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On Peak-Detecting and RMS Feedback and Feedforward Compressors

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ABSTRACT
Differential equations governing the behavior of first-order peak-detecting and RMS feedback and feedforward analog compressors are presented. Based on these equations, the relationship between feedback and feedforward compressor behavior is explored, and simple, accurate digital emulations are provided. Feedback and feedforward gain reduction trajectories are shown to be equivalent by transforming the feedback gain reduction into a feedforward gain reduction having a level-dependent time constant. This time constant has the effect of slowing down the transition into and out of compression, and accounts for much of the difference in compression character between the two architectures.

1. INTRODUCTION

Dynamic range compressors typcially have either a feedback or feedforward architecture, as shown in Fig. 1 and Fig. 2. In a feedforward compressor, the input signal level is estimated in the detector and used by the gain computer to determine the gain reduction applied to the input to form the compressed output. In a feedback compressor, the output signal level is detected and a gain reduction formed. The gain reduction is applied to the input signal to form the compressed output, thus closing the feedback loop.

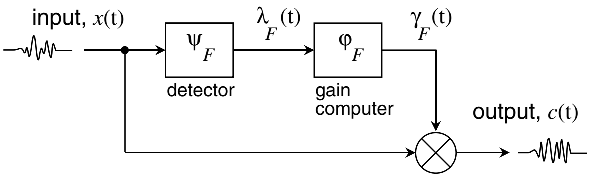

Fig. 1: *Feedforward Compressor. A feedforward compressor is shown with the input signal driving a sidechain level detector ψ_F and gain computer ϕ_F.*

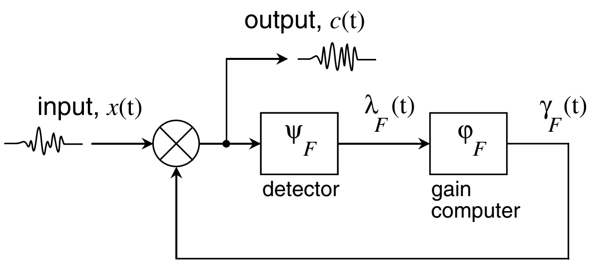

Fig. 2: *Feedback Compressor. A feedback compressor is shown with the sidechain level detector ψ_B and gain computer ϕ_B operating on the output $c(t)$.*

Digital implementation of feedforward compressors is straightforward, and their architecture lends itself to sidechain processing and look-ahead. Feedforward compressors are also able to achieve true limiting. Feedback compressors, by contrast, are known for their fast attack and desirable release, and are often used for analog implementation, as their gain computer needs to be accurate only over a small dynamic range.

In this paper, the relationship between peak-detecting feedback and feedforward analog compressors is explored, and the digital emulation of analog feedback compressors is discussed.

In [1], a hypothesized exponential signal was used to show that a feedback compressor attacks with the detector time constant decreased by a factor equal to the compression ratio. This is in contrast with the feedforward architecture, where the attack and release time constants are roughly equal to the detector time constant. It was pointed out that the release curves for the feedforward and feedback architectures were different. In [2], similar results are obtained for so-called linear and logarithmic compressors having an RMS (root mean square) detector and inverting gain computer.

Here, differential equations governing compressor behavior are developed for the case of first-order peak-detecting and RMS compressors. For the feedback architecture a change of variables is suggested which transforms its differential equation to that of an equivalent feedforward compressor with a transformed gain computer and level-dependent time constant.

This equivalence confirms the results of [1, 2] for a general input and static compression function: the time constant for a feedback compressor is scaled by the instantaneous compression ratio relative to that of a feedforward compressor with the same detector dynamics and static compression curve. It also leads to a simple update for digital implementation of a feedback compressor. Finally it is argued that the level-dependent time constant is responsible for much of the difference in character between the feedforward and feedback compressor architectures.

2. COMPRESSOR ARCHITECTURE

It will be helpful to introduce some notation and definitions.

