Attack and Release Time Constants in RMS-Based Feedback Compressors*

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Mathematical models for feedback and feedforward compressors are developed. A couple of possible configurations are explored: linear-output rms detector with linearly controlled VCA (linear-domain compressor) and logarithmic-output rms detector with exponentially controlled VCA (log-domain compressor). It is shown that the transfer functions of both configurations are equivalent. A formula for transforming the compression ratio of a log-domain compressor to that of a linear-domain compressor is derived. The differences between feedforward and feedback compressor configurations, with regard to time constants and performance, are considered.

0 INTRODUCTION

The core of any compressor consists of two elements, a level detector and an amplifier with variable gain. The output of the level detector is a direct current or voltage that is a representation of the ac input signal. The detector could be a peak detector, an average detector, or a true root-mean-square (rms) detector. The peak detector is usually low cost and can be fast. The signal detected is the maximum level of the signal. The average detector computes the mean level of the signal. It is rather inexpensive and the time constants are comparable to those of the rms detector. The true rms detector is the only detector that directly relates to the power of the signal, independent of the signal waveform. The rms value of an alternating voltage or current is the equivalent direct voltage or current that generates the same amount of real power in a resistive load. If the shape of the incoming signal is known (for example, a sine wave or a square wave), then either the peak or the average detector can be used to calculate the rms level of the signal. Unforunately the waveform of a music source cannot be predicted. This is the reason why peak and average detectors are not necessarily appropriate for audio applications. Any of these detectors could be implemented in the digital domain as well. For instance, the ac signal can be digitized by an analog-to-digital converter (ADC), and the computation takes place in a digital signal processing (DSP) unit or microcontroller. Another way is to digitize the dc output of a true rms detector and use the microcontroller to control the compression ratio. The compressors considered for this paper are based on true rms detectors.

The variable-gain amplifier is usually a three-port device—input, output, and gain control. Any of the ports could be voltage or current connections. The variable-gain amplifier could be a voltage divider with variable shunt impedance (bipolar or JFET transistor, optoresistor), and operational transconductance amplifier (OTA), a voltage-controlled amplifier (VCA), a digitally controlled attenuator, or a multiplier in a DSP. The voltage divider and shunt impedance solution has the drawbacks of high distortion, limited dynamic range, and unpredictable transfer function. The OTA has the advantage of a defined transfer function and a rather better dynamic range. Yet the distortion performance of these devices is not appropriate for professional audio applications [1]. A digitally controlled attenuator is more difficult to implement since it needs a digitized value of the rms level [2]. Another problem is that the attenuation steps are often large and the compressor cannot be controlled between steps. Digitally controlled attenuators are also prone to zipper noise. The DSP-based variable-gain amplifier is essentially a multiplier, and it makes sense in the context of a DSP-based compressor. In this case the variable-gain element is linearly controlled. VCAs are the highest performance variable-gain amplifiers [3], [4]. Recent developments in VCAs [5] show the commitment of the IC manufacturers to improving this popular device. The most popular VCAs have exponential gain control. The advantage is that the exponential transfer

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function is linear in decibels. In order to use these VCAs in compressor applications, the detector should preferably have a logarithmic output. In other words, the dc output of the detector is a logarithmic representation of the ac input signal.

1 RMS DETECTOR

The theory of the rms detector was described previously [6]. The rms value V_{rms} of an input signal $v_{in}(t)$ is defined as

$$V_{rms} = \lim_{T\to\infty} \sqrt{\frac{1}{T}\int_{-\infty}^{T} v_{in}^2(t) \cdot dt}\,. \tag{1}$$

As stated, the rms detector can have a linear or a logarithmic output. A block diagram of each detector is shown in [6]. In a linear output detector, the input signal is squared, then integrated over a finite time, and eventually the square root is extracted. The problem with this approach is that the squaring operation needs a wide dynamic range. For instance, to detect professional audio signals that could be as high as +24 dBu over 80 dB dynamic range, the peak output voltage of the square block swings between 3.03 μV and 303 V. The dynamic range required at the output of the square block is 160 dB. In the digital domain this dynamic range translates into 27-bit words. One way around this limitation is to compress the signal going into the linear rms detector. Then the output of the linear rms detector is multiplied by the compression factor. The integrator is a first-order low-pass filter. The time constant of the integrator determines the ripple as well as the transient response at the detector output.

The logarithmic rms detector is more forgiving in terms of the dynamic range of voltage required. The input signal needs to be rectified because the logarithmic function is defined only for positive values. The next step is to take the logarithm of the rectified signal. The squaring operation is just a multiplication by a factor of 2. Thus the dynamic range required at the output of the square block is reduced to 25.3 dB. The signal is integrated by a first-order log filter as described in [7]. A limiting factor can be the amount of current available for the log integrator. The current in the log filter diode is proportional to the square of the input current. Thus the rms detector based on the log filter requires a wide dynamic range in current instead of voltage.

2 COMPRESSOR TOPOLOGIES

2.1 Linear-Domain Compressor

2.1.1 Feedforward

The block diagram of a linear-domain feedforward compressor is shown in Fig. 1. The rms detector and the VCA are linear devices. The input of the rms detector is prescaled to a reference level V_r. This voltage is also called "reference level." At reference level, the output of the linear rms detector is 1. The voltage at the gain control port G_c is applied to an internal math block. The output voltage is the product of the input voltage multiplied by the gain control voltage. In the case of a linear VCA, used in a compressor application, the transfer function is

$$f(G_c) = \frac{1}{G_c}\,. \tag{2}$$

The VCA output and input voltages have the same polarity.

The output of the linear rms detector is calculated in [6] as

$$V_{rms}(t) = \left\{\int \frac{1}{\tau}\left[\frac{V_{in}(t)}{V_r}\right]^2 \exp\left(\frac{t}{\tau}\right) dt + c\right\}^{1/2} \exp\left(-\frac{t}{2\tau}\right) \tag{3}$$

where τ is the time constant of the integrator and c is a dimensionless constant.

The rms detector output V_{rms} is applied to the gain control port G_c of the linear VCA. Assume that the multiplier factor $k = 1$. The function of the k factor is

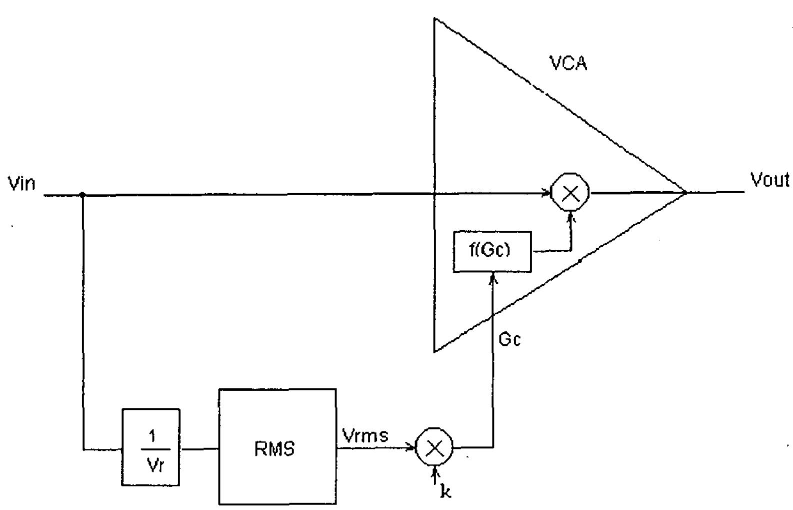


Fig. 1. Feedforward compressor topology.

explained later. The transfer function of the feedforward compressor is calculated as

$$\frac{V_{out}(t)}{V_{in}(t)} = f(V_{rms}) = \frac{1}{V_{rms}(t)}. \tag{4}$$

Substituting V_{rms} from Eq. (3) into Eq. (4) the transfer function becomes

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\int \frac{1}{\tau}\frac{V_{in}^2(t)}{V_r^2}\exp\left(\frac{t}{\tau}\right)dt + c\right]^{-1/2}\exp\left(\frac{t}{2\tau}\right). \tag{5}$$

2.1.2 Feedback

The block diagram of a linear-domain feedback compressor is shown in Fig. 2. The rms detector and the VCA are also linear devices. The transfer function of the linearly controlled VCA is given by Eq. (2). The detector input is the output voltage divided by the reference level V_r. Eq. (3) cannot be applied directly, as in the case of the feedforward compressor, because the detector output V_{rms} is a function of the output voltage. Thus it is necessary to go back one step to the differential equation that describes the functionality of the linear rms detector. From [6], the differential equation of the linear rms detector is calculated as

$$\frac{V_{out}^2(t)}{V_R^2} = \tau\frac{\partial V_{rms}^2(t)}{\partial t} + V_{rms}^2(t). \tag{6}$$

Assuming that the multiplier factor $k = 1$, the output voltage can be calculated from Eq. (4),

$$V_{out}(t) = \frac{V_{in}(t)}{V_{rms}(t)}. \tag{7}$$

Substituting Eq. (7) into Eq. (6) and rearranging the terms, the differential equation can be written as

$$\frac{V_{in}^2(t)}{V_r^2} = \tau V_{rms}^2(t)\frac{\partial V_{rms}^2(t)}{\partial t} + V_{rms}^4(t) \tag{8}$$

noting that

$$x\frac{\partial x}{\partial t} = \frac{1}{2}\frac{\partial x^2}{\partial t}. \tag{9}$$

