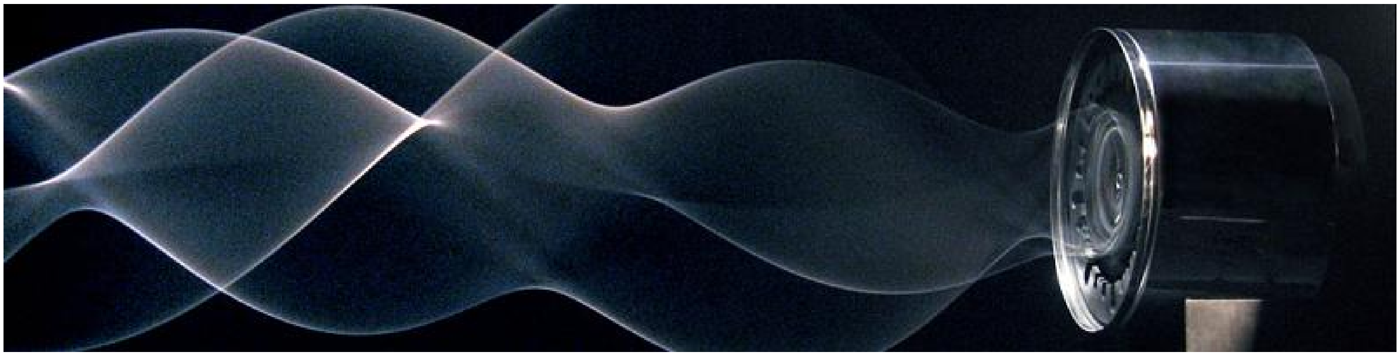
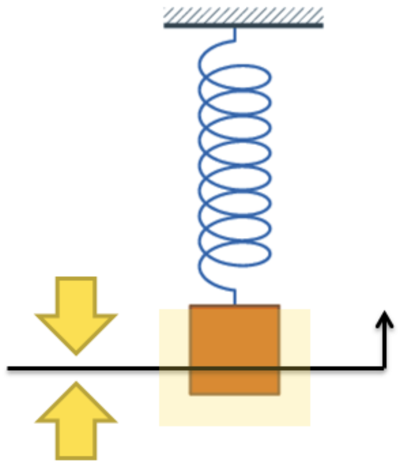
Vibration and Oscillation



