COMP4418

Assignment2

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1. Answer Set Programming

(a)

```
% COMP4418 Assignment 2
% Question 1
#const k = 3.

% encoding
k{c(X) : v(X)}k.
:- not e(X, Y), c(X), c(Y), X != Y.

% Vertices
v(1..6).

% Edges
e(1,2). e(2,1).
e(1,4). e(4,1).
e(1,3). e(3,1).
e(2,4). e(4,2).
e(2,6). e(6,2).
e(2,5). e(5,2).
e(3,4). e(4,3).
e(3,5). e(5,3).
e(3,6). e(6,3).
e(4,5). e(5,4).
e(5,6). e(6,5).

% Display
#show c/1.
```

(b) Outputs for $K \in \{3,4,5,6\}$

```
E:\UNSW_semester_3\COMP4418\clingo>clingo.exe --const k=3 -n 0 E:/COMP4418/assn2/clique.lp
clingo version 5.2.1
Reading from E:/COMP4418/assn2/clique.lp
Solving...
Answer: 1
c(5) c(2) c(4)
Answer: 2
c(5) c(3) c(4)
Answer: 3
c(5) c(6) c(2)
Answer: 4
c(5) c(6) c(3)
Answer: 5
c(1) c(2) c(4)
Answer: 6
c(1) c(3) c(4)
SATISFIABLE

Models       : 6
Calls        : 1
Time         : 0.035s (Solving: 0.03s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.016s

E:\UNSW_semester_3\COMP4418\clingo>clingo.exe --const k=4 -n 0 E:/COMP4418/assn2/clique.lp
clingo version 5.2.1
Reading from E:/COMP4418/assn2/clique.lp
Solving...
UNSATISFIABLE

Models       : 0
Calls        : 1
Time         : 0.000s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.000s

E:\UNSW_semester_3\COMP4418\clingo>clingo.exe --const k=5 -n 0 E:/COMP4418/assn2/clique.lp
clingo version 5.2.1
Reading from E:/COMP4418/assn2/clique.lp
Solving...
UNSATISFIABLE

Models       : 0
Calls        : 1
Time         : 0.000s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.000s

E:\UNSW_semester_3\COMP4418\clingo>clingo.exe --const k=6 -n 0 E:/COMP4418/assn2/clique.lp
clingo version 5.2.1
Reading from E:/COMP4418/assn2/clique.lp
Solving...
UNSATISFIABLE

Models       : 0
Calls        : 1
Time         : 0.000s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.000s

E:\UNSW_semester_3\COMP4418\clingo>
```

2. Answer Set Programming

Output of the logic:

```
E:\UNSW_semester_3\COMP4418\clingo>clingo.exe  -n 0 E:/COMP4418/assn2/Question2.m
clingo version 5.2.1
Reading from E:/COMP4418/assn2/Question2.m
Solving...
Answer: 1
a d
Answer: 2
b d
Answer: 3
c d
SATISFIABLE

Models       : 3
Calls        : 1
Time         : 0.026s (Solving: 0.02s 1st Model: 0.00s Unsat: 0.02s)
CPU Time     : 0.000s
```

Candidate S	Reduct P^S	Stable model
$\{a, b, c, d\}$	d :- a. d :- b. d :- c.	NO
$\{a, b, c\}$	d :- a. d :- b. d :- c.	NO
$\{a, b, d\}$	d :- a. d :- b. d :- c.	NO
$\{a, c, d\}$	d :- a. d :- b. d :- c.	NO
$\{b, c, d\}$	d :- a. d :- b. d :- c.	NO
$\{a, b\}$	d :- a. d :- b. d :- c.	NO
$\{a, c\}$	d :- a. d :- b. d :- c.	NO
$\{a, d\}$	a. d :- a. d :- b. d :- c.	YES
$\{b, c\}$	d :- a. d :- b. d :- c.	NO
$\{b, d\}$	b. d :- a. d :- b. d :- c.	YES
$\{c, d\}$	c. d :- a. d :- b. d :- c.	YES
$\{a\}$	a. d :- a. d :- b. d :- c.	NO
$\{b\}$	b. d :- a. d :- b. d :- c.	NO
$\{c\}$	c. d :- a. d :- b. d :- c.	NO
$\{d\}$	a. b. c. d :- a. d :- b. d :- c.	NO
$\{\}$	a. b. c. d :- a. d :- b. d :- c.	NO