Applying this property to Eq. (8) and rearranging terms, the latter can be written as

$$\frac{\partial V_{rms}^4(t)}{\partial t} + \frac{2}{\tau}V_{rms}^4(t) = \frac{V_{in}^2(t)}{V_r^2}\frac{2}{\tau}. \tag{10}$$

The algorithm for solving this differential equation is described in [6] and [8]. Following the same steps as in [6], we use the following notation:

$$y(t) = V_{rms}^4(t)$$
$$u(t) = \frac{2t}{\tau}. \tag{11}$$

Also notice the identity

$$\frac{\partial}{\partial t}\{y(t)\exp[u(t)]\} = \left[\frac{\partial y(t)}{\partial t} + \frac{2}{\tau}y(t)\right]\exp[u(t)]. \tag{12}$$

Let us multiply the left and right terms of Eq. (10) by $\exp[u(t)]$ and use identity (12),

$$\frac{\partial}{\partial t}\left[y(t)\exp\left(\frac{2t}{\tau}\right)\right] = \frac{V_{in}^2(t)}{V_r^2}\frac{2}{\tau}\exp\left(\frac{2t}{\tau}\right). \tag{13}$$

Differential Eq. (13) can be solved by integrating both terms with respect to time,

$$y(t) = \left[\int \frac{2}{\tau}\frac{V_{in}^2(t)}{V_r^2}\exp\left(\frac{2t}{\tau}\right)dt + c\right]\exp\left(-\frac{2t}{\tau}\right) \tag{14}$$

Substituting $y(t)$ with its definition from Eq. (11), the output V_{rms} of the linear rms detector has the following

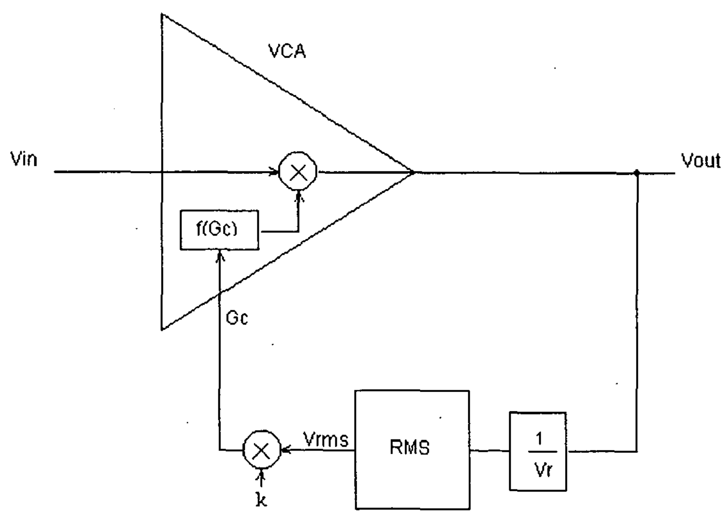


Fig. 2. Feedback compressor topology.

representation:

$$V_{\mathrm{rms}}(t) = \left[\int \frac{2}{\tau} \frac{V_{\mathrm{in}}^2(t)}{V_{\mathrm{r}}^2} \exp\left(\frac{2t}{\tau}\right) \mathrm{d}t + c\right]^{1/4} \exp\left(-\frac{t}{2\tau}\right) \tag{15}$$

where c is a dimensionless constant.

Substituting Eq. (15) into Eq. (4), the transfer function of the feedback compressor can be calculated as

$$\frac{V_{\mathrm{out}}(t)}{V_{\mathrm{in}}(t)} = \left[\int \frac{2}{\tau} \frac{V_{\mathrm{in}}^2(t)}{V_{\mathrm{r}}^2} \exp\left(\frac{2t}{\tau}\right) \mathrm{d}t + c\right]^{-1/4} \exp\left(\frac{t}{2\tau}\right). \tag{16}$$

Notice that the time constant of the rms detector integrator is halved by the feedback.

2.2 Log-Domain Compressor

2.2.1 Feedforward

The block diagram of a log-domain feedforward compressor is shown in Fig. 1. In this case the rms detector is logarithmic and the VCA is exponential. The rms detector input voltage is prescaled to the reference level V_{r}. At reference level, the output of the logarithmic detector is zero. An extra multiplier block is added between the output of the rms detector and the VCA gain control port. Multiplier k defines the compression ratio, which is analyzed later in this paper. Notice that the factor k multiplies a logarithmic representation of the rms detector input signal. A similar multiplier block, in the linear-domain compressor, was not shown because it does not have a direct and obvious correspondence with its counterpart in the log-domain compressor. In the case of the exponentially controlled VCA, the transfer function at the control port is [9]

$$f(G_{\mathrm{c}}) = \exp\left(-\frac{G_{\mathrm{c}}}{V_{\mathrm{T}}}\right) \tag{17}$$

where V_{T} is the thermal voltage equal to 0.0259 V at 27° C.

In [9] the voltage applied to the gain control port is divided by 2, which represents a square-root operation. In real applications the square-root operation in the logarithmic rms detector is transferred to the VCA. In the case of the log-domain compressor, it does not matter where the division by 2 is made. So in order to be consistent with the description of the linear-domain compressor, the square-root operation is done in the rms detector. The output of the logarithmic rms detector is calculated in [6] as

$$V_{\mathrm{rms}}(t) = -\frac{V_{\mathrm{T}}t}{2\tau} + V_{\mathrm{T}} \ln \sqrt{\int \frac{1}{\tau} \frac{V_{\mathrm{in}}^2(t)}{V_{\mathrm{r}}^2} \exp\left(\frac{t}{\tau}\right) \mathrm{d}t + c} \tag{18}$$

where τ is the time constant of the log integrator [7] and c is a constant without units.

The transfer function of the feedforward compressor is calculated as

$$\frac{V_{\mathrm{out}}(t)}{V_{\mathrm{in}}(t)} = f(kV_{\mathrm{rms}}) = \exp\left[-\frac{kV_{\mathrm{rms}}(t)}{V_{\mathrm{T}}}\right]. \tag{19}$$

Substituting Eq. (18) into Eq. (19), the transfer function becomes

$$\frac{V_{\mathrm{out}}(t)}{V_{\mathrm{in}}(t)} = \exp\left[\frac{kt}{2\tau} - k \ln \sqrt{\int \frac{1}{\tau} \frac{V_{\mathrm{in}}^2(t)}{V_{\mathrm{r}}^2} \exp\left(\frac{t}{\tau}\right) \mathrm{d}t + c}\right]. \tag{20}$$

Rearranging terms, the transfer function has the form

$$\frac{V_{\mathrm{out}}(t)}{V_{\mathrm{in}}(t)} = \left[\int \frac{1}{\tau} \frac{V_{\mathrm{in}}^2(t)}{V_{\mathrm{r}}^2} \exp\left(\frac{t}{\tau}\right) \mathrm{d}t + c\right]^{-k/2} \exp\left(\frac{kt}{2\tau}\right). \tag{21}$$

Notice that for $k = 1$, Eq. (21) has the same expression as the transfer function for the linear-domain feedforward compressor.

2.2.2 Feedback

The block diagram of a log-domain feedback compressor is shown in Fig. 2. The rms detector is logarithmic and the VCA is exponentially controlled. The transfer function at the control port of the exponentially controlled VCA is given by definition (17). As in the case of the linear-domain feedback compressor, the input of the rms detector is proportional to the output voltage of the feedback compressor. The output voltage is calculated in Eq. (19) and is a function of the VCA gain, which is a function of the output of the logarithmic rms detector. Thus the differential equation of the logarithmic rms detector has to be solved for a different input signal. The differential equation of the logarithmic output rms detector was calculated in [6] as

$$\frac{V_{\mathrm{out}}^2(t)}{V_{\mathrm{r}}^2} = \tau \frac{\partial \exp[2V_{\mathrm{rms}}(t)/V_{\mathrm{T}}]}{\partial t} + \exp\left[\frac{2V_{\mathrm{rms}}(t)}{V_{\mathrm{T}}}\right]. \tag{22}$$

The output voltage is calculated from Eq. (19) as

$$V_{\mathrm{out}}(t) = V_{\mathrm{in}}(t) \exp\left[-\frac{kV_{\mathrm{rms}}(t)}{V_{\mathrm{T}}}\right]. \tag{23}$$

Substituting Eq. (23) into Eq. (22) and rearranging terms, the rms detector differential equation can be written as

$$\frac{V_{\mathrm{in}}^2(t)}{V_{\mathrm{r}}^2} = \tau \exp\left[\frac{2kV_{\mathrm{rms}}(t)}{V_{\mathrm{T}}}\right] \cdot \frac{\partial \exp[2V_{\mathrm{rms}}(t)/V_{\mathrm{T}}]}{\partial t} + \exp\left[\frac{2(k+1)V_{\mathrm{rms}}(t)}{V_{\mathrm{T}}}\right]. \tag{24}$$

