

# Physics 201 Lab 2

## Air Drag Simulation

Jan 28, 2013

### Equipment

### Initial Set Up

Type the data from Table 1 into the appropriate cells.

By preceding the content of the cell with an equal sign (as in cell A6) we tell Excel that it needs to calculate this formula rather than simply parrot the value back.

Now highlight the selection A7 to A26 (the shorthand notation for this is A7:A26). Hit **Ctrl-D**. This will “drag-down” the formula from cell A7 into all the other cells (see Figure 1).

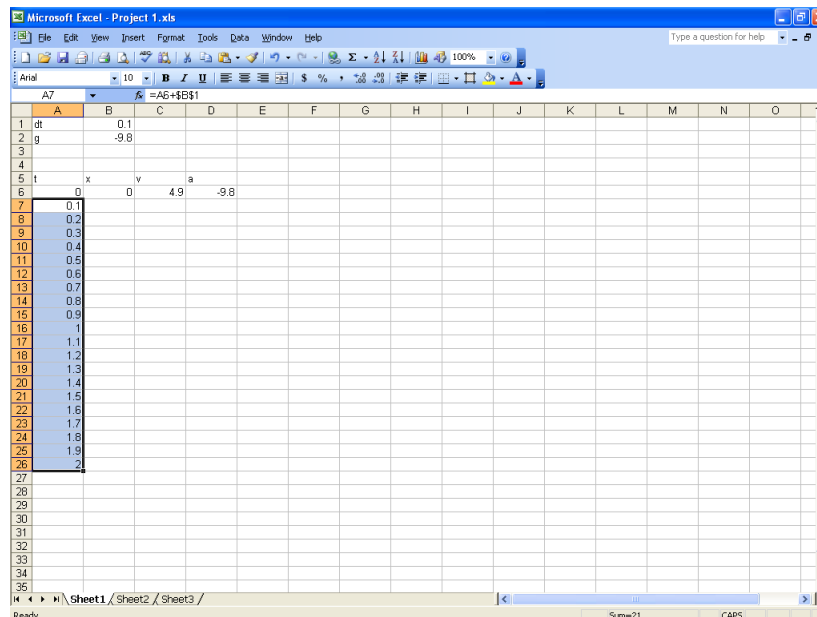


Figure 1: First screen-shot

The dollar signs indicate the difference between a relative and absolute reference. When a dollar sign is involved (as in **\$B\$1**) the reference stays the same no matter where you paste or drag this cell. However, if there is no dollar sign, the reference maintains its relative “distances” in the new cell. For example, highlight cell A15. The formula in it should be **=A14+\$B\$1**. Notice how the first term (A14) is the cell just above the selection (like it was for cell A5), but the second term (**\$B\$1**) is unchanged.

As you may have guessed, the goal is to represent the position of the particle in column B, its velocity in column C, and its acceleration in column D while column A represents the time elapsed.

Type the data from Table 2 into the appropriate cells.

The use of the average in the above formula may seem odd at first. Suffice it to say that when we do this it makes the calculations slightly more accurate than

A1	dt
B1	0.1
A2	g
B2	-9.8
A5	t
B5	x
C5	v
D5	a
A6	0
B6	0
C6	4.9
D6	=\$B\$2
A7	=A6+\$B\$1

Table 1: Initial set up

B7	=B6+AVERAGE(C6:C7)*\$B\$1
C7	=C6+AVERAGE(D6:D7)*\$B\$1
D7	=\$B\$2

Table 2: Formulas for velocity and acceleration

the more straightforward formula  $=C6+D6*\$B\$1$  which follows from  $v = v_0 + at$ . Similar logic holds for how the velocity changes with acceleration.

Drag these formula down into the range B7:D26 as shown in Figure 2.

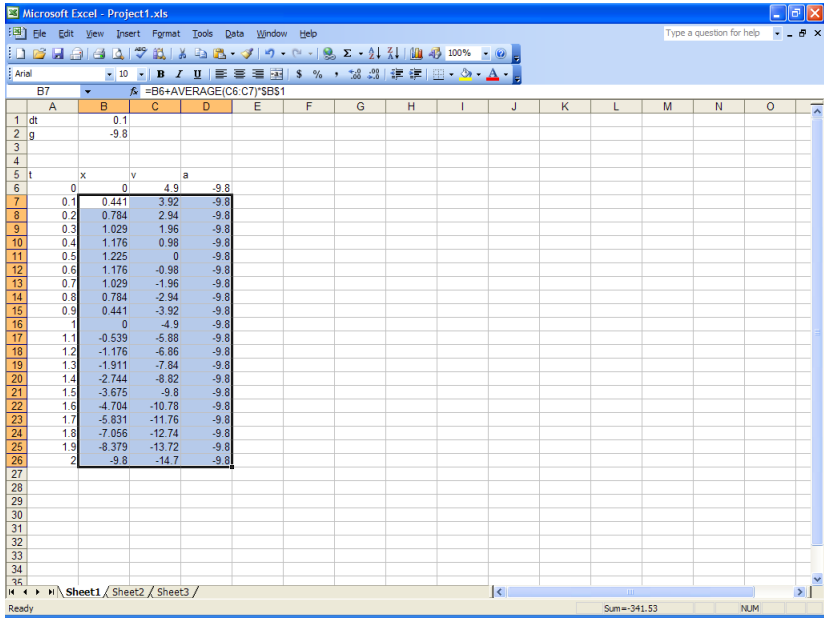


Figure 2: Filling in the calculations

Now we want to use a scatter plot to graph this information. Highlight the range A6:B26 and click on the graphing wizard and choose a scatter plot with connected lines.<sup>1</sup>

You should get something like Figure 3 when you are done. This represents the path of a vertical projectile under constant acceleration.

The initial position is at  $x_0 = 0$ , the initial velocity is  $v_0 = 4.9$ . After 2.0 seconds, the projectile is at  $x = -9.8$  with a velocity of  $v = -14.7$ . The acceleration is a constant  $-9.8$ .

<sup>1</sup>The location of the graphing wizard is different for Office 2003 than for Office 2007. I am using 2003. You may have to search around or ask someone where to find some of these things if you are using 2007.

