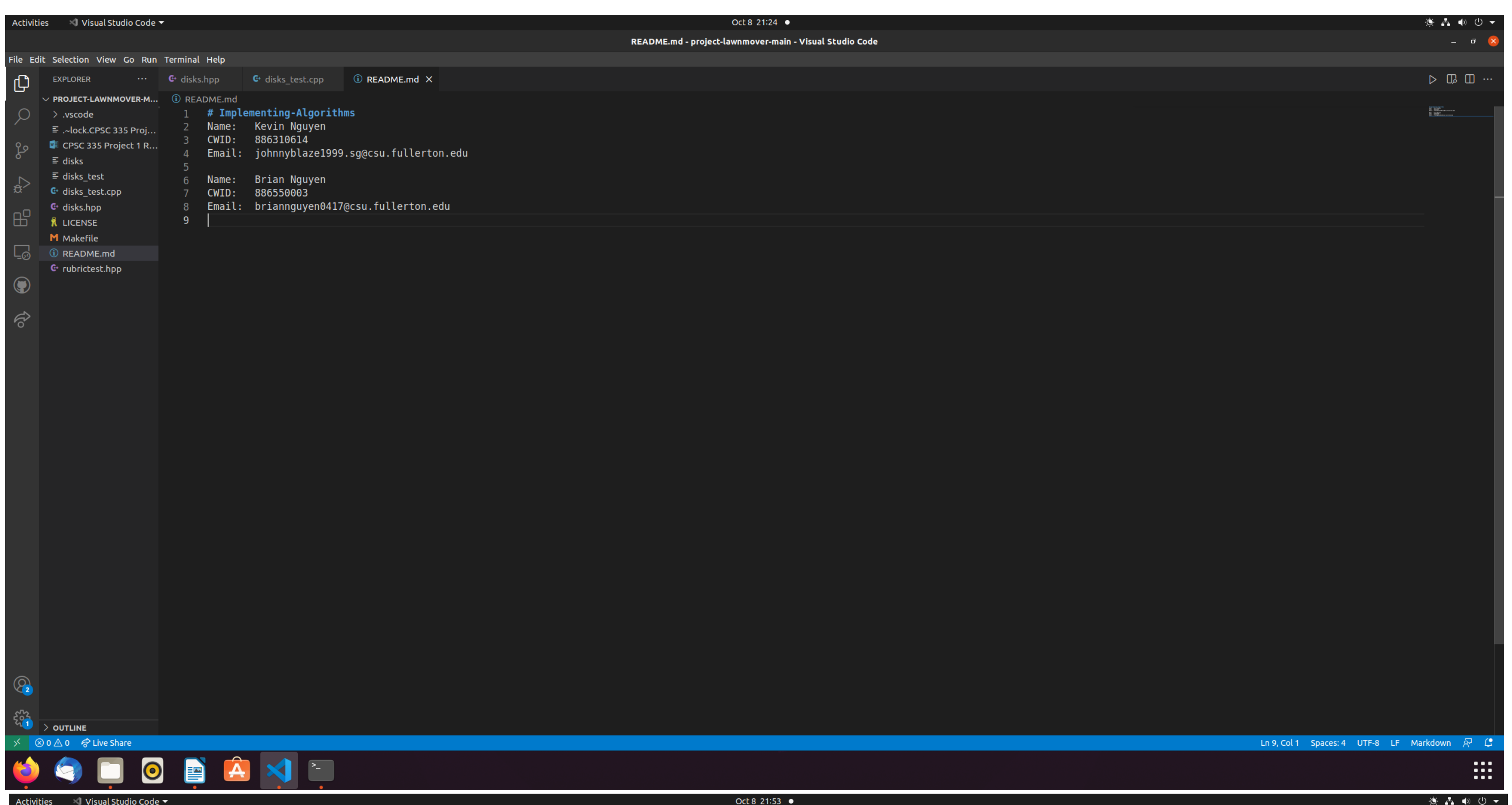

