IMPULSE RESPONSE MEASUREMENT USING GOLAY CODES

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ABSTRACT

This paper describes a digital processing technique for measuring the impulse response of a system. The method utilizes "complementary" codes as described by Golay in 1961 and has been employed by designers of radar, communications and other equipment. This paper discusses the implementation of the technique in a device that measures the impulse response of audio bandwidth systems.

The paper begins by describing the general problem of impulse response measurement using digital methods. This leads to a "unit pulse" response sequence that completely characterizes a bandlimited, linear and time-invariant system. Next, Schroeder's (1979) method of probing the system with pseudo-random noise and correlating the result to obtain an estimate of this response is discussed. Our method is a variation on this scheme wherein the system is probed twice using sequences chosen from among the Golay codes. These codes have the remarkable property that their autocorrelation functions have complementary sidelobes. Finally, the response of a speaker/microphone pair is measured using the proposed method.

INTRODUCTION

The impulse response of a linear, time-invariant system completely describes the behavior of that system. If $h(t)$ is the impulse response of a linear, time-invariant system presented with the input $x(t)$, the output $y(t)$ is expressed by the convolution integral,

$$y(t) = \int x(\tau)h(t-\tau)d\tau$$

The classical method of impulse response measurement is, of course, to generate an impulse, pass it through the system, and record the results. The problem with this approach is that an ideal impulse is not practical and must be approximated by a short, finite pulse that does not drive either the system under test or the measuring apparatus out of its linear, time-invariant range. This limits the amount of power available in the probe signal (the pulse) to compete with the inevitable noise in the measurement of the response.

Fortunately, much of the noise that appears in the measurement is uncorrelated with the probing pulse and can be reduced by coherently averaging the results from a series of measurements. Basically, if the results from N trials are averaged the SNR of the resulting measurement is $\sqrt{N}$ larger than the SNR of each trial. For many real-world problems the SNR available is very much less than that desired in the final estimate and hence very long averaging times are required.

In order to reduce the averaging time required the engineer has three options:

1 Reduce the amplitude of the noise corrupting the measurement.

2 Increase the amplitude of the probe signal. At some point, of course, a larger signal may not be possible or may cause saturation.

3 Use a longer probe signal. However, simply widening the impulse has the effect of *smearing* the response of the system, thereby degrading the measurement.

DIGITAL TECHNIQUES

We introduce $\Pi(x)$, the unit pulse function of width T,

$$\Pi(x) = \begin{cases} 1, & \text{if } -T/2 < x < T/2 \\ 0, & \text{else.} \end{cases}$$

and define the unit pulse response of a system to be $h'(t)$,

$$h'(t) = \Pi(t) * h(t) = \int h(x)\Pi(t-x)dx$$

We note that $h'(t)$ is a good approximation of $Th(t)$ if T is short compared with the response of the system. In fact, the actual impulse response $h(t)$ may be recovered from samples of $h'(t)$ spaced at intervals of T, provided that $h(t)$ is strictly bandlimited to frequencies below $F_c = \frac{1}{2T}$ Hz. Some care must be taken to deal with the smoothing of the response caused by the finite pulse duration, T. The power spectrum of a system whose response is measured in this way will exhibit a rolloff in high frequency response of up to 3.9 db at the Nyquist frequency, F_c. This "droop" is well known in digital audio circles as the result of the "hold" feature of D/A converters.

If $h'(t)$ is bandlimited to frequencies below F_c then we may

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represent it as a sampled sequence,

$$h'_n = h'(nT + d)$$

where the d term accounts for the unknown phase between the pulse and the sampler. The sampling theorem says that if the system under observation, $h(t)$, is properly bandlimited, which we assume to be the case for the remainder of this paper, h'_n contains all the information required to compute $h(t)$, regardless of the value of d. The goal of the measurement, then, is h'_n.

In a typical measurement configuration, a computer is used to trigger the pulse into the system while recording the samples from the A/D converter. The computer waits until the system has returned to steady-state and then repeats the procedure. The final estimate is formed as the average of these responses.