Consider the compression characteristic shown in Fig. 3. The dB output level ℓ_O is seen to equal the dB input level ℓ_I up to a threshold ℓ_T, and increase $1/\rho$ dB for every dB increase in input level thereafter. The quantity ρ is referred to as the *compression ratio*, and the output level as a function of input level, $\ell_O(\ell_I)$, the *compression characteristic* or *static compression curve*.

In general, the compression characteristic is not a piecewise linear function, and the compression ratio ρ—defined as the inverse slope of the compression characteristic—is level-dependent. However, static compression curves are often reminiscent of that depicted in Fig. 3, with no compression (a ratio of one) below a threshold, and a constant compression ratio above a transition region called the “knee”.

2.1. Gain Computer

The *gain reduction* is the gain applied by the compressor relative to that applied when not in compression. For convenience, in the sequel we assume

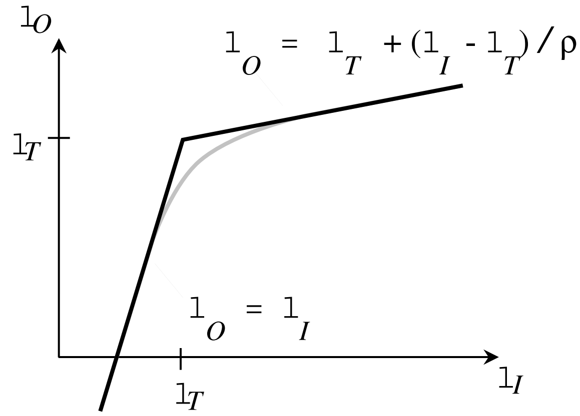

Fig. 3: *Static Compression Curve. An example piecewise linear static compression curve with a compression ratio ρ above the threshold ℓ_T is shown (black) along with a similar static compression curve having a soft knee (gray).*

the gain applied when not in compression, referred to as the *make-up gain*, is one.

With a unity make-up gain, the dB compressor gain is the difference between the dB output and input levels,

$$g = \ell_O - \ell_I. \tag{1}$$

In a feedforward compressor, this gain results from applying the gain computer ϕ_F to an estimate of the input signal level produced by the detector ψ_F. In a feedback compressor, the gain is computed from the estimated output level.

For the piecewise linear compression characteristic under consideration, the output level may be expressed in terms of the input level as follows:

$$\ell_O = \ell_T + (\ell_I - \ell_T)/\rho, \quad \ell_I \geq \ell_T. \tag{2}$$

Substituting for ℓ_O in (1), the dB compressor gain is written in terms of the dB input level,

$$g_F = (1/\rho - 1) \cdot (\ell_I - \ell_T), \quad \ell_I \geq \ell_T. \tag{3}$$

Exponentiating (3), we see that to achieve a constant compression ratio using the feedforward architecture of Fig. 1 (and assuming the detector accurately estimates its input level), the gain computer $\phi_F(\lambda)$ must be

$$\phi_F(\lambda) = \left[\frac{\lambda}{\lambda_T}\right]^{(1/\rho-1)}, \tag{4}$$

where λ and λ_T are the detected input level and threshold in amplitude units.

Expressed in terms of the output level, the dB compressor gain is

$$g_B = (1 - \rho) \cdot (\ell_O - \ell_T), \quad \ell_I \geq \ell_T. \tag{5}$$

Accordingly, the gain computer ϕ_B needed to implement the static compression curve of Fig. 3 in a feedback compressor is

$$\phi_B(\lambda) = \left[\frac{\lambda}{\lambda_T}\right]^{(1-\rho)}, \tag{6}$$

where λ is in this case the detected output level.

Fig. 4 shows feedforward and feedback gain computers for the compression characteristic of Fig. 3. There is unity gain up to the threshold, above which a gain reduction is applied. Since the feedback gain computer operates on the output signal level, its gain reduction takes place over a relatively small dynamic range, particularly for large compression ratios. Because it operates on the input signal level, the feedforward gain computer can generate any output level as a function of input level, and is capable of perfect limiting.