3. Reasoning about Knowledge

(a)

$$\mathbf{OKB} \vDash \mathbf{K}\big(P(n_1) \wedge Q(n_2)\big) \Leftrightarrow$$

$$\vDash \left\|\mathbf{K}\big(P(n_1) \wedge Q(n_2)\big)\right\|_{\mathbf{KB}} \Leftrightarrow$$

$$\vDash \mathbf{RES}[\mathbf{KB}, \|P(n_1) \wedge Q(n_2)\|_{\mathbf{KB}}] \Leftrightarrow$$

$$\vDash \mathbf{RES}[\mathbf{KB}, P(n_1) \wedge Q(n_2)] \stackrel{\text{def}}{=} \begin{cases} TRUE & \mathbf{KB} \vDash P(n_1) \wedge Q(n_2) \\ FALSE & Otherwise \end{cases}$$

Since

$$\forall x(P(x) \Leftrightarrow x = \#1 \vee x = \#2) \wedge \forall x\big(P(x) \Leftrightarrow \neg Q(x)\big)$$

Then

$$P(x) \Leftrightarrow x \in \{\#1, \#2\}$$
$$Q(x) \Leftrightarrow x \in \{\#1, \#2\}^C = \{\#3, \#4, \#5 \ldots\}$$

Therefore, if $\mathbf{OKB} \vDash \mathbf{K}\big(P(n_1) \wedge Q(n_2)\big)$ should be true,

Then

$$n_1 \in \{\#1, \#2\}$$
$$n_2 \in \{\#3, \#4, \#5, \ldots\}$$

Or equivalently:

$$(n_1, n_2) \in \{\#1, \#2\} \times \{\#3, \#4, \#5, \ldots\}$$

(b)

$$\mathbf{RES}[\mathbf{KB}, P(x) \wedge \neg Q(y)] \stackrel{\text{def}}{=} \begin{cases} TRUE & \mathbf{KB} \vDash P(x) \wedge \neg Q(y) \\ FALSE & therwise \end{cases}$$
$$\Leftrightarrow$$
$$\mathbf{KB} \vDash P(x) \wedge \neg Q(y)$$
$$\Leftrightarrow$$
$$\mathbf{KB} \vDash P(x) \wedge P(y)$$

Since

$$P(x) \wedge P(y) = \begin{cases} TRUE & x \in \{\#1, \#2\} \wedge y \in \{\#1, \#2\} \\ FALSE & otherwise \end{cases}$$

Therefore,

$$\mathbf{KB} \vDash \neg \mathbf{K}(P(x) \wedge P(y))$$

Which means

$$\mathbf{RES}[\mathbf{KB}, P(x) \wedge \neg Q(y)]$$

Cannot be determined.

4. Limited Belief

(a)

Proper+ of KB:

$$(\neg F \vee C) \wedge (\neg C \vee \neg U) \wedge (C \vee U)$$
or
$$\mathbf{KB} = \{\neg F \vee C, \neg C \vee \neg U, C \vee U\}$$

(b)

$\mathbf{KB}$	$\{\neg F \vee C, \neg C \vee \neg U, C \vee U\}$
$\mathrm{UP}^-(\mathbf{KB})$	$\{\neg F \vee C, \neg C \vee \neg U, C \vee U\}$
$\mathrm{UP}(\mathbf{KB})$	$\{\neg F \vee C, \neg C \vee \neg U, C \vee U\}$
$\mathrm{UP}^+(\mathbf{KB})$	$\{\neg F \vee C, \neg C \vee \neg U, C \vee U\} \cup$ $\{c \mid c \supseteq \neg F \vee C \ \ or \ \ c \supseteq \neg C \vee \neg U \ \ or \ \ c \supseteq C \vee U\}$

According to the above table, $\mathbf{KB}$ is not obviously inconsistent.
However, $(F \vee U \vee C) \in \mathrm{UP}^+(\mathbf{KB})$
Then $\mathbf{OKB} \ | \approx \mathbf{K_0}(F \vee U \vee C)$ hold. The minimal $k = 0$

(c)

$$\neg(\neg F \wedge \neg U) \Leftrightarrow F \vee U$$
$$F \vee U \notin \mathrm{UP}^+(\mathbf{KB})$$

Then

A. $\neg(\neg F \wedge \neg U) \notin \mathrm{UP}^+(\mathbf{KB})$

In addition:

B. $\mathbf{KB}$ is not obviously inconsistent

According to A and B:

$$\mathbf{OKB} \ | \approx \neg \mathbf{K_0} \neg(\neg F \wedge \neg U)$$

The minimal $k = 0$

5. Reasoning about Action

(a) Proved

$$(a) \Leftrightarrow \mathbf{\Sigma}_0 \vDash \forall x \mathbf{R}[\langle\rangle, [putInBox(x)]InBox(x)]$$
$$\Leftrightarrow \mathbf{\Sigma}_0 \vDash \forall x \mathbf{R}[putInBox(x), InBox(x)]$$
$$\Leftrightarrow \mathbf{\Sigma}_0 \vDash \forall x \mathbf{R}\left[\langle\rangle, \gamma_{in}{}^{a}_{putInBox(x)}\ {}^{b}_{x}\right]$$
$$\Leftrightarrow \mathbf{\Sigma}_0 \vDash \forall x \left(\left(putInBox(x) = putInBox(s)\right) \vee InBox(s)\right)$$

if $x = s$, then

$$\forall x \left(\left(putInBox(x) = putInBox(s)\right) \vee InBox(s)\right) = \mathbf{TRUE}$$

