Music 421A
Winter 2011-2012
Homework #6
Due in one week

Theory Problems

1. (5 pts) Show that *pink noise* has the same average power in any octave. [Hint: "average power" is the same thing as "sample variance". Recall that the sample variance within a frequency band is obtained by integrating the sample power spectral density across that band.]

2. (10 pts) **Processing Gain & Impulse Response Measurements**
To simulate the acoustics of a room, a room impulse response $h(t)$ can be estimated via a speaker and microphone. For this problem, we assume that the measured response $r(t) = h(t) + n(t)$ is corrupted by additive white noise $n(t)$ having zero mean and variance σ^2. By knowing $h(t)$ is a deterministic signal (adds coherently) and $n(t)$ is random (adds non-coherently), we can reduce the effect of the noise and get a better estimate of the impulse response. Practical impulse response measurement techniques typically increase the *processing gain* of the measurement system in this way. The processing gain can be quantified as the signal-to-noise ratio (SNR) gain.

   As a baseline measurement, we first make an impulse response measurement by sending $s(t) = \delta(t)$ to a speaker and recording the response $r(t)$, where $\delta(t)$ is a unit impulse, resulting in $r(t) = (s * h)(t) + n(t) = (\delta * h)(t) + n(t) = h(t) + n(t)$. To receive full credit, you must show your work:

   (a) (3 pts) What is SNR gain of a double-amplitude impulse $2\delta(t)$ when compared to the unit impulse $\delta(t)$?

   (b) (3 pts) What is the SNR gain of 2 repeated measurements $r_1(t)$ and $r_2(t)$ averaged together?

   (c) (4 pts) Length L complementary sequences $a(t)$ and $b(t)$ are constructed so that $(a \star a)(t) + (b \star b)(t) = 2L\delta(t)$. If you record the system with $s(t) = a(t)$ and $s(t) = b(t)$ separately and use this previously mentioned property, what is the SNR gain?

3. (13 pts) **Spectral Smoothing**
Suppose we have used the FFT to compute the spectrum $X(\omega_k)$, $k = 0, 1, \ldots, N-1$ of a length $M = N/2$ signal frame $x_m(n)$ centered at time mR samples using a length M (causal) rectangular window with unit amplitude, and suppose we now smooth this spectrum using the length N smoothing kernel $H = [1, 0, -1/2, 0, 0, \ldots, 0, 0, -1/2, 0]$.

   (a) (5 pts) What is the new effective window in the time domain?

   (b) (5 pts) Given that the maximum COLA hop size for the original window was M, what is the maximum COLA hop size for the new effective window?

(c) (3 pts) Verify your result by plotting the new effective window, overlapped and added to itself at the new maximum COLA hop size, using 5 copies of the window in the overlap-add (corresponding to 5 successive values of m).

Lab Assignment

1. (20 pts) **Resynthesizing colored noise with an LTI filter**
   Colored noise can be modeled as a filtered white noise. Here we estimate the filter given a recording of colored noise so that we can resynthesize the noise by passing white noise through the filter.

   (a) (5 pts) Download the sound file `airplane_noise.wav`[1]. Compute and plot the power spectral density using Welch's method. Set the window length to 2048. In your work, do not use functions in Matlab, such as `pwelch` or `periodogram` (but you may refer to them).

   (b) (5 pts) Now you can get the amplitude response of the noise by taking square-root of the power spectral density. In order to estimate the filter coefficents, you also need the phase response. A trivial solution is just to set the phase response to zero. However, this will produce a non-causal symmetric impulse response, which is not what we want. Here, we derive the phase response from the amplitude response by assuming that the filter has minimum-phase. A nice propoerty of minumum-phase filter is that log-magnitude and phase form a Hilbert transform pair. Thus, the phase response (imaginary part of the log spectrum) can be obtained from the amplitude response (real part of the log spectrum) by Hilbert transform [2]. Using this propoerty, derive the frequecny response of the noise (You can use `hilbert` function in Matlab). For sanity check, plot the amplitude response from the frequency response and compare it to the power spectral density above.

   (c) (5 pts) Finally, you can estimate filter coefficients by fitting the transfer function of a LTI filter to the complex frequency response above. This can be done by `invfreqz` function in Matlab. Using this function, find the appropriate number of cofficients for both numerator and denominator by synthesizing the noise and comparing it to the original recording. Optionally, you can add weighting factor in `invfreqz` (that is, by giving more weight to low frequencies than high frequencies).

   (d) (5 pts) Compute and plot the power spectrum density of the synthesized noise using Welch's method. Compare it to that from the original noise.

[1] `http://ccrma.stanford.edu/~jos/sasp/hw/airplane_noise.wav`
[2] `http://ccrma.stanford.edu/~jos/filters/Hilbert_Transform_Relations.html`