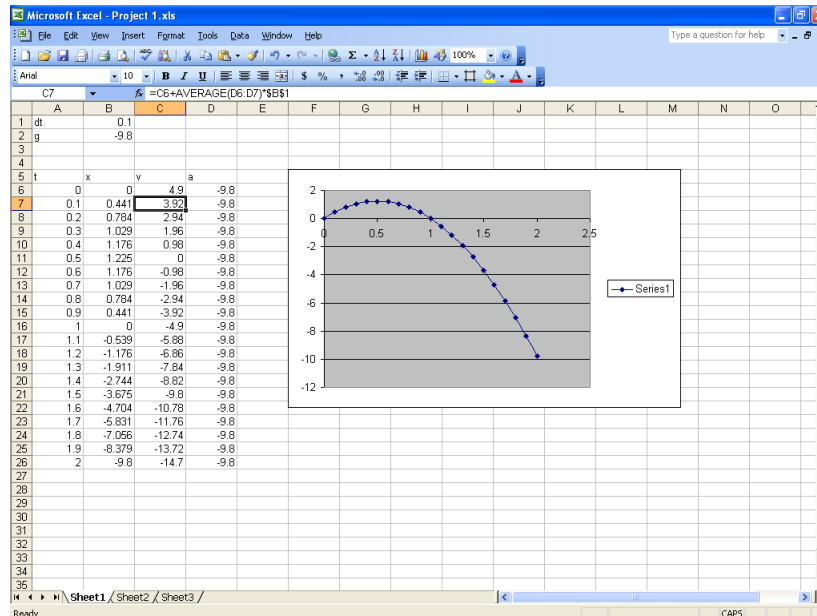


Figure 3: Adding a graph

There are a lot of things you can do to manipulate the look and feel of these graphs: move them around, change their size, delete the legend, change the background color, alter the number of digits displayed on the axes, etc. I won't go through all those details for you—usually you can double-click on something you want to change and a menu will appear with options to choose from. Trust me: it is easy to waste a lot of time playing around with these formats!

Suppose instead we want to model an object dropped from 50 meters high from rest. Change cell B6 to 50 and cell C6 to 0. Your screen should now look like Figure 4.

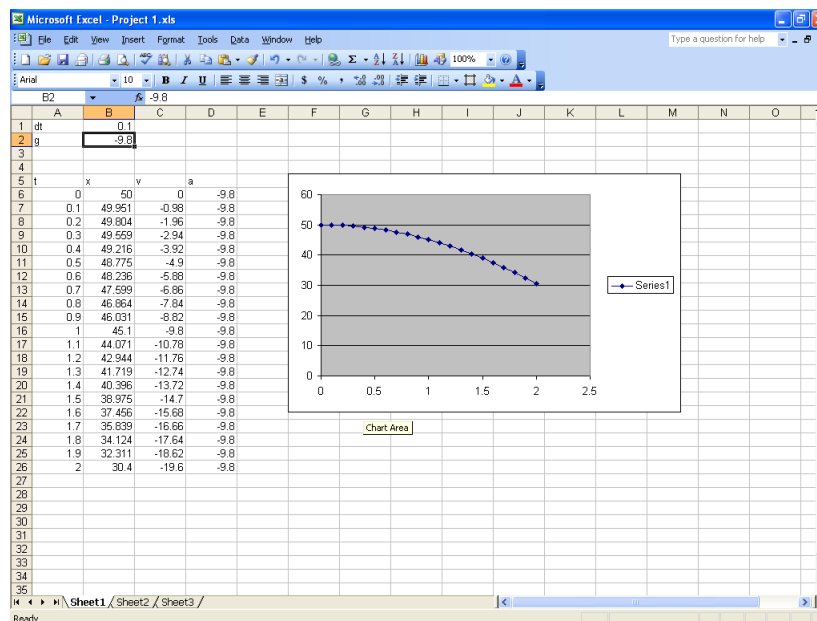


Figure 4: Projectile starting from rest at  $x = 50$

However, we may be interested to know how long it takes the projectile to hit the ground. One way to do this is by changing the time differential in cell B2. Change it to 0.2. Now the chart crosses the  $x$ -axis. See Figure 5.

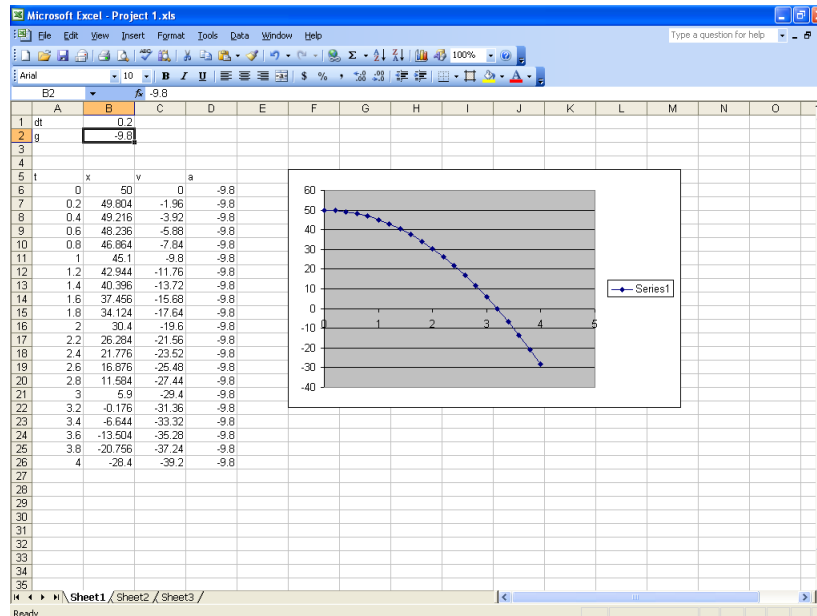


Figure 5: Increasing the time scale

Notice how in row 22 the position in column B is practically zero. The time in column A is 3.2 seconds. So, we reach  $x = 0$  in about 3.2 seconds. Now might be a good time to save the workbook if you haven't already done so.

## One Dimensional Linear Air Drag

We now want to introduce some air resistance into the system. We are going to assume that as the projectile moves faster, the air drag is larger. In other words, the particle will suffer deceleration which is proportional to its speed.

Change the values back according to Table 3.

We are now going to add a new parameter to control the air drag. Type  $\$B\$$  and 0.1 into cells A3 and B3 respectively. This is going to represent the amount of air resistance on the projectile.

Now alter the formula in D7 (not D6) to  $=\$B\$2-\$B\$3*C6\}$ . Drag this formula over the range 'D7:D26. This has the effect of introducing a deceleration which is proportional to velocity. Your screen should now look like Figure 6.

B1	0.1
B6	0
C6	4.9

Table 3: Reset initial parameters

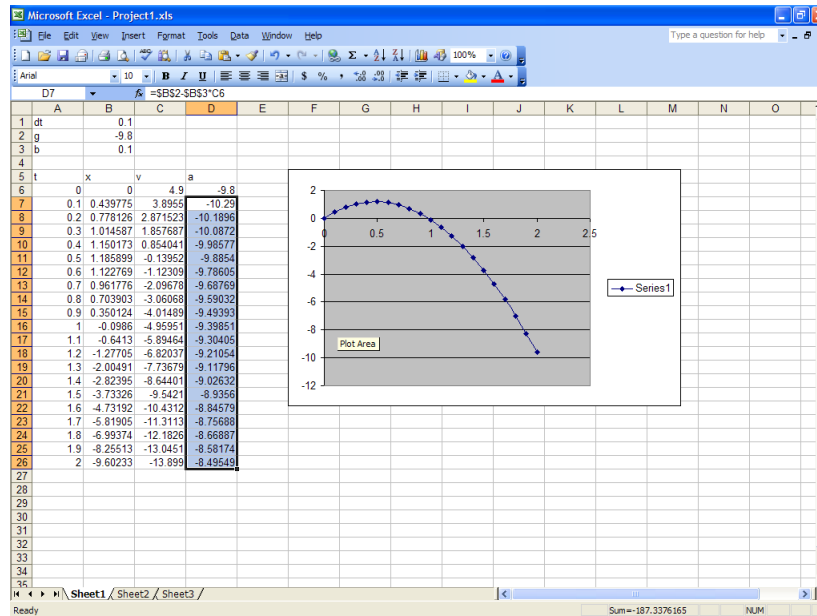


Figure 6: Linear air drag added to calculation

Notice how this is very similar to the previous trajectory. This is telling us that our coefficient of air drag is small. Now increase cell B3 to 1. Your screen should now look like Figure 7.