2.2. Detector

While the gain computer determines the static compression characteristic—the gain applied to steady-state signals—the detector determines the "dynamics" of the compressor—the way in which the gain reduction follows the envelope of the signal. Fig. 5 shows a first-order peak detector which estimates signal level in *attack* and *release* phases. When the applied signal $x(t)$ is greater than the signal level estimate $\lambda(t)$, the capacitor charges through the attack resistor; regardless of input signal level, the capacitor discharges (usually much more slowly) through the release resistor. The result is that the signal level estimate obeys separate first-order differential equations, attacking to the signal level with time constant τ_A, and releasing to zero with time constant τ_R:

$$\frac{\partial \lambda}{\partial t} = \begin{cases} \frac{1}{\tau_A} \cdot [x(t) - \lambda(t)], & x(t) \geq \lambda(t), \\ -\frac{1}{\tau_R} \cdot \lambda(t), & x(t) < \lambda(t), \end{cases} \tag{7}$$

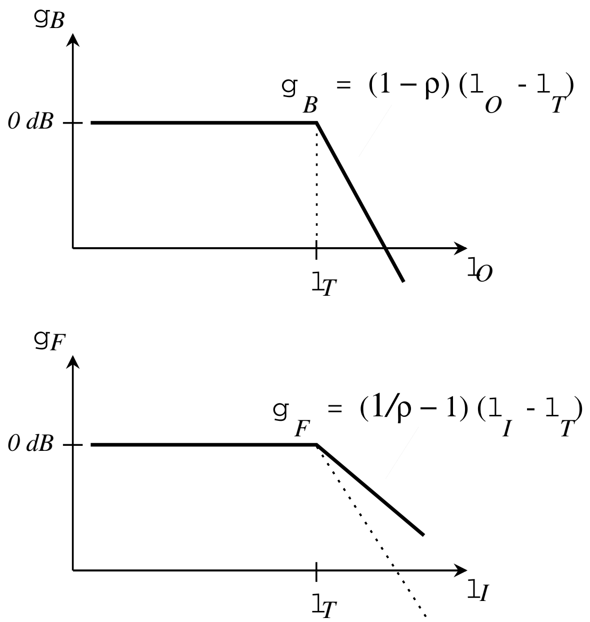

Fig. 4: *Gain Computers. Feedforward and feedback gain computers for the example static compression curve of Fig. 3 are shown as solid lines along with those required to achieve limiting, shown as dashed lines.*

Root mean square (RMS) detectors are also popular, and smooth the square of the applied level. Optical detectors fall into this category, with light being generated in proportion to a function of the applied signal energy. The output of a first-order RMS detector with time constant τ is governed by the following differential equation.

$$\frac{\partial \lambda}{\partial t} = \frac{1}{\tau} \cdot \left[x^2(t) - \lambda(t) \right]. \tag{8}$$

3. GAIN REDUCTION BEHAVIOR

We are now in a position to develop and explore the detector output and gain reduction behavior for feedforward and feedback architectures. We focus on the case of peak detecting compressors, since the results for the RMS detector are identical, as derived in the appendix.

3.1. Feedforward Compression

The gain reduction in a feedforward compressor is simply a sort of smoothed version of the instantaneous signal level translated through the gain computer. During attack, the signal level estimate $\lambda_F(t)$ obeys (cf. (7))

$$\frac{\partial \lambda_F}{\partial t} + \frac{1}{\tau_A}\lambda_F(t) = \frac{1}{\tau_A} x(t). \tag{9}$$

In release, the level estimate follows

$$\frac{\partial \lambda_F}{\partial t} + \frac{1}{\tau_R}\lambda_F(t) = 0. \tag{10}$$

The compressor gain $\gamma_F(t)$ is found by applying the signal level estimate to the gain computer,

$$\gamma_F(t) = \phi_F(\lambda_F(t)). \tag{11}$$

Using the piecewise linear static compression curve of (3) gives

$$\gamma_F(t) = \left[\frac{\lambda_F(t)}{\lambda_T} \right]^{1/\rho - 1}. \tag{12}$$

