$$P(\beta,\gamma) + P(\alpha,\delta) - P(\alpha,\gamma) - P(\beta,\delta) \geq 0$$

$\beta > \alpha$
$\delta > \gamma$

$$P(w_m, w_n) = K(1 - \delta(w_m - w_n))$$

↓ Kronecker delta $\delta(x) = \begin{cases} 1, & x=0 \\ 0, & \text{otherwise} \end{cases}$

$$K(1-\delta(\beta-\gamma)) + K(1-\delta(\alpha-\delta)) - K(1-\delta(\alpha-\gamma)) - K(1-\delta(\beta-\delta)) \geq 0$$

$$-\delta(\beta-\gamma) - \delta(\alpha-\delta) + \delta(\alpha-\gamma) + \delta(\beta-\delta) \geq 0$$

If any pair: (β,γ), (α,δ), (α,γ), (β,δ) has equal values the submodularity constraint won't be satisfied.

example: $\beta = \gamma \Rightarrow \delta > \gamma = \beta > \alpha$

$\delta = 4$, $\gamma = 3$, $\beta = 3$, $\alpha = 2$

$$-\delta(0) - \delta(-2) + \delta(-1) + \delta(-1) \geq 0$$

$$-1 - 0 + 0 + 0 \geq 0$$

$$-1 \not\geq 0$$