Homework 5

Partial Differential Equations, Spring 2023 — David Yang

Logan Chapter 1.7, Problem 6

Consider the PDE

$$u_{xx} - 3u_{xt} - 4u_{tt} = 0 \quad \textbf{for } x \in \mathbb{R} \textbf{ and } t > 0$$

with initial conditions

$$u(x,0) = x^2 \quad \textbf{and} \quad u_t(x,0) = e^x \textbf{ for } x \in \mathbb{R}$$

a) **Calculate the discriminant and classify the PDE as hyperbolic, parabolic, or elliptic.**

Solution. This PDE is a second-order differential equation of the form $Au_{xx} + Bu_{xt} + Cu_{tt} = 0$, with $A = 1$, $B = -3$, and $C = -4$.

The discriminant of the PDE, then, is

$$D = B^2 - 4AC = (-3)^2 - 4(1)(-4) = \boxed{25} > 0.$$

Since this PDE has a positive discriminant, the PDE is $\boxed{\text{hyperbolic}}$. ■

b) **Solve the PDE.**

Solution. ■