Music 421A
Winter 2011-2012
Homework #4
FIR Filter Design, Resolving Spectral Peaks
Due in one week

Theory Problems

1. (5 pts) Derive the ideal impulse response corresponding to the desired amplitude response
$$H(e^{j\omega T}) = \begin{cases} 1, & |\omega| \le \omega_c \\ 0, & \omega_c \le |\omega| \le \pi/T \end{cases} .$$

2. (5 pts) Derive the ideal impulse response corresponding to the desired amplitude response
$$H(e^{j\omega T}) = \begin{cases} 1, & 0 \le \omega_1 \le |\omega| \le \omega_2 \le \pi/T \\ 0, & \text{otherwise} \end{cases}$$
[Hint: Use a Fourier theorem to make use of the answer for the ideal lowpass filter.]

3. (5 pts) When designing an FIR bandpass filter, explain the benefit of choosing the lower cut-off frequency f_1 to be equal to the difference between the Nyquist limit $f_s/2$ and the upper cut-off frequency f_2. In other words, what is the benefit of the constraint $f_1 = f_s/2 - f_2$? (Prove that the claimed benefit is obtained in general for any $f_1 \in (0, f_s/4)$.)

4. (5 pts) Figure 1 shows the impulse response and the corresponding magnitude spectrum of a lowpass filter.

   (a) (3 pts) Without using MATLAB, sketch the magnitude spectrum of the impulse response shown in Figure 2, which was obtained from that in Fig. 1 by negating the odd-numbered samples. Explain how you obtained your answer.

   (b) (2 pts) Verify your answer by plotting the magnitude spectrum of the above impulse response using MATLAB. You can download a `.mat` file `ir.mat`[1] which contains an impulse response vector $h(n)$ shown in Fig. 2.

5. (5 pts) What length Blackman window is required to resolve a sinusoid at 100 Hz and another one at 101 Hz? State your definition of resolution in this context, and draw a sketch showing the two sinusoids and the window transform in the frequency domain, with the window transform being centered on one of the sinusoidal frequencies.