√

(b) Proved

$$(b) \Leftrightarrow \mathbf{\Sigma}_0 \vDash \mathbf{R}[\langle\rangle, \forall y[moveBox(y)]\forall x(InBox(x) \Rightarrow localtion(x) = y)]$$
$$\Leftrightarrow \mathbf{\Sigma}_0 \vDash \mathbf{R}\left[\forall y\left(moveBox(y)\right), \forall x(InBox(x) \Rightarrow location(x) = y)\right]$$
$$\Leftrightarrow \mathbf{\Sigma}_0 \vDash \forall y \forall x \mathbf{R}\left[moveBox(y), \left([putInBox(x)](localtion(x) = y)\right)\right.$$
$$\left.\vee \left(InBox(x) \wedge (location(x) = y)\right)\right]$$
$$\Leftrightarrow \mathbf{\Sigma}_0 \vDash \forall y \forall x (\mathbf{R}[moveBox(y) \cdot putInBox(x), localtion(x) = y] \vee \dots)$$
$$\Leftrightarrow \mathbf{\Sigma}_0 \vDash \forall y \forall x \left(\mathbf{R}\left[putInBox(x), \gamma_{pos}{}^{a}_{moveBox(y)}\ {}^{p}_{y}\right] \vee \dots\right)$$
$$\Leftrightarrow \mathbf{\Sigma}_0 \vDash \forall y \forall x \left(\left(\left(\left(moveBox(y) = moveBox(t)\right) \wedge \mathbf{R}[putInBox(x), InBox(s)]\right) \vee \dots\right) \vee \dots\right)$$
$$\Leftrightarrow \mathbf{\Sigma}_0 \vDash \forall y \forall x \left(\left(\left(moveBox(y) = moveBox(t)\right) \wedge \mathbf{R}[putInBox(x), InBox(s)]\right) \vee \dots\right)$$

if $x = s$ and $y = t$, then

$$\forall y \forall x (\mathbf{R}[putInBox(x), InBox(x)] \vee \dots)$$
$$\Leftrightarrow \forall y \forall x \left(\mathbf{R}\left[\langle\rangle, \gamma_{in}{}^{a}_{putInBox(x)}\ {}^{b}_{x}\right] \vee \dots\right)$$
$$\Leftrightarrow \forall y \forall x \left(\left(putInBox(x) = putInBox(s)\right) \vee InBox(s) \vee \dots\right) \Leftrightarrow \mathbf{TRUE}$$

√

(c) Proved

$$[shakeBox]\mathbf{K}\exists x InBox(x) \Longleftrightarrow$$
$$\left(\mathbf{SF}(shakeBox) \Rightarrow \mathbf{K}\left(\mathbf{SF}(shakeBox) \Rightarrow [shakeBox]\exists x InBox(x)\right)\right)$$
$$\wedge \left(\neg\mathbf{SF}(shakeBox) \Rightarrow \mathbf{K}\left(\neg\mathbf{SF}(shakeBox) \Rightarrow [shakeBox]\exists x InBox(x)\right)\right)$$

$$\Rightarrow$$

$$\mathbf{K}\big(\mathbf{SF}(shakeBox) \Rightarrow [shakeBox]\exists x InBox(x)\big)$$

$$\vee\ \mathbf{K}\big(\neg\mathbf{SF}(shakeBox) \Rightarrow [shakeBox]\exists x InBox(x)\big)$$

$$\Rightarrow$$

$$\mathbf{K}\big(\mathbf{SF}(shakeBox) \Rightarrow [shakeBox]\exists x InBox(x)\big) \qquad \mathbf{P}$$

Since

$$\Big(\Box\mathbf{SF}(a) \Leftrightarrow \big((a = shakeBox) \Rightarrow \exists x InBox(x)\big)\Big) \in \boldsymbol{\Sigma}_{dyn}$$

and

$$\big(\Box\mathbf{Poss}(a) \Leftrightarrow \mathbf{TRUE}\big) \in \boldsymbol{\Sigma}_{dyn}$$

Therefore

$$\boldsymbol{\Sigma}_0 \wedge \boldsymbol{\Sigma}_{dyn} \wedge \mathbf{O}\boldsymbol{\Sigma}_{dyn} \models \mathbf{P} \quad \surd$$

(d) Disproved