ASSIGNMENT 1 - SOLUTIONS

PROBLEM 1: DCF

"Firm A" is a large, stable company generating steady free cash flows of $10M per year, such that:

CF_0 = $10,000,000
g = 0%
r = 10%

For simplicity's sake, we will assume that it is currently the start of year 0, and that all cash flows arrive at the start of each year. Thus, year 1's cash flows are worth ($CF_1/(1+0.10)^1$) today, at the start of year 0.

To solve this problem, begin by determining free cash flows in each year. We know that in year 0, free cash flows are $10,000,000. These cash flows have a constant, zero percent growth rate, and will remain at this level forever, given by:

$$CF_N = CF_0*(1+g)^N \quad \rightarrow \quad CF_N = CF_0*(1+0)^N \quad \rightarrow \quad CF_N = CF_0 = \$10,000,000$$

Then, we know that we have a constant 10% discount rate, applied to each year's free cash flows using the following equation, to determine the cash flow's discounted present value (DPV) at the start of year 0:

$$DPV_0 = \frac{CF_N}{(1+r)^N}$$

Finally, we know that the terminal value of the free cash flows at year 20 are given by the free cash flows in the following year, divided by the difference between r and g:

$$TV_N = \frac{CF_{N+1}}{(r-g)} \quad \rightarrow \quad TV_{20} = \frac{CF_{21}}{(r-g)}$$

The value in year zero of the terminal value at year N is given by:

$$TV_0 = \frac{TV_N}{(1+r)^N} \quad \rightarrow \quad TV_0 = \frac{CF_{21}}{(r-g)(1+r)^{20}}$$

Using these equations, we are able to generate the following table of cash flows, their DPVs, and the terminal value at the year calculated and its value in year 0.

CF_0	10,000,000
g	0%
r	10%

Year	**Free cash flows**	**DPV_0**	**TV_n**
0	$ 10,000,000	$ 10,000,000	$ 14,864,363
1	$ 10,000,000	$ 9,090,909	
2	$ 10,000,000	$ 8,264,463	
3	$ 10,000,000	$ 7,513,148	
4	$ 10,000,000	$ 6,830,135	
5	$ 10,000,000	$ 6,209,213	
6	$ 10,000,000	$ 5,644,739	
7	$ 10,000,000	$ 5,131,581	
8	$ 10,000,000	$ 4,665,074	
9	$ 10,000,000	$ 4,240,976	
10	$ 10,000,000	$ 3,855,433	
11	$ 10,000,000	$ 3,504,939	
12	$ 10,000,000	$ 3,186,308	
13	$ 10,000,000	$ 2,896,644	
14	$ 10,000,000	$ 2,633,313	
15	$ 10,000,000	$ 2,393,920	
16	$ 10,000,000	$ 2,176,291	
17	$ 10,000,000	$ 1,978,447	
18	$ 10,000,000	$ 1,798,588	
19	$ 10,000,000	$ 1,635,080	
20	$ 10,000,000	$ 1,486,436	$ 100,000,000
21	$ 10,000,000	$ 1,351,306	

With this, we can solve the problems below.

A. What are the total (cumulative) free cash flows generated by Firm A during years 1-20?

Method 1: Sum the “free cash flows” column from years 1 through 20.

Method 2: Multiply the constant cash flow value per year by 20, the number of years.

The answer is $200,000,000

B. How much are year 10’s free cash flows worth in year 0, assuming the discount rate “r” above?

$$DPV_0 = \frac{CF_{10}}{(1+r)^{10}} = \frac{\$10{,}000{,}000}{(1+0.10)^{10}}$$

The answer is $3,855,433 (as shown in column “DPV_0”, row “10”)

C. How much are year 10's free cash flows worth in year 10?

Because the question asks the value of the cash flows in the same year they are generated, the answer is simply equal to the cash flows in year 10, or **$10,000,000**. Using the equations:

$$\text{DPV}_0 = \frac{CF_N}{(1+r)^N} \text{ is a special case (where t=0) of } \text{DPV}_t = \frac{CF_N}{(1+r)^{N-t}}$$

$$\text{DPV}_{10} = \frac{CF_{10}}{(1+r)^{10-10}} = \text{CF}_{10} = \mathbf{\$10{,}000{,}000}$$

D. What is the terminal value of the cash flows during years 21 and onward, in year 20?

$$\text{TV}_{21+@20} = \frac{CF_{21}}{(r-g)} = \frac{\$10{,}000{,}000}{(0.10-0.00)} = \mathbf{\$100{,}000{,}000}$$

E. What is the terminal value of the cash flows during years 21 and onward, in year 0?

$$\text{TV}_{21+@0} = \frac{TV_{20}}{(1+r)^{20}} = \frac{\$100{,}000{,}000}{(1+0.10)^{20}} = \mathbf{\$14{,}864{,}363}$$

