Course name:
Empirical software engineering DAT246/DIT278 (formerly DAT245)

Date:
August 28, 2018

Time:
am (08:30 –12:30)

Responsible teachers:
Richard Torkar, Lucas Gren, and David Mattos.

Teacher will visit the exam room.
Twice (at 09:30 and at 11:30)

Telephone number:
0739-882010 (Lucas Gren)

Examiner:
Richard Torkar

Aids:
Approved calculator by Chalmers.

Grades:
Maximum points: 35
Chalmers:
Grade 3: 18 – 23
Grade 4: 24 – 30
Grade 5: 31 – 35
GU:
Grade G: 18 - 30
Grade VG: 31 - 35

Appeal of Grading:
September 25th at 11.00 – 12:00 (Lucas' office in Jupiter, floor 4, room 413)

Time and place for presentation of solutions:
September 26th at 11.00 – 12:00 (Lucas' office in Jupiter, floor 4, room 413)

“In science, progress is possible. In fact, if one believes in Bayes' theorem, scientific progress is inevitable as predictions are made and as beliefs are tested and refined.”

— Nate Silver

Grades:
Maximum points: 35
Chalmers:
Grade 3: 18 – 23
Grade 4: 24 – 30
Grade 5: 31 – 35
GU:
Grade G: 18 - 30
Grade VG: 31 - 35

Question 1 (6pt)

In a set of experiments I (Lucas) am working on at the moment we wanted to assess if obsolete requirements, that are explicitly stated to be excluded from the effort estimates, have any impact on the estimates for the remaining requirements.

We randomly assigned the following three requirements specifications to three independent groups of students, i.e. a student was either given Task A, B, or C:

Task A

Your task is to estimate how long (in terms of weeks) it will take to implement the following requirements.

R1: The system shall receive uncompressed data and shall compress and save the data to desired JPEG size

R2: The maximum delay from a call answer is pressed to opened audio paths is XX ms

R3: The system shall have support for Time Shift (playback with delay)

R4: The system shall have a login function that consists of a username and a password.

The total estimated effort is: ______________ week(s)

Task B

Your task is to estimate how long (in terms of weeks) it will take to implement the following requirements.

R1: The system shall receive uncompressed data and shall compress and save the data to desired JPEG size

R2: The maximum delay from a call answer is pressed to opened audio paths is XX ms

R3: The system shall have support for Time Shift (playback with delay)

R4: The system shall have a login function that consists of a username and a password.

R5: It shall be possible to dedicate a host buffer in RAM that is configurable between XX to XX MB for HDD

The total estimated effort is: ______________ week(s)

Task C

Your task is to estimate how long (in terms of weeks) it will take to implement the following requirements.

R1: The system shall receive uncompressed data and shall compress and save the data to desired JPEG size

R2: The maximum delay from a call answer is pressed to opened audio paths is XX ms

R3: The system shall have support for Time Shift (playback with delay)

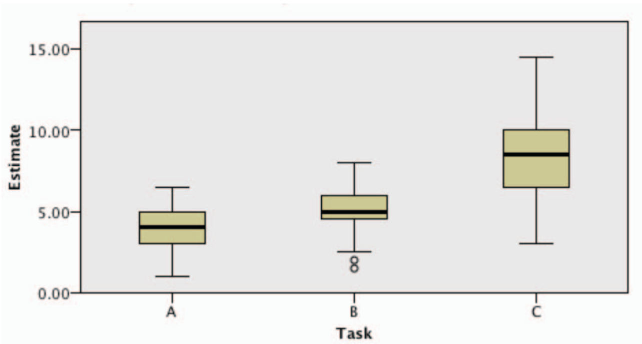
R4: The system shall have a login function that consists of a username and a password.