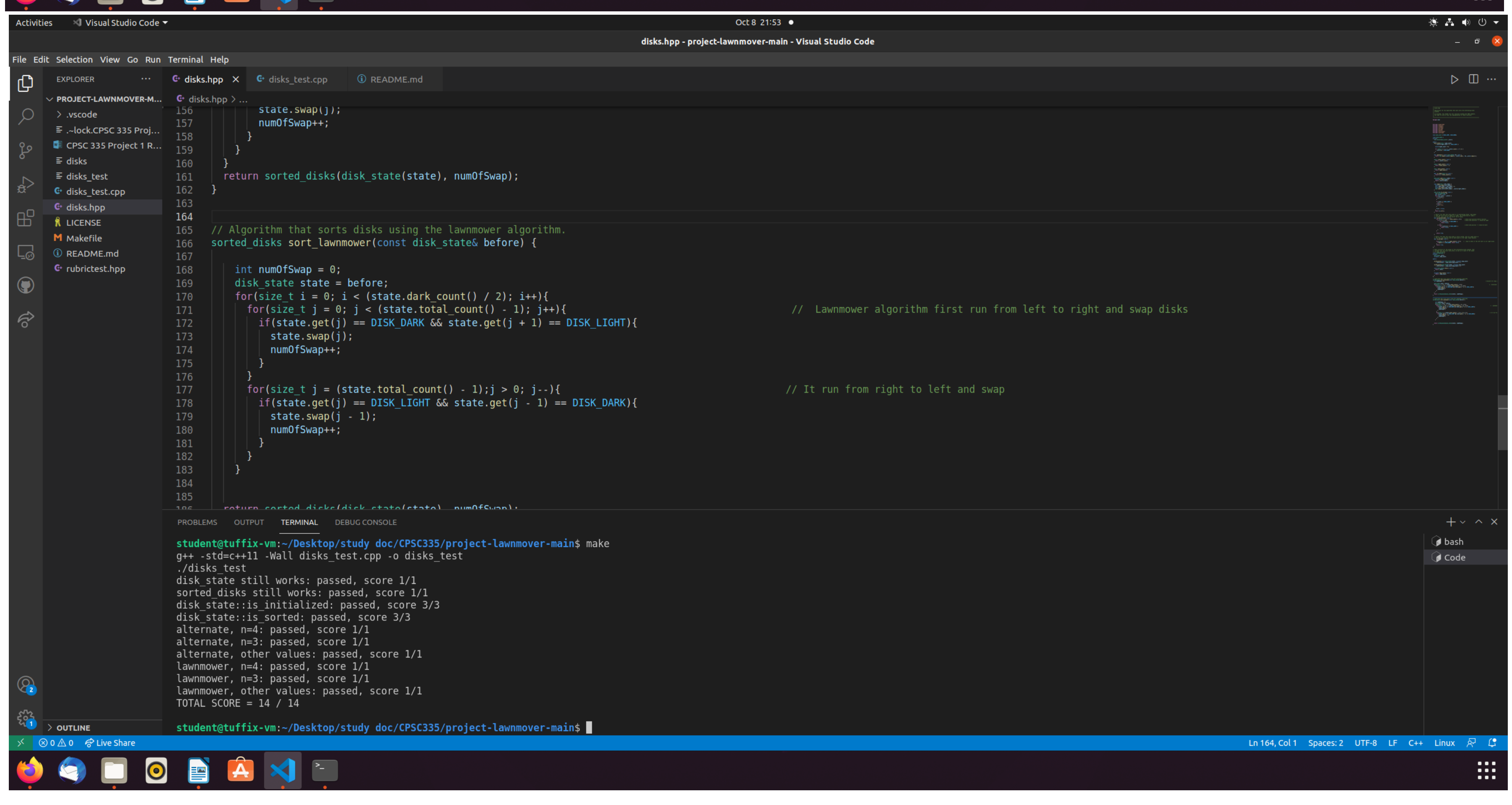

Pseudo code – Proof:

1. Alternate Algorithm

```
E  numOfswap = 0                                   -> 1 tu
C  for i = 0 to n do                               -> n - 0 + 1 => n+1 tu
B    for j = 0 to n - 1 do                         -> n - 1 + 1 => n tu
A      If ( get(j) == DARK && get(j+1) == LIGHT )  -> 3 tu
         swap (j)                                  -> 1 tu
         numOfswap ++                              -> 1 tu
       endif
     endfor
   endfor
D  return numofswap                                -> 1 tu
```

Step count: $A = 3 + \max(2, 0) = 3 + 2 = 5$ sc

$B = n * 5 = 5n$

$C = (n+1)(5n) = 5n^2 + 5n + D + E = 5n^2 + 5n + 2$

S.C = $\boxed{5n^2 + 5n + 2}$

* $5n^2 + 5n + 2 \leq O(n^2)$

$c = 5 + 5 + 2 = 12$

$n_0 = 1 \Rightarrow 5 \cdot 1 + 5 \cdot 1 + 2 \leq 12 \cdot (1^2)$

$12 \overset{?}{\leq} 12$ ✓ yes

By definition, $5n^2 + 5n + 2 \in O(n^2)$

* $\lim_{n \to \infty} 5n^2 + 5n + 2 \leq O(n^2)$

$\rightarrow \lim_{n \to \infty} 5 + \cancel{\frac{5}{n}}^{\to 0} + \cancel{\frac{2}{n^2}}^{\to 0} = L$

$5 + 0 \quad 0 = L$

$L = 5 \geq 0 \Rightarrow$ By limit theorem, $5n^2 + 5n + 2 \in O(n^2)$

2. Lawnmower Algorithm

```
F  numofswap = 0                                       1 tu
E  for i = 0 to n/2 do                                 n/2 + 1 - 0 = (n-2)/2 tu
D    for j = 0 to n - 1 do                             n - 1 - 0 + 1 = n tu
C      if ( get(j) == DARK && get(j+1) == LIGHT )      3 tu
         swap (j)                                      1 tu
         numofswap ++                                  1 tu
       endif
     endfor
   endfor
B  for j = n - 1 down to 0 do                          (0 - n - 1)/(-1) + 1 = n+1+1 = n+2 tu
A    if ( get(j) == LIGHT && get(j-1) == DARK )        3 tu
       swap ( j - 1 )                                  2 tu
       numofswap ++                                    1 tu
     endif
   endfor
G  return numofswap                                 -> 1 tu
```

Stepcount:

$A = 3 + \text{Max}(3, 0) = 3 + 3 = 6$

$B = (n+2)(6) = 6n + 12$

$C = 3 + \text{Max}(2, 0) = 3 + 2 = 5$

$D = n * 5 = 5n$

$E = \frac{n-2}{2} \times 5n = \frac{5n^2 - 10n}{2} = \frac{5}{2}n^2 - 5n$

$S.C = \frac{5}{2}n^2 - 5n + 6n + 12 + F + G$

$= \frac{5}{2}n^2 + n + 14$

* $\frac{5}{2}n^2 + n + 14 \leq O(n^2)$

$c = \frac{5}{2} + 1 + 14 = \frac{35}{2}$

$n_0 = 1 \Rightarrow \frac{5}{2} \cdot (1) + (1) + 14 \leq \frac{35}{2} \cdot (1)^2$

$\frac{35}{2} \overset{?}{\leq} \frac{35}{2}$ ✓ yes

By definition, $\frac{5}{2}n^2 + n + 14 \in O(n^2)$

$\text{limit}_{n \to \infty} \frac{5}{2}n^2 + n + 14 \in O(n^2)$

$\rightarrow \text{limit}_{n \to \infty} \frac{5}{2}\frac{n^2}{n^2} + \frac{n}{n^2} + \frac{14}{n^2} = L$

$\rightarrow \text{limit}_{n \to \infty} \frac{5}{2} + \cancel{\frac{1}{n}}^{\to 0} + \cancel{\frac{14}{n^2}}^{\to 0} = L$

$L = \frac{5}{2} \geq 0$

By limit theorem, $\frac{5}{2}n^2 + n + 14 \in O(n^2)$