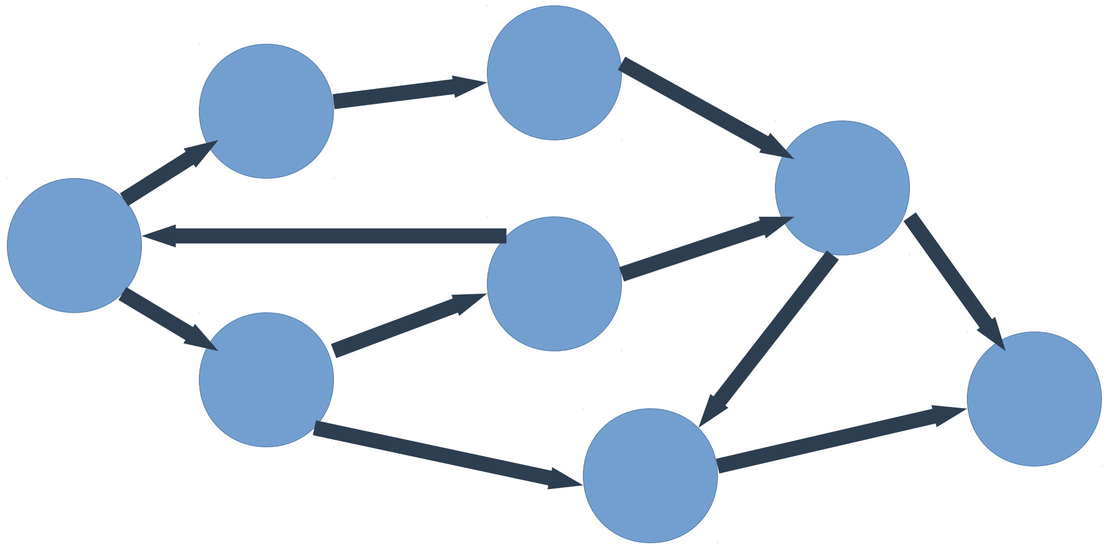

Planning ahead: using Markov decision processes to optimize your life

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Bio

Eric Cotner is a former theoretical physicist who studied exotic dark matter candidates such as Q-balls and black holes during his graduate school years. More recently, he has turned his attention to data science; specifically prescriptive analytics. He works as a data scientist at American Tire Distributors in Huntersville, where he focuses on supply chain and logistics problems such as optimizing delivery routes and modeling inter-warehouse product flow. In his free time, he enjoys hiking, astronomy, and scuba diving. He holds a PhD/Masters/Bachelors in Physics from UCLA/UCLA/UCSD.

Outline

- How to quantify risk/reward?
- Value of sequential actions
- What is a Markov decision process?
- An example MDP
- Solving MDP's using `pymdptoolbox`
- Working with more complex problems
- Reinforcement learning

How to quantify risk/reward/value?

- **Reward: some thing of value gained (money, game points, reddit karma, personal satisfaction, etc.) by achieving certain goals**
- **Risk: (reward) x (probability of getting that reward)**
  - Example: roll a 6-side die; {1,4}=-$2.15, {2,3,5,6}=+$1.34; risks associated with these two outcomes are (-$2.15)x(2/6)= -$0.72 and (+$1.34)x(4/6)=+$0.89
- **Value: sum of all risks; i.e. the expected/average reward**
  - Previous example: value = (-$0.72) + (+$0.89) = +$0.17