3.2. Feedback Compression

In a feedback compressor, the detector operates on the gain-reduced signal $\gamma_B(t) \cdot x(t)$, and, during attack, the signal level estimate $\lambda_B(t)$ follows

$$\frac{\partial \lambda_B}{\partial t} = \frac{1}{\tau_A} \left[\gamma_B(t) \cdot x(t) - \lambda_B(t) \right]. \tag{13}$$

The gain $\gamma_B(t)$ is the gain computer operating on the level estimate $\lambda_B(t)$, and may be written as

$$\gamma_B(t) = \left[\frac{\lambda_B(t)}{\lambda_T} \right]^{1-\rho}. \tag{14}$$

Substituting into (13), and rearranging terms gives an expression only in terms of the level estimate and the input signal,

$$\frac{\partial \lambda_B}{\partial t} + \frac{1}{\tau_A}\lambda_B(t) = \frac{1}{\tau_A} \left[\frac{\lambda_B(t)}{\lambda_T} \right]^{1-\rho} x(t). \tag{15}$$

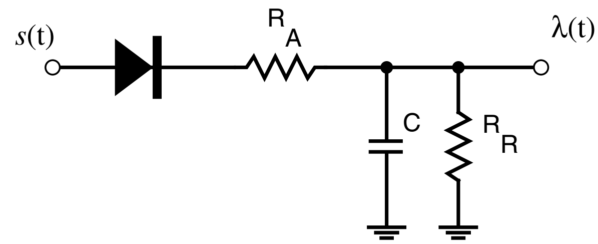

Fig. 5: *Peak Detector Circuit.*

The differential equation (15) turns out to be a so-called Bernoulli equation [3, page 20]; it may be converted to a linear differential equation by the change of variables

$$\mu(t) = \lambda_T \cdot \left[\frac{\lambda_B(t)}{\lambda_T}\right]^{\rho}. \tag{16}$$

The time derivative of the transformed level estimate $\mu(t)$ is

$$\frac{\partial\mu(t)}{\partial t} = \rho\lambda_T \cdot \left[\frac{\lambda_B(t)}{\lambda_T}\right]^{\rho-1} \frac{\partial\lambda_B}{\partial t}. \tag{17}$$

Writing (15) in terms of the transformed level and rearranging terms gives the desired linear first-order differential equation for the transformed level estimate (valid in attack $x(t) \geq \lambda_T$),

$$\frac{\partial\mu}{\partial t} + \frac{\rho}{\tau_A}\mu(t) = \frac{\rho}{\tau_A}x(t). \tag{18}$$

In release, the detector runs open loop, decaying to zero with a time constant τ_R,

$$\frac{\partial\lambda_B}{\partial t} + \frac{1}{\tau_R}\lambda_B(t) = 0. \tag{19}$$

Multiplying both sides of this equation by $[\lambda_B(t)/\lambda_T]^{\rho-1}$ and using (16) gives the differential equation for the transformed level estimate, valid in release.

$$\frac{\partial\mu}{\partial t} + \frac{\rho}{\tau_R}\mu(t) = 0. \tag{20}$$

Finally, the gain computer may also be written in terms of the transformed level estimate,

$$\gamma_B(t) = \left[\frac{\lambda_B(t)}{\lambda_T}\right]^{1-\rho} = \left[\frac{\mu(t)}{\lambda_T}\right]^{1/\rho-1}. \tag{21}$$

3.3. Architecture Comparison

To compare the behavior of peak-detecting feedback and feedforward compessors, it is helpful to reproduce the equations determining the time evolution of their respective gain reductions when given the same input and configured to have the same static compression curves. The feedforward compressor gain follows

$$\gamma_F(t) = \left[\frac{\lambda_F(t)}{\lambda_T}\right]^{1/\rho-1}, \tag{22}$$

with

$$\frac{\partial\lambda_F}{\partial t} = \begin{cases} \frac{1}{\tau_A} \cdot [x(t) - \lambda_F(t)], & x(t) \geq \lambda_F(t), \\ -\frac{1}{\tau_R} \cdot \lambda_F(t), & x(t) < \lambda_F(t). \end{cases} \tag{23}$$

