Music 320
Autumn 2011–2012
Homework #6
DFT, Convolution, Correlation
150 points
Due in one week (11/10/2011) by 11:59pm

Theory Problems

1. (20 pts) [Convolution] For $x = [1, 2, 3, 2]$ and $h = [3, -1, 2, 1]$, find $(x * h)_n$ and $(x \star h)_n$. Note that they are both cyclic, not linear. Hint: knowing x is even can help you.

2. (15 points) [Convolution] The *impulse* or "unit pulse" signal is defined by

$$\delta(n) \triangleq \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

   For example, $\delta = [1, 0, 0, 0]$ for $N = 4$.

   (a) Verify that the impulse signal is the *identity element* under convolution using the impulse signal $\delta = [1, 0, 0, 0]$ and the input signal $x = [1, -1, 1, -1]$. That is, show that $x * \delta = x$.

   (b) Show that $x * \text{Shift}_1(\delta) = \text{Shift}_1(x)$, where $\textsc{Shift}_{1,n}(x) \triangleq x(n-1)$.

   (c) Find $(x * [1, 1, 0, 0, \ldots])_n$.
   (Hint: use linearity of convolution and the preceding results)

3. (15 pts) [Simple Sample Rate Conversion] Say we are given some digital signal $x_1(n)$ that was sampled at some sampling rate f_s. But, upon examining the DFT of $x_1(n)$, we find that the spectrum is only non-zero for f where $\frac{-f_s}{3} < f < \frac{f_s}{3}$, giving a total bandwidth of $\frac{2f_s}{3}$. Let's say that we wish to minimize the data flow for signal $x_1(n)$ without losing any information, so we wish to create a new signal $x_2(n)$ which is equivalent to $x_1(n)$ except resampled at rate $\frac{2f_s}{3}$.

   We cannot accomplish this simply by downsampling, as we wish to downsample by a factor of $\frac{3}{2}$, and we can only downsample by integral factors. We could solve the problem by running $x_1(n)$ through a D/A converter and then resampling it, but this is not ideal.

Find a way to create the signal $x_2(n)$ without resorting to any kind of D/A conversion (this includes sinc interpolation). Explain what effect your method has in both the time and frequency domains. If the order of your steps matters, please be sure to explain this as well. See Chapter 7 for a start on upsampling and downsampling.

Lab Assignments

NOTE: Please go back to the old naming conventions for the lab assignment:

For all lab assignments, submit your M-file scripts, functions, and figures in one zip file through coursework[1]. Within coursework, upload the zip file using the Drop Box menu.

The zip file should be named with your last name, first name and homework number. Each problem should be named with your last name, first name, homework number, and the problem number. So, for John Doe's zip file, the file should be titled `doe_john_hw6.zip`. For John Doe's answer to problem 2 on homework 6, the file would be titled `doe_john_hw6_q2.m`. Also, at the beginning of each script, include the following comment:

```
% Your Name / Lab # - Question #
```

For problems with question(s), include your answer(s) in the body of the script files as comments.

1. (30 pts) [Windows] Matlab has functions for creating windows, including the followings:

   - `boxcar`
   - `bartlett`
   - `hann (hanning)`

   (a) Write a script to plot these windows (of length 128) both in the time domain and their magnitude spectra using your `plotspec` function. For magnitude spectra, zero-pad your windows with a zpf of 8.

   (b) Describe the characteristics of each window spectrum (main lobe width, relative height of sidelobes, etc.) and what consequence it might have on a sinusoid you are analyzing.

[1] http://coursework.stanford.edu

2. (20 pts) [Windowed and zero-padded sinusoid] Apply a window of your choice to a 1 second sinusoid at 16.0625 Hz, using a sampling rate of 128 Hz.

   (a) Plot your windowed sinusoid in the time domain with the title indicating the window type.

   (b) Zero-pad your windowed signal with a zpf of 8, and plot the magnitude spectra of the windowed, zero-padded signal with the title indicating the window type.

3. (50 points) You are a recording engineer/audio detective. You wanted to test the sound of three different microphones on one of the three violinists in a string quartet. You asked your assistant (Blair) to record a little with each of the three microphones placed next to a respective player. While Blair was preparing the recordings for you, she stupidly mixed up the ordering of the files. As a result, you don't know which recording corresponds to which microphone. As a matter of life or death, you need to solve this mystery!

   To help recover the unknown ordering, you were able to recover a few seconds of each performer playing the same music individually at another time. Use this evidence (`cleanSignals`) along with the unordered (`mixedSignals`) to solve the mystery. See the following Matlab starter code and data file:

   ```
   Lab7_3.m
   data.mat
   ```