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Development of a Control Model for a Four Wheel Mecanum Vehicle

In this paper, a refined control model for a Mecanum-wheeled mobile robot is developed and presented. Available control models in literature for Mecanum-wheeled mobile robots are based on a simplification which defines the contact point of the wheel on the ground as the point in the center of the wheel, which does not vary. This limits the smoothness of motion of a mobile robot employing these wheels and impacts the efficiency of locomotion of mobile robots using Mecanum wheels. The control model proposed in this paper accounts for the fact that the contact point in fact changes position down the axle of the wheel as the angle roller moves on the ground. The developed control model is verified with experimental results. Using the refined model, control of Mecanum-wheeled mobile robot is made more predictable and accurate.
[DOI: 10.1115/1.4005273]

Keywords: Mecanum wheel, kinematics, and dynamics modeling

1 Introduction

The Mecanum or Ilon wheel was patented in 1973 [1]. The design was used on utility vehicles such as forklifts and material handling vehicles. Control methods have been derived for this type of vehicle [2]. Mecanum wheeled mobile robots are suitable for applications in congested environments as they allow for greater flexibility in mobility [3]. Modeling of a Mecanum-wheeled mobile robot for the purposes of robotic control has since been investigated in other works [4–8]. These works are based on the simplification that defines the contact point of the wheel with the ground to be positioned on the circumference of the wheel, but in the center of the wheel's width.

When the wheel in motion is examined closely it is observed that the contact point moves from one side of the wheel to the other as the contact point moves down the angled roller. This fact is not mentioned at all in Ref. [5], and while it is mentioned in Ref. [4], it is not added to the mathematical analysis of the problem. A more recent paper [9] pointed this out. The analysis of the model proposed in this paper is different to the one proposed in Ref. [9].

2 Modeling The Platform

A Mecanum wheel is a wheel with a number of rollers arranged around a hub at a predefined angle θ, usually ± 45 deg, to the rotational plane of the wheel as can be seen in Fig. 1.

2.1 Wheel Forces. When a Mecanum wheel rotates there is a force applied down the roller axis, generated by the torque applied by the motor shaft, this force acts at the same angle θ to the wheel plane. The rollers rotate freely about their respective axes as they make contact with the ground. As the Mecanum wheel rotates, its contact point with the ground moves from one side of the wheel to the other due to angled rollers. The direction of motion of the contact point depends on the direction of rotation of the wheel, the mounting of the wheel on the robot frame and the angle θ. For a wheel configuration shown in Fig. 1, anticlockwise rotation will result in the contact point moving from the center of the wheel toward the reader.

In Fig. 2 a local co-ordinate system is defined with two orthogonal axes, x and y. Positive rotation is defined as anticlockwise around the origin. The wheels are made and attached such that the forces produced by the wheels act in the directions of the dotted lines in Fig. 2. These wheel forces can then be split into x and y components and summed to get the Eqs. (1) and (2) below. These two equations define the total force applied to the vehicle, the subscript T is applied to show that these are total forces

$$F_{Tx}i = \sum_{w=1}^{4} F_{xw}i$$

$$F_{Ty}j = \sum_{w=1}^{4} F_{yw}j$$

where **i** and **j** are unit vectors in the x and y directions, respectively and w represents the wheel number. If there is pure translation, i.e., no rotational motion, then the direction of motion, which makes an angle α with the x-axis can be shown to be

$$\alpha = arctan\frac{F_{Ty}}{F_{Tx}} \tag{2.1}$$

If there is rotational motion then moments are taken around the vehicle center at (0,0), which results in an equation for torque, τ, which will result in rotation of the vehicle. The resulting torque can be expressed with the following equation:

$$\tau = (-F_{x1} - F_{x2} + F_{x3} + F_{x4})l_y + \big((F_{y1} - F_{y2} - F_{y3} + F_{y4})\big)l_x \tag{2.2}$$

where l_x and l_y are half the wheel base and track, respectively and are the radii at which the forces are applied by the wheels.

