



hurdlr: An R Package for Zero-inflated and Extremely Over-dispersed Count Data

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Abstract

The abstract of the article.

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1. Introduction

Count data is one of the most common and important forms of data encountered in every field of study. While some count data can be modeled easily with binomial or poisson likelihoods, many times the data is gathered such that a disproportionate amount of zeros appear relative to non-zeros. For these, a class of models called zero-inflated models were created with the reasoning that the counts came from two separate data-generating mechanisms; one that generated counts according to some discrete probability distribution (such as Poisson), and another that generated the excess amount of zeros.

In this paper, we present an R package dedicated to modeling zero-inflated count data, with an emphasis on developing new methods to account for both zero-inflation and extreme over-dispersion simultaneously. The models in this package utilize Bayesian estimation techniques. Parameter estimation under a Bayesian framework is suitable because of the model's hierarchical structure. Model parameters were given uninformative priors and updated one-at-a-time using a Markov chain Monte Carlo (MCMC) algorithm.

2. Models for count data

2.1. Zero-inflated models

Zero-inflated models are widely used for count data exhibiting a significantly higher amount of zeros than would otherwise be observed given some typical count distribution. These are simply two-component mixture models that can separate the excess zeros in the data from the underlying (parametric) count distribution. A common choice for the count distribution is Poisson, but is typically replaced with negative binomial if there is over-dispersion in the data.

2.2. Hurdle models

Hurdle models are similar to zero-inflated models in that they also account for the high occurrence of zeros in data. The difference is that for hurdle models, *all* zero-count observations are assumed to have been generated by a mechanism separate of the non-zeros. We begin by trying to fit the negative binomial hurdle model, with the following distributional components:

$$h_1(y_i|m_i, r) = \begin{cases} p_i & \text{if } y_i = 0, \\ (1 - p_i) \cdot f(y_i|m_i, r) & \text{if } y_i > 0, \end{cases} \quad (1)$$

where for each observation i , $0 \leq p_i \leq 1$ and $f(y_i|m_i, r)$ is the negative binomial probability function, with mean m_i and overdispersion parameter r , truncated at 0.

3. Double-hurdle models

In some cases, few but very large count values causes the data to be so extremely overdispersed that a simple negative binomial hurdle model proves to be inadequate.

Properly fitting such an extremely right-skewed distribution is the goal of the double-hurdle model. In this model, a second hurdle is created to represent the transition from a distribution of “typical” counts to a distribution of larger counts. Just as in the hurdle model, we assume separate data-generating processes for each of the components. The first hurdle represents the divide between the zero-counts and the “typical” non-zero counts. Larger counts can be thought of as coming from a heavy-tailed distribution g if the second hurdle is crossed. The double-hurdle model has the following form for observation i :

$$h_2(y_i|\boldsymbol{\theta}) = \begin{cases} p_i & \text{if } y_i = 0, \\ (1 - q_i)(1 - p_i)f(y_i|m_i, r) & \text{if } 1 \leq y_i < \mu, \\ q_i(1 - p_i)g(y_i|\mu, \sigma, \xi) & \text{if } y_i \geq \mu, \end{cases} \quad (2)$$

where $\boldsymbol{\theta}$ is the (possibly) vector-valued set of model parameters, p_i is the probability that observation i is governed by a zero-generating process, and q_i is the probability that observation i is drawn from the large-count distribution conditional on it being non-zero. The typical-count component is left-truncated at 0 and right-truncated at μ to maintain separability in the likelihood then normalized to ensure that each component is a density.

Balderama et al 2016 considered the Generalized Pareto distribution (GPD) to describe the third component of the count data. If $y \sim \text{GPD}(\mu, \sigma, \xi)$, then y has density function

$$g(y|\mu, \sigma, \xi) = \frac{\sigma^{1/\xi}}{\left(\sigma + \xi \times (y - \mu)\right)^{\frac{1}{\xi}+1}} \quad (3)$$

for $y \geq \mu$ and $\xi \neq 0$. Since we are dealing with count data, we use a discretized version of the GPD distribution function, defined by taking the difference between the cumulative densities at $y + 0.5$ and $y - 0.5$.

Notice that when $\mu = \infty$, (??) simplifies to (??). Also notice that setting $\mu = 1$ essentially removes the second component, thus all non-zero counts are modeled under the large-count distribution. Balderama et al 2016 defined this as the GPD-hurdle model, a new single-hurdle alternative to the negative binomial distribution for extremely right-skewed count data.

4. Implementing hurdlr

5. Discussion

References

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