Music 320
Autumn 2011–2012
Homework #4
Signal Metrics, DFT, Fourier Theorems
155 points
Due in one week (10/27/2011) at 11:59pm

Theory Problems

1. (10 pts) [Orthogonal Projections] Find the orthogonal projection of the signal $y[n] = n$ (i.e., $y = [0, 1, 2, 3]$), onto each of the signals x_k, $k = 0, 1, 2, 3$, where

$$x_k(n) = (W_4^k)^n = e^{\frac{j2\pi kn}{4}}$$

Example:

$$y_{x_0}[n] \triangleq \frac{\langle y, x_0 \rangle}{\|x_0\|^2} x_0[n] = \frac{6}{4} x_0[n] = [1.5, 1.5, 1.5, 1.5]$$

2. (10 pts) [Orthogonal Projections] Add the orthogonal projections together from the previous problem. That is, find the sum

$$s[n] \triangleq y_{x_0}[n] + y_{x_1}[n] + y_{x_2}[n] + y_{x_3}[n]$$

Explain the significance of the above operations in the spectrum analysis context. What are we doing from a geometric perspective?

3. (30 points) If $Y(k)$ denotes the kth element of the length N DFT of y, show that:

   (a) $\text{im}\{y\} = 0 \Longleftrightarrow Y(k) = \overline{Y[N-k]}$ (DFT{real} is *Hermitian*)
   (b) $\text{re}\{y\} = 0 \Longleftrightarrow Y(k) = -\overline{Y[N-k]}$ (*anti-Hermitian*)
   (c) y even $\Longleftrightarrow Y$ even
   (d) y odd $\Longleftrightarrow Y$ odd
   (e) y real, even $\Longleftrightarrow Y$ real,even
   (f) y real, odd $\Longleftrightarrow Y$ imag, odd
   (g) y imag, even $\Longleftrightarrow Y$ imag, even
   (h) y imag, odd $\Longleftrightarrow Y$ real, odd

4. (15 pts) [Even and odd parts] A sequence $x[n]$ is said to be *even* if $x[-n] = x[n]$, and *odd* if $x[-n] = -x[n], n = 0, 1, 2, \ldots, N-1$, where indexing is carried out modulo N. Find the even and odd parts of the following sequences:

(a) $[0, 1, 2, 1]$

(b) $[0, 1, 0, -1]$

(c) $[0, 1, 2, 3]$

(d) $x(n) = n^2,\ n = 0, 1, 2, ..., N-1$

5. (30 pts) [DFT of a sinusoid] For the simple sinusoid

$$x(n) = \cos(2\pi f n T)$$

where $T = 1$ (sampling rate f_s set to 1) and FFT size of $N = 8$, find and plot (without using `Matlab`)

(a) $|X(\omega_k)|$ when $f = f_s/4$

(b) $|X(\omega_k)|$ when $f = f_s/4 + 0.5/N$

(c) Explain the difference in results you obtained in the previous problems, and describe how you can avoid the spectral leakage found in the spectrum of (b).

Lab Assignments NOTE: change of file naming conventions!

For all lab assignments, submit your M-file scripts, functions, and figures in one zip file through coursework[1]. Within coursework, upload the zip file using the Drop Box menu.

The zip file should be named with your initials and the homework number. Within the zip file, please have a single folder named hw4. Within the hw4 folder, you can title each question as q1, q2, etc. So, for John Doe's zip file, the file should be titled `jd_hw4.zip`. For John Doe's answer to question 2 on homework 4, the file would be titled `hw4/q2.m`. For any required functions, please leave the functions as titled by their function name.

1. (10 pts) [Plotting magnitude spectrum] Write a function that will quickly plot a magnitude spectrum of a signal.

```
function plotspec (x, fs)

% function plotspec (x, fs)
% A function to quickly plot the spectrum of a time domain signal
%
% plotspec(x)
```

[1] http://coursework.stanford.edu

```
%   when no sampling rate is specified, normalize frequency
%
% plotspec(x, fs)
%   when a sampling rate is given, set the frequency axis in [Hz]
%
% Your Name / Lab 4-1
```

(a) Use `[dB]` scale for magnitude spectrum.

(b) If there is no input for `fs`, use normalized frequency [-0.5, 0.5] for the horizontal axis.

(c) If necessary, use the following structure to support optional arguments.

```
if nargin == 1
    ...
else
    ....
end
```

(d) Label your plot carefully (using `title`, `xlabel`, and `ylabel`).

2. (40 points) [Spectrum of a "time-slice"] Write a function that plots the magnitude spectrum of an input signal at a specific time t.

```
function plotspec_st (x, fs, time, Nf)

% function plotspec_st (x, fs, time, Nf)
% A function to quickly plot the spectrum of a signal
% segment centered at a given time
%
% x: input signal (assume a row vector)
% fs: sampling rate of x
% time: the time at which you want to see the spectrum
% Nf: frame (or slice) size
%
% Your Name / Lab ##
```

Remember to do the following:

(a) Apply a hann window (same size) to the sliced short signal.

(b) Zero-pad your windowed signal with zpf of 8.

(c) Plot the magnitude spectra of the windowed, zero-padded signal using your `plotspec` function.

3. (10 points) Use the following Matlab command sequence to generate a chirp signal going up:

```
w1=100; w2=3000; %(Hz)
T=3; %(sec)
fs=8000; %(Hz)
dT=1/fs;
t=(0:dT:T);
up = chirp(t,w1,T,w2);
```

Using your function from the previous problem, plot the spectrum of the `up` signal just generated at 0.1 [sec], 1.5 [sec], and 2.9 [sec]. Submit these three plots in one figure (as a Matlab figure file); in the title of each plot, indicate the time it was taken. Do they verify that your function works as expected?