COMP4418 Assignment 1

Chunnan Sheng

Student Code: z5100764

1. Propositional Inferences

(a) Yes

p	q	r	p + qr	p + q	p + r	(p + q)(p + r)
0	0	0	0	0	0	0
0	0	1	0	0	1	0
0	1	0	0	0	0	0
0	1	1	1	1	1	1
1	0	0	1	1	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

(b) Yes

P	q	q -> p	p -> (q -> p)
0	0	1	1
0	1	0	1
1	0	1	1
1	1	1	1

(c) No

p	q	p -> q	-p -> -q
0	0	1	1
0	1	1	0
1	0	0	1
1	1	1	1

(d) Yes

P	q	p -> q	-p -> -q	(p -> q)(-p -> -q)	-p <-> -q
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	1	1	1	1

(e) Yes

p	q	r	-q -> -p	-r -> -q	(-q -> -p)(-r -> -q)	P -> r
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	1	0	0	1
0	1	1	1	1	1	1
1	0	0	0	1	0	0
1	0	1	0	1	0	1
1	1	0	1	0	0	0
1	1	1	1	1	1	1

(f) Yes

1. $\neg\big((p \wedge q) \vee (p \wedge r)\big)$ (negation of the right side)
2. $\neg(p \wedge q) \wedge \neg(p \wedge r)$ (negation of the right side)
3. $(\neg p \vee \neg q) \wedge (\neg p \vee \neg r)$ (negation of the right side)
4. p
5. $\neg q \wedge (\neg p \vee \neg r)$ (3 and 4)
6. $q \vee r$
7. $r \wedge (\neg p \vee \neg r)$ (5 and 6)
8. $r \wedge \neg r$ (4 and 7)
9. $\square$

(g) No

1. $\neg(p \Rightarrow q)$ (negation of the right side)
2. $\neg(\neg p \vee q)$ (negation of the right side)
3. $p \wedge \neg q$ (negation of the right side)
4. p (left side)
5. $p \wedge \neg q$ (4 and 5)

(h) Yes

1.	$\neg((\neg q \vee r) \wedge (q \vee \neg r)) \vee ((\neg p \vee r) \wedge (p \vee \neg r))$	(the right side)
2.	$(\neg q \vee r) \wedge (q \vee \neg r) \wedge ((p \wedge \neg r) \vee (\neg p \wedge r))$	(negation of the right side)
3.	$(\neg q \vee r) \wedge (q \vee \neg r) \wedge (p \vee r) \wedge (\neg p \vee \neg r)$	(negation of the right side)
4.	$(p \vee \neg q) \wedge (\neg p \vee q)$	(left side)
5.	$(\neg q \vee r) \wedge (q \vee \neg r) \wedge (p \vee r) \wedge (\neg p \vee \neg r) \wedge (p \vee \neg q) \wedge (\neg p \vee q)$	(3 and 4)
6.	$(\neg p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge (p \vee \neg q \vee \neg r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q \vee \neg r) \wedge (p \vee q \vee r)$	(Complete Conjunctive Normal Form)
7.	$\square$	

(i) Yes

1.	$(\neg p \vee q) \wedge (p \vee \neg q)$	(the right side)
2.	$\neg((\neg p \vee q) \wedge (p \vee \neg q))$	(negation of the right side)
3.	$(p \wedge \neg q) \vee (\neg p \wedge q)$	(negation of the right side)
4.	$((p \wedge \neg q) \vee \neg p) \wedge ((p \wedge \neg q) \vee q)$	(negation of the right side)
5.	$((p \vee \neg p) \wedge (\neg q \vee \neg p)) \wedge ((p \vee q) \wedge (\neg q \vee q))$	(negation of the right side)
6.	$(\neg p \vee \neg q) \wedge (p \vee q)$	(negation of the right side)
7.	$p \vee q$	(one CNF item of right side)
8.	$\neg p$	(one CNF item of left side)
9.	q	(7 and 8)
10.	$\neg q$	(one CNF item of left side)
11.	$\square$	(9 and 10)

