A1 $f < \dfrac{f_s}{2} = 50\ kHz$

A2 $|G(j\omega)| = \dfrac{1}{2}$ för $\omega = 3\sqrt{3}$ rad/s

A3 $T = \dfrac{2\pi}{3\omega_0}$ s

A4 $k = 4$ och 28

A5 $y[0] = 2$

A6 $H(j\omega) = \dfrac{j\omega}{j\omega + 9}$

A7 C_k ii)   icke periodisk
              iv)   en diskret sekvens

A8 $X_1 - C$ $X_3 - D$
     $X_2 - B$ $X_4 - A$

A9   iv)   Polernas belopp < 1

A10 $\Omega = \omega T = 0{,}05\,\pi$ rad

B11

$x(t) = e^{-t} u(t) \quad \overset{\mathcal{L}}{\longmapsto} \quad X(s) = \dfrac{1}{s+1}$

$y(t) = e^{-2t} \cos(3t) u(t) \quad \overset{\mathcal{L}}{\longmapsto} \quad Y(s) = \dfrac{s+2}{(s+2)^2 + 9}$

$H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{(s+2)(s+1)}{(s+2)^2 + 9} = \dfrac{s^2 + 3s + 2}{s^2 + 4s + 13}$

$H(s) = \dfrac{s^2 + 4s + 13 - 4s - 13 + 3s + 2}{s^2 + 4s + 13} = 1 + \dfrac{-s - 11}{(s+2)^2 + 9}$

$H(s) = 1 - \dfrac{s + 11}{(s+2)^2 + 9} = 1 - \dfrac{s + 2 + 9}{(s+2)^2 + 9} =$

$= 1 - \dfrac{s + 2}{(s+2)^2 + 3^2} - 3 \cdot \dfrac{3}{(s+2)^2 + 3^2}$

Impulssvar:

$h(t) = \mathcal{L}^{-1}\{H(s)\} = \delta(t) - e^{-2t}\left(\cos 3t + 3 \sin 3t\right) u(t)$

B12

$$y[n] - \frac{1}{2}y[n-1] = x[n] + \frac{7}{6}x[n-1]$$

Z-transf.

$$Y(z)\left(1 - 0.5z^{-1}\right) = X(z)\left(1 + \frac{7}{6}z^{-1}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{7}{6}z^{-1}}{1 - 0.5z^{-1}} = \frac{z + \frac{7}{6}}{z - 0.5}$$

$$x[n] = \left(-\frac{1}{3}\right)^n u[n] \xrightarrow{\ z\ } X(z) = \frac{1}{1 + \frac{1}{3}z^{-1}} = \frac{z}{z + \frac{1}{3}}$$

$$Y(z) = H(z) \cdot X(z) = \frac{\left(z + \frac{7}{6}\right)z}{(z - 0.5)\left(z + \frac{1}{3}\right)}$$

$$\frac{Y(z)}{z} = \frac{z + \frac{7}{6}}{(z - 0.5)\left(z + \frac{1}{3}\right)} = \left\{P.B.U\right\} = \frac{A}{z - 0.5} + \frac{B}{z + \frac{1}{3}}$$

$$z + \frac{7}{6} = A\left(z + \frac{1}{3}\right) + B(z - 0.5)$$

$$z = \frac{1}{2} \Rightarrow \frac{1}{2} + \frac{7}{6} = A\left(\frac{1}{2} + \frac{1}{3}\right); \quad \frac{10}{6} = A\left(\frac{5}{6}\right) \Rightarrow A = 2$$
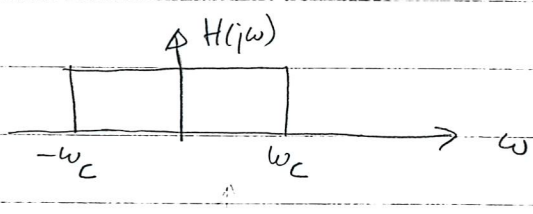
$$z = -\frac{1}{3} \Rightarrow -\frac{1}{3} + \frac{7}{6} = B\left(-\frac{1}{3} - \frac{1}{2}\right); \quad \frac{5}{6} = B\left(-\frac{5}{6}\right) \Rightarrow B = -1$$

$$Y(z) = 2 \cdot \frac{z}{z - 0.5} - \frac{z}{z + \frac{1}{3}}$$

$$y[n] = Z^{-1}\left\{Y(z)\right\} = \left(2 \cdot \left(\frac{1}{2}\right)^n - \left(-\frac{1}{3}\right)^n\right) u[n]$$
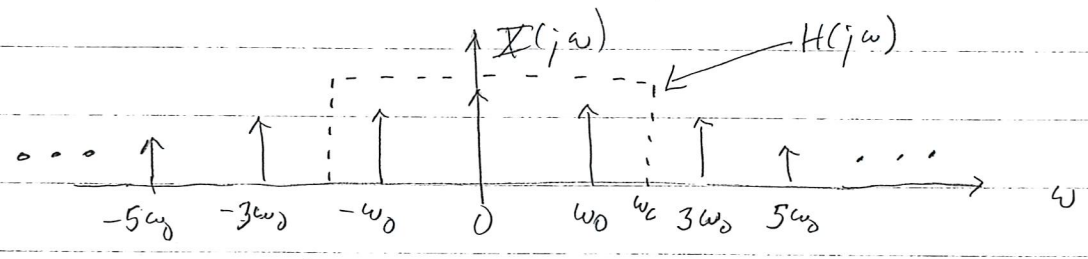
B13

$$h(t) = \frac{\sin(100\pi t)}{\pi t} \quad \xleftarrow{\quad \mathcal{F} \quad} \quad H(j\omega) = \begin{cases} 1, & |\omega| < 100\pi \\ 0, & |\omega| > 100\pi \end{cases}$$

$\omega_c = 100\pi$

Fourierserien: Endast grundton + DC-värde skall passera $H(j\omega)$

$$\omega_0 < \omega_c < 3\omega_0$$

$$\omega_0 = \frac{\pi}{L} \qquad , \qquad \omega_c = 100\pi$$

$\omega_0 < \omega_c$	$3\omega_0 > \omega_c$
$\frac{\pi}{L} < 100\pi$	$\omega_0 > \frac{\omega_c}{3}$
$L > \frac{1}{100}$	$\frac{\pi}{L} > \frac{100\pi}{3}$
	$L < \frac{3}{100}$

Svar: $\quad \frac{1}{100} < L < \frac{3}{100} \qquad [s]$