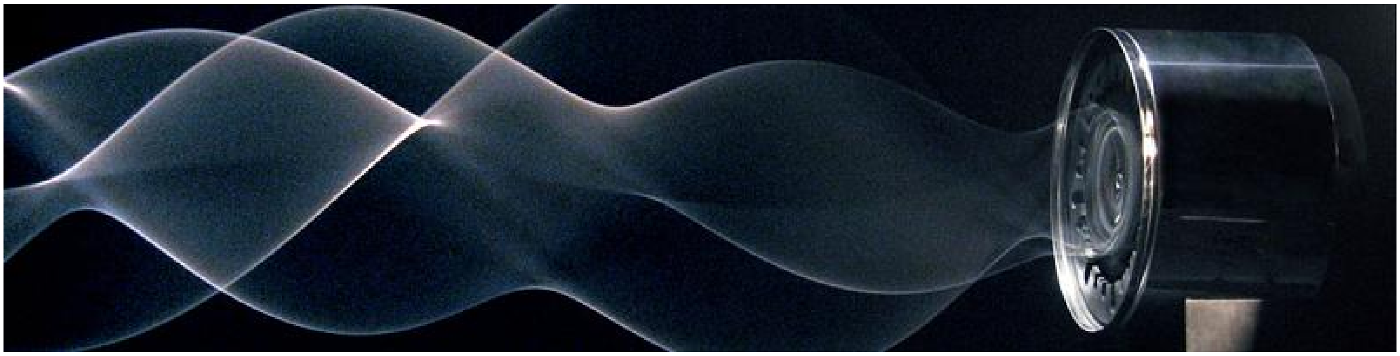
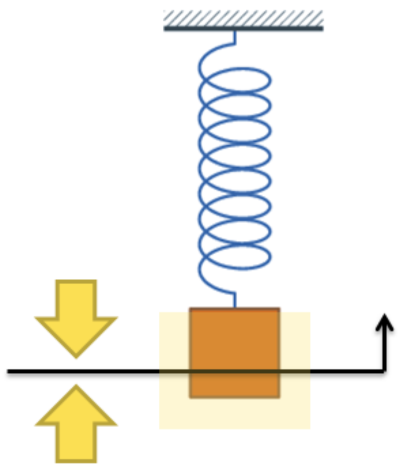
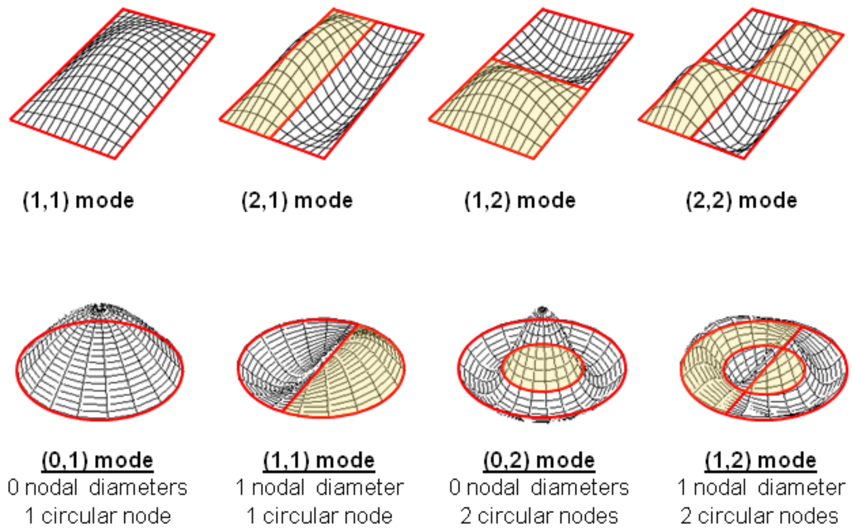
Vibration and Oscillation



Physical systems will vibrate near and around points of equilibrium

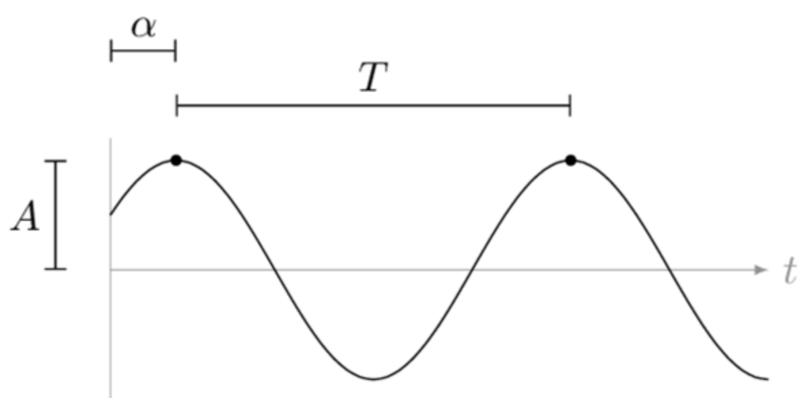


Vibration in multiple dimensions is complicated and depends on boundaries



http://www.kettering.edu/~drussell/demos.html

Simple harmonic motion is the motion of vibration with a specific mathematical form



Define:

$$f = 1/T$$
$$\omega = 2\pi f$$
$$\phi = -\alpha\omega$$

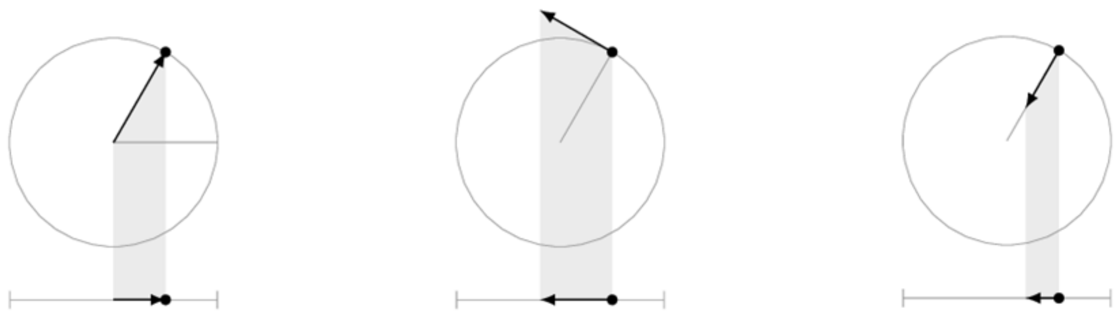
$$\psi(t) = A\cos(\omega t + \phi)$$

A fortunate fluke: the projection of uniform circular motion is simple harmonic motion

$$r = A$$
$$\theta = \omega t$$

$$v = \frac{2\pi r}{T} = A\omega$$

$$a = \frac{v^2}{r} = A\omega^2$$



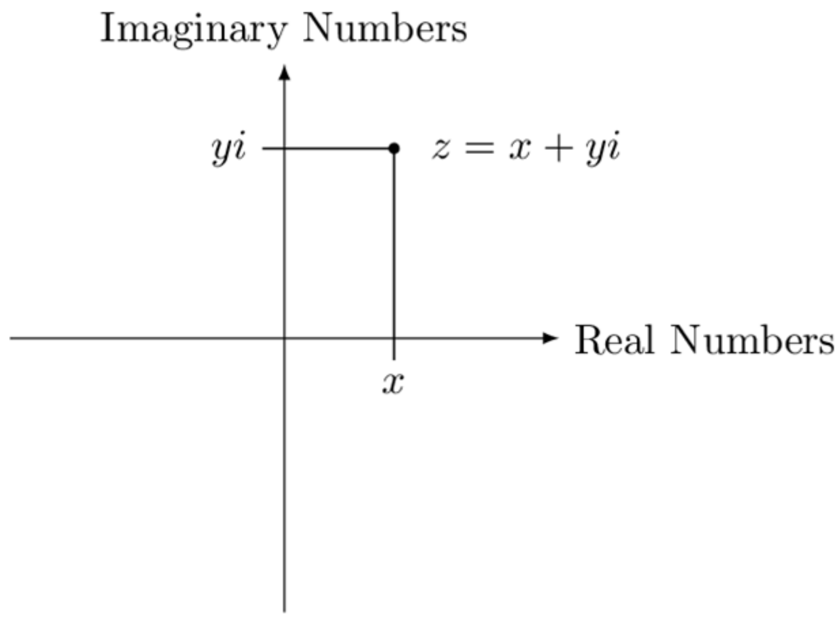
$$x = A\cos(\omega t)$$
$$x_{max} = A$$

$$v = A\omega\sin(\omega t)$$
$$v_{max} = A\omega$$

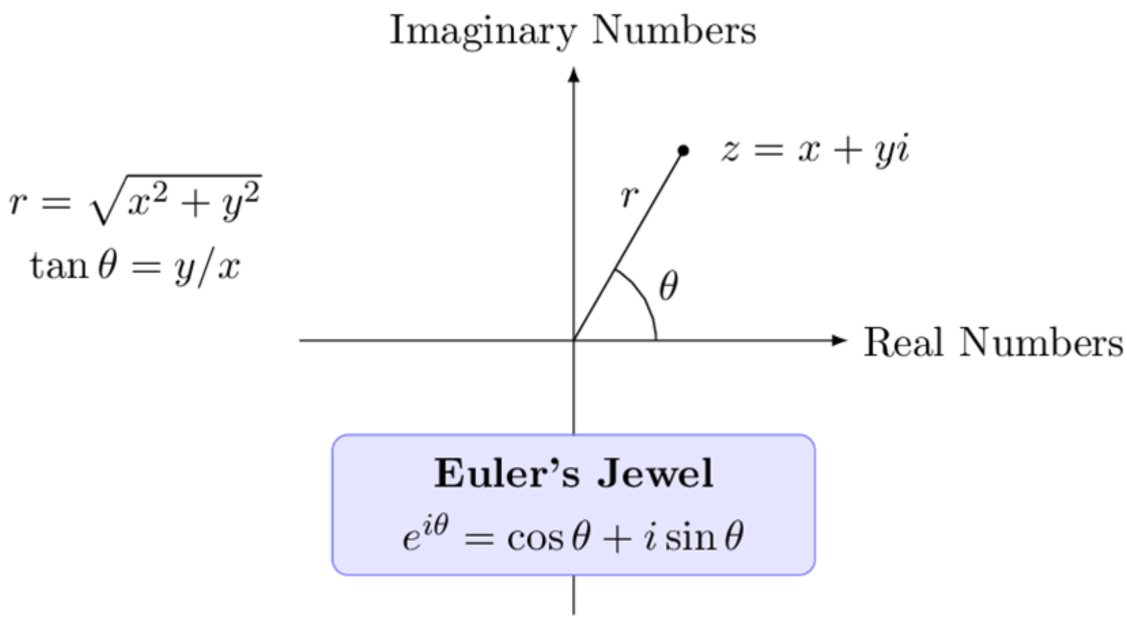
$$a = -A\omega^2\cos(\omega t)$$
$$a_{max} = A\omega^2$$

We can use complex numbers to model vibrations — they are called "phasors"

$$i = \sqrt{-1}$$
$$i^2 = -1$$



The reason we can get away with using complex numbers is in Euler's jewel



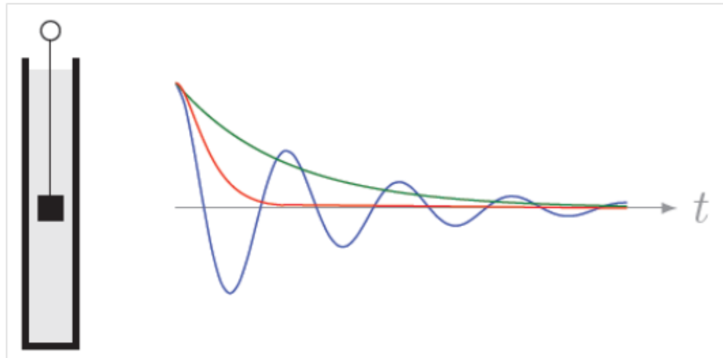
Euler's Jewel

$$e^{i\theta} = \cos\theta + i\sin\theta$$

An oscillation is said to be "damped" when energy is drained from the system

$$F_{\text{drag}} = -cv$$

$$\gamma = c/m.$$



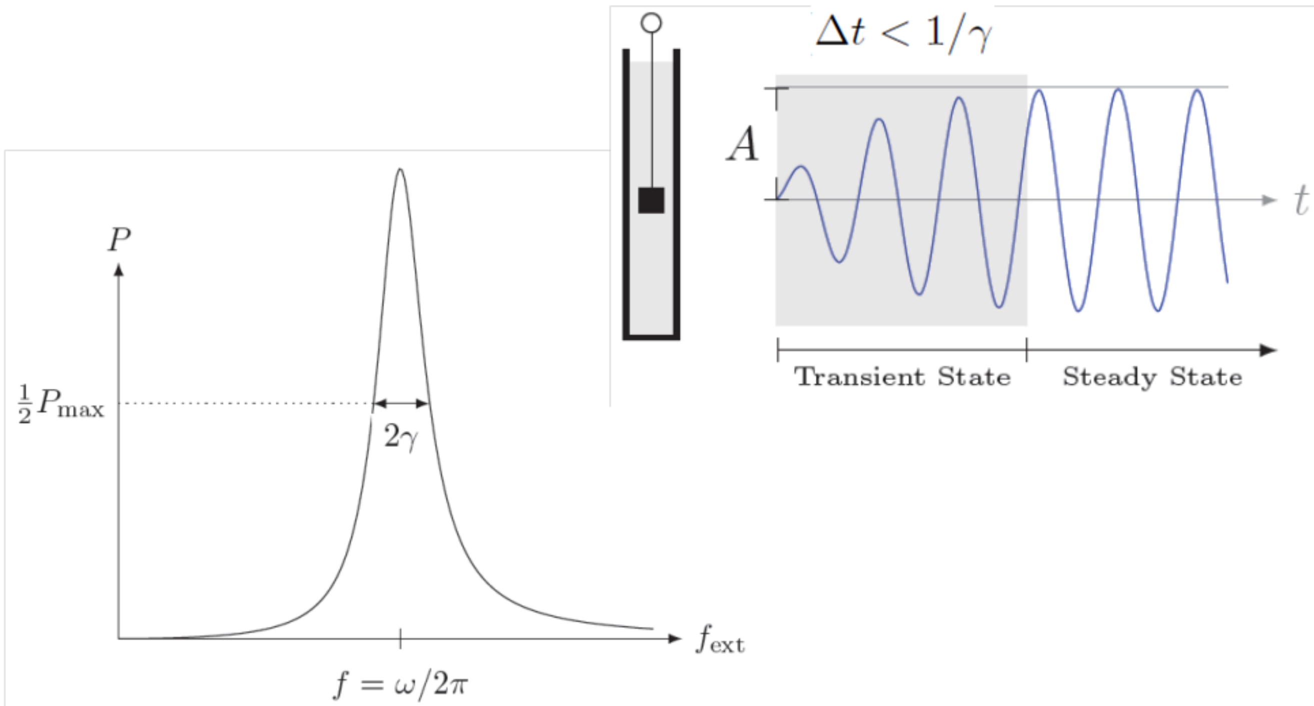
$$A(t) = A_0 \exp(-\tfrac{1}{2}\gamma t)$$

Light damping $(\gamma < 2\omega)$

Heavy damping $(\gamma > 2\omega)$

Critical damping $(\gamma = 2\omega)$

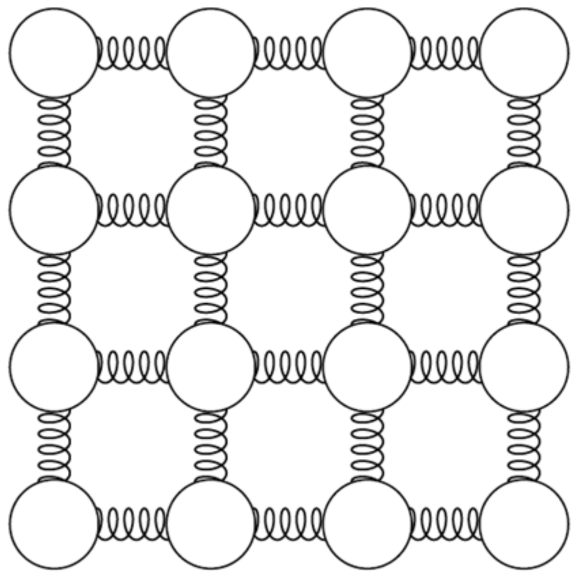
If energy is put into the system it is "driven"; some driving frequencies create resonance



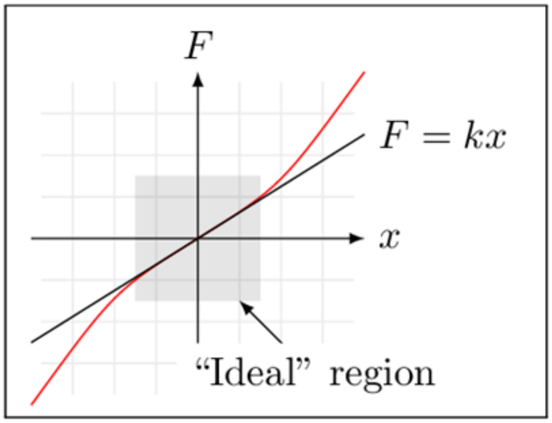
Elasticity

The three phases of matter are characterized by how they respond to external force

As solids deform, they "push back" — this elasticity is often shown with springs



An ideal spring is the simplest elastic system and is defined by Hooke's Law



For an ideal spring:

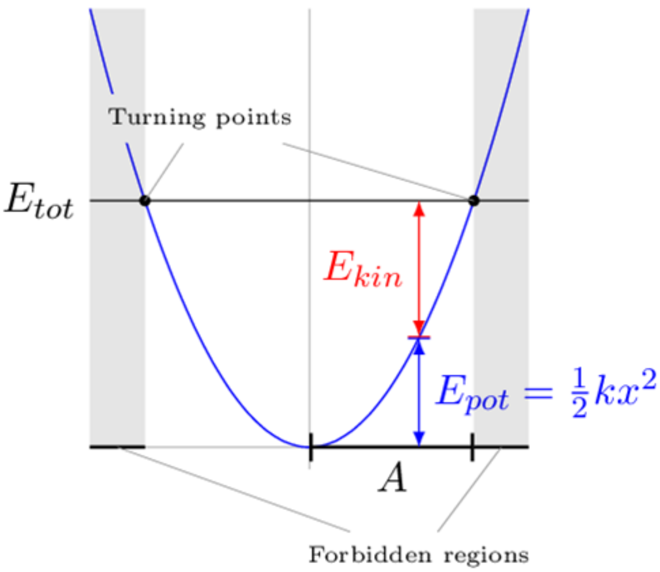
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

When the restorative force of a system around an equilibrium state is linear in its disturbance (i.e., if the disturbance is doubled, so is the force of restoration), then result will be simple harmonic motion:

$$\psi(t) = A\cos(\omega t + \phi)$$

The amplitude A and phase shift ϕ generally depend on the initial state of the disturbance, but the angular frequency ω is usually a characteristic of the system itself.

