Data Mining and Machine learning

Naïve Bayes classifier

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Building a Bayesian Classifier

- Assume you want to predict output Y which assumes n_Y different values $v_1, v_2, \ldots v_{ny}$.
- Assume there are m input attributes called $X_1, X_2, \ldots X_m$
- Break dataset into n_Y smaller datasets called $DS_1, DS_2, \ldots DS_{ny}$.
- Define DS_i = Records in which $Y=v_i$
- For each DS_i , learn Density Estimator M_i to model the input distribution among the $Y=v_i$ records. There are several approaches but the easier is the Naïve Density estimator.
- M_i estimates P($X_1, X_2, \ldots X_m \mid Y=v_i$)
- Estimate P($Y=v_i$). as fraction of records with $Y=v_i$.
- For a new prediction:

$$Y^{\text{predict}} = \underset{v}{\operatorname{argmax}}\, P(Y = v \mid X_1 = u_1 \cdots X_m = u_m)$$

$$= \underset{v}{\operatorname{argmax}}\, P(X_1 = u_1 \cdots X_m = u_m \mid Y = v) P(Y = v)$$

Naïve Bayes Classifier

In the case of the naive Bayes Classifier, the random variables X1, ….. ,Xm are considered independent, hence the previous equation can be simplified as:

$$Y^{\text{predict}} = \underset{v}{\text{argmax}} P(Y = v) \prod_{j=1}^{m} P(X_j = u_j | Y = v)$$

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If you have a lot of input attributes that product will underflow in floating point math. You should use logs:

$$Y^{\text{predict}} = \underset{v}{\text{argmax}} \left(\log P\,(Y = v) + \sum_{j=1}^{m} \log P\,(X_j = u_j | Y = v) \right)$$

Naïve Bayes classifier

Although Naïve Bayes classifier is more suitable for categorical features it can be applied when there are continuous attributes in the dataset. There are two options:

a) Discretize the continuous attributes using methods such as, Equal width discretization, Equal frequency discretization and, Entropy with MDLP discretization. After that, apply NB on the discretized data. Scikit learn has two function BernoulliNB (it assumes that all the predictors are binary) and MultinomialNB (it assumes that the predictors can assume mores than two different values).

b) Assume a Gaussian distribution for the continuous predictors Scikit learn has a GaussianNB function. The option a) is preferred by many.

Example

X1	X2	X3	Y
0	0	1	0
0	1	0	0
1	1	0	0
0	0	1	1
1	1	1	1
0	0	1	1
1	1	0	1

Example: Cont

To which class will be assigned the record (0,0,1)?

$$P(Y=0)=3/7 \qquad P(Y=1)=4/7$$

$$P(X_1=0,X_2=0,X_3=1/Y=0)=P(X_1=0/Y=0)P(X_2=0/Y=0)P(X_3=1/Y=0)=$$

$$(2/3)(1/3)(1/3)=2/27$$

$$P(X_1=0,X_2=0,X_3=1/Y=1)=P(X_1=0/Y=1)P(X_2=0/Y=1)P(X_3=1/Y=1)=$$

$$(2/4)(2/4)(3/4)=3/16$$

X1=0,X2=0, x3=1 will be assigned to class 1

Example 2. (continuous and discrete attributes)

X1	X2	X3	X4	Y
0	0	1	3.15	0
0	1	0	8.17	0
1	1	0	5.72	0
0	0	1	7.16	1
1	1	1	9.32	1
0	0	1	12.81	1
1	1	0	15.48	1

Naïve Bayes for Diabetes

Without Discretization

```
# Calculo de las probabilidades posteriores y de las clases predichas
clf = GaussianNB()
clf.fit(X1,y1)
proba=pd.DataFrame(clf.predict_proba(X1))
pred=clf.predict(X1)
print 'Accuracy by Resubstitution',(pred==y1).sum()/float(768)
```

Accuracy by Resubstitution 0.76171875

```
# Calculo de las probabilidades posteriores y de las clases predichas
```

With Discretization; Equal width

```
from sklearn.preprocessing import KBinsDiscretizer
est = KBinsDiscretizer(n_bins=10, encode='ordinal', strategy='uniform')
est.fit(X)
Xt = est.transform(X)
clf=MultinomialNB()
clf.fit(Xt,y1)
scores = cross_val_score(clf, X1, y1, cv=10)
print("Accuracy by cross validation: %0.3f (+/- %0.3f)" % (scores.mean(), scores.std()))
```

Accuracy by cross validation: 0.654 (+/- 0.048)

