Homework #1: Impulse Response Measurement
Due Date: May 15, 2012

Problem 1. **[20 Points]**

1(a) **[5 points]** Show that the sequences $A(t) = [1, 1]$ and $B(t) = [1, -1]$ form a Golay pair, that is, show that the sum of their autocorrelations is a scaled impulse.

1(b) **[15 points]** Show that if $A(t)$ and $B(t)$ are Golay, then so are the concatenated sequences $[A(t), B(t)]$ and $[A(t), -B(t)]$.

Problem 2. **[40 points]**

The Matlab function `RESPONSE = hmeasure(SIGNAL)` returns the noisy response of an unknown system $h(t)$ to an input signal, clipped to have a maximum absolute level of 1.0,

$$r(t) = s(t) * h(t) + n(t). \tag{1}$$

2(a) **[10 points]** Using the recursion above, write a Matlab script to generate Golay codes of length 2^n, starting with $A(t) = 1$, $B(t) = 1$.

2(b) **[20 points]** Write a Matlab script to measure the system $h(t)$ using Golay codes of length 2^n, for n in the range $n = [0, 1, \ldots, 10]$. Estimate the impulse response using length 1, length 32 and length 1024 Golay codes. Turn in plots of your impulse response estimates.

2(c) **[10 points]** Using the fact that the impulse response $h(t)$ begins with a series of zeros, estimate the signal-to-noise ratio improvement using length 32 and length 1024 Golay codes relative to using length 1 Golay codes.

Problem 3. **[40 Points]**

Consider the cascade of n identical first-order allpass filters

$$G_n(z) = \left(\frac{\rho + z^{-1}}{1 + \rho z^{-1}}\right)^n. \tag{2}$$

3(a) **[10 points]** Write a Matlab script to generate $g_n(t)$, the impulse response of the allpass cascade $G_n(z)$. Turn in a plot of the impulse response for $\rho = 0.5$ and $n = 64$.

3(b) **[10 points]** Find a value of ρ which approximately minimizes the maximum absolute impulse response tap level for $n = 64$,

$$\rho^* = \text{Argmin}_\rho \{\max_t |g_{64}(t)|\}. \tag{3}$$

You can do this numerically.

3(c) **[15 points]** Write a script which uses $g_{64,\rho^*}(t)$ to measure the system $h(t)$ from Problem 1 above. Remember that the maximum absolute input level can be at most one.

3(d) **[5 points]** Compute the measurement signal-to-noise ratio improvement compared with that of using a unit pulse to measure the system. How does the measurement signal-to-noise ratio improvement compare to that of the Golay codes?

Problem 4. **[50 points]**

Estimate the impulse response of Memorial Church, using different responses recorded last Monday.

4(a) **[25 points]** The files `gcC4A02105a.wav` and `gcC4A02105b.wav` contain a pair of Golay Codes of length $L = 2^{16}$. These were separately played back in the space and recoded using a microphone with a *figure-8* polar pattern. The responses recorded are provided (`gcAresponse.wav` and `gcBresponse`).

Load the original codes and the recorded responses into Matlab and estimate the Impulse Response of the church (`irhatg`). You will need to compute the correlation between pairs of signals. Instead of using Matlab's `corr()` function you should use `fftfilt()` to compute the correlation between the signals (remember that $x[-n] * y[n] = x[n] \star y[n]$). Note that you will probably need to zero pad the recorded responses to compute the correlation.

Turn in the Matlab code you wrote to estimate the impulse response.

You should use `ftgram()` (Matlab function provided) to display the impulse response and its associated spectrogram. This function will help you plot the IR using perceptually relevant time and frequency axis. To do so, once `irhatg` is computed, execute the following command in Matlab:

```
ftgram(irhatg, fs, 'music', 'waveform', true, 'dbrange', 80);
```

Turn in the generated figure.

4(b) **[25 points]** On Monday we also recorded the response of the church to a linear sine-sweep. The original signal (`ssl_48_18.wav`) and the recorded response (`ssl_response`) are provided. Note that `ssl_response.wav` is a stereo file, with the first (left) and second (right) channels corresponding to the recordings captured by the *figure-8* and *omni-directional* microphones respectively.

Turn in the spectrogram of the recorded response. Is it what you expected to see or is there something unexpected? If so, what could be the cause? (**note**: there was no signal clipping while playing the signal).

Using the technique discussed in class, estimate the impulse response of the church (`irhatl`). You should estimate two IRs, each one using one of the microphones. In other words, `irhatl` should be a 2 column matrix with the first one being the IR estimated with the left channel and the second column corresponding to the IR estimated with the right channel.

Turn in the Matlab code you wrote to estimate the impulse response. Also turn in a figure of the estimated impulse response and its spectrogram made with `ftgram`. In this case, display the response for both channels. Use the following command to generate the figure:

```
ftgram(irhatl, fs, 'rir', 'waveform', true, 'dbrange', 80, 'preroll', 100);
```

Analyze the spectrogram of the impulse response estimated using the *omni-directional* microphone. Point out at least two large reflections and identify whether or not the reflection might be coming from the rear of the church.