Stanford University **CS 183 – Startup**

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PROBLEM SET 4

Due: Wednesday, May 16, 11:59PM

Note: Each student must submit their solutions to the form at http://bit.ly/IE8MK9 for grading.
To earn partial credit, please submit your work to Lore (Coursekit) in .xls, .pdf, .doc, or .docx format
Unless otherwise specified, **answer to 2 decimal places**, omit symbols, and express percentages as "0.XX"

This problem set explores the following concepts:

- Distribution channels, cost per user acquisition and customer lifetime value
- Exponential growth and viral coefficients

Problem 1 – Distribution Channels

You are the founder of a startup providing a content-focused service online. The product has roughly zero variable cost per user, but acquiring users unfortunately requires effort and spending. You are currently evaluating your potential different distribution channels, with the likely candidates being search engine marketing (SEM), direct mail and using an outsourced call center.

You've evaluated the metrics of prior customers obtained through each of these distribution channels in statistically significant tests, and want to decide how much to spend on each channel.

Starting with the SEM channel, you have the following customer data:

CPM for your best campaign:	**$0.35**
Click through rate (CTR) for your best campaign:	**0.04%**
Signup rate, per click:	**35%**
Conversion rate (to paying user):	**6%**
Churn (attrition) rate per month:	**10%**
Monthly plan price (only one plan exists on your site):	**$8.00**

For this SEM channel, please answer the following:

A. What is the average customer lifetime, in months?

$$Retention = \frac{1}{Churn\ rate} = \frac{1}{.1} = 10\ months$$

B. What is the average revenue per month from each user who signs up?
(Please submit your answer as a number with two decimal places, like "X.YZ" or "0.YZ")

$$Average\ Revenue = Revenue\ per\ paying\ customer * percent\ paying$$
$$= \$8 * \%6 = \$0.48\ per\ month$$

C. What is the average customer lifetime value (CLV)?
(Please submit your answer as a number with two decimal places, like "X.YZ" or "0.YZ")

$$CLV = Average\ Revenue * Lifetime$$
$$= \$.048 * 10 = \$4.8$$

D. What is the cost per click (CPC) for this campaign?
(Please submit your answer as a number with two decimal places, like "X.YZ" or "0.YZ")

$$CPC = \frac{CPM}{CTR*1000} = \$0.35*0.04\% = \$0.875$$

E. What is the cost per customer acquisition for this campaign?
(Please submit your answer as a number with two decimal places, like "X.YZ" or "0.YZ")

$$CPA = \frac{CPC}{Conversion\ rate} = \frac{\$0.875}{.35} = \$2.5$$

F. Is CLV > CPA for this campaign?
(Please submit your answer as 0 or 1, where "0" indicates "false" and "1" "true")
***Yes**, $4.8 > $2.5*

G. What is the ROI on ad spend in this SEM campaign?

(Please submit your answer as a number with two decimal places, like "X.YZ" or "0.YZ")

$$ROI = \frac{(CLV-CPA)}{CPA} = \frac{(\$4.8-\$2.5)}{\$2.5} = 92\%$$

You heard one or two other startups in your space have had success with direct mail and call center distribution channels, and want to consider these options as well. The metrics you are able to pull for customers obtained through the direct mail channel, in tests your company has run, are:

Cost per customer acquisition:	**$5.20**
Conversion rate (to paying user):	**10%**
Churn (attrition) rate per month:	**12.5%**

H. What is the average customer lifetime value (CLV) for customers acquired through direct mail?
(Please submit your answer as a number with two decimal places, like "X.YZ" or "0.YZ")

$$Lifetime = \frac{1}{Churnrate} = \frac{1}{12.5\%} = 8\ months$$

$$AvgRev = Converstion\ rate * Rev/month$$
$$= \$8 * 10\% = \$.8$$

$$CLV = Lifetime * Avg\ Revenue = \$.8 * 8 = \$6.4$$

I. What is the ROI on ad spend for this direct mail campaign?

(Please submit your answer as a number with two decimal places, like "X.YZ" or "0.YZ")

$$ROI = \frac{(CLV - CPA)}{CPA} = \frac{(\$6.4 - \$5.2)}{\$5.2} = 23\%$$

Finally, the metrics for a cohort of users that you obtained through a 3rd party call center are:

Cost per hour per call representative:	**$24.00**
Calls per hour made:	**80**
Conversion rate from call to signup:	**2%**
Conversion rate (to paying user):	**8%**
Churn (attrition) rate per month:	**5%**

J. What is the cost per user acquisition associated with this distribution channel?
(Please submit your answer as a number with two decimal places, like "X.YZ" or "0.YZ")

$$Cost/call = \frac{cost/hr}{calls/hr} = \frac{\$24}{80} = \$.3$$

$$CPA = \frac{cost\ per\ call}{conversion\ rate} = \frac{\$.3}{.02} = \$15$$

K. What is the average customer lifetime value (CLV) through the call center channel?
(Please submit your answer as a number with two decimal places, like "X.YZ" or "0.YZ")

$$Lifetime = \frac{1}{Churnrate} = \frac{1}{5\%} = 20\ months$$

$$AvgRev = Converstion\ rate * Rev/month$$
$$= \$8 * 8\% = \$.64$$

$$CLV = Lifetime * Avg\ Revenue$$
$$= \$.64 * 20 = \$12.8$$

L. What is the ROI on spend through this call center distribution channel?
(Please submit your answer as a number with two decimal places, like "X.YZ" or "0.YZ")

$$ROI = \frac{(CLV - CPA)}{CPA} = \frac{(\$12.8 - \$15)}{\$15} = -15\%$$

Problem 2 – Viral/SEM distribution

Building on problem 1, you ran some more customer acquisition tests, and have refined data on each channel's user acquisition costs and conversion and retention metrics. Based on this new data, you've decided to use a focused SEM campaign. The SEM metrics from these tests are:

CPM for your best campaign:	**$0.50**
Click through rate (CTR) for your best campaign:	**0.05%**
Signup rate, per click:	**25%**
Conversion rate (to paying user):	**5%**
Churn (attrition) rate per month:	***TBD***
Monthly plan price (only one plan exists on your site):	**$8.00**

Given that you ran these acquisition tests just a few weeks ago, you don't have an accurate churn rate for the type of users now being obtained through this optimized SEM campaign, which is targeting different sites and using slightly different keywords.