For the dynamic case Newton's Second Law allows the following equations to be applied:

$$\begin{aligned} m\ddot{x} &= F_x - c_x\dot{x} \\ m\ddot{y} &= F_y - c_y\dot{y} \\ I_o\ddot{\phi} &= \tau - c_z\dot{\phi} \end{aligned} \tag{2.4}$$

Contributed by the Dynamic Systems Division of ASME for publication in the Journal of Dynamic Systems, Measurement, and Control. Manuscript received May 17, 2010; final manuscript received July 27, 2011; published online December 2, 2011. Assoc. Editor: Qian Wang.

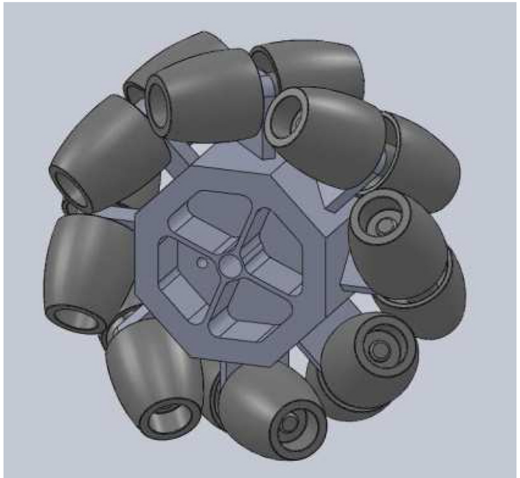

Fig. 1 Mecanum wheel with rollers

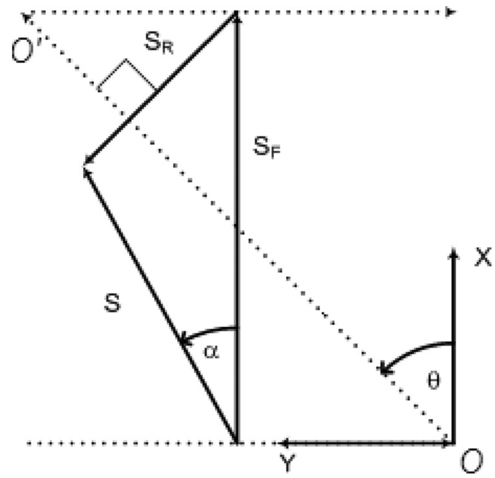

Fig. 3 Wheel displacement vectors

where $c_{x,y,z}$ are the viscous damping factors for motion in the three dimensions, m is the mass of the vehicle and I_o is the moment of inertia of the mobile robot about the (0,0) point. Because this is not a spring system, there is no displacement term and in the steady state case all accelerations are zero

$$\begin{aligned}\dot{\hat{x}} &= \frac{F_x}{c_x} = \frac{F_{x1}+F_{x2}+F_{x3}+F_{x4}}{c_x}\\ \dot{\hat{y}} &= \frac{F_y}{c_y} = \frac{F_{y1}+F_{y2}+F_{y3}+F_{y4}}{c_y}\\ \dot{\hat{\phi}} &= \frac{\tau}{c_z} = \frac{(-F_{x1}-F_{x2}+F_{x3}+F_{x4})l_y+(F_{y1}-F_{y2}-F_{y3}+F_{y4})l_x}{c_z}\end{aligned} \tag{2.5}$$

In the steady state case the velocities are shown with a caret (^) to differentiate them from the case when the vehicle is accelerating. This implies that in the steady state, transition only case, the vehicle velocity and, by implication, the wheels velocities, are proportional to the forces applied by each wheel:

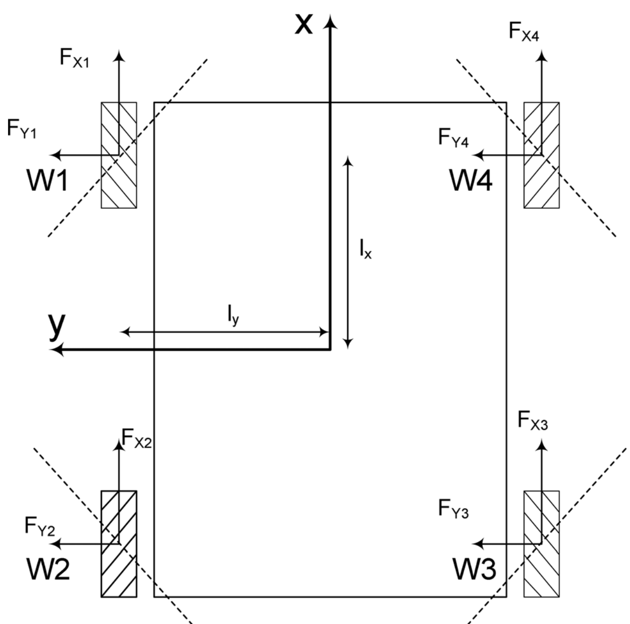

Fig. 2 Forces acting on a Mecanum wheel vehicle

$$V \propto \sum_{w=1}^{4} F_w$$

and so, because this is a rigid body, for translational motion

$$V = V_w \qquad w = 1 \rightarrow 4$$

2.2 Wheel Velocity. Consider Fig. 3, the dotted lines represent the path taken by the contact point of the wheel with the ground when the vehicle is moving straight forward (in the x-axis direction), note that this contact point moves back and forth along the wheel axis. Points O and O' are the points at the inner and outer edge of the roller, respectively (refer to Fig. 2 with respect to the top right hand wheel). The horizontal dotted lines are discontinuities where the contact point transfers from one roller to the next, however, for the purposes of calculation the vector origin is put halfway along this line. The diagonal dotted line is in the same direction as the roller axis.

The vertical solid line represents the displacement vector due only to wheel rotation S_F, in the positive x-direction. The solid line S_R represents the displacement due to rolling, in the direction orthogonal to the roller axis. S represents the resultant displacement of the wheel.

The angle α is the direction of the resultant displacement vector, and is measured from the x-axis, θ is the angle that the roller makes with the x-axis. The above displacement vectors translate to the velocity vectors when differentiated with respect to time, resulting in Fig. 4.

In Fig. 4, ω is the wheel rotational velocity, positive rotation results in a positive velocity vector $\omega r\mathbf{i}$ along the X axis, r is the wheel radius at the point of contact with the ground and $\mathbf{i}$

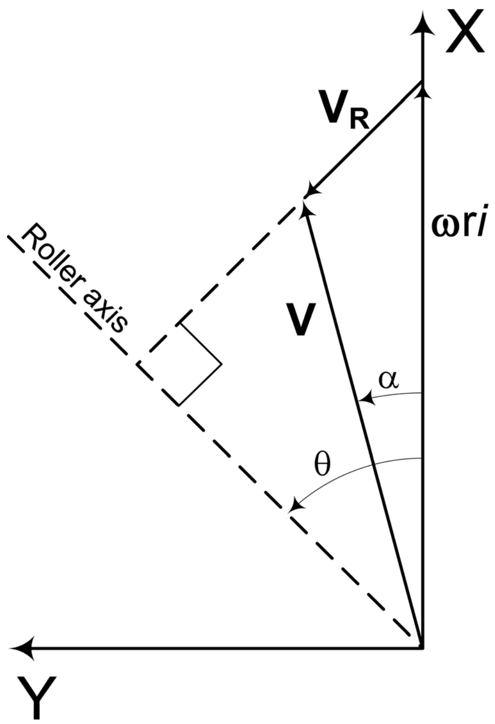

Fig. 4 Wheel velocity vectors

is a unit vector in the X direction. Since the shape of the rollers is such that the overall shape of the wheel is elliptical, the cross section of the wheel is a circle [10]. The radius of the wheel as the contact point with the ground moves from the edge of the wheel to the center varies with respect to the angular rotation of the wheel (δ) by a factor f'. The values of f' are also affected by the roller angle θ and the number of rollers per wheel, n. These two parameters are assumed constant in this paper. The values of this factor, f', are shown graphically in Fig. 5 (assuming no slip of the wheel). The effect of this factor can be averaged and a correction factor f can be used for simplification.