Notice the following identities:

$$\frac{\partial \exp[2V_{\mathrm{rms}}(t)/V_{\mathrm{T}}]}{\partial t} = \frac{1}{V_{\mathrm{T}}}\exp\left[\frac{2V_{\mathrm{rms}}(t)}{V_{\mathrm{T}}}\right]\frac{\partial[2V_{\mathrm{rms}}(t)]}{\partial t} \tag{25}$$

$$\frac{\partial \exp[2(k+1)V_{\mathrm{rms}}(t)/V_{\mathrm{T}}]}{\partial t} = \frac{k+1}{V_{\mathrm{T}}}\exp\left[\frac{2kV_{\mathrm{rms}}(t)}{V_{\mathrm{T}}}\right]\exp\left[\frac{2V_{\mathrm{rms}}(t)}{V_{\mathrm{T}}}\right]\frac{\partial[2V_{\mathrm{rms}}(t)]}{\partial t}\,. \tag{26}$$

Using these identities, Eq. (24) can be written as

$$\frac{V_{\mathrm{in}}^2(t)}{V_{\mathrm{r}}^2} = \frac{\tau}{k+1}\frac{\partial \exp[2(k+1)V_{\mathrm{rms}}(t)/V_{\mathrm{T}}]}{\partial t} + \exp\left[\frac{2(k+1)V_{\mathrm{rms}}(t)}{V_{\mathrm{T}}}\right]. \tag{27}$$

In order to solve this differential equation, let us use the following notation:

$$z(t) = \exp\left[\frac{2(k+1)V_{\mathrm{rms}}(t)}{V_{\mathrm{T}}}\right]$$
$$v(t) = \frac{(k+1)\,t}{\tau}\,. \tag{28}$$

This differential equation is solved the same way as the differential equation for the linear-output rms detector. Substituting $z(t)$ in Eq. (27) and multiplying both sides by $\exp[z(t)]$, the differential equation can be written as

$$\frac{\partial}{\partial t}\{z(t)\exp[v(t)]\} = \frac{k+1}{\tau}\frac{V_{\mathrm{in}}^2(t)}{V_{\mathrm{r}}^2}\exp[v(t)]\,. \tag{29}$$

Both sides of Eq. (29) are integrated, and substituting $z(t)$ from Eq. (28), the following equation is obtained:

$$\exp\left[\frac{2(k+1)V_{\mathrm{rms}}(t)}{V_{\mathrm{T}}}\right] = \left\{\int\frac{k+1}{\tau}\frac{V_{\mathrm{in}}^2(t)}{V_{\mathrm{r}}^2}\exp\left[\frac{(k+1)t}{\tau}\right]\mathrm{d}t + c\right\}\exp\left[-\frac{(k+1)t}{\tau}\right] \tag{30}$$

where c is a dimensionless constant.

Eq. (30) is solved for $V_{\mathrm{rms}}(t)$ as follows:

$$V_{\mathrm{rms}}(t) = -\frac{V_{\mathrm{T}}t}{2\tau} + \frac{V_{\mathrm{T}}}{k+1}\ln\sqrt{\int\frac{k+1}{\tau}\frac{V_{\mathrm{in}}^2(t)}{V_{\mathrm{r}}^2}\exp\left[\frac{(k+1)t}{\tau}\right]\mathrm{d}t + c}\,. \tag{31}$$

The transfer function of the feedback compressor is calculated by substituting Eq. (31) into Eq. (19),

$$\frac{V_{\mathrm{out}}(t)}{V_{\mathrm{in}}(t)} = \left\{\int\frac{k+1}{\tau}\frac{V_{\mathrm{in}}^2(t)}{V_{\mathrm{r}}^2}\exp\left[\frac{(k+1)t}{\tau}\right]\mathrm{d}t + c\right\}^{-k/2(k+1)}\exp\left(\frac{kt}{2\tau}\right). \tag{32}$$

Notice that in the case of the feedback compressor the log integrator time constant is divided by the factor $(k + 1)$. For $k = 1$, the tranfer function of the log-domain feedback compressor is identical to the transfer function of the linear-domain feedback compressor, Eq. (16).

3 COMPRESSION RATIO

The compression ratio is defined as the rate of change at the input of the compressor divided by the rate of change at the output of the compressor, in decibels. For instance, a 10-dB change at the input of a 2:1 compressor translates into a 5-dB change at the output. This is illustrated in Fig. 3, a plot of the compressor output versus the compressor input, both in decibels. The mathematical definition of the compression ratio is

$$C_{\mathrm{r}} = \frac{\partial \mathrm{dB}[V_{\mathrm{in}}(t)]}{\partial \mathrm{dB}[V_{\mathrm{out}}(t)]} \approx \frac{\Delta \mathrm{dB}[V_{\mathrm{in}}(t)]}{\Delta \mathrm{dB}[V_{\mathrm{out}}(t)]}\,. \tag{33}$$

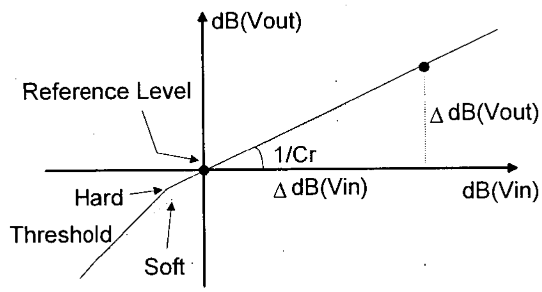


Fig. 3. Compressor transfer function.

The infinitesimal difference and the finite difference are equal if the compression ratio is not a function of the input voltage, such as 2:1 or 1:3. In some audio applications the compressor is on above a threshold. The threshold can be "hard knee" or "soft knee," as shown in Fig. 3. In the cased of a "soft knee" compressor, the last two terms of Eq. (33) are not equal and only the infinitesimal difference should be used.

3.1 Log-Domain Compressor

3.1.1 Feedforward

Let us assume that the input voltage has the expression

$$V_{\text{in}}(t) = V_{\text{p}} f(t) \tag{34}$$

where V_{p} is the peak level and $f(t)$ is the time-dependent part of the input signal.

The transfer function of the feedforward compressor, Eq. (21), can be simplified as

$$\frac{V_{\text{out}}(t)}{V_{\text{in}}(t)} = \left\{ \exp\left(-\frac{t}{\tau}\right) \left[\int \frac{1}{\tau} \frac{V_{\text{in}}^2(t)}{V_{\text{r}}^2} \exp\left(\frac{t}{\tau}\right) \mathrm{d}t \right] + c \exp\left(-\frac{t}{\tau}\right) \right\}^{-k/2} . \tag{35}$$

Notice that the constant c makes a contribution to the transient response only. If the time is greater than three time constants, the contribution of the second summation term in parentheses is minimal. Therefore for $t \Rightarrow \infty$, the output voltage is calculated as

$$V_{\text{out}}(t) = V_{\text{in}}(t) \left\{ \exp\left(-\frac{t}{\tau}\right) \left[\int \frac{1}{\tau} \frac{V_{\text{in}}^2(t)}{V_{\text{r}}^2} \exp\left(\frac{t}{\tau}\right) \mathrm{d}t \right] \right\}^{-k/2} . \tag{36}$$

Substituting Eq. (34) into Eq. (36) of the feedforward compressor, the latter becomes

$$V_{\text{out}}(t) = V_{\text{in}}^{1-k}(t) \left\{ \frac{1}{\tau} \frac{\exp(-t/\tau)[\int f^2(t) \exp(t/\tau)\, \mathrm{d}t]}{V_{\text{r}}^2 f^2(t)} \right\}^{-k/2} . \tag{37}$$

The output voltage in decibels is

$$\mathrm{dB}[V_{\text{out}}(t)] = (1-k)\, \mathrm{dB}[V_{\text{in}}(t)] + \mathrm{dB}\left\{ \left[\frac{\int \frac{1}{\tau} \cdot \frac{f^2(t)}{V_{\text{r}}^2} \exp\left(\frac{t}{\tau}\right) \mathrm{d}t}{f^2(t) \exp\left(\frac{t}{\tau}\right)} \right]^{-k/2} \right\} . \tag{38}$$

The compression ratio for a feedforward compressor can be calculated from Eq. (38) as

$$C_{\text{r}} = \frac{\partial \mathrm{dB}[V_{\text{in}}(t)]}{\partial \mathrm{dB}[V_{\text{out}}(t)]} = \frac{1}{1-k} \tag{39}$$

or

$$k = 1 - \frac{1}{C_{\text{r}}} . \tag{40}$$

Table 1 summarizes the compressor functionality as a function of gain factor k.

3.1.2 Feedback

The transfer function of the feedback compressor, Eq. (32), can be simplified as

$$\frac{V_{\text{out}}(t)}{V_{\text{in}}(t)} = \left\{ \frac{\int \frac{(k+1)}{\tau} \frac{V_{\text{in}}^2(t)}{V_{\text{r}}^2} \exp\left[\frac{(k+1)t}{\tau}\right] \mathrm{d}t}{\exp\left[\frac{(k+1)t}{\tau}\right]} + c \exp\left[-\frac{(k+1)t}{\tau}\right] \right\}^{-k/2(k+1)} . \tag{41}$$

As in the case of the feedforward compressor, constant c makes a contribution during the transient response only. Notice that the rms detector time constant is divided by the factor $(k + 1)$. If time is greater than three times the equivalent time constant τ, the contribution of the second summation term in parentheses is minimal. Therefore for $t \Rightarrow \infty$, the output voltage is calculated as

$$V_{\text{out}}(t) = V_{\text{in}}(t) \left\{ \frac{\int \frac{(k+1)}{\tau} \frac{V_{\text{in}}^2(t)}{V_{\text{r}}^2} \exp\left[\frac{(k+1)t}{\tau}\right] \mathrm{d}t}{\exp\left[\frac{(k+1)t}{\tau}\right]} \right\}^{-k/2(k+1)} . \tag{42}$$

Table 1.

