Music 320
Autumn 2011–2012
Homework #5
DFT, Spectrogram, Convolution
145 points
Due in one week (11/03/2011) by 11:59pm

Theory Problems

1. (10 pts) [Fourier Theorems] Design a *complex frequency shifter* which accepts a complex input signal and shifts its spectrum up by 100 Hz (adds 100 Hz to all frequency components, positive and negative). Denote the sampling interval by T. (Hint: Consider the dual of the shift theorem.) For what input signals is the output signal real?

2. (30 pts) [DFT] Define $\omega_k T \triangleq 2\pi k/N$. Find the length $N = 8$ DFTs

$$X_i(k), \quad k = 0, 1, \ldots, 7,$$

for the following sequences (without using `Matlab`):

   (a) $x_1 = [1, 0, 0, 0, 0, 0, 0, 0]$
   (b) $x_2 = [0, 1, 0, 0, 0, 0, 0, 0]$
   (c) $x_3 = [1, 1, 1, 1, 1, 1, 1, 1]$
   (d) $x_4 = [2, 1, 0, 0, 0, 0, 0, 1]$
   (e) Express $X_4(k)$ in terms of $X_1(k)$ and $X_2(k)$.

3. (20 pts) [Convolution] For the signals $x(n) = [0, 0, 1, 0, 0, 0, 1, 0]$ and $h(n) = [1, 1, 1, 1, 1, 0, 0, 0]$ compute:

   (a) $y(n) = (x * h)_n$
   (b) $y(n) = (x \circledast h)_n$

Lab Assignments

Note: follow the naming conventions of the last Homework

1. (25 pts) [Zero padding] The zero padding factor is the ratio of the length of the zero-padded signal (N) to the length of the original signal(M):

$$\text{zpf} = \frac{N}{M}$$

where zpf is the zero padding factor, N is the length of the zero-padded signal (also equal to the number of bins in the spectrum), and M is the length of the original signal (or *frame* to be analyzed).

   (a) Create a 1 second sinusoid at 16.0625 Hz using a sampling rate of 128 Hz.

   (b) Using your `plotspec` function from the previous homework (HW 4), plot the magnitude spectrum of the sinusoid. Is the result what you expected?

   (c) Zero-pad your sinusoid by a factor of zpf = 2, and plot its magnitude spectrum. Repeat while increasing the zero padding factor until your spectrum is interpolated enough so that the peak magnitude bin frequency corresponds to the correct frequency of the sinusoid. Turn in your final plot as a Matlab figure file. Specify the value of zpf you used.

2. (40 points) [Short-time Fourier transform] Write a function that generates a spectrogram of an input signal.

```
function myspecgram (x, fs, Nf)

% function myspecgram (x, fs, Nf)
% A function to plot the spectrogram of input signal
% using hann window and zpf = 8
%
% x: input signal (assume a row vector)
% fs: sampling rate of x
% Nf: frame size
%
% Your Name / Lab ##
```

   Please note:

   (a) Use the hann window and zpf=8.

   (b) To produce an image from a matrix, use `image` or `imagesc` function. Try with various colormap and choose one for your plot.

3. (20 points) Use the following Matlab command sequence to generate a chirp signal going up:

```
w1=100; w2=3000; %(Hz)
T=3; %(sec)
fs=8000; %(Hz)
dT=1/fs;
t=(0:dT:T);
up = chirp(t,w1,T,w2);
```

Using your function from the previous problem, plot the spectrum of the `up` signal. Turn in your plot as a Matlab figure file. Do they verify that your function works as expected?