CS 135 Fall 2017
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Assignment 9

Due:	Monday, December 4th, 9:00 pm
Language level:	Intermediate Student with Lambda
Allowed recursion:	See notes below
Files to submit:	`ca.rkt`, `rectangles.rkt`, `bonus.rkt`
Warmup exercises:	HtDP *Without using explicit recursion:* 9.5.2, 9.5.4
Practice exercises:	HtDP 20.2.4, 24.3.1, 24.3.2

- **Make sure you read the OFFICIAL A09 post on Piazza** for the answers to frequently asked questions.
- Most sub-questions will give specific instructions about what sorts of recursion are allowed (if any), and what built-in functions may be used. If no restrictions are applied, you may assume that you have full access to all forms of recursion, all techniques learned this term, and all built-in functions in Intermediate Student with Lambda.
- Your solutions must be entirely your own work.
- Solutions will be marked for both correctness and good style as outlined in the Style Guide.

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方法的问题随你怎
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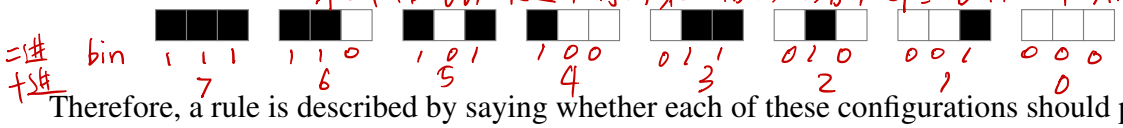
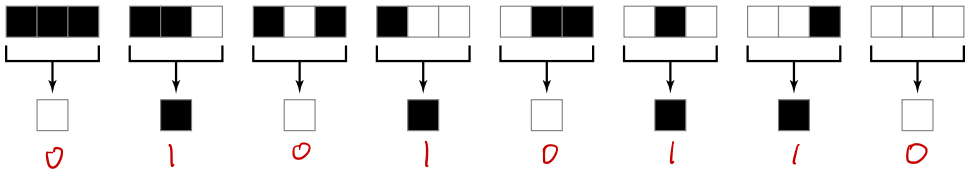
1. A one-dimensional elementary cellular automaton (CA) is a computational machine that evolves a row of "cells", each coloured black or white, according to a fixed "rule". In each successive generation, the rule determines the colour of a cell based on the colours of that cell and its immediate left and right neighbours in the previous generation.

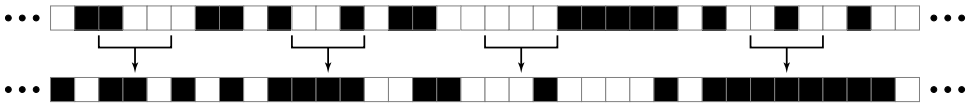
   A cell and its two neighbours can be in eight possible configurations.



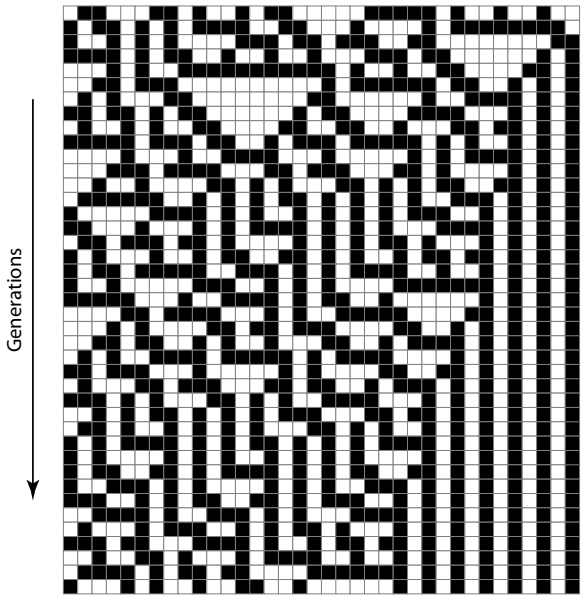
   Therefore, a rule is described by saying whether each of these configurations should produce a black or white cell in the next generation. Here is one possible rule.



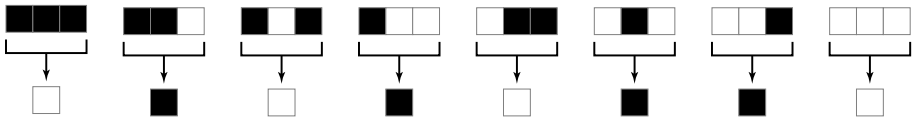
   Then, given a row of black and white cells, the rule is applied to each trio of cells to generate the cells in a new row. A few examples are highlighted below, though of course the rule is applied everywhere. (Don't worry for now about what happens at the start or end of the row; we'll work around that later.)



