

Logistic regression

(a) Given that

$$J(\theta) = \frac{1}{m} \sum_{i=1}^m \log(1 + e^{-y^{(i)} \theta^T x^{(i)}})$$

we can get

$$\frac{\partial J(\theta)}{\partial \theta_i} = \frac{1}{m} \sum_{k=1}^m \frac{-y^{(k)} x_i^{(k)}}{1 + e^{y^{(k)} \theta^T x^{(k)}}}$$

then

$$\frac{\partial J(\theta)}{\partial \theta_i \partial \theta_j} = \frac{1}{m} \sum_{k=1}^m \frac{x_i^{(k)} x_j^{(k)} e^{y^{(k)} \theta^T x^{(k)}}}{(1 + e^{y^{(k)} \theta^T x^{(k)}})^2}$$

which is H_{ij}

so

$$\begin{aligned} Z^T H Z &= \sum_{i=1}^n \sum_{j=1}^n z_i H_{ij} z_j \\ &= \frac{1}{m} \sum_{k=1}^m \frac{\sum_{i=1}^n \sum_{j=1}^n z_i x_i^{(k)} x_j^{(k)} z_j e^{y^{(k)} \theta^T x^{(k)}}}{(1 + e^{y^{(k)} \theta^T x^{(k)}})^2} \end{aligned}$$

known that

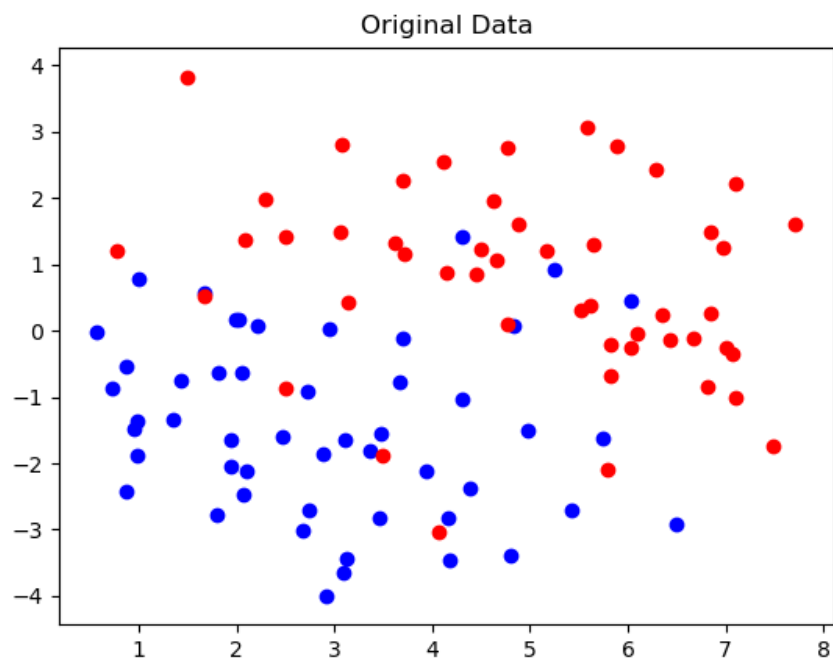
$$\sum_{i=1}^n \sum_{j=1}^n z_i x_i^{(k)} x_j^{(k)} z_j = (X^T Z)^2 \geq 0$$

$$\frac{e^{y^{(k)} \theta^T x^{(k)}}}{(1 + e^{y^{(k)} \theta^T x^{(k)}})^2} > 0$$

we can easily get

$$Z^T H Z \geq 0$$

(b) Firstly, we plot the original data



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