Documentation for @circular

August 20, 2001

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function c = circular(x,y,units,axial)

Description Constructor for the circular class. This class encapsulates the properties of circular data. It provides methods of descriptive statistics as well as methods for hypothesis testing. See @circular/docs/usage.pdf for a brief introduction or @circular/docs/reference.pdf for a list of functions and their calling sequences.

Input Variables

- x A list of polar angles. (in radians or degrees depending on 'units' argument)
- y A corresponding list of radii. (in radians or degrees depending on 'units' argument)

units 'RAD' or 'DEG'. RAD is assumed by default. Note that all internal calculations are done with RADIANS, only what is passed to the outside is affected by this.

axial 1 or 0. Specifying whether these data should be treated as axial data.

Output Variables

c A @Circular object Contact: bart@salk.edu for more information.

```
BK - 27.7.2001 - last change Date:2001/08/2104:07:27 by Author:bart\ Revision:1.5
```

function c = subsasgn(c,subscript,value)

Description Subscript assignment for the @circular object.

```
BK - 27.7.2001 - Last Change Date:2001/07/3002:06:58 by Author:bart\ Revision:1.2
```

- GROUPS
- AXIAL

function value= subsref(c, subscript)

Description Subsref for Circular objects.

```
BK - 27.7.2001 - Last Change Date:2001/08/0201:02:03 by Author:bart\ Revision:1.3
```

- X
- Y
- RADIANS
- DEGREES

- R
- MEAN
- MEDIAN
- N
- AXIAL

function display(c)

Description Display a circular object.

```
BK - 27.7.2001 - last change Date: 2001/08/2103:50:31 by Author: bart
Revision: 1.4
```

function varargout = plot(c,options, start,nrSpokes)

Description Plot a circular set of data.

Input Variables

c Circular data

re appropriate for the arrows (a.colour 'r', for instance) 2. A mode string: 'MEAN' Add a vector showing the mean

'MEAONLY' Show only the mean

'MEDIAN' Add a vector showing the median.

'SUM' Add a vector showing the sum vector.

start For the calculation of the median only: where to start on the circle.

nrSpokes The number of spokes on the polar plot.

Output Variables

h A vector of handles of all the vectors that were plotted.

```
BK - 27.7.2001 - last change Date: 2001/08/2104: 47: 19 by Author: bart
Revision: 1.7
```

function [cMedian,m,phi,r] = median(c,start)

Description Determine the circular median.

Input Variables

c The circular object

[start] Angle at which to start counting (CounterClockWise). Defaults to zero. Can be a circular object or a number in the units of c.

Output Variables

cMedian The median direction, as a circular object.

m The median direction as a 2-vector

phi The median angle.

r The length of the median vector

```
BK - 28.7.2001 - last change Date: 2001/08/0201: 02: 03 by Author: bart
Revision: 1.2
```

function deg = deg(c)

Description Returns the angles in degrees.

Input Variables

c Circular object

Output Variables

```
deg Degrees
```

```
{\rm BK} - 29.7.2001 - last change Date:2001/07/3018:52:47 by Author:bart\ Revision:1.2
```

function rad = rad(c)

Description Returns the angles in radians.

Input Variables

c Circular object

Output Variables

```
rad Radians
```

```
BK - 27.7.2001 - last change Date:2001/07/2901:14:00 by Author:bart\ Revision:1.1
```

$function \ [\ p,\!U\] = \ vtest \ (c\,,theta)$

Description Do a one-sample V test on these circular data. I.e. test whether this sample is concentrated around

the angle theta. This tests the null hypothesis that this sample

is drawn from a random/uniform distribution, but it uses prior (!) information that the concentration

would be around theta if it existed. This makes it somewhat more powerful than the Rayleigh test, but you

cannot choose the theta on the basis of the data!

Corrects for grouping/binning if c.groups is set to the number of bins, and deals appropriately

with axial data if c.axial = 1.