Consider that the components along the roller axis of the $\omega r\mathbf{i}$ and V vectors are equal, hence

$$\omega rf\cos\theta = \mathrm{V}\cos(\theta-\alpha) \tag{1}$$

$$\Rightarrow \omega rf = V\frac{\cos(\theta-\alpha)}{\cos\theta} \quad \theta \neq \frac{\pi}{2}+n\pi; \quad n\in\mathbb{Z} \tag{2}$$

When $=\frac{\pi}{2}+n\pi$, the rotation of the wheel results in no translational motion. When $\theta = n\pi$ the wheels become "standard' 90 deg omni-wheels, but in the configuration of a Mecanum vehicle this will allow only forward motion, omni-directionality is lost. Additionally, any external force in the Y direction will result in motion in that direction as the rollers will allow free motion in those directions.

Equation (6) results in an equation for the magnitude of the velocity in the direction α

$$V = \omega rf\frac{\cos\theta}{\cos(\theta-\alpha)} \quad (\theta-\alpha) \neq \frac{\pi}{2}+n\pi; \quad n\in\mathbb{Z} \tag{3}$$

When $(\theta-\alpha) = \pi/2$, the rotational speed of the wheel must be zero, but any value of translational motion is valid as this is driven by other wheels on the vehicle.

2.3 Wheel Velocities for Translational Motion. The equations for the forces acting on the vehicle and the vehicle velocity can be used to calculate the wheel velocities (ω) required to move the vehicle in any particular direction α at any particular speed V. Assume that there is enough motor torque to overcome friction in the drive train and rolling friction and that the vehicle is in steady state. It is known from Eq. (3) that, $V_w \propto F_T$ and from Eq. (4) that, $V_w = V$, because this is a rigid body. Using Eq. (6) we get an equation for wheel rotational velocity ω_w

$$\omega_w = \frac{V\cos(\theta_w-\alpha)}{r_w f\cos\theta_w} \quad \theta \neq \frac{\pi}{2}+n\pi; \quad n\in\mathbb{Z} \tag{4}$$

2.4 Wheel Velocities for Rotational Motion. It is now possible to define equations for rotation of the vehicle around any point in the plane. In this case the angle α defining the direction of motion of an individual wheel is no longer the same. It varies depending on the position of the center of rotation. For Eq. (8) to be applied an additional calculation must be made to determine α_w, $w = 1 \rightarrow 4$.

At an arbitrary point in time it is desired that the velocity of the origin of the vehicle be $V = iV_x + jV_y$ and the rotational velocity be $\dot{\phi}$ around the resulting instantaneous centre of rotation (ICR) [11]. The radius to the ICR in Fig. 6 is then