The above procedure is simple and effective and is commonly used. We had intended to use it in order to measure the direction-variant responses of human ears. However, we found that our rather noisy environment required uncomfortably long averaging times, on the order of one or two minutes for each response. Although it would certainly have been possible to use more powerful devices than the tweeters and spark-gap devices that we employed, the high peak sound levels might have been uncomfortable for the subject.

As it turns out, there are better methods. In fact it is possible to measure the unit pulse response sequence, h_k using continuous or nearly continuous probe signals.

SCHROEDER'S METHOD

In his 1979 paper [1], Schroeder describes a method based on maximal length noise sequences which have the property that their normalized, circular autocorrelation function

$$r_m = \frac{1}{L}\sum_k a_k a_{k+m}$$

is two valued:

$$r_m = \begin{cases} 1, & \text{if } m = 0 \ (modL) \\ \text{-}1/\text{L}, & \text{if } m \neq 0 \ (modL) \end{cases}$$

In this method, the system is probed continuously with a periodic maximal length sequence. The output from the system is therefore also periodic and is sampled and averaged coherently to form an estimate, in the presence of noise, of the response to one period of the sequence. This response is then correlated with the probe sequence, usually using the FFT/multiply/IFFT method, although in a recent paper [2] Borish gives an efficient time domain aloorithm that uses a fast Hadamard transform. The result is the unit pulse response sequence, h'_k, convolved with the autocorrelation function, r_m.

Note that this autocorrelation function consists of a unit sample function coupled with a "pedestal" of height $\frac{-1}{L}$ and length L. The pedestal is undesirable since it goes through the system also and affects the measured response. Fortunately, the pedestal is quite low and wide for large values of L and therefore represents energy near DC only. In fact, the DFT of the pedestal is a sampled sinc function with zeroes at multiples of the sample rate divided by L. Therefore the error in response due to the pedestal is the inverse transform of the product of this sinc function with the transform of the system under test. For systems where the gain near DC is significant this error is undesirable.

GOLAY CODES

Our method is based on the use of Golay (complementary) pairs of binary codes [3]. These codes have the remarkable property that their autocorrelation functions have complementary sidelobes. That is, the sum of the autocorrelation sequences is exactly zero everywhere except at the origin.

This allows us to recover sidelobe-free estimates of the unit sample response with the following procedure:

- probe the system with the first code, correlating the result with that code. This yields the desired response convolved with the autocorrelation of the first code.
- probe the system with the second code, correlating the result with that code. This yields the desired response convolved with the autocorrelation of the second code.
- combine the two results. Since the sum of the autocorrelations of the two codes is a scaled unit sample function, this final addition produces the desired unit sample response.

To see how this works we first define a pair of sequences a_n, b_n of length L,

$$a_n = \pm 1 \ for\ 1 \ \leq n \leq L;\ 0\ for\ all\ other\ n.$$

$$b_n = \pm 1 \ for\ 1 \leq n \leq L;\ 0\ for\ all\ other\ n.$$

and assume that they are a Golay pair, that is,

$$\sum_j (a_j a_{n+j} + b_j b_{n+j}) = 2L \text{ if } n = 0, \text{ else } 0.$$

To probe the system we first pass the a_n sequence through an ideal D/A converter using a fixed sampling interval T to obtain the probing (input) signal,

$$a(t) = \sum_j a_j \Pi(t - jT)$$

Since this probing signal is just a combination of shifted and scaled pulses the output from the system is

$$y(t) = h'(t) * a(t) = \sum_j a_j h'(t - jT)$$

which is sampled by an ideal A/D converter, operating with the same sampling interval, T, as before to produce the output sample sequence,

$$y_k = y(kT) = \sum_j a_j h'((k-j)T + d) = \sum_j a_{k-j} h'(jT + d)$$

where the d term again accounts for the unknown phase between A/D converter timing and that of the D/A. This sequence is correlated with the probe sequence a_n to produce the correlator result,

$$ca_n = \sum_k a_k y_{n+k} = \sum_k a_{k-n} y_k$$

Substituting the equation for y_k yields

$$ca_n = \sum_k a_{k-n} \sum_j a_{k-j} h'(jT + d)$$

We interchange the order of summation and rearrange terms to see that the result after correlation is the sequence,

$$ca_n = \sum_j h'(jT + d) \sum_k a_{k-n} a_{k-j}$$

The system is now probed again, this time using the sequence b_n. The output is sampled and correlated as before to produce another sequence,

$$cb_n = \sum_j h'(jT + d) \sum_k b_{k-n} b_{k-j}$$

which we combine with the previous result to form the final result:

$$\begin{aligned} c_n &= ca_n + cb_n \\ &= \sum_j h'(jT + d) \sum_k [a_{k-n} a_{k-j} + b_{k-n} b_{k-j}] \end{aligned}$$