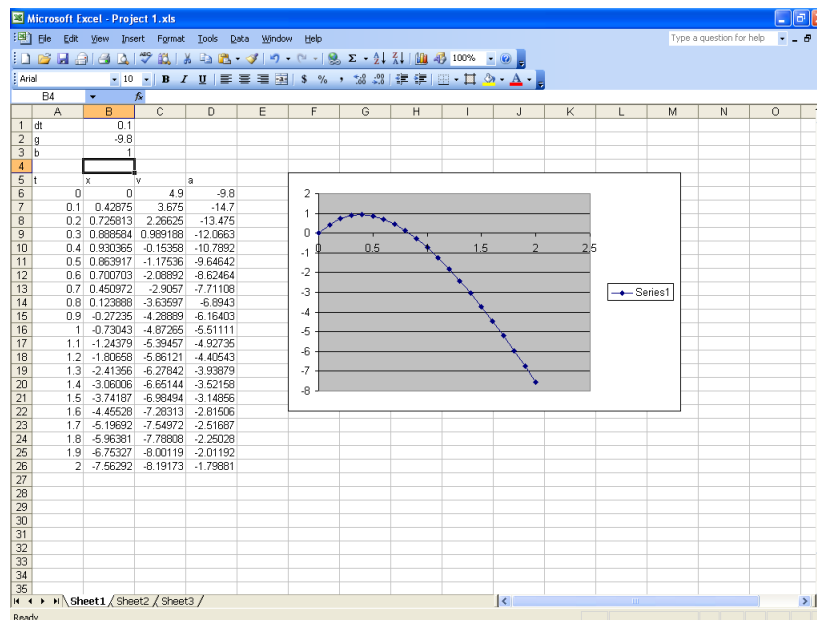


Figure 7: Linear air drag turned on

Notice some differences:

- The trajectory peaks earlier (near 0.4 rather than 0.5)
- The projectile hits the ground earlier (near 0.8 rather than 1.0)
- The second half of the trajectory is much more linear—this indicates that the projectile is moving with something close to constant velocity, a.k.a. terminal velocity.

These differences are easier to see if you “freeze the scales” on the graph. Double-click on the numbers on the vertical axis and find the boxes that allow you to change the scale. Deselect the boxes and change the minimum to -12 and the maximum to 2. Do the same to the horizontal axis. Change the minimum to 0 and the maximum to 2.5.

Now your screen should look Figure 8 which is now comparable to Figure 6. The differences are more apparent. Temporarily change the  $\$B\$$  value to 0. You should get back to the screen shown in Figure 3. Reset the  $\$B\$$  value back to 1.

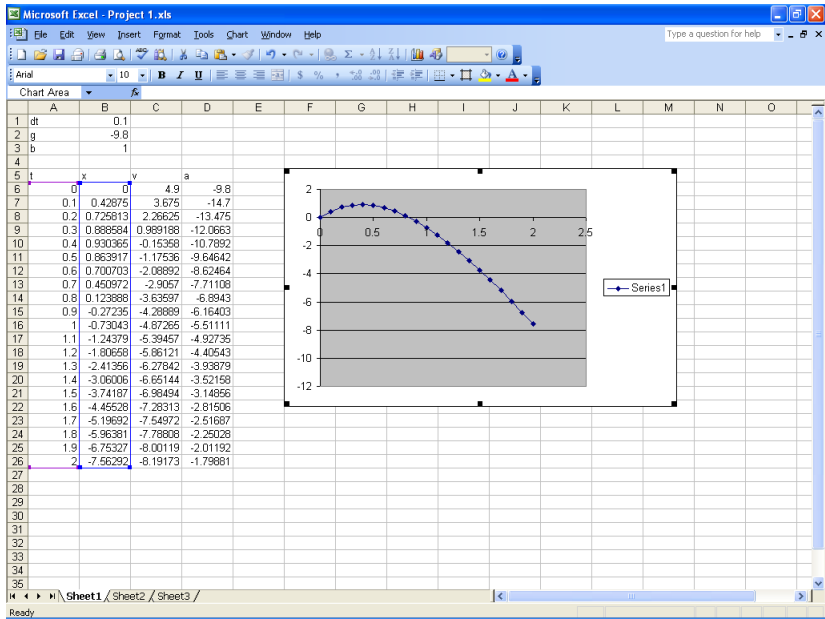


Figure 8: Vertical scale set to compare with Figure ??

The terminal velocity can be read directly from column C. It looks like the value is trending somewhere less than -8.2. Can we get a better estimate? Yes, if we extend the calculations. Highlight cells A26:D26 and drag it down to row 100. Your screen should look like Figure 9.

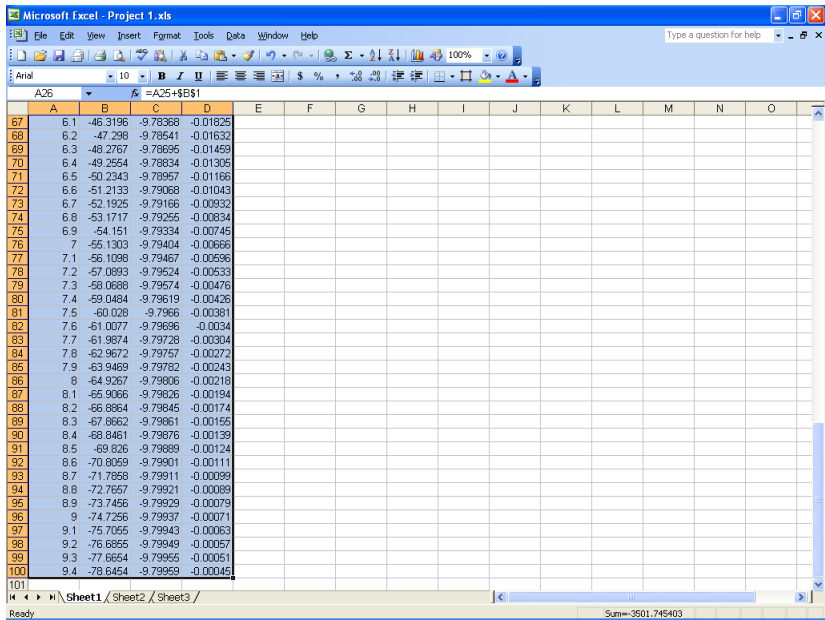


Figure 9: Extending the calculation

Apparently the terminal velocity is closer to -9.8. Notice how the acceleration in column D is practically zero at this point which is indicating that the projectile is near terminal velocity. When does the particle hit terminal velocity? Technically, never. The acceleration is always non-zero. But for this project let's agree that we will declare terminal velocity hit when the magnitude of the acceleration is less than 0.01. The acceleration just isn't large enough to make a significant different anymore. In this case, we simply scroll up until we see that number in column D. See Figure 10. This happens in row 73. We can say that terminal velocity is effectively reached in 6.7 seconds.