Gain Factor k	Compression Ratio	Function
$1 < k < \infty$	$-\infty < C_{\text{r}} \leq 0$	Negative compression ratio output decreases as input increases; compressor sounds like playing a tape backward
$k = 1$	$C_{\text{r}} = \infty$	Infinite compression: output is set to reference level independent of input level; perfect automatic gain control
$0 < k < 1$	$1 < C_{\text{r}} < \infty$	Compressor
$k = 0$	$C_{\text{r}} = 1$	No compression
$-\infty < k < 0$	$0 < C_{\text{r}} < 1$	Expander: output change greater than input change

Substituting the definition of the input signal, Eq. (34), into Eq. (42), the output voltage becomes

$$V_{out}(t) = V_{in}^{1/(k+1)}(t) \left\{ \frac{\int \frac{(k+1)}{\tau} \frac{f^2(t)}{V_r^2} \exp\left[\frac{(k+1)t}{\tau}\right] \mathrm{d}t}{f^2(t) \exp\left[\frac{(k+1)t}{\tau}\right]} \right\}^{-k/2(k+1)} . \quad (43)$$

The output voltage is expressed in decibels as

$$\mathrm{dB}[V_{out}(t)] = \frac{1}{k+1}\mathrm{dB}[V_{in}(t)] + \mathrm{dB}\left[\left\{ \frac{\int \frac{(k+1)}{\tau} \frac{f^2(t)}{V_r^2} \exp\left[\frac{(k+1)t}{\tau}\right] \mathrm{d}t}{f^2(t) \exp\left[\frac{(k+1)t}{\tau}\right]} \right\}^{-k/2(k+1)} \right] . \quad (44)$$

The compression ratio for a feedback compressor can be caculated from Eq. (44) as

$$C_r = \frac{\partial \mathrm{dB}[V_{in}(t)]}{\partial \mathrm{dB}[V_{out}(t)]} = k + 1 \quad (45)$$

or

$$k = C_r - 1 . \quad (46)$$

Table 2 summarizes the compressor functionality as a function of gain factor k.

3.2 Linear-Domain Compression Ratio

In a log-domain compressor the compression ratio is determined by the multiplier k. The question is, how can the compression ratio be controlled in the context of an linear-domain compressor?

Let us define the following notation:

V_{rmsLL} Output of rms detector in linear-domain compressor

V_{rmsLE} Output of rms detector in log-domain compressor.

3.2.1 Feedforward

In Section 2 it was shown that the transfer function of the compressor based on a linear detector and linear. VCA matches the transfer function of the compressor based on a logarithmic detector and exponential VCA for $k = 1$. If both transfer functions match for any value of k, then the gain control function $f(G_c)$ of the linearly controlled VCA has a different form. Let us equate both transfer functions, Eqs. (4) and (21),

$$f_1(V_{rmsLL}(t)) = \left[\sqrt{\int \frac{1}{\tau} \frac{V_{in}^2(t)}{V_r^2} \exp\left(\frac{t}{\tau}\right) \mathrm{d}t + c}\right]^{-k/2} \exp\left(\frac{kt}{2\tau}\right) \quad (47)$$

where f_1 is the new gain control function of the linear VCA.

Substituting the linear rms detector output, Eq. (3), into Eq. (47), function f_1 has the form

$$f_1(V_{rmsLL}(t)) = \left(\frac{1}{V_{rmsLL}}\right)^k . \quad (48)$$

Thus the compression ratio transformation from the log-domain feedforward compressor to the linear-domain feedforward compressor is implemented by raising the VCA control voltage to the power of k.

3.2.2 Feedback

It is necessary to check whether raising the linear VCA control port to the power of k is a solution for the feedback configuration also. Unfortunately the transfer functions of the linear-domain and the log-domain feedback compressors cannot be equated like in the case of the feedforward compressor. The problem is that the factor k is associated with the log filter time constant, as shown in Eq. (32). The solution is to solve the transfer function of the linear-domain feedback compressor using $f_1(G_c)$ given by Eq. (48), instead of $f(G_c)$ as defined in

Table 2.

Gain Factor k	Compression Ratio	Function
$0 < k < \infty$	$1 < C_r < \infty$	Compressor: infinite compression not possible because it requires infinite gain
$k = 0$	$C_r = 1$	No compression
$-1 < k < 0$	$0 < C_r < 1$	Expander: output change greater than input change
$k = -1$	$C_r = 0$	Infinite expander: unstable setting
$-\infty < k < -1$	$-\infty < C_r < 0$	Negative compression ratio: output decreases as input increases; compressor sounds like playing a tape backward

Eq. (2). So the output voltage is calculated from Eqs. (4) and (48) as

$$V_{\mathrm{out}}(t) = \frac{V_{\mathrm{in}}(t)}{V_{\mathrm{rms}}^{k}(t)} . \tag{49}$$

Substituting Eq. (49) into Eq. (6) and applying the property of the derivative function as shown in Eq. (9), the differential equation of the feedback compressor becomes

$$\frac{\partial V_{\mathrm{rms}}^{2(k+1)}(t)}{\partial t} + \frac{k+1}{\tau} V_{\mathrm{rms}}^{2(k+1)}(t) = \frac{V_{\mathrm{in}}^2(t)}{V_{\mathrm{r}}^2} \frac{k+1}{\tau} . \tag{50}$$

Eq. (50) is solved the same way as Eq. (10). The solution is

$$V_{\mathrm{rms}}(t) = \left\{ \int \frac{k+1}{\tau} \frac{V_{\mathrm{in}}^2(t)}{V_{\mathrm{r}}^2} \exp\left[\frac{(k+1)t}{\tau}\right] dt + c \right\}^{1/2(k+1)} \exp\left(-\frac{t}{2\tau}\right) . \tag{51}$$

Substituting the rms detector output level, Eq. (51), into Eq. (49), the transfer function is calculated as

$$\frac{V_{\mathrm{out}}(t)}{V_{\mathrm{in}}(t)} = \left\{ \int \frac{k+1}{\tau} \frac{V_{\mathrm{in}}^2(t)}{V_{\mathrm{r}}^2} \exp\left[\frac{(k+1)t}{\tau}\right] dt + c \right\}^{-k2(k+1)} \exp\left(\frac{kt}{2\tau}\right) . \tag{52}$$

Eq. (52) matches the transfer function of the log-domain feedback compressor, Eq. (32).

The compression ratio transformation implemented by raising the VCA control voltage to the power of k works for both feedforward and feedback linear-domain compressors.

4 PERFORMANCE

So far all the results are based on a generic time-varying input signal. It was demonstrated that both compressor topologies are identical. Thus the performance analysis is independent of the rms detector or VCA type. In order to quantify the performance of each compressor topology, the input signal needs to be defined more specifically, such as sine or square. Let us specify the input voltage as

$$V_{\mathrm{in}}(t) = V_{\mathrm{A}} \cos(\omega t) \tag{53}$$

where V_{A} is the peak level and ω is the signal frequency in radians per second.

4.1 Feedforward

Substituting the definition of the input signal, Eq. (53), into the transfer function of a feedforward compressor, Eq. (21),

$$\frac{V_{\mathrm{out}}(t)}{V_{\mathrm{in}}(t)} = \left[\frac{V_{\mathrm{A}}^2}{2\tau V_{\mathrm{r}}^2} E(t) + c\right]^{-k/2} \exp\left(\frac{kt}{2\tau}\right) \tag{54}$$

where

$$E(t) = \int \exp\left(\frac{t}{\tau}\right) dt + \int \cos(2\omega t) \exp\left(\frac{t}{\tau}\right) dt \tag{55}$$

and the following trigonometric property is used:

$$\cos^2(\omega t) = \frac{1 + \cos(2\omega t)}{2} . \tag{56}$$

The first integral in Eq. (55) is solved as

$$\int \exp\left(\frac{t}{\tau}\right) dt = \tau \exp\left(\frac{t}{\tau}\right) . \tag{57}$$

and the second integral is solved as

$$\begin{aligned} \int \cos(2\omega t) \exp\left(\frac{t}{\tau}\right) dt &= \tau \exp\left(\frac{t}{\tau}\right) \cos(2\omega t) + 2\omega\tau \int \exp\left(\frac{t}{\tau}\right) \sin(2\omega t)\, dt \\ &= \tau \exp\left(\frac{t}{\tau}\right) \cos(2\omega t) + 2\omega\tau \left[\tau \exp\left(\frac{t}{\tau}\right) \sin(2\omega t) - 2\omega\tau \int \cos(2\omega t) \exp\left(\frac{t}{\tau}\right) dt\right] . \end{aligned} \tag{58}$$

