We use the associative property of convolutions and prove that the convolution of two Gaussian Kernels $(k(x,y))$ with std. dev. 'σ' is equivalent to one Gaussian Kernel with std. dev. '$\sqrt{2}\sigma$'.

$$\therefore \quad k(x,y) = \frac{1}{2\pi\sigma^2} e^{\frac{-(x^2+y^2)}{2\sigma^2}} = \frac{1}{2\pi\sigma^2} \cdot e^{-\frac{x^2}{2\sigma^2}} \cdot e^{-\frac{y^2}{2\sigma^2}}$$

$$\therefore \quad k \otimes k = F^{-1}\left[F(k(x,y))\, F(k(x,y))\right]$$

$\Rightarrow$ Using the Fourier Transform of convolutions property. (F is Fourier Transform)

$$\therefore \; F(k(x,y)) = \frac{1}{2\pi\sigma^2} \cdot \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right) \exp\left(-i\,2\pi(ux+vy)\right) dx\, dy$$

$$= \frac{1}{2\pi\sigma^2} \cdot \int_{-\infty}^{\infty} \exp\left(\frac{-x^2}{2\sigma^2}\right) \exp\left(-i 2\pi ux\right) dx \int_{-\infty} \exp\left(\frac{-y^2}{2\sigma^2}\right) \exp\left(-2\pi i vy\right) dy$$

$$= \frac{1}{2\pi\sigma^2} \cdot \int_{-\infty}^{\infty} \exp\left(\frac{-x^2}{2\sigma^2}\right)\left(\cos 2\pi ux - i \sin 2\pi ux\right) dx \cdot \int_{-\infty}^{\infty} \exp\left(\frac{-y^2}{2\sigma^2}\right)\left(\cos 2\pi vy - i\sin 2\pi vy\right) dy$$

Now, sin is an odd function, so its integral over $(-\infty,\infty)$ will be zero. Also, cos is an even funct. $\therefore \int_{-\infty}^{\infty}$ even funct $= 2\int_0^{\infty}$ even funct.

$$= \frac{1}{2\pi\sigma^2} \cdot \int_{-\infty}^{\infty} \cos 2\pi ux \exp\left(\frac{-x^2}{2\sigma^2}\right) dx \int_{-\infty} \cos 2\pi vy \exp\left(\frac{-y^2}{2\sigma^2}\right) dy$$

Now, $\displaystyle\int_0^{\infty} e^{-at^2} \cos(bt)\, dt = \frac{1}{2}\sqrt{\frac{\pi}{a}} \exp\left(-\frac{b^2}{4a}\right)$

we have for the above form, $a = \frac{1}{2\sigma^2}$, $b = 2\pi u$ or $2\pi v$

$$\therefore F(k(x,y)) = \frac{1}{\cancel{2\pi\sigma^2}} \cdot \cancel{\sqrt{2\pi}\sigma} \cdot \exp\left(-\frac{4\pi^2u^2}{4/2\sigma^2}\right) \cdot \cancel{\sqrt{2\pi}\sigma} \cdot \exp\left(-\frac{4\pi^2v^2}{4/2\sigma^2}\right)$$

$$\therefore F(k(x,y)) = \exp\left(-2\pi^2\sigma^2(u^2+v^2)\right) \quad \ldots (1)$$

Fourier Tranform of a 2D Gaussian Kernel with variance σ^2

$\therefore \quad F(h(x,y)) \cdot F(h(x,y)) = \exp(-2\pi^2\sigma^2(u^2+v^2)) \exp(-2\pi^2\sigma^2(u^2+v^2))$

$$= \underbrace{\exp(-2\pi^2 \times 2\sigma^2 \times (u^2+v^2))}$$

From (1) ... This part is similar to the Fourier Transform of a gaussian with std. dev $= \sqrt{2}\sigma$

$\therefore \quad F^{-1}(F(h(x,y)) \cdot F(h(x,y))) = F^{-1}(\exp(-2\pi^2 \cdot 2\sigma^2 \times (u^2+v^2)))$

$\therefore \quad h(x,y) \circledast h(x,y)$ with variance σ^2 $= \dfrac{1}{2\pi(\sqrt{2}\sigma)^2} \times e^{-\frac{(x^2+y^2)}{2(\sqrt{2}\sigma)^2}}$ $\Rightarrow$ Gaussian kernel with variance $(2\sigma^2)$

Hence proved