From the definition of Golay sequences, the second summation in the above equation is equal to $2L$, if $n = j$, and 0 otherwise. Hence c_n consists of a scaled, sampled unit pulse response,

$$c_n = 2Lh'(nT + d)$$

NOISE ANALYSIS

A detailed analysis of the effects of various noise mechanisms on the results from this method is beyond the scope of this paper. However, if we assume that the noise can be modeled as stationary white noise of amplitude σ injected before the sampler, then the effect is easy to calculate.

Assume the noise appears as independant random samples with variance σ^2. The resulting noise in the final estimate $c(n)$ likewise consists of independant samples, with variance $2L\sigma^2$. This is because the effect of the correlation is to add together $2L$ independent samples for each output point. The amplitude of the noise in the output is therefore $\sqrt{2L}$ times that of the input. The output noise is also spectrally white, not surprising since the correlation process is just like passing the noise through a system whose response has a flat power spectrum. This is insured because the average autocorrelation of this response is a unit sample function, from the Golay property, therefore its power spectrum is flat.

From the preceeding calculations we can state the improvement in SNR available using this method as the ratio of the gain in the unit pulse response, $2L$, to the increased gain of the noise, $\sqrt{2L}$. Of this improvement factor, $\sqrt{2L}$, the $\sqrt{2}$ part can be traced directly to the fact that the system is probed twice. Hence the actual gain in available SNR using this method as opposed to single pulses is $\sqrt{L}$. This is basically the same SNR enhancement available with Schroeder's technique where a maximal length sequence of length $L-1$ provides an enhancement of $\sqrt{L-1}$.

IMPLEMENTATION DETAILS

In his 1961 paper [3] Golay gives two main methods for construction of code pairs. Only one will be discussed here, the appending method. Start with a simple Golay pair $\{a_k\}, \{b_k\}$:

$$a_k = \{+1 \ \ +1\}$$
$$b_k = \{+1 \ \ -1\}$$

Note that a_k and b_k are a Golay pair since their autocorrelation sequences, $a_k * a_{-k} = \{+1 \ +2 \ +1\}$ and $b_k * b_{-k} = \{-1 \ +2 \ -1\}$ are complementary. These two sequences can be appended to form the first member of a larger Golay pair:

$$a' = a \mid b = \{+1 \ +1 \ +1 \ -1\}$$

The complement to this code is obtained by inverting the second half:

$$b' = a \mid \bar{b} = \{+1 \ +1 \ -1 \ +1\}$$

This procedure may be repeated recursively to produce code pairs of any desired length $L = 2^n$. It is not difficult to show that each code pair has the desired property, that is, that the sum of the autocorrelation sequences is a scaled unit sample-function:

$$a_k * a_{-k} + b_k * b_{-k} = \{0 \ 0 \ \ldots \ 0 \ 2L \ 0 \ 0 \ \ldots \ 0\}$$

The probe signal is injected into the system under test and the output is digitized using an A/D converter. This is safe enough if the sample rate, usually identical for the probe signal and the A/D, is higher than twice the highest frequency that the system under test can pass with significant gain. If this is not the case then a higher sampling rate is recommended since the only alternative (to avoid aliasing distortion) is to bandlimit the output of the system before sampling. The resulting measurement is the unit sample response of the system under test *convolved with the unit sample response of the bandlimiting filter.* Although in some applications the effect of this filter can be ignored (or even removed), there are other situations in which it is best to simply oversample the response, that is, use a very high rate.

