Typos in Logan's *Applied Partial Differential Equations, 3rd ed.* Solutions

David Yang

Contents

1 Chapter 1

1.2 Chapter 1, Section 2

Exercise 7. *Solve the initial boundary value problem*

$$u_t + cu_x = \lambda u, \;\; x, t > 0$$

$$u(x, 0) = 0, \; x > 0, \; u(0, t) = g(t), \; t > 0.$$

Solution. We should find that $\phi(t) = e^{-\lambda t/c} g(-t/c)$ (notice the negative in the exponent). This follows from the fact that if $\lambda(-ct)e^{\lambda t} = g(t)$, then substituting $t = -t/c$ gives us the negative in the exponent. This gives us $u(x, t) = g(t - x/c)e^{-\lambda x/c}$, in $0 \leq x < ct$, which matches the solutions. ■

Exercise 12. *Find a formula that implicitly defines the solution $u = u(x, t)$ of the initial value problem for the reaction-advection equation*

$$u_t + cu_x = -\frac{\alpha u}{\beta + u}, \;\; x \in \mathbb{R}, \; t > 0,$$

$$u(x, 0) = f(x), \;\; x \in \mathbb{R}$$

Here, v, α, β are positive constants. Show from the implicit formula that you can always solve for u in terms of x and t.

Solution. $f(x)$ should be $f(x - ct)$, as we need to change back to $x - t$ coordinates. ■

1.3 Chapter 1, Section 3

Exercise 2. *Let $u = u(x, t)$ satisfy the heat flow model*

$$u_t = ku_{xx}, \;\; 0 < x < l, \; t > 0$$

$$u(0, t) = u(l, t) = 0, \;\; t > 0,$$

$$u(x, 0) = u_0(x), 0 \leq x \leq l.$$

Show that

$$\int_0^l u(x, t)^2 \, dx \leq \int_0 u_0(x)^2 \, dx, \; t \geq 0.$$

Hint: Let $E(t) = \int_0^l u(x, t)^2 \, dx$ and show that $E'(t) \leq 0$. What can be said about $u(x, t)$ if $u_0(x) = 0$?

Solution. We have that $E(t) \leq E(0) = \int_0^l u_0(x)^2 \, dx$. This does not affect the solution, though, as if $u_0 \equiv 0$, then $E(0) = 0$ still.

■

Exercise 6. *Heat flow in a metal rod with a unit internal heat source is governed by the problem*

$$u_t = ku_{xx} + 1, \; 0 < x < 1, \; t > 0,$$

$$u(0, t) = 0, \; u(1, t) = 1, \; t > 0.$$

What will be the steady-state temperature in the bar after a long time? Does it matter that no initial condition is given?

Solution. The answer should be

$$u(x) = -\frac{1}{2k}x^2 + \left(1 + \frac{1}{2k}\right)x.$$

Originally, there is a $=$ sign rather than a $+$. ■

1.5 Chapter 1, Section 5

Exercise 5. *The total energy of the string governed by equation (1.37) with boundary conditions (1.40) is defined by*

$$E(t) = \int_0^\ell \left(\frac{1}{2}\rho_0 u_t^2 + \frac{1}{2}\tau_0 u_x^2\right) dx.$$

Show that the total energy is constant for all $t \geq 0$. Hint: Multiply (1.37) by u_t and note that $(u_t^2)_t = 2u_t u_{tt}$ and $(u_t u_x)_x = u_t u_{xx} + u_{tx} u_x$. Then show that

$$\frac{d}{dt}\int_0^\ell \rho_0 u_t^2\, dx = [2\tau_0 u_t u_x]_0^\ell - \frac{d}{dt}\int_0^\ell \tau_0 u_x^2\, dx.$$

Solution. The hint should include an extra factor of 2 (colored in red). ■

Exercise 8. *At the end ($x = 0$) of a long tube ($x \geq 0$) the density of air changes according to the formula $\tilde{\rho}(0, t) = 1 - \cos 2t$ for $t \geq 0$, and $\tilde{\rho}(0, t) = 0$ for $t < 0$. Find a solution to the wave equation in the domain $x > 0, -\infty < t < \infty$, in the form of a right-traveling wave that satisfies the given boundary condition. Take $c = 1$ and plot the solution surface.*

Solution. We should have that

$$\tilde{\rho}(0, t) = F(-ct) = 1 - \cos(2t).$$

This tells us that

$$F(t) = 1 - \cos(2(t - x/c)).$$

Notice that the factors of 2 are inside the cos term rather than outside (as a coefficient). ■

2 Chapter 2

2.5 Chapter 2, Section 5

Exercise 3. *Using Duhamel's principle, find a formula for the solution to the initial value problem for the convection equation*

$$u_t + cu_x = f(x, t),\ x \in \mathbb{R},\ t > 0;\ \ u(x, 0) = 0,\ x \in \mathbb{R}.$$

Solution. The solution contains no typos, but should be split into two solutions (pertaining to Exercises 3 and 4.) ■

Exercise 4. *Solve the problem*

$$u_t + 2u_x = xe^{-t},\ x \in \mathbb{R},\ t > 0;\ \ u(x, 0) = 0,\ x \in \mathbb{R}.$$

Solution. The given answer of

$$u(x, t) = -(x - 2t)(e^{-t} - 1) - 2te^{-t} + 2(1 - e^{-t})$$

is correct, but may be more clear in its simplest form, which is the result one should get after integration by parts:

$$u(x, t) = (-x - 2)e^{-t} + x - 2t + 2.$$

■

A Completed Problems

The following list consists of problems I have done. The list is non-exhaustive, but contains problems that I did not find typos in.

A.1 Chapter 1

- Section 1.1: 1-11
- Section 1.2: 1-11
- Section 1.3: 2, 3, 4, 6, 9
- Section 1.5: 3, 4, 5, 8, 9
- Section 1.7: 6
- Section 1.9: 1, 2, 5

A.2 Chapter 2

- Section 2.1: 1, 2, 6, 7
- Section 2.2: 2, 3, 4, 6, 7
- Section 2.4: 1-4
- Section 2.5: 1-5