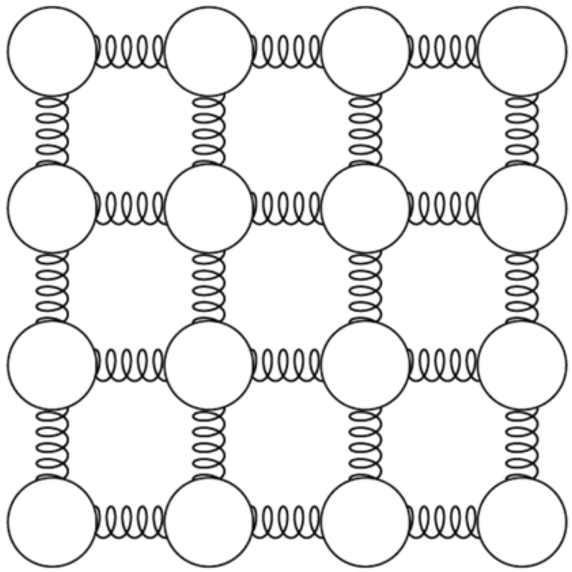
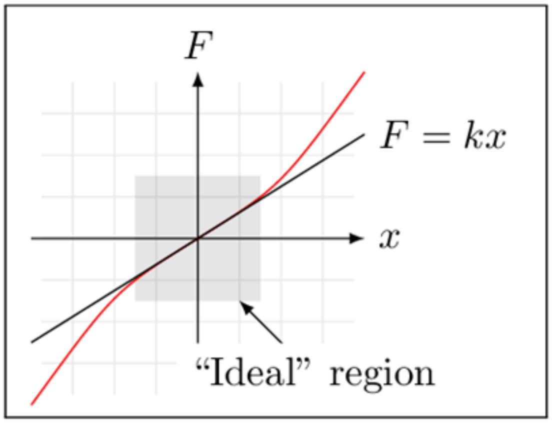
Elasticity

The three phases of matter are characterized by how they respond to external force

As solids deform, they "push back" — this elasticity is often shown with springs



An ideal spring is the simplest elastic system and is defined by Hooke's Law



For an ideal spring:

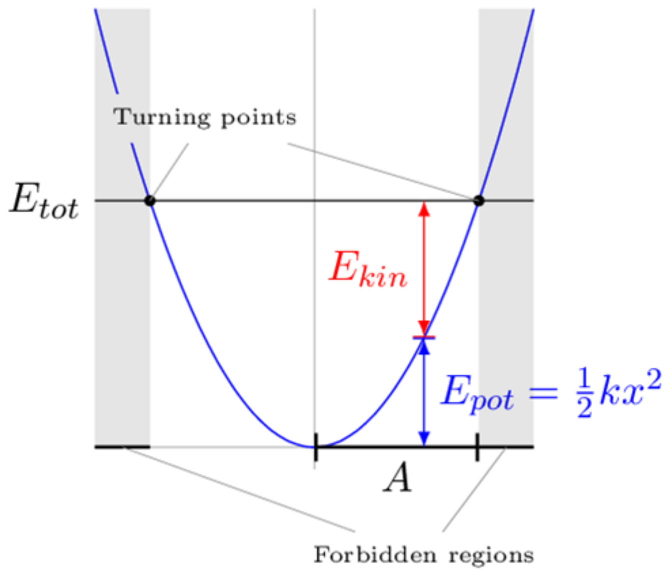
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

When the restorative force of a system around an equilibrium state is linear in its disturbance (i.e., if the disturbance is doubled, so is the force of restoration), then result will be simple harmonic motion:

$$\psi(t) = A\cos(\omega t + \phi)$$

The amplitude A and phase shift ϕ generally depend on the initial state of the disturbance, but the angular frequency ω is usually a characteristic of the system itself.