Physical systems will vibrate near and around points of equilibrium



Equilibrium

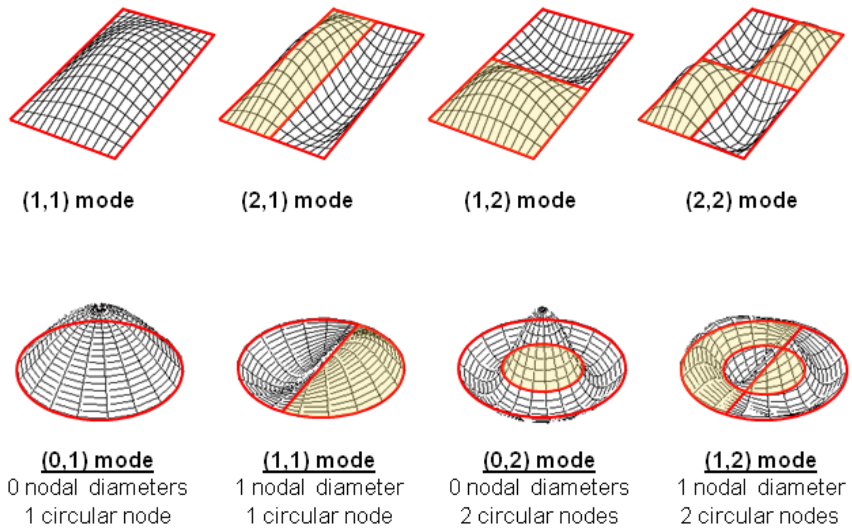
+

Displacement

=

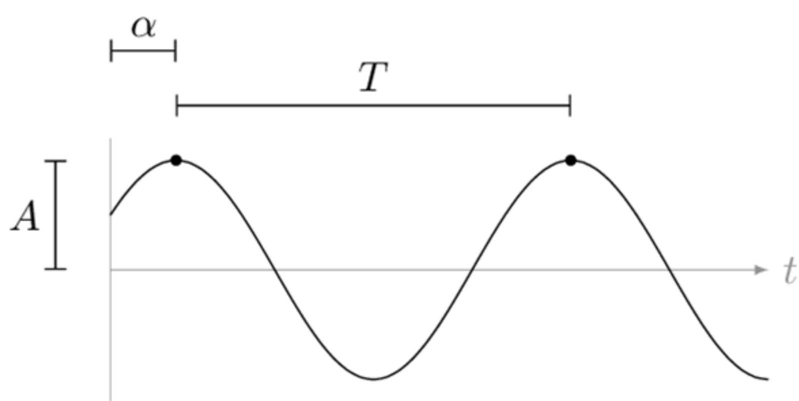
Vibration

Vibration in multiple dimensions is complicated and depends on boundaries



http://www.kettering.edu/~drussell/demos.html

Simple harmonic motion is the motion of vibration with a specific mathematical form



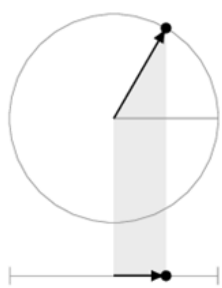
Define:

$$f = 1/T$$
$$\omega = 2\pi f$$
$$\phi = -\alpha\omega$$

$$\psi(t) = A\cos(\omega t + \phi)$$

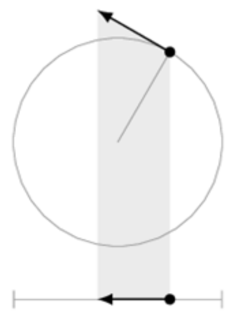
A fortunate fluke: the projection of uniform circular motion is simple harmonic motion