R5: It shall be possible to dedicate a host buffer in RAM that is configurable between XX to XX MB for HDD

Please note that **R5** should **NOT** be implemented

The total estimated effort is: ______________ week(s)

We plotted the data using boxplots:



a) Just by eyeballing the boxplot, do you think we have hetero- or homoscedasticity? (Briefly explain why) (1pt)
b) Could we run multiple t-tests comparing the three different groups? Why or why not? (2pt)
c) State the null and the alternative hypothesis if you would choose to run an ANOVA (1pt, 0.5 for each).
d) Describe how you would analyze and decide if the task group is explaining estimation variance (assuming all the assumptions you need are met). What type of test would run in order to see which task groups that are significantly different from each other? (2pt)

Question 2 (5pt)

Backtracking a bit, we need to make sure the assumptions of our tests are met. In the case of the study presented in Question 1 the Kolmogorov-Smirnov tests of normality for the groups Task A and B were:

- Test Statistic for group Task A=0.128, p=0.040,
- Test Statistic for group Task B=0.209, p=0.000, and
- Test Statistic for group Task C=0.108, p=0.198.

a) What is the null hypothesis of the Kolmogorov-Smirnov test? (It's interpreted in the same way as the Shapiro-Wilk test) (1pt)

b) What can be said about normality (using a confidence level of 95%) for the p-values in the Kolmogorov-Smirnov tests for group A and B above? (1pt)

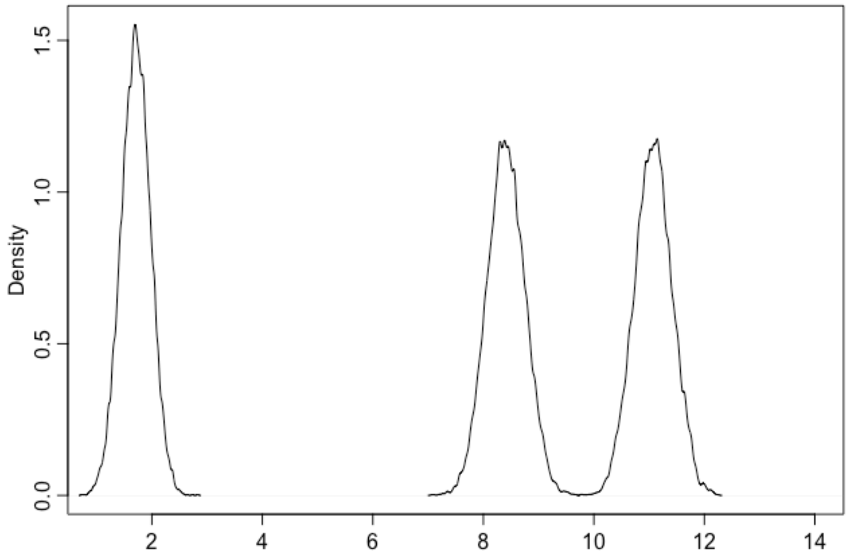
c) What can be said about normality (using a confidence level of 95%) in the case above, for group C? (1pt)

d) What can we do if we run into problems with our normality assumption? Name two things a researcher can do (2pt, 1pt for each suggestion).

Question 3 (8pt)

We replicated the study presented in the previous two Questions in an industry setting. Instead of checking the model for assumption the way we did before, we used a Bayesian way of defining the parameters. You don't need to know any details about Bayesian statistics, just that we then can obtain the *true* distributions of our parameters given how we believe it behaves backed up by our data.

The plot obtained for task groups A (left), B (middle), and C (right):



	Mean	StdDev	|0.89	0.89|
Task A	1.71	0.27	1.29	2.14
Task B	8.41	0.34	7.87	8.97
Task C	11.08	0.34	10.55	11.66

a) Based on the plot, do you believe that three different groups are normally distributed? Why? (1pt)
b) Why would you expect (according to what theorem) that these sampled mean values would be normally distributed? (1pt).

