1) BIQUADRATIC INTERPOLATION

$$V(x,y) = \sum_{i=0}^{2}\sum_{j=0}^{2} a_{ij} x^i y^j$$

$$= a_{00} + a_{01}y + a_{02}y^2 + a_{10}x + a_{11}xy + a_{12}xy^2 + a_{20}x^2 + a_{21}x^2y + a_{22}x^2y^2$$

Let $X = \begin{bmatrix} 1 & y & y^2 & x & xy & xy^2 & x^2 & x^2y & x^2y^2 \end{bmatrix}$

and, Let $A = \begin{bmatrix} a_{00} \\ a_{01} \\ a_{02} \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{20} \\ a_{21} \\ a_{22} \end{bmatrix}$

Then $V(x,y) = \underset{1\times 9}{X} \cdot \underset{9\times 1}{A}$

Now, in order to obtain coefficients, we need multiple instances,

(All 9 neighbour pixels $x_1 - x_9$, $y_1 - y_9$, $V_1 - V_9$)

$V_1 = V(x_1, y_1)$

$V_2 = V(x_2, y_2)$

$\vdots$

$V_9 = V(x_9, y_9)$

$$\therefore \underbrace{\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_9 \end{bmatrix}}_{V} = \underbrace{\begin{bmatrix} 1 & y_1 & y_1^2 & \cdots & x_1^2 y_1^2 \\ 1 & y_2 & y_2^2 & \cdots & x_2^2 y_2^2 \\ \vdots & & & & \\ 1 & y_9 & y_9^2 & \cdots & x_9^2 y_9^2 \end{bmatrix}}_{X} \underbrace{\begin{bmatrix} a_{00} \\ a_{01} \\ \vdots \\ a_{22} \end{bmatrix}}_{A}$$

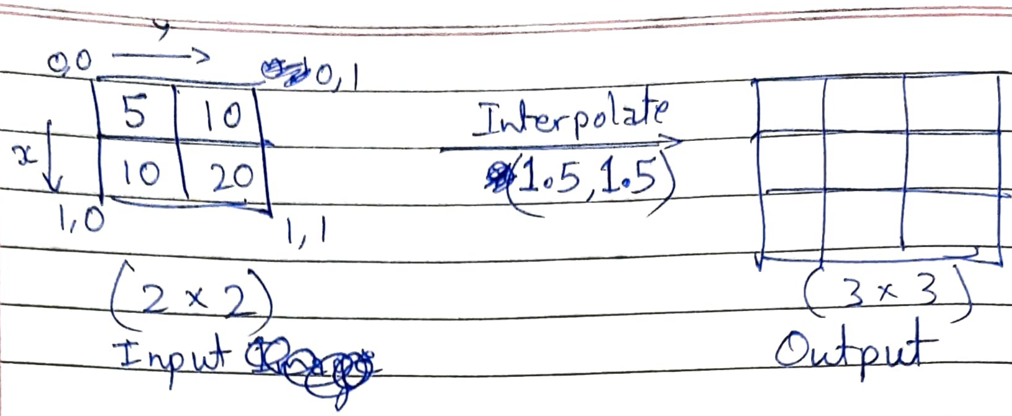
$\therefore \quad V = X . A$

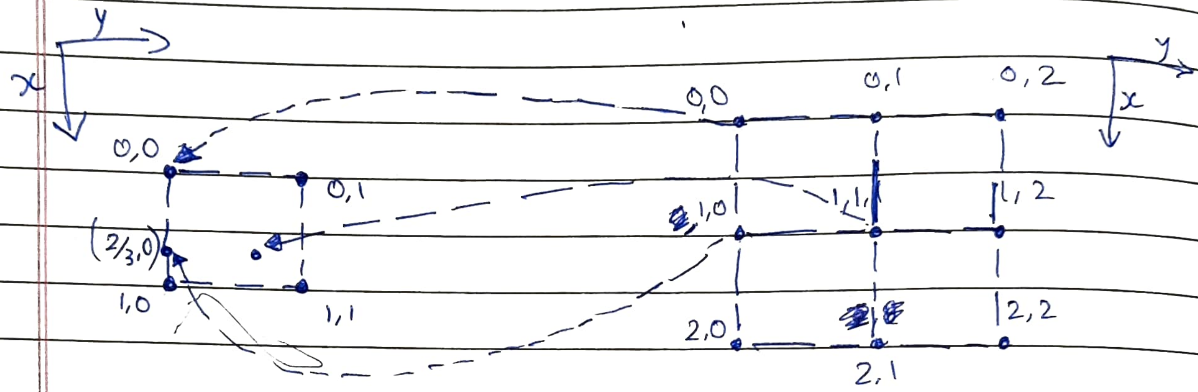
OR $\quad X^{-1} V = X^{-1} X . A$

$\Rightarrow \quad X^{-1} V = I . A$

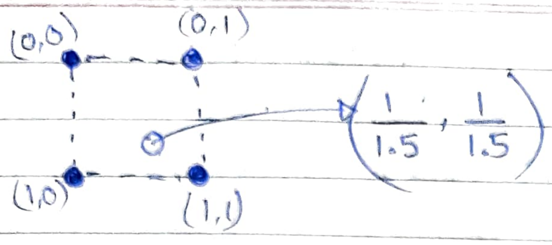
$\Rightarrow \quad \boxed{A = X^{-1} V}$

2)





Output coordinate	$\xrightarrow{\text{MAP}}$	Input Coordinate
0,0	$\left(\frac{0}{1.5}, \frac{0}{1.5}\right) =$	$(0, 0)$
0,1	$\left(\frac{0}{1.5}, \frac{1}{1.5}\right) =$	$(0, 2/3)$
0,2	$\left(\frac{0}{1.5}, \frac{2}{1.5}\right) =$	$(0, 4/3)$
1,0		$(2/3, 0)$
1,1		$(2/3, 2/3)$
1,2		$(2/3, 4/3)$
2,0		$(4/3, 0)$
2,1		$(4/3, 2/3)$
2,2		$(4/3, 4/3)$



4 Neighbors : (0,0), (1,0), (0,1), (1,1)

Bilinear Interpolation:

$$V(x,y) = ax + by + cxy + d$$

Writing down as matrix:-

$$\begin{bmatrix} 0 & 0 & (0)(0) & 1 \\ 1 & 0 & (1)(0) & 1 \\ 0 & 1 & (0)(1) & 1 \\ 1 & 1 & (1)(1) & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 10 \\ 20 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 10 \\ 10 \\ 20 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 1 & -1 & -1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 10 \\ 10 \\ 20 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \end{bmatrix}$$

$\Rightarrow V(x,y) = 5(x + y + xy + 1)$

$\therefore$ pixel value for $(1,1)$ maps to $V(2/3, 2/3)$

$$V(2/3, 2/3) = 5\left(\frac{2}{3} + \frac{2}{3} + \frac{4}{9} + 1\right)$$

$$= 5\left(\frac{12 + 4 + 9}{9}\right) = \frac{125}{9}$$

$$= 13.89$$

$$\approx \boxed{14}$$

Similarly calculating for other output pixels, we get:-

5	8	7
8	14	11
7	11	9