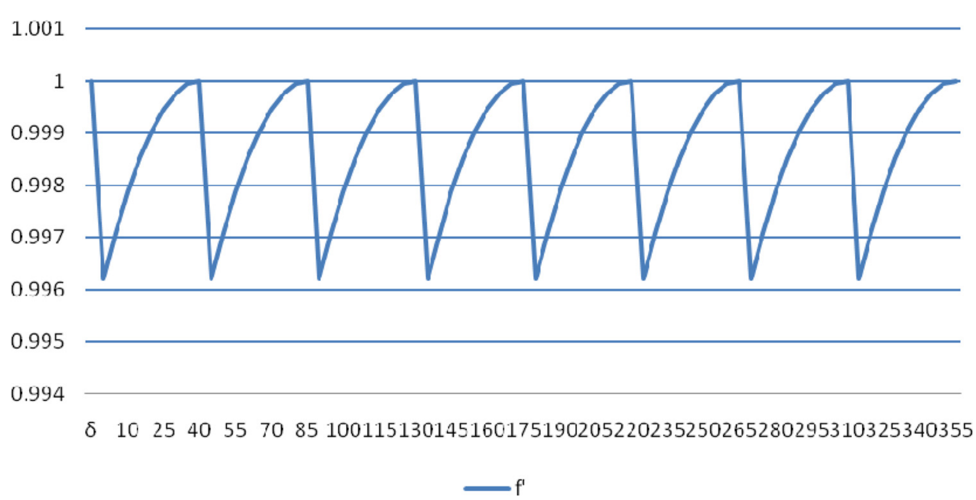

Fig. 5 Factor by which the radius of the wheel changes as the wheel contact point is moved from the edge of the wheel to the center of the wheel

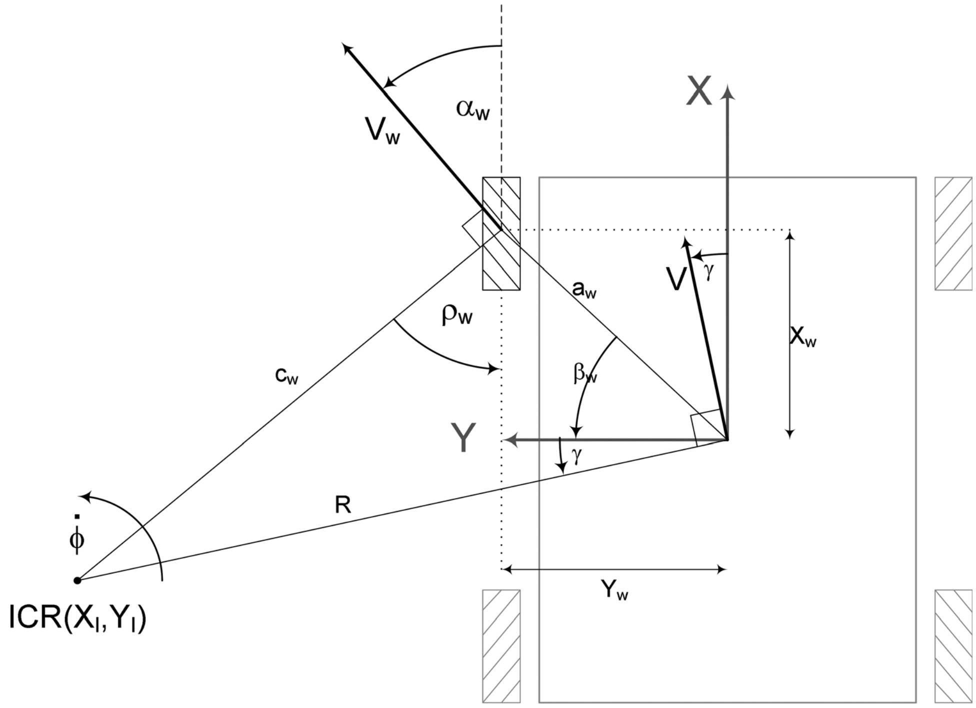

Fig. 6 Vehicle and ICR geometry

$$R = \frac{V}{\dot{\phi}} \tag{5}$$

and

$$\gamma = \mathrm{a}\tan\frac{V_y}{V_x} \tag{6}$$

which allows us to define the position of the ICR

$$X_I = -R\sin\gamma \tag{7}$$

$$Y_I = R\cos\gamma \tag{8}$$

Consider Fig. 6, we again define a local reference frame with its origin at the center of the vehicle. For the purposes of the geometric calculation, we now fix the contact point of the wheel with the ground at the center of the wheel's width.