   We can produce interesting pictures by iterating this process. We draw a sequence of rows of black and white squares, where each row is generated by applying the rule to the row above it.



Because the rule depends on making a binary (black vs. white) choice for each of the eight possible trios of input squares, there are precisely $2^8 = 256$ possible rules. We encode a rule as a number between 0 and 255 (inclusive) by assigning the number 0 to white squares and 1 to black squares and converting the rule's outputs into a binary number, as in the diagram below, where we learn that the rule we're working with is number 86.



$$\begin{array}{cccccccc} 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 2^7\times 0 & +\ 2^6\times 1 & +\ 2^5\times 0 & +\ 2^4\times 1 & +\ 2^3\times 0 & +\ 2^2\times 1 & +\ 2^1\times 1 & +\ 2^0\times 0 \\ 128\times 0 & +\ 64\times 1 & +\ 32\times 0 & +\ 16\times 1 & +\ 8\times 0 & +\ 4\times 1 & +\ 2\times 1 & +\ 1\times 0 \quad = 86 \end{array}$$

Constructing images showing the generations of a CA's operation is a prime example of generative recursion: each successive row is generated from the row before it through a general computation that doesn't follow purely from the structure of the data. In this question you will build up to a Racket function to execute a CA, producing a list of rows where each row follows from the previous one through the application of a given rule.

(a) Write a function *apply-rule* that consumes four natural numbers *a*, *b*, *c*, and *r*. The numbers *a*, *b*, and *c* are either 0 or 1, representing consecutive white and black squares in a row, from left to right. The number *r* is between 0 and 255, and encodes a CA rule as described above. The function should apply the given CA rule and produce either 0 or 1 depending on whether the resulting square should be white or black. Based on the diagram above, we would expect (*apply-rule* 1 1 1 86) ⇒ 0, (*apply-rule* 1 1 0 86) ⇒ 1, and so on.

The best approach is first to convert *a*, *b*, and *c* into a single natural number between 0 and 7, as shown below. (Hint: treat the three numbers as bits in a three-bit binary number).

7 6 5 4 3 2 1 0

If the three cell colours produce some combined number *v*, we then check whether the *v*th bit is set in *r*. Note that in Racket, one way to perform this test would be to check (*odd?* (*floor* (*/ r* (*expt* 2 *v*)))).

(b) Using the function *apply-rule*, write a *next-row* that consumes two values: a list of 0s and 1s of length at least 1, representing a row in a CA, and a rule number between 0 and 255. It produces a new list of 0s and 1s, of the same length as the input list, containing the result of applying the given rule at every position in the original list.

The consumed list does not contain enough information to determine what should happen to the first and last cells. You should assume that all cells that are not explicitly encoded in the list are white.

You may find the functions *append*, *second*, and *third* useful in this question. A reasonable approach is for *next-row* to be a small wrapper around a structurally recursive helper function.

(c) Write a higher-order function *iterate*. It consumes three values: a function f, a base value b, and a natural number n, and produces a list of length n containing $(b, f(b), f(f(b)), f(f(f(b))), \ldots, f^{(n-1)}(b))$, where each $f^{(i)}(b)$ is the result of applying f i times to b. For example, (*iterate sqr* 2 4) would produce the list '(2 4 16 256).

Your function should use explicit recursion, and not use any helper functions defined globally or in a **local**. Each element of the resulting list should be generated *from the one before it* (meaning that your function will use generative recursion). That is, don't generate $f^{(i)}(b)$ by starting from b every time and applying f to it i times; a given call to *iterate* should require only $n-1$ applications of f in total. (Hint: in your recursive call, both n and b should change.)

(d) Write a function *run-automaton*. It consumes three values: an initial non-empty row of 0s and 1s, a rule number, and a number of generations n. It produces a list of n lists, where the first sub-list is the consumed row, and each subsequent row is the result of applying the CA rule to the row before it.

Your function should be a one-liner: aside from the header of the function, it should consist of a single short line of code with no explicit recursion. Do not write any additional helper functions and do not use **local**. You might wish to use **lambda**. You may, of course, use the functions you have written for previous parts of this question.

You can write tests for this function by manually computing a few (short) rows of CA generations for a few different rules. You could also try using an online CA simulator like the one at `http://www.mattlag.com/html5/automata.html` to create tests; try choosing small values for the numbers of columns and iterations, and a larger number (around 25) for the Cell Size.

Place your solution in a file named `ca.rkt`.

