Simulation of FSAE Acceleration Event

Notes on motor modeling, optimal gearbox transmission ratio and optimal battery voltage

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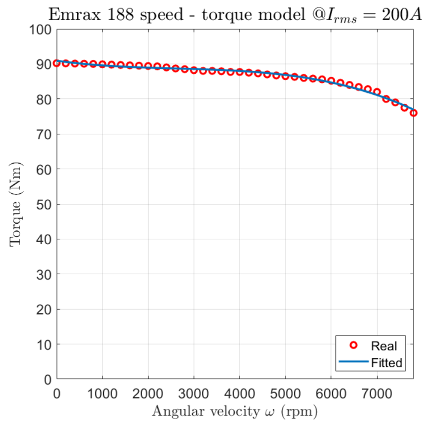
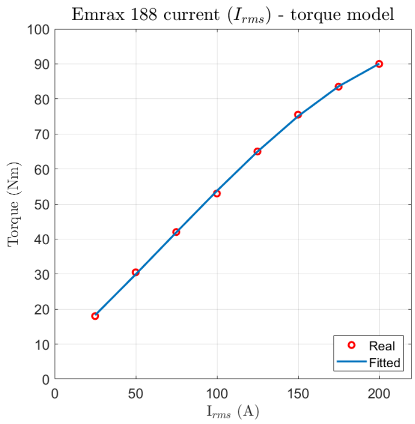
1 Emrax 188 model

Emrax 188 is a permanent magnet synchronous motor (PMSM). It operates in ***High Voltage*** mode when it is powered with a DC battery voltage comprised in $280 \div 400$ V; it can provide a maximum of 90 Nm (when the effective current I_{rms} is 200 A) and it can spin up to 6500 rpm (no field weakening).
The angular speed - torque characteristic curve corresponding to $I_{rms} = 200$ A was fitted in MATLAB using a third order polynomial starting from the one reported in Emrax catalogue. The same operation was done for the effective current - torque curve, as shown below.



(a)



(b)

Figure 1: Emrax 188 characteristic curves

If the motor is supplied with a current I_{rms} lower than 200 A then the speed-torque curve will be shifted downwards.

2 Simulation of Acceleration Event

The FSAE Acceleration Event consists of a straight line with a length of 75 m to be completed in the minimum time. Top FSAE Electric teams (with 4 in-wheel motors) are able to complete an acceleration run in about $3.2 \div 3.4$ seconds.
The Acceleration Event is here considered because it is the one in which the vehicle must be pushed to the limit in terms of power, torque and (possibly) max speed.

2.1 Motor and battery pack electrical parameters

Emrax 188 motor is able to spin at a max rotational velocity which depends on the DC voltage that the battery pack can provide when powering the motor. This

specific load speed S ranges between 15 rpm/Vdc (at full load) and 19 rpm/Vdc (with no load).
Setting as a target the max desired rotational speed of the motor $\omega_{mot,max}$ (that then determines the max vehicle speed), the voltage that the battery must be delivering to the motor is:

$$V_{batt,accel} = \frac{\omega_{mot,max}}{S} \tag{1}$$

The max power $P_{batt,max}$ that the battery pack can deliver is limited by FSAE rules to 80 kW. Considering that in a real case the battery power can be limited to 78 kW in order to avoid penalties, the **max DC current** $I_{batt,accel}$ that the battery can produce during the acceleration event is:

$$I_{batt,accel} = \frac{P_{batt,max}}{V_{batt,accel}} = \frac{78000}{V_{batt,accel}} \tag{2}$$

The max voltage of the battery pack, at full charge, is:

$$V_{batt,FullCharge} = V_{batt,accel} + I_{batt,accel} R_{tot}, \tag{3}$$

where $R_{tot} = n_{modules} \cdot R_{module}$ is the total resistance of the battery pack and it is equal to the number of modules (electrically in series) times the resistance of a single module (4.25 mΩ at 30°C).
The **Rinehart inverter catalogue** explains that the available AC motor voltage (line to line) from the driver output terminals is a function of the battery DC voltage. In particular, the **effective line to line motor voltage** is:

$$V_{mot,rms} = \frac{V_{batt,accel}}{\sqrt{2}} \tag{4}$$

The **mechanical power** that the motor is able to deliver is:

$$P_{mot} = \sqrt{3} \cdot V_{mot,rms} \cdot I_{mot,rms} \cdot cos(\phi) \cdot \eta_{mot} \tag{5}$$

where $\cos(\phi)$ is the motor power factor and η_{mot} is the efficiency of the motor.
The DC current $I_{batt,accel}$ coming from the battery is split between the 2 inverters in a symmetric way during the acceleration event. Another expression can be derived for the mechanical motor power considering the battery pack parameters:

$$P_{mot} = \frac{V_{batt,accel} \cdot I_{batt,accel} \cdot \eta_{inv} \cdot \eta_{mot}}{2} \tag{6}$$

The efficiency of the inverter η_{inv} is around 0.97. The motor power factor $\cos(\phi)$ is not known, but for synchronous motors under load it is around 0.8. By equating the previous two equations it is possible to find the effective motor current $I_{mot,rms}$:

$$I_{mot,rms} = \frac{I_{batt,accel} \cdot \eta_{inv}\sqrt{6}}{6cos(\phi)} \tag{7}$$

2.2 Motor and vehicle mechanical parameters

The top speed that the vehicle can reach is:

$$u_{max} = \frac{\omega_{mot,max} \cdot \pi \cdot r_{tire}}{30 \cdot \tau_{GBX}}, \tag{8}$$

where r_{tire} is the tire radius and τ_{GBX} is the transmission ratio of the gearbox.
The **real max motor torque** $C_{mot,max}$ can be calculated using the fitted curve shown in Figure 1 (b) since $I_{mot,rms}$ is known.
The **max torque at a rear wheel** $C_{wheel,max,real}$ is:

$$C_{wheel,max,real} = C_{mot,max} \cdot \tau_{GBX} \cdot \eta_{GBX} \tag{9}$$

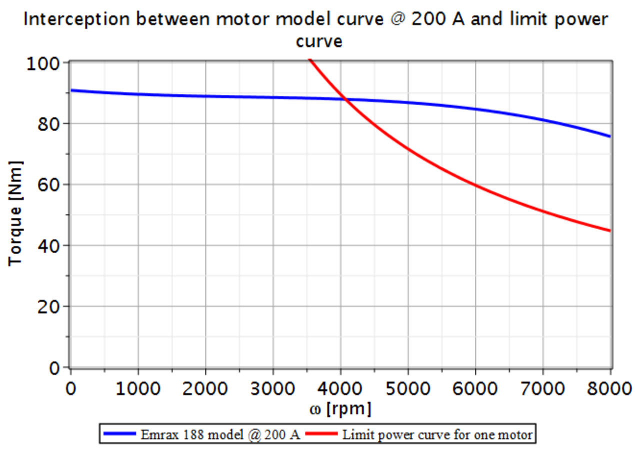
where η_{GBX} is the efficiency of the gearbox (not known, but it can be assumed that it is about 0.93).
The **maximum torque** that can be provided to a rear tire before it slips was calculated using **Pacejka magic formula** (see report for tire data analysis) and it is:

$$C_{wheel,max,pacejka} \cong 400Nm \tag{10}$$

However, as it will be shown, a lower amount of torque can be provided to the wheel in real conditions.
The max power limitation imposed by FSAE rules determines also the max power that a motor can deliver. An upper bound to the electric power that can be supplied to the two motors is around 75 kW. As a consequence, the **max power for a single motor** is roughly **37.5 kW**. So, for one motor, the relation describing the limiting power is:

$$P_{mot,max} = 37500 = \frac{C_{mot} \cdot \omega_{mot,rpm} \cdot \pi}{30} \Longrightarrow C_{mot} = \frac{358098.6}{\omega_{mot,rpm}} \tag{11}$$

For example, if the motor could be powered at $I_{mot,rms} = 200$ A (never true in reality), then the **power limit** produces a hyperbola that also sets a **limit to the max rotational speed at which the motor can deliver the max torque**:



The **real motor torque profile** is therefore:

$$C_{mot}(t) = \begin{cases} C_{mot,max}, & \text{if } 0 \leq \omega_{mot,rpm} < \omega_{limitPower} \\ \frac{358098.6}{\omega_{mot,rpm}}, & \text{if } \omega_{limitPower} \leq \omega_{mot,rpm} \leq \omega_{mot,max} \end{cases} \tag{12}$$

