediatrics warns that kids' literacy and language skills suffer with screen use, and MRI scans of their brains appear to back up the findings.

The study: Forty-seven 3- to 5-year-olds took a test to measure their cognitive abilities, and their parents were asked to answer a detailed survey about screen time habits. Questions included: How frequently do they use that screen? What type of content are they viewing? And is there an adult sitting with the child talking about what they're watching? The answers were scored against a set of screen time guidelines put out by the American Academy of Pediatrics. The kids also had their brains scanned in an MRI machine.

Brain changes: The scans revealed that kids who spent more time in front of screens had what the authors call lower "white matter integrity." White matter can be roughly thought of as the brain's internal communications network—its long nerve fibers are sheathed in fatty insulation that allows electrical signals to move from one area of the brain to another without interruption. The integrity of that structure—how well organized the nerve fibers are, and how well developed the myelin sheath is—is associated with cognitive function, and it develops as kids learn language.

Lead author John Hutton of Cincinnati Children's Hospital told MIT Technology Review there's a clear link between higher screen use and lower white matter integrity in the children his team studied. That structural change appears to be reflected in the results of the cognitive test the kids took as well, which showed high screen time associated with lower levels of language and literacy skills. "The effect size is substantial, as these findings also rigorously controlled for multiple comparisons across the brain," Hutton says.

The big caveats: It's a small and preliminary study. "It's absolutely not clear that screen time causes differences in brain development and there are many factors that could explain the association found here," Signe Lauren Bray, a researcher at the University of Calgary who was not involved in the study, said via email. Bray has done fMRI studies on kids brains, and pointed to other work [2] that suggested kids who spent more time in front of screen tended to display more symptoms of ADHD. But that study also suggested that the symptoms could be the very reason why kids were spending time in front of screens in the first place. Socioeconomics could also play a role, she said, and screen time effects could disappear if those factors were taken into account.

Just tell me how much screen time is okay for my kid already. Unfortunately, that's not so easy. "It's hard to say what the 'safe' age or amount of screen time is," Hutton says. "My motto is 'Screen-free until three'—this at least gets kids to preschool with a solid anchor in the real world, where their basic sense of connection with caregivers and early language skills have solidified."

It's a small study, but also big. "While relatively small for a behavioral study, this is actually a fairly large MRI study, especially involving young children, [and] the first to explore associations between screen time and brain structure," Hutton says. Next steps include more tests on kids and efforts to figure out how parents' screen use might influence their children.

The take-home message: “Caution is warranted,” Hutton says. “Children are not small grown-ups, and their needs change with development.”

[1] Hutton JS, Dudley J, Horowitz-Kraus T, DeWitt T, Holland SK. Associations Between Screen-Based Media Use and Brain White Matter Integrity in Preschool-Aged Children. JAMA Pediatr. Published online November 04, 2019. doi:https://doi.org/10.1001/jamapediatrics.2019.3869
[2] Tamana, Sukhpreet K., et al. "Screen-time is associated with inattention problems in preschoolers: Results from the CHILD birth cohort study." PloS one 14.4 (2019): e0213995.

Question 1) The first highlight section indicates that there is a clear link between higher screen use and lower white matter integrity. The third highlight also discusses the relation between screen time and symptoms of ADHD. One of the problems in research communication is to mistakenly assess correlation and causation. **(6pts)**

a) Define the concepts of correlation and causation and explain the difference between them and how they are related**(3pts- 1pt for each definition + 1 pt for the difference and relation).**
b) The study [1] indicates a strong correlation between screen use time and lower white matter integrity. Give three possible explanations for this observation in terms of causality (we discussed five of those in class) **(3pts - 1pt for each)**

Question 2) Based on the description of the study and the second highlight. Answer the following questions **(7 pts)**

a) Is this an experimental or observational study? Justify **(1 pt)**
b) What is the importance of sample size in both experimental and observational studies such as survey **(4 pts – 2 for each)**
c) The study [1] discuss correlations in the collected data. Is it possible to also assess causality based on the type of study conducted? Justify **(2pts)**

Question 3) One important aspect in both observational and experimental studies is randomization. In the course, we saw three types of randomization. Explain and exemplify Random Sampling, Stratified Sampling and Convenience Sampling **(1 for explanation +1 for example for each. Total 6 pts)**

Questions 4 and 5 are based on the study:
Regnell, Björn, Per Runeson, and Thomas Thelin. "Are the perspectives really different?– further experimentation on scenario-based reading of requirements." *Empirical Software Engineering* 5.4 (2000): 331-356.