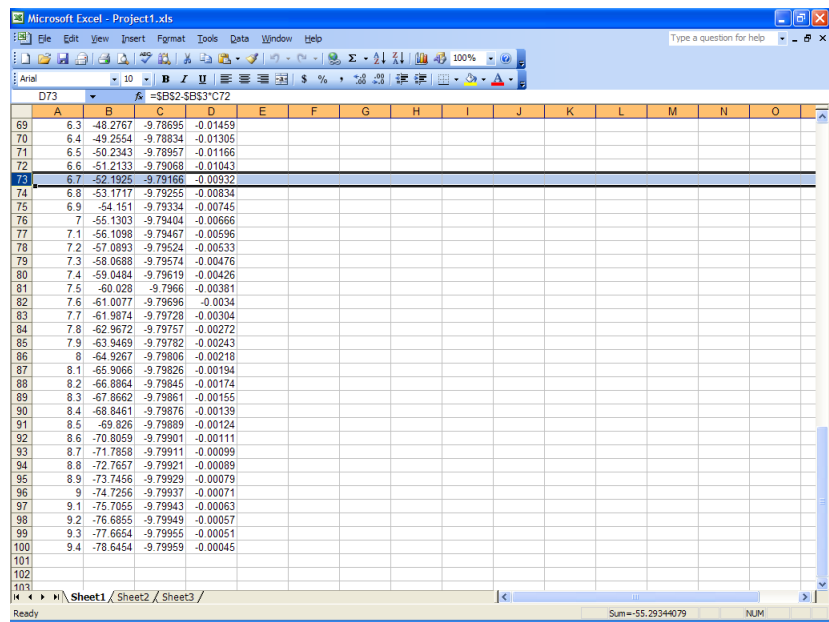


Figure 10: Finding terminal velocity

Finally, change the  $\$B\$$  value to ten. Your screen should look something like Figure 11.

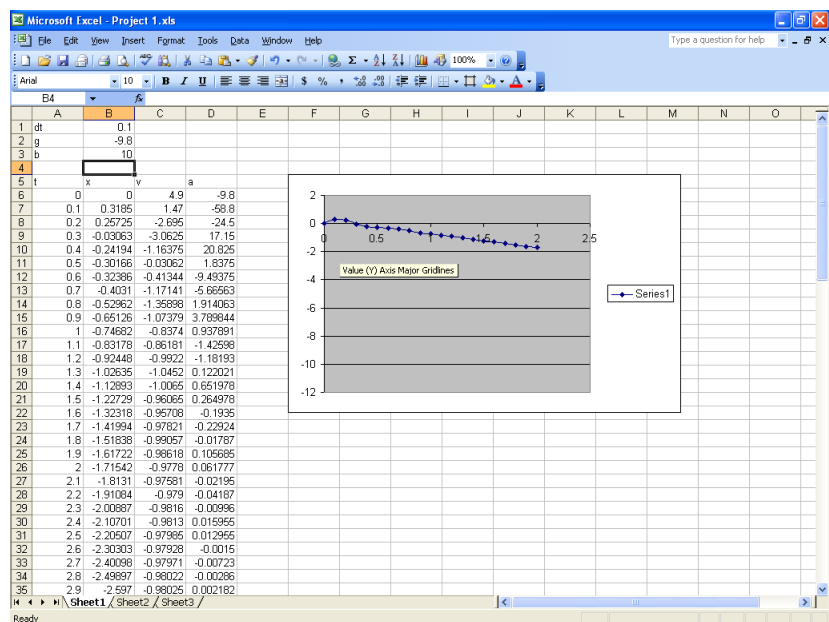


Figure 11: Extreme air drag—calculation broken

Notice how the graph looks funny in the time period 0.3 to 0.7? This is because we have pushed the spreadsheet approach too far. The calculations are not precise enough and we are getting erroneous results. One foolproof way to fix this problem is to reduce our time differential in cell B1. Change it to 0.02. You should now see something like Figure 12.

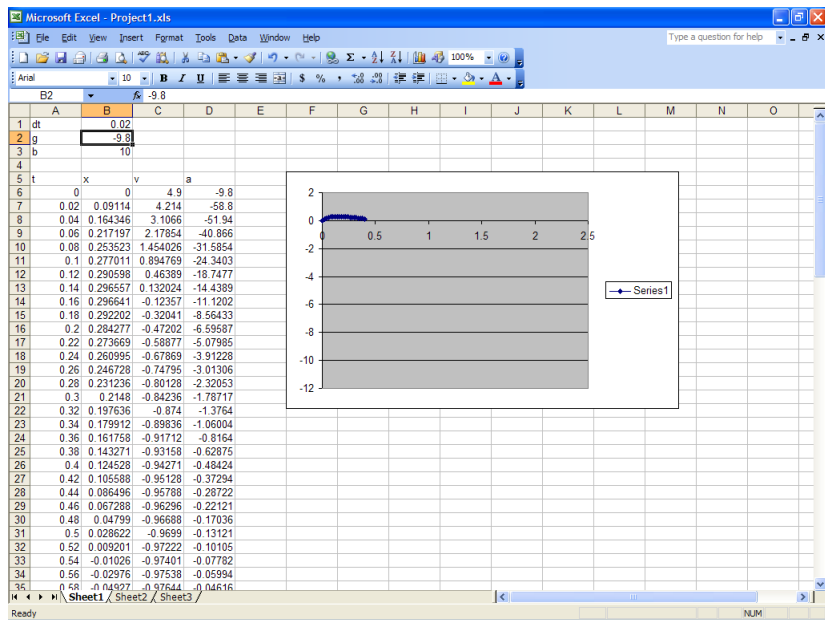


Figure 12: Extreme air drag—calculation fixed

This is better, but our graph is still only linked down to row 26. Go ahead and click on the graph. Your screen should look like Figure 13.

