Course name:
Empirical software engineering DAT246/DIT278 (formerly DAT245)

Date:
April 5, 2018

Time:
am (08:30 –12:30)

Responsible teachers:
Richard Torkar, David Mattos, Lucas Gren and Katja Tuma

Teacher will visit the exam room.
Twice (at 09:30 and at 11:00)

Telephone number:
0723526147 (David Mattos)

Examiner:
Richard Torkar

Aids:
Approved calculator by Chalmers.

Grades:
Maximum points: 35
Chalmers:
Pass : 18
Grade 3: 18 – 23
Grade 4: 24 –30
Grade 5: 31 –35
GU:
Grade G: 18
Grade VG: >= 31

Appeal of Grading:
April 26th at 13.00 – 15:00 (David's office in Jupiter, floor 4, room 457)

Time and place for presentation of solutions:
April 26th at 13.00 – 15:00 (David's office in Jupiter, floor 4, room 457)

"He uses statistics as a drunken man uses lamp posts -
for support rather than for illumination."
– Andew Lang

Grades:
Maximum points: 35
Chalmers:
Pass : 18
Grade 3: 18 – 23
Grade 4: 24 –30
Grade 5: 31 –35
GU:
Grade G: 18
Grade VG: >= 31

Question 1 (6pt)

The χ-bandit problem refers to the problem of optimizing a parameter (determining the best value) in the continuous space when the distribution of the rewards is unknown. This is an important problem in machine learning applications as it allows the optimization and tuning of hyper-parameters. For example, the weights of a neural network are determined through training with a training dataset. However, determining the number of layers or number of blocks in each layer is a hyper-parameter tuning problem.

A group of researchers developed a new algorithm called HOO (Hierarchical Optimistic Optimization) to solve the problem of the χ-bandit problem. They claim that their algorithm converges closer to the solution when compared to another algorithm called BAST (Bandit Algorithm for Smooth Trees).

To check whether this algorithm HOO is better than the BAST, you decided to run an experiment. In this experiment, you run each algorithm 1000 times using a labeled data set of random functions to optimize with the true correct value. The dataset contains approximately 100.000 functions. So, you decide to select 1000 functions randomly from the dataset. Your response variable is the absolute difference between the output of the algorithm (what the algorithm thinks it is the correct value) and the true correct value. Therefore, the best algorithm is the one that minimizes the average distance. Below are the results of the experiment:

For the HOO:

- Average distance: $\bar{X}_{HOO}$
- Variance of the distance: s^2_{HOO}
- The means are distributed normally

For the BAST:

- Average distance: $\bar{X}_{BAST}$
- Variance of the distance: s^2_{BAST}
- The means are distributed normally

a) Explain why you cannot decide which algorithm is better based only in the average distance metric. (1pt)
b) What are the four assumptions of an equal variance independent samples t-test? (2pt, 0.5 for each)
c) State the null and the alternative hypothesis (1pt, 0.5 for each)
d) Describe (including all the formulas with the question notation symbols), how would you analyze and decide if the algorithm HOO is better than the BAST, with a 99% confidence interval. (2pt)

Context for question 2, 3 and 4

Questions 2, 3 and 4 are adapted from on a recently published study in software engineering.

The questions are independent of being familiar with this study, therefore we reproduce only the abstract of the research to contextualize the questions.

The tables were adapted from the paper, to facilitate the understanding.

A. Niknafs and D. Berry, *The impact of domain knowledge on the effectiveness of requirements engineering activities*, vol. 22, no. 1. Empirical Software Engineering, 2017.

Abstract:

"One factor that seems to influence an individual's effectiveness in requirements engineering activities is her knowledge of the problem being solved, i.e., domain knowledge. While in-depth domain knowledge enables a requirements analyst to understand the problem easier, she can fall for tacit assumptions and fail to consider issues that she believes to be obvious.

This paper investigates the impact of domain knowledge on requirements engineering activities. Its main research question is "How does one form the most effective team, consisting of some mix of domain ignorants and domain awares, for a requirements engineering activity involving knowledge about the domain of the computer-based system whose requirements are being determined by the team?" For completeness, a number of other factors, such as educational background, are considered for their effect on teams' effectiveness.