Energy in an ideal spring is parabolic; the amplitude is tied to the total energy



In general,

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

But,

$$x = A \Rightarrow v = 0$$

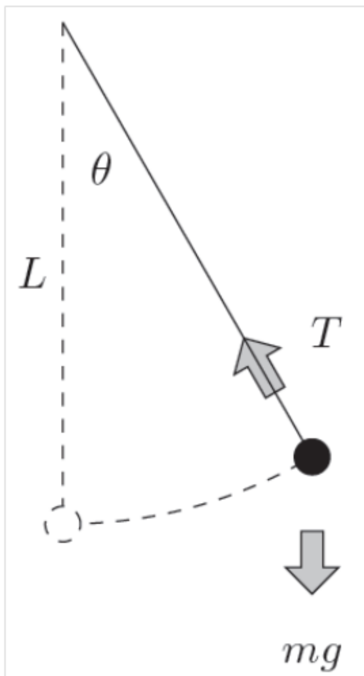
So,

$$E = \frac{1}{2}kA^2$$

Thus,

$$A = \sqrt{\frac{2E}{k}}$$

The forces need not be perfectly linear to apply these concepts: e.g., the pendulum

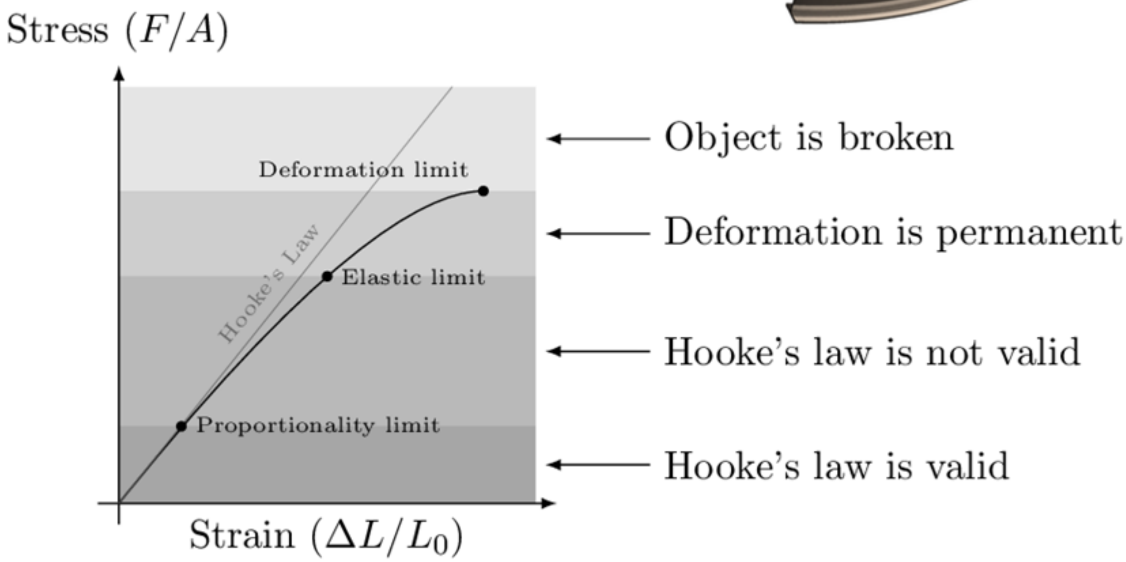


$$\tau = -mgL\sin\theta$$

$$\sin\theta \approx \theta$$

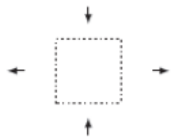
$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

The deformation that results from small stress is linear (Hooke's law again)



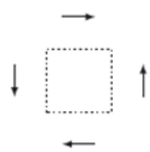
Stress is from multiple forces on an object — can be classified as three basic types

Normal



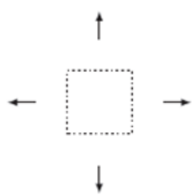
$$\frac{F}{A} = Y\left(\frac{\Delta L}{L_0}\right)$$

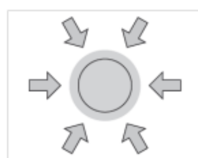
Shear



$$\frac{F}{A} = S\left(\frac{\Delta x}{L_0}\right)$$

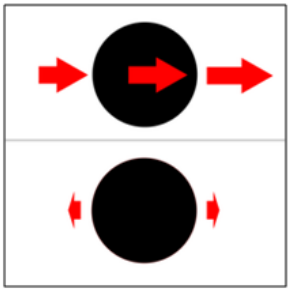
Pressure

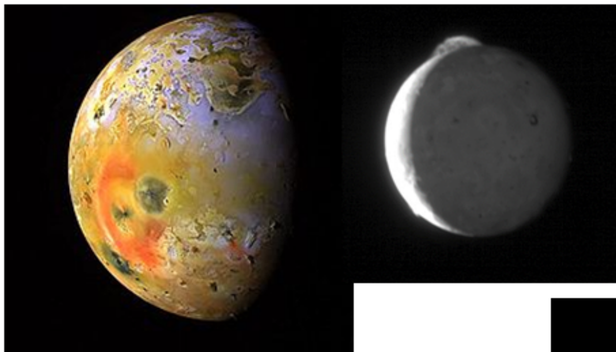


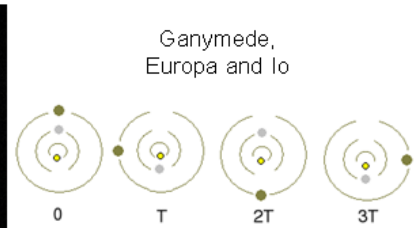


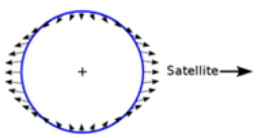
$$\Delta P = -B\left(\frac{\Delta V}{V_0}\right)$$

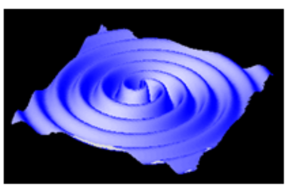
Newton's law of gravity creates stress on an astronomic scale: tides and gravity waves

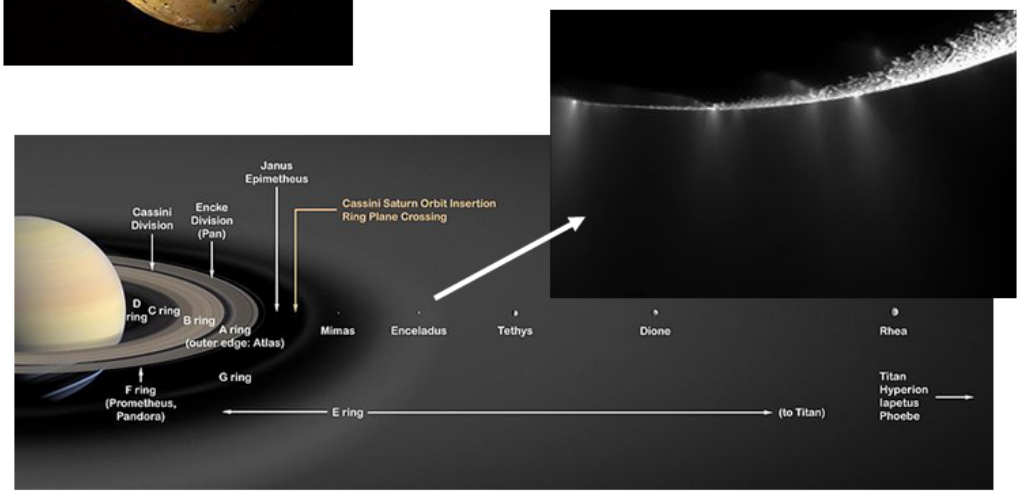




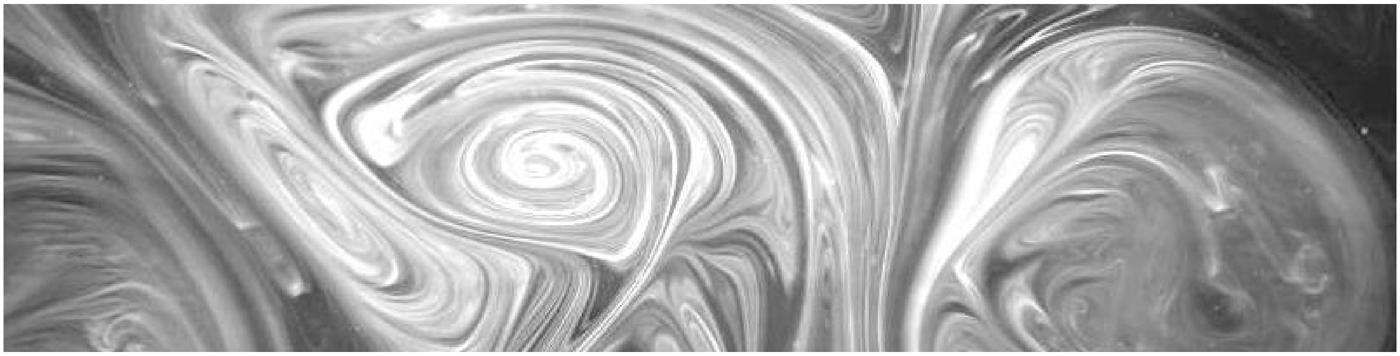




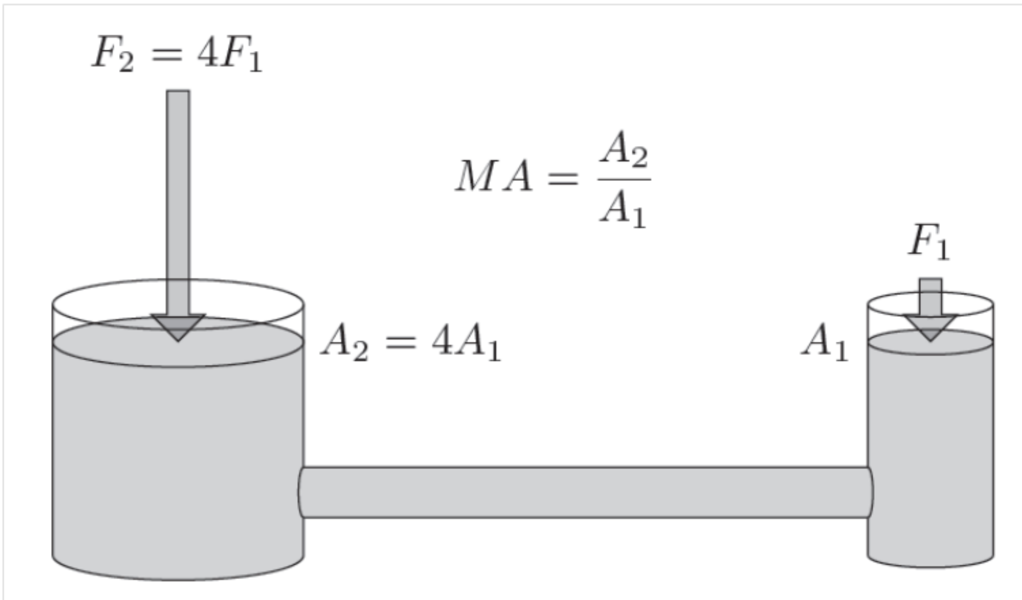




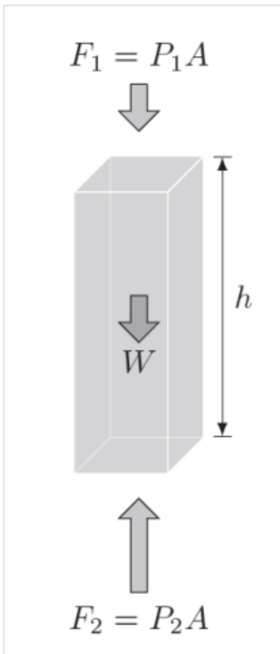
Fluids



Pascal's principle: any pressure change will flow through the entire fluid equally



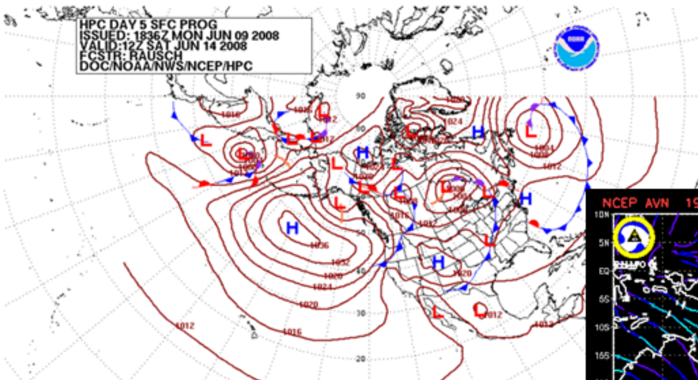
Archimedes' principle: vertical pressure differentials provides the force of buoyancy

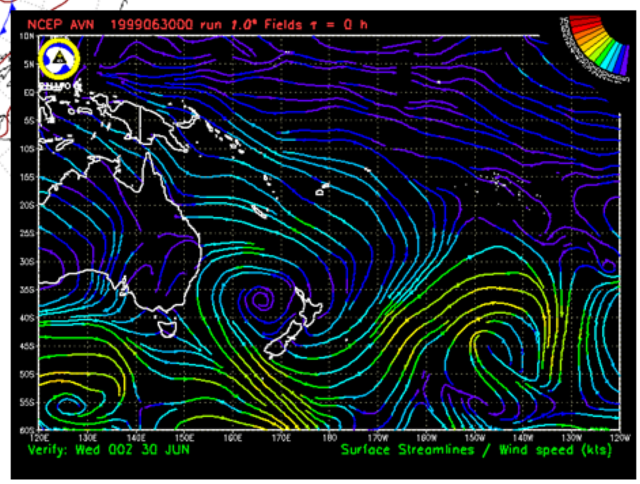


$$\Delta P = \rho g h$$

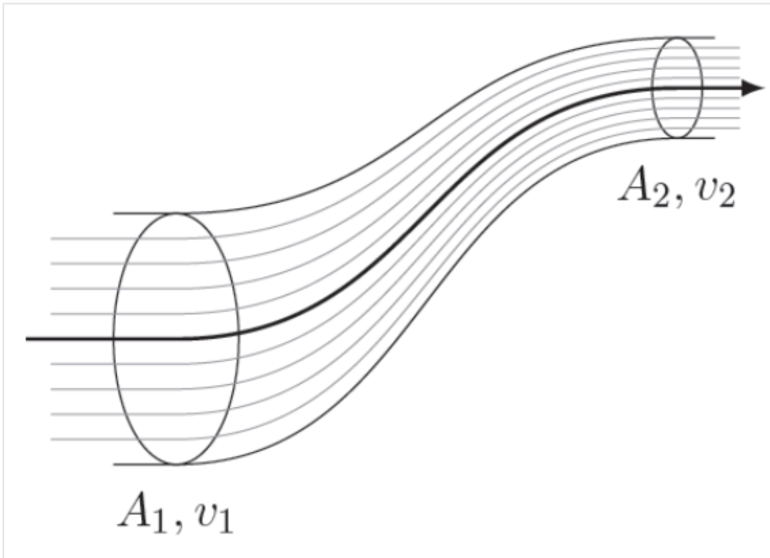
$$B = \rho V g$$

Pressure differentials cause flow; steady flow moves along streamlines of current





The steady flow of an ideal fluid is both incompressible and non-viscous

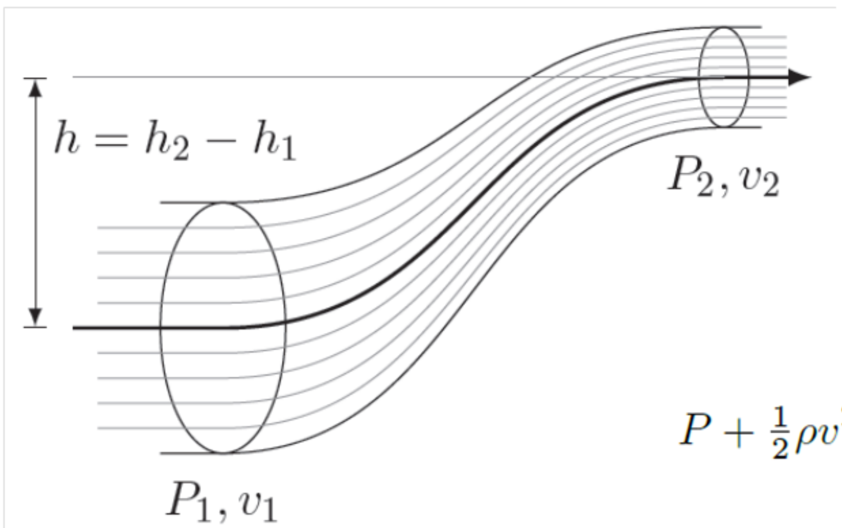


$$Q = Av$$

$$\Delta m/\Delta t = \rho v A$$

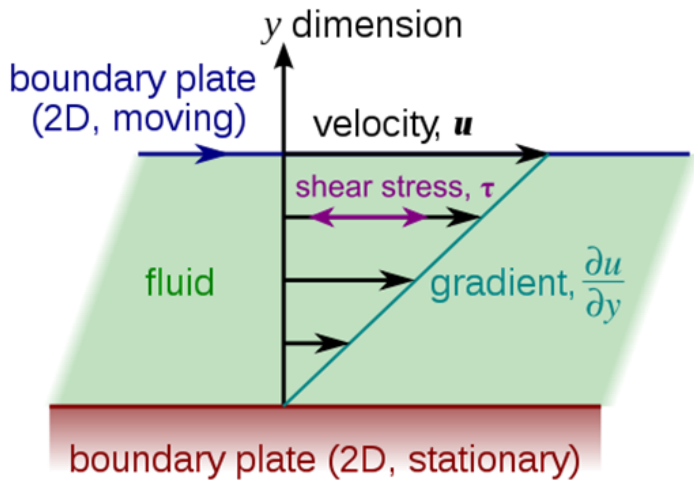
$$\rho A_1 v_1 = \rho A_2 v_2 \implies A_1 v_1 = A_2 v_2$$

Bernoulli's equation describes the pressure differentials in a ideal fluid that is flowing



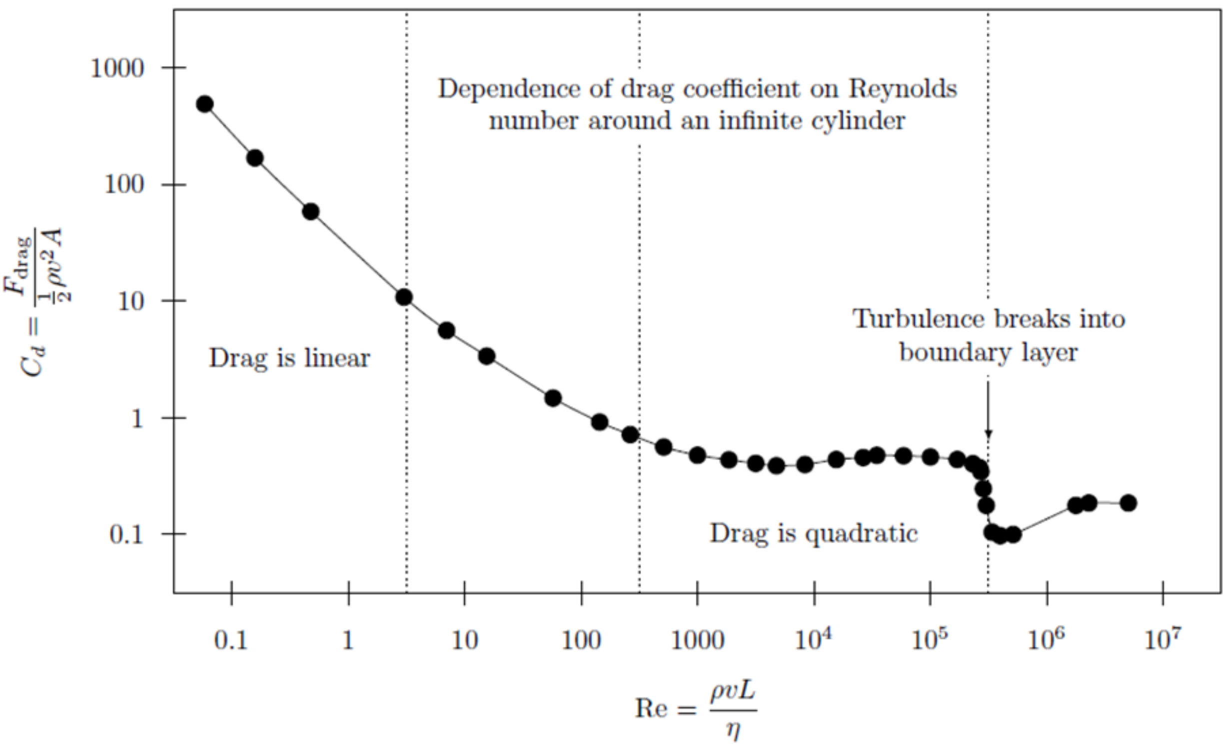
$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{const.}$$