F. What is the total value of this company in year 0, measured by ($\text{DPV}_{0\text{-}20}$ + TV_{21+})?

To solve this problem, you first sum the DPVs of cash flows in years zero through 20, and add them to the terminal value of years 21+, as calculated in part E above

$$\text{DPV}_{0\text{-}20} + \text{TV}_{21+@0} = \$95{,}135{,}637 + \$14{,}864{,}363 = \mathbf{\$110{,}000{,}000}$$

Note that due to the constant growth rate and discount rate used in this problem, this answer is equal to the solution you would have arrived at by simply calculating:

$$\text{DPV}_0 + \text{TV}_{1+} = \text{DPV}_0 + \frac{CF_1}{(r-g)^1(1+r)^0} = \$10{,}000{,}000 + \frac{\$10{,}000{,}000}{(0.10-0.00)^1} = \$10{,}000{,}000 + \$100{,}000{,}000$$

G. What percentage of this total company value is generated in years 11 and onward?

The answer is achieved by summing the terminal value and the DPV of the cash flows in years 11-20, and dividing by the total value from Question F, above.

$$\text{Percentage in years 11 and onward} = \frac{(TV_0 + DPV_{11-20})}{(\$110{,}000{,}000)} = \frac{(\$14{,}684{,}363 + \$23{,}689{,}966)}{(\$110{,}000{,}000)} = \mathbf{35.0\%}$$

Problem 2: DCF

"Firm Z" is a small, rapidly growing tech startup, with the following cash flow forecasts:

CF_0 = $1,000,000

$g_{0\text{-}5}$ = 40% **$g_{6\text{-}10}$ = 25%** **$g_{11\text{-}20}$ = 15%** **g_{21+} = 5%**

r = 20%

To clarify the growth rates, the $g_{0\text{-}5}$ growth rate would be applied to determine the CF growth from years 0-1, 1-2, 2-3, 3-4, 4-5 and 5-6. To determine CF growth from years 6-7, $g_{6\text{-}10}$ should be applied.

This problem is very similar to Problem 1, with the difference being a non-zero, changing growth rate. The problem is solved by applying the growth rates as follows:

$CF_1 = CF_0 * (1+g_{0\text{-}5})$

$CF_2 = CF_1 * (1+g_{0\text{-}5})$

$CF_3 = CF_2 * (1+g_{0\text{-}5})$

$CF_4 = CF_3 * (1+g_{0\text{-}5})$

$CF_5 = CF_4 * (1+g_{0\text{-}5})$

$CF_6 = CF_5 * (1+\mathbf{g_{0\text{-}5}})$ ← the growth rate during year 5 determines CF_6

$CF_7 = CF_8 * (1+\mathbf{g_{6\text{-}10}})$ ← the year 6 growth rate determines CF_7, etc

The following table of free cash flows, DPVs and TV is obtained:

Year	Growth rate	Cash flows	DPV	TV
0	40%	$ 1,000,000	$ 1,000,000	$ 16,165,178
1	40%	$ 1,400,000	$ 1,166,667	
2	40%	$ 1,960,000	$ 1,361,111	
3	40%	$ 2,744,000	$ 1,587,963	
4	40%	$ 3,841,600	$ 1,852,623	
5	40%	$ 5,378,240	$ 2,161,394	
6	25%	$ 7,529,536	$ 2,521,626	
7	25%	$ 9,411,920	$ 2,626,694	
8	25%	$ 11,764,900	$ 2,736,140	
9	25%	$ 14,706,125	$ 2,850,146	
10	25%	$ 18,382,656	$ 2,968,902	
11	15%	$ 22,978,320	$ 3,092,606	
12	15%	$ 26,425,068	$ 2,963,747	
13	15%	$ 30,388,829	$ 2,840,258	
14	15%	$ 34,947,153	$ 2,721,914	
15	15%	$ 40,189,226	$ 2,608,501	
16	15%	$ 46,217,610	$ 2,499,813	
17	15%	$ 53,150,251	$ 2,395,654	
18	15%	$ 61,122,789	$ 2,295,835	
19	15%	$ 70,291,207	$ 2,200,176	
20	15%	$ 80,834,888	$ 2,108,502	$ 619,734,143
21	5%	$ 92,960,121	$ 2,020,647	