Input Variables

c Circular data

theta The expected direction (in the units of c!)

Output Variables

- p The p-value to reject the null hypothesis.
- U The statistic.

TESTED

В

```
BK - 27.7.2001 - last change Date:2001/08/2103:44:44 by Author:bart\ Revision:1.4
```

function skew = skew(c)

Description The skewness of the data in a circular object.

Input Variables

c A circular object

Output Variables

s The skewness. Not tested on data whether the values are correct. Batschelet p44.

```
BK - 27.7.2001 - last change Date:2001/08/0201:02:03 by Author:bart\ Revision:1.2
```

function bool = isdeg(c)

Description Return whether the units are in degrees.

Input Variables

c Circular data.

Output Variables

bool Yes or No

```
BK - 27.7.2001 - last change Date:2001/08/2104:47:18 by Author:bart\ Revision:1.3
```

function kurt = kurtosis(c)

Description The Kurtosis of the data in a circular object.

Input Variables

c A circular object

Output Variables

kurt The kurtosis.

Note Not tested on data whether the values are correct.

```
BK - 27.7.2001 - last change Date:2001/08/2104:47:18 by Author:bart\ Revision:1.3
```

function [k,theta] = vonmises(c)

Description Fit the circular data to a von Mises distribution. Use maximum likelihood estimators with

minimal bias.

```
Von Mises PDF: f(phi) = 1/(2pi Io(k) exp(k*(cos(phi-theta)))
```

This is symmetric around the maximum obtained at phi = theta and k is a measure of concentration: the larger k, the narrower the peak around the mean.

Input Variables

c Circular data

Output Variables

k The concentration parameter

theta The mean angle.

```
BK - 28.7.2001 - last Change DAte : by Author : bart Revision : 1.3
```

function [r,p,U] = rankcorr(c1,c2);

Description Rank correlation coefficients for two sets of circular data or for circular and linear data.

Input Variables

- c1 Circular data
- c2 Circular data object or linear data in a vector.

Output Variables

- r The correlation coefficient.
- p The p-value associated with the signficance of the r.
- R The statistic for p

Note Circular-Circular uses the methods of Batschelet p.185

Circular-Linear uses the methods of Batschelet p.195

These methods cannot be applied to grouped data, and ties are not really dealt with in the ranking (i.e. breaking is random in c2, ordered in c1)

```
BK - 29.7.2001 - last change Date:2001/08/2103:45:57 by Author:bart\ Revision:1.4
```

$function \ k = vonmisestable(n,r)$

Description Given a number of samples n and the mean length of the vector of a sample, returns the

maximum likelihood estimate of the concentration parameter (kappa) of the von Mises distribution

Input Variables

- n Number of samples
- r Mean vector length

Output Variables

k Von Mises concentration parameter.

Note Solved by finding a solution to the Bessel equations. Batschelet gives a table up to k=10, apparently,

higher values are uncommon (extremely peaked), hence the search for k in [0,100] should suffice, but could

be extended in this file.

I have not been able to test the precise values yet, although it works for B101.

```
BK - 16.8.2001 - last change Date:2001/08/2104:47:19 by Author:bart\ Revision:1.2
```

function P = ptable;

Description Table P from Batschelet. For use with pFromCritical.

Input Variables

void No Input

Output Variables

P The whole table.

```
{\rm BK} - 3.12.2000 - Last change Date:2001/08/1808:46:32 by Author:bart\ Revision:1.2
```

function H = htable;

Description Table H from Batschelet. For use with pFromP

Note 'p' is always in the range [1 0.001]. P-values larger than 0.1 are rounded to 1, values smaller than 0.001 are 0.001.

Input Variables

void

Output Variables

H The table

```
BK - 3.12.2000 - Last change Date: 2001/08/1808: 46: 33 by Author: bart\ Revision: 1.2
```

$function \ p = pFromCritical(stat, n, tableName)$

Description Determine a p-value given a value of a statistic, the number of samples and the name of the table for this statistic. Note that the Table should have sample numbers in its first column, p-values in its first row and corresponding critical values in all other entries. For an example, see ptable.m. From left to righ the significance level should increase $p = [1\ 0.5\ 0.01\ 0.001]$ for instance.