Perspective-Based Reading (PBR) is a scenario-based inspection technique where several reviewers read a document from different perspectives (e.g. user, designer, tester). The reading is made according to a special scenario, specific for each perspective. The basic assumption behind PBR is that the perspectives find different defects and a combination of several perspectives detects more defects compared to the same amount of reading with a single perspective.

The study is conducted in an academic environment using graduate students as subjects. Each perspective applies a specific modelling technique: use case modelling for the user perspective, equivalence partitioning for the tester perspective and structured analysis for the design perspective. Two types of inspection objects are used: an automatic teller machine

ATM (17 pages containing 29 defects) and a parking garage system PG (16 pages containing 30 defects)

The experiment is planned with a total of 30 subjects randomized into 6 experimental groups, considering the different types of documents and perspectives, giving 5 subjects per group.

The study proposes the following two research questions:

1. Do the perspectives detect different defects?
2. Is one perspective superior to another?

There are two aspects of superiority that are measure in this experiment:

1. effectiveness, i.e. how high fraction of the existing defects are found (detection rate)
2. efficiency, i.e. how many defects are found per time unit

Question 4) Based on the text above answer the questions regarding this study? **(5 pts)**

a) What are the experimental units? **(1 pt)**
b) What is the experimental subject? **(1 pt)**
c) What are the two response variables? **(1 pt)**
d) How many factors do we have? **(1 pt)**
e) How many levels in each factor? **(1 pt)**

Question 5) (19 pts)
The researchers conducted an ANOVA for the efficiency and for the detection rate and found the following incomplete ANOVA tables:

ANOVA table for the Efficiency

	Sum of Squares	Degrees of freedom	Mean Square	F-value
Perspective (PERSP)	1751			
Document type (DOC)	1640			
PERSP * DOC	2229			
Residual				
Total	34147			

ANOVA table for the RATE

	Sum of Squares	Degrees of freedom	Mean Square	F-value
Perspective (PERSP)	0.012			
Document type (DOC)	0.011			
PERSP * DOC	0.004			
Residual				
Total	0.199			

a) Why did the researchers conduct an ANOVA instead of making a direct comparison between the different groups? **(1 pt)**
b) Create the linear models (similar to the R notation) that represent each ANOVA conducted **(2pt, 1 for each model)**
c) What are the four assumptions of this linear model (same assumptions as the ANOVA)? Justify how to assess if they are satisfied **(4pts – 0.5 for each + 0.5 for each for justifying)**
d) Complete both the ANOVA tables **(4 pts, 2 pts for each)**
e) At the confidence level of 95%, are the perspective, document type and interaction statistically significant for both the rate and the efficiency **(4 pts, 0.5 for each and 1pt for listing the F critical that is being used)**
f) What is the effect size for the perspective, document type and interaction for both the rate and the efficiency? **(1 pts 0.5 pt for rate and 0.5 pt for efficiency if all effect sizes are correct)**
g) How could the researchers compare the superiority of one of the perspectives over the other? **(1 pt)**
h) Interpret the result of this experiment in the context described earlier **(2 pts)**

Question 6 (3pts) There are three ways, and three ways only, to ethically encourage participation in a study (Senese 1997). What are the three ways? Describe them briefly! **(3pts, 1pt for each)**

Question 7 (5 pt)
In empirical software engineering we distinguish between four main types of validity threats
a) Name and exemplify them **(1pt each)**
b) Provide an example of how lowering one type of validity threat could increase another type of validity threat? **(1 pt)**

Question 8 (5 pt)
Abstract: The detection of fraud and other systematic data irregularities in clinical trials is an important issue. While awareness of the problem is growing and willingness to combat it is clear, there still appears to be a lack of detection procedures suitable for routine implementation by trial coordinators. The shortage is particularly acute for discrete data, since the majority of methods which are available have been developed for continuous responses. In this paper, we examine the suitability of existing methods for discrete outcomes and propose a new technique for questionnaire data in both an informal graphical mode and as a randomization test. This method exploits the underlying correlation structure of a questionnaire and the difficulty in fabricating such details. A data set concerning a trial of a novel drug for treatment of schizophrenia, in which the Brief Psychiatric Rating Scale was used to assess patient mental health, is used for illustration.