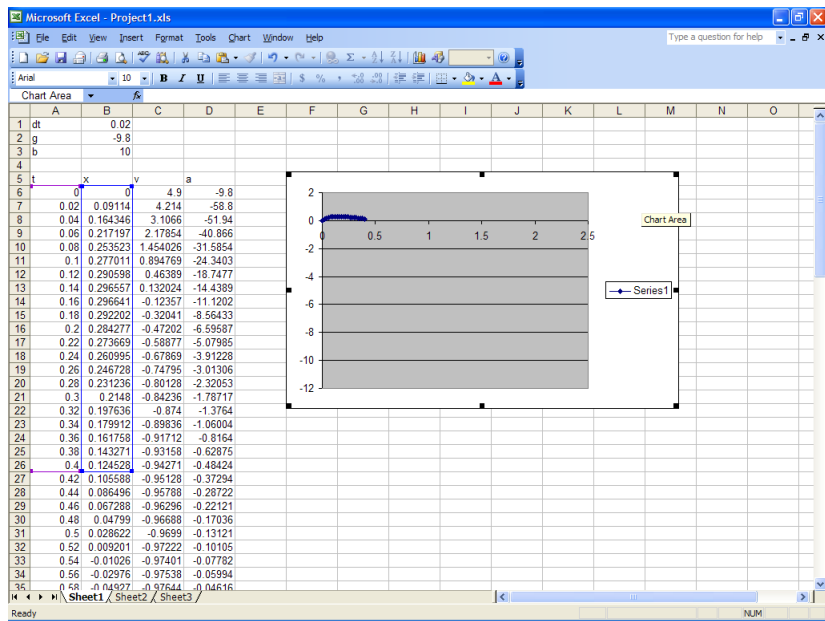


Figure 13: Highlighting the graphed data

Move your cursor until it is just on the bottom corner of the blue box in column B. Click and drag this box down to row 100. The purple box should follow. Scroll back up to the top again. You should now have Figure 14.



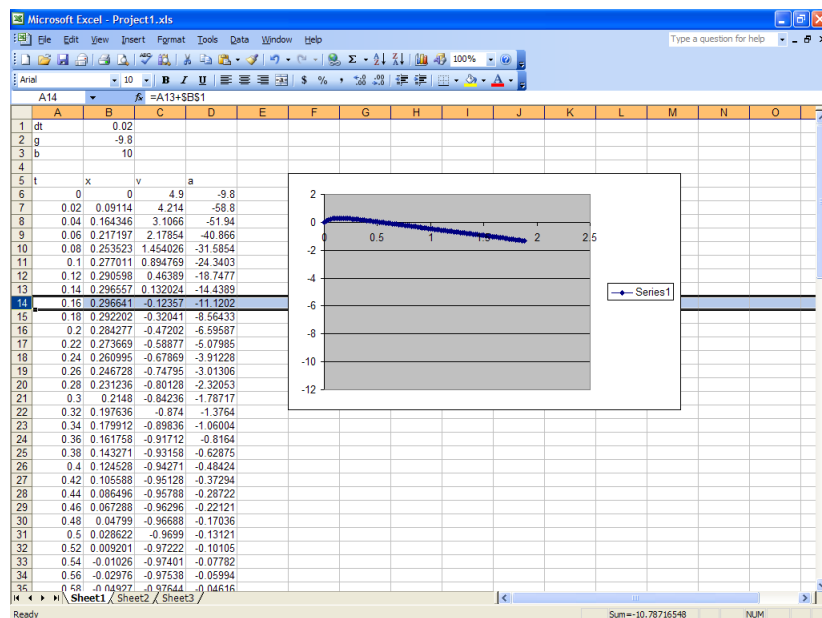


Figure 14: Graph with extended data represented

Notice that we hit  $x = 0$  in between  $t = 0.52$  and  $t = 0.54$ . (Maybe we should say  $t = 0.53$ ?) The particle hits terminal velocity at  $t = 0.70$  with a value of  $v = -0.98$ . We can also tell that the maximum height achieved is  $x = 0.30$  at  $t = 0.16$  from row 14.

Rename the sheet by double-clicking on the name **Sheet1**. Change the name to **1D linear drag**. Go ahead and save the workbook.

## Quadratic Air Drag

We have modeled a very simplistic form of air resistance. The linear form is appropriate for small objects like raindrops. For larger objects the drag tends to be dominated by the square of the velocity. Truthfully, real air drag is a combination of both effects and more. It is possible to combine these effects using the techniques here, but I will let you explore those possibilities on your own.

Quadratic air drag is simple to incorporate into our spreadsheet. First copy the sheet. Do this by dragging the sheet name while holding the CTRL key. Rename it **1D quadratic drag**. Change the time differential  $dt$  to 0.02 and the  $\$B\$$  value back to 0. Go to cell D7 and change the equation to  $=\$B\$2-\$B\$3*\text{SIGN}(C6)*C6^2\}$ .

What we have done here is square the value in cell C6: the velocity. We have also multiplied by  $\text{SIGN}(C6)$  which is just +1 or -1 depending on the sign of whatever is in cell C6. This way the air drag always opposes the direction of the velocity. (Why didn't we have to multiply by  $\text{SIGN}(C6)$  in step 2?) Drag this equation down to row 100 and scroll back to the top. Your screen should look Figure 15.

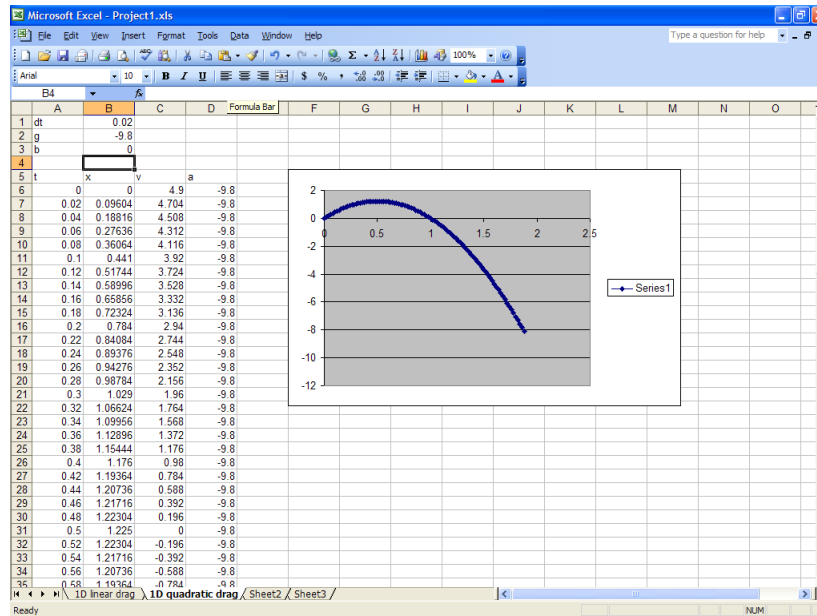


Figure 15: Quadratic air drag turned off

Again, this is not any different than no air drag at all. What value of  $\$B\$$  produces a reasonable curve? Try 1. You should have a graph like Figure 16.