Viscosity is fluid friction; leads to air drag and requires extra pressure for flow

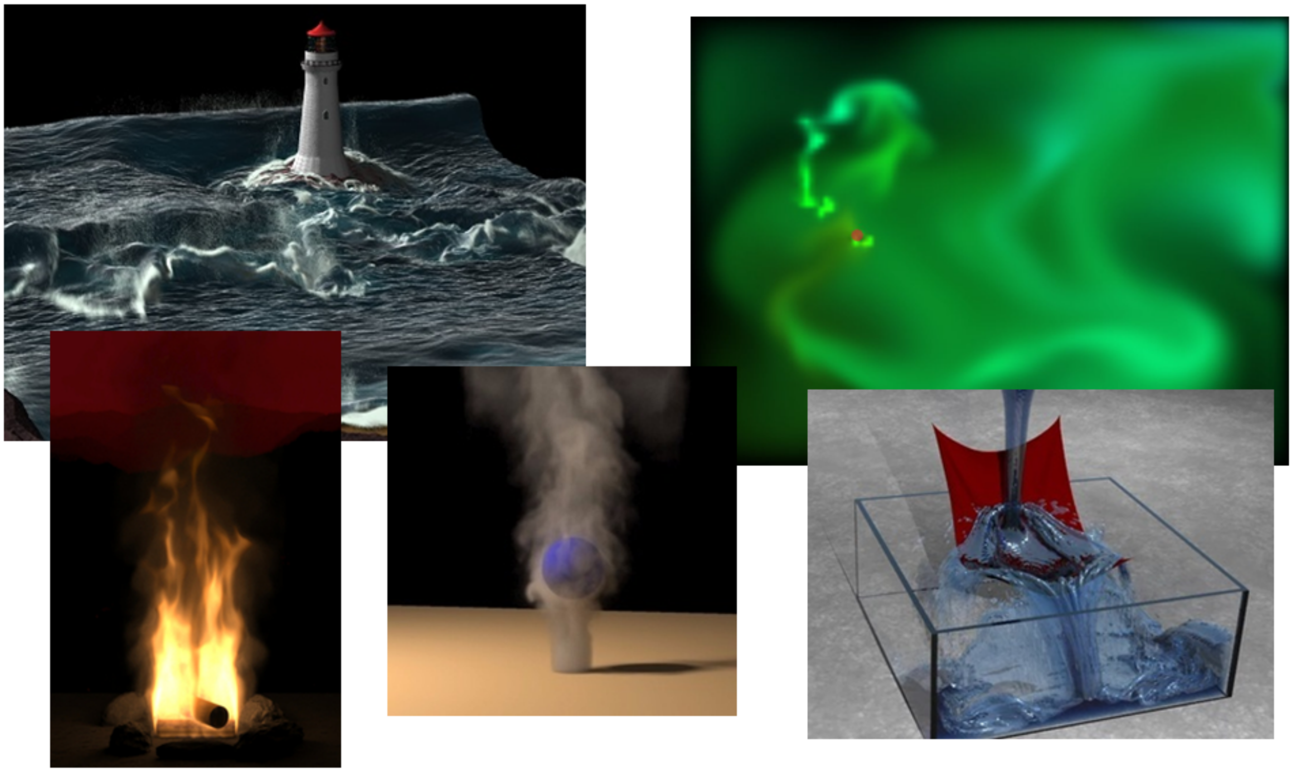


$$F = \eta A v / d \qquad F = 6\pi\eta R v \qquad \Delta P = \frac{8\eta L Q}{\pi r^4}$$

Turbulence driven by speed and size of the object, viscosity and density of the fluid

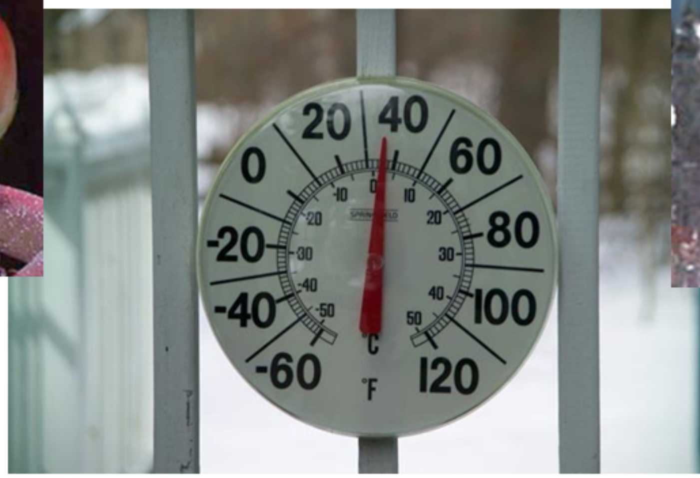


Navier-Stokes equations go beyond Bernoulli, but are very difficult to solve



Heat and Temperature

Of the many possible temperature scales, the Celsius and Kelvin scales are preferred



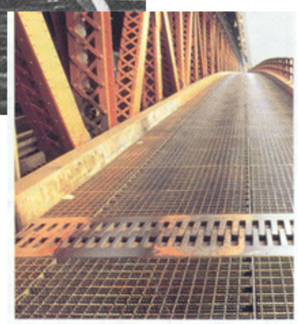
$$F = (9/5)C + 32$$

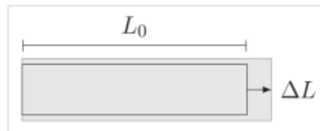
$$C = (5/9)(F - 32)$$

$$K = C + 273.15$$

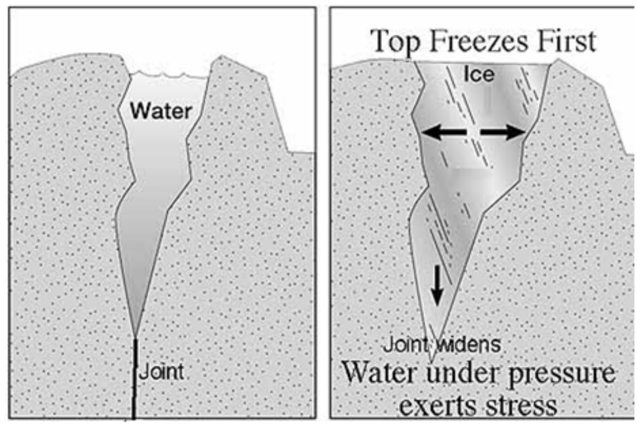
$$C = K - 273.15$$

Thermometric properties are linear in small increments; simplest is thermal expansion

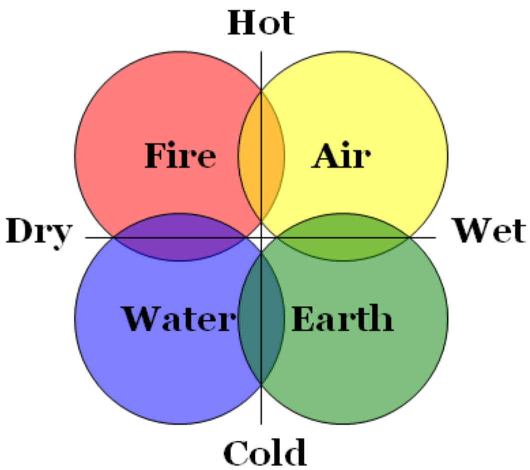




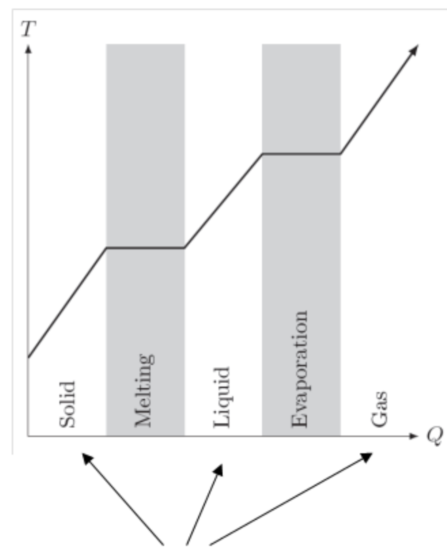
$$\frac{\Delta L}{L_0} = \alpha \Delta T$$



The difference between temperature and heat: heat capacity and internal energy



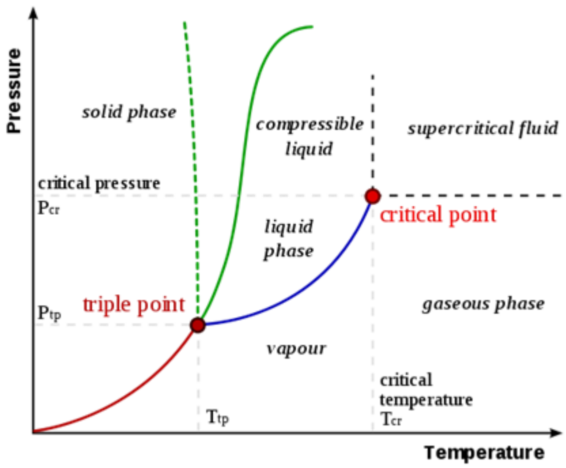
$$1 \text{ cal} = 4.186 \text{ J}$$

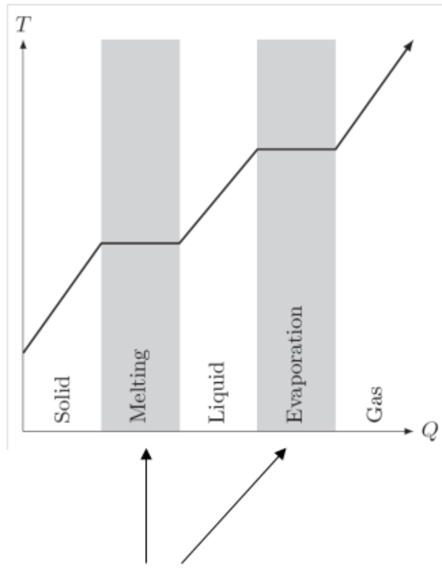


$$Q = C\Delta T$$

$$Q = cm\Delta T$$

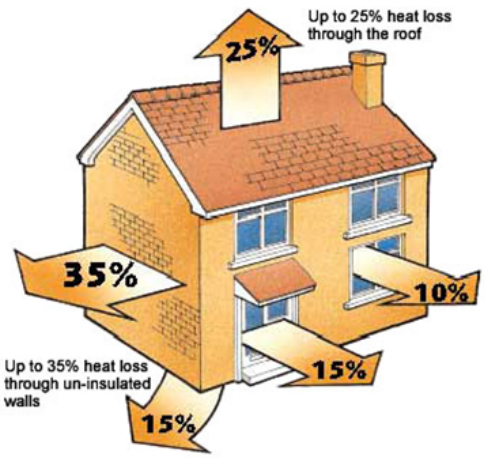
Phase changes also show the distinction between heat energy and temperature





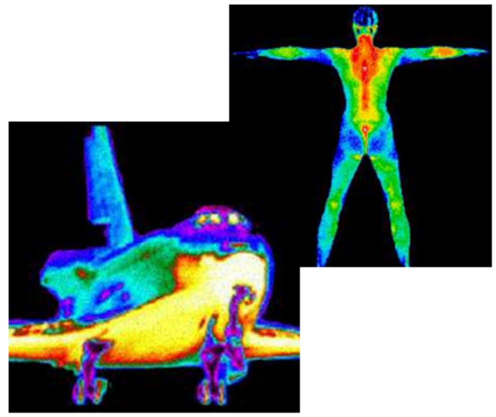
$$Q = mL$$

Heat energy is transported via conduction (solids) and convection (fluids)



$$P = \frac{Q}{t} = \frac{kA}{L}(\Delta T) \qquad R = \frac{L}{k}$$

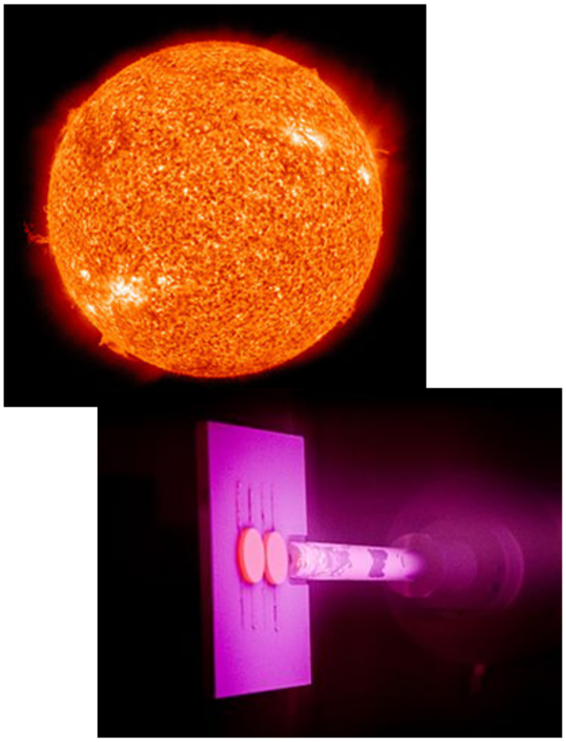
The electromagnetic field is also able to transport heat energy via radiation

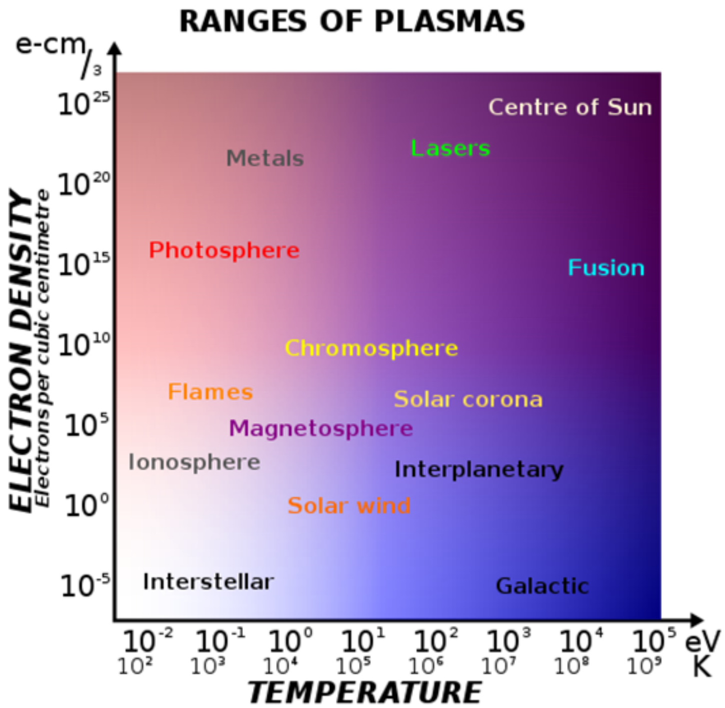


$$P = \frac{Q}{t} = \epsilon \sigma A T^4$$

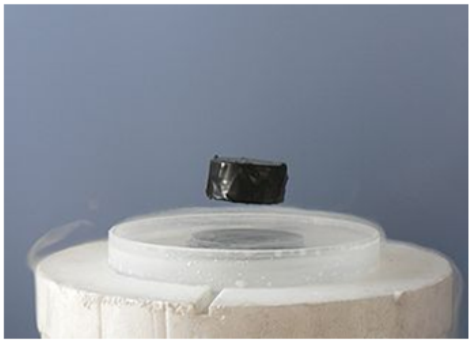
5.67×10^{-8}

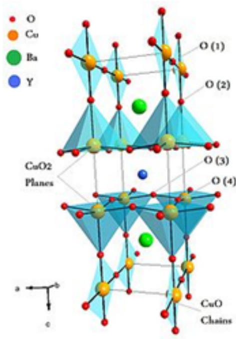
Very high temperatures can rip apart the atoms in a gas to create plasma

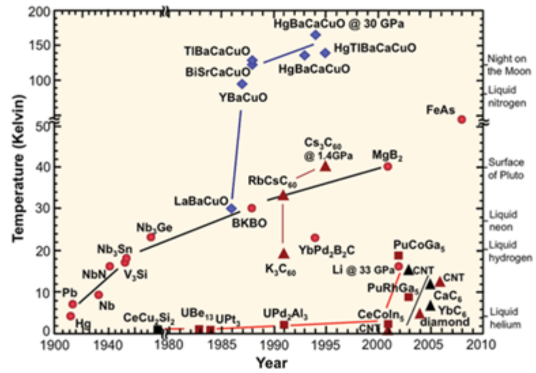


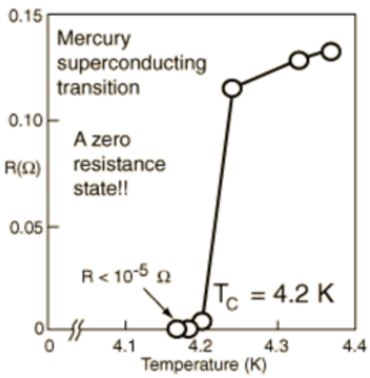


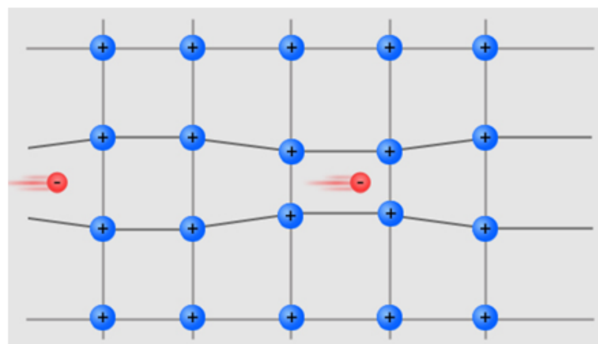
Very low temperatures reveal a new world of superconductivity and other effects