Source: Taylor, Rosemary N., Damian J. McEntegart, and Eleanor C. Stillman. "Statistical techniques to detect fraud and other data irregularities in clinical questionnaire data." Drug information journal 36.1 (2002): 115-125.

In fraud they make a distinction between omission/commission and fabrication, falsification and plagiarism. **Explain** and **exemplify** the differences. **(1 pt for each correct)**

APPENDIX A

Standard deviation of the sample mean:

$$\frac{\sigma}{\sqrt{n}}$$

Variance of a sample:

$$s^2 = \frac{1}{n-1}\sum(X_i - \bar{X})^2$$

Pooled variance of a sample:

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$

Z-test statistic:

$$z = \overline{(X} - \mu)\ /\sigma$$

T test statistic:

One-sample t-test

$$T = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$df = n - 1$

Two-sample t-test

$$T = \frac{\bar{X}_1 - \bar{X}_2 - diff}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$df = n_1 + n_2 - 2$

One-way analysis of variance structuring help

The commonly used normal linear models for a completely randomized experiment are:

$y_{i,j} = \mu_j + \varepsilon_{i,j}$ (the means model)

or

$y_{i,j} = \mu + \tau_j + \varepsilon_{i,j}$ (the effects model)

where

$i = 1, \ldots, I$ is an index over experimental units
$j = 1, \ldots, J$ is an index over treatment groups
I_j is the number of experimental units in the jth treatment group
$I = \sum_j I_j$ is the total number of experimental units
$y_{i,j}$ are observations
μ_j is the mean of the observations for the jth treatment group
μ is the grand mean of the observations
τ_j is the jth treatment effect, a deviation from the grand mean
$\sum \tau_j = 0$
$\mu_j = \mu + \tau_j$
$\varepsilon \sim N(0, \sigma^2)$, $\varepsilon_{i,j}$ are normally distributed zero-mean random errors.

Means model calculations:

	Lists of Group Observations							
	1	2	$\cdots$	j	$\cdots$	J		
1	y_{11}	y_{12}				y_{1J}		
2	y_{21}	y_{22}				y_{2J}		
3								
$\vdots$								
i				y_{ij}				
$\vdots$								
	$y_{I_1 1}$	$\cdots$						
		$y_{I_2 2}$	$\cdots$					
	Group Summary Statistics						**Grand Summary Statistics**	
# Observed	I_1	I_2	$\cdots$	I_j	$\cdots$	I_J	# Observed	$I = \sum I_j$
Sum				$\sum_i y_{ij}$			Sum	$\sum_j \sum_i y_{ij}$
Sum Sq				$\sum_i (y_{ij})^2$			Sum Sq	$\sum_j \sum_i (y_{ij})^2$
Mean	m_1	$\cdots$		m_j	$\cdots$	m_J	Mean	m
Variance	s_1^2	$\cdots$		s_j^2	$\cdots$	s_J^2	Variance	s^2

Comparing model to summaries: $\mu = m$ and $\mu_j = m_j$. The grand mean and grand variance are computed from the grand sums, not from group means and variances.

One-way ANOVA for means model:

Source of variation	Sums of squares Explanatory SS[4]	Sums of squares Computational SS[5]	Degrees of freedom DF	Mean square MS	F
Treatments	$\sum_{Treatments} I_j (m_j - m)^2$	$\sum_j \frac{(\sum_i y_{ij})^2}{I_j} - \frac{(\sum_j \sum_i y_{ij})^2}{I}$	$J - 1$	$\frac{SS_{Treatment}}{DF_{Treatment}}$	$\frac{MS_{Treatment}}{MS_{Error}}$
Error	$\sum_{Treatments} (I_j - 1) s_j^2$	$\sum_j \sum_i y_{ij}^2 - \sum_j \frac{(\sum_i y_{ij})^2}{I_j}$	$I - J$	$\frac{SS_{Error}}{DF_{Error}}$	
Total	$\sum_{Observations} (y_{ij} - m)^2$	$\sum_j \sum_i y_{ij}^2 - \frac{(\sum_j \sum_i y_{ij})^2}{I}$	$I - 1$		

MS_{Error} is the estimate of variance corresponding to σ^2 of the model.

