Control of Omniwheeled robot through Kinematic Modeling

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ABSTRACT

1 Background

2 Kinematic Model

Constraint on Swedish wheels are described in (1)

$$[\sin(\alpha+\beta+\gamma) - cos(\alpha+\beta+\gamma)(-l)\cos(\beta+\gamma)]\,\mathbf{R}(\theta)\,\dot{\xi}_I - r\dot{\phi}\cos\gamma = 0 \tag{1}$$

where α is the angle of the wheel's center from the positive x axis in the robots frame of reference, β is the mounting angle of the wheel with respect to α, γ is the angle of the rollers with respect to the wheels plane, l is the distance from the robot's center of mass to the center of the wheel, $\mathbf{R}(\theta)$ is a rotation matrix described in (2), $\dot{\xi}_I$ is the state velocities in the global frame described in (3), r is the radius of the wheel, and $\dot{\phi}$ is the angular velocity of the wheel.

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \; \dot{y} \; \dot{\theta} \end{bmatrix}^T \tag{3}$$

Using (1) and the geometry of the robot, the kinematics can be derived. The wheels being used are standard 90 degree Swedish wheels meaning $\gamma = 0$, and since the wheels are mounted radially $\beta = 0$. For each wheel i the constraint simplifies to (4)

$$[\sin(\alpha_i) + l cos(\alpha_i)]\,\mathbf{R}(\theta)\,\dot{\xi}_I - r\dot{\phi} = 0 \tag{4}$$

where $\alpha_i = \frac{2i\pi - \pi}{4}$ for $i = 1..4$. Making the substitution the kinematics of the robot in a global frame are described by (5)

$$\begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \\ \dot{\phi}_4 \end{bmatrix} = \frac{1}{r}\mathbf{B}(\theta)\,\dot{\xi}_I \tag{5}$$

$$\mathbf{B}(\theta) = \frac{\sqrt{2}}{2} \begin{bmatrix} \cos(\theta)+\sin(\theta) & -\cos(\theta)+\sin(\theta) & l \\ \cos(\theta)-\sin(\theta) & \cos(\theta)+\sin(\theta) & l \\ -\cos(\theta)-\sin(\theta) & \cos(\theta)-\sin(\theta) & l \\ -\cos(\theta)+\sin(\theta) & -\cos(\theta)-\sin(\theta) & l \end{bmatrix} \tag{6}$$

In order to develop a controller it is more useful to use the inverse kinematic model described by (7)

$$\dot{\xi}_I = r\mathbf{B}^{-1}(\theta) \begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_3 \\ \dot{\phi}_4 \end{bmatrix} = r\mathbf{B}^{-1}(\theta)\, u \tag{7}$$

where u is the control vector, made of the angular velocities of each wheel. Finally, the inverse kinematic model can be converted into a discrete time form, making it more suitable for control on a discrete system. This takes the form of (**??**).

$$\xi_{i+1} = \xi_i + r\mathbf{B}^{-1}(\theta)_i\, u_{i+1} \cdot T \tag{8}$$

where i denotes the current time step, and T denotes the length of each time step.

References