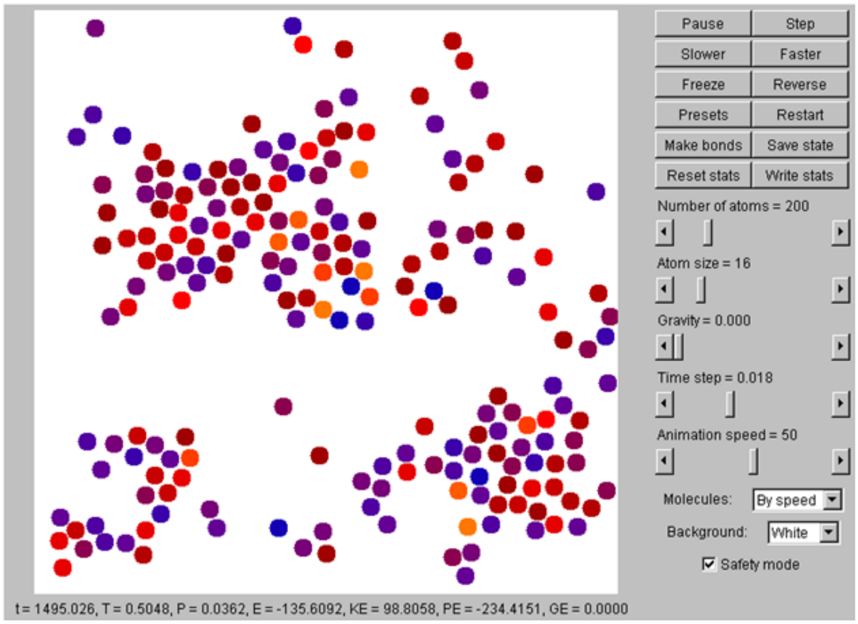






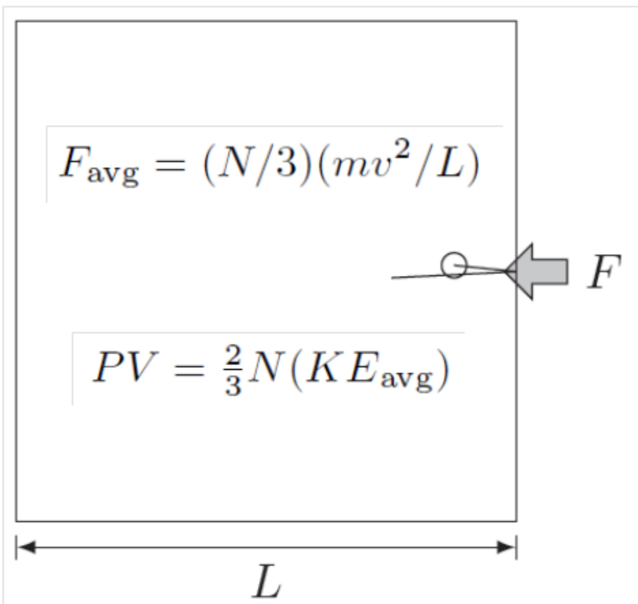
Kinetic Theory

The kinetic hypothesis identifies heat energy with the random mechanical energy of molecules



http://physics.weber.edu/schroeder/software/MDApplet.html

The simplest thermal system is one with no molecular interaction at all — an ideal gas



$$E = \frac{1}{2}kT$$

$$U = \frac{3}{2}NkT$$

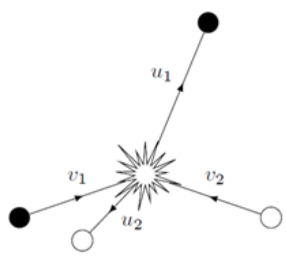
$$PV = NkT$$

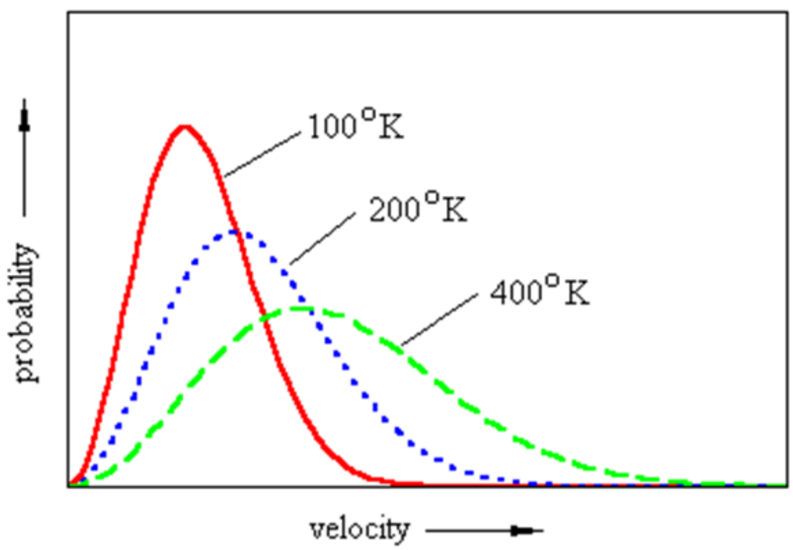
$$k = R/N_A$$

The Maxwell-Boltzman distribution depicts the molecular speeds in an ideal gas

$$v_{\text{avg}} = \sqrt{3kT/M}$$

$$N \propto v^2 \exp(-\tfrac{1}{2}mv^2/kT)$$





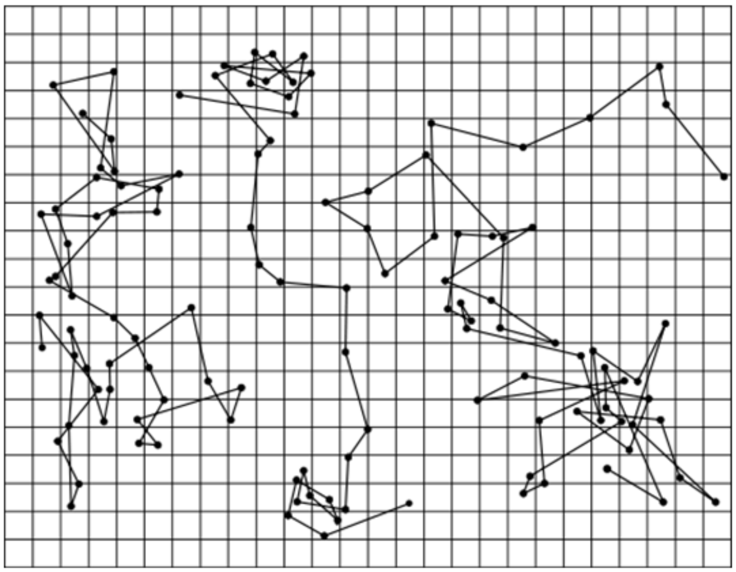
Entropy measures the distribution of internal energy in a system

E_1	E_2	Ω_1	Ω_2	Ω
0	5	0	25	0
1	4	1	16	16
2	3	8	9	72
3	2	27	4	108
4	1	64	1	64
5	0	125	0	0

$$\Omega \propto E^{d/2}$$

$$S = k \ln \Omega$$

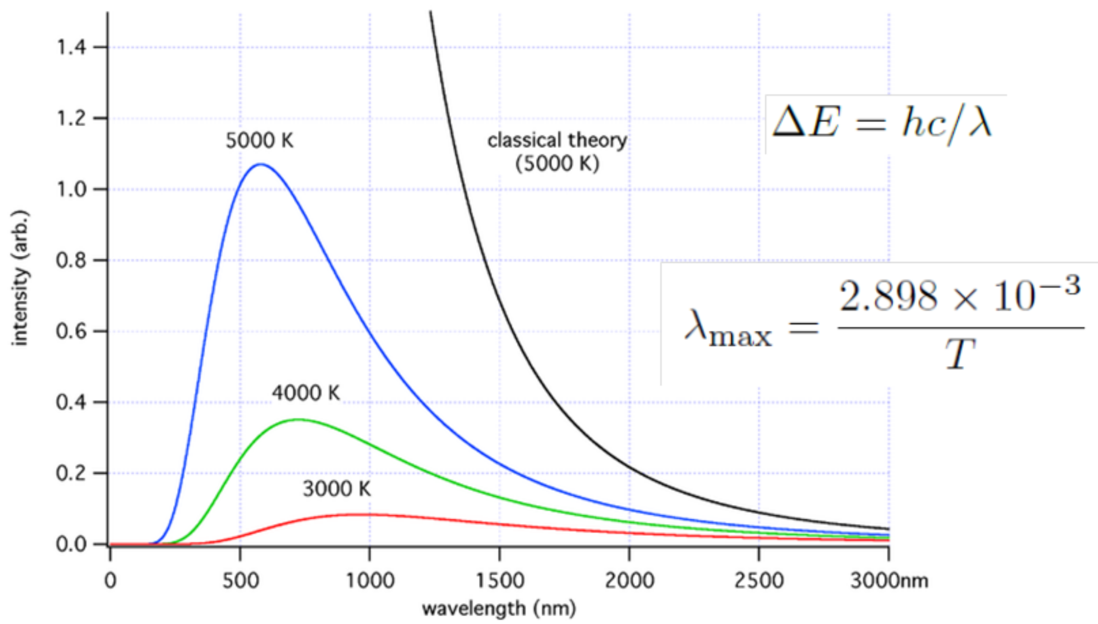
Fick's law of diffusion and Brownian motion also follows from the kinetic hypothesis



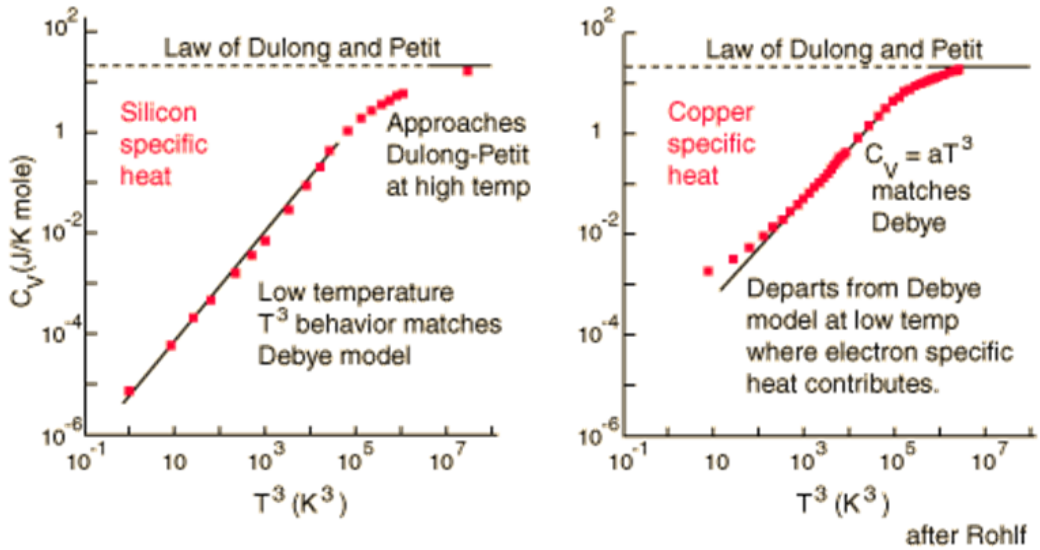
$$\frac{M}{t} = DA\frac{\Delta C}{\Delta x}$$

$$D = kT/6\pi\eta r$$

Statistical mechanics based on Newton's laws is not as successful as it should be

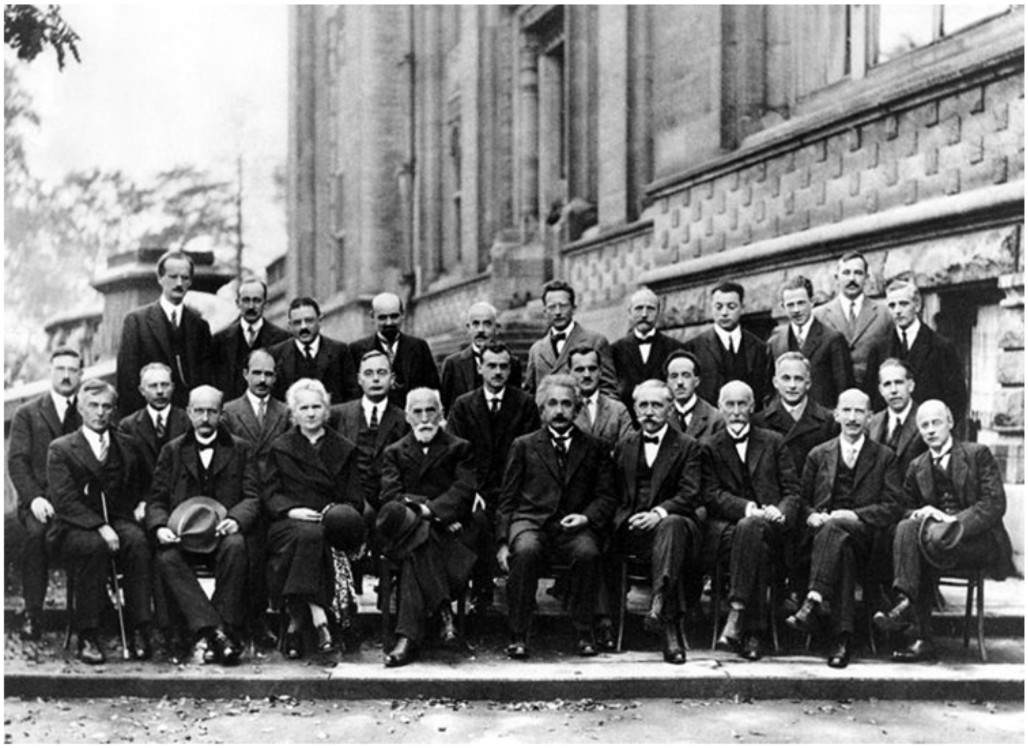


The root cause of this failure appears to be "frozen" degrees of freedom

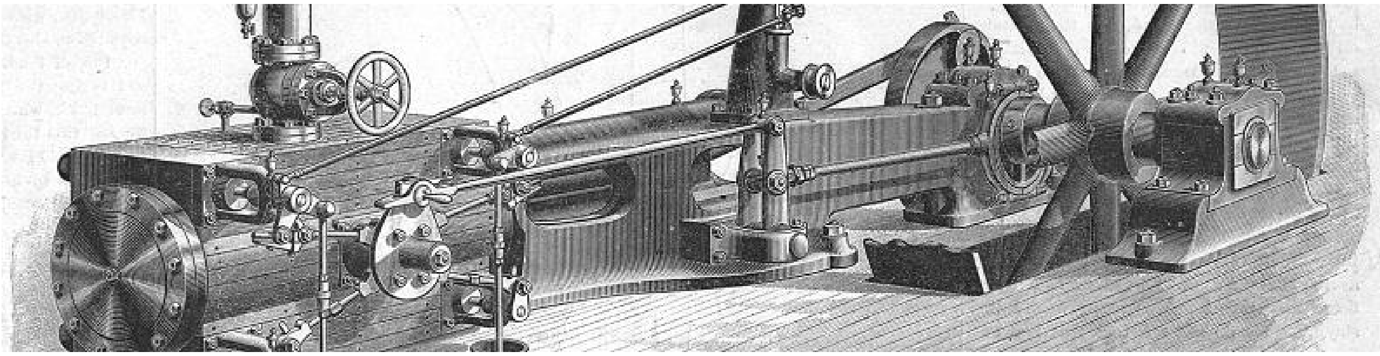


$$\cancel{C/n = 3R \sim 25}$$

These considerations paved the way to a new theory of quantum mechanics



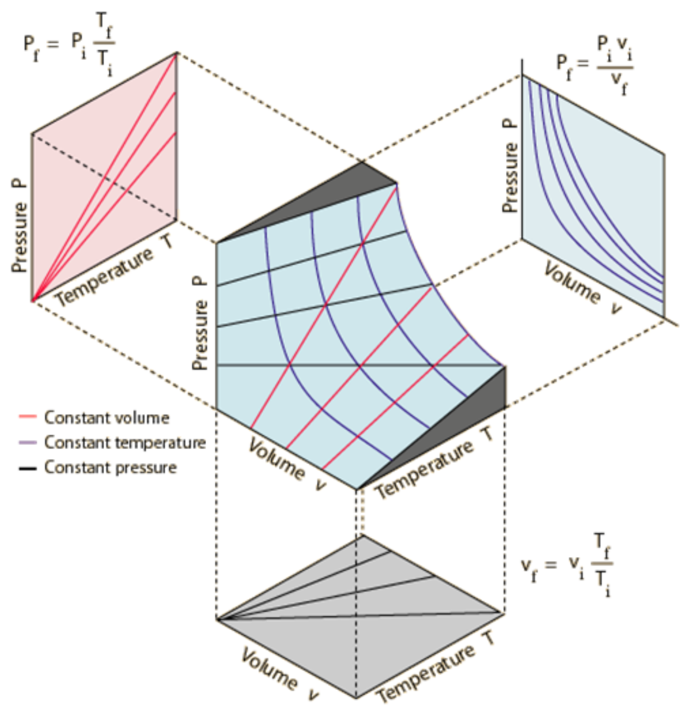
Thermodynamics



The first law: a heat engine balances internal energy, heat and work

$$\Delta U = Q - W$$

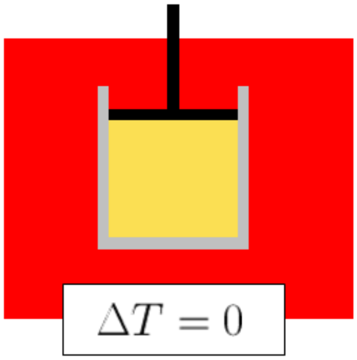
The ideal gas provides the simplest system to consider in thermodynamics

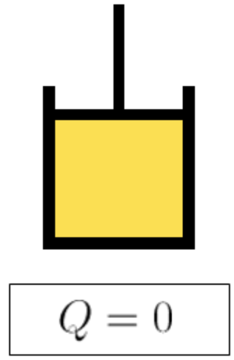


$$PV = nRT$$

$$U = \frac{3}{2}nRT$$

Work done by an ideal gas within isothermal and adiabatic process can be calculated





