Physics 201 Examples 11

Mar 13, 2013

1. An evil genius wants to shrink the Moon in order to steal it. He accidentally shrinks its below it Schwarzschild radius and the Moon is lost forever in a mini-black hole. What is this radius?

0.109 millimeters

2. The acceleration due to gravity varies depending on height according to Newton's law of gravitation ($F = GMm/r^2$). To a first approximation, it can be shown that the revised acceleration due to gravity is given by

3.6 seconds

$$g' = g(1 - 2h/r_e)$$

where h is the height of the object from the surface of the earth and r_e is the radius of the earth. Use this formula to answer the following question.

Suppose we are in Einstein's elevator—a cube 100 meters on each side in free-fall just above the surface of the Earth. Suppose our best precision across this frame is 1 millimeter. How long can we consider this an inertial frame?

3. A synchronous satellite "parked" in orbit over the equator is used to relay microwave transmissions between stations on the ground. How much is the frequency shift at the satellite's receiver if the frequency of the transmission from Earth is exactly 9.375 GHz? (Ignore Doppler effects.)

2.77 hertz

4. Sirius and a small companion revolve about one another. Astronomers have determined that the mass of the companion is roughly equal to the mass of our Sun. A spectral line from the companion is shifted in frequency by 0.07% compared to the frequency of the same spectral line in the laboratory. Assuming that this is a gravitational red shift, estimate the radius of the companion of Sirius.

2,100 kilometers

5. The gravitational precession for a circular orbit will have the effect of slightly decreasing its period. Calculate the number of seconds missing from one Earth year as a consequence of this precession.

1.07 seconds

6. Consider a bullet is that is fired and hits a target 10 meters away in 0.020 seconds. Due to gravity, the bullet must be fired at a slight angle up. Suppose a ball thrown at the same target takes 2.0 seconds. Clearly the curvature of the two trajectories are different, but you will show that the curvature in space-time is the same. This is one of the justifications for reinterpreting the action of gravity as a kind of "curvature in time".

(a) 25,500 meters and 2.55 meters
(b) 9.2×10^{15} meters

The curvature of these two trajectories can be compared using the following formula for the radius of curvature (valid only at the top of the trajectory):

$$r = d^2/8h$$

In this formula, d represents the range of the projectile and h represents its maximum height. (a) Calculate the curvature of the trajectories for the bullet

and ball. (b) Repeat using the distance in time, ct, instead of d within the radius of curvature calculation.

7. An example of a gravitational lens is the "Einstein Cross" depicted in Figure 1[1] located in the Pegasus constellation. The four outer points of light are images of a quazar 8.0 billion light years away that is directly behind some galaxy. The light from the quazar bends around the galaxy and reaches earth from four different directions. The angular deflection of the images is about 7.8×10^{-6} radians. Assume the deflecting galaxy is in the middle and estimate the mass of the deflecting galaxy.

7.8×10^{41} kilograms

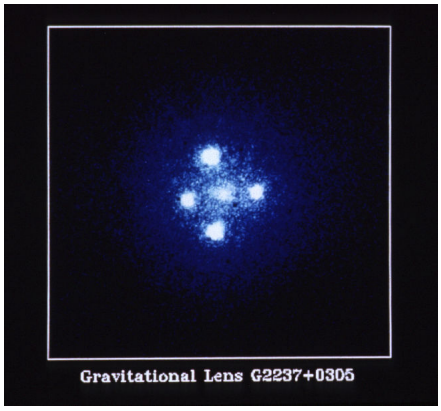

Figure 1: Gravitational Lens G2237 + 0305, Einstein Cross

8. (a) Estimate the radius of a neutron star with twice the mass of the Sun. Use the fact that its density ought to be comparable to that of the atomic nucleus: 3×10^{17} kg/m^3. (b) Light can orbit around an object at 1.5 times its Schwarzschild radius. Does this neutron star possess such a "photon sphere"?

(a) 14.7 kilometers
(b) No

9. Calculate the Shapiro delay between an airplane flying at an altitude of 5.0 kilometers and the radio tower directly below. Since these are terrestrial distances, you may use the first approximation for the altitude dependence of gravity $g' = g(1 - 2h/r_e)$ in the apparent speed of light formula to calculate this time delay. (Because of this, the average apparent speed of light over the distance is the average of the value at the bottom and top of the trip.)

3×10^{-24} seconds

10. Calculate the density of a black hole with a mass of ten solar masses. Assume the volume of the black hole is a sphere with a radius equal to the Schwarzschild radius.

1.5×10^{18} kg/m^3

11. Consider a photon of light generated by the Sun and reaching the Earth. As it leaves the Sun's gravitational well, its frequency is red-shifted and as it enters the Earth's gravitational well it gets "blue-shifted". Compare the magnitude of these two effects.

Sun's red-shift is over 3000 times larger than the Earth's blue-shift

[1] Source: http://hubblesite.org/newscenter/archive/releases/1990/20/