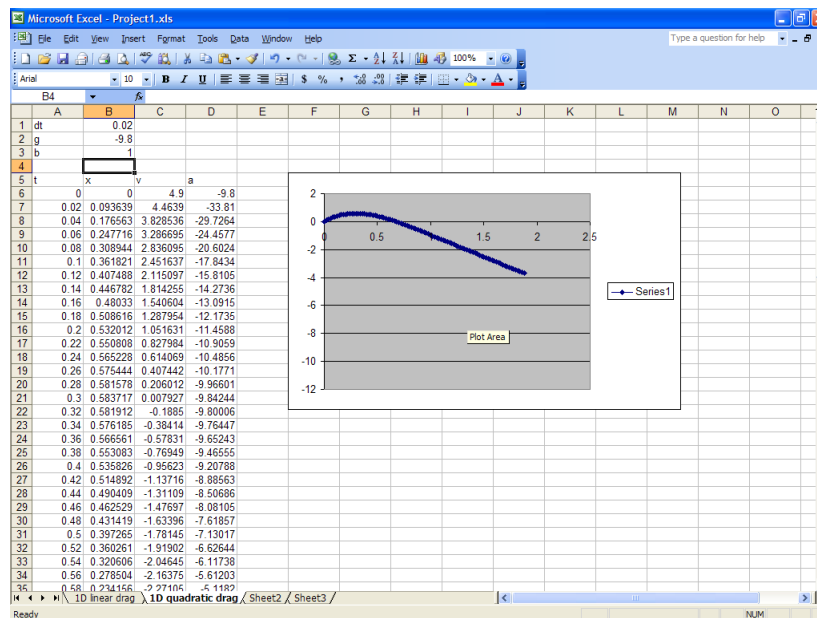


Figure 16: Quadratic air drag turned on

The stats for this curve are as follows:

- The maximum height achieved is  $x = 0.58$  at  $t = 0.30$ .
- We hit  $x = 0$  near  $t = 0.68$ .
- Terminal velocity is  $-3.13$ .
- The particle hits terminal velocity at  $t = 1.50$ .

Apparently this type of air drag is less effective than the linear form since it reaches a higher height, the particle is in the air longer, and ends with a larger terminal velocity.

## Projectile in Two Dimensions

Let's start over on **Sheet2**. If you don't have another sheet available, you can add one by going up to the menu bar **Insert** and click on **Worksheet**. Rename this sheet **2D linear drag**. (Don't forget to hit **Enter** when you are done). Also you can delete them by right-clicking on their name and clicking on **Delete**.

Set up the sheet as in Table 4, similar to what we had before.

Now add the formulas listed in Table ??.

And drag them down to row 26. High-light the range **B6:C26** and build a scatter graph like before.

Move the graph to the top of screen as shown in Figure 17.

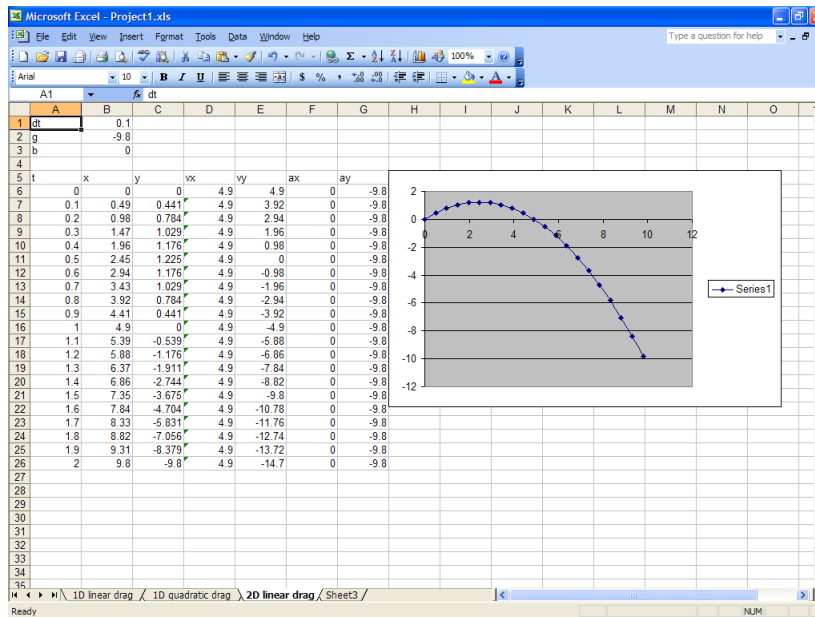


Figure 17: Two dimensional projectile

This looks just like before, but there is an important difference now. The horizontal axis no longer represents time. It represents the displacement in the  $x$ -direction. The vertical axis is different too. It now represents the displacement in the  $y$ -direction. This is what the trajectory of the particle would look like to a person watching from the side.

Let's change the parameters a bit to see what happens. In cell **C6**, let the initial  $y$ -displacement be 50. See Figure 18.

A1	dt
B1	0.1
A2	g
B2	-9.8
A3	b
B3	0
A5	t
B5	x
C5	y
D5	vx
E5	vy
F5	ax
G5	ay
A6	0
B6	0
C6	0
D6	4.9
E6	4.9
F6	0
G6	=\$B\$2

Table 4: Initial set up

A7	=A6+\$B\$1
B7	=B6+AVERAGE(D6:D7)*\$B\$1
C7	=C6+AVERAGE(E6:E7)*\$B\$1
D7	=D6+AVERAGE(F6:F7)*\$B\$1
E7	=E6+AVERAGE(G6:G7)*\$B\$1
F7	0
G7	=\$B\$2

Table 5: Definition of velocity and acceleration

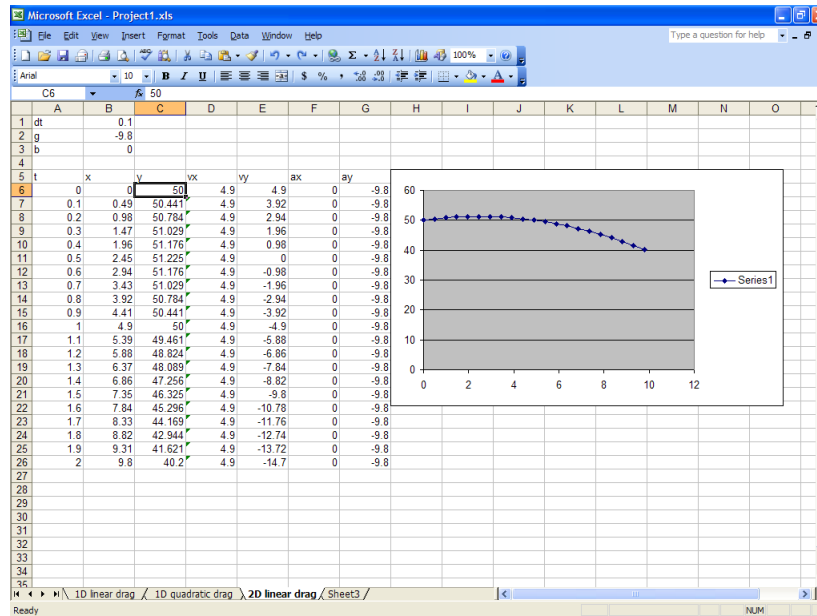


Figure 18: Projectile launched from 50 meter height

Again, change the time differential to 0.2 in order to extend the graph a bit as in Figure 19.