In order to assess the effect sizes we still created a linear model based on the parametric assumption and obtained the following result:

```
Call:
lm(formula = estimate ~ 1 + as.factor(task), data = y5)

Residuals:
   Min          1Q            Median         3Q       Max
-5.1200       -0.7308         0.2083         0.7692  2.3800
```

```
Coefficients:
                      Estimate Std. Error  t value   Pr(>|t|)
Task A                4.7308    0.2581     18.329    < 2e-16 ***
Task B                2.0609    0.3725     5.532     4.83e-07 ***
Task C                3.8892    0.3687     10.550    2.93e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.316 on 72 degrees of freedom
Multiple R-squared:  0.6077,      Adjusted R-squared:  0.5968
F-statistic: 55.76 on 2 and 72 DF,  p-value: 2.35e-15
```

c) Is there a significant overall effect? Why? (1pt)
d) Are there significant differences between the treatment (task) groups? Why? (Hint: the coefficients of the model do not tell you that, look at the plot above and argue for your case) (2pt)
e) What is effect size? (1pt)
f) What is the effect size above? What does that mean in relation to our research? (2pt).

Question 4 (4 pt)

In a frequentist approach, the concept of statistical power is often discussed. This concept is closely related to errors type I and type II.

a) What is statistical power? (1pt)

b) What is error type I, error type II? (1 pt)

c) How are they related when the sample size is constant? (1 pt)

d) How can you keep error type I controlled while reducing error type II in an experiment? (1pt)

Question 5 (5 pt)

a) In survey research, it is important that the sample is large enough. What problems can we encounter if it is not large enough? (2pt)

b) In a questionnaire, questions can be expressed as open or closed. Define and explain the pros and cons to each category. (3pt)

Question 6 (3 pt)

There are three ways, and three ways only, to ethically encourage participation in a study (Senese 1997). What are the three ways? Describe them briefly! (1pt for each)

Question 7 (2 pt)

When designing a survey one needs to take the reliability and validity of the survey instrument into consideration. **Explain** what reliability and validity means in this case (1pt each)

Question 8 (2pt)

In a study, researchers wanted to investigate the impact of three techniques (Factor *A*) and four different domain problems (Factor *B*) in the accuracy of an estimation method. A full factorial design was conducted with independent samples.

Component	Sum of squares (x 1000)	Degrees of freedom	Mean square (x 1000)	F- Computed	F-Table
Y		48			
$\overline{Y}_{..}$		1			
$Y - \overline{Y}_{..}$		47			
A	1033				
B	922.4				
AB	250.1				
e	800.7				

a) Complete the ANOVA table (5% significance level) (2pt).

APPENDIX A

Standard deviation of the sample mean:

$$\frac{\sigma}{\sqrt{n}}$$

Variance of a sample:

$$s^2 = \frac{1}{n-1}\sum(X_i - \bar{X})^2$$

Pooled variance of a sample:

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$

Z-test statistic:

$$z = (\overline{X} - \mu)\ /\sigma$$

T test statistic:

One-sample t-test

$$T = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}$$

df = *n* - 1

Two-sample t-test

$$T = \frac{\overline{X}_1 - \overline{X}_2 - diff}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$df = n_1 + n_2 - 2$

One-way analysis of variance structuring help

The commonly used normal linear models for a completely randomized experiment are:

$y_{i,j} = \mu_j + \varepsilon_{i,j}$ (the means model)

or

$y_{i,j} = \mu + \tau_j + \varepsilon_{i,j}$ (the effects model)

where

$i = 1, \ldots, I$ is an index over experimental units
$j = 1, \ldots, J$ is an index over treatment groups
I_j is the number of experimental units in the jth treatment group
$I = \sum_j I_j$ is the total number of experimental units
$y_{i,j}$ are observations
μ_j is the mean of the observations for the jth treatment group
μ is the grand mean of the observations
τ_j is the jth treatment effect, a deviation from the grand mean
$\sum \tau_j = 0$
$\mu_j = \mu + \tau_j$
$\varepsilon \sim N(0, \sigma^2)$, $\varepsilon_{i,j}$ are normally distributed zero-mean random errors.

