Music 320
Autumn 2011–2012
Homework #8
Z-transform, FIR & IIR Filters
135 points
Due in two weeks (12/01/2011) by 11:59pm

Theory Problems

1. (20 points) Consider the filter

$$y(n) = x(n) - x(n-1)$$

which is identical to the simplest low-pass filter except that adjacent input samples are subtracted rather than added. Derive the amplitude response and the phase response. How has the response changed? Would you call this a low-pass filter, high-pass filter, or something else? In the time domain, we may call it a *first-order difference*.

2. (10 points) For the two input sequences

$$x_1(n) = [1, 1, 1, 1, 1, 1, 1, 1]$$

and

$$x_2(n) = [1, -1, 1, -1, 1, -1, 1, -1]$$

find the output $y(n)$ using the first-order difference filter given in the previous problem. How would you relate your answers to the results you got in the previous problem?

3. For the following filter:

$$H(z) = \frac{6z^2 - 6z}{1 - 5z + 6z^2}$$

   (a) (5 pts) Draw the direct-form-II realization.
   (b) (5 pts) Draw the transposed direct-form-II realization.
   (c) (5 pts) Find the partial fraction expansion.
   (d) (5 pts) Draw a realization as parallel one-pole sections.

4. (30 points) [Partial Fraction Expansion] Express the following transfer functions as a sum of one pole filters using *partial fraction expansion* (PFE):

(a)

$$H_1(z) = \frac{-2}{1 - \frac{4}{3}z^{-1} + \frac{1}{3}z^{-2}}$$

(b)

$$H_2(z) = \frac{4 - \frac{7}{2}z^{-1}}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}}$$

(c)

$$H_3(z) = \frac{1 - j2 + (\frac{7}{4} - j\frac{9}{4})z^{-1}}{1 + (\frac{3}{4} - j)z^{-1} - j\frac{3}{4}z^{-2}}$$

5. (15 points) [Inverse z Transform] Give the impulse response of each of the filters in the previous problem by inverting the z transform. (*Hint*: The z transform is linear, so you do this by inverting the one pole filters you found with PFE)

Lab Assignments

Follow the same file naming convention of the previous lab.

1. (20 points) Simple FIR Digital Filter Design

   (a) (5 points) Use the Matlab function `fir1` to design a 10th order FIR lowpass filter that cuts off at one-fourth the sampling rate. Plot the impulse response.

   (b) (5 points) Use `freqz` to display the amplitude and phase response of this filter.

   (c) (10 points) Generate 4096 samples of a white noise signal using `randn` and apply the FIR filter to it. With your sound volume TURNED WAY DOWN (at first), listen to the input and output signals. Plot the magnitude of a length 8192 FFT of the input and output signals.

   Turn in your Matlab code.

2. (20 points) [Convolution measurement] A second-order IIR filter is given like below

$$y(n) = 0.3024\,x(n) - 0.3024\,x(n-2) + 1.749\,y(n-1) - 0.9244\,y(n-2)$$

   Using the `jobs.wav` sound file (downloaded with the pdf), write a script which applies the given filter to the `jobs.wav` in the following ways:

(a) directly using the difference equation above

(b) using `conv`

(c) using `filter`

(d) using `fftfilt`

Measure the run time of each operation using `tic` and `toc` function. The usage of `tic` and `toc` function is as follows.

```
tic
...perform your calculation...
toc // returns an elapsed time.
```

Compare the performances of these operations when $N = 128$, 1024, and 8092. Turn in your code, and the run time results for each N.

Note that you need a finite impulse response `h` to calculate them in (a), (b) and (d), which is contrary to the given filter. But, you can get an approximated finite impulse response like this.

```
h = filter(b,a,[1; zeros(N-1,1)])
```

Use the impulse response `h` or filter coefficients `a` and `b` as arguments of Matlab functions above. Also, make sure that the filtered outputs are the same by listening to them.