$$r = A$$
$$\theta = \omega t$$



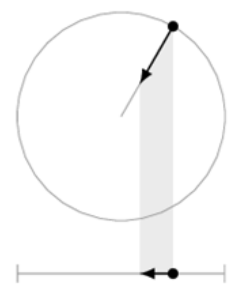
$$x = A\cos(\omega t)$$
$$x_{max} = A$$

$$v = \frac{2\pi r}{T} = A\omega$$



$$v = A\omega\sin(\omega t)$$
$$v_{max} = A\omega$$

$$a = \frac{v^2}{r} = A\omega^2$$

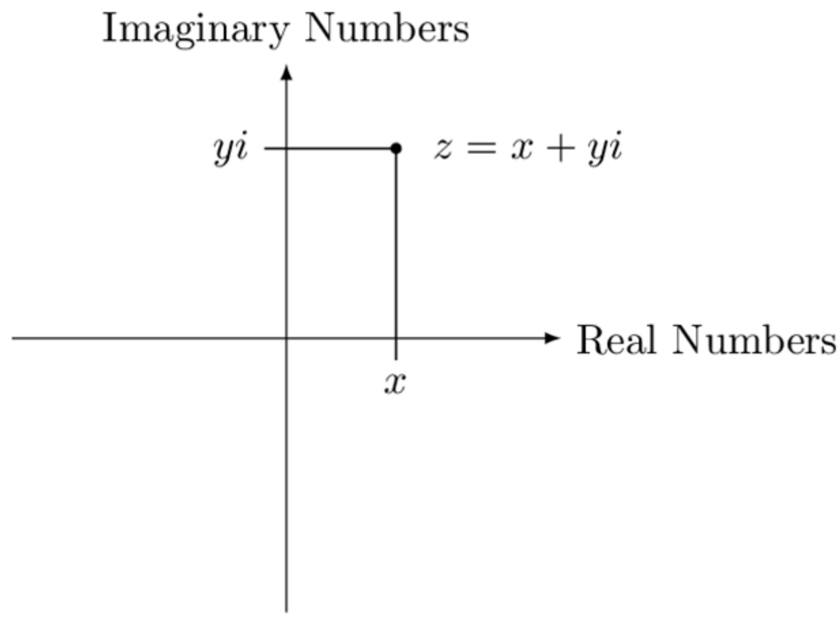


$$a = -A\omega^2\cos(\omega t)$$
$$a_{max} = A\omega^2$$

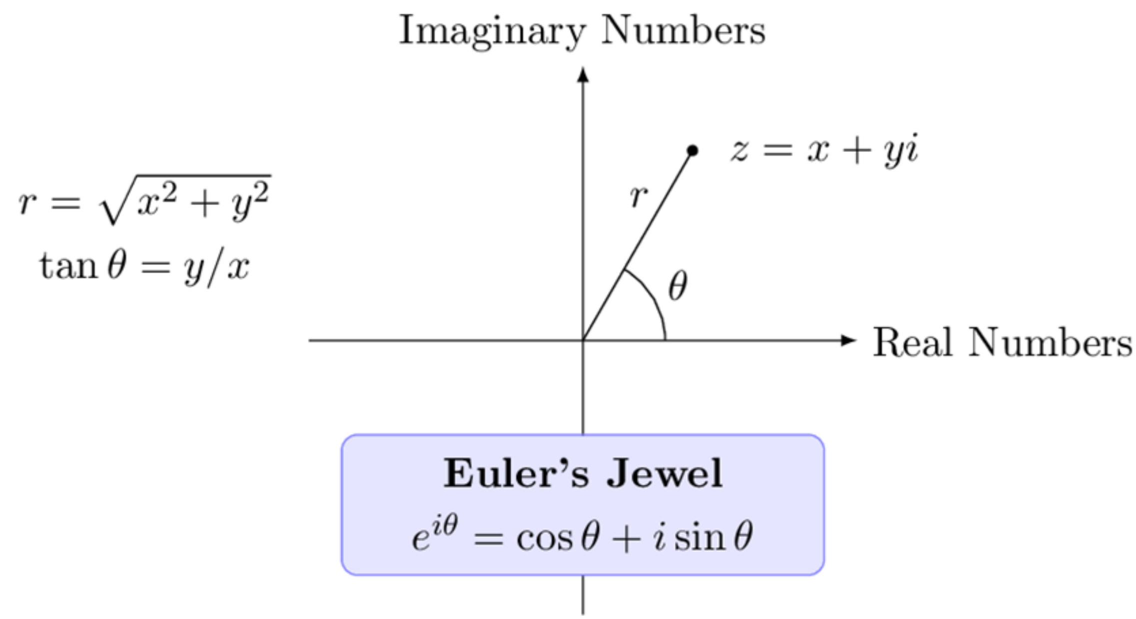
We can use complex numbers to model vibrations — they are called "phasors"

$$i = \sqrt{-1}$$

$$i^2 = -1$$



The reason we can get away with using complex numbers is in Euler's jewel



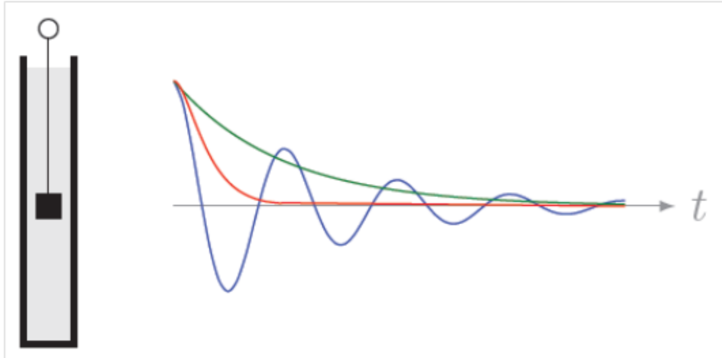
Euler's Jewel

$$e^{i\theta} = \cos\theta + i\sin\theta$$

An oscillation is said to be "damped" when energy is drained from the system

$$F_{\text{drag}} = -cv$$

$$\gamma = c/m.$$



$$A(t) = A_0 \exp(-\tfrac{1}{2}\gamma t)$$

Light damping $(\gamma < 2\omega)$

Heavy damping $(\gamma > 2\omega)$

Critical damping $(\gamma = 2\omega)$

If energy is put into the system it is "driven"; some driving frequencies create resonance