Means model calculations:

	Lists of Group Observations					
	1	2	$\cdots$	j	$\cdots$	J
1	y_{11}	y_{12}				y_{1J}
2	y_{21}	y_{22}				y_{2J}
3						
$\vdots$						
i				y_{ij}		
$\vdots$						
	$y_{I_1 1}$	$\cdots$				
		$y_{I_2 2}$	$\cdots$			

	Group Summary Statistics							Grand Summary Statistics
# Observed	I_1	I_2	$\cdots$	I_j	$\cdots$	I_J	# Observed	$I = \sum I_j$
Sum				$\sum_i y_{ij}$			Sum	$\sum_j \sum_i y_{ij}$
Sum Sq				$\sum_i (y_{ij})^2$			Sum Sq	$\sum_j \sum_i (y_{ij})^2$
Mean	m_1	$\cdots$		m_j	$\cdots$	m_J	Mean	m
Variance	s_1^2	$\cdots$		s_j^2	$\cdots$	s_J^2	Variance	s^2

Comparing model to summaries: $\mu = m$ and $\mu_j = m_j$. The grand mean and grand variance are computed from the grand sums, not from group means and variances.

One-way ANOVA for means model:

Source of variation	Sums of squares Explanatory SS[4]	Sums of squares Computational SS[5]	Degrees of freedom DF	Mean square MS	F
Treatments	$\sum_{Treatments} I_j (m_j - m)^2$	$\sum_j \frac{(\sum_i y_{ij})^2}{I_j} - \frac{(\sum_j \sum_i y_{ij})^2}{I}$	$J-1$	$\frac{SS_{Treatment}}{DF_{Treatment}}$	$\frac{MS_{Treatment}}{MS_{Error}}$
Error	$\sum_{Treatments} (I_j - 1) s_j^2$	$\sum_j \sum_i y_{ij}^2 - \sum_j \frac{(\sum_i y_{ij})^2}{I_j}$	$I-J$	$\frac{SS_{Error}}{DF_{Error}}$	
Total	$\sum_{Observations} (y_{ij} - m)^2$	$\sum_j \sum_i y_{ij}^2 - \frac{(\sum_j \sum_i y_{ij})^2}{I}$	$I-1$		

MS_{Error} is the estimate of variance corresponding to σ^2 of the model.

The tables in the course book are based on the effects model:

Table 8.5. Analysis of variance table for one-factor experiments

COMPONENT	SUM OF SQUARES	PERCENTAGE VARIATION	DEGREES OF FREEDOM	MEAN SQUARE	F CALCULATION	F TABLE
Y	$SSY = \sum Y_{ij}^2$		N			
$\bar{Y}_{..}$	$SSO = N\mu^2$		1			
$Y - \bar{Y}_{..}$	$SST = SSY - SSO$	100	N-1			
A	$SSA = \sum r_j \alpha_i^2$	$100\left(\frac{SSA}{SST}\right)$	k-1	$MSA = \frac{SSA}{k-1}$	$\frac{MSA}{MSE}$	$F_{[1-\alpha;a-1,(N-k)]}$
e	$SSE = SST - SSA$	$100\left(\frac{SSE}{SST}\right)$	N-k	$MSE = \frac{SSE}{(N-k)}$		

Table 10.4 Analysis of variance table for two factors

Component	Sum of squares	Degrees of freedom	Mean square	F- Computed.	F-Table
Y	$SSY = \sum Y_{ij}^2$	abr			
$\overline{Y}..$	$SSO = abr\mu^2$	1			
$Y - \overline{Y}..$	$SST = SSY - SSO$	abr-1			
A	$SSA = br\sum \alpha_i^2$	a-1	$MSA = \frac{SSA}{a-1}$	$\frac{MSA}{MSE}$	$F_{[1-\alpha;(a-1),ab(r-1)]}$
B	$SSB = ar\sum \beta_j^2$	b-1	$MSB = \frac{SSB}{b-1}$	$\frac{MSB}{MSE}$	$F_{[1-\alpha;(b-1),ab(r-1)]}$
AB	$SSAB = r\sum \alpha\beta_{ij}^2$	(a-1)(b-1)	$MSAB = \frac{SSAB}{(a-1)(b-1)}$	$\frac{MSAB}{MSE}$	$F_{[1-\alpha;(a-1)(b-1),ab(r-1)]}$
e	$SSE = \sum e_{ijk}^2$	ab(r-1)	$MSE = \frac{SSE}{ab(r-1)}$		

abr = I (in means model) = the total number of experimental units (people) in the study in this case.
r = number of replications (individuals) in each group.