Input Variables

stat The value of the statistic.

n The number of samples

tableName The name of the table.

Output Variables

p The p-value associated with this statistic (conservative)

```
BK - 16.8.2001 - last change Date:2001/08/2103:41:11 by Author:bart\ Revision:1.2
```

function X = xtable;

Description Table X from Batschelet. For use with pFromCritical.

Input Variables

void

Output Variables

X The table

```
BK - 16.8.2001 - last change Date:2001/08/2103:40:55 by Author:bart\ Revision:1.2
```

function W = wtable;

Description Table W from Batschelet.

Input Variables

void

Output Variables

W The table

```
BK - 16.8.2001 - last change Date:2001/08/2103:40:55 by Author:bart\ Revision:1.2
```

function Q = qtable;

Description Table Q from Batschelet.

Input Variables

void

Output Variables

Q The table

```
BK - 16.8.2001 - last change Date:2001/08/2103:40:55 by Author:bart\ Revision:1.2
```

function I = itable;

Description Table I from Batschelet. For use with pFromCritical

Input Variables

void

Output Variables

I The table

```
{\rm BK} - 16.8.2001 - last change Date:2001/08/2103:40:55 by Author:bart\ Revision:1.2
```

function H = hlargen;

Description Table 4.2.1 from Batschelet, for Rayleigh, For use with pFromCritical. test with large N.

Input Variables

void

Output Variables

H The table.

```
BK - 16.8.2001 - last change Date:2001/08/2104:47:19 by Author:bart\ Revision:1.2
```

function p = pFromP(stat,n,tableName)

Description Determine a p-value given a value of a statistic, the number of samples and the name of the table for this statistic. Note that the Table should have values for the statistic in its first column and it is assumed that the higher the statistic the beter. The first row should contain the number of samles and the p-values fill all other other entries of the table. For an example, see htable.m

Input Variables

stat The value of the statistic.

n The number of samples

tableName The name of the table.

Output Variables

p The p-value associated with this statistic (conservative)

```
BK - 16.8.2001 - last change Date:2001/08/1808:46:56 by Author:bart\ Revision:1.1
```

function O = otable;

Description Table O from Batschelet. For use with pFromCritical

Input Variables

void

Output Variables

O The table

```
BK - 16.8.2001 - last change Date:2001/08/1808:46:32 by Author:bart\ Revision:1.1
```

function L = ltable;

Description Table L from Batschelet. For use with pFromCritical

Input Variables

void

Output Variables

L The table

```
BK - 16.8.2001 - last change Date:2001/08/2103:40:55 by Author:bart\ Revision:1.2
```

function N = ntable;

Description Table N from Batschelet. For use with pFromCritical

Input Variables

void

Output Variables

N The table

```
BK - 16.8.2001 - last change Date:2001/08/2103:40:55 by Author:bart\ Revision:1.2
```

function [mr,sr] = meanr(c,phiStart,phiStop)

Description Determine the mean length of the vectors between angles phiStart and phiStop. This ignores directionality inside this range of angles.

Input Variables

c A circular data object

phiStart Angle to start including vectors (DEG or RAD, depending on units of the circular data object)

phiStop Angle to stop including vectros (DEG or RAD)

mr Mean length of the vectors in the range [phiStart,phiStop)

sr The standard deviation

 ${\tt BK}$ - 15.8.2001 - last change Date:2001/08/2104:47:19 by $Author:bart\ Revision:1.2$

function test (in)

Description This script tests most of the methods of the @Cricular class with examples given in Batschelet1981.