Two controlled experiments test a number of hypotheses derived from the question, including mainly that for a computer-based system in a particular domain, a team consisting of a mix of requirements analysts that are both ignorant and aware of the domain, is more effective at requirement idea generation than a team consisting of only analysts that are aware of the domain.

The results show no significant effect of the mix by itself on effectiveness in requirement idea generation. However, the results do show surprising significant effects of the educational background and of the mix combined with the educational background. Combining the results, the main conclusion is that the presence of a domain ignorant with a computer science or software engineering background improves the effectiveness of a requirement idea generation team."

The tables below show the notation used for the dependent and the independent variables in this study. Both experiments were conducted with a total of 40 teams.

Independent variables about a team:

Abbreviation	**Explanation**	**Values (Levels)**
MIX	Mix of domain familiarities	0,1,2,3
EXP	Experience of the team	Low, Medium, High
CR	Creativity score	Low, Medium, High
EDU	Education level of the team	With Computer Science background, Without Computer Science background

Dependent variables:

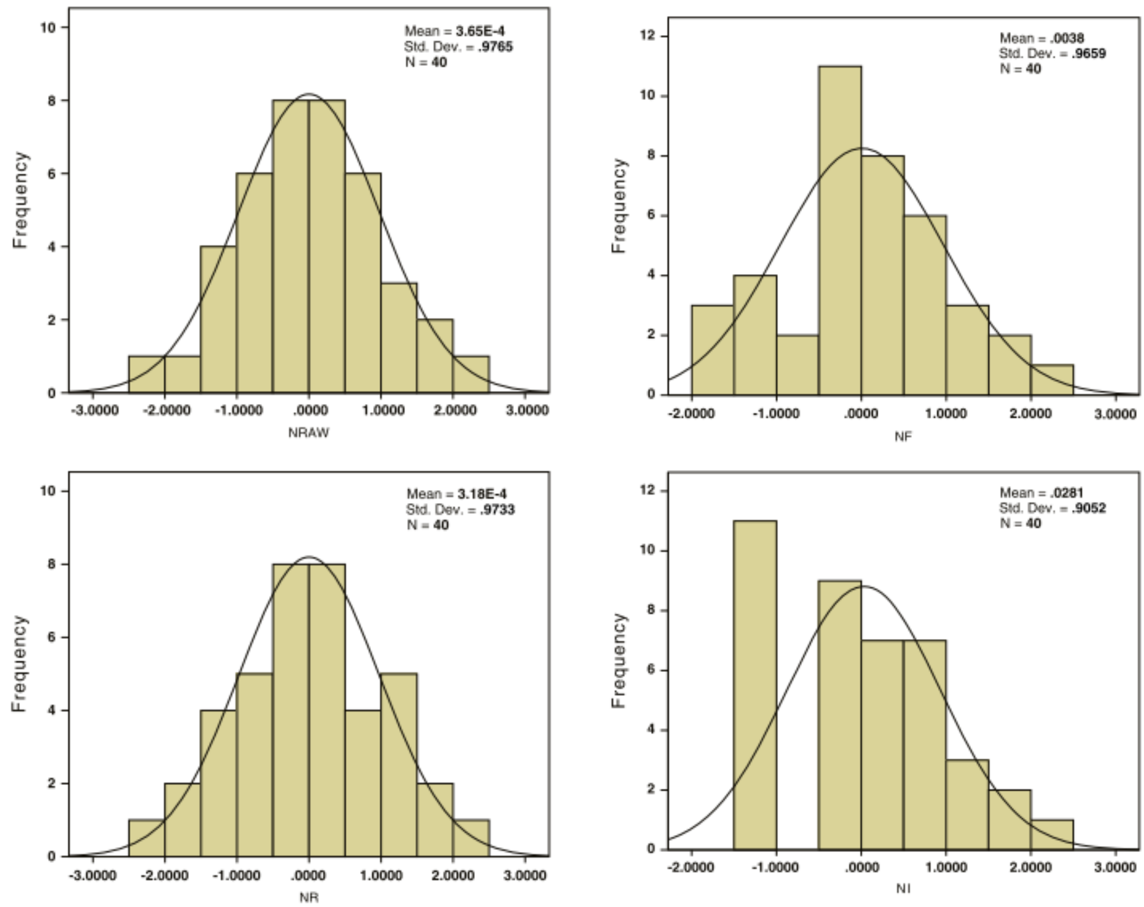
Abbreviation	**Explanation**	**Values (Levels)**
NRAW	Normalized raw number of ideas	numeric
NR	Normalized average number of relevant ideas	numeric
NF	Normalized average number of feasible ideas	numeric
NI	Normalized average number of innovative ideas	numeric