The motor velocity $\omega_{limitPower}$ also corresponds to a vehicle velocity $u_{limitPower}$ above which torque must be reduced.
The equation of motion for the vehicle dynamics along the longitudinal x axis is:

$$m\frac{du(t)}{dt} = F_{x,rl}(t) + F_{x,rr}(t) - F_{x,aero}(t) \tag{13}$$

where m is the mass of the vehicle (the target is 200 kg), $u(t)$ is the longitudinal vehicle speed, $F_{x,rl}(t)$ and $F_{x,rr}(t)$ are the longitudinal tire forces for the two rear tires, while $F_{x,aero}(t)$ is the aerodynamic drag, which from literature data can be approximated with $F_{x,aero}(t) \cong 1.55 \cdot u(t)^2$.

$$F_{x,rl}(t) = F_{x,rr}(t) = \begin{cases} C_{mot,max} \cdot \tau_{GBX} \cdot \eta_{GBX}, & \text{if } 0 \leq u < u_{limitPower} \\ \frac{358098.6 \cdot \pi \cdot r_{tire}}{u \cdot \tau_{GBX} \cdot 30}, & \text{if } u_{limitPower} \leq u < u_{max} \end{cases} \tag{14}$$

Equation (13) can be integrated by dividing it in two parts, the first of which corresponding to $u(t) \in [0, u_{limitPower}]$ and the second to $u(t) \in [u_{limitPower}, u_{max}]$.
In many cases that will be shown the max motor torque $C_{mot,max}$ is so low that the **limiting power condition is** actually **never reached**, and the **vehicle eventually gets to the top speed** u_{max} before completing the 75 m.
As a result, the overall **Acceleration Event can be divided in three main phases**:

- **First phase**: motor torque $= C_{mot,max}$, vehicle speed $\in [0, u_{limitPower}]$;
- **Second phase**: motor torque $= \frac{358098.6}{\omega_{mot,rpm}}$ = limited by max power, vehicle speed $\in [u_{limitPower}, u_{max}]$;
- **Third phase**: motor torque = can be $C_{mot,max}$ or limited by power, it depends, vehicle speed saturated $= u_{max}$.

Pay attention because, depending on the selected battery voltage $V_{batt,accel}$ and transmission ratio τ_{GBX}, **not all the three acceleration phases may be present**.

2.3 Simulations

Many tests were done varying the battery voltage $V_{batt,accel}$ and the transmission ratio τ_{GBX}. The times corresponding to each of the three acceleration phases were summed in order to determine the optimal parameters.
One of the main unknowns for the simulations is the **specific load speed of the motor** S, which can range in [15,19] rpm/Vdc. Since, as it will be shown, in all simulations the motor works at about 2/3 of its max torque, S was set to **17.5 rpm/Vdc**. Other simulations were performed also with S = 15 rpm/Vdc and S = 16 rpm/Vdc.
Consider that the rolling resistance of tires was neglected because unknown and difficult to estimate and that the aerodynamic model was derived from literature data. As a consequence the absolute value of results may not be very realistic, but the analysis can be used for comparisons of different solutions.
The main results are summarized below.

S = 17,5 rpm/Vdc		
$V_{batt,accel}$ = 331,4V, $I_{mot,rms}$ = 116,5A, C_{mot} = 61 Nm		
τ [-]	total time [s]	final speed [km/h]
4	4,0295	110,9
4,4	3,9363	102,1
4,5	3,9325	98,6
4,6	3,9335	96,5
5	3,9820	88,8

(a)

S = 17,5 rpm/Vdc		
$V_{batt,accel}$ = 348,6V, $I_{mot,rms}$ = 110,8A, C_{mot} = 59 Nm		
τ [-]	total time [s]	final speed [km/h]
4	4,0950	109,5
4,4	3,9410	107,1
4,5	3,9240	102,2
4,6	3,9142	100,4
4,7	**3,9100**	**98,1**
4,8	3,9155	96,4
5	3,9220	94,2

(b)