The name of the test is given, followed by two columns of numbers. On the left is what the Circular Toolbox calculated, on the right is the value given in Batschelet. Note that sometimes the p-values are different because I did not type in the complete tables (i.e. every p-value above 0.1 will be 1, but also sometimes Batschelet states that the effect is significant at the level x, which just means that p_1x . In the test report you will see x.

Do not put this test function in @circular: it will call the wrong functions for corrcoef. To have a look at some of the properties of the correlation coefficients, call test('cc')

BK - August 2001, last chagne Date:2001/08/2103:42:45 by $Author:bart\ Revision:1.2$

function [p,R] = rao(c)

Description Do a one-sample Rao test on these circular data. This tests the null hypothesis that this sample

is drawn from a random/uniform distribution. Where Rayleigh and V test are only powerful for

unimodal distributions, this Rao tests also works for multimodal distributions.

The data cannot be grouped

Input Variables

c Circular data

Output Variables

- p The p-value to reject the null hypothesis.
- $R \;\;$ The Rao statistic in degrees or radians, depending on the units of c.

BK - 27.7.2001 - last change Date:2001/08/2103:45:37 by $Author:bart\ Revision:1.2$

function [p,X,df] = chisq(c,e,df)

Description Do a Chi-squared test on these circular data. This tests the null hypothesis that this sample

is drawn from a random/uniform distribution. By specifying a particular model distribution

in 'e', the goodness of fit of the sampling with a theoretical distribution can be tested.

Note that if you use the samples to determine the parameters of the model distribution e,

the degrees of freedom should be adjusted to reflect this. Specify the number of parameters

in e you estimated from the data as the df argument

The data must be grouped

Input Variables

- c Circular data
- [e] Expected frequencies (optional, defaults to flat distribution)
- [df] Number of parameters of the expected distribution estimated from the data.

Output Variables

- p The p-value to reject the null hypothesis.
- X The Chi squared statistic
- df The degrees of freedom of the Chi.

```
BK - 27.7.2001 - last change Date: 2001/08/2104: 47: 18 by Author: bart\ Revision: 1.2
```

function bool= isgrouped(c)

Description Determine whether these circular data are grouped.

Input Variables

c Circular data object

Output Variables

```
bool 1 or 0.
```

Note This depends ont he user specifyig this, there is no way to calculate this.

```
BK - 29.7.2001 - last change Date: 2001/07/3018:52:47 by Author:bart\ Revision:1.2
```

function [p,K] = kuipers(c,model)

Description Do a one or two-sample Kuipers test on these circular data. I.e. test the null hypothesis that this sample

is drawn from the given distribution. If the theoretical distribution is not specified, the homogeneous

distribution is assumed, hence this becomes a test of non-randomness or directedness.

Data should not be grouped (or groupsize; 5 degrees) (If they are grouped, use Chisquared test,

but Kuipers is more powerful)

Note I cannot get the numbers right as in B78, but they are very close...

Input Variables

c Circular data

model Model distribution to test again. Optional, defaults to homogeneous. Others must be given as a circular data object,hence this becomes a two-sample test

Output Variables

- p The p-value to reject the null hypothesis.
- R The Kuipers statistic.

```
BK - 29.7.2001 - last change Date:2001/08/2103:48:02 by Author:bart\ Revision:1.3
```

function [p,U] = watson(c,model)

Description Do a one-sample Watson U test on these circular data. I.e. test the null hypothesis that this sample

is drawn from the given distribution. If the theoretical distribution is not specified, the homogeneous

distribution is assumed, hence this becomes a test of non-randomness or directedness.

Data should not be grouped (If they are grouped, use Chisquared test, but Watson is more powerful, especially

for small sample sizes.)

Input Variables

c Circular data

model Model distribution to test again. Optional, defaults to homogeneous. Others not implemented yet.