(j) Yes

1.	$\neg p \vee q$	(one CNF of left side)
2.	$\neg q \vee r$	(one CNF of right side)
3.	$\neg p \vee r$	(right side)
4.	$p \wedge \neg r$	(negation of right side)
5.	$q \wedge \neg r$	(1 and 4)
6.	$r \wedge \neg r$	(2 and 5)
7.	$\square$	

2. Logic Puzzle

a)

Huey is younger than the boy in the green tee-shirt

$$boy\_name(huey)$$
$$t\_shirt\_colour(green)$$
$$\forall x \forall a \forall b \ [name(x) \wedge colour(x, green) \wedge age(huey, a) \wedge age(x, b) \Rightarrow a < b]$$

The five year-old wore the tee-shirt with the camel design

$$t\_shirt\_design(camel)$$
$$\forall x \ [boy\_name(x) \wedge age(x, 5) \Leftrightarrow boy\_name(x) \wedge design(x, camel)]$$

Dewey's tee-shirt was yellow

$$boy\_name(dewey)$$
$$t\_shirt\_colour(yellow)$$
$$colour(dewey, yellow)$$

Louie's tee-shirt bore the giraffe design

$$boy\_name(louie)$$
$$t\_shirt\_design(giraffe)$$
$$design(louie, giraffe)$$

The panda design was not featured on the white tee-shirt

$$t\_shirt\_design(panda)$$
$$t\_shirt\_colour(white)$$
$$\forall x \ [boy\_name(x) \Rightarrow \neg(design(x, panda) \wedge colour(x, white))]$$

---

b)

This problem is solvable and has an unique solution.

1. $\forall x \ [boy\_name(x) \Leftrightarrow x \in \{dewey, huey, louie\}]$ **premise**
2. $\forall x \ [boy\_name(x) \wedge age(x, a) \Rightarrow a \in \{4,5,6\}]$ **premise**
3. $\forall x \ [t\_shirt\_colour(x) \Leftrightarrow x \in \{yellow, green, white\}]$ **premise**
4. $\forall x \ [t\_shirt\_design(x) \Leftrightarrow x \in \{camel, panda, giraffe\}]$ **premise**
5. $\boldsymbol{colour(dewey, yellow)}$ **premise**
6. $colour(huey, green) \vee colour(huey, white)$ **resolution of 1,3,5**
7. $\forall x \forall a \forall b \ [boy\_name(x) \wedge colour(x, green) \wedge age(huey, a) \wedge age(x, b) \Rightarrow a < b]$ **premise**

Proof by contradiction start

8. If $colour(huey, green)$, then
9. $\forall a \forall b \ [age(huey, a) \wedge age(huey, b) \Rightarrow a < b]$ **resolution of 1,2,7,8**

10. $\forall a \forall b \ [age(huey, a) \wedge age(huey, b) \Rightarrow a = b]$ **premise**
11. 9 and 10 contradict
12. □

Proof by contradiction end

13. $\boldsymbol{colour(huey, white)}$ **resolution of 8,10,11,12**
14. $\boldsymbol{colour(louie, green)}$ **resolution of 1,3,7,13**
15. $\boldsymbol{design(louie, giraffe)}$ **premise**
16. $design(huey, panda) \vee design(huey, camel)$ **resolution of 1,4,15**

Proof by contradiction start

17. If $design(huey, panda)$,
18. then $design(huey, panda) \wedge colour(huey, white)$ **resolution of 13,17**
19. $\forall x \ [boy\_name(x) \Rightarrow \neg(design(x, panda) \wedge colour(x, white))]$ **premise**
20. $\neg(design(huey, panda) \wedge colour(huey, white))$
21. □ **resolution of 18,20**