abr-1 = (a-1) + (b-1) + (a-1)(b-1) + ab(r-1)

Table 9.4. Analysis of variance by one factor and one block variable

Component	Sum of Squares	Degrees of Freedom	Mean Square	F-Computed	F-Table
Y	$SSY = \sum y_{ij}^2$	ab			
$\overline{Y}..$	$SS0 = ab\mu^2$	1			
$Y - \overline{Y}..$	$SST = SSY - SS0$	ab-1			
A	$SSA = b\sum \alpha_i^2$	a-1	$MSA = \frac{SSA}{a-1}$	$\frac{MSA}{MSE}$	$F_{[1-\alpha;(a-1);(a-1)(b-1)]}$
B	$SSB = a\sum \beta_j^2$	b-1	$MSB = \frac{SSB}{b-1}$		
e	$SSE = \sum e_{ij}^2$	(a-1)(b-1)	$MSE = \frac{SSE}{(a-1)(b-1)}$		

Table 9.5. Analysis of variance by one factor and two block variables

$$SS_{total} = \sum_{i=1}^{a}\sum_{j=1}^{b}(y_{ijk} - \overline{y}_{..})^2 = \sum_{i=1}^{p}\sum_{j=1}^{p} y_{ijk}^2 - \frac{y_{...}^2}{p^2}$$

$$SS_{row} = \sum_{i=1}^{p} p(\overline{y}_{i..} - \overline{y}_{..})^2 = \sum_{i=1}^{p} \frac{R_i^2}{p} - \frac{y_{...}^2}{p^2}$$

$$SS_{trt} = \sum_{j=1}^{p} p(\overline{y}_{.j.} - \overline{y}_{..})^2 = \sum_{j=1}^{p} \frac{T_j^2}{p} - \frac{y_{...}^2}{p^2}$$

$$SS_{col} = \sum_{k=1}^{p} p(\overline{y}_{..k} - \overline{y}_{..})^2 = \sum_{k=1}^{p} \frac{C_k^2}{p} - \frac{y_{...}^2}{p^2}$$

Source of Variation	Sum of Squares	d.f.	Mean Square	F Ratio
Treatments	SS_{trt}	$p-1$	MS_{trt}	$\frac{MS_{trt}}{MS_E}$
Rows	SS_{row}	$p-1$	MS_{row}	
Columns	SS_{col}	$p-1$	MS_{col}	
Error	SS_E	$(p-1)(p-2)$	MS_E	
Total	SS_{total}	p^2-1		

ANOVA for nested design (corrected, i.e. the course book is wrong):

Correction to the formulas

Table 11.4. Analysis of variance for the data of example 12.1

Source of variation	Sum of squares	Degrees of freedom	Mean square	F-Computed	F-Table
A	$SSA = br\sum_{i=1}^{a}(\bar{y}_{i..} - \bar{y}_{...})^2$	a–1	$MSA = \frac{SSA}{a-1}$	$\frac{MSA}{\quad}$	$F_{[1-\alpha;(a-1),ab(r-1)]}$
B within A	$SSB(A) = r\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{y}_{ij.} - \bar{y}_{i..})^2$	a(b–1)	$MSB(A) = \frac{SSB}{a(b-1)}$	$\frac{MSB(A)}{\quad}$	$F_{[1-\alpha;a(b-1),ab(r-1)]}$
Error	$SSE = \sum_{k=1}^{r}\sum_{i=1}^{a}\sum_{j=1}^{b}$	ab(r–1)	$MSE = \frac{SSE}{ab(r-1)}$		
Total	$SST = \sum_{k=1}^{r}\sum_{i=1}^{a}\sum_{j=1}^{b}(y_{ijk} - \bar{y}_{...})^2$	abr–1			

The correct SSE is:

$$\sum_i \sum_j \sum_k (Y_{ijk} - \bar{Y}_{ij.})^2$$

And, to get the F value you of course divide by MSE.