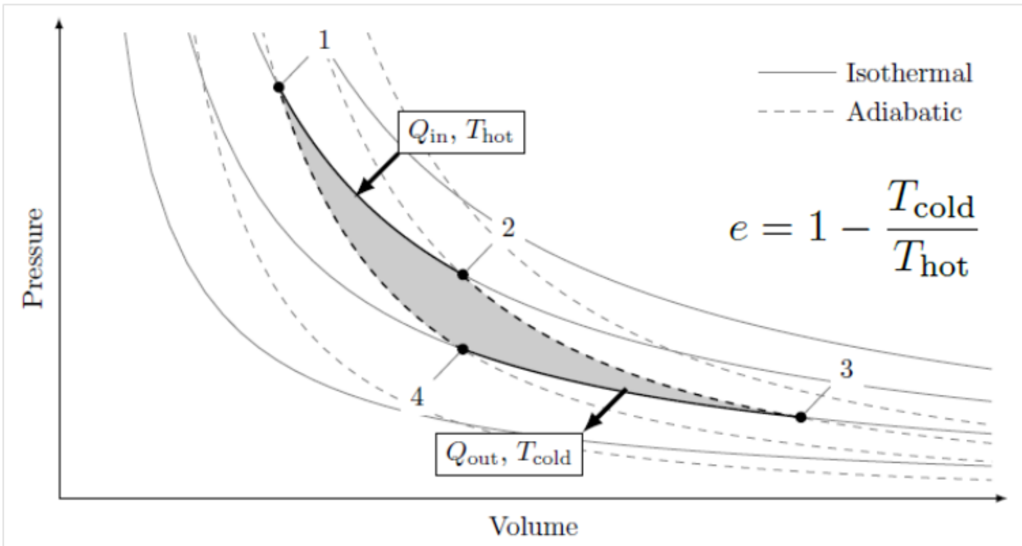
$$W_{\text{isoth}} = nRT \ln(V/V_0)$$

$$PV = \text{const.}$$

$$W_{\text{adiab}} = -\frac{3}{2} nR\Delta T$$

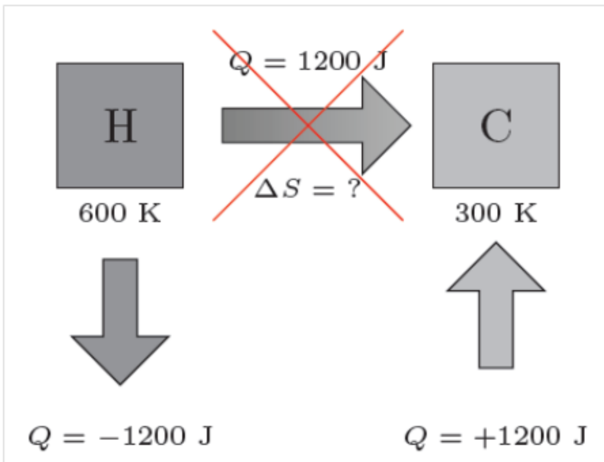
$$PV^{\gamma} = \text{const.}$$
$$\gamma = 5/3$$

The Carnot heat engine is reversible and is therefore the upper bound on efficiency



Another perspective on entropy: energy irreversibly lost to mechanical work

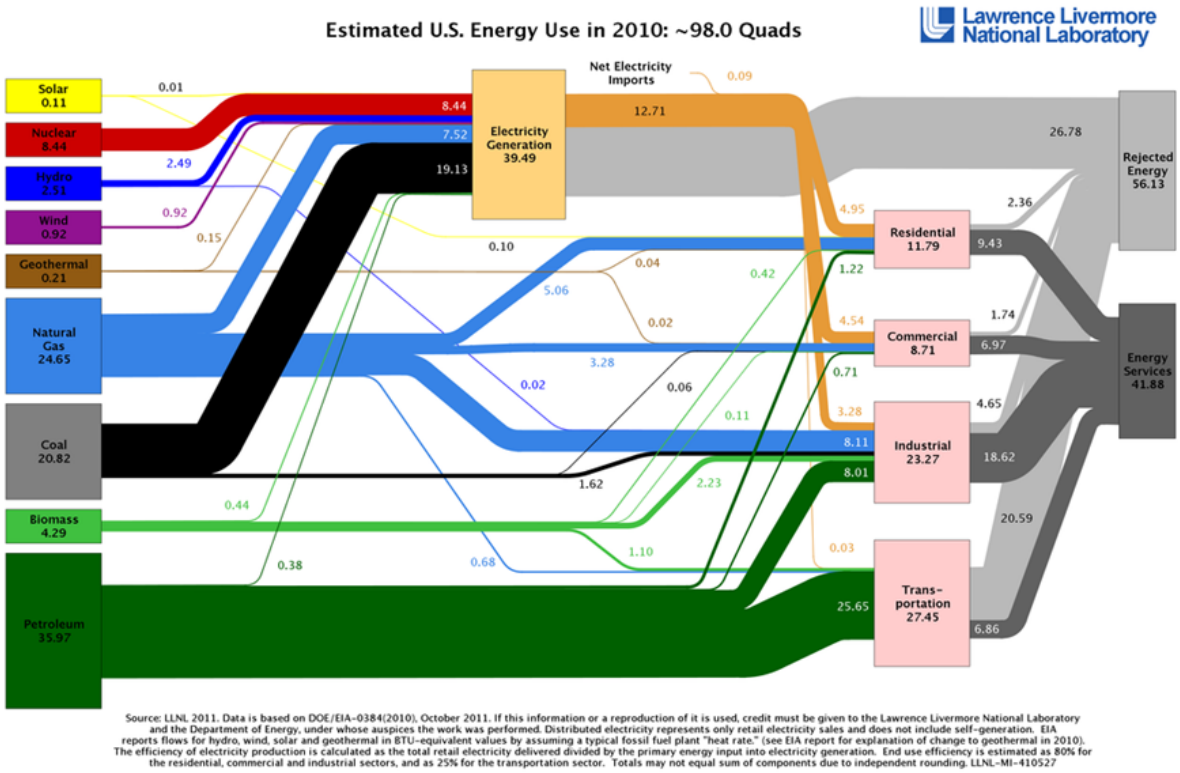
Any decrease in system entropy must come from a larger increase somewhere else



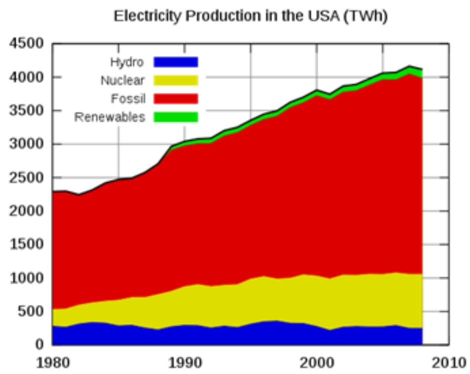
$$\Delta S = Q/T$$

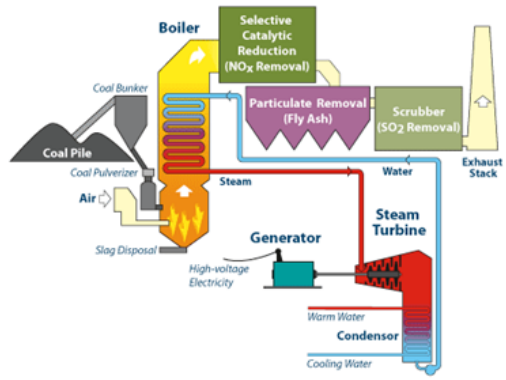
$$\boxed{\Delta S \geq 0}$$

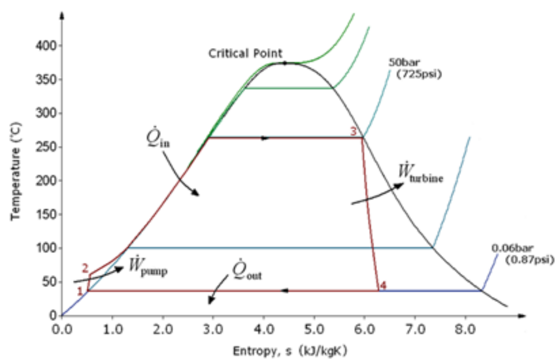
Practical questions about the most efficient use of global energy is challenging



Nearly all electricity generated in the US comes ultimately from steam engines

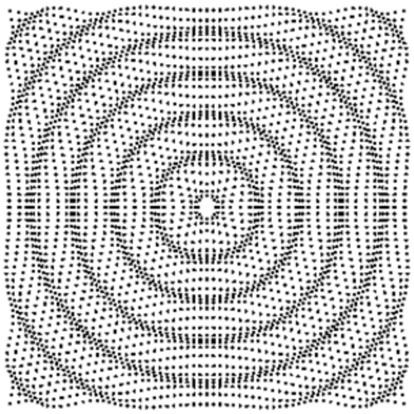


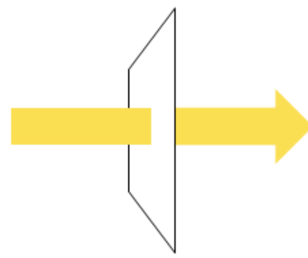




Radiation: Particles, Waves, Rays

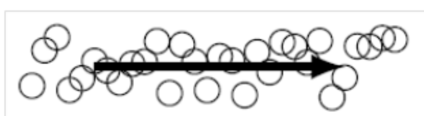
Energy flow and intensity is related to the amplitude and power of the driving source

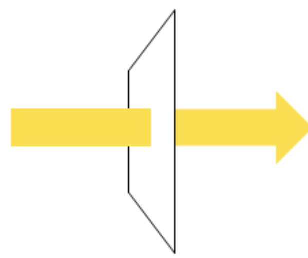




$$I = \frac{P}{4\pi r^2}$$

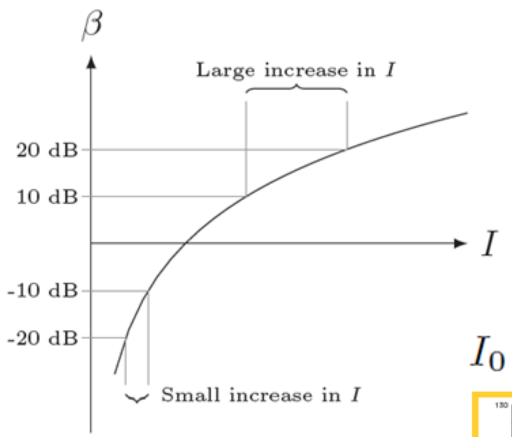
$$I = P/A$$



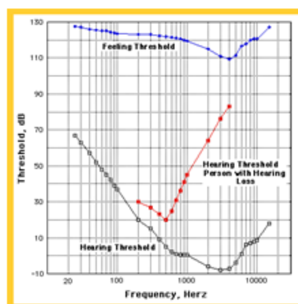


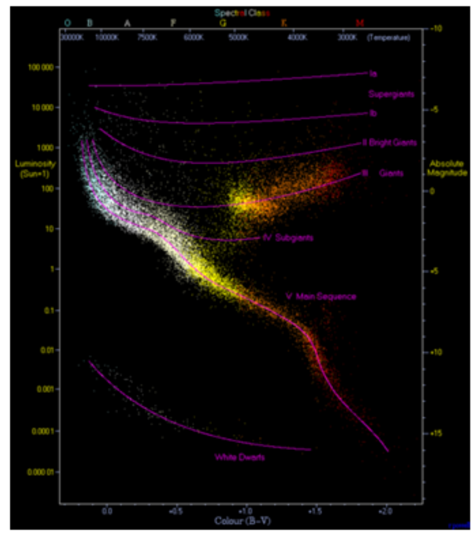
$$I = nE/t$$

"Intensity level" recalibrates the definition of intensity to our physiological sensation



$$I_0 = 1.0 \times 10^{-12}$$



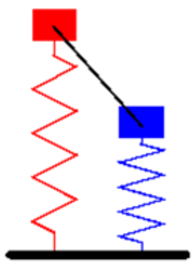


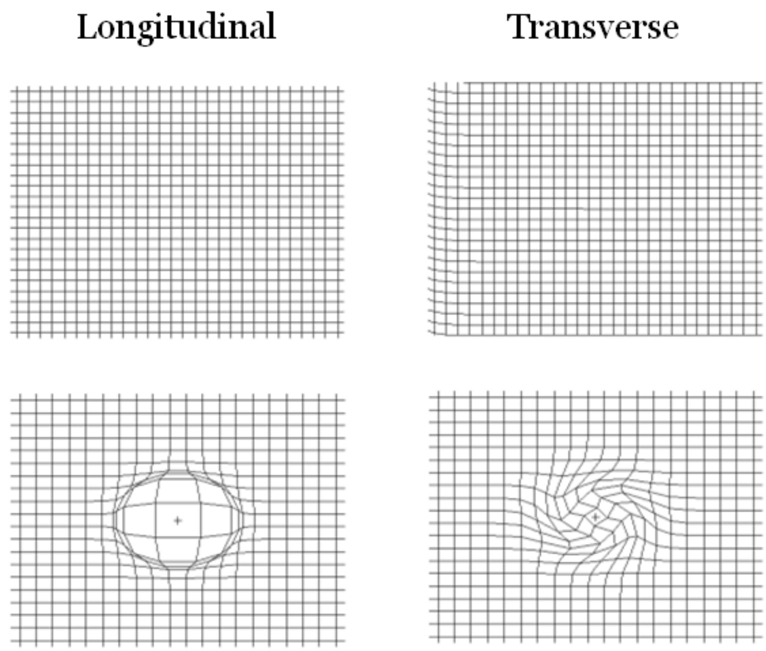
$$\beta = 10\log_{10}(I/I_0)$$

$$I = 10^{(0.1\beta - 12)}$$

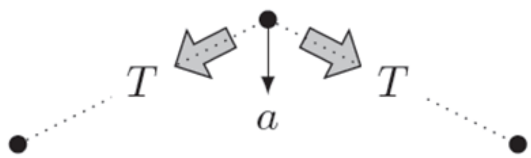
$$m = -2.5\log_{10}(I/I_0)$$

Waves require a source and a medium made up of coupled oscillators near equilibrium





Any disturbance in the medium acts as a damped oscillator as energy radiates away



$$v = \sqrt{T/\mu}$$

Solid

$$v = \sqrt{Y/\rho}$$

Liquid

$$v = \sqrt{B/\rho}$$

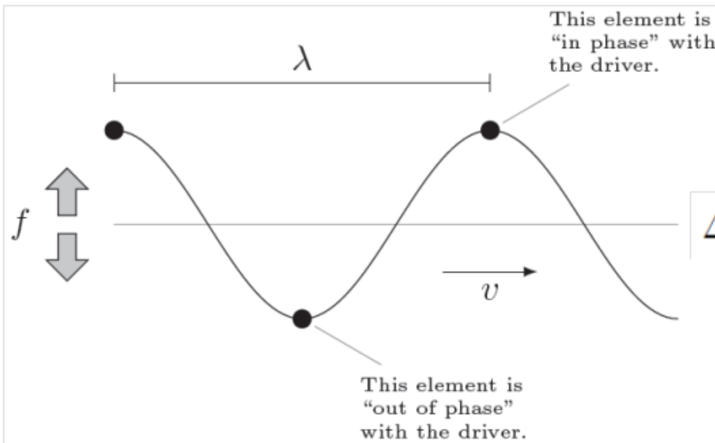
Gas

$$v = \sqrt{\gamma k T/M}$$

Air

$$v = 331 + 0.6T_C$$

All the elements of the medium match the oscillation of the source with a phase shift



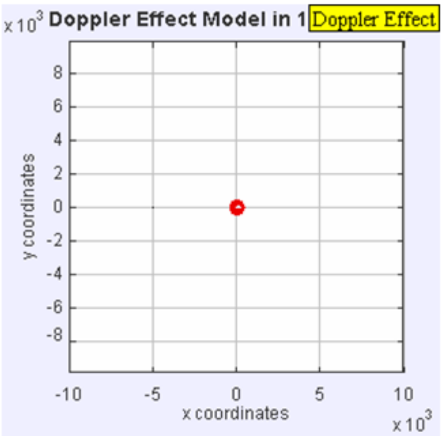
$$\Delta\phi = -(2\pi f/v)\Delta x$$

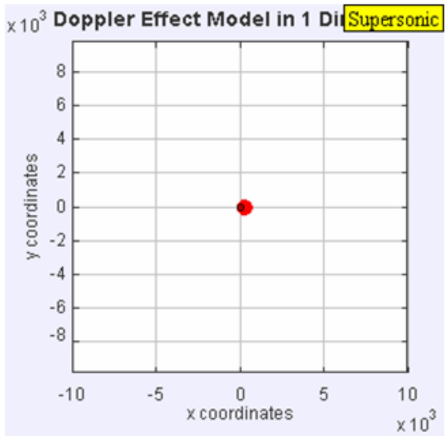
$$\psi = A\cos\left(2\pi f t - \frac{2\pi}{\lambda}x\right)$$

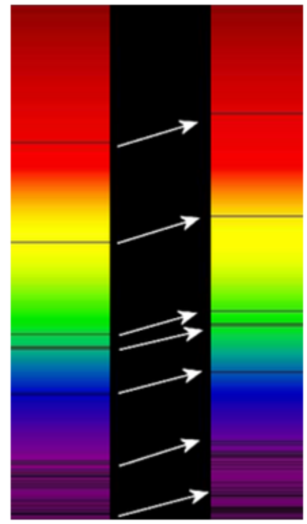
$$v = f\lambda$$

If the source is moving, the observed frequency suffers a Doppler shift

$$f\lambda = v_r \pm v_s$$







$$f_o = f\left(\frac{v_r}{v_r \pm v_s}\right)$$

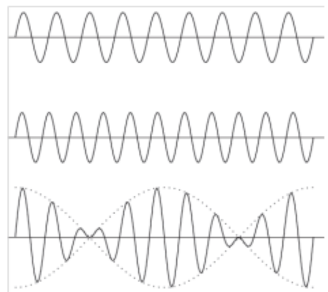
$$\sin\alpha = v_r/v_s$$

When the frequencies of two sources differ, non-sinusoidal beats result

$$\psi_1 = \cos(2\pi f_1 t)$$

$$\psi_2 = \cos(2\pi f_2 t)$$

$$\psi = \psi_1 + \psi_2$$

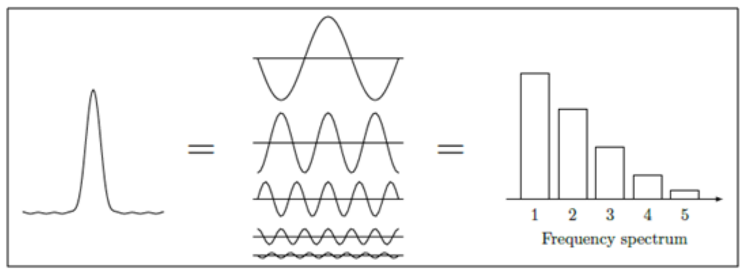


$$\psi(t) = A(t)\cos(2\pi f_{\text{avg}} t)$$

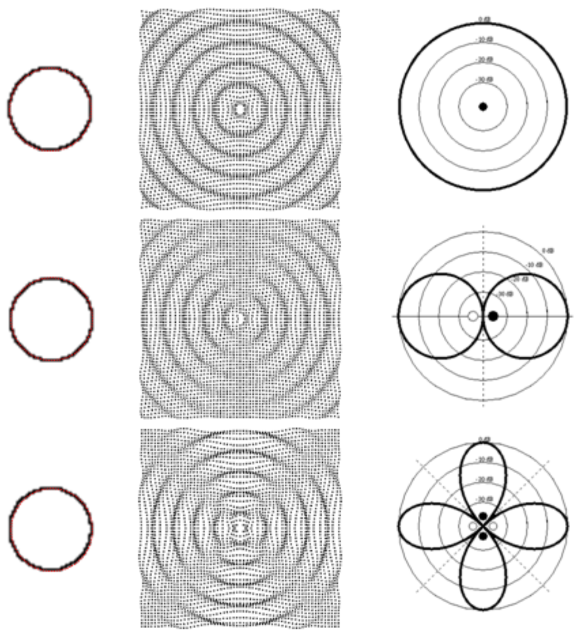
$$A(t) = 2\cos(\pi \Delta f t)$$

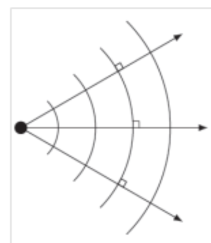
$$f_{\text{beat}} = \Delta f = f_1 - f_2$$

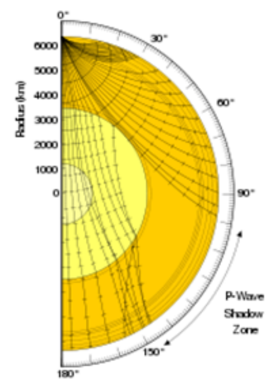
Fourier Analysis



Energy propagates along "rays" which move perpendicular to the overall wave fronts