A. What are the total (cumulative) free cash flows generated by Firm B during years 1-5?

Summation of free cash flows in the table above: **$15,323,840**

B. What are the total (cumulative) free cash flows generated by Firm B during years 1-20?

Summation of free cash flows in the table above: **$543,664,318**

C. What is the discounted present value (DPV) of the cash flows generated during years 1-10?

Summation of free cash flows in the table above: **$21,833,266**

D. What is the discounted present value (DPV) of the cash flows generated during years 11-20?

Summation of free cash flows in the table above: **$25,727,005**

E. What is the terminal value of the cash flows during years 21 and onward, in year 0?

$$TV_{21+@0} = \frac{CF_{21}}{(r-g_{21+})} = \frac{\$92{,}960{,}121}{(0.20-0.05)} = \$619{,}734{,}143$$

$$TV_{21+@0} = \frac{\$619{,}734{,}143}{(1+0.20)^{20}} = \mathbf{\$16{,}165{,}178}$$

F. What is the total value of this company in year 0, measured by ($DPV_{0\text{-}20} + TV_{21+}$)?

$48,560,271 + $16,165,178 = **$64,725,449**

G. What percentage of this total company value is generated in years 0-5?

$$\frac{\$9{,}129{,}758}{\$64{,}725{,}449} = \mathbf{14.1\%}$$

H. What percentage of this total company value is generated during years 0-10?

Similarly, **35.3%**

I. What percentage of this total company value is generated during years 11 and onward?

Similarly (or by subtracting the answer to Question H from 100%), **64.7%**

Problem 3: Perfect Competition

"Firm C" is competing in a market of perfect competition, in equilibrium with zero barriers to entry. *(all units in $)*

The market's aggregate demand curve is given by: P = 2300 – 4*Q

The market's aggregate supply curve is given by: P = 200 + 3*Q

Firm C's marginal cost curve is given by: MC = 200 + 100*Q

A. What is the market's equilibrium price?

The market's equilibrium price occurs where the demand and supply curves intersect:

$$P_{demand}(Q) = P_{supply}(Q)$$
$$2300 - 4*Q = P = 200 + 3*Q$$
$$7*Q = 2100 \rightarrow Q = 300$$
$$P = 2300 - 4*300 = 2300 - 1200 \rightarrow \mathbf{P = 1100}$$

B. What is the market's equilibrium quantity?

Solved above, **Q = 300**

C. Knowing only that this perfectly competitive market is in equilibrium, what are Firm C's net profits?

By definition, firms in perfectly competitive markets face are identical, and face increasing competition as long as industry profits are greater than zero. As more competitors enter the market, the supply curve will shift outward (greater quantity produced at any given price), driving equilibrium price down. This will continue until all industry profits are eliminated. Thus, for Firm C, by definition, **net profits = 0.**

D. At what quantity does Firm C choose to produce, assuming it produces at all?

To maximize profits, Firm C will produce such that marginal cost is equal to marginal revenue.

MC = MR

MR = P in a perfectly competitive market (output level doesn't influence price)

$$MC = P = 1100$$
$$MC = 1100 = 200 + 100*Q$$
$$100*Q = 900 \rightarrow \mathbf{Q = 9}$$

E. What is Firm C's average total cost at this production level?

Firms produce where ATC = P in a perfectly competitive market in equilibrium. **ATC = 1100.**

Problem 4: Perfect Competition

"Firm D" is a monopoly with 100% market share, facing the same demand curve as in Problem 3. *(all units in $)*

Firm D's marginal cost curve is given by: **MC = 175 + 0.5*Q**

A. At what quantity does Firm D choose to produce?

As in Problem 3, the demand curve is given by:

P_{demand} = 2300 – 4*Q

Firm D's total revenue at any value of Q is:

Revenue = P_{demand}*Q = 2300*Q – 4*Q^2

The marginal revenue is determined by taking the derivative of revenue with respect to Q:

MR = 2300 – 8*Q

To maximize profits, firm D will produce where MR = MC:

2300 – 8*Q = 175 + 0.5*Q

8.5*Q = 2125 → **Q = 250**

B. What is the market price at this production level?

P_{demand} = 2300 – 4*$Q_{produced}$ → $P_{equilibrium}$ = 2300 – 1000

P = 1300

C. What are Firm D's revenues?

Revenues = P*Q = 1300*250 → **Rev = 325,000**

D. What is Firm D's producer surplus? (i.e. the highest fixed costs Firm D can have and still break even)

If Firm D is exactly at break-even (total revenues equal total costs):

Revenues = Total marginal costs (at current production level) + Fixed costs

Fixed costs = Revenues – Total marginal costs

MC = 175 + 0.5*Q for any value of Q. To determine total marginal costs, we must add up the marginal cost of each item produced, up to the quantity of production. More simply, we take:

$\int_0^{Q=250} 175 + 0.5 * Q$ → ([(175*Q)+0.25*Q^2] evaluated at Q=250, minus at Q=0

Total marginal cost = (175)(250)+(0.25)(250^2) = 59,375

Fixed costs = 325,000 – 59,375

Fixed costs = 265,625