The Auto-mpg dataset

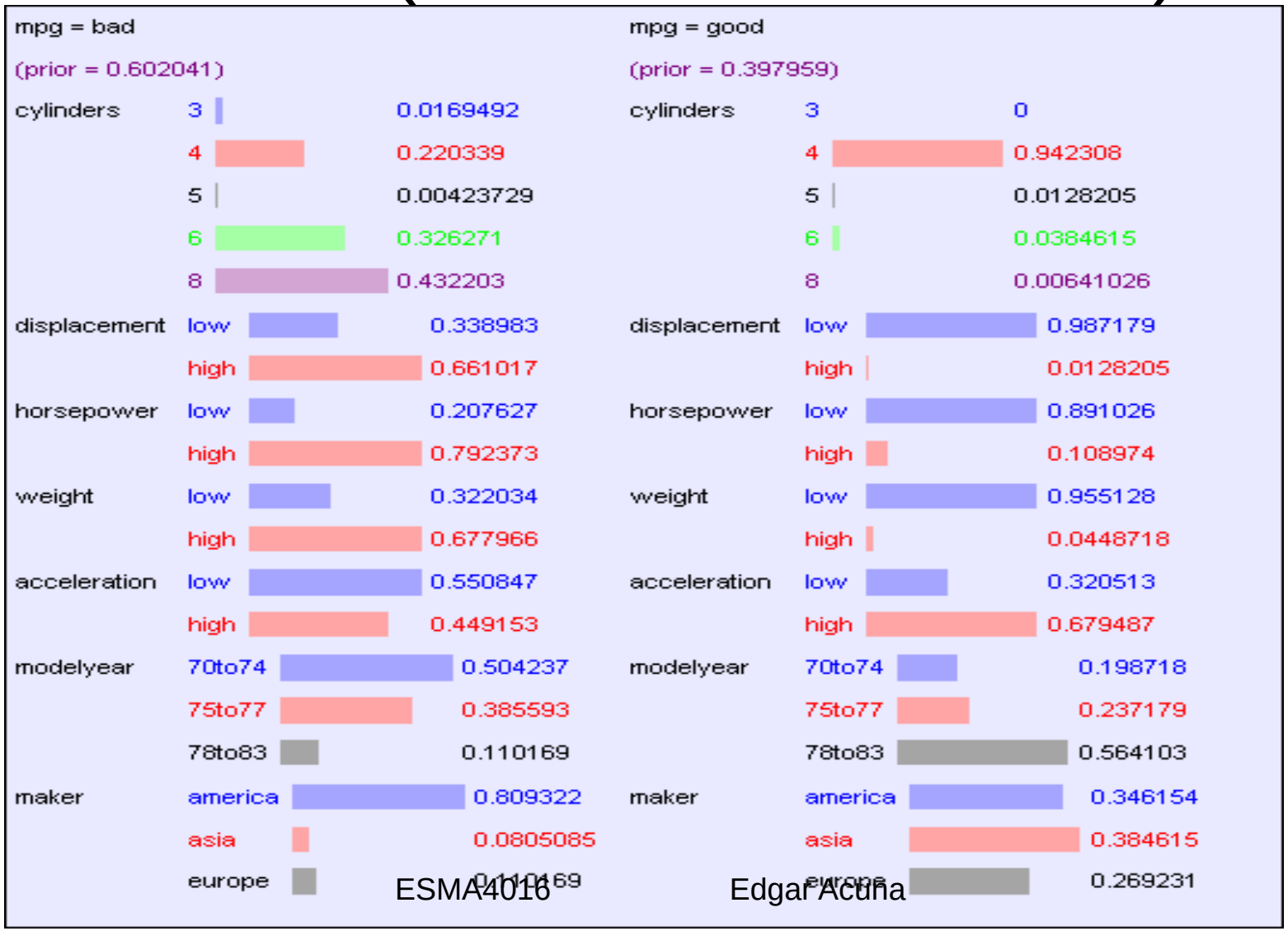
Donor: Quinlan,R. (1993)
Number of Instances: 398 minus 6 missing=392 (training: 196 test: 196):
Number of Attributes: 9 including the class attribute7. Attribute Information:
1. mpg: continuous (discretizado bad<=25,good>25)
2. cylinders: multi-valued discrete
3. displacement: continuous (discretizado low<=200, high>200)
4. horsepower: continuous ((discretizado low<=90, high>90)
5. weight: continuous (discretizado low<=3000, high>3000)
6. acceleration: continuous (discretizado low<=15, high>15)
7. model year: multi-valued discrete (discretizado 70-74,75-77,78-82
8. origin: multi-valued discrete
9. car name: string (unique for each instance)
Note: horsepower has 6 missing values

Available at: https://archive.ics.uci.edu/ml/machine-learning-databases/auto-mpg/auto-mpg.data