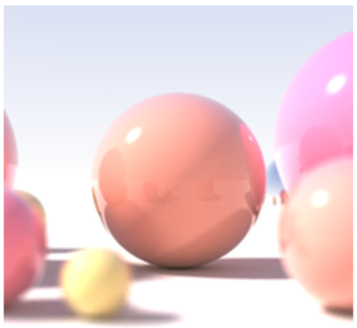


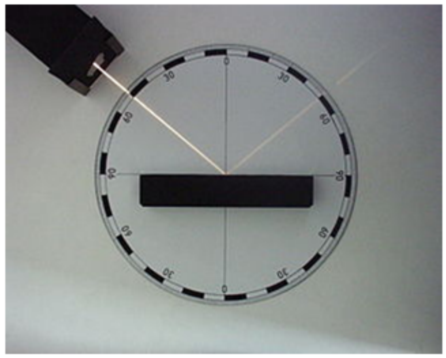


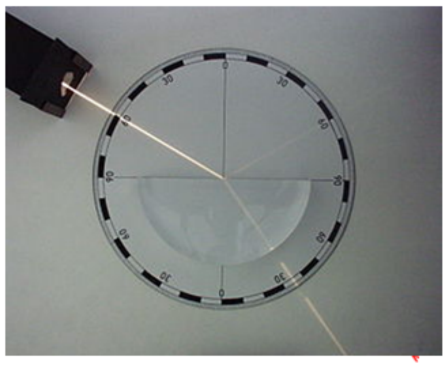


Geometric Optics

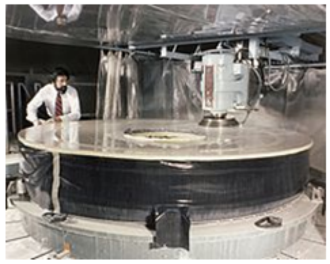
Geometric optics ignores the wave nature of light and is based on geometric laws

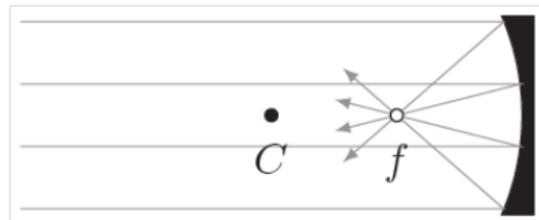




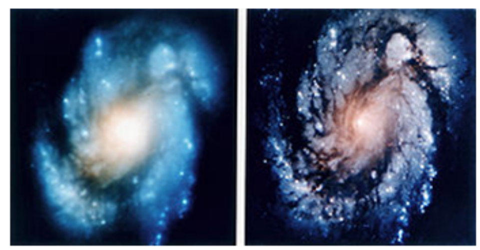


A curved mirror will act as a lens creating a magnified image based on its focal length

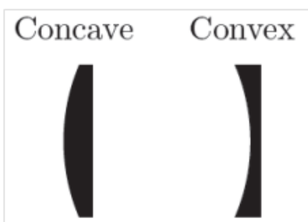




$$f = R/2$$



In a ray diagram, we only need a few key rays to identify the image location



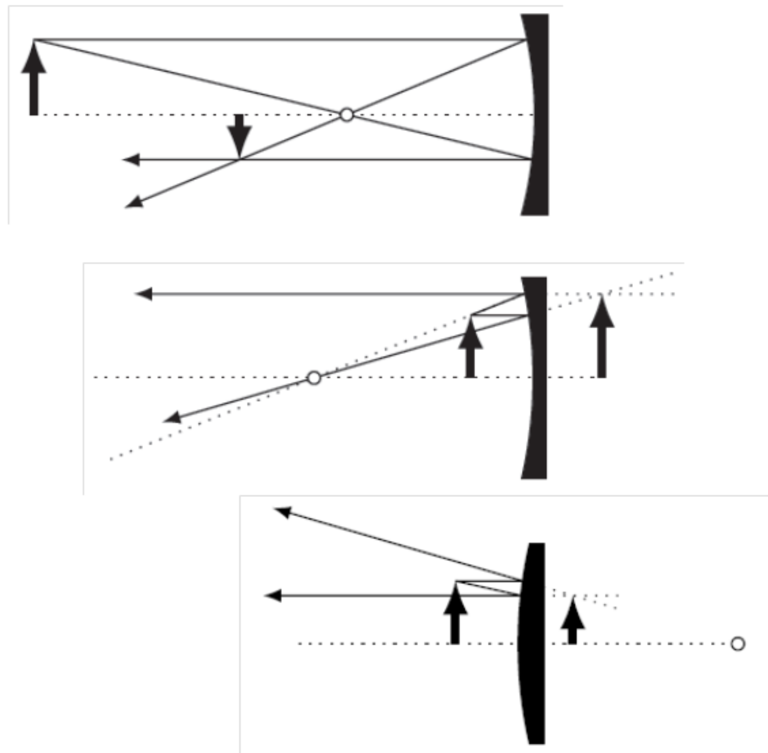
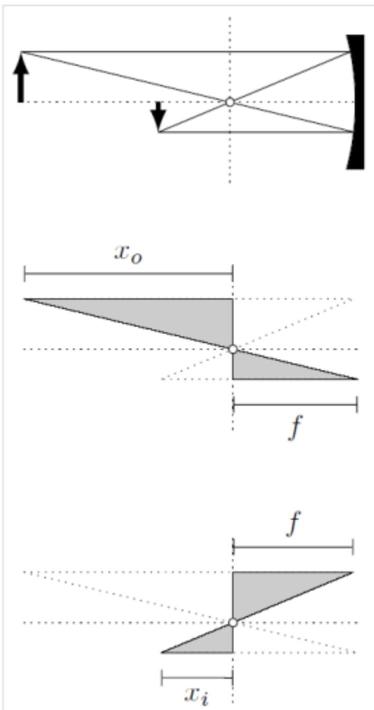


Image distance and magnification is related to object distance and lens focal length



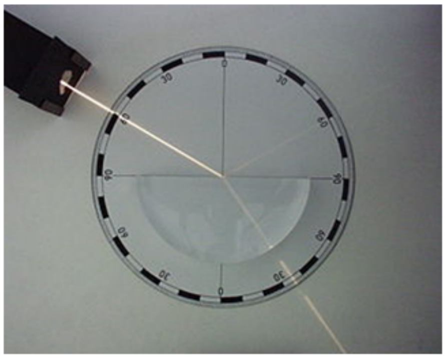
$$x_o x_i = f^2$$

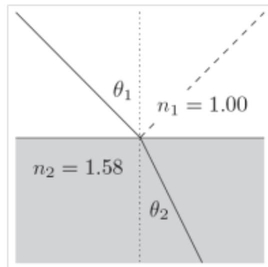
$$\frac{h_i}{h_o} = \frac{f}{x_o}$$

$$\frac{h_i}{h_o} = \frac{x_i}{f}$$

$$\boxed{\begin{gathered}\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \\ m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}\end{gathered}}$$

Snell's law governs the angles involved in refraction and is related to the speed of light

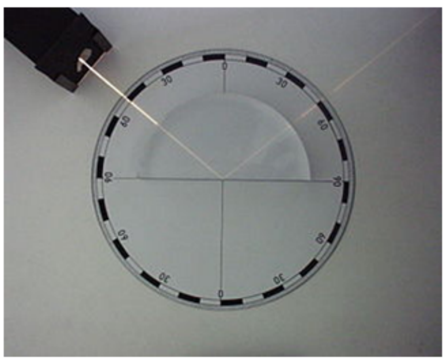


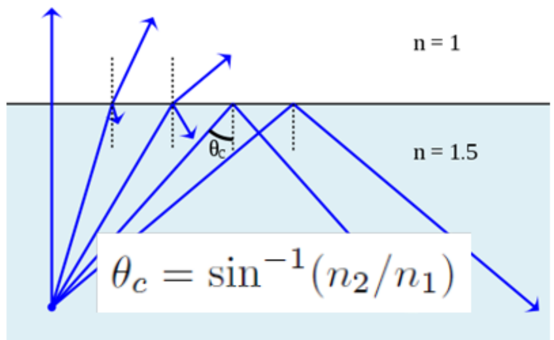


$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

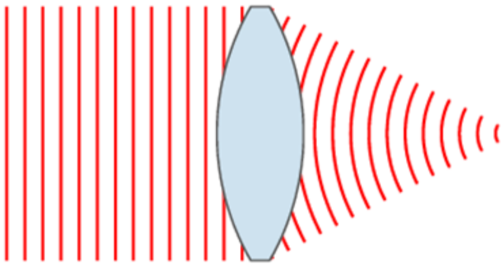
$$v = c/n$$

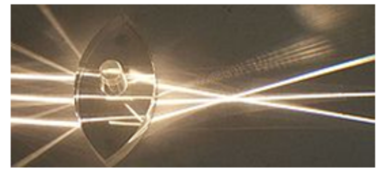
$$c = 3 \times 10^8 \text{ m/s}$$

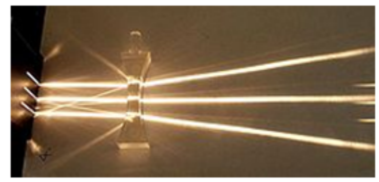




Transparent lenses obey same geometric formulas with a different sign convention

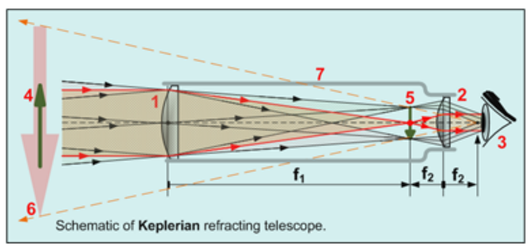






$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

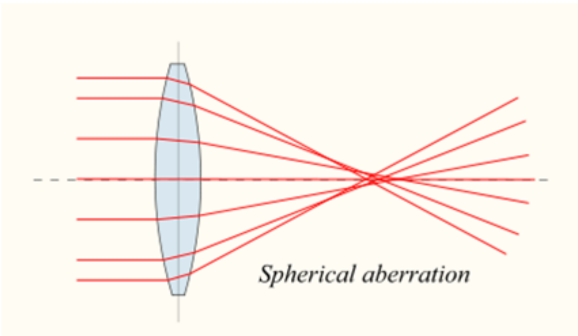
$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

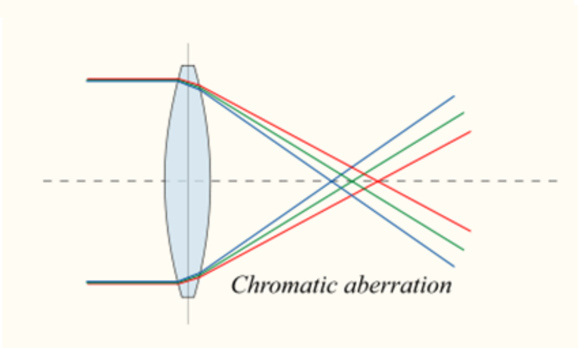


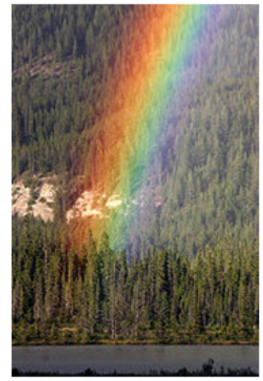
Schematic of **Keplerian** refracting telescope.

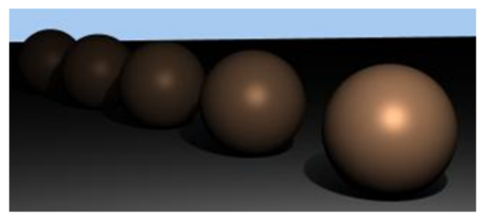
$$m = f_{\mathrm{obj}}/f_{\mathrm{eye}}$$

Most transparent materials are dispersive which explains prisms and rainbows

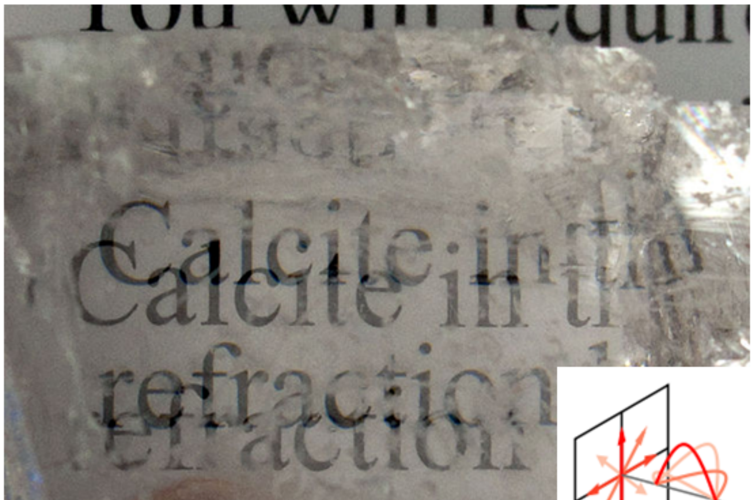


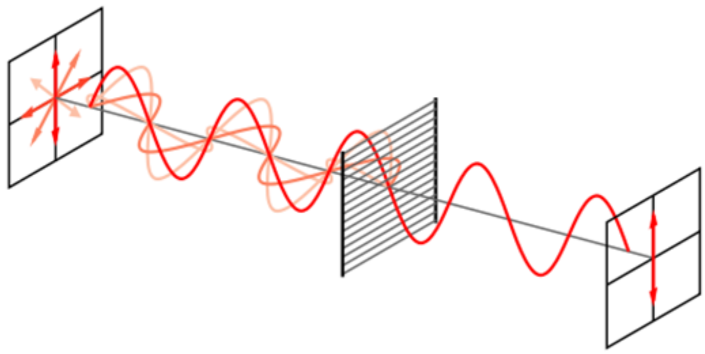






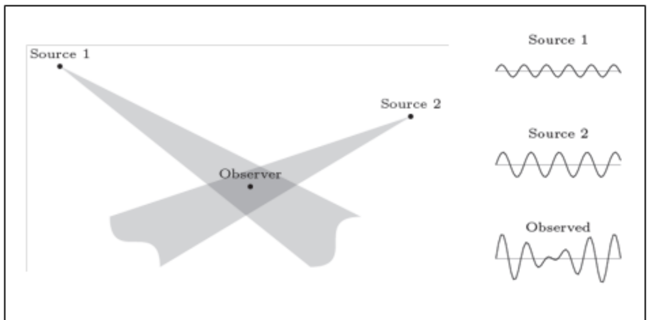
Double refraction and the problem of polarization



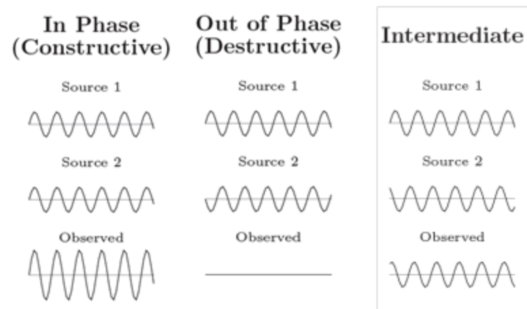


Wave Interference

When the disturbances from two sources combine the instantaneous amplitudes add



$$\psi(t) = \psi_1(t) + \psi_2(t)$$

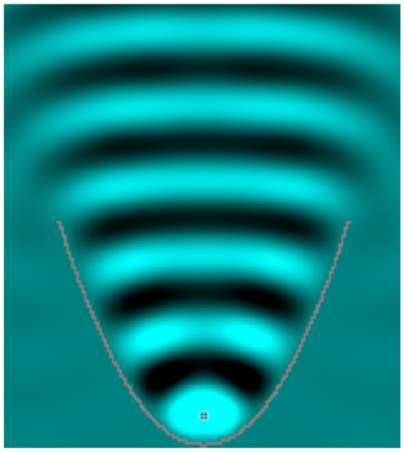
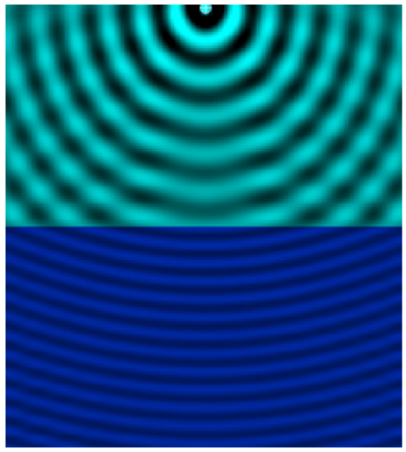
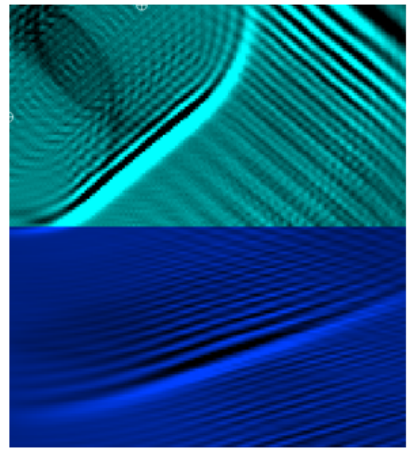


$$\Delta\phi = -(2\pi/\lambda)\Delta x$$

$$\Delta\ell = n\lambda$$

$$\Delta\ell = (n + \tfrac{1}{2})\lambda$$

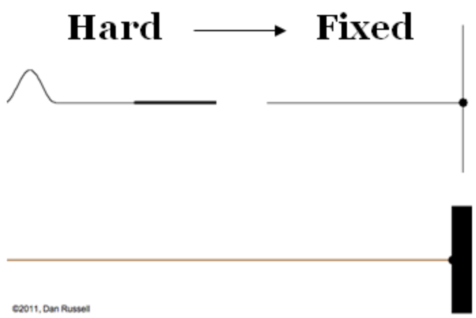
Reflection and refraction of waves provide a mechanical model for light and sound

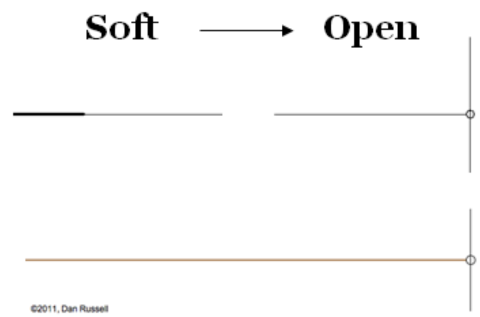




http://www.falstad.com/ripple/

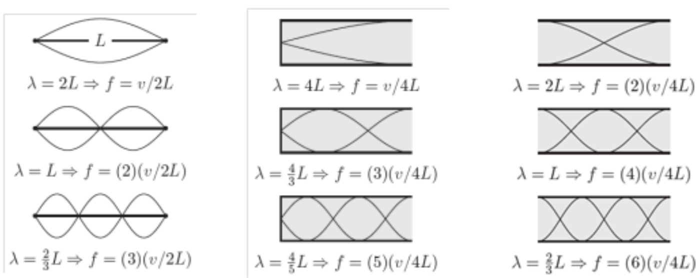
http://www.falstad.com/mathphysics.html

Reflection off a boundary acts as a second source that can produce standing waves