Just for fun: It's natural to borrow the drawing library from Assignment 05 to turn the result of *run-automaton* into an actual image. If you want to try that, download the auxiliary file `drawinglib-a09.rkt` into the same directory as `ca.rkt` and add the line (*require* "drawinglib-a09.rkt") at the top of your solution. The file provides a function *draw-ca* that consumes two values: a list of lists, as produced by *run-automaton*, and a natural number giving the side length of a square in the grid. The function produces an image of the grid like the diagrams here (without the grey grid lines). As before, use *draw-ca* for interactive testing, but do not include any applications of that function in your submission.

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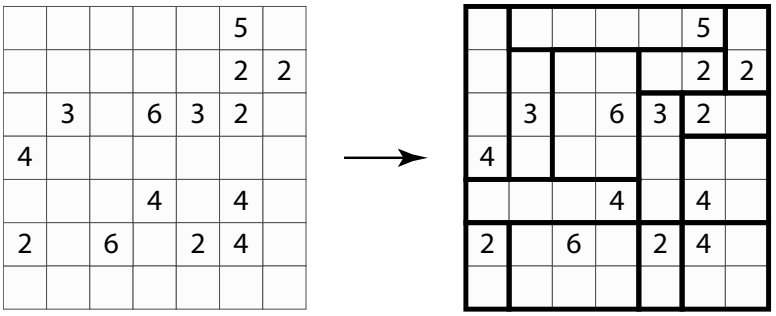
给你列出来了

2. A *rectangle puzzle* is a simple logic puzzle played on a grid of squares, where some of the squares contain positive integers. The goal is to divide the grid into rectangles so that

   - Every square in the grid belongs to exactly one rectangle
   - Every rectangle contains exactly one numbered cell, and the number is equal to the area of the rectangle.

   Below, we see an initial puzzle grid on the left, and the corresponding solution on the right. You may wish you build up your intuition for these puzzles by printing the puzzle on the left, and solving it yourself with a pencil. You can also try Rectangle puzzles online at http://www.chiark.greenend.org.uk/~sgtatham/puzzles/js/rect.html.

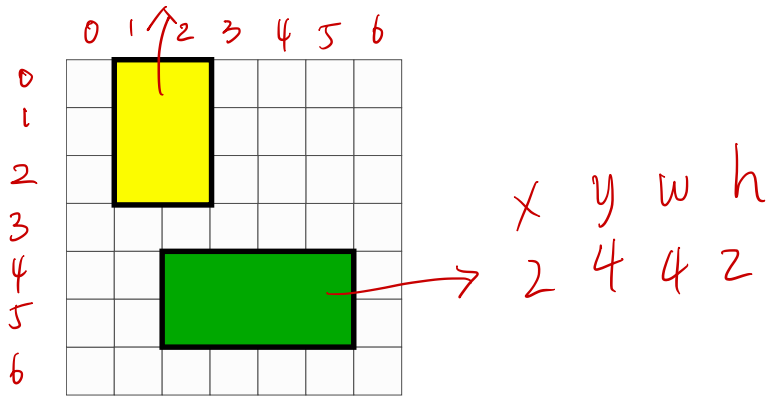


   In this question, you will develop Racket functions that can consume an empty rectangle puzzle and find the list of rectangles that solve it. In order to do so, let us develop a few necessary data definitions to talk about the grid, its cells, and rectangles.

```
(define-struct cell (num used?))
;; A Cell is a (make-cell Nat Bool)
;; A Grid is a (listof (listof Cell))
;; requires: the grid contains a non-empty list of non-empty lists,
;; all the same length.
(define-struct rect (x y w h))
;; A Rect is a (make-rect Nat Nat Nat Nat)
(define-struct state (grid rects))
;; A State is a (make-state Grid (listof Rect))
```

   A *Cell* (not at all related to the cells in the cellular automaton question!) holds information about one grid square: the number written in the square (we use 0 to denote that the square does not have a number in it) and whether that square currently belongs to any rectangle.

   A *Rect* describes a rectangle whose top-left corner is given by the coordinates (x, y), and extends to the right by width w and downward by height h. The top-left corner of the grid is considered to be at $(0, 0)$. For example, in the diagram below, the yellow rectangle is defined by (*make-rect* 1 0 2 3), and the green rectangle by (*make-rect* 2 4 4 2).



To begin, download the files `rectanglelib.rkt` and `rectangle.rkt`. Immediately modify the header in `rectangle.rkt` with your name and student ID. Then write the functions below.

(a) Write a higher-order Racket function *map2d* that behaves like the two-dimensional analogue of *map*. It consumes a function *f* and a list of lists of values, and produces a new list of lists in which *f* has been applied to every element of the input. For example, (*map2d add1* '((3 4 5) (10 9 8))) would produce '((4 5 6) (11 10 9)).