Question 2 (5pt)

Before the analysis of the data from the experiments the authors tested for the normality of the dependent variables (after a normalization transformation).
To do that they plotted the data and performed a Shapiro-Wilk test for each dependent variable. The results of the tests are and the plots are below:

Table 9 Test of Normality of the Dependent Variables after Normalization

Dependent Variable	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistic	df	p	Statistic	df	p
NRAW	.041	40	.200	.997	40	1.000
NR	.054	40	.200	.994	40	.998
NF	.106	40	.200	.984	40	.844
NI	.165	40	.008	.919	40	.007



For the plots: (top-left corner is the NRAW, top-right corner is the NF, bottom-left corner NR and bottom-right corner is the NI dependent variables)

a) What is the null hypothesis of the Shapiro-Wilk test? (1pt)

b) What can be said about normality (using a confidence level of 95%) if the p-value in the Shapiro Wilk test is equal to 0.1? (1pt)

c) What can be said about normality (using a confidence level of 95%) in this case above, if the p-value in the Shapiro Wilk test is equal to 0.01? (1pt)

d) How would you classify the normality of the each of the four dependent variables? Justify each decision (2pt, 0.5 for each)

Question 3 (10 pt)

After the analysis of normality the researchers decided to run a three-way ANOVA for the factors MIX, EXP and EDU.
For this question, we are only interested in the response variable NR (normalized average of relevant ideas)

a) Why did the authors decide to run an ANOVA instead of just running multiple t-tests? (1pt)

b) What are the four assumptions of an independent groups ANOVA for a full factorial design? (2pt, 0.5 for each)

c) Complete the ANOVA table for the variable **NR only** below using a confidence level of 95%.
Notes: you can see the number of levels for each factor in the table with the variables of the study (In the context part for question 2 and 3) (4pt = 1pt for mean value, 1pt for the F-calculated, 1pt for the F-table, 1pt for the effect size)
The symbol * indicates interaction between factors

Factors	Sum of Squares	Df	Mean Square	F-calculated	F-table	Effect-size
MIX	1.879	3				
EXP	0.288	2				
EDU	4.069	1				
MIX*EXP	3.677	6				
MIX*EDU	0.080	3				
EXP*EDU	0.160	2				
MIX*EXP*EDU	4.662	6				
Residuals	24.466	26				
Total		39				

d) Discuss the obtained effect size. Are they small, medium or large? (1 pt)
e) If the authors wanted to detect not only if a factor is statistically but also the difference between the levels, what should they do? (1pt)
f) Discuss the results from the test. (1 pt)

Question 4 (4 pt)

During the study, the authors discuss the concept of statistical power. This concept is closely related to errors type I and type II.

a) What is statistical power? (1pt)

b) What is error type I, error type II? (1 pt)

c) How are they related? (1 pt)

d) How can you keep error type I controlled while reducing error type II in an experiment? (1pt)

Question 5 (5 pt)

a) In survey research, it is important that the sample is large enough. What problems can we encounter if it is not large enough? (2pt)

b) In a questionnaire, questions can be expressed as open or closed. Define and explain the pros and cons to each category. (3pt)

Question 6 (3 pt)

There are three ways, and three ways only, to ethically encourage participation in a study (Senese 1997). What are the three ways? Describe them briefly! (1pt for each)

Question 7 (2 pt)

When designing a survey one needs to take the reliability and validity of the survey instrument into consideration. **Explain** what reliability and validity means in this case (1pt each)

APPENDIX A

Standard deviation of the sample mean:

$$\frac{\sigma}{\sqrt{n}}$$

Variance of a sample:

$$s^2 = \frac{1}{n-1}\sum(X_i - \bar{X})^2$$

Pooled variance of a sample:

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$

Z-test statistic:

$\boldsymbol{z = \overline{(X} - \mu)\ /\sigma}$

T test statistic:

One-sample t-test

$$\boldsymbol{T = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}}$$

df = *n* - 1

Two-sample t-test

$$\boldsymbol{T = \frac{\overline{X}_1 - \overline{X}_2 - diff}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}}$$

$df = n_1 + n_2 - 2$

One-way analysis of variance structuring help

The commonly used normal linear models for a completely randomized experiment are:

$y_{i,j} = \mu_j + \varepsilon_{i,j}$ (the means model)

or

$y_{i,j} = \mu + \tau_j + \varepsilon_{i,j}$ (the effects model)

where

$i = 1, \ldots, I$ is an index over experimental units
$j = 1, \ldots, J$ is an index over treatment groups
I_j is the number of experimental units in the jth treatment group
$I = \sum_j I_j$ is the total number of experimental units
$y_{i,j}$ are observations
μ_j is the mean of the observations for the jth treatment group
μ is the grand mean of the observations
τ_j is the jth treatment effect, a deviation from the grand mean
$\sum \tau_j = 0$
$\mu_j = \mu + \tau_j$
$\varepsilon \sim N(0, \sigma^2)$, $\varepsilon_{i,j}$ are normally distributed zero-mean random errors.

Means model calculations:

	Lists of Group Observations					
	1	2	$\cdots$	j	$\cdots$	J
1	y_{11}	y_{12}				y_{1J}
2	y_{21}	y_{22}				y_{2J}
3						
$\vdots$						
i				y_{ij}		
$\vdots$						
	$y_{I_1 1}$	$\cdots$				
		$y_{I_2 2}$	$\cdots$			

	Group Summary Statistics						Grand Summary Statistics	
# Observed	I_1	I_2	$\cdots$	I_j	$\cdots$	I_J	# Observed	$I = \sum I_j$
Sum				$\sum_i y_{ij}$			Sum	$\sum_j \sum_i y_{ij}$
Sum Sq				$\sum_i (y_{ij})^2$			Sum Sq	$\sum_j \sum_i (y_{ij})^2$
Mean	m_1	$\cdots$		m_j	$\cdots$	m_J	Mean	m
Variance	s_1^2	$\cdots$		s_j^2	$\cdots$	s_J^2	Variance	s^2

Comparing model to summaries: $\mu = m$ and $\mu_j = m_j$. The grand mean and grand variance are computed from the grand sums, not from group means and variances.

One-way ANOVA for means model:

Source of variation	Sums of squares Explanatory SS[4]	Sums of squares Computational SS[5]	Degrees of freedom DF	Mean square MS	F
Treatments	$\sum_{Treatments} I_j (m_j - m)^2$	$\sum_j \frac{(\sum_i y_{ij})^2}{I_j} - \frac{(\sum_j \sum_i y_{ij})^2}{I}$	$J - 1$	$\frac{SS_{Treatment}}{DF_{Treatment}}$	$\frac{MS_{Treatment}}{MS_{Error}}$
Error	$\sum_{Treatments} (I_j - 1) s_j^2$	$\sum_j \sum_i y_{ij}^2 - \sum_j \frac{(\sum_i y_{ij})^2}{I_j}$	$I - J$	$\frac{SS_{Error}}{DF_{Error}}$	
Total	$\sum_{Observations} (y_{ij} - m)^2$	$\sum_j \sum_i y_{ij}^2 - \frac{(\sum_j \sum_i y_{ij})^2}{I}$	$I - 1$		

MS_{Error} is the estimate of variance corresponding to σ^2 of the model.

The tables in the course book are based on the effects model:

Table 8.5. Analysis of variance table for one-factor experiments

COMPONENT	SUM OF SQUARES	PERCENTAGE VARIATION	DEGREES OF FREEDOM	MEAN SQUARE	F CALCULATION	F TABLE
Y	$SSY = \sum Y_{ij}^2$		N			
$\bar{Y}_{..}$	$SSO = N\mu^2$		1			
$Y - \bar{Y}_{..}$	$SST = SSY - SSO$	100	N-1			
A	$SSA = \sum r_j \alpha_i^2$	$100\left(\frac{SSA}{SST}\right)$	k-1	$MSA = \frac{SSA}{k-1}$	$\frac{MSA}{MSE}$	$F_{[1-\alpha;a-1,(N-k)]}$
e	$SSE = SST - SSA$	$100\left(\frac{SSE}{SST}\right)$	N-k	$MSE = \frac{SSE}{(N-k)}$		

