# Feasible optimality of vegetation patterns in river basins

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[1] We examine mechanisms leading to organization of vegetation patterns within the channel network structure of a semi-arid New Mexico river basin under the controlling influence of water stress. We compare the actual pattern of water stress within the basin to patterns resulting from two algorithms of local stress optimization which proceed from an initial fully random vegetation distribution. Here we show that the distribution of vegetation and basin water stress derived from an algorithm that maintains local optimization within the network flow path exhibits considerably better agreement with the actual distribution than one that ignores the network structure of the basin. These results suggest the pattern of actual vegetation observed within the basin corresponds to a condition of feasible optimality in which organization is constrained by the stochastic nature of local interactions mediated by the network configuration. The principles of such organization have important consequences regarding the interaction between land cover change and hydrological dynamics in river basins, as well as the biogeographical evolution of INDEX TERMS: 1851 Hydrology: Plant ecology; 1836 Hydrology: Hydrologic budget (1655); 1824 Hydrology: Geomorphology (1625); 1848 Hydrology: Networks; 1866 Hydrology: Soil moisture. Citation: Caylor, K. K., T. M. Scanlon, and I. Rodriguez-Iturbe (2004), Feasible optimality of vegetation patterns in river basins, Geophys. Res. Lett., 31, L13502, doi:10.1029/2004GL020260.

## 1. Introduction

[2] Recent years have seen dramatic advances in the quantitative description of the geomorphologic structure of river basins [Rodriguez-Iturbe and Rinaldo, 1997]. In particular, the consideration of a river network as a dendritic structure characterized by minimum total energy dissipation (i.e., optimal channel networks) has provided a means of connecting the observed self-affinity and scaling in river basins to the physical dynamics of open, dissipative systems. However, it has been shown that observed scaling properties of river networks are obtained by suboptimal minimization schemes which restrict their search for optimality to local minima that are accessible from initial conditions [Rigon et al., 1998]. The limitation of natural processes that search for optima which are dynamically accessible within the constraints of initial and boundary conditions has been termed feasible optimality, a condition also observed in systems like Ising ferromagnets [Swift et al., 1997] and the aggregation of metal balls within open, dissipative electrical systems [Marani et al., 1998]. Here we consider the argument that minimization of plant water

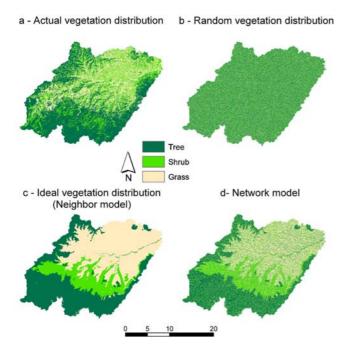
stress plays a fundamental role in the organization of vegetation pattern in semi-arid river basins, but that the suite of possible patterns obtained is similarly constrained to a suboptimal configuration due to local minima enforced by the structure of the river network. To this end, we analyze and simulate the spatial distribution of water stress arising from different vegetation configurations in the Upper Rio Salado basin, located near the Sevilleta Long-term Ecological Research (LTER) site in central New Mexico.

### 2. Methods

[3] The Upper Rio Salado basin covers an area of 464 km<sup>2</sup>, and its elevation ranges from 1985 meters above sea level (a.s.l.) to 2880 m a.s.l. The high degree of relief, heterogeneous soils, semi-arid climate, and diverse vegetation types present in the selected sub-basin make it a suitable candidate for examining the hypothesis that the river basin structure acts as a template for the spatial distribution of vegetation which self-organizes through local stress optimization within the network flow paths of the basin. During the growing season (May-September) the Upper Rio Salado basin receives a mean rainfall depth that varies according to elevation, ranging between 220-325 mm/season. Here we represent the rate of rainfall as a marked Poisson process of storm arrivals in time with rate  $\lambda$  $(day^{-1})$ , each storm having a depth h (mm), where h is modeled as an exponentially distributed random variable with mean  $\alpha$  (mm). Twelve years of daily rain gauge data (1990-2001) recorded in the Sevilleta research area and locations near to the Rio Salado basin are used to develop relationships between elevation and each of the rainfall process parameters. The stream network and basin flow paths are derived from 30-meter resolution USGS digital elevation models [Gesch et al., 2002]. The geomorphic structure of the basin is determined by the flow direction for each location within the basin, which is derived from the distribution of elevations and slopes contained within the digital elevation model data. Patterns of soil texture and land cover within the Rio Salado basin are used to parameterize a model of stochastic soil moisture dynamics. Vegetation pattern is taken from the USGS National Land Cover Dataset [Vogelmann et al., 2001], which varies between grassland, shrubland, or forest. A total of eight parameters are required for each of the three vegetation types, the most important of which are rooting depth, leaf area index, and maximum stomatal conductance. A further seven parameters are used to characterize the soil, including most critically soil porosity and vertical saturated hydraulic conductivity. The values of soil parameters vary according to the distribution of three different soil textures - loam, sandy loam, and silty loam, whose spatial variation is taken from the USDA STATSGO soil database [USDA, 1994]. In