Do not use any explicit recursion or helper functions to write *map2d*. Instead, use a short one-line function body based on two uses of *map* and a single **lambda**.

(b) Write a function *construct-puzzle* that consumes a (*listof* (*listof Nat*)) and produces a *State* representing the initial state of a puzzle. Your function should use *map2d* to create the initial *Grid*, and the initial *State* should have no rectangles in it.

(c) Write a predicate *solved?* that consumes a *State* and produces a boolean value that indicates whether the puzzle described by the state is fully solved. We consider a puzzle to be solved if and only if all of its cells are used. (This isn't ideal, in that the puzzle could be lying to us—perhaps we simply marked all the cells as used without bothering to find any rectangles. But we'll assume that we're honest, and that if we mark a cell as used, it's because we put a rectangle in the state's list of rectangles that uses it.)

(d) Write a function *get-first-unused* that consumes a *Grid* and finds the topmost, leftmost cell in the grid that isn't marked as used. That is, the cell you find should be in the topmost row that contains unused cells, and it should be the leftmost unused cell in that row. The function should produce a (*list Nat Nat*) containing the *x* and *y* coordinates of the first unused cell. You can assume that if this function is being called, the grid contains at least one unused cell.

(e) Write a function *neighbours* that consumes a *State* and produces a list of new states that might legitimately follow from the given state after adding a single new rectangle. If no legal rectangles can be added from the current state, produce the empty list.

To find a state's neighbours, first find the topmost, left unused cell in the grid. Construct all rectangles that fit within the bounds of the grid and that have the unused cell at their top-left corners. Now identify the subset of those rectangles that are legal in

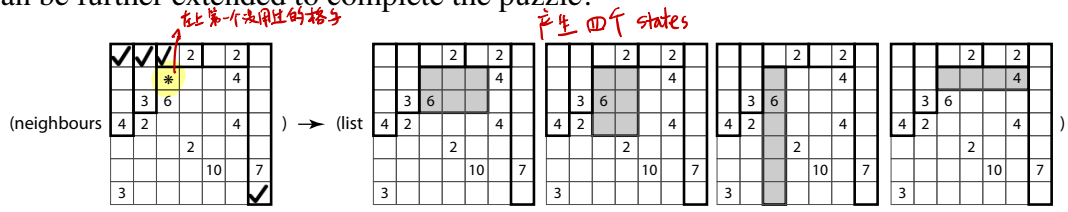
这一问就是这个作业最难的一个问题了，通过当前 state，找到所有下一个可能的 states，注意，我们会用左上开始第一个出现的没用过的格子开始找例有可能，也就是说，你要找的是所有包含左上开始第一个没用过的地方的可能的下一个 state

the puzzle: rectangles that do not contain any already used cells, and that contain a single numbered cell, where the number equals the area of the rectangle. For each such rectangle, construct a new neighbour state in which all cells inside the rectangle have been marked as used, and where the rectangle itself has been added to the list of the state's rectangles (making up a potential partial solution to the puzzle). You can output these neighbour states in any order.

每个产生的可能的 state 还要把对应格子里面 used? 变成 true，同时要加一个新的 rect 到 state 里面

这道题你很难一气做出来，你肯定需要若干个 helpers，这样可以把你的 logic 拆开，容易一步步做，一定要认真分析你需要哪几步，方法有很多但是 local functions 和 local constant 都会对你很有帮助

The diagram below visualizes an example of computing a state's neighbours. The marked cell is the leftmost, topmost unused cell in the source state; the neighbours are the four states on the right with the new rectangles shaded. Only one of those neighbours can be further extended to complete the puzzle!



This step is, by far, the most complicated one in this question. It's likely your solution will depend on a number of helper functions, which you should use liberally, either at the top level or inside a **local** (or both).

(f) Now let's put it all together. Write a function *solve-rectangle-puzzle* that consumes a (*listof* (*listof Nat*)) describing an initial puzzle. It attempts to solve the puzzle, producing either the list of rectangles that describe a solution if one exists, or *false* if no solution can be found.

Your solution must perform a graph search with backtracking on an implicit graph, as suggested in Module 12 of the course notes. In `rectanglelib.rkt` we provide a higher-order function *search* that performs the backtracking search for you; all you have to do is plug in the initial state to search from, and functions to enumerate a state's neighbours and determine whether a given state is a goal state, all of which are solutions to previous sub-questions. This function can therefore be fairly short.

Place your solution in a file named `rectangle.rkt`.

This concludes the list of questions for which you need to submit solutions. Do not forget to always check your email for the basic test results after making a submission.