S = 17,5 rpm/Vdc		
$V_{batt,accel}$ = 360V, $I_{mot,rms}$ = 107,2A, C_{mot} = 57 Nm		
τ [-]	total time [s]	final speed [km/h]
4	4,1660	109,5
4,5	3,9540	107,1
4,7	3,9215	102,2
4,8	3,9145	100,4
4,9	3,9127	98,1
5	3,9155	96,4
5,1	3,9220	94,2
5,5	3,9805	87,7

(c)

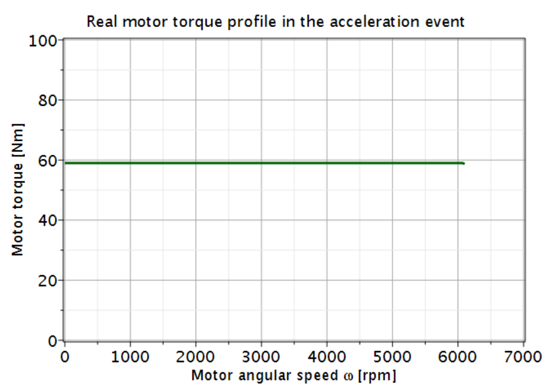
Figure 2: Results of simulations for comparison of acceleration times

2.3.1 Optimal parameters

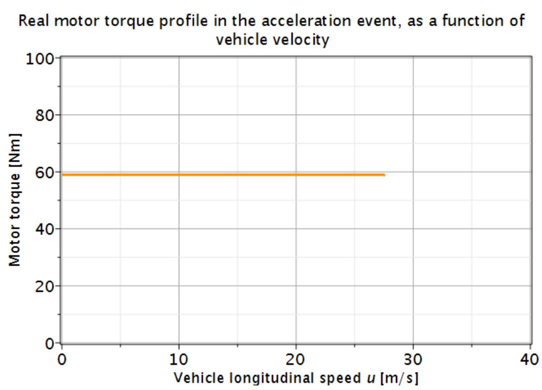
The **minimum time** corresponds to a **battery voltage** $V_{batt,accel}$ of about **350 V** and a **transmission ratio** τ_{GBX} of **4.7**.
Other simulations (not reported here) were performed with motor specific load speed of 15 and 16 rpm/Vdc. They yield an optimal transmission ratio comprised in [4.5,5] and an optimal battery voltage between 350 V and 400 V, depending on the simulation.
The main parameters of the optimal solution are summarized in the following table.

Parameter	Value	Notes
$V_{batt,accel}$ [V dc]	350	
$I_{batt,accel}$ [A dc]	224	
n° of modules [-]	108	can be changed
$V_{batt,FullCharge}$ [V dc]	452	can be changed
$I_{mot,rms}$ [A]	111	
$\omega_{max,mot}$ [rpm]	6100	
$C_{max,mot}$ [Nm]	59	
$C_{max,wheel}$ [Nm]	258	
τ_{GBX} [-]	4,7	
Top speed [km/h]	≈ 100	
Acceleration time [s]	3,91	

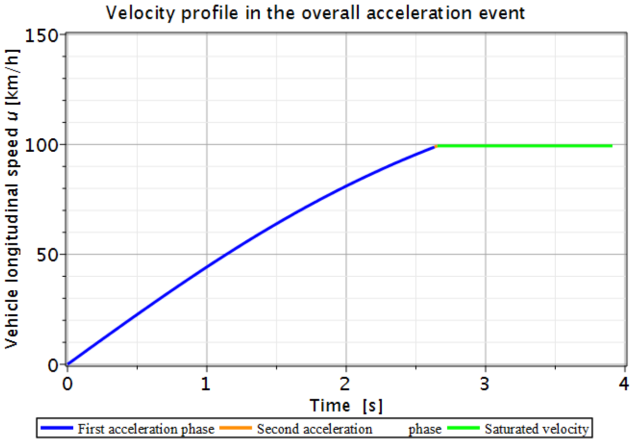
In this case the motor angular velocity $\omega_{limitPower}$ at which the limit power is reached results to be 6070 rpm, which is very close to the max motor speed of 6100 rpm. This means that in practice the motor can deliver the max torque (59 Nm) throughout the entire acceleration event.



(a)



(b)


Velocity profile in the overall acceleration event
Vehicle longitudinal speed u [km/h]
150
100
50
0
0
1
2
3
4
Time [s]
First acceleration phase
Second acceleration phase
Saturated velocity