Output Variables

- p The p-value to reject the null hypothesis.
- U The Watson U statistic.

```
BK - 27.7.2001 - last change Date:2001/08/2103:43:58 by Author:bart\ Revision:1.3
```

function [p,F] = watsonwilliams(varargin);

Description Do a Watson-Williams n-sample test to determine whether the means of these circular data distributions are significantly different. No posthoc test is available

Note Assumption for this test is that all data drawn from a von Mises distributions with the same parameter of concentration (k) and the concentration of the averaged distribution is sufficiently large k_i .2. (This is tested).

Input Variables

- c1 A circular data object.
- c2 A second circular data object.
- cn The n-th circular data object.

- p The p-value
- F The F-statistic.

```
BK - 27.7.2001 - last change Date:2001/08/2103:43:32 by Author:bart\ Revision:1.4
```

function s = sum(c)

Description Calculate the sum or resultant vector of this dataset.

Input Variables

c A circular data object

Output Variables

s A circular data object representing the sum of the data in c.

```
BK - 29.7.2001 - last change Date: 2001/07/3018:52:48 by Author:bart\ Revision:1.2
```

function s = plus(c1,c2);

Description Add the data in two circular objects.

Input Variables

- c1 A circular data object
- c2 A circular data object

Output Variables

s A circular data object representing the union of the data in c1 and c2. Units are the units of c1.

Note Axiality is inherited if both are axial

```
BK - 29.7.2001 - last change Date:2001/07/3018:52:48 by Author:bart\ Revision:1.2
```

function [p,u,w] = ranksum(c1,c2)

Description Do a two-sample Ranksum test on these circular data. I.e. test the null hypothesis that these samples

are drawn from a distribution with the same mean value.

Data should not be grouped: ties in ranking will be broken randomly!.

Note The U value is correctly calculated for the example on B120, the corresponding P-value.

which uses code copied from Matlab ranksum.m, is somehwat lower than the value in Table \boldsymbol{U}

of Batschelet.

Input Variables

- c1 Circular data
- c2 Circular data

- p The p-value to reject the null hypothesis.
- U The U statistic, as used by Batschelet, table U.
- w The ranksum, the p-value is calculated on this basis, with Mathworks code.

```
BK - 29.7.2001 - last change Date:2001/07/3018:52:48 by Author:bart\ Revision:1.2
```

function [p,F,df1,df2] = ftest(c1,c2)

Description Do a two-sample RF-test test on these circular data. I.e. test the null hypothesis that these samples

are drawn from a von Mises distribution with the same k value (i.e. the same parameter of concentration)

Two tailed, divide p by two for one tailed.

Input Variables

- c1 Circular data
- c2 Circular data

Output Variables

- p The p-value to reject the null hypothesis.
- F The F statistic

```
BK - 29.7.2001 - last change Date:2001/08/0201:02:03 by Author:bart\ Revision:1.2
```

function d = distance(phi,theta)

Description Calculates the angular distance between two angles. This is a non-circular quantity in the range

[0, 180] (and linear

Input Variables

phi A circular data object

theta An angle, a column vector of angles or a circular data object. Units must be the same as the units of phi.

Output Variables

d An angular distance, in the units of phi.

```
BK - 29.7.2001 - last change Date: 2001/08/2103:50:12 by Author:bart\ Revision:1.3
```

function [r,p] = corrcoef(c1,c2);

Description Correlation coefficient for two sets of circular data.

Both r's range from 0 to 1.

Input Variables

- c1 Circular data
- c2 Circular data

- r The circular correlation coefficient. r
- p The p-value associated with the signficance of the r.

Note This correlation measure is only valid for c1 and c2 uniform. Could do a rayleigh test to

test this.

See also rankcorr, veccorr for correlation coefficients with fewer assumptions on the data.

```
BK - 29.7.2001 - last change Date:2001/08/0201:02:03 by Author:bart\ Revision:1.3
```

function [r,p,X] = veccorr(c1,c2);

Description Parametric correlation coefficients for two sets of circular data or for circular and linear data.