Each wheel is then defined by its position X_w and Y_w and the orientation of its rollers θ. Let β_w be the angle from the Y-axis to the ground contact point of wheel w, then

$$\beta_w = \arctan\frac{X_w}{Y_w} \tag{9}$$

Let a_w be the distance from the origin to the contact point of wheel w, then

$$a_w = \sqrt{X_w^2 + Y_w^2} \tag{10}$$

In the case of a Mecanum vehicle all a_w should be equal and constant as the geometry of the vehicle is fixed and symmetrical. β_w can be precalculated and will remain constant. Using Eqs. (9), (10), (13), and (14) it is now possible to calculate the distance from the wheel contact point to the ICR. Let c_w be this distance

$$c_w = \sqrt{a_w^2 + R^2 - 2a_w R\cos(\beta_w + \gamma)} \tag{11}$$

The resultant wheel velocity V_w must act perpendicular to the line c_w which makes an angle ρ_w with the X-axis, note that in Fig. 5 ρ_w is negative, which follows since $X_I - X_w$ will be negative

$$\rho_w = \arctan\left(\frac{Y_I - Y_w}{X_I - X_w}\right) \tag{12}$$

$$\Rightarrow \alpha_w = \frac{\pi}{2} + \rho_w \tag{13}$$

Given Eq. (9), the instantaneous resultant velocity of the wheel, V_w, orthogonal to the line c_w, can be shown to have a magnitude of.

$$|V_w| = c_w|\dot{\phi}| \tag{14}$$

Equation (14) can now be substituted in Eq. (4) to calculate ω_w, resulting in

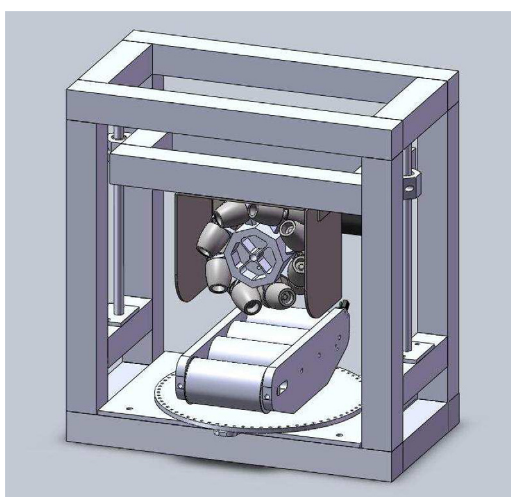

Fig. 7 Mecanum wheel test platform

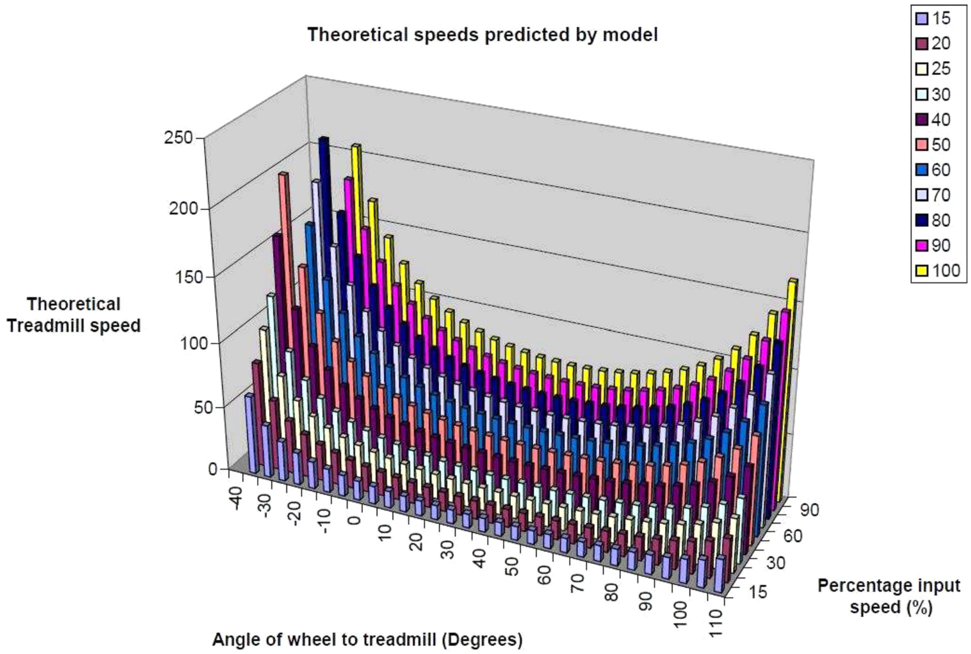

Fig. 8 Theoretical wheel speeds

$$\omega_w = \frac{c_w \dot{\phi} \cos(\theta_w - \alpha_w)}{rf \cos \theta_w} \qquad \theta \neq \frac{\pi}{2} + n\pi; \quad n \in \mathbb{Z} \tag{15}$$