For a feedback compressor having the same piecewise linear compression characteristic, the compressor gain is

$$\gamma_B(t) = \left[\frac{\mu(t)}{\lambda_T}\right]^{1/\rho-1}, \tag{24}$$

where the transformed detector output $\mu(t)$ follows

$$\frac{\partial\mu}{\partial t} = \begin{cases} \frac{\rho}{\tau_A} \cdot [x(t) - \mu(t)], & x(t) \geq \mu(t), \\ -\frac{\rho}{\tau_R} \cdot \mu(t), & x(t) < \mu(t), \end{cases} \tag{25}$$

The equations for the feedforward and feedback compressor gains are identical, save a factor of the compression ratio ρ scaling the attack and release time constants. In other words, a feedforward compressor having a piecewise linear static compression curve and first-order peak detector will produce the same output as a feedback compressor sharing its static compression curve and detector, but with time constants made smaller by the compression ratio. This is seen in Fig. 6 which shows feedback and feedforward compressor gains in response to a signal with a stepped amplitude.

It can be argued that the equations above approximately hold in the neighborhood of any point in time for a general static compression curve if the compression ratio ρ were replaced by its instantaneous value determined by the level at that time. In this way, feedforward-style equations may be used to simulate a feedback compressor. Denoting by T the sampling interval, the compressor gain may be updated as follows:

$$\gamma_B(t_0 + T) = \left[\frac{\mu(t_0 + T)}{\lambda_T}\right]^{1/\rho-1}, \tag{26}$$

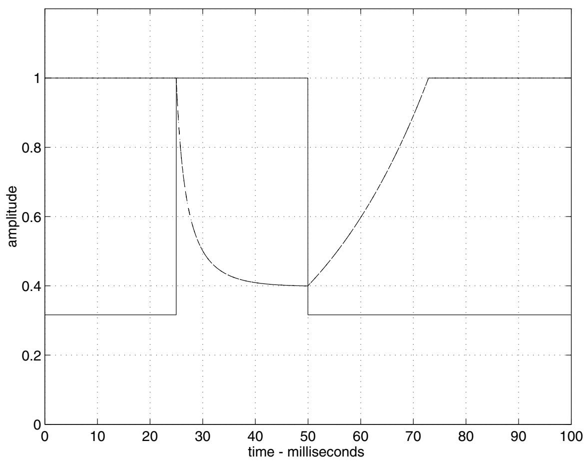

Fig. 6: *Signal Envelope, Feedforward and Feedback Compressor Gains. Overlapping feedforward (dashed) and feedback compressor (dash-dot) gain trajectories are shown along with the input signal envelope (solid). The compressor gain computers were adjusted to have the same piecewise linear static compression curve.*

with

$$\mu(t_0+T) = \mu(t_0) + \begin{cases} \dfrac{T\rho(\mu(t_0))}{\tau_A} \cdot [x(t_0) - \mu(t_0)], \\ -\dfrac{T\rho(\mu(t_0))}{\tau_R} \cdot \mu(t_0), \end{cases} \tag{27}$$

where the upper expression is used in attack, $x(t) \geq \mu(t)$, and the lower one either just in release, or at every sample.

While feedback and feedforward peak detecting compressors can be made equivalent in the presence of a constant compression ratio, they cannot be made the same if the compression ratio is level-dependent. This may be seen by considering the scenario of Fig. 7 showing a compressor releasing through a static compression function having a knee over which the compression ratio $\rho(\lambda)$ transitions from its large signal value to one. As a feedforward compressor releases, the compressor gain release slows as a result of the decreasing compression ratio through the knee. A feedback compressor, on the other hand, will experience additional slowing of the gain release since its detector time constant also slows as the compression ratio decreases. In general, over concave sections of the static compression curve, feedback compressor gain trajectories will decelerate on release and accelerate on attack compared with the gain trajectories of "equivalent" feedforward compressors which have fixed time constants.