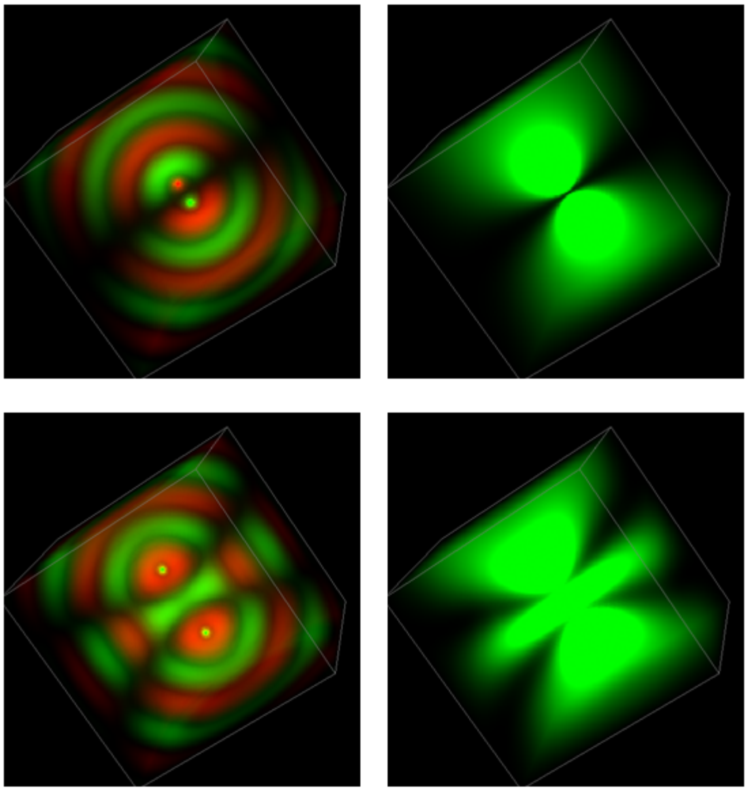
http://www.acs.psu.edu/drussell/Demos/

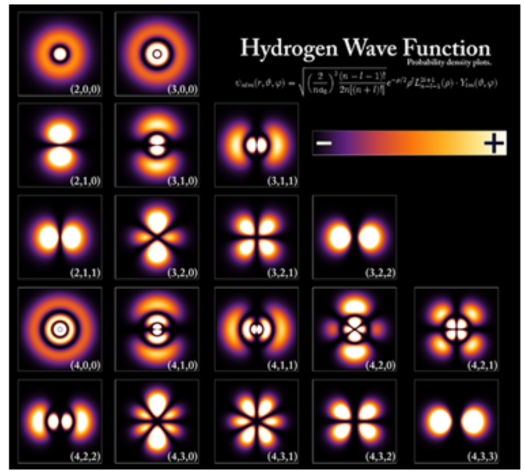


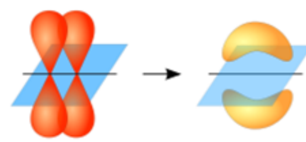
$$f_n = n\frac{v}{2L}$$

$$f_n = (2n-1)\frac{v}{4L}$$

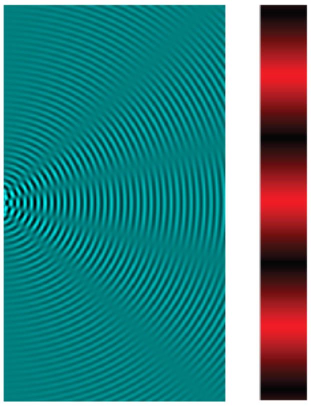
Three-dimensional standing waves are used to build microphones, antenna, and atoms

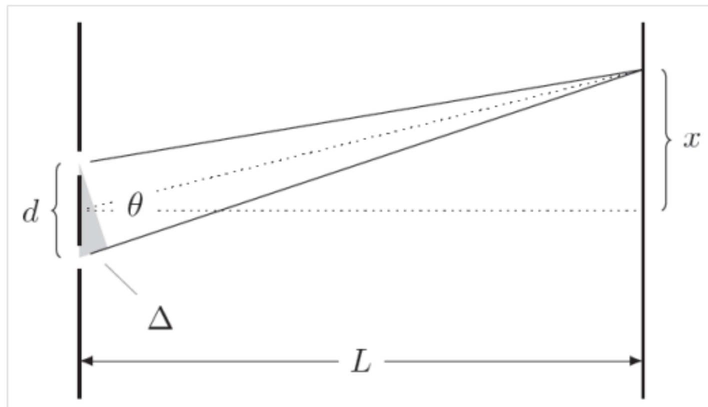






Double slit experiment provides definitive evidence for the wave nature of light

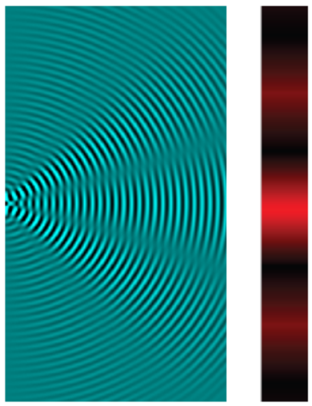




$$\sin\theta = n\lambda/d$$

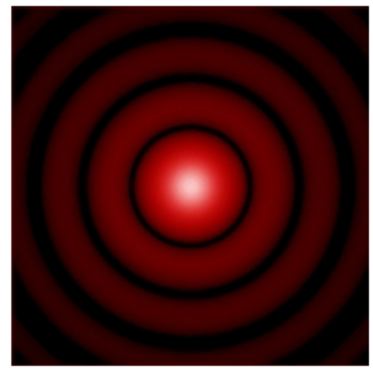
$$\Delta\ell = d\sin\theta$$

Single slit diffraction can be understood as the interference of a series of single sources



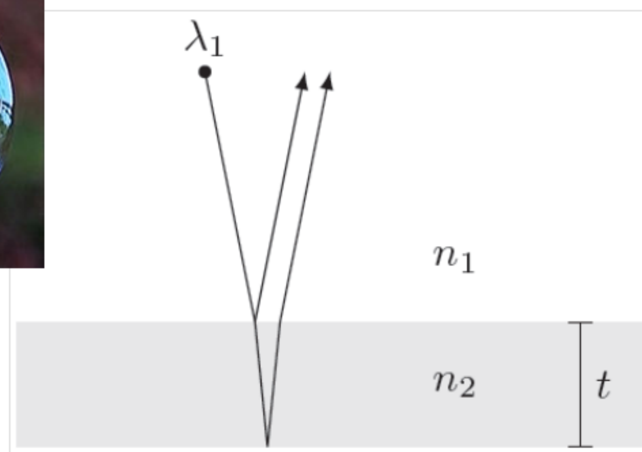
$$\theta = n\lambda/a$$

$$I = \left[\left(\frac{\lambda}{\pi a\theta}\right)\sin\left(\frac{\pi a\theta}{\lambda}\right)\right]^2$$

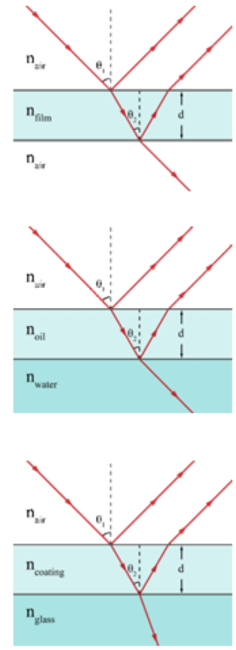


$$\theta_{\mathrm{min}} = 1.22\lambda/D$$

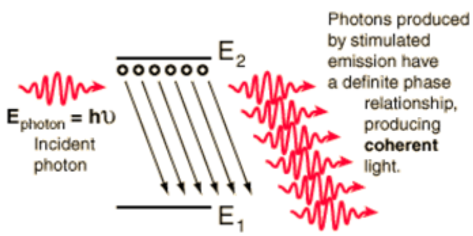
Iridescence in nature is usually from thin-film interference

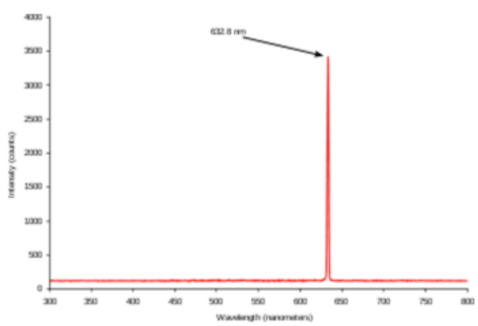


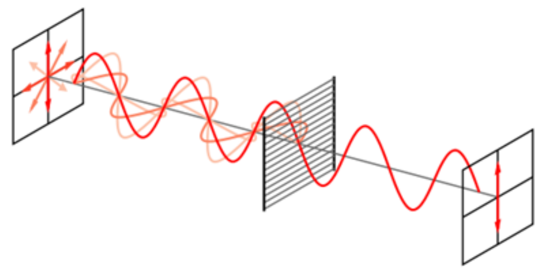
$$2t = (m + \tfrac{1}{2})(n_1/n_2)\lambda_1$$



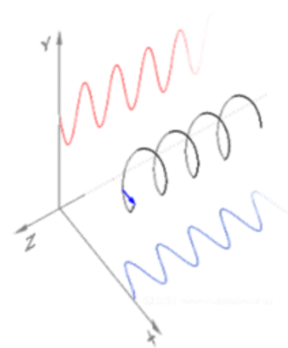
The difference between incandescent and laser light is the coherence of the waves





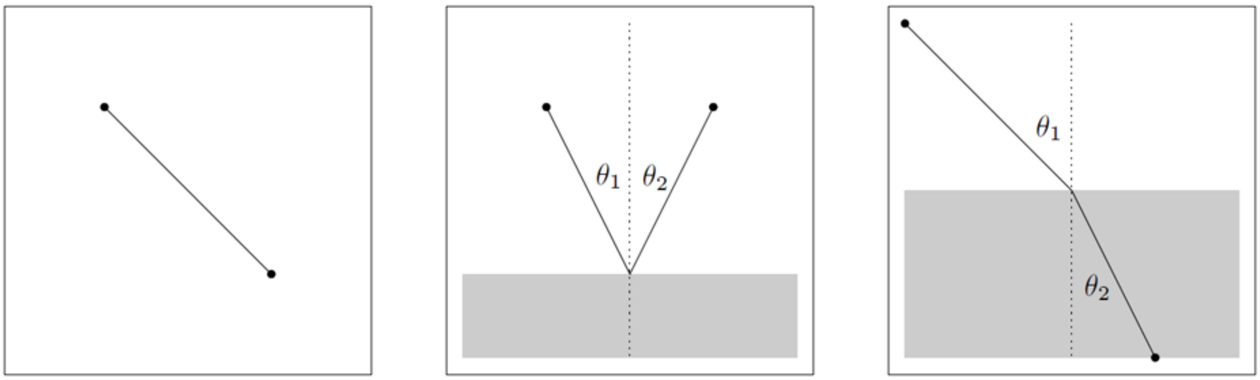


$$I = I_0 \sin^2 \theta$$



Quantum Optics and Least Action

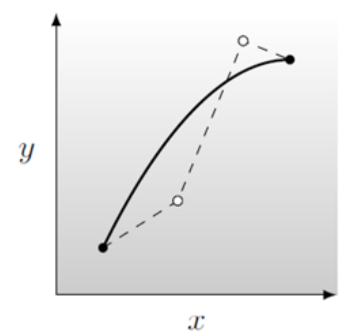
Fermat's principle: the path a ray of light takes the least amount of time to travel

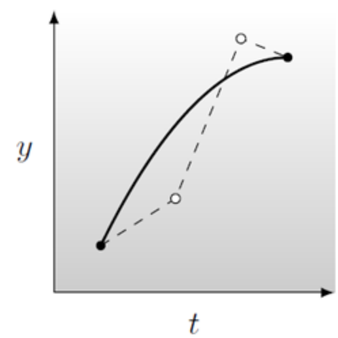


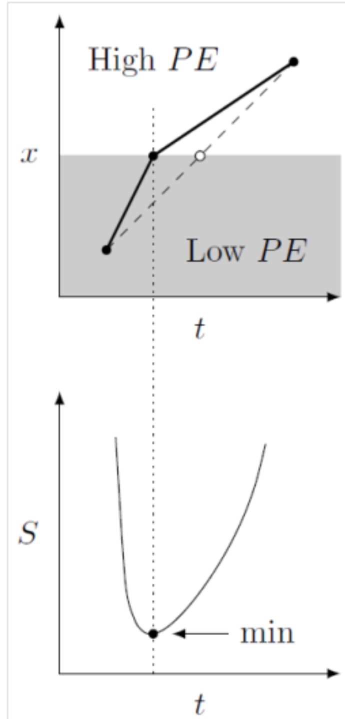
$$v = c/n$$

$$\ell = \sum n\Delta x$$

Hamilton's principle: the path a physical particle is the path of least action







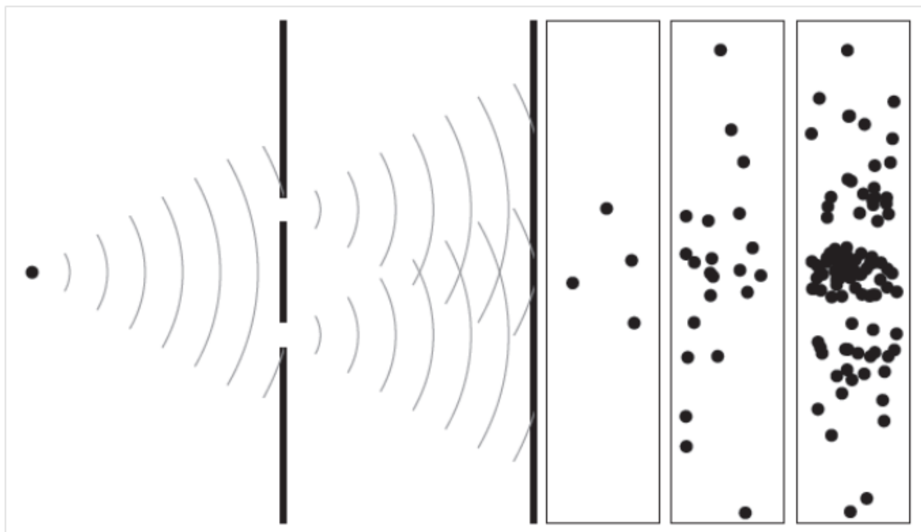
$$S_0 = \sum mv\Delta x$$

$$E_{\text{tot}} = \frac{1}{2}mv^2 + PE$$

$$S = \sum L\Delta t$$

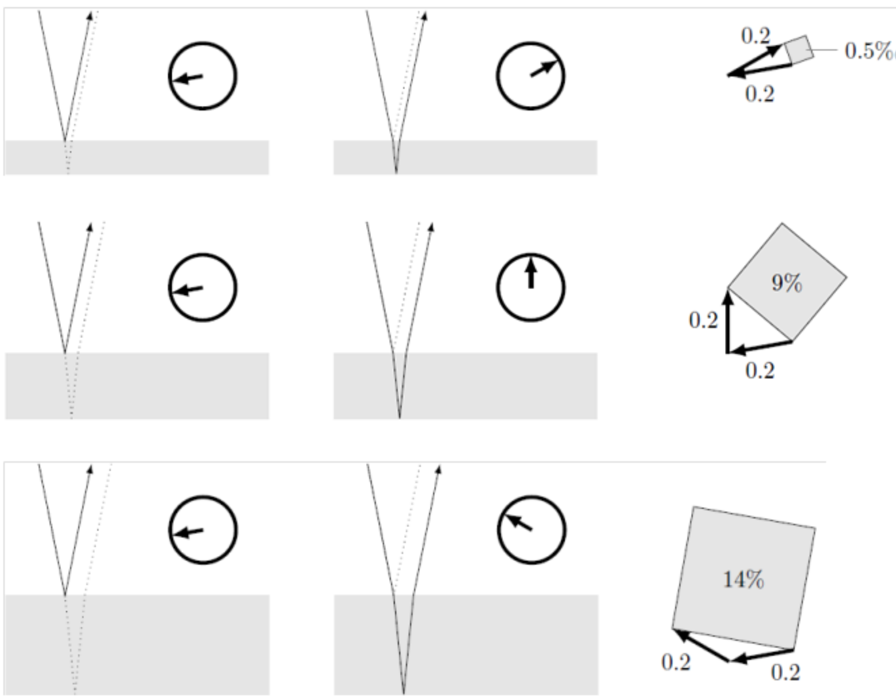
$$L = KE - PE$$

Feynman's rebuilds optics via least action and particles of light (photons)



$$\psi = \exp(iS/\hbar)$$

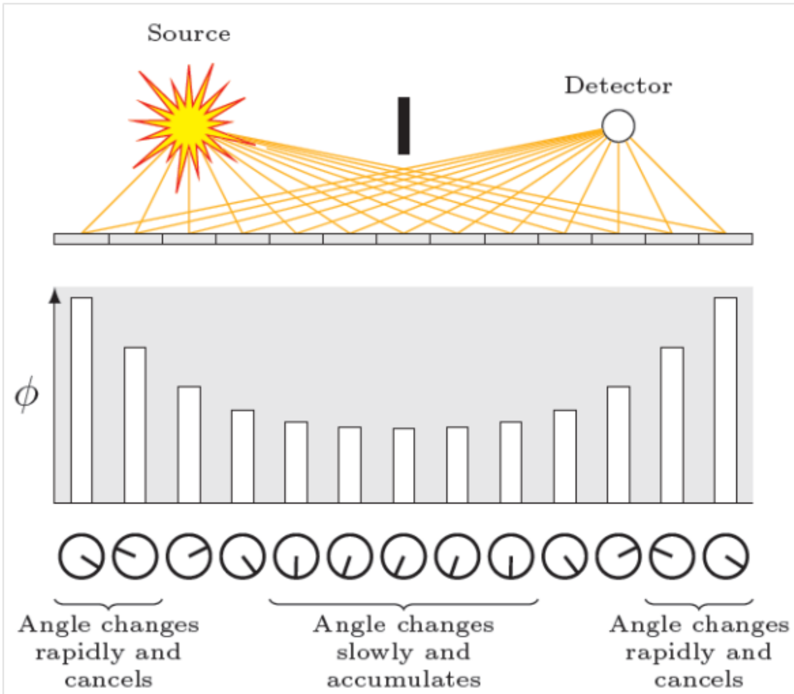
The “grand principle” of QED: phasors characterize the probability of the action

