How to solve force problems involving Newton's second law, $\sum F = ma$...

Step 1 : Classify the problem

- If the system is not moving, it is in static equilibrium.
- If the motion of the system is constant, it is in kinetic equilibrium.
- If the motion is accelerating, it is not in equilibrium.
- Circular motion is never in equilibrium.

Step 2 : Identify the forces

If there is more than one part to the system, pick one and label it (e.g., #1). Do not mix up the forces acting on different parts of the system. Now ask yourself...

Is there weight?
Draw an arrow straight down.

Is there support?
Draw an arrow pointing perpendicular to the surface.

Is there tension?
Draw an arrow along the string.

Is there kinetic friction?
Draw an arrow parallel to the surface opposing the motion.

Is there static friction?
Draw an arrow parallel to the surface to create equilibrium.

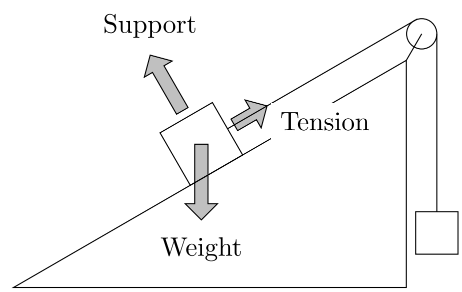

Figure 1: Inclined plane with pulley

Step 3 : Extract the equations

Orient your coordinate system. Each part in the system may have a different orientation of coordinates.

- If the system is not in equilibrium, align your coordinates along the acceleration. If the object is in uniform circular motion the acceleration is toward the center of the circle.
- If the system is in equilibrium, align your coordinates along your most unknown force (usually support or tension).
- When in doubt, go ahead and use the standard horizontal/vertical orientation.

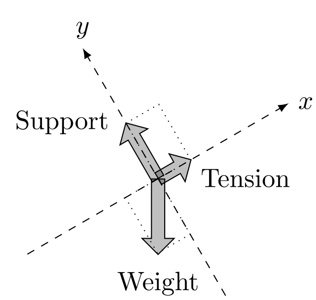

Figure 2: Free-body diagram from Figure 1

If the system is in equilibrium (either static or kinetic), all the forces must balance. If the system is not in equilibrium, the forces are not in balance. By design, the net force will now be aligned with the coordinates (either the x or y).

Every coordinate for every part will yield an equation. If there is a net force along the direction, we use $\sum F = ma$.[1] If the acceleration is down or left, put in a negative sign ($-a$). Otherwise, we use $\sum F = 0$. In either case we begin by calculating the components of all the forces against our coordinate system.

Start with the x-direction. If there is no motion along this direction, all the force components add to zero. Be sure to pay attention to the signs (forces pointing

[1] Remember: m is the mass of this particular part of the system. If the mass is in uniform circular motion we have $a = v^2/r$. For a block-and-tackle problem you may need to distinguish the acceleration of each part of the system.

left are negative, forces pointing right are positive). Use $F_x = F\cos\theta$ if necessary. Remember to always measure the angle θ from the positive x-direction. The sum of all these components is $\sum F$. Set this equal to either zero or ma as appropriate.

Repeat for the y-direction. Use $F_y = F\sin\theta$ if necessary. Up is positive and down is negative. The sum of all these components is $\sum F$. Set this equal to either zero or ma as appropriate.

Repeat this for every part in the system. You will end up with $2n$ equations, but some may be trivial like $0 = 0$.

Step 4 : Solve the equations

Make sure to substitute $W = mg$ for the weight and $F = \mu_k N$ for the kinetic friction. There is no formula for support, tension, or static friction. You now have a system of equations. Solve it. Start with the most trivial equation first (maybe it's the shortest, or one side is zero, etc.) Simplify this and substitute it into your other equations. Continue until something pops out.

Step 5 : Answer the question

You should have enough information to solve for and answer any question about the forces and the acceleration of any the part of the system.

But you may be asked for other quantities like speed or displacement. You will need to use the equations of constant acceleration to do this. You now know the acceleration—look for the other two quantities you are given. The quantity in the question is your fourth quantity. Find the equation (listed below) with these four quantities and solve.

- $v = v_0 + at$
- $x = \frac{1}{2}(v + v_0)(t)$
- $x = v_0 t + \frac{1}{2}at^2$
- $x = vt - \frac{1}{2}at^2$
- $v^2 = v_0^2 + 2ax$