Music 320
Autumn 2011–2012
Homework #3
Complex Sinusoids, Geometric Signal Theory, DFT
150 points
Due in one week (10/20/2011) by 11:59pm

Theory Problems

1. (10 pts) Find the following sum

$$\sum_{k=0}^{N-1} e^{\frac{j2\pi k}{N}}$$

   You may find it helpful to remember

$$\sum_{i=n_1}^{n_2} r^i = \frac{r^{n_1} - r^{(n_2+1)}}{1-r}$$

2. (10pt) [Inner Products] For the complex vectors $x = (1, j, 1-j)$ and $y = (1+j, -1+j, -j)$ in the 3-D complex space $\mathbf{C}^3$, find the inner products
   (a) $\langle x, x \rangle$
   (b) $\langle y, y \rangle$
   (c) $\langle x, y \rangle$
   (d) $\langle y, x \rangle$

3. (20 pts) [Projections] For the complex vectors $x = (1, j, 1-j)$ and $y = (1+j, -1+j, -j)$, find the
   (a) the projection of $y \in C^N$ onto $x \in C^N$, and the coefficient of projection
   (b) the projection of $x \in C^N$ onto $y \in C^N$, and the coefficient of projection

4. (20 pts) [Signal Metrics] For the vectors in three dimensional space $y_1 = (1, 2, -3)$ and $y_2 = (1, j, 1-j)$, find the
   (a) mean μ_y
   (b) total signal energy $\mathcal{E}_y$

(c) average signal power $\mathcal{P}_y$

(d) sample variance σ_y^2

(e) Euclidean (L^2) norm $\|y\|_2$

(f) Chebyshev (L^∞) norm $\|y\|_\infty$

5. (15 pts) [Inner Products and the DFT] Define $x(n) = [j, -1, -j, 1]$ for $n = 0, 1, 2, 3$.

   (a) Find the inner product of $x(n)$ with the complex sinusoid $e^{\frac{j\pi n}{2}}$, or

   $$< x(n), e^{\frac{j\pi n}{2}} >$$

   (b) Without Matlab, compute the 4 point DFT ($N = 4$)of $x(n)$

   $$X(k) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}}$$

   (c) Explain how the previous 2 parts are related to each other.

6. (10 pts) [DFT of an impulse]

   (a) Find the length 16 DFT of the unit-amplitude, zero-centered impulse:

   $$x(n) = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$

   (b) Find the length 16 DFT of the unit-amplitude impulse centered at $n = 3$:

   $$x(n) = [0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$

7. (10 pts) [DFT of a pair of impulses]

   (a) Find the length 8 DFT of a pair of symmetric unit-amplitude impulses:

   $$x(n) = [0, 0, 1, 0, 0, 0, 1, 0]$$

   (b) Find the length 8 DFT of a pair of anti-symmetric unit-amplitude impulses:

   $$x(n) = [0, 0, 1, 0, 0, 0, -1, 0]$$

Lab Assignments

For all lab assignments, submit your M-file scripts, functions, and figures in one zip file through coursework[1]. Within coursework, upload the zip file using the Drop Box menu.

[1]http://coursework.stanford.edu

The zip file should be named with your last name, first name and homework number. Each problem should be named with your last name, first name, homework number, and the problem number. So, for John Doe's zip file, the file should be titled `doe_john_hw2.zip`. For John Doe's answer to problem 2 on homework 2, the file would be titled `doe_john_hw2_q2.m`. Also, at the beginning of each script, include the following comment:

```
% Your Name / Lab # - Question #
```

For problems with question(s), include your answer(s) in the body of the script files as comments.

1. (30 pts) [Additive synthesis] Using your `additive` function (from previous homework), try to produce the following sounds:

   (a) Square wave (`square.wav`)

   (b) Triangle wave (`triangle.wav`)

   (c) Sawtooth wave (`sawtooth.wav`)

   Write a script which

   (a) prepares parameters (`f` and `Z`) for each sound,

   (b) utilizes your additive function to save the results as wave files (with names given above), and

   (c) plots every waveform for the first five periods.

   Please note:

   (a) Your sounds should be 2 seconds long.

   (b) Use 441 `[Hz]` for f or fundamental frequency and 44100 `[Hz]` for your sampling frequency f_s.

   (c) Make sure your function works fast enough, and your sounds are not clipped. If necessary, modify your function.

   (d) Verify your result with plot. Put the three waveforms in one figure, and name them appropriately.

   (e) Submit your script and function (no plots). Include your name, lab and problem number. For each sound, how many partials do you need to obtain a faithful production of the waveform? How do they sound? Also, describe the characteristics of your frequency components.

2. (25 pts) Your task for this problem is to write a Matlab function to test the beating effect when adding two sinusoids. In this case, one of the sinusoidal signals will have a fixed frequency f_c, while the other is a "chirp"—that is, its frequency sweeps linearly from $f_c + f_1$ to $f_c + f_2$. The sum can be expressed as

$$x(t) = A\cos(2\pi f_c t) + B\cos(2\pi(f_c + f_\Delta(t))t).$$

where f_Δ sweeps linearly from frequency f_1 to f_2. We will keep f_1 and f_2 much smaller than f_c, so that both sinusoids are always close together in frequency.

There are generally four separate sounding cases when considering the addition of two sinusoids:

(a) The two sinusoids are of the same frequency and sound identical

(b) The two frequencies are very close and beating occurs

(c) The two frequencies are farther apart and create a rough sounding tone, but separate pitches are not yet distinguishable

(d) The two frequencies are far apart from one another and sound as two distinct tones

Write a Matlab function that implements this synthesis method and saves the result as an audio file. The syntax of your function should be:

```
function y_beat = beat_sweep(A, B, fc, f_delt, fs, dur, name)

% function y_beat = beat_sweep(A, B, fc, f_delt, fs, dur, name)
%
% A: amplitude of the center frequency cosine
% B: amplitude of the sweeping frequency cosine
% fc: center frequency in Hz
% f_delt: [f1 f2] vector for frequency sweeping
% fs: sampling rate in Hz
% dur: total duration of the signal in seconds
% name: name of the output audio file
% Your Name / Lab#-Q#
```

Test your function with f_c= 100Hz, 300Hz, 1000Hz and 3000Hz. For each of these frequencies,

(a) With $f_\Delta = [0, f_2]$, find the value of f_2 that is approximately just after the roughness zone.

(b) With $f_\Delta = [f_1, f_1]$, select f_1 so as you are hearing the transition from beats to roughness. Plot a few periods of your signal that let you clearly see these beats (or submit a script that creates the plot using your function).

Comment on your results.