The tables in the course book are based on the effects model:

Table 8.5. Analysis of variance table for one-factor experiments

COMPONENT	SUM OF SQUARES	PERCENTAGE VARIATION	DEGREES OF FREEDOM	MEAN SQUARE	F CALCULATION	F TABLE
Y	$SSY = \sum Y_{ij}^2$		N			
$\bar{Y}_{\bullet\bullet}$	$SSO = N\mu^2$		1			
$Y - \bar{Y}_{\bullet\bullet}$	$SST = SSY - SSO$	100	N-1			
A	$SSA = \sum r_j \alpha_i^2$	$100\left(\frac{SSA}{SST}\right)$	k-1	$MSA = \frac{SSA}{k-1}$	$\frac{MSA}{MSE}$	$F_{[1-\alpha;a-1,(N-k)]}$
e	$SSE = SST - SSA$	$100\left(\frac{SSE}{SST}\right)$	N-k	$MSE = \frac{SSE}{(N-k)}$		

Table 10.4 Analysis of variance table for two factors

Component	Sum of squares	Degrees of freedom	Mean square	F- Computed.	F-Table
Y	$SSY = \sum Y_{ij}^2$	abr			
$\overline{Y}..$	$SSO = abr\mu^2$	1			
$Y - \overline{Y}..$	SST = SSY– SSO	abr-1			
A	$SSA = br\sum \alpha_i^2$	a-1	$MSA = \frac{SSA}{a-1}$	$\frac{MSA}{MSE}$	$F_{[1-\alpha;(a-1),ab(r-1)]}$
B	$SSB = ar\sum \beta_j^2$	b-1	$MSB = \frac{SSB}{b-1}$	$\frac{MSB}{MSE}$	$F_{[1-\alpha;(b-1),ab(r-1)]}$
AB	$SSAB = r\sum \alpha\beta_{ij}^2$	(a-1)(b-1)	$MSAB = \frac{SSAB}{(a-1)(b-1)}$	$\frac{MSAB}{MSE}$	$F_{[1-\alpha;(a-1)(b-1),ab(r-1)]}$
e	$SSE = \sum e_{ijk}^2$	ab(r-1)	$MSE = \frac{SSE}{ab(r-1)}$		

abr = I (in means model) = the total number of experimental units (people) in the study in this case.
r = number of replications (individuals) in each group.

abr-1 = (a-1) + (b-1) + (a-1)(b-1) + ab(r-1)

Table 9.4. Analysis of variance by one factor and one block variable

Component	Sum of Squares	Degrees of Freedom	Mean Square	F-Computed	F-Table
Y	$SSY = \sum y_{ij}^2$	ab			
$\overline{Y}..$	$SS0 = ab\mu^2$	1			
$Y - \overline{Y}..$	$SST = SSY - SS0$	ab-1			
A	$SSA = b\sum \alpha_i^2$	a-1	$MSA = \frac{SSA}{a-1}$	$\frac{MSA}{MSE}$	$F_{[1-\alpha;(a-1);(a-1)(b-1)]}$
B	$SSB = a\sum \beta_j^2$	b-1	$MSB = \frac{SSB}{b-1}$		
e	$SSE = \sum e_{ij}^2$	(a-1)(b-1)	$MSE = \frac{SSE}{(a-1)(b-1)}$		

Table 9.5. Analysis of variance by one factor and two block variables

$$SS_{total} = \sum_{i=1}^{a}\sum_{j=1}^{b}(y_{ijk} - \overline{y}_{..})^2 = \sum_{i=1}^{p}\sum_{j=1}^{p} y_{ijk}^2 - \frac{y_{...}^2}{p^2} \qquad SS_{row} = \sum_{i=1}^{p} p(\overline{y}_{i..} - \overline{y}_{..})^2 = \sum_{i=1}^{p} \frac{R_i^2}{p} - \frac{y_{...}^2}{p^2}$$

$$SS_{trt} = \sum_{j=1}^{p} p(\overline{y}_{.j.} - \overline{y}_{..})^2 = \sum_{j=1}^{p} \frac{T_j^2}{p} - \frac{y_{...}^2}{p^2} \qquad SS_{col} = \sum_{k=1}^{p} p(\overline{y}_{..k} - \overline{y}_{..})^2 = \sum_{k=1}^{p} \frac{C_k^2}{p} - \frac{y_{...}^2}{p^2}$$

Source of Variation	Sum of Squares	d.f.	Mean Square	F Ratio
Treatments	SS_{trt}	$p-1$	MS_{trt}	$\frac{MS_{trt}}{MS_E}$
Rows	SS_{row}	$p-1$	MS_{row}	
Columns	SS_{col}	$p-1$	MS_{col}	
Error	SS_E	$(p-1)(p-2)$	MS_E	
Total	SS_{total}	p^2-1		

