1 Variables

$$\alpha = \frac{\sqrt{2m(U_0 - E)}}{\hbar}$$
$$k = \frac{\sqrt{2mE}}{\hbar}$$

2 Even case

2.1 Wavefunction

$$\psi(x) = \begin{cases} Ce^{\alpha x} & x < -\frac{L}{2} \\ B\cos(kx) & -\frac{L}{2} \leq x \leq \frac{L}{2} \\ Ce^{-\alpha x} & x > \frac{L}{2} \end{cases} \tag{1}$$

2.2 Continuity condition

$$\alpha = k\tan\frac{kL}{2} \tag{2}$$
$$B = \frac{Ce^{-\alpha L/2}}{\cos\frac{kL}{2}} \tag{3}$$

2.3 Norm $= 1$

$$1 = \int_{-\infty}^{-\frac{L}{2}} \left(Ce^{\alpha x}\right)^2 \mathrm{d}x + \int_{-\frac{L}{2}}^{\frac{L}{2}} \left(B\cos(kx)\right)^2 \mathrm{d}x + \int_{\frac{L}{2}}^{\infty} \left(Ce^{-\alpha x}\right)^2 \mathrm{d}x \tag{4}$$
$$\implies C = \sqrt{\alpha k e^{L\alpha}\left(\frac{1+\cos(Lk)}{L\alpha k + \alpha\sin(Lk) + k\cos(Lk) + k}\right)} \tag{5}$$

2.4 Solving for E

$$E\sec^2\left(\frac{L\sqrt{2mE}}{2\hbar}\right) = U \tag{6}$$

3 Odd case

3.1 Wavefunction

$$\psi(x) = \begin{cases} Ce^{\alpha x} & x < -\frac{L}{2} \\ B\sin(kx) & -\frac{L}{2} \leq x \leq \frac{L}{2} \\ -Ce^{-\alpha x} & x > \frac{L}{2} \end{cases} \tag{7}$$

3.2 Continuity condition

$$\alpha = -k\cot\frac{kL}{2} \tag{8}$$
$$B = \frac{-Ce^{-\alpha L/2}}{\sin\frac{kL}{2}} \tag{9}$$

3.3 Norm $= 1$

$$1 = \int_{-\infty}^{-\frac{L}{2}} \left(Ce^{\alpha x}\right)^2 \mathrm{d}x + \int_{-\frac{L}{2}}^{\frac{L}{2}} \left(B\sin(kx)\right)^2 \mathrm{d}x + \int_{\frac{L}{2}}^{\infty} \left(-Ce^{-\alpha x}\right)^2 \mathrm{d}x \tag{10}$$

$$\implies C = \sqrt{\alpha k e^{L\alpha}\left(\frac{1-\cos\left(Lk\right)}{L\alpha k - \alpha\sin\left(Lk\right) - k\cos\left(Lk\right) + k}\right)} \tag{11}$$

3.4 Solving for E

$$E\csc^2\left(\frac{L\sqrt{2mE}}{2\hbar}\right) = U \tag{12}$$