Table 10.4 Analysis of variance table for two factors

Component	Sum of squares	Degrees of freedom	Mean square	F- Computed.	F-Table
Y	$SSY = \sum Y_{ij}^2$	abr			
$\overline{Y}..$	$SSO = abr\mu^2$	1			
$Y - \overline{Y}..$	$SST = SSY - SSO$	abr-1			
A	$SSA = br\sum \alpha_i^2$	a-1	$MSA = \frac{SSA}{a-1}$	$\frac{MSA}{MSE}$	$F_{[1-\alpha;(a-1),ab(r-1)]}$
B	$SSB = ar\sum \beta_j^2$	b-1	$MSB = \frac{SSB}{b-1}$	$\frac{MSB}{MSE}$	$F_{[1-\alpha;(b-1),ab(r-1)]}$
AB	$SSAB = r\sum \alpha\beta_{ij}^2$	(a-1)(b-1)	$MSAB = \frac{SSAB}{(a-1)(b-1)}$	$\frac{MSAB}{MSE}$	$F_{[1-\alpha;(a-1)(b-1),ab(r-1)]}$
e	$SSE = \sum e_{ijk}^2$	ab(r-1)	$MSE = \frac{SSE}{ab(r-1)}$		

abr = I (in means model) = the total number of experimental units (people) in the study in this case.
r = number of replications (individuals) in each group.

abr-1 = (a-1) + (b-1) + (a-1)(b-1) + ab(r-1)

Table 9.4. Analysis of variance by one factor and one block variable

Component	Sum of Squares	Degrees of Freedom	Mean Square	F-Computed	F-Table
Y	$SSY = \sum y_{ij}^2$	ab			
$\overline{Y}..$	$SS0 = ab\mu^2$	1			
$Y - \overline{Y}..$	$SST = SSY - SS0$	ab-1			
A	$SSA = b\sum \alpha_i^2$	a-1	$MSA = \frac{SSA}{a-1}$	$\frac{MSA}{MSE}$	$F_{[1-\alpha;(a-1);(a-1)(b-1)]}$
B	$SSB = a\sum \beta_j^2$	b-1	$MSB = \frac{SSB}{b-1}$		
e	$SSE = \sum e_{ij}^2$	(a-1)(b-1)	$MSE = \frac{SSE}{(a-1)(b-1)}$		

Table 9.5. Analysis of variance by one factor and two block variables

$$SS_{total} = \sum_{i=1}^{a}\sum_{j=1}^{b}(y_{ijk} - \overline{y}_{..})^2 = \sum_{i=1}^{p}\sum_{j=1}^{p} y_{ijk}^2 - \frac{y_{...}^2}{p^2}$$

$$SS_{row} = \sum_{i=1}^{p} p(\overline{y}_{i..} - \overline{y}_{..})^2 = \sum_{i=1}^{p}\frac{R_i^2}{p} - \frac{y_{...}^2}{p^2}$$

$$SS_{trt} = \sum_{j=1}^{p} p(\overline{y}_{.j.} - \overline{y}_{..})^2 = \sum_{j=1}^{p}\frac{T_j^2}{p} - \frac{y_{...}^2}{p^2}$$

$$SS_{col} = \sum_{k=1}^{p} p(\overline{y}_{..k} - \overline{y}_{..})^2 = \sum_{k=1}^{p}\frac{C_k^2}{p} - \frac{y_{...}^2}{p^2}$$

Source of Variation	Sum of Squares	d.f.	Mean Square	F Ratio
Treatments	SS_{trt}	$p-1$	MS_{trt}	$\frac{MS_{trt}}{MS_E}$
Rows	SS_{row}	$p-1$	MS_{row}	
Columns	SS_{col}	$p-1$	MS_{col}	
Error	SS_E	$(p-1)(p-2)$	MS_E	
Total	SS_{total}	p^2-1		

ANOVA for nested design (corrected, i.e. the course book is wrong):