ANOVA for nested design (corrected, i.e. the course book is wrong):

Correction to the formulas

Table 11.4. Analysis of variance for the data of example 12.1

Source of variation	Sum of squares	Degrees of freedom	Mean square	F-Computed	F-Table
A	$SSA = br\sum_{i=1}^{a}(\bar{y}_{i..} - \bar{y}_{...})^2$	a–1	$MSA = \frac{SSA}{a-1}$	$\frac{MSA}{\quad}$	$F_{[1-\alpha;(a-1),ab(r-1)]}$
B within A	$SSB(A) = r\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{y}_{ij.} - \bar{y}_{i..})^2$	a(b–1)	$MSB(A) = \frac{SSB}{a(b-1)}$	$\frac{MSB(A)}{\quad}$	$F_{[1-\alpha;a(b-1),ab(r-1)]}$
Error	$SSE = \sum_{k=1}^{r}\sum_{i=1}^{a}\sum_{j=1}^{b}$	ab(r–1)	$MSE = \frac{SSE}{ab(r-1)}$		
Total	$SST = \sum_{k=1}^{r}\sum_{i=1}^{a}\sum_{j=1}^{b}(y_{ijk} - \bar{y}_{...})^2$	abr–1			

The correct SSE is:

$$\sum_i \sum_j \sum_k (Y_{ijk} - \bar{Y}_{ij\cdot})^2$$

And, to get the F value you of course divide by MSE.

ANOVA for mixed design (Blocked Factorial Design)

Component	Sum of squares	Degrees of freedom	Mean square	F- Computed	F-Table (α=0.99)
Between groups	$SS_{between-groups}$	3			
Groups or ABC	$SSABC = 2^k r\, C_{ABC}^2$	1	MSABC=SSABC	$\frac{MSABC}{MSEbetween-groups}$	$F_{[1-\alpha;\,1,\,2]}$
Error	$SSE_{between-groups}$= Difference	2	$MSEbetween-groups = \frac{SSEbetween-groups}{2}$		
Within groups					
A	$SSA = 2^k r\, C_A^2$	2^kr-4	MSA= SSA	$\frac{MSA}{MSEwithin-groups}$	$F_{[1-\alpha;\,1,\,2^k r-10]}$
B	$SSB = 2^k r\, C_B^2$	1	MSB = SSB	$\frac{MSB}{MSEwithin-groups}$	
C	$SSC = 2^k r\, C_C^2$	1	MSC = SSC	$\frac{MSC}{MSEwithin-groups}$	
AB	$SSAB = 2^k r\, C_{AB}^2$	1	MSAB =SSAB	$\frac{MSAB}{MSEwithin-groups}$	
AC	$SSAC = 2^k r\, C_{AC}^2$	1	MSAC = SSAC	$\frac{MSAC}{MSEwithin-groups}$	
BC	$SSBC = 2^k r\, C_{BC}^2$	1	MSBC= SSBC	$\frac{MSBC}{MSEwithin-groups}$	
Error	$SSE_{within-groups}$= Difference	2^kr-10			

$$R_A^2 = \frac{SS_A}{SS_{Total}}$$
$$R_B^2 = \frac{SS_B}{SS_{Total}}$$

$$R_{AB}^2 = \frac{SS_{AB}}{SS_{Total}}$$ Etc.

APPENDIX B (*t* Distribution)

One Sided	75%	80%	85%	90%	95%	97.5%	99%	99.5%	99.75%	99.9%	99.95%
Two Sided	50%	60%	70%	80%	90%	95%	98%	99%	99.5%	99.8%	99.9%
1	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
120	0.677	0.845	1.041	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

APPENDIX C (*F* Distribution)