Notice that the last term of Eq. (58) is the original integral. So Eq. (58) can be solved for the second integral term of Eq. (55)

$$\int \cos(2\omega t) \exp\left(\frac{t}{\tau}\right) dt = \tau \exp\left(\frac{t}{\tau}\right) \frac{\cos(2\omega t) + 2\omega\tau \sin(2\omega t)}{1 + (2\omega\tau)^2} . \tag{59}$$

Substituting the integral results, Eqs. (57) and (59), into Eq. (55), $E(t)$ is evaluated,

$$E(t) = \tau \exp\left(\frac{t}{\tau}\right) G(\omega t, \tau) \tag{60}$$

where $G(\omega t, \tau)$ is defined as

$$G(\omega t, \tau) = 1 + \frac{\cos(2\omega t) + 2\omega\tau \sin(2\omega t)}{1 + (2\omega\tau)^2} . \tag{61}$$

Finally, substituting Eqs. (40) and (60) into Eq. (54), the transfer function of the feedforward compressor is expressed as a function of the input signal and compression ratio,

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\frac{(V_A/\sqrt{2})^2}{V_r^2} G(\omega t, \tau) + c \exp\left(-\frac{t}{\tau}\right)\right]^{(1-C_r)/2C_r} . \tag{62}$$

The transfer function of a compressor is basically its gain function. The transfer function in Eq. (62) has a few terms, each with its own significance. The first term in parentheses is the steady-state solution—ripple and dc. The ripple is given by $G(\omega t, \tau)$, as shown in Eq. (61). The ripple is the residual alternating current left after the first-order low-pass filter in the rms detector. It modulates the compressor gain and is a major source of distortion. The dc term would be the compressor ideal gain if the rms detector low-pass filter had been ideal. The second summation term in parentheses decays with time and has a significant contribution only during the transient response.

4.1.1 Ideal Gain Transfer Function

The ideal gain transfer function is the steady-state solution of the transfer function, Eq. (62), in the presence of an rms detector with ideal integrator, or no ripple. In this case the transfer function (62) can be written as

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\frac{(V_A/\sqrt{2})^2}{V_r^2}\right]^{(1-C_r)/2C_r} . \tag{63}$$

The transfer function was rearranged to show that the compressor gain is proportional to the rms value of the input signal. Indeed, the rms value of a sine is its peak value divided by the square root of 2. For commonly used time constants, in the range of tens of milliseconds, this formula is quite safe to use when calculating the compressor gain. In this case the error caused by ripple is small. For instance, for a time constant of 35 ms, the peak value of $G(\omega t, \tau)$ at 100 Hz is about 1.02.

4.1.2 Ripple and Harmonic Distortion

As mentioned before, the ripple modulates the compressor gain causing distortion. Substituting, Eqs. (53) and (61) into Eq. (62), the output voltage steady-state solution is

$$V_{out}(t) = V_A \cos(\omega t)\left\{\frac{(V_A/\sqrt{2})^2}{V_r^2}[1 + \cos(2\omega t - \phi)\cos\phi]\right\}^{(1-C_r)/2C_r} \tag{64}$$

where

$$\tan\phi = 2\omega\tau . \tag{65}$$

The Taylor series of a binomial is

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots . \tag{66}$$

If x is small, a good approximation is to consider only the first two terms of the Taylor series. In this case this can be done if $\cos\phi$ is small. The greatest value of $\cos\phi$ is at low frequencies or small time constants. For commonly used time constants this approximation can be made. For instance, for a time constant of 35 ms, $\cos\phi$ at 20 Hz is about 0.1 and less for higher frequencies. Thus taking into account the Taylor series, Eq. (66), the output voltage can be approximated as

$$V_{out}(t) = V_A\left(\frac{V_A/\sqrt{2}}{V_r}\right)^{(1-C_r)/C_r} \cos(\omega t)\left[1 + \frac{1 - C_r}{2C_r}\cos\phi\cos(2\omega t - \phi)\right] . \tag{67}$$

Recall from trigonometry the following property of cosine functions:

$$\cos a \cos b = \frac{\cos(a + b) + \cos(a - b)}{2} . \tag{68}$$

This trigonometric property is applied to Eq. (67),

$$V_{out}(t) = V_A\left(\frac{V_A/\sqrt{2}}{V_r}\right)^{(1-C_r)/C_r}\left\{\cos(\omega t) + \frac{1 - C_r}{4C_r}\cos\phi\,[\cos(3\omega t - \phi) + \cos(\omega t - \phi)]\right\} . \tag{69}$$

The spectrum of the output voltage contains the fundamental and the third harmonic of the fundamental. The total harmonic distortion is the square root of the sum of the squares of all harmonics divided by the square of the fundamental. If all the harmonics are normalized to the amplitude of the fundamental, then the total harmonic distortion is the square root of the sum of the squares of all harmonics. In this case only the third harmonic exists, and the total harmonic distortion is equal to the normalized third-harmonic amplitude. This is because only the first term of the Taylor series is considered. If more terms are taken into account, then the spectrum of the output signal is the sum of the odd harmonics. This can be demonstrated by noticing that the input signal $\cos(\omega t)$ multiples a $\cos^n(2\omega t)$ term [see Eq. (64)]. This can be written as

$$\begin{aligned}\cos(\omega t)\cos^n(2\omega t) &= \cos(\omega t)\sum_m a_m \cos(m\cdot 2\omega t)\\ &= \sum_m a_m \cos(\omega t)\cos(m\cdot 2\omega t)\\ &= \sum_m \frac{a_m}{2}\{\cos[(2m+1)\,\omega t] + \cos[(2m-1)\,\omega t]\}\end{aligned} \tag{70}$$

where the a_m are a series of real coefficients.

Thus the result of the multiplication is a sum of odd harmonics. Notice another trigonometric property of the cosine function:

$$\cos(\omega t) - b\cos(\omega t - \phi) = \sqrt{1 + b^2 - 2b\cos\phi}\,\cos(\omega t + \phi_1) \tag{71}$$

where ϕ_1 is defined as

$$\tan\phi_1 = \frac{b\sin\phi}{1 - b\cos\phi}\,. \tag{72}$$

Applying the cosine property, Eq. (71), to Eq. (69) and normalizing the spectrum, the frequency content of the output voltage is

$$V_{\text{out}}(\omega t) = \cos(\omega t + \phi_1) - \frac{(C_r - 1)/4C_r}{\sqrt{[(3C_r + 1)/4C_r]^2 + (2\omega\tau)^2}}\cos(3\omega t - \phi) \tag{73}$$

where ϕ_1 is defined as

$$\tan\phi_1 = \frac{[(C_r - 1)/4C_r]\,(2\omega\tau)}{[(3C_r + 1)/4C_r] + (2\omega\tau)^2}\,. \tag{74}$$

The distortion of the feedforward compressor is equal to the coefficient of the third harmonic,

$$\text{THD}_{\text{FF}} = \frac{|(C_r - 1)/4C_r|}{\sqrt{[(3C_r + 1)/4C_r]^2 + (2\omega\tau)^2}}\,. \tag{75}$$

Eq. (75) is plotted in Fig. 4 for a 20:1 compression ratio. It is interesting to note that the distortion at low frequencies is flat. At high frequencies the distortion decreases with frequency at a rate of 6 dB per octave. In a real compressor the distortion meter measures THD + N, or total harmonic distortion plus noise. Therefore when the harmonic distortion is lower than the noise floor, the distortion curve is flat also, this time due to the noise of the compressor. Since the distortion is flat and then decreases at a rate of 6 dB per octave, it has a −3-dB point at

$$\omega_{\text{FF}(-3\,\text{dB})} = \frac{1}{2\tau}\,\frac{3C_r + 1}{4C_r}\,. \tag{76}$$

For a 35-ms time constant this point moves from 1.7 Hz at infinite compression to 2 Hz at 2:1 compression and to 2.3 Hz at no compression. So the −3-dB corner frequency is around 2 Hz. Below the corner frequency the distortion predicted by Eq. (75) is 33% at infinite compression and 14% at 2:1 compression. At 1 kHz the distortion drops to 0.07% at infinite compression and to about 0.03% at 2:1 compression.

Fig. 5 shows the fast Fourier transform (FFT) of a 500-Hz output signal given by Eq. (64) for an rms detector time constant $\tau = 350\ \mu$s and infinite compression. The input signal is a 500-Hz cosine. Notice that only the third harmonic is in the output signal spectrum. The distortion calculated from FFT is 11%. The same distortion calculated by Eq. (75) is 10.7%. Lowering the rms detector time constant to $\tau = 100\ \mu$s, more odd terms show up in the FFT, as shown in Fig. 6. The distortion calculated from the FFT is 31.7%, and since Eq. (75) takes into account only the third harmonic, it predicts 25.6%. Notice that no even harmonics are present in the output signal spectrum, as demonstrated in Eq. (70).

4.1.3 Transient Response

Transient behavior occurs when the input signal changes from one level to another. Let us define the following input signals around $t = 0$:

$$\begin{aligned} t_{0-}&:\quad V_{\text{in}}(t) = V_A\cos(\omega t)\\ t_{0+}&:\quad V_{\text{in}}(t) = V_B\cos(\omega t)\end{aligned} \tag{77}$$

where t_{0-} is the time just before $t = 0$ and t_{0+} just after $t = 0$.