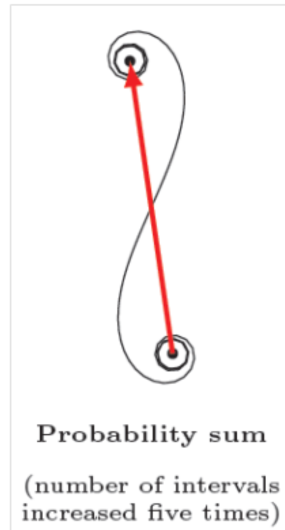


$$S = hf\Delta t$$

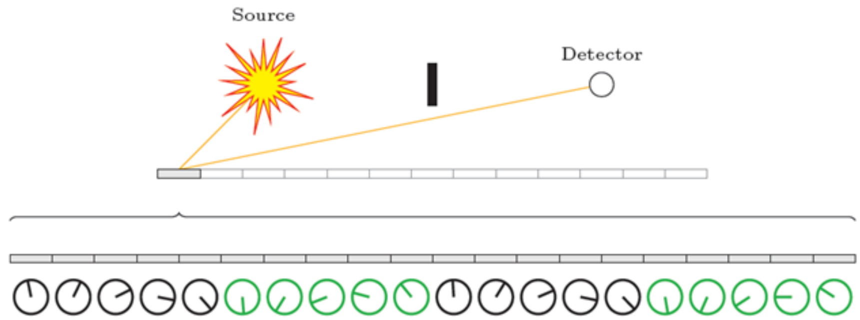
$$\phi = S/h = f\Delta t$$

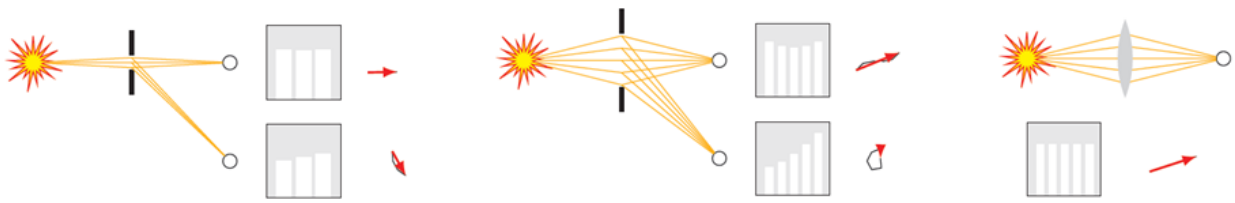
The "general rule" of QED: the true path is the phasor sum of all possible paths



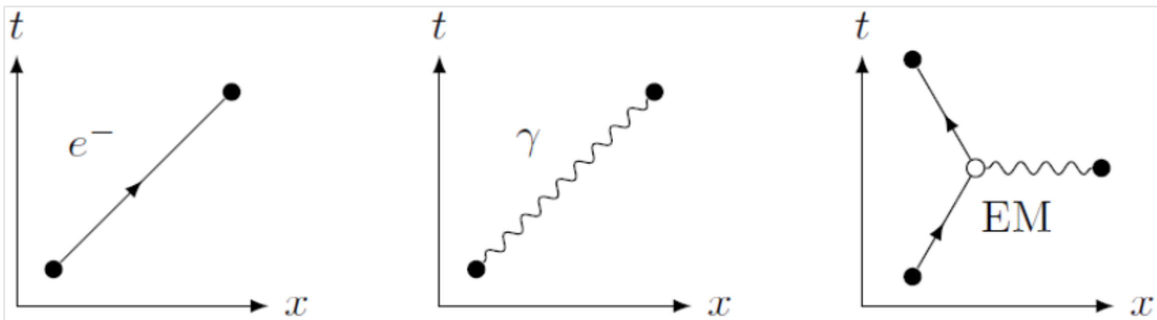


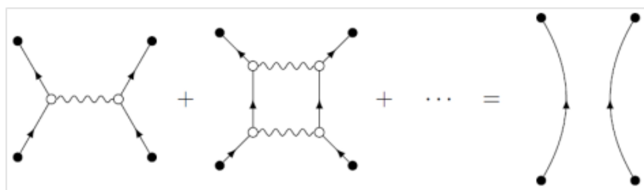
Incredibly, these ideas explain all of optics and quantum mechanics

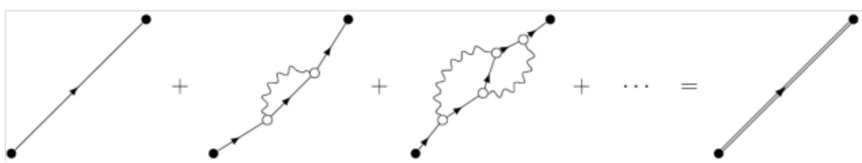




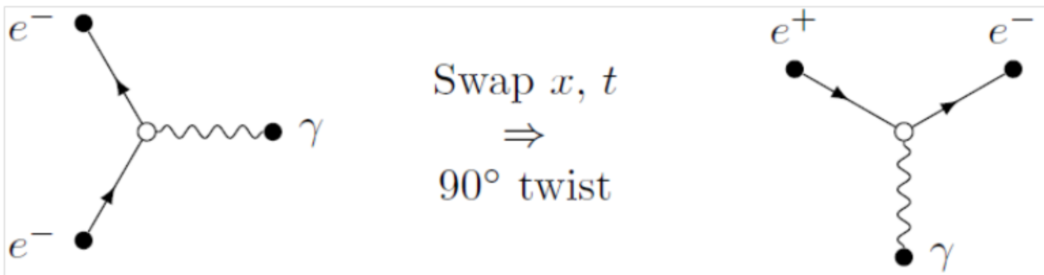
Electromagnetism comes from charged particles exchanging photons of light

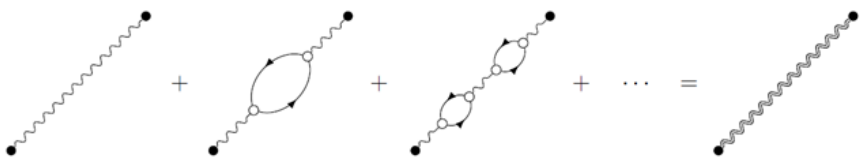


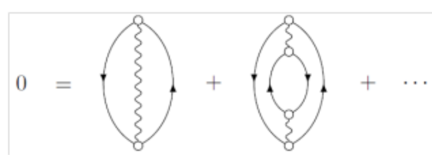




What's new: anti-matter annihilates matter in a burst of light and vice versa





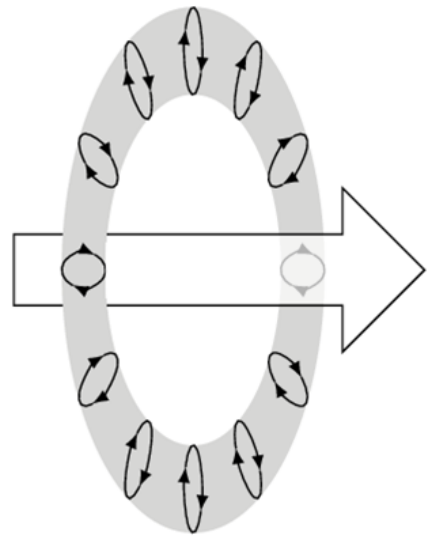


- With Feynman diagrams, truly anything goes. Including moving backward in time.
- As one plays around with these diagrams, one gets used to imagining them in a malleable way because each of these internal changes are indistinguishable.
- QED is also relativistically valid and in relativity time and space are joined in space-time. Understanding this, Feynman asked the question: what happens if we rotate these diagrams 90°? Essentially this swaps the role of space and time. The two diagrams are no longer equivalent, but it does beg the question what is it?
- In particular, our basic interaction diagram: an electron absorbs a photon. On its side, this diagram starts with just a photon and ends with two electrons, one moving up in time (normal) and another moving backward in time.
- Feynman recognized this process as the creation of anti-matter. Energy goes in (as a photon) and is converted into an electron (the normal one) and its anti-matter twin the positron (an electron with positive charge).
- In this way the theory provides a simple explanation for the existence of anti-matter and why every particle must have an anti-particle twin.
- Whether or not we interpret anti-matter as time-reversed matter, the fact is that a photon can dissolve into matter/anti-matter pairs and vice versa. This means that we must consider so-called **bubble diagrams** as possible contributions to our overall sum over history.
- And it is precisely these bubble diagrams that explain the slight deviations from classical field theory such as the gyromagnetic moment of the electron to astonishing accuracy.
- We can even draw a self-contained bubble diagram without any incoming or outgoing particles. The vacuum is a sea full of virtual matter-antimatter pairs. In QED even the vacuum is a many-body problem!

Limits of Classical Mechanics

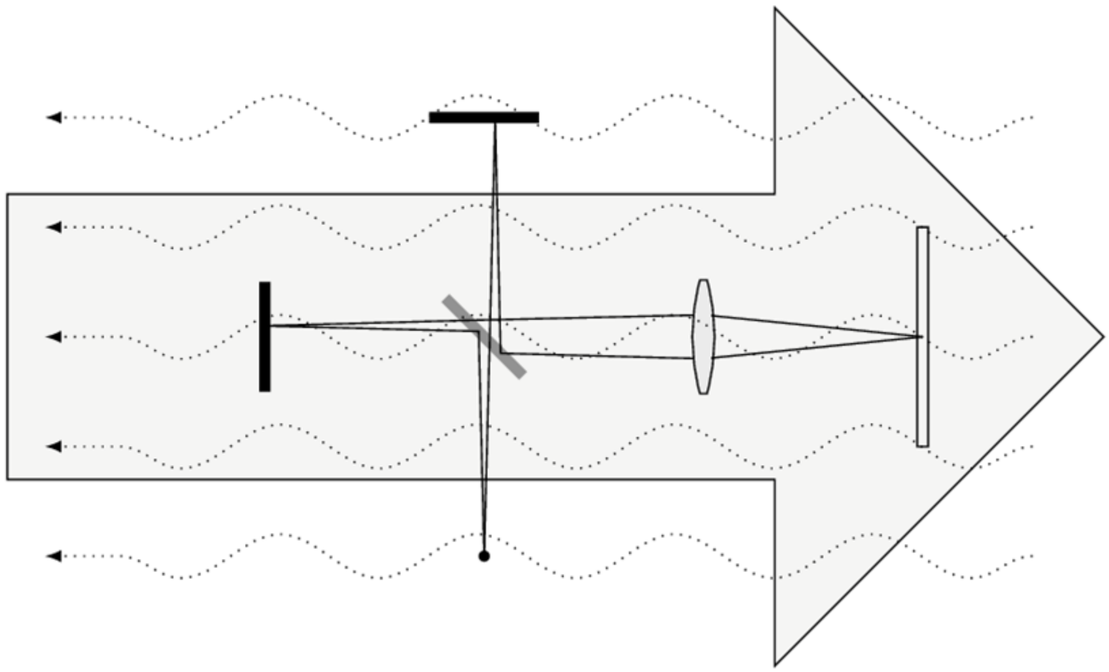
EM theory predicts that light will carry momentum — perhaps the ether is a fluid?

- Ether is solid?
  - Supports waves
  - Supports polarization
  - Does <u>not</u> support momentum transfer
- Ether is fluid?
  - Supports waves
  - Supports momentum transfer
  - Does <u>not</u> support polarization

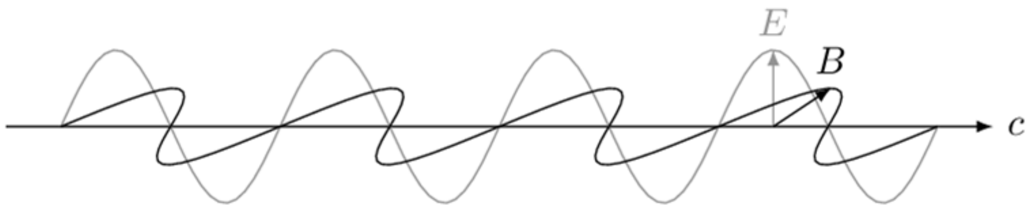


Atomic model?

Michelson and Morley experiment should but does not detect any "ether wind"



Special relativity: the ether is dead — but must rethink space-time mechanics

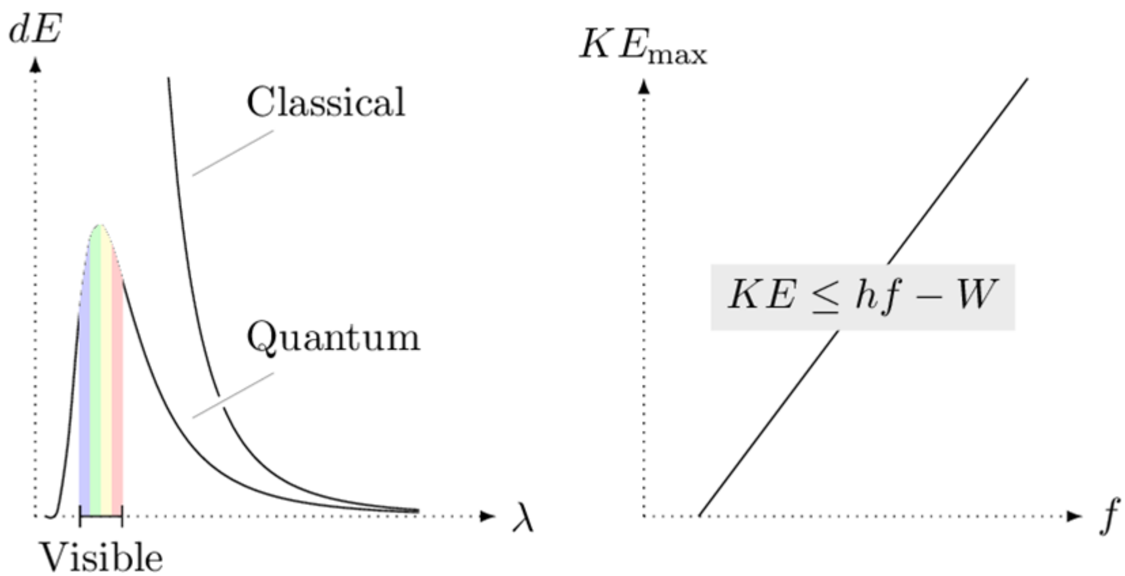


$v = c$?

$$v_{ac} = \frac{v_{ab} + v_{bc}}{1 + v_{ab}v_{bc}/c^2}$$

$$E^2 = p^2c^2 + m^2c^4$$

Quantum mechanics discovered at the same time as relativity but for different reasons



Problems in low temperature kinetic theory: specific heats of solids, also "superstuff"

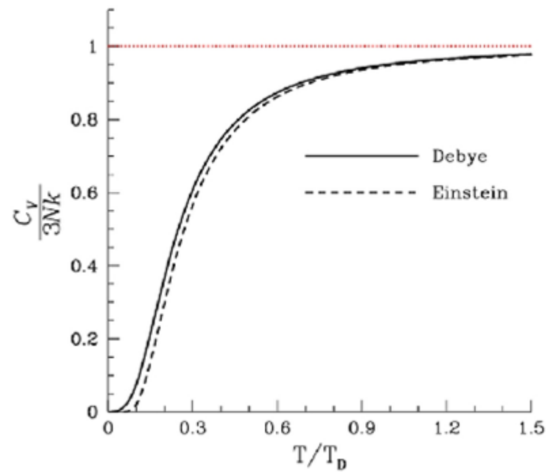


Image credit: http://en.wikipedia.org/wiki/File:DebyeVSEinstein.jpg

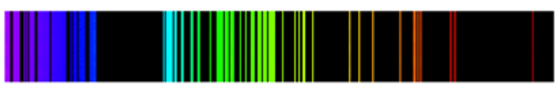
- Superstuff happens when fermions act like bosons
  - Superconductivity: Resistance drops to zero
  - Superfluid: Viscosity drops to zero
- Typically only occurs at very low temperatures
  - LN_2 temperature (77 K) superconductor was a goal
  - Achieved in 1987 (YBCO)
  - Highest known superconducting temperature is 135 K

Problems in atomic theory: atom should be unstable, spectroscopy inexplicable

Hydrogen

Image credit: http://upload.wikimedia.org/wikipedia/commons/4/4c/Emission_spectrum-H.png

Iron



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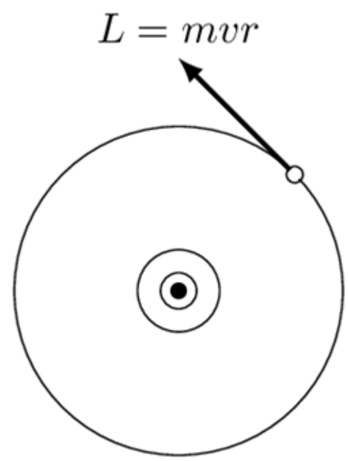
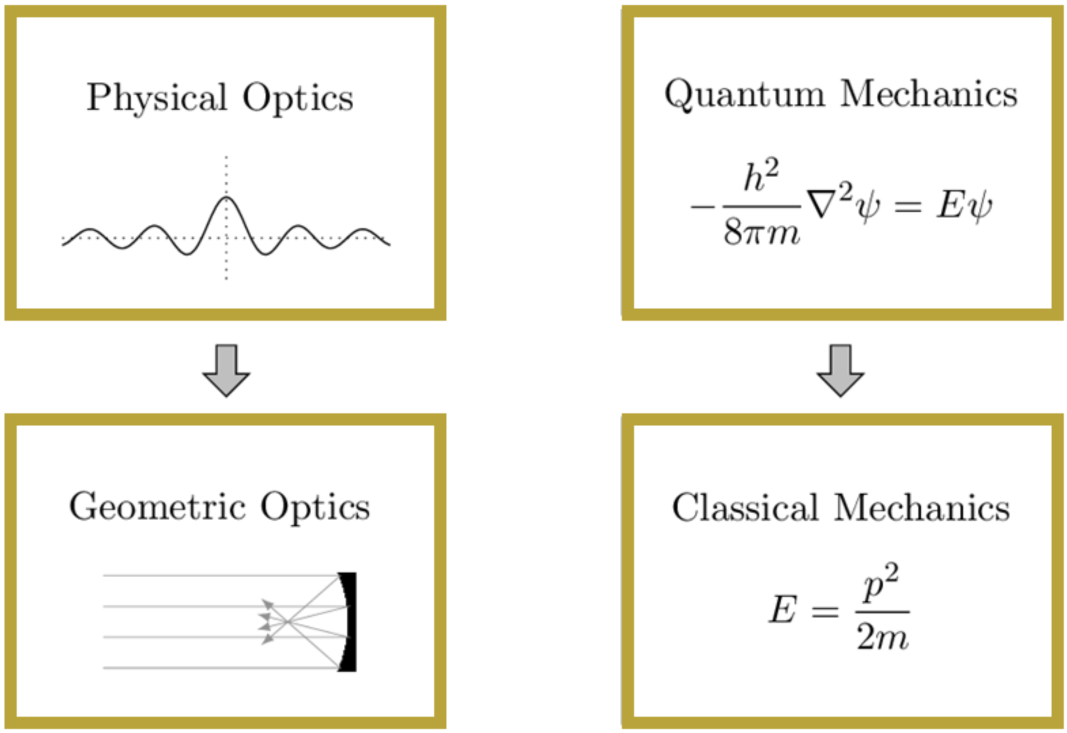


Quantum mechanics has wave-particle duality in a way similar to optics

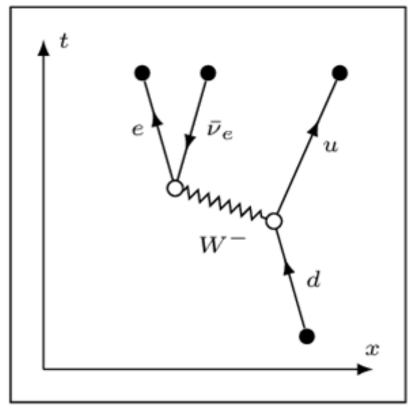


Quantum field theory is the "natural" combination of special relativity with QM

Spin 1 particles

Spin 1/2 particles	Photon γ 0	Gluon g 0	Weak W $\sim$ 80000
Electron e 0.511	-1	0	
Neutrino ν_e > 0	0	0	
Quark d 4.8	-1/3	g	
Quark u 2.4	2/3	g	

Couplings



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