Music 320
Autumn 2011–2012
Homework #9
Filters & Turkey Analysis
135 points
Due next FRIDAY (12/09/2011) by 11:59pm - no late submissions!

Theory Problems

1. (15 pts) [Symmetric Filter Chain]

   Given a filter of the form $y(n) = x(n) + 2x(n-1) + 3x(n-2) + 2x(n-3) + x(n-4)$

   (a) (5 pts) Find an expression for the group delay

   (b) (5 pts) Find an expression for the phase delay

   (c) (5 pts) Find and expression for the phase and group delay for a chain of N of these filters

2. (25 pts) [Butterworth filter]

   A continuous time second-order high-pass Butterworth filter with normalized cutoff frequency $\omega_c = 0.5$ has the following frequency response in the S-domain:

   $$H(s) = \frac{s^2}{s^2 + \frac{1}{\sqrt{2}}s + \frac{1}{4}}$$

   (a) (10 pts) Sketch the bode plots of the filter (magnitude and phase). Use a log scale for the frequency axis, dB for the magnitude axis and degree for the phase axis.

   (b) (10 pts) One common method for discretizing a continuous-time filter is by using the Bilinear Transform $s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$
   Find the discrete-time frequency response $H(z)$ of the filter $H(s)$ and its corresponding difference equation, assuming $T = 1$

   (c) (5 pts) Plot the poles and zeros of the filter in the Z-plane

3. (45 points) [Thanksgiving Turkey] You wake up on Thanksgiving, put your turkey into a 350 degree oven ($T_{oven} = 350$), go back to sleep, wake up again, and...oh no! You forgot to write down what time you put the Turkey in the oven! You must figure out when (t_{cooked}) the turkey will reach the desired temperature $T_{cooked} = 160$. To do so, you measure the temperature $T_1 = 100$ at $t_1 = 0$ and $T_2 = 110$ at $t_2 = 30$ where T is in degrees farenheit and t is in minutes.

   Using these measurements can now solve for t_{cooked} by approximating the system as a leaky integrator (one-pole) filter $y[n] = (1-\lambda)x[n] + \lambda y[n-1]$. The input $x[n]$ is the oven temperature over time (assumed constant), while the output $y[n]$ is the turkey temperature over time.

Using this information, solve for the following:

(a) (5 points) Draw the signal flow diagram of the turkey/oven system.

(b) (5 points) Write the transfer function of the turkey/oven system.

(c) (5 points) Plot the poles and zeros of the turkey/oven system.

(d) (15 points) Write the turkey temperature trajectory (step response with the input at 350 instead of 1).

(e) (5 points) Solve for the forgetting factor λ

(f) (5 points) Solve for the turkey time constant τ.

(g) (5 points) Solve for the time t_{cooked} when the turkey will reach T_{cooked}.

Lab Assignments

Follow the same file naming convention of the previous lab.

1. (30 pts) The purpose of this problem is to introduce you to a couple of useful Matlab commands for filter design.

   (a) (10 pts) (5 points) Use `butter` to design a digital second order low-pass filter that cuts off at fs/3. Design a filter with the same cutoff frequency using `cheby1`. Turn in the coefficients for both filters.

   (b) (10 pts) Use `freqz` to display the amplitude and phase response of both filters designed in part 1a (plot at least 2048 points). Plot both filters in the same figure using `hold` and comment briefly on the spectral differences between the filters.

   (c) (5 pts) Repeat parts 1a and 1b for a high-pass filter with cutoff frequency of fs/6.

   (d) (5 pts) Repeat parts 1a and 1b for a band-pass filter with a low cutoff frequency of fs/6 and high cutoff frequency of fs/3.

   Turn in your Matlab code

2. (20 pts) [Allpass Filter Group Delay]

   Form the allpass filters `b = [rho 1]; a = [1 rho];` for different values of `rho = [-0.5, 0, 0.5];`. For each filter, compute the group delay by using `diff()` on `angle()` of the transfer function formed by `freqz(b, a, 0:1/(nbins-1):pi)` (you probably don't have to do `unwrap()`, but verify if you need to). Compare that group delay to the output of `grpdelay()`. Plot the poles and zeros of the system using `zplane()` (note the different way `zplane()` treats row and column vectors).

   Turn in your Matlab code.