Music 320
Autumn 2011–2012
Homework #1
Complex Numbers, Polynomials, Trigonometry
55 points
Due in one week (10/6/2011) at 11:59pm

Theory Problems

1. (10 pts) Find the roots of the following polynomials ($j = \sqrt{-1}$):

   1. $x^2 + 2x + 1$
   2. $5x^2 + 6x + 1$
   3. $5x^2 - 2x + 1$
   4. $x^3 + 2x^2 + x$
   5. $ax^2 + bx + j$
   6. $jx^2 + jx + j$

2. (10 pts) For the complex number $z = x + jy$, where x and y are real, find:

   (a) real part
   (b) imaginary part
   (c) modulus
   (d) phase
   (e) complex conjugate
   (f) reciprocal in rectangular form
   (g) reciprocal in polar form

3. (10 pts) Derive the identities

$$\begin{aligned} \cos(a+b) &= \cos(a)\cos(b) - \sin(a)\sin(b) \\ \sin(a+b) &= \sin(a)\cos(b) + \cos(a)\sin(b) \end{aligned}$$

   using Euler's identity and the basic rule of exponents

$$e^{j(a+b)} = e^{ja}e^{jb}.$$

4. (5 pts) Using DeMoivre's formula, find $(3/5 + j4/5)^{100}$ in polar form.

5. (10 pts) Convert the following expressions to both Cartesian and polar forms (a, b, c, and d are real). Be sure to include *all* possible solutions.

   (a) $(1+j)^2$
   (b) $(a+jb)/(c+jd)$
   (c) $e^{j\pi} + 1$
   (d) $\sqrt{1+j}$
   (e) $e^{e^{j\theta}}$
   (f) $(-1)^{1/10}$
   (g) $\ln(j)$
   (h) j^j
   (i) $\tan(\frac{1+j}{1-j})$

6. (5 pts) If a complex number z is multiplied by $-j$, by how many degrees is z rotated in the complex plane? Is the rotation clockwise or counterclockwise? What is the rotation in radians?

7. (5 pts) Plot the complex numbers $e^{j2\pi k/8}$ in the complex plane for $k = 0, 1, ..., 7$. [Hint: use Euler's identity to find the Cartesian coordinates for each complex number.] On the same plot, draw the unit circle $|z| = 1$.

Lab Assignments

1. For this assignment, the lab is simply to get comfortable with Matlab. Spend some time using the help function (`>> help functionName`) on each of the functions listed below. You should code an example using each of the operators/functions. There is nothing to be turned in for this Lab.

   (a) **operators**: `* .* + - / ./ ' .' : ; ^ .^`

   (b) **math constants**: `1i, 1j, pi, exp(1)`

   (c) **simple math functions**: `angle, conj, abs, real, imag, min, max, sum , exp, log, log10, sin, cos, tan, asin, acos, atan, sqrt`

   (d) **math concepts**: vector/matrix vs. scalar operators, creating vectors and matrices

   (e) **generators**: `ones, zeros, eye, rand, randn, linspace`

   (f) **plotting**: `plot, figure, subplot, xlabel, ylabel, title, legend, grid, axis, hold`

   (g) **audio functions**: `wavread, wavwrite, sound, soundsc`

   (h) **general programming concepts**: functions, plotting, command line vs. scripting vs. functions, control statements (loops and conditional statements using `==, =, <, >, <=, >=`)

   (i) **other useful commands**: `help, clear all, clc, close all, size, length, % (for comments), whos`

   (j) **less useful, but come up**: `eps, format, fliplr, flipud, pause`

   (k) **storing your work**: `disp, print, saveas, save, diary`