Task 3 — Exercise Sheet 01

$$\left((f * g) * h\right)(x) = \int (f * g)(u)\, h\,(x-u)\, du$$

$$= \int_u \int_w f(w)\, g(u-w)\, dw\; h\,(x-u)\, du$$

$$= \int_u \int_w f(w)\, g(u-w)\, h\,(x-u)\, dw\, du$$

$$= \int_w \int_u f(w)\, g(u-w)\, h(x-u)\, du\, dw \qquad \Rightarrow \text{Switch Integral order}$$

$$= \int_w f(w) \left[\int_u g(u-w)\, h(x-u)\, du \right] dw$$

$$\underline{\text{Let } \quad u = v + w} \qquad \Rightarrow \text{Use Shift invariance}$$

$$= \int f(w) \left[\int_v g(v)\, h\,((x-w)-v)\, dv \right] dw$$

$$= \int f(w) \left[(g * h)(x-w) \right] dw$$

$$= \left(f * (g * h)\right)(x)$$

Hence proved

Convolution is Associative