ANOVA for mixed design (Blocked Factorial Design)

Component	Sum of squares	Degrees of freedom	Mean square	F- Computed	F-Table (α=0.99)
Between groups	$SS_{between-groups}$	3			
Groups or ABC	$SSABC = 2^k r\, C_{ABC}^2$	1	MSABC=SSABC	$\frac{MSABC}{MSEbetween-groups}$	$F_{[1-\alpha;\, 1,\, 2]}$
Error	$SSE_{between-groups}$= Difference	2	$MSEbetween-groups = \frac{SSEbetween-groups}{2}$		
Within groups					
A	$SSA = 2^k r\, C_A^2$	$2^k r$-4	MSA= SSA	$\frac{MSA}{MSEwithin-groups}$	$F_{[1-\alpha;\, 1,\, 2^k r-10]}$
B	$SSB = 2^k r\, C_B^2$	1	MSB = SSB	$\frac{MSB}{MSEwithin-groups}$	
C	$SSC = 2^k r\, C_C^2$	1	MSC = SSC	$\frac{MSC}{MSEwithin-groups}$	
AB	$SSAB = 2^k r\, C_{AB}^2$	1	MSAB =SSAB	$\frac{MSAB}{MSEwithin-groups}$	
AC	$SSAC = 2^k r\, C_{AC}^2$	1	MSAC = SSAC	$\frac{MSAC}{MSEwithin-groups}$	
BC	$SSBC = 2^k r\, C_{BC}^2$	1	MSBC= SSBC	$\frac{MSBC}{MSEwithin-groups}$	
Error	$SSE_{within-groups}$= Difference	$2^k r$-10			

$$R_A^2 = \frac{SS_A}{SS_{Total}}$$

$$R_B^2 = \frac{SS_B}{SS_{Total}}$$

$$R_{AB}^2 = \frac{SS_{AB}}{SS_{Total}}$$ Etc.

APPENDIX B (*t* Distribution)

One Sided	75%	80%	85%	90%	95%	97.5%	99%	99.5%	99.75%	99.9%	99.95%
Two Sided	50%	60%	70%	80%	90%	95%	98%	99%	99.5%	99.8%	99.9%
1	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
120	0.677	0.845	1.041	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

APPENDIX C (*F* Distribution)

TABLE C.5
Upper Percentage Points of the *F* Distribution

df for de-nomi-nator	α	*df for numerator* 1	2	3	4	5	6	7	8	9	10	11	12
1	.25	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32	9.36	9.41
	.10	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	60.2	60.5	60.7
	.05	161	200	216	225	230	234	237	239	241	242	243	244
2	.25	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38	3.39	3.39
	.10	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.40	9.41
	.05	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4
	.01	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4
3	.25	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.44	2.45	2.45
	.10	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.22
	.05	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74
	.01	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	27.1
4	.25	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08
	.10	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.91	3.90
	.05	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91
	.01	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.4
5	.25	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
	.10	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.28	3.27
	.05	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.71	4.68
	.01	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.96	9.89
6	.25	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.78	1.77	1.77	1.77	1.77
	.10	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.92	2.90
	.05	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00
	.01	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72
7	.25	1.57	1.70	1.72	1.72	1.71	1.71	1.70	1.70	1.69	1.69	1.69	1.68
	.10	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.68	2.67
	.05	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57
	.01	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.54	6.47
8	.25	1.54	1.66	1.67	1.66	1.66	1.65	1.64	1.64	1.63	1.63	1.63	1.62
	.10	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.52	2.50
	.05	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28
	.01	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.73	5.67
9	.25	1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.58
	.10	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.40	2.38
	.05	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07
	.01	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.18	5.11
10	.25	1.49	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.56	1.55	1.55	1.54
	.10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.30	2.28
	.05	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91
	.01	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71

