Typos in Logan's *Applied Partial Differential Equations, 3rd ed* Solutions

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1 Chapter 1

1.2 Chapter 1, Section 2

Exercise 7. *Solve the initial boundary value problem*

$$u_t + cu_x = \lambda u,\ \ x, t > 0$$

$$u(x, 0) = 0,\ x > 0,\ u(0, t) = g(t),\ t > 0.$$

Solution. We should find that $\phi(t) = e^{-\lambda t/c} g(-t/c)$ (notice the negative in the exponent). This follows from the fact that if $\lambda(-ct)e^{\lambda t} = g(t)$, then substituting $t = -t/c$ gives us the negative in the exponent. This gives us $u(x, t) = g(t - x/c)e^{-\lambda x/c}$, in $0 \le x < ct$, which matches the solutions. ■

Exercise 12. *Find a formula that implicitly defines the solutionu* $= u(x, t)$ *of the initial value problem for the reaction-advection equation*

$$u_t + cu_x = -\frac{\alpha u}{\beta + u},\ \ x \in \mathbb{R},\ t > 0,$$

$$u(x, 0) = f(x),\ \ x \in \mathbb{R}$$

Here, v, α, β *are positive constants. Show from the implicit formula that you can always solve for* u *in terms of* x *and* t*.*

Solution. $f(x)$ should be $f(x - ct)$, as we need to change back to $x - t$ coordinates. ■

1.3 Chapter 1, Section 3

Exercise 2. *Let* $u = u(x, t)$ *satisfy the heat flow model*

$$u_t = ku_{xx},\ \ 0 < x < l,\ t > 0$$

$$u(0, t) = u(l, t) = 0,\ \ t > 0,$$

$$u(x, 0) = u_0(x), 0 \le x \le l.$$

Show that

$$\int_0^l u(x, t)^2\, dx \le \int_0 u_0(x)^2\, dx,\ t \ge 0.$$

Hint: Let $E(t) = \int_0^l u(x, t)^2\, dx$ *and show that* $E'(t) \le 0$*. What can be said about* $u(x, t)$ *if* $u_0(x) = 0$*?*

Solution. We have that $E(t) \le E(0) = \int_0^l u_0(x)^2\, dx$. This does not affect the solution, though, as if $u_0 \equiv 0$, then $E(0) = 0$ still.

■

Exercise 6. *Heat flow in a metal rod with a unit internal heat source is governed by the problem*

$$u_t = ku_{xx} + 1,\ 0 < x < 1,\ t > 0,$$

$$u(0,t) = 0,\ u(1,t) = 1,\ t > 0.$$

What will be the steady-state temperature in the bar after a long time? Does it matter that no initial condition is given?

Solution. The answer should be

$$u(x) = -\frac{1}{2k}x^2 + \left(1 + \frac{1}{2k}\right)x.$$

Originally, there is a $=$ sign rather than a $+$. ∎

1.5 Chapter 1, Section 5

Exercise 5. *The total energy of the string governed by equation (1.37) with boundary conditions (1.40) is defined by*

$$E(t) = \int_0^{\ell} \left(\frac{1}{2}\rho_0 u_t^2 + \frac{1}{2}\tau_0 u_x^2\right) dx.$$

Show that the total energy is constant for all $t \geq 0$. Hint: Multiply (1.37) by u_t and note that $(u_t^2)_t = 2u_t u_{tt}$ and $(u_t u_x)_x = u_t u_{xx} + u_{tx} u_x$. Then show that

$$\frac{d}{dt}\int_0^{\ell} \rho_0 u_t^2\, dx = [2\tau_0 u_t u_x]_0^{\ell} - \frac{d}{dt}\int_0^{\ell} \tau_0 u_x^2\, dx.$$

Solution. The hint should include an extra factor of 2 (colored in red). ∎

Exercise 8. *At the end ($x = 0$) of a long tube ($x \geq 0$) the density of air changes according to the formula $\tilde{\rho}(0,t) = 1 - \cos 2t$ for $t \geq 0$, and $\tilde{\rho}(0,t) = 0$ for $t < 0$. Find a solution to the wave equation in the domain $x > 0, -\infty < t < \infty$, in the form of a right-traveling wave that satisfies the given boundary condition. Take $c = 1$ and plot the solution surface.*

Solution. We should have that

$$\tilde{\rho}(0,t) = F(-ct) = 1 - \cos(2t).$$

This tells us that

$$F(t) = 1 - \cos(2(t - x/c)).$$

Notice that the factors of 2 are inside the cos term rather than outside (as a coefficient). ∎