Energy in an ideal spring is parabolic; the amplitude is tied to the total energy



In general,

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

But,

$$x = A \Rightarrow v = 0$$

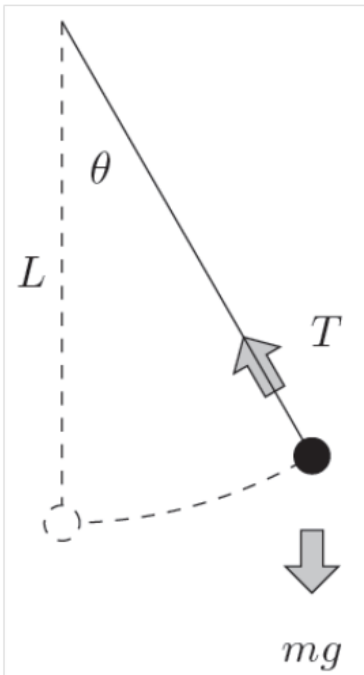
So,

$$E = \frac{1}{2}kA^2$$

Thus,

$$A = \sqrt{\frac{2E}{k}}$$

The forces need not be perfectly linear to apply these concepts: e.g., the pendulum

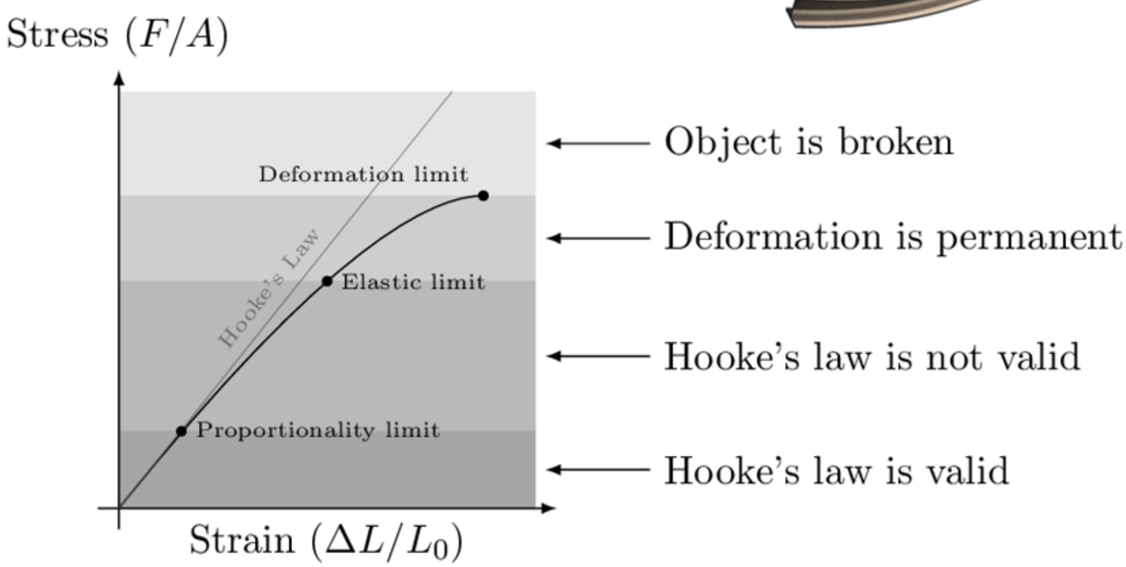


$$\tau = -mgL\sin\theta$$

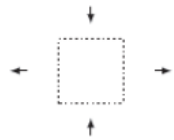
$$\sin\theta \approx \theta$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

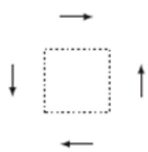
The deformation that results from small stress is linear (Hooke's law again)



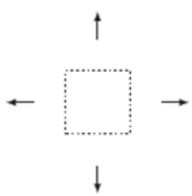
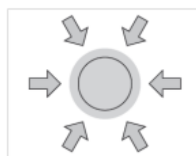
Stress is from multiple forces on an object — can be classified as three basic types

Normal  

$$\frac{F}{A} = Y\left(\frac{\Delta L}{L_0}\right)$$

Shear  

$$\frac{F}{A} = S\left(\frac{\Delta x}{L_0}\right)$$

Pressure  

$$\Delta P = -B\left(\frac{\Delta V}{V_0}\right)$$

Newton’s law of gravity creates stress on an astronomic scale: tides and gravity waves