Correction to the formulas

Table 11.4. Analysis of variance for the data of example 12.1

Source of variation	Sum of squares	Degrees of freedom	Mean square	F-Computed	F-Table
A	$SSA = br\sum_{i=1}^{a}(\bar{y}_{i..} - \bar{y}_{...})^2$	a–1	$MSA = \frac{SSA}{a-1}$	MSA	$F_{[1-\alpha;(a-1),ab(r-1)}$
B within A	$SSB(A) = r\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{y}_{ij.} - \bar{y}_{i..})^2$	a(b–1)	$MSB(A) = \frac{SSB}{a(b-1)}$	MSB(A)	$F_{[1-\alpha;a(b-1),ab(r-1)}$
Error	$SSE = \sum_{k=1}^{r}\sum_{i=1}^{a}\sum_{j=1}^{b}$	ab(r–1)	$MSE = \frac{SSE}{ab(r-1)}$		
Total	$SST = \sum_{k=1}^{r}\sum_{i=1}^{a}\sum_{j=1}^{b}(y_{ijk} - \bar{y}_{...})^2$	abr–1			

The correct SSE is:

$$\sum_i \sum_j \sum_k (Y_{ijk} - \bar{Y}_{ij.})^2$$

And, to get the F value you of course divide by MSE.

ANOVA for mixed design (Blocked Factorial Design)

Component	Sum of squares	Degrees of freedom	Mean square	F- Computed	F-Table (α=0.99)
Between groups	$SS_{between-groups}$	3			
Groups or ABC	$SSABC = 2^k r\, C_{ABC}^2$	1	MSABC=SSABC	$\frac{MSABC}{MSEbetween-groups}$	$F_{[1-\alpha;\, 1,\, 2]}$
Error	$SSE_{between-groups}$= Difference	2	$MSEbetween-groups = \frac{SSEbetween-groups}{2}$		
Within groups					
A	$SSA = 2^k r\, C_A^2$	$2^k r-4$	MSA= SSA	$\frac{MSA}{MSEwithin-groups}$	$F_{[1-\alpha;\, 1,\, 2^k r-10]}$
B	$SSB = 2^k r\, C_B^2$	1	MSB = SSB	$\frac{MSB}{MSEwithin-groups}$	
C	$SSC = 2^k r\, C_C^2$	1	MSC = SSC	$\frac{MSC}{MSEwithin-groups}$	
AB	$SSAB = 2^k r\, C_{AB}^2$	1	MSAB =SSAB	$\frac{MSAB}{MSEwithin-groups}$	
AC	$SSAC = 2^k r\, C_{AC}^2$	1	MSAC = SSAC	$\frac{MSAC}{MSEwithin-groups}$	
BC	$SSBC = 2^k r\, C_{BC}^2$	1	MSBC= SSBC	$\frac{MSBC}{MSEwithin-groups}$	
Error	$SSE_{within-groups}$= Difference	$2^k r-10$			

$$R_A^2 = \frac{SS_A}{SS_{Total}}$$

$$R_B^2 = \frac{SS_B}{SS_{Total}}$$

$$R_{AB}^2 = \frac{SS_{AB}}{SS_{Total}}$$ Etc.

APPENDIX B (*t* Distribution)

One Sided	75%	80%	85%	90%	95%	97.5%	99%	99.5%	99.75%	99.9%	99.95%
Two Sided	50%	60%	70%	80%	90%	95%	98%	99%	99.5%	99.8%	99.9%
1	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
120	0.677	0.845	1.041	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

APPENDIX C (*F* Distribution)

TABLE C.5
Upper Percentage Points of the *F* Distribution

df for denominator	α	df for numerator 1	2	3	4	5	6	7	8	9	10	11	12
1	.25	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32	9.36	9.41
	.10	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	60.2	60.5	60.7
	.05	161	200	216	225	230	234	237	239	241	242	243	244
2	.25	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38	3.39	3.39
	.10	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.40	9.41
	.05	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4
	.01	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4
3	.25	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.44	2.45	2.45
	.10	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.22
	.05	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74
	.01	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	27.1
4	.25	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08
	.10	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.91	3.90
	.05	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91
	.01	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.4
5	.25	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
	.10	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.28	3.27
	.05	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.71	4.68
	.01	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.96	9.89
6	.25	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.78	1.77	1.77	1.77	1.77
	.10	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.92	2.90
	.05	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00
	.01	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72
7	.25	1.57	1.70	1.72	1.72	1.71	1.71	1.70	1.70	1.69	1.69	1.69	1.68
	.10	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.68	2.67
	.05	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57
	.01	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.54	6.47
8	.25	1.54	1.66	1.67	1.66	1.66	1.65	1.64	1.64	1.63	1.63	1.63	1.62
	.10	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.52	2.50
	.05	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28
	.01	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.73	5.67
9	.25	1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.58
	.10	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.40	2.38
	.05	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07
	.01	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.18	5.11
10	.25	1.49	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.56	1.55	1.55	1.54
	.10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.30	2.28
	.05	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91
	.01	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71

