Music 421A
Winter 2011-2012
Homework #1
Introduction to Windows and Spectral Analysis
31 points
Due in one week (Thursday, January 19, 2012, at 5:00pm)

Theory Problems

1. (5 pts) Give the expression of a rectangular window which, when convolved with itself, gives the length-M triangular window defined as (M can be assumed to be odd)

$$w_\Lambda(n) = \begin{cases} 1 - \frac{|n|}{(M-1)/2}, & -\frac{(M-1)}{2} \leq n \leq \frac{M-1}{2} \\ 0, & \text{otherwise.} \end{cases}$$

   Note that, under this definition, the triangular window includes zeros on each end of the window (*i.e.*, there are only $M-2$ nonzero samples in the window). See the Bartlett window section[1] of the text for further discussion. Windows are normally defined to include only non-zero values, so that $M-1$ in the above definition would normally be $M+1$.

2. (10 points)(1 pt per item part or subpart) Consider the window transform of the length-M triangular window. Find the following in terms of M. [Hint: remember that convolution in the time domain corresponds to pointwise multiplication in the frequency-domain, and vice versa.]

   (a) Main-lobe width in normalized radian frequency (radians per sample) [Normalized radian frequency ωT ranges from $-\pi$ to π).]

   (b) Main-lobe width in Hertz (Hz) when the sampling rate is $f_s = 44100$ (Frequency f in Hz ranges from $-f_s/2$ to $f_s/2$.)

   (c) Side-lobe width in normalized radian frequency

   (d) Sidelobe width in Hz for $f_s = 44100$ Hz

   (e) First sidelobe level, in dB AND in linear amplitude relative to peak = 1. [Hint: You may assume M is large.]

   (f) Roll-off rate, in dB per octave. [Hint: You may assume M is large.]

   (g) The minimum frequency difference for two sinusoids to be resolvable by the window. Give your answer in normalized radian frequency (radians per sample), radian frequency (radians per second), and Hz (cycles per second), always assuming $f_s = 44100$.

3. (7 points) What is the length of the triangular window which can resolve two sinusoids whose frequencies are ω_1 and ω_2 respectively?

   (a) (5 pts) Give your answer in seconds, in terms of ω_1 and ω_2.

   (b) (2 pts) Give your answer in samples, assuming $\omega_1 = 2\pi 440$ rad/s, and $\omega_2 = 2\pi 490$ rad/s, and the sampling rate is 44100 Hz (CD quality).

[1] `http://ccrma.stanford.edu/~jos/sasp/Matlab_Bartlett_Window.html`

Lab Assignments

Lab assignments are due 24 hours after the theory problems (typically Friday at 5:00pm):

1. (4 pts) Using MATLAB, plot the magnitude and phase response of the triangular window given in problem 1, using the below code. Note: you can copy and paste this code from this PDF.

```
M=15;
Nfft = 1024;
n=-(M-1)/2:(M-1)/2;
wtrian = 1-abs(n)*2/(M-1);
nzeros = (Nfft-length(wtrian)-1)/2;
wtrian = fftshift([zeros(1,nzeros+1) wtrian zeros(1,nzeros)]);
Wtr=fftshift(fft(wtrian));
figure;
title('Magnitude and phase response of triangular window-M=15');
subplot(211);
plot([0:2*pi/Nfft:2*pi*(1-1/Nfft)]-pi,20*log10(abs(Wtr)/max(abs(Wtr))));
xlabel('Normalized frequency');
ylabel('Magnitude (dB)');
axis([-pi pi ylim]);grid;
subplot(212);
plot([0:M/Nfft:M*(1-1/Nfft)]-M/2,angle(Wtr));
xlabel('Normalized frequency');
ylabel('Phase');axis([-pi pi ylim]);
grid;
```

   Once you have printed out the plots using the code, triple label the frequency axis by hand in the following units. You should label the minimum frequency, the maximum frequency, and the frequencies corresponding to the first zero crossing of the window transform magnitude.

   (a) radians per sample
   (b) radians per second
   (c) Hz (cycles per second)

   Note: Don't worry about the units MATLAB automatically gives you (which are the numeric indices of the frequency samples). We will address that when we get to zero padding.

2. (5 pts) Using MATLAB, execute the following code:

```
N = 1024; % window length
w = (.42 - .5*cos(2*pi*(0:N-1)/(N-1)) + .08*cos(4*pi*(0:N-1)/(N-1)))';
% Blackman window
```

A Blackman window is now stored in w. Then write matlab code to do the following:

(a) Create a 1000 Hz sinusoid (cosine) of the same duration N samples, assuming a sampling rate of 8192 Hz. Give it amplitude of 0.6 and a phase 0.25*2*π radians.

(b) Window the sinusoid with the Blackman window. (Implement via Time Domain multiplication.)

(c) In separate subplots, plot the window, the sinusoid, and the windowed sinusoid.

(d) Plot the log-mag-spectrum, normalizing the peak to 0 dB. Use the fft() function in matlab with zero padding (set a larger number than N as an argument of the matlab function, e.g. fft(x,4*N)).

(e) Find the worst-case side-lobe level graphically.

(f) (Bonus) Change the window function to

```
w = boxcar(N);
```

which is a convenient way of making an N sample rectangle window. Do the log-mag-spectrum plot again for a 1024 Hz sinusoid and also a 1023 Hz sinusoid. These two new plots look very different from each other. Why? We will cover this soon...