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3. **5% Bonus**:

   In a classic math puzzle, a father dies leaving behind an estate of 35 camels. His will asks to divide the camels among his three children so that the eldest child gets $1/2$ of the camels, the middle child gets $1/3$, and the youngest gets $1/9$. The children are stuck: they don't want to resort to messy fractional camels!

The town sage devises a solution. She borrows a camel to bring the total to 36, and then distributes $36/2 = 18$, $36/3 = 12$, and $36/9 = 4$ camels to the three children. They receive 34 camels in total; the sage returns the borrowed camel and keeps the final remaining animal as a fee, leaving everyone happy.

Let's ignore the borrowing step, and simply say that 36 camels can be divided into fractions of $1/2$, $1/3$, and $1/9$, with two camels left over. Other combinations of numbers yield similar division problems; for example, 54 camels can be divided into fractions of $1/2$, $1/3$, and $1/9$ with three camels left over ($54/2 + 54/3 + 54/9 + 3 = 54$). We can also extend the problem to any number of children; for example, with five children, 1320 camels can be divided into $1/3$, $1/4$, $1/5$, $1/8$, and $1/11$ with one camel left over.

Write a function *camels* that produces similar division equations. Specifically, the function consumes two *Nat*s: the number of brothers ($k > 0$), and the number of camels that should be left over at the end of the division process ($r > 0$). The function should find *all* sets of natural numbers $t_1, t_2, \ldots, t_k$ and N that satisfy:

$$\frac{N}{t_1} + \frac{N}{t_2} + \ldots + \frac{N}{t_k} + r = N$$

$$t_1 \leq t_2 \leq \ldots \leq t_k$$

$$N \text{ is a multiple of } t_i \text{ for all } i$$

Your function should produce a (*listof* (*listof Nat*)), where each sublist is of the form (*list* t_1 t_2 … t_k N). So, for example, the following might be a valid test:

```
(check-expect (camels 2 1) '((2 3 6) (2 4 4) (3 3 3)))
```

Because $6/2 + 6/3 + 1 = 6$, $4/2 + 4/4 + 1 = 4$, $3/3 + 3/3 + 1 = 3$, and there are no other valid equations of this type.

Submit your solution in the file `bonus.rkt`.

Enhancements: *Reminder—enhancements are for your interest and are not to be handed in.*

Consider the function (*euclid-gcd*) from slide 7-16. Let f_n be the nth Fibonacci number. Show that if $u = f_{n+1}$ and $v = f_n$, then (*euclid-gcd u v*) has depth of recursion n. Conversely, show that if (*euclid-gcd u v*) has depth of recursion n, and $u > v$, then $u \geq f_{n+1}$ and $v \geq f_n$. This shows that in the worst case the Euclidean GCD algorithm has depth of recursion proportional to the logarithm of its smaller input, since f_n is approximately ϕ^n, where ϕ is about 1.618.

You can now write functions which implement the RSA encryption method (since Racket supports unbounded integers). In Math 135 you will see fast modular exponentiation (computing $m^e \bmod t$). For primality testing, you can implement the little Fermat test, which rejects numbers for which

$a^{n-1} \not\equiv 1 \pmod{n}$, but it lets through some composites. If you want to be sure, you can implement the Solovay–Strassen test. If $n-1 = 2^d m$, where m is odd, then we can compute $a^m \pmod{n}$, $a^{2m} \pmod{n}, \ldots, a^{n-1} \pmod{n}$. If this sequence does not contain 1, or if the number which precedes the first 1 in this sequence is not -1, then n is not prime. If n is not prime, this test is guaranteed to work for at least half the numbers $a \in \{1, \ldots, n-1\}$.

Of course, both these tests are probabilistic; you need to choose random a. If you want to run them for a large modulus n, you will have to generate large random integers, and the built-in function *random* only takes arguments up to 4294967087. So there is a bit more work to be done here.

For a real challenge, use Google to find out about the AKS Primality Test, a deterministic polynomial-time algorithm for primality testing, and implement that.

Continuing with the math theme, you can implement the extended Euclidean algorithm: that is, compute integers a, b such that $am + bn = \gcd(m, n)$, and the algorithm implicit in the proof of the Chinese Remainder Theorem: that is, given a list $(a_1, \ldots, a_n)$ of residues and a list $(m_1, \ldots, m_n)$ of relatively coprime moduli ($\gcd(m_i, m_j) = 1$ for $1 \leq i < j \leq n$), find the unique natural number $x < m_1 \cdots m_n$ (if it exists) such that $x \equiv a_i \pmod{m_i}$ for $i = 1, \ldots, n$.