Zero-Jitter Task Chains via Algebraic Rings

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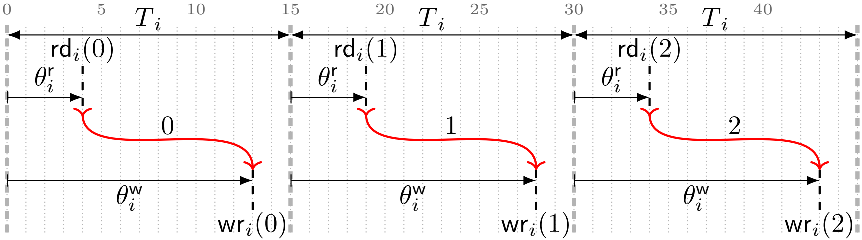
IEEE Transactions on Computers, 2023

WaP @RTSS 2023, December 6th 2023
Taipei, Taiwan

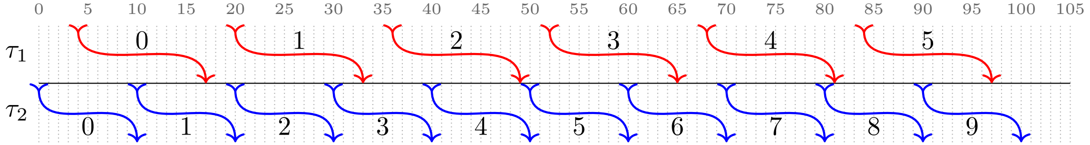
Model of a LET task τ_i



- A LET *task* τ_i is composed by *periodic jobs* (curvy arrows)
  - in LET, we only care of the read and write instants

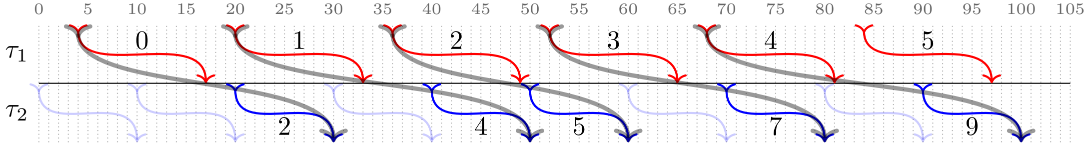
θ_i^{r}	*read phasing* of τ_i, relative to the *period* T_i (aka the *offset*)
θ_i^{w}	*write phasing* of τ_i, rel. to *period* T_i ($\theta_i^{\mathsf{w}} - \theta_i^{\mathsf{r}}$ is aka the *deadline*)
$\mathsf{rd}_i(j) = j\,T_i + \theta_i^{\mathsf{r}}$	*read instant* of job j of τ_i
$\mathsf{wr}_i(j) = j\,T_i + \theta_i^{\mathsf{w}}$	*write instant* of job j of τ_i

Chain of 2 LET tasks



- Chain of LET tasks
  - each task reads data written by the previous one
  - data is on **shared memory**
    - τ_1 may over-write data before τ_2 reads (if $T_1 < T_2$)
    - τ_2 may read again the same data (if $T_2 < T_1$)

Chain of 2 LET tasks



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  - each task reads data written by the previous one
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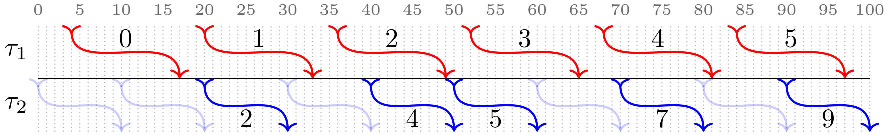
- Chains of jobs have **variable** read-to-write delay

			rd→wr delay
(j_1, j_2)	$\mathrm{rd}_1(j_1)$	$\mathrm{wr}_2(j_2)$	$\mathrm{wr}_2(j_2) - \mathrm{rd}_1(j_1)$
$(0,2)$	4	30	$26 = 13 + \mathbf{3} + 10$
$(1,4)$	20	50	$30 = 13 + \mathbf{7} + 10$
$(2,5)$	36	60	$24 = 13 + \mathbf{1} + 10$
$(3,7)$	52	80	$28 = 13 + \mathbf{5} + 10$
$(4,9)$	68	100	$32 = 13 + \mathbf{9} + 10$
...	...	...	...