Which is a general equation giving wheel velocity ω_w in terms of the position of the ICR (X_I, Y_I) and rotational velocity around this point, ultimately in terms of velocity V and rotational velocity $\dot{\phi}$. The application of this equation follows a matrix approach similar to [12].

3 Testing the Model

3.1 Single Wheel Tests. A test platform was designed to run tests on a single Mecanum wheel. The wheel motion was constrained on a belted treadmill, which was in turn mounted on a turn table as shown in Fig. 7.

The tester is equipped with quadrature encoders on the wheel and the treadmill to measure relative speeds. Figure 7 shows theoretical vehicle (treadmill) speeds at varying wheel speeds (ω) and angles (α) as calculated by Eq. (7). At -45 deg and 135 deg the predicted vehicle speeds tend towards infinity as the equation reaches an asymptote. This corresponds to when the vehicle is moving at 45 deg and the wheel rotational velocity is zero, but the wheel translational velocity is nonzero. Some of the data points approaching this asymptote have been removed in Fig. 8 making it clearer to view.

Tests were performed at various speeds up to an arbitrary maximum speed determined by the available torque of the motor.

While the results shown in Fig. 9 do not match the predictions exactly there is some correlation. The primary reason for the mismatch is a known problem with the equipment. As the wheel-treadmill angle is changed, the wheel exerts more and more force