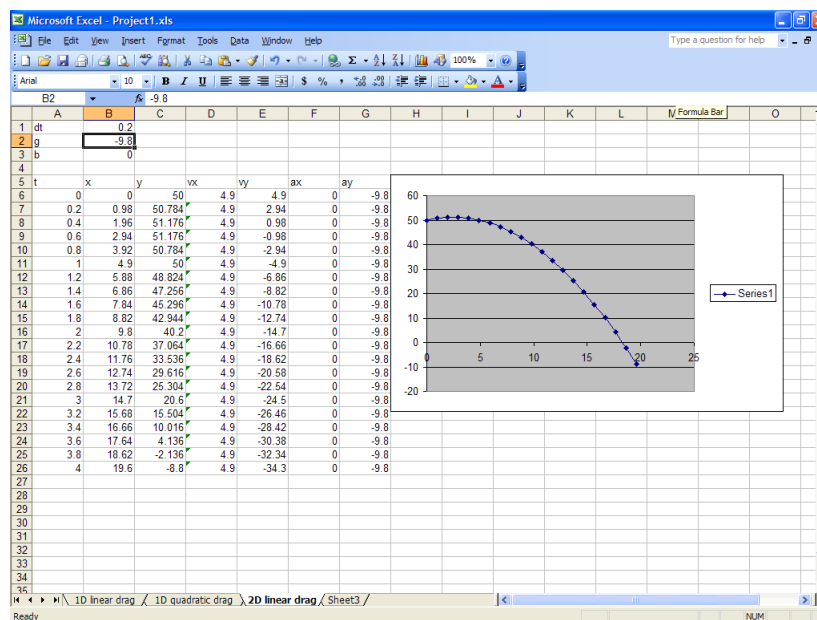


Figure 19: Finding the range of the projectile

It looks like the range of the projectile is somewhere between 17.64 and 18.62 (closer to 18.62) because this is where the  $y$  position flips sign—see column C. What is the velocity when it strikes the ground? The horizontal velocity is always 4.9, and the vertical velocity is about -32.

Now change the initial vertical velocity to be -4.9 in cell E6. You should now have Figure 20.

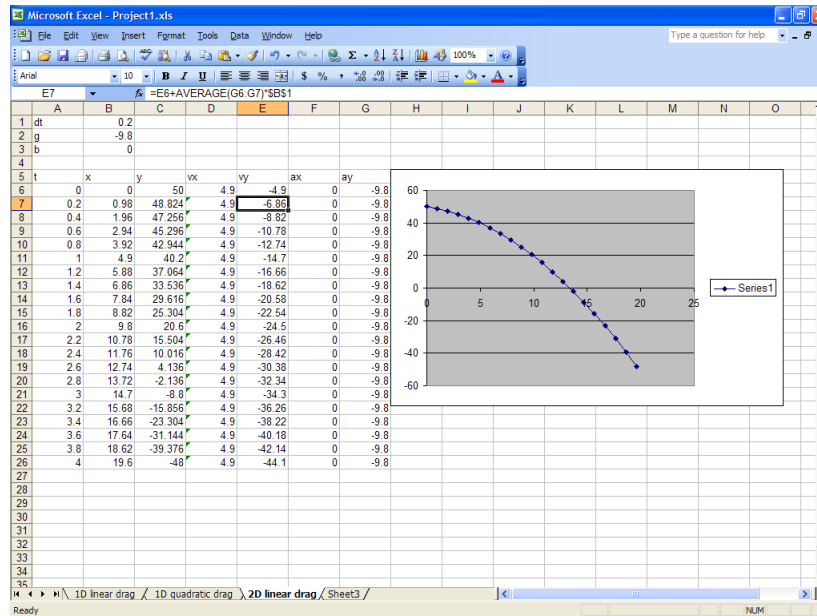


Figure 20: Projectile with initial velocity downward

The range is shorter: somewhere between 2.6 and 2.8 (see rows 19:20). What is the velocity when it strikes the ground? As before, the horizontal velocity is always 4.9. The vertical velocity is -32. This is the same as the previous situation. Do you see how this curve is nothing but the subset of the previous curve after it reaches its peak and crosses  $y = 50$  again?

Okay, drag down the formulas to row 100 and set the initial  $y$  position to zero. Extend the graph to capture this data as before. Reset the time differential to 0.1. Change the horizontal and vertical velocities to 45. Your screen should look like Figure 21.

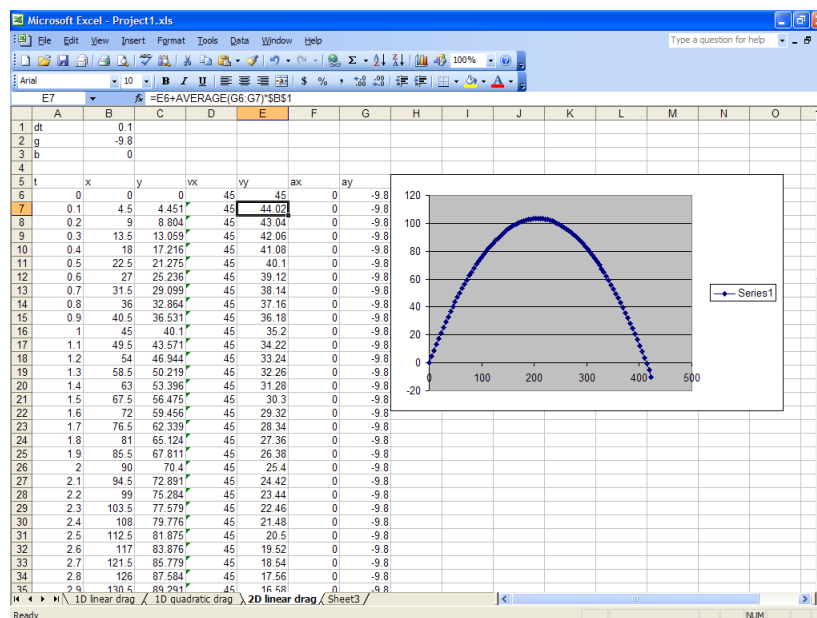


Figure 21: Linear air drag turned off

This is something like a home run hit in baseball. Now would be a good time to “freeze the scales” on both the horizontal and vertical axes of the graph.

In order to determine the terminal velocities, we are going to want to know the magnitude of the velocity and acceleration. We can do this systematically by adding two columns to our spreadsheet. Type the data from Table 6 (you may need to move the graph).

Drag these formulas down to row 100. You should now have a screen like Figure 22. These two new columns represent the magnitude of the velocity and acceleration.