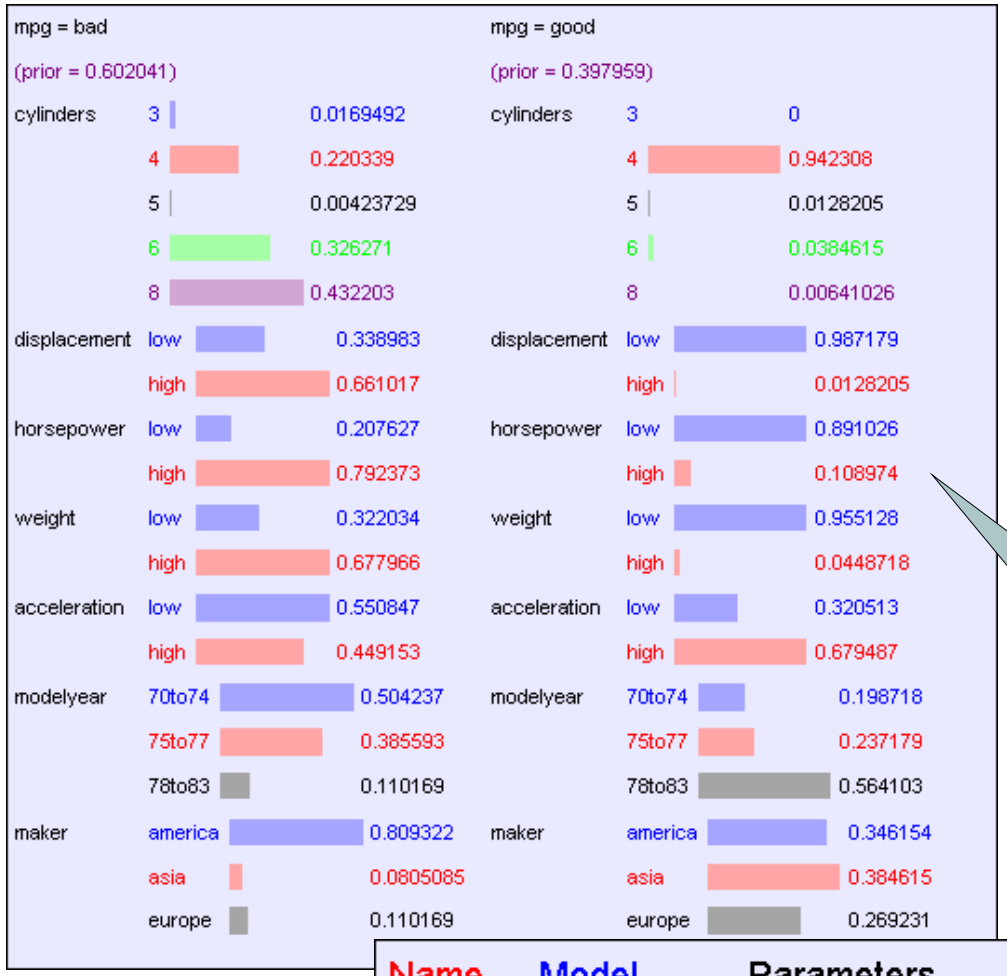
The auto-mpg dataset

18.0 8 307.0 130.0 3504. 12.0 70 1 "chevrolet chevelle malibu"
15.0 8 350.0 165.0 3693. 11.5 70 1 "buick skylark 320"
18.0 8 318.0 150.0 3436. 11.0 70 1 "plymouth satellite"
16.0 8 304.0 150.0 3433. 12.0 70 1 "amc rebel sst"
17.0 8 302.0 140.0 3449. 10.5 70 1 "ford torino"
……..
27.0 4 140.0 86.00 2790. 15.6 82 1 "ford mustang gl"
44.0 4 97.00 52.00 2130. 24.6 82 2 "vw pickup"
32.0 4 135.0 84.00 2295. 11.6 82 1 "dodge rampage"
28.0 4 120.0 79.00 2625. 18.6 82 1 "ford ranger"
31.0 4 119.0 82.00 2720. 19.4 82 1 "chevy s-10"

Resultados del clasificador NB para “MPG”: 392 records (Prof. A. Moore, CMU)



BC Results: “MPG”: 392 records



The Classifier learned by “Naive BC”

Name	Model	Parameters	FracRight		
Model1	bayesclass	density=joint submodel=gauss gausstype=general	0.885256	+/-	0.0247796
Model2	bayesclass	density=naive submodel=gauss gausstype=general	0.852372	+/-	0.0400495

Advantages/Disadvantages of NB

Probabilities zeros can affect the NB classifier. A correction factor is applied to the computation of probabilities to solve this problem.

- The discretization process seems to affect the performance of the classifier.
- Naïve Bayes is very cheap. It does not have problem working with 10,000 attributes.
- Naïve Bayes is a particular case of Bayesian networks