The compressor gain just before $t = 0$ is the steady-state solution of the transfer function for the input level at t_{0-},

$$\frac{V_{\text{out}}(t)}{V_{\text{in}}(t)} = \left[\frac{(V_A/\sqrt{2})^2}{V_r^2} G(\omega t, \tau)\right]^{(1-C_r)/2C_r} \quad (78)$$

When the input signal changes at $t = 0$, the compressor takes a finite time to react and its gain is still the one at t_{0-}. Thus the gain calculated in Eq. (78) is equated with the transfer function of the feedforward compressor, Eq. (62), for the input signal as defined for t_{0+},

$$\left[\frac{(V_A/\sqrt{2})^2}{V_r^2} G(\omega 0, \tau)\right]^{(1-C_r)/2C_r} = \left[\frac{(V_B/\sqrt{2})^2}{V_r^2} G(\omega 0, \tau) + c\right]^{(1-C_r)/2C_r} . \quad (79)$$

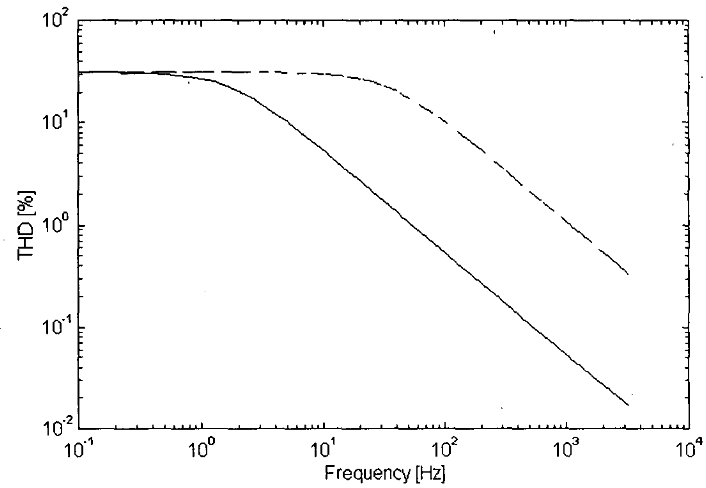


Fig. 4. THD versus frequency. ——— feedforward compressor; – – – feedback compressor. Compression ratio 20; rms detector time constant 35 ms.

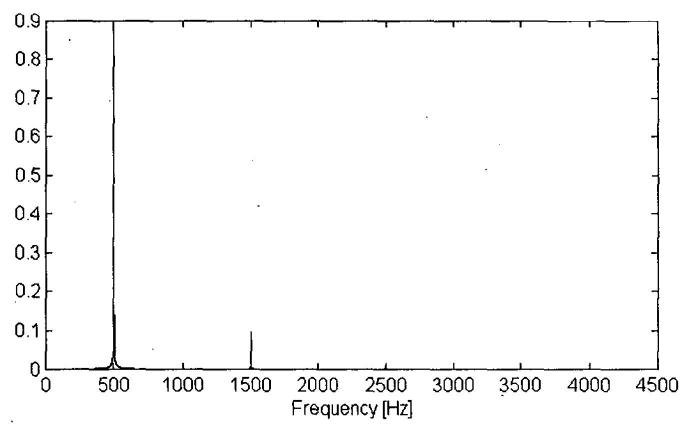


Fig. 5. FFT of feedforward compressor output; rms detector time constant 350 μs.

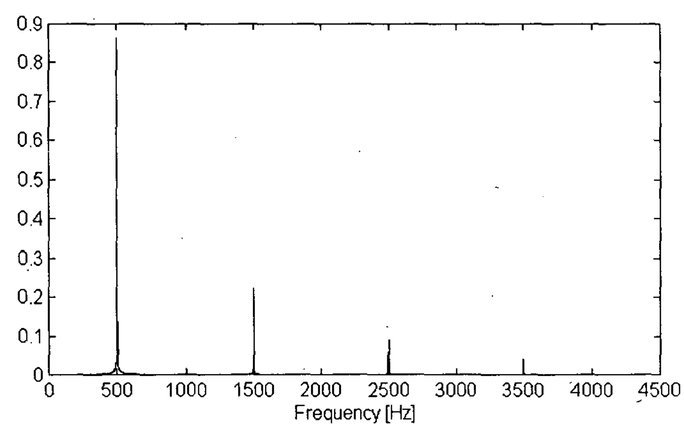


Fig. 6. FFT of feedforward compressor output; rms detector time constant 100 μs.

Eq. (79) is solved for the constant c as

$$c = \frac{G(\omega 0, \tau)}{V_r^2}\left[\left(\frac{V_A}{\sqrt{2}}\right)^2 - \left(\frac{V_B}{\sqrt{2}}\right)^2\right]$$

$$= \frac{1}{2}\frac{G(\omega 0, \tau)}{V_r^2}(V_A^2 - V_B^2) . \quad (80)$$

Constant c is proportional to the difference between the rms values of the signals before $t = 0$ and after $t = 0$. The factor $G(\omega 0, \tau)$ is added to match the phase of the ripple before and after $t = 0$. Substituting constant c in the transfer function of the feedforward compressor, Eq. (62), and ignoring the effect of the ripple, the transient transfer function becomes

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\frac{1}{2}\frac{V_B^2}{V_r^2} + \frac{1}{2}\frac{V_A^2 - V_B^2}{V_r^2}\exp\left(-\frac{t}{\tau}\right)\right]^{(1-C_r)/2C_r} . \quad (81)$$

This transfer function is the same for attack and release times. A plot of the attack and release transfer function is shown in Fig. 7. The shape of the attack and release envelope is the shape of a decaying and raising exponential function with a time constant τ.

4.2 Feedback

The transfer function for the feedback compressor is determined in the same manner as for the feedforward compressor. The input signal definition, Eq. (53), is substituted into Eq. (32),

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\frac{(k+1)^2 V_A^2}{2\tau V_r^2}D(t) + c\right]^{-k/2(k+1)} \exp\left(\frac{kt}{2\tau}\right) \quad (82)$$

where $D(t)$ has the form

$$D(t) = \int \exp\left[\frac{t}{\tau}\cdot(k+1)\right]dt + \int \cos(2\omega t)\exp\left[\frac{t}{\tau}(k+1)\right]dt . \quad (83)$$

Solving the integrals in Eq. (83), $D(t)$ is written as

$$D(t) = \frac{\tau}{k+1}\exp\left[\frac{t}{\tau}(k+1)\right]H(\omega t, \tau) \quad (84)$$

where $H(\omega t, \tau)$ is defined as

$$H(\omega t, \tau) = 1 + \frac{\cos(2\omega t) + 2\omega\tau/(k+1)\sin(2\omega t)}{1 + [2\omega\tau/(k+1)]^2} . \quad (85)$$

Substituting Eqs. (46) and (84) into Eq. (82), the transfer function of the feedback compressor can be expressed as a function of the input signal and the compression ratio as

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\frac{(V_A/\sqrt{2})^2}{V_r^2}H(\omega t, \tau) + c\exp\left(-\frac{t}{\tau/C_r}\right)\right]^{(1-C_r)/2C_r} . \quad (86)$$

The same notes that applied to the feedforward compressor are valid for the feedback configuration. The major difference is that the time constant is divided by the compression ratio.

4.2.1 Ideal Transfer Function

The ideal transfer function of the feedback compressor is the steady-state solution of the transfer function, Eq. (86), with an ideal low-pass filter in the rms detector, or no ripple,

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\frac{(V_A/\sqrt{2})^2}{V_r^2}\right]^{(1-C_r)/2C_r} . \quad (87)$$

The feedback compressor gain is proportional to the rms value of the input signal. If the equivalent time constant, that is, the rms detector time constant divided by the compression ratio, is in the range of tens of milliseconds, Eq. (87) can be used safely to calculate the gain of the feedback compressor.

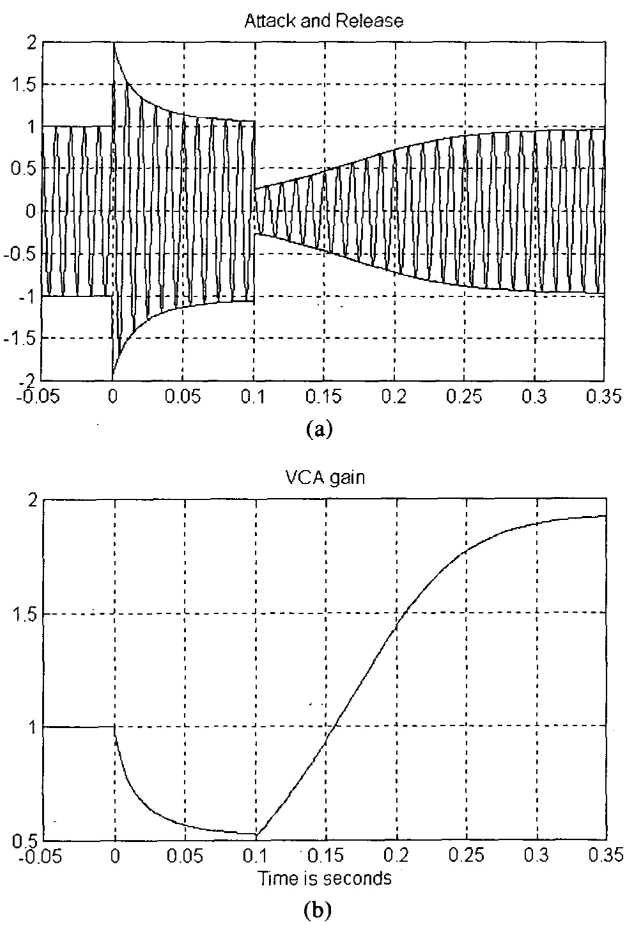


Fig. 7. Feedforward compressor. (a) Attack and release gain modulation. (b) VCA gain. rms detector time constant 35 ms; reference level 1 V peak; signal frequency 100 Hz; compression ratio 20:1; signal level 6 dB above reference level during attack, 6 dB below reference level during release.