H5	v
I5	a
H6	=SQRT(D6^2+E6^2)
I6	=SQRT(F6^2+G6^2)

Table 6: Magnitude of velocity and acceleration

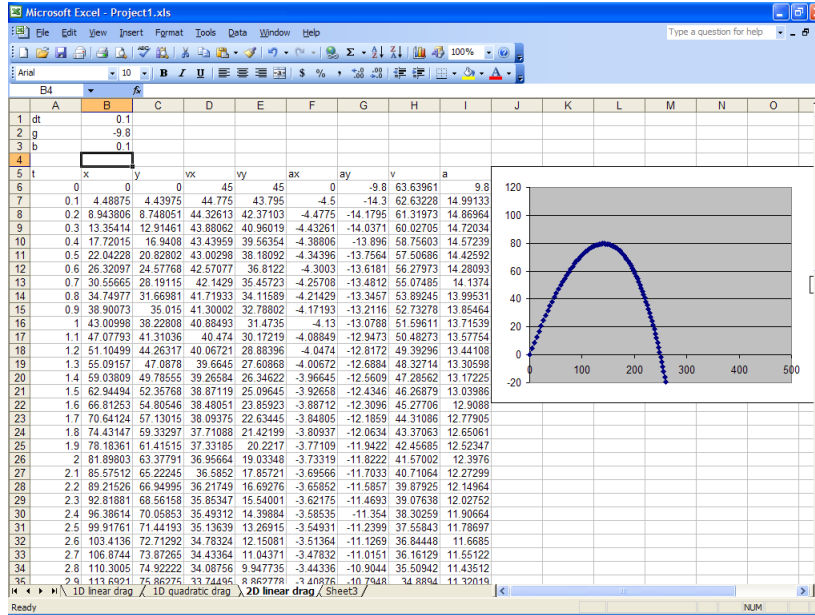


Figure 22: Magnitude of velocity and acceleration included

We are now ready introduce some air resistance. Change the formulas in F7 to  $=0-\$B\$3*D6$  and G7 to  $=\$B\$2-\$B\$3*E6$ . Drag them down as usual. That's it—we have just added a linear air drag term. Now change the drag coefficient in cell B3 to 0.1. You should have something like Figure 23.

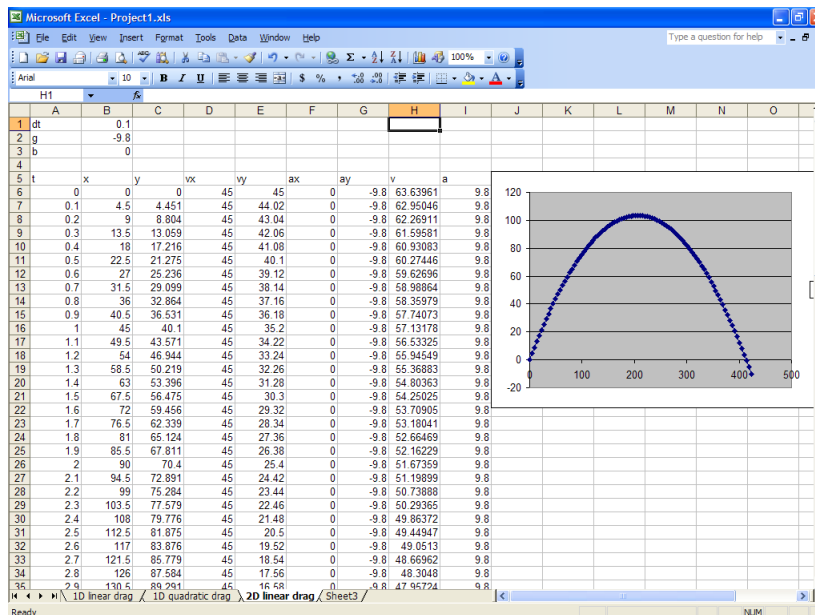


Figure 23: Linear air drag turned on

This has a huge effect. More than you might expect from the first section in the project. The range is reduced from something like 420 to 250. Also notice the asymmetry in the trajectory: the ball falls down much more vertically than it rises.

Change the drag coefficient to one. See Figure 24.

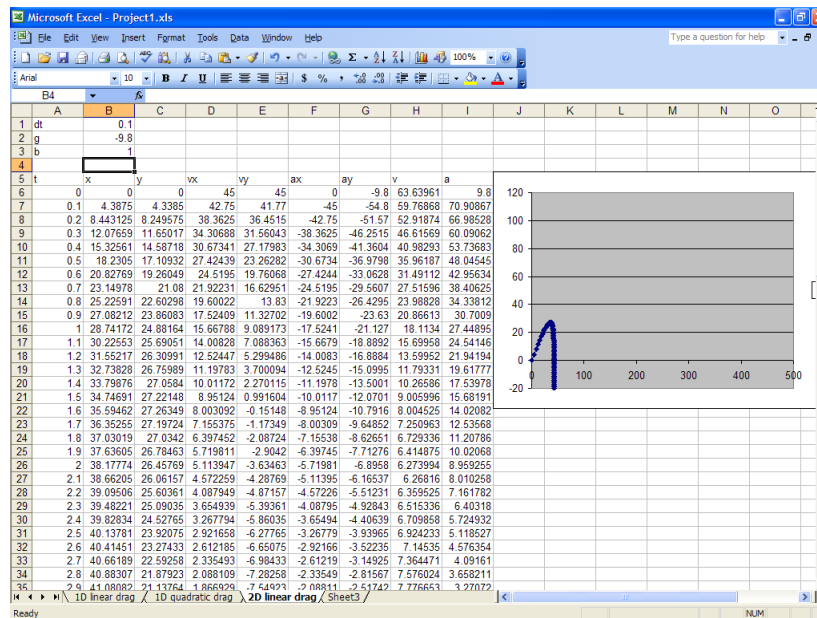


Figure 24: Extreme drag (like water instead of air)

Now there is hardly any range at all (less than 50 meters) and the ball falls nearly straight down. Terminal velocity of  $-9.8$  is reached in 8.1 seconds (see row 87). This much resistance is more like what one would expect from water rather than air.

## Questions

- Raindrops are small and therefore experience an air drag that is linear. Assume a drag coefficient of 30. What is the terminal velocity of a drop from rest and how long does it take to reach that value? (Hint: Reduce the time differential to 0.005).
- A basketball is dropped from rest. The air drag is quadratic with coefficient 0.023. What is its terminal velocity and how long does it take to reach that value? How does this compare with the raindrops in Question 1?
- Let's suppose a pro golfer can hit a tee shot 300 meters. Assume this is under conditions where there is linear air drag with a coefficient of 0.10. If the ball is launched at 45 degrees, what is the initial components of the velocity? (Only two significant figures required.) What is the maximum height, time of flight, and range of the trajectory? Use a time differential of 0.1 for this scenario.
- Use the previous spreadsheet scenario to determine the range of the ball if there was no air drag at all. Use a time differential to 0.2 to answer. How does this compare to the range equation  $R = (v^2/g) \sin 2\theta$ ?