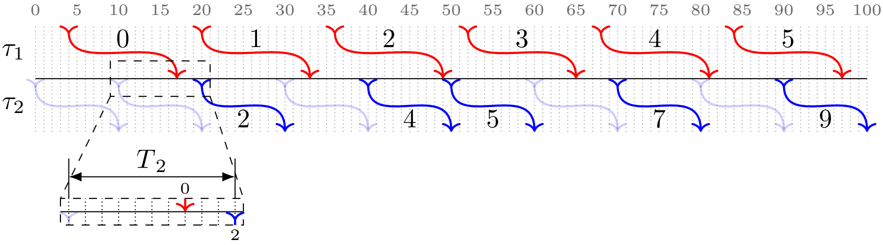
Variability of the input-output delay

- The source of variability is $\mathsf{rd}_2(j_2) - \mathsf{wr}_1(j_1)$. In the example: $3, 7, 1, 5, 9, 3, 7, \ldots$



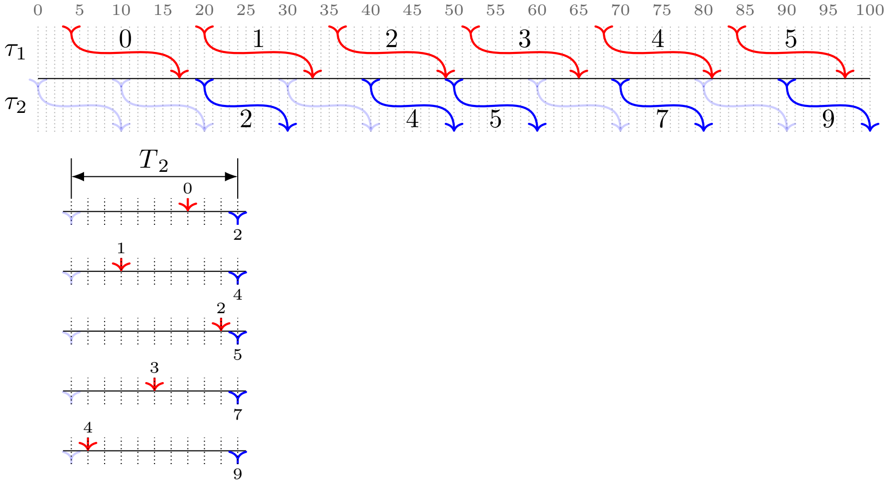
Variability of the input-output delay

- $\mathrm{rd}_2(j_2) - \mathrm{wr}_1(j_1) < T_2$ ($= 10$ in the example), always. Let's zoom in ...



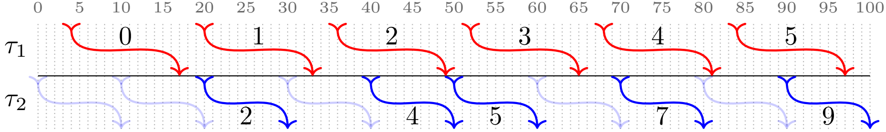
Variability of the input-output delay

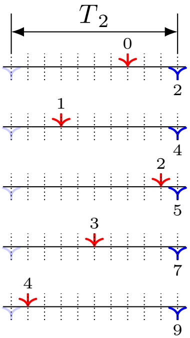
- . . . and take $\mathsf{rd}_2(j_2)$ as reference for all $\mathsf{wr}_1(j_1) \to \mathsf{rd}_2(j_2)$ delays.

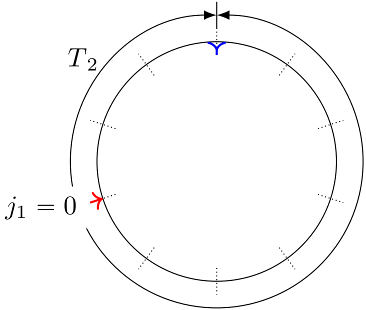


Variability of the input-output delay

- A repetitive requence: let's bend all the T_2-long segments over the (algebraic) ring $\mathbb{Z}/T_2\mathbb{Z}$.

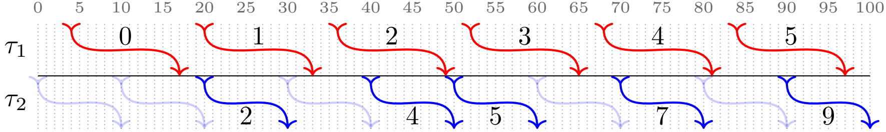


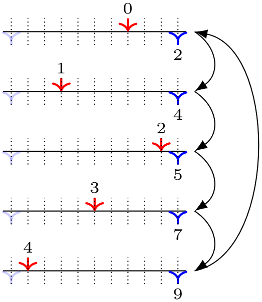


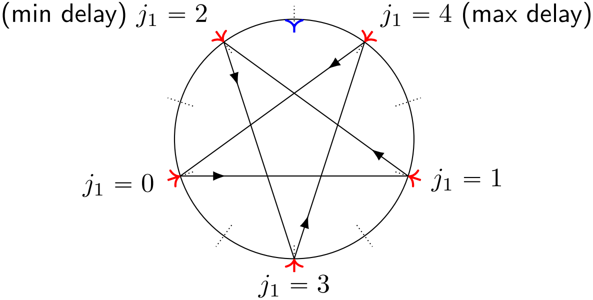


Variability of the input-output delay

- By inverting over the ring $\mathbb{Z}/T_2\mathbb{Z}$, max/min delay is found **without** unrolling the schedule.







Variability of the input-output delay

- We can **eliminate the jitter** of a 2-tasks chain, by adding a copier task.