Music 320
Autumn 2011–2012
Homework #2
Sinusoids, Complex Sinusoids
155 points
Due in one week (10/13/2011)

Theory Problems

1. (15 pts) [Sinusoids] Define $x(t)$ as

$$x(t) = 2\sin(\omega_0 t + \frac{\pi}{4}) + \cos(\omega_0 t)$$

   (a) Express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$, where ϕ is in radians.

   (b) Find a complex valued signal $\tilde{x}$ such that $x(t) = Re\{\tilde{x}(t)\}$

2. (15 pts) The phase of a sinusoid can be related to time shift as follows:

$$x(t) = A\cos(2\pi f_0 t + \phi) = A\cos(2\pi f_0(t - t_1))$$

   In the following parts, assume that the period of the sinusoidal wave is $T_0 = 8$ sec

   (a) When $t_1 = -2$ sec, the value of the phase is $\phi = \pi/2$. Explain whether this is true or false.

   (b) When $t_1 = 3$ sec, the value of the phase is $\phi = 3\pi/4$. Explain whether this is true or false.

   (c) When $t_1 = 7$ sec, the value of the phase is $\phi = \pi/4$. Explain whether this is true or false.

3. (15 pts) [Sinusoids]

   (a) For a sinusoid with a period $T_0 = 8.0$ seconds, what is the frequency f_0 in Hz?

   (b) Define $x(t)$ as

$$x(t) = A\sin(w(t - \tau))$$

   Write an expression for the phase in terms of the frequency and time delay.

   (c) For $x(t)$ defined as above, find the phase at $t = 0$ for a time delay of $\tau = .25$ seconds and frequencies of 2 Hz and 3 Hz.

4. (35 pts) [Complex Sinusoids] Define the discrete-time generalized sinusoid $x[n] = X z_0^n$ for $n = 0, 1, 2, \ldots$ where

$$X = 2e^{j\pi/4}$$

$$z_0 = 0.9e^{j\pi/8}$$

(a) (5 pts) What is the amplitude of this sinusoid? What is the phase in radians? What is the phase in cycles? What is the phase in degrees?

(b) (5 pts) What is the time constant τ of decay (in samples)?

(c) (5 pts) What is the 60 dB decay time T_{60} in time constants?

(d) (5 pts) What is T_{60} in samples?

(e) (5 pts) What is the 80 dB decay time T_{80} in time constants?

(f) (5 pts) What is T_{80} in samples?

(g) (5 pts) If the sampling rate is 800 Hz, what are τ and T_{60} in seconds, and what is the frequency of the sinusoid in Hz?

Lab Assignments

For all lab assignments, submit your M-file scripts, functions, and figures in one zip file through coursework[1]. Within coursework, upload the zip file using the Drop Box menu.

The zip file should be named with your last name, first name and homework number. Each problem should be named with your last name, first name, homework number, and the problem number. So, for John Doe's zip file, the file should be titled `doe_john_hw2.zip`. For John Doe's answer to problem 2 on homework 2, the file would be titled `doe_john_hw2_q2.m`. Also, at the beginning of each script, include the following comment:

```
% Your Name / Lab # - Question #
```

For problems with question(s), include your answer(s) in the body of the script files as comments.

1. (10 pts) Write a Matlab script that generates a sinusoidal wave.
   (a) Its length must be 5 seconds, with 44100 samples per second (CD quality).
   (b) Its amplitude level must be -6 `[dB]`.
   (c) Use 440 `[Hz]` for its frequency.
   (d) Your script must be able to save the wave as a sound file named `'mysound.wav'` and play it. Note that you do not need to submit the sound file: just submit your script.

   (hint: `wavwrite` and `sound` can come handy)

2. (20 pts) Define the discrete-time generalized sinusoid $x(n) = Xz_0^n$ for $n = 0, 1, 2, ...$, where

$$X = 2e^{j\pi/4}$$

$$z_0 = 0.9e^{j\pi/8}$$

   (a) Plot $\text{re}\{Xz_0^n\}$ and $\text{im}\{Xz_0^n\}$ versus n.
   (b) Plot Xz_0^n as a collection of points in the complex plane (imaginary part versus real part).
   (c) Mark the time constant τ of decay on the plots.

[1] http://coursework.stanford.edu

(d) Mark the 60 dB decay time T_{60} on the plots.

3. (25 pts) Additive synthesis, the sum of cosine waves, can be given by

$$y(t) = \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \phi_k)$$

where A_k, f_k, and ϕ_k mean the peak amplitude, frequency, and initial phase of kth sinusoidal component. Also, N is the number of frequency and amplitude components. Write a Matlab function that implements this synthesis method and saves the result as an audio file. The syntax of your function should be:

```
function y = additive(f, Z, fs, dur, name)

% function y = additive(f, Z, fs, dur, name)
% f: vector of frequencies in Hz
% Z: vector of complex amplitudes A*exp(j*phi)
% fs: sampling rate in Hz
% dur: total duration of the signal in seconds
% name: name of the output audio file
% f and Z must be of the same length:
% Z(1) corresponds to f(1) and so on.
% Your Name / Lab 2-2
```

Remember:

(a) Your function must be able to take any length of `f` and `Z`, as long as they are of the same length.

(b) Note that `Z` is a vector of complex amplitudes (that is, phasors), not real numbers.

(c) Try to make it run as fast as possible: can you implement this without using any loop in your code?

(Hints: Obtain the real sinusoid by taking the real part of a complex sinusoid. For a fast implementation, think about vector and matrix multiplication instead of loops)

4. (20 pts) For each of the following complex signals $x[n]$, plot the first $10,000$ samples in the Z-plane:

(a) $x[n] = e^{j\frac{\pi}{4}}(0.999e^{j\frac{\pi}{N}})^n$
(N = 512)

(b) $x[n] = e^{j\frac{\pi}{2}}(0.999e^{-j\frac{\pi}{N}})^n$
(N = 2048)

(c) $x[n] = (e^{j\frac{\pi}{N}})^n + (0.9995e^{(-j\frac{4\pi}{N})})^n$
(N = 1024)