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**Figure 1.** (a) Actual vegetation distribution in the upper Rio Salado river basin. (b), Random vegetation pattern representing equal amounts of each land cover type. (c), Ideal (optimal) vegetation pattern that minimizes water stress at each location. (d), The pattern of vegetation that arises from local optimization within the network flow path starting from an initially random condition.

the upper part of the basin, loam and silty loam are most common. Sandy loam is found in the lower portion of the basin, particularly along the channel network. A detailed presentation of the derivation of the necessary soil and vegetation parameters for the study basin is given elsewhere (K. K. Caylor et al., On the coupled geomorphological and ecohydrological organization of river basins, submitted to *Advanced Water Resources*, 2004, hereinafter referred to as Caylor et al., submitted manuscript, 2004).

[4] The climate, soil, and vegetation dynamics are linked to plant water stress through the probability distribution of soil moisture and the components of the water balance. The approach used here follows that of Rodriguez-Iturbe et al. [1999], and relies upon an analytical solution for a stochastic differential equation of the relative soil moisture, s [Laio et al., 2001]. Recent work on stochastic soil moisture dynamics across hillslopes has demonstrated that topography can lead to significant spatial gradients in soil moisture due to lateral soil moisture redistribution, but that such patterns only develop in relatively humid climates [Ridolfi et al., 2003]. Consequently, in this analysis of a semi-arid river basin we have chosen to neglect the impact of lateral soil moisture redistribution. Vegetation water stress patterns within the Rio Salado basin are determined using a dynamic water stress index [Porporato et al., 2001],  $0 \le \theta \le 1$ . This index is a function of the average degree of stress experienced by vegetation during a growing season, the average number of excursions below the critical soil moisture level at which the plant begins to close stomata in response to water stress, and the mean duration of such excursions.

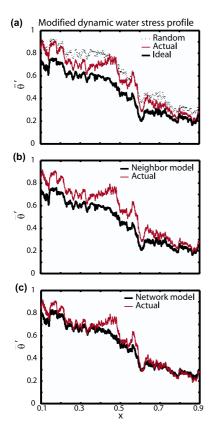
Data from weather stations proximate to the Rio Salado basin indicate that the average winter snowfall increases from <13 cm at 2000 meters (Magdalena, NM; NCDC COOP Station # 295353) to over 127 cm at 2400 meters (Pietown, NM; NCDC COOP Station #296812). We therefore adopt a modified dynamic water stress ( $\theta'$ ) according to Caylor et al. (submitted manuscript, XXXX) which scales the statistical steady state stress condition according to the duration of the mean first passage time between an initial condition associated with the spring snow melt and the statistical steady state mean [cf. Rodriguez-Iturbe et al., 2001]. We determine the average value of the steady state modified dynamic water stress,  $\theta'$ , for all points located at the same distance x from the basin outlet, where x is measured through the network and  $0 < x \le 1$  through normalization by the maximum upstream distance from the outlet. This modified dynamic water stress profile,  $\theta'(x)$ , is therefore an average quantity conditioned on the network width function, which is itself a well-studied metric of basin geomorphology [Marani et al., 1994].

[5] We investigate the extent to which the current organization of vegetation (Figure 1a) is optimized with respect to dynamic water stress through a comparison of the existing vegetation pattern to two alternative hypothetical vegetation distributions. The first is the distribution of vegetation that results from a random assignment of vegetation type at each location within the basin (Figure 1b). The second hypothetical pattern is the one arising from the specification of the vegetation type that exhibits the lowest dynamic water stress at each location within the basin (Figure 1c). From the sole point of view of water stress we could say that the pattern in Figure 1c represents an optimal or ideal distribution of vegetation. The hypothesis of feasible optimality is explored using two simple cellular automata approaches to model the statistical steady state conditions of a vegetation mosaic initiated from a random condition containing 1/3 each of trees, shrub and grass. In both models, the initial random vegetation mosaic is modified through the iteration of local interactions that occur between adjacent locations. These interactions are defined such that vegetation replacement can occur when the stress at a randomly chosen location is less than the stress in an adjacent location. Following the initial random placement of vegetation, a cell at location i within the basin is randomly selected and allowed to replace an adjacent location n with probability  $1 - (\theta_i'/(\theta_i' + \theta_n'))$ . If a successful replacement occurs, the procedure is repeated with a new neighbor chosen from the locations adjacent to cell n, otherwise a new random location i is chosen. Of critical importance is the manner by which the two models differ in how the adjacent neighborhood of each location is defined. Our first approach (the 'neighbor' model) is based on local interactions that occur between each location and a random neighbor chosen from any of the locations adjacent to the cell under consideration. Our second approach (the 'network' model) is constrained such that interactions only occur between each location and its direct downstream neighbor. In both models the procedure of probabilistic replacement is repeated until the initially random mosaic of vegetation evolves to a steady state condition in which no further replacements occur ( $\sim 10^7$  steps).

