Homework 6

Partial Differential Equations, Spring 2023 David Yang

HW 6 Problems

1. **Find a solution to the diffusion PDE**

$$u_t - u_{xx} = 0 \text{ for } x \in \mathbb{R},\, t > 0$$

with initial value

$$u(x,0) = e^{-x^2/4} \text{ for } x \in \mathbb{R}.$$

2. **Show that your solution to # 1 satisfies the property that, for all $t > 0$,**

$$\int_{-\infty}^{\infty} u(x,t)\,dx = \int_{-\infty}^{\infty} u(x,0)\,dx.$$

In other words, $\int_{-\infty}^{\infty} u(x,t)\,dx$ is a *conserved quantity* (constant with respect to t).

3. (a) **If u solves the diffusion equation on the infinite domain ($x \in \mathbb{R}$), with bounded initial value $u(x,0) = \phi(x)$ that has the property that**

$$\lim_{x\to-\infty} \phi(x) = a \text{ and } \lim_{x\to\infty} \phi(x) = b \quad \text{(a, b constants)}.$$

What is the value of $\lim_{t\to\infty} u(x,t)$?

(b) **Review Eq 2.5 on page 82 of Logan, which is a solution for the PDE**

$$w_t = kw_{xx} \text{ for } x \in \mathbb{R},\, t > 0$$

$$w(x,0) = 0 \text{for } x < 0\,;\; w(x,0) = 1 \text{for } x > 0.$$

What is $\lim_{t\to\infty} w(x,t)$ and does this agree with your result in 3(a)?