A. What can your monthly user churn rate be, at maximum, to make ad spend through this SEM customer acquisition channel break-even (i.e. ROI = 0%)?
(Please submit your answer as a number with two decimal places, like "X.YZ" or "0.YZ")

$$CPC = \frac{CPM}{CTR*1000} = \frac{\$0.5}{0.0005*1000} = \$1$$

$$CPA = \frac{CPC}{Conversion\ rate} = \frac{\$1}{.25} = \$4$$

$$Monthly\ rev = (Rev/mo)*Conversion\ rate$$
$$= \$8 * .05 = \$.4$$

$$Months = \frac{CPA}{Monthly\ rev} = \frac{\$4}{\$.4} = 10\ months$$

$$Churn = \frac{1}{Months} = \frac{1}{10} = .1$$

B. If you assume your average customer lifetime is 2 years, what is the ROI on these ads?
(Please submit your answer as a number with two decimal places, like "X.YZ" or "0.YZ")

$$CLV = Lifetime * Avg\ Revenue$$
$$= 24 * \$.4 = \$9.6$$

$$ROI = \frac{(CLV - CPA)}{CPA} = \frac{(\$9.6 - \$4)}{\$4} = 140\%$$

Your senior VP of marketing comes to your cubicle with some good and bad news. First, following the expiration and renegotiation of your contract with your content provider, the assumption of the variable cost per user as zero is no longer reasonable. The content contract has switched over to a per-user format, and every customer on your site is costing you **30 cents per month**, on average. The good news is based on analysis of the latest SEM customer cohorts, their churn rate implies an average customer lifetime of **30 months**.

C. What is the gross margin of your product under this new license agreement?
That is, what is your average gross profit per user, divided by the average revenue per user?

(Please submit your answer as a number with two decimal places, like "0.YZ")

$$\textit{Gross revenue} = (\textit{rev}/\textit{mo}) * \textit{Conversion rate}$$
$$= \$8 * .05 = \$.4$$

$$\textit{Gross profit} = \frac{\textit{Gross revenue} - \textit{Cost}/\textit{user}}{\textit{Gross revenue}}$$

$$= \frac{\$.4 - \$.3}{\$.4} = 25\%$$

D. What is now the CLV of customers acquired through this SEM channel?
(Please submit your answer as a number with two decimal places, like "X.YZ" or "0.YZ")

$$\textit{ARPU} = (\textit{rev}/\textit{mo}) * \textit{Conversion rate}$$
$$= \$8 * .05 = .4$$

$$\textit{CLV} = \textit{ARPU} * \textit{Gross profit} * \textit{Months}$$

$$= .4 * .25 * 30 = \$3$$

E. What is the new ROI associated with ad spending through this SEM channel?
(Please submit your answer as a number with two decimal places, like "X.YZ" or "0.YZ")

$$\textit{RIO} = \frac{\textit{CLV} - \textit{CPA}}{\textit{CPA}} = \frac{\$3 - \$4}{\$4} = -25\%$$

You are not thrilled with these new SEM and content licensing numbers, so you talk with your product designer to see what can be done. Together, you decide that a few viral features could be added to the product. They wouldn't be core product features or unleash exponential growth, but could help reduce the effective CPA by getting each customer acquired to invite a few friends.

The product manager designs some features which are then added to the product. There is some "viral" activity now by the newly acquired users, with the following metrics:

Email invitations sent per user (on average, total):	**2.5**
Email invitation open rate (by recipient):	**60%**
Opened email conversion rate (to user signup):	**30%**

Based on initial data, these invited users have usage, subscription conversion and retention metrics identical to those of users acquired through SEM.

F. What is the new, effective CPA for each user acquired through the SEM channel? i.e., what is the amount spent per SEM acquisition, divided by the number of users ultimately obtained?
(Please submit your answer as a number with two decimal places, like "X.YZ" or "0.YZ")

$$\textit{Viral users} = \textit{emails} * \textit{open rate} * \textit{Signup rate}$$
$$= 2.5 * .6 * .3 = .45$$

$$\textit{Total users} = 1 + \textit{Viral users} = 1.45$$

$$CPA_{eff} = \frac{CPA}{Total\ users} = \frac{\$4}{1.45} = \$2.76$$

G. What is the new ROI on SEM ad spend, with these new viral features?
(Please submit your answer as a number with two decimal places, like "X.YZ" or "0.YZ")

$$ROI = \frac{(CLV - CPA_{eff})}{CPA_{eff}} = \frac{(\$3 - \$2.76)}{\$2.76} = 8.75\%$$

H. If this is your only possible acquisition channel, can you acquire customers profitably?
(Please submit your answer as 0 or 1, where "0" indicates "no" and "1" "yes")

Yes. *CLV ($3) > CPA ($2.76)*