The samples from the A/D contain three terms:

$$y'_k = y(kT) + n(kT) + q_k$$

where $y(kT)$ is the instantaneous output from the system at time kT, $n(kT)$ is a noise term and q_k accounts for the quantization noise introduced by the A/D converter. In order to form a better estimate of $y(kT)$ the system is probed repeatedly and the output samples for each time value (kT) are averaged over N probings. If the noise term $n(kT)$ is uncorrelated from probe to probe this averaging will reduce the amplitude of this noise by a factor $\sqrt{N}$. One must be a bit careful about the quantization noise term, q_k, since it depends on $y(kT) + n(kT)$ and therefore may not be independant from probe to probe. This topic has been studied by several authors (see [4], for example). The bottom line is that the quantization noise WILL NOT average away if it is large compared to the "ambient" noise $n(kT)$. However if the amplitude of the ambient noise exceeds the quantization step size then the resulting quantization noise becomes uncorrelated from probe to probe and does average away.

After averaging, the response from the system is correlated with the probing code. Since the code consists of +1's and -1's this is a simple matter of addition or subtraction, according to the code, of L consecutive response points for each output point. Actually one can do better by rewriting the correlation in recursive form, yielding about $\frac{L}{2}$ adds/subs per output point. For large L even fewer operations may be realized using the FFT/multiply/IFFT or other "fast convolution" methods, however these operations may now include multiplies and not just add/sub's. Hence in many situations the simple correlation is preferred using "hard-wired" add and subtract instructions in the code to perform it.

Finally, the resulting correlated responses from each code are combined (summed) point by point to produce an estimate of the unit pulse response of the system.

EXAMPLE

A test system was constructed using a speaker, microphone and pre-amp to provide a linear, time-invariant system to demonstrate this technique. The sampling period, T, was 20 microseconds. The output from the pre-amp was digitized by a 16 bit A/D converter that was synchronized to the probing pulses.

The system was probed using a series of 1000 unit pulse probe signals. The resulting average response is shown in Figure 1. The system contains a delay corresponding to the propagation time from the speaker to the microphone, hence the early part of this response contains noise only.

Figure 2 gives the computed response using the Golay method with code length L equal to 64. The system is again probed 1000 times using codes of positive and negative pulses of the same amplitude as in the unit pulse case. Because 64 times as many pulses are used in this method the measured response is 64 times greater than in the unit pulse case. According to theory, the noise in this response should be only 8 times greater in amplitude than for the other case, leading to an improvement in SNR of 8 in approximately the same observation time. Our actual measured noise is slightly higher than expected, presumably due to the fact that some of the noise is at such a low frequency that samples of it cannot be assumed to be uncorrelated from probe to probe. The interval between probings was about 20 milliseconds.

CONCLUSION

We have presented a method of impulse response measurement using Golay codes. The method is generally comparable to Schroeder's M-sequence technique but with certain differences. Chief among these is that no reliance is made upon the special properties of circular convolution. There is therefore no requirement to probe the system under test with a periodic sequence. Relatively short probing codes (with respect to the duration of the system response) may be used, for instance, when the required SNR enhancement is modest and the system under test contains slow response components.

REFERENCES

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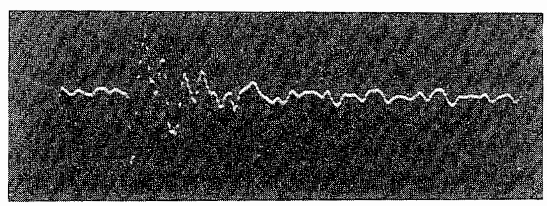

Figure 1. Averaged unit pulse response from speaker/microphone pair. The "scalloped" look is an artifact of the display.

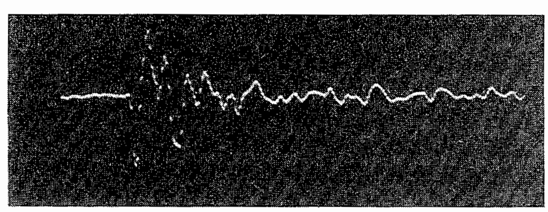

Figure 2. Computed unit pulse response using length-64 Golay codes.