Proof by contradiction end

22. $\boldsymbol{design(huey, camel)}$ **resolution of 16,17,18,19,20,21**
23. $\boldsymbol{design(dewey, panda)}$ **resolution of 1,4,15,22**
24. $\forall x \ [boy\_name(x) \wedge age(x, 5) \Leftrightarrow boy\_name(x) \wedge design(x, camel)]$ **premise**
25. $age(huey, 5) \Leftrightarrow design(huey, camel)$ **resolution of 1,24**
26. $\boldsymbol{age(huey, 5)}$ **resolution of 22,25**
27. $\forall x \forall b \ [boy\_name(x) \wedge colour(x, green) \wedge age(x, b) \Rightarrow 5 < b]$ **resolution of 7,26**
28. $\forall b \ [age(louie, b) \Rightarrow 5 < b]$ **resolution of 1,14,27**
29. $[age(louie, 4) \Rightarrow 5 < 4] \wedge [age(louie, 5) \Rightarrow 5 < 5] \wedge [age(louie, 6) \Rightarrow 5 < 6]$ **resolution of 2,28**
30. $\neg age(louie, 4) \wedge \neg age(louie, 5)$ **5<4 is false, 5<5 is false**
31. $\boldsymbol{age(louie, 6)}$ **resolution of 2,30**
32. $\boldsymbol{age(dewey, 4)}$ **1,2,26,31**

Name	Dewey	Huey	Louie
Age	4	5	6
Colour	Yellow	White	Green
Design	Panda	Camel	Giraffe

3. Automated Theorem Proving

This automated program is developed via C++ programming language. Please execute the **Makefile** in the directory to compile the source code first so that this program can be executed.

Makefile and Source files:
Formula.cpp
Formula.h
FormulaBuilder.cpp
FormulaBuilder.h
Sequent.cpp
Sequent.h
assn1q3.cpp
Makefile

Please **double quote** the arguments while executing the program on Linux, Unix or Mac OS, for example:

```
wagner % ./assn1q3 "[p iff q] seq [(q iff r) imp (p iff r)]"
true
Proofs:
1.      [r, p, q] seq [r]                                                                   Rule P1
2.      [r, p, q] seq [p, q, r]                                                             Rule P1
3.      [r, p, q] seq [r]  - AND -  [r, p, q] seq [p, q, r]                                 AND
4.      [r, p iff q, p, q] seq [r]                                                          Rule P6b
5.      [q, p] seq [q, r]                                                                   Rule P1
6.      [p] seq [p, q, r]                                                                   Rule P1
7.      [q, p] seq [q, r]  - AND -  [p] seq [p, q, r]                                       AND
8.      [p, p iff q] seq [q, r]                                                             Rule P6b
9.      [r, p iff q, p, q] seq [r]  - AND -  [p, p iff q] seq [q, r]                        AND
10.     [p, q iff r, p iff q] seq [r]                                                       Rule P6b
11.     [p, q, r] seq [p]                                                                   Rule P1
12.     [q, r] seq [q, p]                                                                   Rule P1
13.     [p, q, r] seq [p]  - AND -  [q, r] seq [q, p]                                       AND
14.     [q, r, p iff q] seq [p]                                                             Rule P6b
15.     [q, p, r] seq [q, r, p]                                                             Rule P1
16.     [r] seq [q, r, p]                                                                   Rule P1
17.     [q, p, r] seq [q, r, p]  - AND -  [r] seq [q, r, p]                                 AND
18.     [r, p iff q] seq [q, r, p]                                                          Rule P6b
19.     [q, r, p iff q] seq [p]  - AND -  [r, p iff q] seq [q, r, p]                        AND
20.     [q iff r, r, p iff q] seq [p]                                                       Rule P6b
21.     [p, q iff r, p iff q] seq [r]  - AND -  [q iff r, r, p iff q] seq [p]               AND
22.     [q iff r, p iff q] seq [p iff r]                                                    Rule P6a
23.     [p iff q] seq [(q iff r) imp (p iff r)]                                             Rule P5a
```