TABLE C.5
Upper Percentage Points of the *F* Distribution

df for de-nomi-nator	α	*df for numerator*											
		1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
1	.25	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32	9.36	9.41
	.10	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	60.2	60.5	60.7
	.05	161	200	216	225	230	234	237	239	241	242	243	244
2	.25	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38	3.39	3.39
	.10	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.40	9.41
	.05	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4
	.01	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4
3	.25	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.44	2.45	2.45
	.10	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.22
	.05	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74
	.01	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	27.1
4	.25	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08
	.10	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.91	3.90
	.05	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91
	.01	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.4
5	.25	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
	.10	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.28	3.27
	.05	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.71	4.68
	.01	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.96	9.89
6	.25	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.78	1.77	1.77	1.77	1.77
	.10	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.92	2.90
	.05	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00
	.01	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72
7	.25	1.57	1.70	1.72	1.72	1.71	1.71	1.70	1.70	1.69	1.69	1.69	1.68
	.10	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.68	2.67
	.05	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57
	.01	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.54	6.47
8	.25	1.54	1.66	1.67	1.66	1.66	1.65	1.64	1.64	1.63	1.63	1.63	1.62
	.10	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.52	2.50
	.05	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28
	.01	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.73	5.67
9	.25	1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.58
	.10	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.40	2.38
	.05	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07
	.01	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.18	5.11
10	.25	1.49	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.56	1.55	1.55	1.54
	.10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.30	2.28
	.05	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91
	.01	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71

TABLE C.5 (continued)

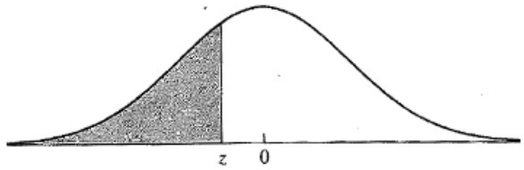
df for denominator	α	df for numerator											
		1	2	3	4	5	6	7	8	9	10	11	12
11	.25	1.47	1.58	1.58	1.57	1.56	1.55	1.54	1.53	1.53	1.52	1.52	1.51
	.10	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.23	2.21
	.05	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79
	.01	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.46	4.40
12	.25	1.46	1.56	1.56	1.55	1.54	1.53	1.52	1.51	1.51	1.50	1.50	1.49
	.10	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.17	2.15
	.05	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69
	.01	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.22	4.16
13	.25	1.45	1.55	1.55	1.53	1.52	1.51	1.50	1.49	1.49	1.48	1.47	1.47
	.10	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.12	2.10
	.05	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60
	.01	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	4.02	3.96
14	.25	1.44	1.53	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.46	1.45
	.10	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.08	2.05
	.05	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53
	.01	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.86	3.80
15	.25	1.43	1.52	1.52	1.51	1.49	1.48	1.47	1.46	1.46	1.45	1.44	1.44
	.10	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.04	2.02
	.05	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48
	.01	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67
16	.25	1.42	1.51	1.51	1.50	1.48	1.47	1.46	1.45	1.44	1.44	1.44	1.43
	.10	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	2.01	1.99
	.05	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42
	.01	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.62	3.55
17	.25	1.42	1.51	1.50	1.49	1.47	1.46	1.45	1.44	1.43	1.43	1.42	1.41
	.10	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.98	1.96
	.05	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38
	.01	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.52	3.46
18	.25	1.41	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.42	1.41	1.40
	.10	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.96	1.93
	.05	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34
	.01	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.43	3.37
19	.25	1.41	1.49	1.49	1.47	1.46	1.44	1.43	1.42	1.41	1.41	1.40	1.40
	.10	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.94	1.91
	.05	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31
	.01	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.36	3.30
20	.25	1.40	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.39
	.10	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.92	1.89
	.05	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28
	.01	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23

TABLE C.5 (continued)

df for denominator	α	df for numerator											
		1	2	3	4	5	6	7	8	9	10	11	12
22	.25	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.38	1.37
	.10	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.88	1.86
	.05	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23
	.01	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12
24	.25	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.37	1.36
	.10	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.85	1.83
	.05	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.21	2.18
	.01	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03
26	.25	1.38	1.46	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.37	1.36	1.35
	.10	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.84	1.81
	.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15
	.01	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02	2.96
28	.25	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34
	.10	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.81	1.79
	.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12
	.01	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90
30	.25	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.35	1.34
	.10	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77
	.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
	.01	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84
40	.25	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31
	.10	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.73	1.71
	.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
	.01	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66
60	.25	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.29
	.10	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66
	.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
	.01	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50
120	.25	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.27	1.26
	.10	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.62	1.60
	.05	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.87	1.83
	.01	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34
200	.25	1.33	1.39	1.38	1.36	1.34	1.32	1.31	1.29	1.28	1.27	1.26	1.25
	.10	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66	1.63	1.60	1.57
	.05	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80
	.01	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.34	2.27
∞	.25	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.24
	.10	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55
	.05	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75
	.01	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18

APPENDIX D (*Z* Distribution)

TABLE A.2 Cumulative normal distribution (z table)



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
−3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

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