Music 320
Autumn 2011–2012
Homework #7
LTI, Filters
140 points
Due in one week (11/17/2011) by 11:59pm

Theory Problems

1. (30 points) [Linearity and Time-Invariance] Show the linearity and/or time-invariance (or not) of each filter given below.

   (a) $y(n) = x(n) + x(n-1)$

   (b) $y(n) = 2x(n) - x(n-2)$

   (c) $y(n) = 3\sqrt{x(n)}$

   (d) $y(n) = x(n) - 0.5nx(n-1)$

   (e) $y(n) = x(n) - y(n-1)$

   (f) $y(n) = \frac{x^2(n)}{2n}$

2. (15 points) [System Diagram]. Draw both Direct Form I and Direct Form II signal flow graphs for the filters determined by the difference equations below:

   (a) (Allpole IIR case)

   $$y(n) = \frac{1}{2}x(n) + \frac{1}{3}y(n-1) + \frac{1}{4}y(n-2)$$

   (b) (Pole-zero IIR case)

   $$y(n) = x(n) + 0.5x(n-1) - 0.8y(n-1) - 0.5y(n-2)$$

   (c) What is the *order* of each of the above two filters?

3. (20 points) [IIR Filters] Consider the filter

   $$y(n) = 0.5x(n) + 0.5x(n-1) + 0.8y(n-1)$$

   Find and sketch the first 10 samples of

   (a) the impulse response.

(b) the output when the input is given by

$$x(n) = [1, -1, 1, 1, -1, -1, 0, 0, 0, 0]$$

4. (35 points) [Frequency Response Analysis] For the filter defined by the difference equation

$$y(n) = 0.5x(n) + 0.5x(n-1) - 0.5y(n-1)$$

   (a) (5 points) Find the transfer function.

   (b) (15 points) Find and sketch the amplitude response. Give specific values at $\omega T = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$, and π.

   (c) (15 points) Find and sketch the phase response. Give specific values at $\omega T = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$, and π.

Lab Assignments

Follow the same file naming convention of the previous lab.

```
% Your Name / Lab #-Question #
```

For problems with question(s), include your answer(s) in the body of the script files as comments.

1. (20 points) Verify the *linearity* of the filter in problem 1(a) by the following steps:

   (a) Generate 60 samples of the sinusoid $x_0(n) = 0.5\cos(0.2\pi n)$

   (b) Zero-pad to length $N = 100$, forming the signal $x_1(n)$ using the Matlab statement "`x1 = [x0(:)',zeros(1,40)]`" or equivalent.

   (c) Using Matlab's `filter` function, filter the input signal $x_1(n)$ to obtain $y_1(n)$.

   (d) Plot x_1 in `subplot(2,1,1)` and y_1 in `subplot(2,1,2)`.

   (e) What kind of convolution has been performed (cyclic or acyclic)?

(f) Would all output samples have been returned if we did not zero pad?

(g) Filter the input signal $x_2(n) = 2\cos(0.1\pi n)$ to obtain $y_2(n)$.

(h) Filter the input signal $x_3(n) = 0.5x_1(n) + 2x_2(n)$ to obtain $y_3(n)$.

(i) Compare $y_3(n)$ to $0.5y_1(n) + 2y_2(n)$ by plotting both signals on the same plot using `hold on` command in Matlab.

Turn in your Matlab code and answers.

2. (20 points) Verify the *time-invariance* of the filter in problem 1(b) using the following steps:

   (a) Filter the input signal $x(n) = \cos(0.25\pi n)$ to obtain $y(n)$.

   (b) Time shift (delay) the original input signal $x(n)$ by 3 samples to get $x_{s,3}(n) = \textsc{Shift}_3\{x\} = \cos(0.25\pi(n-3))$.

   (c) Filter $x_{s,3}(n)$ to obtain $y_{s,3}(n)$.

   (d) Compare $y_{s,3}(n)$ to $y(n)$ by plotting both signals on the same plot using `hold on` command in Matlab.

   (e) What would you have to do to make them the same?

   Turn in your Matlab code and answers.