This correlation measure works for large n, whether or not c1 and c2 are uniformly distributed

(as in corrcoef) and there is no problem dealing with ties (as in rankcorr). Uses the chi-squared

distribution to test significance.

Input Variables

- c1 Circular data
- c2 Circular data object or linear data (as a vector)

Output Variables

- r The circular correlation coefficient
- p The p-value associated with the signficance of the r.
- X The Chisquared statistic.

```
BK - 29.7.2001 - last change Date:2001/08/2104:47:19 by Author:bart\ Revision:1.4
```

function [p,Tsquared]= hotelling(c1,c2)

Description Hotelling one and two sample test. Determines whether the origin of the bivariate sample

is significantly different from the origin. Or whether two bivariate samples are different from each other. This is a parametric test and it assumes that

the parent populations are bivariate normal, with the same variance and covariance. Moreover the

two samples should be independent and neither should be grouped.

Input Variables

- c1 A circular object
- c2 Another circular object

- p The probability that the center of the c distribution is at (0,0)
- T The T-squared statistic for this test (it has an F(2,n-2) distribution.

```
BK - 29.7.2001 - last change Date:2001/08/2104:47:18 by Author:bart\ Revision:1.4
```

function [p,U]= mardia(c1,c2)

Description Marida non-parameteric two sample bivariate test. Determines whether the centres of two bivariate samples

differ significantly from each other Assumes that the parent populations are continuous and identical except for

the possible location shift. The two samples should be independent and not grouped.

Uses ranksum to do the final test.

Input Variables

- c1 A circular object
- c2 Another circular object

Output Variables

- p The probability that the center of the c distribution is at (0,0)
- w The ranksum statistic

```
BK - 29.7.2001 - last change Date:2001/08/2104:47:18 by Author:bart\ Revision:1.4
```

function d = minus(c1,c2)

Description Subtract two circular data sets from each other. If either is a single vector, this vector is subtracted

from all the data in the other set. If both are vectors or matrices, they should be of the same size.

Units are inherited from c1 axiality only if c1 and c2 are axial.

Input Variables

- c1 A circular data object.
- c2 A circular data object.

Output Variables

cm A circular data object.

Note c1+c2 does something quite different: it adds data, not vectors!

```
BK - 29.7.2001 - last change Date:2001/08/0200:45:20 by Author:bart\ Revision:1.1
```

function v = x(c)

Description Return the x-components of the circular data.

Input Variables

c a circular data object.

Output Variables

v The x-components of these vectors.

```
BK - 29.7.2001 - last change Date:2001/08/0200:43:29 by Author:bart\ Revision:1.1
```

function v = y(c)

Description Return the y-components of the circular data.

Input Variables

c a circular data object.

Output Variables

v The x-components of these vectors.

```
BK - 29.7.2001 - last change Date:2001/08/0200:43:29 by Author:bart\ Revision:1.1
```

function cn = normalise(c)

Description Normalise the vectors in this circular data object.

Input Variables

c A circular data object.

Output Variables

cn A new cicrcular data object whose vectors all have length 1.

```
BK - 1.8.2001 - last change Date:2001/08/0200:45:02 - by Author:bart\ Revision:1.1
```

function [mPhi,mR,meanC,mConf,mS,p] = mstd(c,alpha)

Description Determine the circular mean and its (1-alpha)

with the rayleigh test for non-randomness (i.e. is there a mean?)

Corrects for grouping/binning if c.groups is set to the number of bins, and deals appropriately

with axial data if c.axial = 1.

Input Variables

c The circular object

alpha The alpha value for the confidence interval. Optional, defaults to 95

Output Variables

```
phi The mean angle.
```

r The length of the mean vector

meanC The nean circular data object.

conf The x

- s The standard deviation
- p P-value of the Rayleigh test

Note The calculation of the confidence interval assumes linear statistics, hence only makes sense for

small dispersions.

```
BK - 28.7.2001 - last change Date:2001/08/2104:47:19 by Author:bart\ Revision:1.6
```