Homework 8

Partial Differential Equations, Spring 2023 — David Yang

HW 8 Problem

Consider the 1st order linear initial value PDE problem:

$$tu_t + u_x = 0 \text{ for } t > 0, x \in \mathbb{R}$$

$$u(x, 0) = f(x) \text{ for } x \in \mathbb{R}.$$

(a) **Apply the Method of Characteristics. Your goal is to find a characteristic value $\xi(x, t)$ so that any function of the form $u(x, t) = f(\xi)$ satisfies the PDE.**

Solution. We begin by applying the Method of Characteristics. We get that

$$t_\tau = t, \;\; x_\tau = 1, \text{ and } U_\tau = 0.$$

Solving for t and x by integrating with respect to τ, we find that

$$t = \xi e^\tau \;\; x = \tau + \xi.$$

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(b) **Set $f(x) = x$ as the initial value for the PDE given above. Show that the form of the solution you found in (a) does not satisfy this initial value.**

Remark: In fact, no solutions exist for this PDE that solve the initial value $u(x, 0) = x$. This PDE is ill-posed for $u(x, 0) = x$.

(c) **Set $f(x) = 1$ as the initial value for the PDE given above. Now let ξ be the characteristic variable you found in (a). For what values of the constants a and b does the function $u(x, t) = a + b\xi$ also solve the PDE and satisfy the initial value $u(x, 0) = 1$?**

Is the PDE with the initial value $f(x) = 1$ *well-posed* or *ill-posed*? Why?