Typos in Logan's *Applied Partial Differential Equations, 3rd ed.* Solutions

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# Contents

# 1 Chapter 1

## 1.2 Chapter 1, Section 2

**Exercise 7.** *Solve the initial boundary value problem*

$$u_t + cu_x = \lambda u,\ \ x, t > 0$$

$$u(x, 0) = 0,\ x > 0,\ u(0, t) = g(t),\ t > 0.$$

*Solution.* We should find that $\phi(t) = e^{-\lambda t/c} g(-t/c)$ (notice the negative in the exponent). This follows from the fact that if $\lambda(-ct)e^{\lambda t} = g(t)$, then substituting $t = -t/c$ gives us the negative in the exponent. This gives us $u(x, t) = g(t - x/c)e^{-\lambda x/c}$, in $0 \le x < ct$, which matches the solutions. ∎

**Exercise 12.** *Find a formula that implicitly defines the solution$u = u(x, t)$ of the initial value problem for the reaction-advection equation*

$$u_t + cu_x = -\frac{\alpha u}{\beta + u},\ \ x \in \mathbb{R},\ t > 0,$$

$$u(x, 0) = f(x),\ \ x \in \mathbb{R}$$

*Here, $v, \alpha, \beta$ are positive constants. Show from the implicit formula that you can always solve for $u$ in terms of $x$ and $t$.*

*Solution.* $f(x)$ should be $f(x - ct)$, as we need to change back to $x - t$ coordinates. ∎

## 1.3 Chapter 1, Section 3

**Exercise 2.** *Let $u = u(x, t)$ satisfy the heat flow model*

$$u_t = ku_{xx},\ \ 0 < x < l,\ t > 0$$

$$u(0, t) = u(l, t) = 0,\ \ t > 0,$$

$$u(x, 0) = u_0(x), 0 \le x \le l.$$

*Show that*

$$\int_0^l u(x, t)^2\, dx \le \int_0 u_0(x)^2\, dx,\ t \ge 0.$$

*Hint: Let $E(t) = \int_0^l u(x, t)^2\, dx$ and show that $E'(t) \le 0$. What can be said about $u(x, t)$ if $u_0(x) = 0$?*

*Solution.* We have that $E(t) \le E(0) = \int_0^l u_0(x)^2\, dx$. This does not affect the solution, though, as if $u_0 \equiv 0$, then $E(0) = 0$ still. ∎

**Exercise 6.** *Heat flow in a metal rod with a unit internal heat source is governed by the problem*

$$u_t = ku_{xx} + 1,\ 0 < x < 1,\ t > 0,$$

$$u(0, t) = 0,\ u(1, t) = 1,\ t > 0.$$

*What will be the steady-state temperature in the bar after a long time? Does it matter that no initial condition is given?*

*Solution.* The answer should be

$$u(x) = -\frac{1}{2k}x^2 + \left(1 + \frac{1}{2k}\right)x.$$

Originally, there is a $=$ sign rather than a $+$. ∎

### 1.5 Chapter 1, Section 5

**Exercise 5.** *The total energy of the string governed by equation (1.37) with boundary conditions (1.40) is defined by*

$$E(t) = \int_0^\ell \left(\frac{1}{2}\rho_0 u_t^2 + \frac{1}{2}\tau_0 u_x^2\right) dx.$$

*Show that the total energy is constant for all $t \geq 0$. Hint: Multiply (1.37) by $u_t$ and note that $(u_t^2)_t = 2u_t u_{tt}$ and $(u_t u_x)_x = u_t u_{xx} + u_{tx} u_x$. Then show that*

$$\frac{d}{dt}\int_0^\ell \rho_0 u_t^2\, dx = [2\tau_0 u_t u_x]_0^\ell - \frac{d}{dt}\int_0^\ell \tau_0 u_x^2\, dx.$$

*Solution.* The hint should include an extra factor of 2 (colored in red). ■

**Exercise 8.** *At the end ($x = 0$) of a long tube ($x \geq 0$) the density of air changes according to the formula $\tilde{\rho}(0,t) = 1 - \cos 2t$ for $t \geq 0$, and $\tilde{\rho}(0,t) = 0$ for $t < 0$. Find a solution to the wave equation in the domain $x > 0, -\infty < t < \infty$, in the form of a right-traveling wave that satisfies the given boundary condition. Take $c = 1$ and plot the solution surface.*

*Solution.* We should have that

$$\tilde{\rho}(0,t) = F(-ct) = 1 - \cos(2t).$$

This tells us that

$$F(t) = 1 - \cos(2(t - x/c)).$$

Notice that the factors of 2 are inside the cos term rather than outside (as a coefficient). ■

## 2 Chapter 2

### 2.5 Chapter 2, Section 5

**Exercise 3.** *Using Duhamel's principle, find a formula for the solution to the initial value problem for the convection equation*

$$u_t + cu_x = f(x,t),\ x \in \mathbb{R},\ t > 0;\ \ u(x,0) = 0,\ x \in \mathbb{R}.$$

*Solution.* The solution contains no typos, but should be split into two solutions (pertaining to Exercises 3 and 4.) ■

**Exercise 4.** *Solve the problem*

$$u_t + 2u_x = xe^{-t},\ x \in \mathbb{R},\ t > 0;\ \ u(x,0) = 0,\ x \in \mathbb{R}.$$

*Solution.* The given answer of

$$u(x,t) = -(x - 2t)(e^{-t} - 1) - 2te^{-t} + 2(1 - e^{-t})$$

is correct, but may be more clear in its simplest form, which is the result one should get after integration by parts:

$$u(x,t) = (-x - 2)e^{-t} + x - 2t + 2.$$

■

# A Completed Problems

The following list consists of problems I have done. The list is non-exhaustive, but contains problems that I did not find typos in.

## A.1 Chapter 1

- Section 1.1: 1-11
- Section 1.2: 1-11
- Section 1.3: 2, 3, 4, 6, 9
- Section 1.5: 3, 4, 5, 8, 9
- Section 1.7: 6

## A.2 Chapter 2

- Section 2.5: 1-5