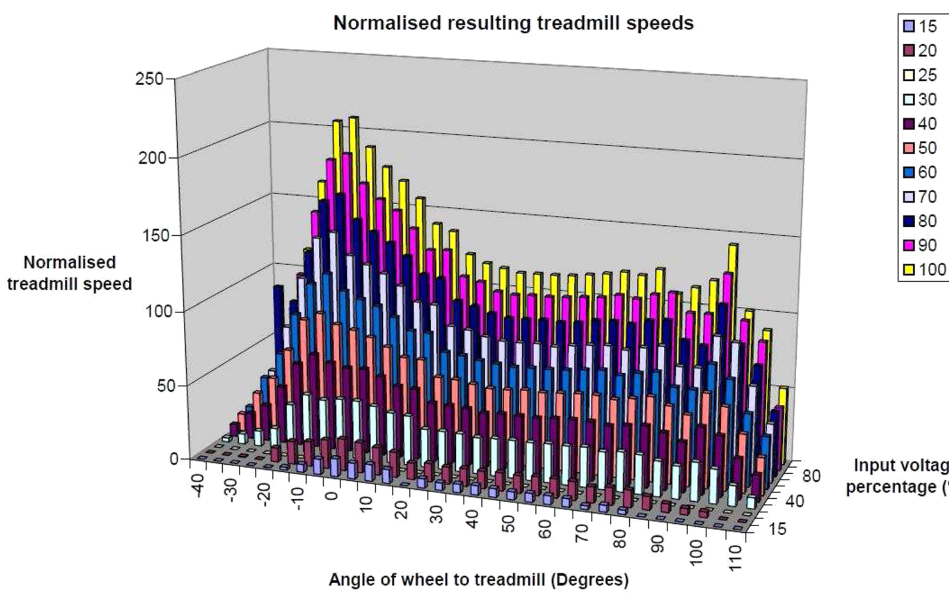

Fig. 9 Test results

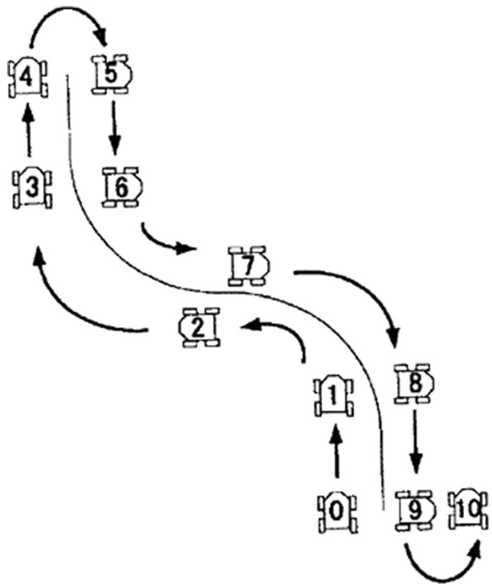

Fig. 10 Benchmark locus

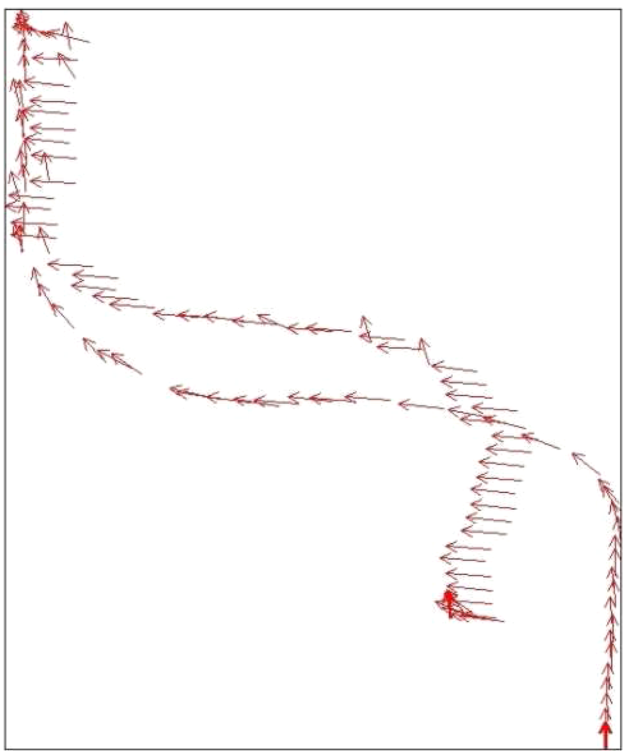

Fig. 11 Laser scanner results

sideways on the belt, this results in the belt rubbing against the end plates of the treadmill. This problem requires modification of the test equipment; where-after more tests will be conducted as part of an ongoing research project.

3.2 Vehicle Tests. A locus described in Ref. [13], shown in Fig. 10 was used to test the application of Eq. (15) on a custom Mecanum vehicle. The test described was performed with no feedback as yet; all wheel motors are controlled with open loop controllers.

Position and direction data was captured using a Sick LMS200 laser scanner. A flat board was mounted longitudinally on the platform to provide directional information. Data were captured from the laser scanner as the platform performed the preprogrammed locus. Figure 11 illustrates the motion of the vehicle as it completes the test locus. The locus begins at the bottom right hand corner of the Fig. 11. The bold arrows in the figure are position and heading markers, the laser scan did not contain directional data so some of the arrows are 180 deg out of alignment with the true direction of the vehicle. All the motion required to demonstrate omni-directionality can be seen. Once feedback control is implemented it should follow the locus more closely.

4 Conclusion

A mathematical model relating the rotational velocity of a Mecanum wheel to its instantaneous velocity has been derived, taking into account that the position of the ground/wheel contact point moves within the width of the wheel as the wheel turns.

Tests were conducted to show that the model works for a single wheel and a vehicle using four wheels.

5 Future Work

More tests will be run on the single wheel tester to eliminate some of the errors seen in the current results.

Feedback control will be implemented on the vehicle to allow it to follow a path more closely.

Acknowledgment

Many thanks to Russ Ether for his valuable criticism of the content, and specifically the equations found, in this paper.

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