TABLE C.5 (continued)

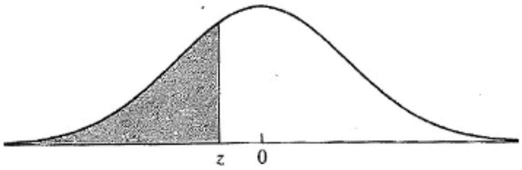
df for de-nomi-nator	α	*df for numerator* 1	2	3	4	5	6	7	8	9	10	11	12
	.25	1.47	1.58	1.58	1.57	1.56	1.55	1.54	1.53	1.53	1.52	1.52	1.51
11	.10	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.23	2.21
	.05	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79
	.01	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.46	4.40
	.25	1.46	1.56	1.56	1.55	1.54	1.53	1.52	1.51	1.51	1.50	1.50	1.49
12	.10	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.17	2.15
	.05	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69
	.01	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.22	4.16
	.25	1.45	1.55	1.55	1.53	1.52	1.51	1.50	1.49	1.49	1.48	1.47	1.47
13	.10	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.12	2.10
	.05	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60
	.01	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	4.02	3.96
	.25	1.44	1.53	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.46	1.45
14	.10	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.08	2.05
	.05	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53
	.01	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.86	3.80
	.25	1.43	1.52	1.52	1.51	1.49	1.48	1.47	1.46	1.46	1.45	1.44	1.44
15	.10	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.04	2.02
	.05	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48
	.01	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67
	.25	1.42	1.51	1.51	1.50	1.48	1.47	1.46	1.45	1.44	1.44	1.44	1.43
16	.10	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	2.01	1.99
	.05	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42
	.01	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.62	3.55
	.25	1.42	1.51	1.50	1.49	1.47	1.46	1.45	1.44	1.43	1.43	1.42	1.41
17	.10	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.98	1.96
	.05	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38
	.01	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.52	3.46
	.25	1.41	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.42	1.41	1.40
18	.10	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.96	1.93
	.05	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34
	.01	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.43	3.37
	.25	1.41	1.49	1.49	1.47	1.46	1.44	1.43	1.42	1.41	1.41	1.40	1.40
19	.10	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.94	1.91
	.05	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31
	.01	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.36	3.30
	.25	1.40	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.39
20	.10	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.92	1.89
	.05	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28
	.01	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23

TABLE C.5 (continued)

df for denominator	α	df for numerator 1	2	3	4	5	6	7	8	9	10	11	12
22	.25	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.38	1.37
	.10	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.88	1.86
	.05	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23
	.01	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12
24	.25	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.37	1.36
	.10	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.85	1.83
	.05	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.21	2.18
	.01	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03
26	.25	1.38	1.46	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.37	1.36	1.35
	.10	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.84	1.81
	.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15
	.01	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02	2.96
28	.25	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34
	.10	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.81	1.79
	.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12
	.01	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90
30	.25	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.35	1.34
	.10	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77
	.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
	.01	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84
40	.25	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31
	.10	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.73	1.71
	.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
	.01	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66
60	.25	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.29
	.10	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66
	.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
	.01	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50
120	.25	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.27	1.26
	.10	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.62	1.60
	.05	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.87	1.83
	.01	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34
200	.25	1.33	1.39	1.38	1.36	1.34	1.32	1.31	1.29	1.28	1.27	1.26	1.25
	.10	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66	1.63	1.60	1.57
	.05	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80
	.01	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.34	2.27
∞	.25	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.24
	.10	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55
	.05	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75
	.01	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18

APPENDIX D (*Z* Distribution)

TABLE A.2 Cumulative normal distribution (*z* table)



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
−3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641