[1] `https://ccrma.stanford.edu/~jos/sasp/hw/ir.mat`

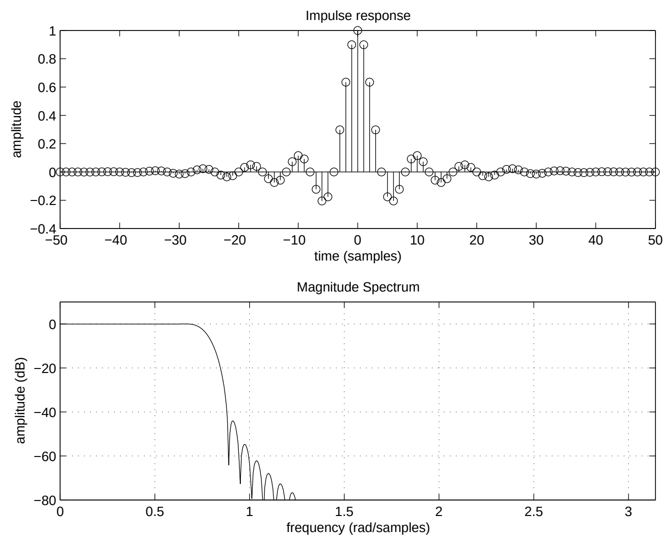

Figure 1: Impulse response of a lowpass filter and its magnitude spectrum

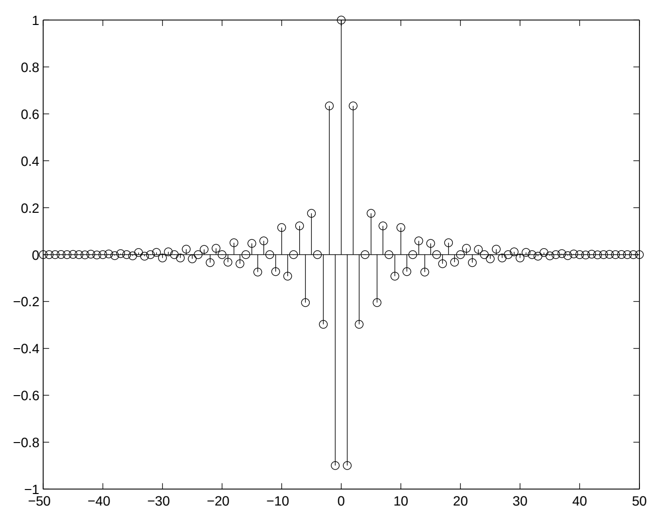

Figure 2: Impulse response

Lab Assignments

1. (5 pts) Design a real, linear-phase, FIR bandpass filter using `firpm()` in Matlab with the following specifications: Sampling rate $f_s = 100$ Hz, pass-band from 20 Hz to 30 Hz, stop-band from 0 to 10 Hz and 40 to 50 Hz, $\delta_s = 0.01$ (-40 dB) ripple in the stop-band, and $\delta_p = 0.02$ ripple in the pass-band, which is unity gain. The filter thus has transition bands from 10 to 20 Hz, and from 30 to 40 Hz. Turn in a listing of your Matlab code, and the result of its execution (e.g., using the `diary` command), which should include a print-out of the filter length, a listing of the filter coefficients, and a plot of the filter amplitude response on a dB vertical scale. [Hint: Start with '`help firpmord`' in Matlab.]

2. (13 pts) Download the sound file `noisypeaches.wav`[2] containing speech embedded in white noise.

   (a) (5 pts) Plot the spectrogram of `noisypeaches.wav` to help you understand its spectral content but there is no need to submit it. Design a low pass filter using the window method with a Kaiser window of length 100 and β=10. The cut-off frequency of the filter should be 4 kHz. Plot its impulse response and magnitude of frequency response.

   (b) (3 pts) Apply this filter to the noisy speech signal either by the FFT method of simple filtering. Listen and describe the result compared to the original.

   (c) (5 pts) Now downsample the original noisy speech signal by a factor of two by simply throwing away every other sample. Listen to the result and compare it to the original higher sampling rate. Repeat the same downsampling scheme on the lowpass filtered speech signal and again, compare with its higher sampling rate version. Why does the latter pair (lowpass filtered) sound more similar than the first pair (unfiltered)?

3. (10 pts)

   In this problem, you will compare the ability of the Hann (a.k.a. "Raised Cosine") window to that of the Hamming window to resolve two sinusoids of significantly different amplitudes[3] and with "significantly different" frequencies[4]. To that end, write a matlab script to perform the following:

   (a) (2 pts) Create a 64-sample sum of two cosines, the first with unity amplitude and normalized frequency 1/8 (cycles per sample), and the second with amplitude 0.001 and normalized frequency 3/8 (cycles per sample).

[2] `https://ccrma.stanford.edu/~jos/sasp/hw/noisypeaches.wav`

[3] Here we mean different by a few orders of magnitude.

[4] Here by "significantly different" we mean spaced by more than a few side-lobe widths.

(b) (3 pts) Window this signal with a 64-sample Hann window (created using the matlab function `Hann()`)[5], and compute the resulting spectrum. For this problem, you may compute the spectrum of a signal using the matlab `freqz()` function, by setting $b = x$ (where x is your input signal vector), and $a = 1$. Finally, plot the magnitude spectrum in dB on a gridded figure with normalized frequency (in cycles per sample) along the x-axis.

(c) (3 pts) Repeat the above procedure for a 64-sample Hamming window, overlaying the spectrum with that of the Hann window.

(d) (2 pts) Which of the windows does a better job of resolving the two sinusoids? What drawback does this window have versus the other in regard to side-lobe levels?

[5]Throughout this problem, always normalize the window to read 0 dB at dc