DIP ASSIGNMENT-3

(2) $x(n,m) = \begin{cases} 1, & n = 256 \\ 0, & \text{otherwise} \end{cases}$ Given

(Input Image)

$N = M = 512$

We need to compute 2D DFT $\{x(n,m)\} = X(k,l)$

By definition,

$$X(k,l) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} x(n,m)\, e^{-jk\frac{2\pi}{N}n}\, e^{-jl\frac{2\pi}{M}m}$$

Since $x(n,m) = 0 \;\forall\; n = \{1, 2, \ldots 255, 257, \ldots, 512\}$

and $x(n,m) = 1$ for $n = 256$

$$\Rightarrow X(k,l) = \sum_{m=0}^{M-1} e^{-jk\frac{2\pi}{N}(256)}\, e^{-jl\frac{2\pi}{M}m}$$

$$e^{-jk\frac{2\pi}{N}(256)} = e^{-jk\frac{2\pi}{512}(256)} \quad (N = 512)$$

$$= \left(e^{-j\pi}\right)^k$$

$$= (-1)^k$$

$$\Rightarrow X(k,l) = \sum_{m=0}^{M-1} (-1)^k\, e^{-jl\frac{2\pi}{M}m}$$

$$= (-1)^k \sum_{m=0}^{M-1} e^{-jl\frac{2\pi}{M}m}$$

$$= (-1)^k \left[\sum_{m=0}^{511} e^{-jl\frac{2\pi}{512}m}\right] \quad \text{---(1)}$$

This resembles DC signal.

Consider $x_1(m) = 1 \quad \forall\ m \in \{0, 1, \ldots, 511\}$ (DC signal)

$M = 512$

$$DFT\{x_1(m)\} = X_1(l)$$

$$X_1(l) = \sum_{m=0}^{M-1} x_1(m)\, e^{-j l \frac{2\pi}{M} m}$$

$$= \boxed{\sum_{m=0}^{511} 1.e^{-j l \frac{2\pi}{512} m}}$$

We know that DFT of DC signal implies $X_1(0)$ = sum of magnitudes $\forall m$

and $X_1(l) = 0 \quad (\forall l \neq 0)$

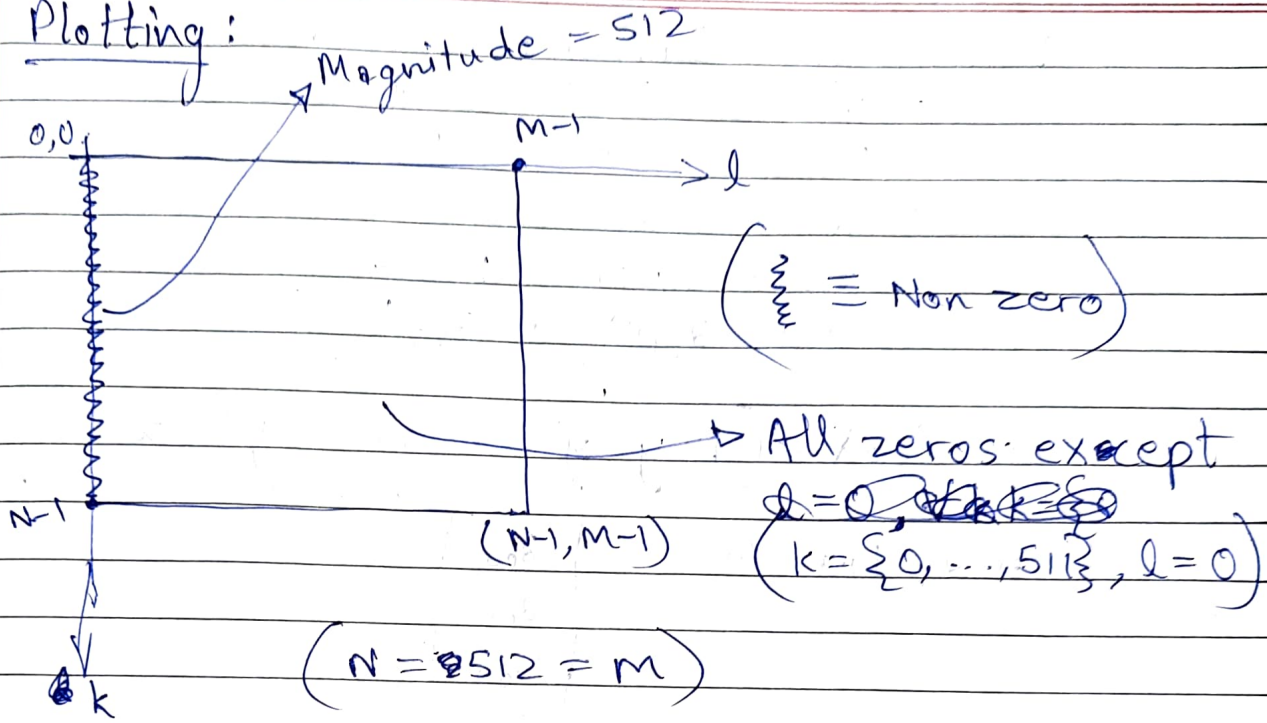
$$\therefore X_1(l) = \begin{cases} 512, & l = 0 \\ 0, & \text{otherwise} \end{cases}$$

$\therefore$ (1) becomes,

$$X(k,l) = (-1)^k \cdot X_1(k,l)$$

$$\boxed{X(k,l) = \begin{cases} (-1)^k \cdot 512, & l = 0 \\ 0, & l \neq 0 \end{cases}}$$

Plotting :



DFT : $X(k, l)$ =

$$\begin{bmatrix} 512, 0, 0, \cdots, 0 \\ -512, 0, \cdots, 0 \\ 512, 0, \cdots, 0 \\ -512, 0, \cdots, 0 \\ \vdots \\ 512, 0, \cdots, 0 \\ -512, 0, \cdots, 0 \end{bmatrix}_{512 \times 512}$$