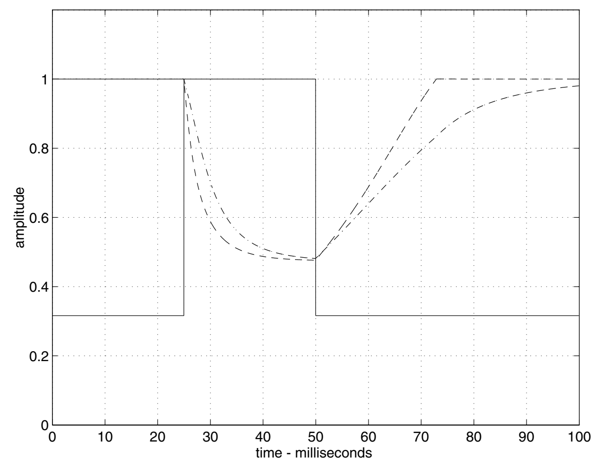

Fig. 7: *Signal Envelope, Feedforward and Feedback Compressor Gains. Feedforward (dashed) and feedback compressor (dash-dot) gain trajectories are shown along with the input signal envelope (solid). The compressor gain computers were adjusted to have the same static compression curve with a soft knee.*

4. SUMMARY

Using a change of variables, the equations governing the time evolution of the gain reduction in a feedback compressor were converted to those describing a feedforward compressor with a time constant proportional to the compression ratio. This result makes it clear that feedforward and feedback compressors can be made equivalent when the compression ratio is constant. However, in the presence of a changing compression ratio, feedback compressors will take longer to get into or out of compression than equivalent feedforward compressors.

Finally, these results were derived for the case of first-order peak- and RMS-detecting compressors. They are likely only applicable to the case of higher-order, program-dependent detectors when the time constants are disparate.

5. APPENDIX: RMS DETECTOR

Here, we show that feedforward and feedback compressors using first-order RMS detectors enjoy the same equivalence as do those employing peak detectors.

In a feedforward configuration, the detector output $\lambda_F(t)$ follows

$$\frac{\partial \lambda_F}{\partial t} + \frac{1}{\tau}\lambda_F(t) = \frac{1}{\tau}x^2(t). \tag{28}$$

The gain computer requires an additional square root to compensate for the square signal level tracked in the detector,

$$\gamma_F(t) = \left[\frac{\lambda_F(t)}{\lambda_T^2}\right]^{(1/\rho-1)/2}. \tag{29}$$

In feedback,

$$\frac{\partial \lambda_B}{\partial t} + \frac{1}{\tau}\lambda_B(t) = \frac{1}{\tau_A}\left[\frac{\lambda_B(t)}{\lambda_T^2}\right]^{(1-\rho)/2} x^2(t). \tag{30}$$

This, too, is a Bernoulli equation, and a similar substitution can be made to convert it to a linear differential equation:

$$\mu(t) = \lambda_T^2 \cdot \left[\frac{\lambda_B(t)^2}{\lambda_T}\right]^{\rho}. \tag{31}$$

We have

$$\frac{\partial \mu}{\partial t} + \frac{\rho}{\tau}\mu(t) = \frac{\rho}{\tau}x^2(t). \tag{32}$$

The feedback gain computer may also be written in terms of the transformed square level estimate,

$$\gamma_B(t) = \left[\frac{\mu(t)}{\lambda_T^2}\right]^{(1/\rho-1)/2}. \tag{33}$$

6. REFERENCES

[1] A. J. Oliveira, "A feedforward side-chain limiter/compressor/de-esser with improved flexibility," *Journal of the Audio Engineering Society*, vol. 37, no.4, pp. 226–240, April 1989.

[2] Fred Floru, "Attack and release time constants in RMS-based feedback compressors," *Journal of the Audio Engineering Society*, vol. 47, no. 10, pp. 788–, October 1999.

[3] Carl M. Bender and Steven A. Orzag, *Advanced Mathematical Methods for Scientists and Engineers*, McGraw Hill, New York, 1979.