4.2.2 Ripple and Harmonic Distortion

As in the case of the feedforward compressor, the ripple factor $H(\omega t, \tau)$ modulates the feedback compressor gain and causes distortion. Substituting the definition of the input signal [Eq. (53)] and the ripple [Eq. (85)] into the transfer function [Eq. (86)], the output voltage is calculated as

$$V_{out}(t) = V_A \cos(\omega t) \left\{ \frac{(V_A/\sqrt{2})^2}{V_r^2} [1 + \cos(2\omega t - \theta) \cos\theta] \right\}^{(1-C_r)/2C_r} \quad (88)$$

where

$$\tan\theta = 2\omega \frac{\tau}{C_r} . \quad (89)$$

Eq. (88) can be expanded into a Taylor series using the binomial series Eq. (66),

$$V_{out}(t) = V_A \left(\frac{V_A/\sqrt{2}}{V_r}\right)^{(1-C_r)/2C_r} \cos(\omega t) \left[1 + \frac{1 - C_r}{2C_r} \cos\theta \cos(2\omega t - \theta)\right] . \quad (90)$$

Using the property of the cosine function (68), Eq. (90) is rewritten as

$$V_{out}(t) = V_A \left(\frac{V_A/\sqrt{2}}{V_r}\right)^{(1-C_r)/2C_r} \left\{\cos(\omega t) + \frac{1 - C_r}{4C_r} \cos\theta\, [\cos(3\omega t - \theta) + \cos(\omega t - \theta)]\right\} . \quad (91)$$

The spectrum of the output voltage contains the fundamental and the third harmonic. The more terms of the Taylor series are considered, the more odd harmonics are added to the sum. For commonly used time constants, in the range of tens of milliseconds, the third harmonic is a good approximation. Applying the cosine property, Eq. (71), to Eq. (91) and normalizing the spectrum to the fundamental, the frequency content of the output voltage is

$$V_{out}(\omega t) = \cos(\omega t + \theta_1) - \frac{(C_r + 1)/4C_r}{\sqrt{[(3C_r + 1)/4C_r]^2 + (2\omega\tau/C_r)^2}} \cos(3\omega t - \theta) \quad (92)$$

where θ_1 is defined as

$$\tan\theta_1 = \frac{[(C_r - 1)/4C_r](2\omega\tau/C_r)}{[(3C_r + 1)/4C_r] + (2\omega\tau/C_r)^2} . \quad (93)$$

The feedback compressor, as well as the feedforward compressor, has a built-in inherent phase shift or delay θ_1. The delay is zero at dc, or if there is no compression, $C_r = 1$, or if the time constant is zero. The distortion of the feedback compressor is equal to the coefficient of the third harmonic,

$$\mathrm{THD}_{FB} = \frac{|(C_r - 1)/4C_r|}{\sqrt{[(3C_r + 1)/4C_r] + (2\omega\tau/C_r)^2}} . \quad (94)$$

Eq. (94) is also plotted in Fig. 4 for a 20:1 compression ratio. The distortion at low frequencies is also flat. At high frequencies, distortion decreases with frequency at a rate of 6 dB per octave. The distortion −3-dB corner frequency for the feedback compressor is

$$\omega_{FB(-3\,dB)} = \frac{C_r}{2\tau} \frac{3C_r + 1}{4C_r} . \quad (95)$$

For a similar rms detector time constant the feedback compressor −3-dB frequency corner is multiplied by the compression ratio. This is especially important at high compression ratios and because the higher distortion plateau is extended to higher frequencies. For instance, for a 35-ms time constant at a compression ratio of 20:1 the corner frequency for the feedforward compressor is 1.7 Hz compared to 35 Hz for the feedback configuration. However, if the rms detector time constant is adjusted by multiplying it by the compression ratio, then there is no difference in performance between feedforward and feedback compressors.

4.2.3 Transient Response

The input signals for the transient response are defined in Eq. (77). The feedback compressor gain right before the input signal transition is

$$\frac{V_{out}(t)}{V_{in}(t)} = \left[\frac{(V_A/\sqrt{2})^2}{V_r^2} H(\omega t, \tau)\right]^{(1-C_r)/2C_r} . \quad (96)$$

When the input signal changes in level at $t = 0$, the compressor gain requires finite time to change. Therefore the gain at t_{0+} equals the gain at t_{0-},

$$\left[\frac{(V_A/\sqrt{2})^2}{V_r^2} H(\omega 0, \tau)\right]^{(1-C_r)/2C_r} = \left[\frac{(V_B/\sqrt{2})^2}{V_r^2} H(\omega 0, \tau) + c\right]^{(1-C_r)/2C_r} . \quad (97)$$

Eq. (97) is solved for the constant c as

$$c = \frac{H(\omega 0, \tau)}{V_r^2} \cdot \left[\left(\frac{V_A}{\sqrt{2}}\right)^2 - \left(\frac{V_B}{\sqrt{2}}\right)^2\right] = \frac{H(\omega 0, \tau)}{2V_r^2}(V_A^2 - V_B^2) . \tag{98}$$

The ripple value at $t = 0$, $H(\omega t, \tau)$, is only necessary to match the phase of the ripple at the transition.

Substituting the constant c in Eq. (86) and neglecting the ripple, the transient transfer function has the form

$$\frac{V_{out}(t)}{V_{in}(t)} = \frac{R_2}{R_1}\left[\frac{V_B^2}{2V_r^2} + \frac{V_A^2 - V_B^2}{2V_r^2}\exp\left(-\frac{t}{\tau/C_r}\right)\right]^{(1-C_r)/2C_r} . \tag{99}$$

The transient response of the feedback compressor is similar to the feedfoward compressor, with the difference that the rsm detector time constant is divided by the compressor ratio. The same attack and release curves shown in Fig. 7, can be obtained by setting the rms detector time constant to $\tau = 35$ ms times C_r. In this case the compression ratio is 20:1 and the rms time constant is $\tau = 700$ ms.

The difference between the transient responses of the feedforward and the feedback compressors is shown in Fig 8. The rms time constant and the compression ratio are the same, $\tau = 35$ ms and $C_r = 20$, respectively. Notice that the feedback compressor is 20 times faster than the feedforward compressor.

4.3 Other Factors that Influence Performance

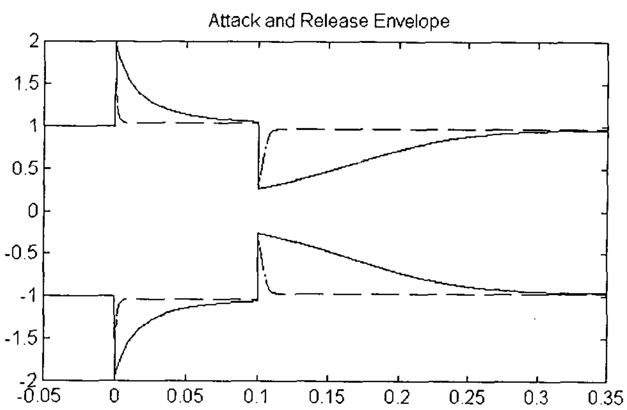
There are second-order effects that could influence the compressor performance. For instance, it was shown that the total harmonic distortion of a compressor, due to detector ripple, is the sum of odd harmonics only. In reality, a nonideal VCA can contribute even harmonics to the spectrum. A not quite symmetrical rectifier in the rms detector adds unwanted harmonics to the compressor spectrum as well. In DSP implementations of the ADCs and DACs add harmonics of their own. Ultimately the noise of the devices, the VCA, and the rms detector operational amplifiers and resistors add to the distortion plus noise number and limit the dynamic range of the detector or compressor.

The transient response can also be slightly different than the one predicted by theory. The timing capacitors have finite equivalent series resistances (ESRs) anywhere from fractions of an ohm to a few ohms. The mathematical model is very complex and is not shown in this paper. The result is a faster rms detector. Other effects of the ESR are ripple increase and eventually higher distortion.