TABLE C.5 (continued)

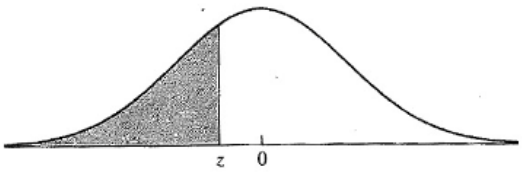
df for de-nomi-nator	α	*df for numerator* 1	2	3	4	5	6	7	8	9	10	11	12
	.25	1.47	1.58	1.58	1.57	1.56	1.55	1.54	1.53	1.53	1.52	1.52	1.51
11	.10	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.23	2.21
	.05	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79
	.01	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.46	4.40
	.25	1.46	1.56	1.56	1.55	1.54	1.53	1.52	1.51	1.51	1.50	1.50	1.49
12	.10	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.17	2.15
	.05	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69
	.01	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.22	4.16
	.25	1.45	1.55	1.55	1.53	1.52	1.51	1.50	1.49	1.49	1.48	1.47	1.47
13	.10	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.12	2.10
	.05	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60
	.01	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	4.02	3.96
	.25	1.44	1.53	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.46	1.45
14	.10	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.08	2.05
	.05	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53
	.01	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.86	3.80
	.25	1.43	1.52	1.52	1.51	1.49	1.48	1.47	1.46	1.46	1.45	1.44	1.44
15	.10	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.04	2.02
	.05	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48
	.01	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67
	.25	1.42	1.51	1.51	1.50	1.48	1.47	1.46	1.45	1.44	1.44	1.44	1.43
16	.10	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	2.01	1.99
	.05	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42
	.01	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.62	3.55
	.25	1.42	1.51	1.50	1.49	1.47	1.46	1.45	1.44	1.43	1.43	1.42	1.41
17	.10	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.98	1.96
	.05	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38
	.01	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.52	3.46
	.25	1.41	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.42	1.41	1.40
18	.10	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.96	1.93
	.05	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34
	.01	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.43	3.37
	.25	1.41	1.49	1.49	1.47	1.46	1.44	1.43	1.42	1.41	1.41	1.40	1.40
19	.10	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.94	1.91
	.05	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31
	.01	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.36	3.30
	.25	1.40	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.39
20	.10	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.92	1.89
	.05	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28
	.01	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23

TABLE C.5 (continued)

df for denominator	α	df for numerator 1	2	3	4	5	6	7	8	9	10	11	12
	.25	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.38	1.37
22	.10	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.88	1.86
	.05	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23
	.01	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12
	.25	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.37	1.36
24	.10	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.85	1.83
	.05	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.21	2.18
	.01	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03
	.25	1.38	1.46	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.37	1.36	1.35
26	.10	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.84	1.81
	.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15
	.01	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02	2.96
	.25	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34
28	.10	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.81	1.79
	.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12
	.01	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90
	.25	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.35	1.34
30	.10	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77
	.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
	.01	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84
	.25	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31
40	.10	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.73	1.71
	.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
	.01	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66
	.25	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.29
60	.10	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66
	.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
	.01	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50
	.25	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.27	1.26
120	.10	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.62	1.60
	.05	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.87	1.83
	.01	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34
	.25	1.33	1.39	1.38	1.36	1.34	1.32	1.31	1.29	1.28	1.27	1.26	1.25
200	.10	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66	1.63	1.60	1.57
	.05	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80
	.01	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.34	2.27
	.25	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.24
∞	.10	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55
	.05	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75
	.01	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18

APPENDIX D (*Z* Distribution)

TABLE A.2 Cumulative normal distribution (*z* table)



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
−3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641