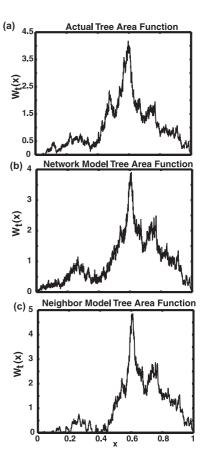
[6] The ability of each modeling approach to appropriately characterize the observed pattern of vegetation distribution within the Rio Salado basin is confirmed through a comparison of the steady state modified dynamic water stress profile described above, and the steady state patterns of tree distribution. The distribution of tree vegetation resulting from our model simulations is analyzed through the normalized tree area function,  $W_t(x)$ , which is the distribution of locations containing tree vegetation at each distance from the outlet x, measured through the basin flow paths in constant intervals of  $\Delta x$ . The resulting function is the probability distribution of tree vegetation within the basin, such that  $\sum_{i=1}^{1} W_t(x) \Delta x = 1$ .

### 3. Results

[7] A comparison of the three alternative vegetation patterns in Figure 1 (actual, random, and ideal) suggests that the actual pattern of vegetation distribution (Figure 1a) contains elements of both the highly organized ideal pattern (Figure 1b), as well the characteristic variation associated with the random pattern (Figure 1c). The stress profiles of the two hypothetical distributions as well as the actual dynamic water stress profile are presented in Figure 2a.



**Figure 2.** Modified dynamic water stress profiles for the Upper Rio Salado basin. (a), Actual vegetation pattern (red line), a random vegetation pattern (dashed line) and the optimal vegetation pattern (solid line). (b), Actual vegetation pattern (red line) and the neighbor model at statistical steady state (solid line). (c), Actual vegetation pattern (red line) and the network model at statistical steady state (solid line).



**Figure 3.** Tree area functions  $W_t(x)$  for the Upper Rio Salado basin. (a), Actual tree area function. (b), Tree area function for the network model at statistical steady state. (c), Tree area function for the neighbor model at statistical steady state.

The current vegetation pattern is well-constrained by the two 'extremes' of vegetation organization so the basin tends to experience an intermediate level of water stress that is neither random nor ideal. This suggests that vegetation patterns in water-controlled ecosystems may be organized around an optimal configuration in terms of water stress, but they are also subject to important and decisive random contingencies of an altogether different character. Conceptually, this is not different than the notion of feasible optimality at work in the organization of the drainage network itself.

[8] The results of our cellular automata modeling reveal that the steady state configuration of the neighbor model (Figure 1c) is identical to that of the ideal basin. In contrast, the steady state configuration of the network model (Figure 1d) is markedly different in character, preserving elements of both the ideal condition and the type of spatial variation exhibited in the actual vegetation pattern (Figure 1a). In Figures 2b–2c we compare the modified dynamic water stress profile for each of the two models at steady state condition with the actual modified dynamic water stress profile for the basin. The stress profile of the neighbor model (Figure 2b) is identical to that of the ideal vegetation (Figure 2a), which is consistently lower than that of the actual basin. In contrast, the agreement between the stress profile for the network model and that for the actual

basin is much better. As in the case of the water stress profile, the tree area function of the network model (Figure 3b) provides a better agreement with the actual pattern (Figure 3a) than does the neighbor model (Figure 3c). In particular, while the neighbor model predicts the absence of tree vegetation at a number of locations in the basin, the network model is consistent with the actual distribution of tree vegetation, which is maintained throughout the basin profile. These results support the hypothesis that in water controlled ecosystems, the drainage network acts as a template for the spatial distribution of vegetation which self-organizes through local stress optimization within the network flow paths of the basin. Moreover, such organization links the geomorphological, ecological and hydrological dynamics operating in the river basin.

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