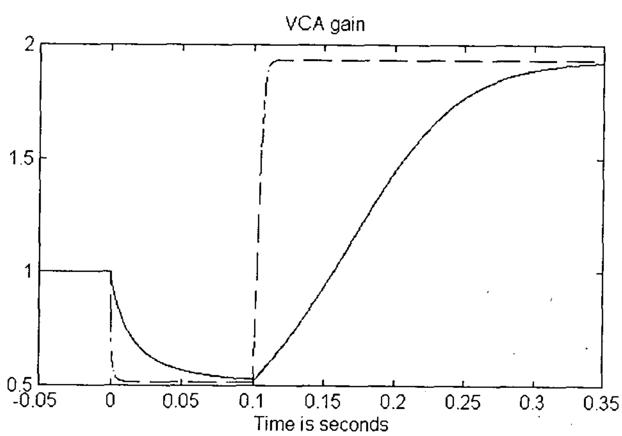
The integrator of the logarithmic output rms detector may be based on a log filter [7]. The structure of the log filter is similar to the linear RC model, but the resistor is replaced by a diode. The diode exhibits dynamic conductance that is a function of the current flowing through it. More current translates into increased conductance. A resistor in series with the diode limits the maximum conductance and substantially complicates the mathematical model of the rms detector. The result is an rms detector with slower attack time. The series resistor is a "natural" component since all diodes have finite series resistors. Also, improper layout design of an rms detector IC metal mask could add unnecessary series resistors.

A linear based compressor can suffer from limitations such as current and voltage availability and ultimately power dissipation. If the filter in the linear rms detector is made of a 3.5-kΩ resistor and a 10-μF capacitor (35-ms time constant), the peak current could be as high as 86 mA when 303 V is applied to the resistor (corresponding to +24 dBu corresponding). This translates to 24-W peak power required from the power supply.

The logarithmic rms detector can suffer from current limitations as well. The current flowing through the logarithmic filter diode is proportional to the square of the input current [7]. A change of +20 dB in the input voltage can translate into a +40-dB current jump in the



(a)



(b)

Fig. 8. Feedforward and feedback compressors. (a) Attack and release gain modulation. (b) VCA gain ——— feedforward; – – – feedback. rms detector time constant 35 ms; reference level 1 V peak; signal frequency 100 Hz; compression ratio 20:1; signal level 6 dB above reference level during attack, 6 dB below reference level during release.

log filter diode. If the reference level current is set to 8.5 μA and the input resistor is 10 kΩ [10], the maximum peak current for a +24-dBu input signal is 3 mA. A proper rms detector design can provide that much current. However, if the logarithmic rms detector input resistor has a lower value, then current clipping can occur. The power dissipation in this case is not an issue, since the maximum voltages are around two diode drops, or about 1.3 V.

5 COMPRESSOR WITH THRESHOLD

In many applications the compressor is implemented with threshold. The input signal is compressed above threshold only. Below threshold the gain of the VCA is 1. The threshold circuit is usually part of the k multiplier block, as shown in Figs. 1 and 2. The transfer function of a compressor with threshold is shown in Fig. 3. The "soft knee" gives the compressor a nice sonic transition.

The transient response of an above threshold compressor is plotted in Fig. 9. (For clarity, only the signal envelope is shown.) The threshold is set to 1 V peak (dash–dot line). The rms detector output is shown in Fig. 9(b). Only the signal above the threshold line is applied to the VCA. If the rms output is below threshold, the VCA gain is 1.

Before $t = 0$, the input signal is below threshold and the VCA gain is 1. Between 0 and 150 ms the input signal is 6 dB above the threshold. The rms detector output increases, but it does not change the VCA gain until it reaches the threshold. Thus until the rms detector output reaches the threshold, the VCA gain is 1 and the compressor puts out the whole input signal. This is an important difference between compressors with and without threshold. In an amplifier or driver protection application the duration of time the protected device has full power applied is very important, and it can determine the compressor time constant. Between 150 and 400 ms the signal drops from 6 to 3 dB above threshold. The compressor output is similar to the one shown in Fig. 8. At 400 ms the input signal drops to 6 dB below threshold and the rms detector output slowly decreases. Although the input signal is below threshold, the VCA gain is modulated by the rms detector, and the output signal is compressed until the rms detector output reaches the threshold.

6 CONCLUSION

At the beginning of the paper it was demonstrated that a compressor based on a logarithmic output rms detector and an exponentially controlled VCA is identical to the one based on a linear output rms detector and a linearly controlled VCA. Therefore all compressor implementations, hardware and software, share similar performance. Table 3 summarizes the performance differences between feedforward and feedback compressor topologies.

A few conclusions can be drawn from Table 3. The performance of the feedback compressor is directly affected by the compression ratio. For a similar transient response, the rms detector time constant in the feedback compressor needs to be multiplied by the compression ratio. This is easy in fixed compression ratio applications. In feedback compressors with adjustable compression ratios the circuit necessary to keep the same time constant is quite complex. The distortion of the feedback compressor is also multiplied by the compression ratio. Thus a feedforward configuration is more appropriate for a compressor.

Theoretically, infinite compression ratio is impossible in feedback compressors. However, a compression ratio of 20 or more approximates an infinite compressor. The major drawback is that the distortion at 1 kHz is 1.4% compared to only 0.07% for the feedforward configuration. In order to maintain the same performance, the rms detector timing capacitor has to be increased 20 times. Thus, the capacitor value can increase to a couple of hundred microfarads. The capacitor is more expensive and requires more PCB area.

In a feedback compressor topology the dynamic range at the input of the rms detector is smaller than the dynamic range at the input of the rms detector in the feedforward compressor topology. The reason is that the

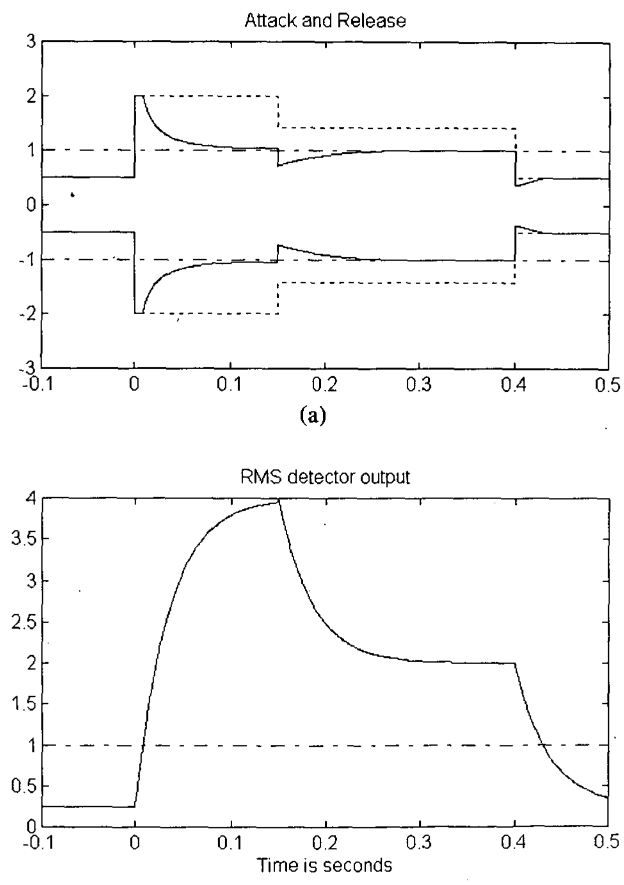


Fig. 9. Feedforward compressor with threshold. (a) —— attack and release gain modulation; — — — input signal peak amplitude; — ·— threshold. (b) —— linear rms detector output; — ·— threshold. Threshold set to 1 V peak. rms detector time constant 35 ms; compression ratio 20:1; signal level 6 dB below threshold (−100 ms, 0 ms), 6 dB above threshold (0 ms, 150 ms), 3 dB above threshold (150 ms, 400 ms), and 6 dB below threshold (400 ms, 500 ms).

Table 3.

	Feedforward	Feedback	Notes
THD at low frequencies or fast time constant	$\frac{C_r - 1}{3C_r + 1}$	$\frac{C_r - 1}{3C_r + 1}$	Distortion is equal at low frequencies
THD at higher frequencies or slow time constant	$\frac{C_r - 1}{8C_r\omega\tau}$	$\frac{C_r - 1}{8\omega\tau}$	Feedback compressor THD is C_r times feedforward THD
THD −3-dB frequency	$\frac{1}{2\tau}\frac{3C_r + 1}{4C_r}$	$\frac{C_r}{2\tau}\frac{3C_r + 1}{4C_r}$	Feedback −3-dB point is C_r times feedforward compressor −3-dB point
Time constant	τ	$\frac{\tau}{C_r}$	For equal rms detector time constants, feedback compressor time constant is divided by C_r

output voltage is already compressed. Thus the feedback compressor is more appropriate in applications that require a large dynamic range, which the rms detector cannot handle.

The feedback configuration is suitable for expander applications. The compressor becomes an expander when output level changes are greater than input level changes. In this case the compression ratio is between 0 and 1. The distortion performance is better than the feedforward expander. This can be explained by the fact that the gain factor k is smaller for a feedback configuration compared to a feedfoward configuration. Therefore there is less ripple passed from the rms detector to the VCA and as a consequence less distortion. The timing capacitor is multiplied by $1/C_r$ also. Thus a smaller timing capacitor is needed. Since the rms detector time constant is divided by the compression, the same disadvantage occurs in applications where the compression ratio is variable.

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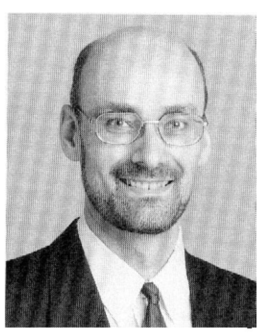

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