Sequential decisons/actions

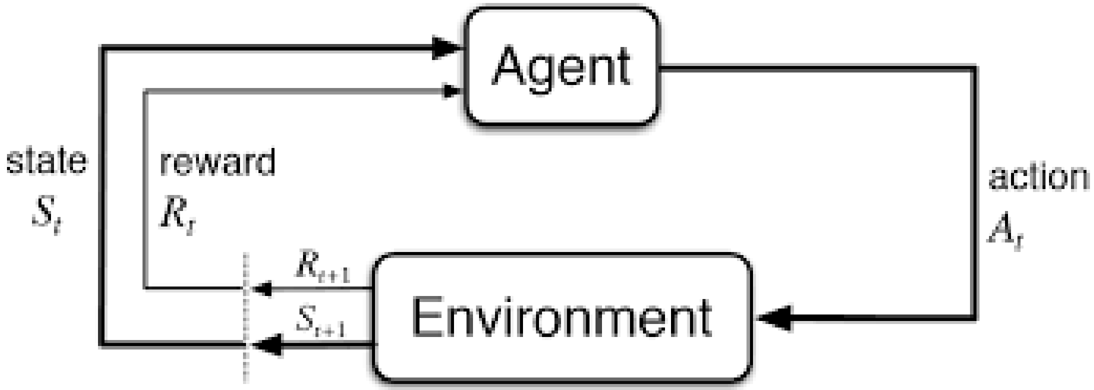
- **Previous example's usefulness is limited**
  - What if you need to plan multiple steps ahead?
  - What if rewards change with time or are random?
  - What if there is more than one available course of action?
  - What if actions don't always result in intended consequences?
- **Value is (discounted) sum of expected future rewards:**

$$v_t = \mathbb{E}\left[\sum_{t'=t}^{T} \gamma^{t'-t} R_{t'}\right]$$

- **How do you determine the optimal sequence of actions to take?**
- **What is the value associated with these actions?**

What is a Markov decision process?

- **Markov decision process (MDP) is a mathematical formalism for solving sequential decision problems**
  - It is called *Markov* because it satisfies the *Markov* property: all states and rewards do *not* depend on past history
- **MDP's are defined in terms of *states*, *actions*, *rewards*, and *transitions*, which are fixed by the nature of the problem/environment**
  - Capital letters denote random variables, lower case denote actual samples
- **Things to solve for are the *policy* (tells you which action to take) and *value function(s)* (how much reward you expect to get from taking an action)**

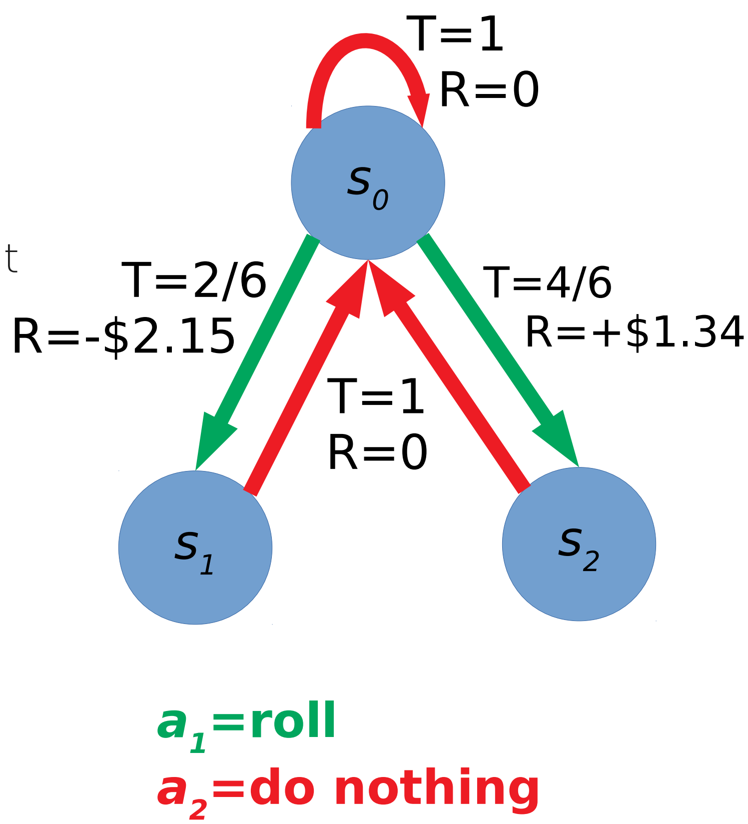


States and actions

- ***States*** **represent the current state of the environment**
  - Usually denoted by symbols s, S_t (sometimes x in control theory); set of all states: $\mathcal{S} = \{s_1, s_2, \ldots\}$
  - Previous die example: s_1=1, s_2=2, …, s_6=6 (or s_1={1,4}, s_2={2,3,5,6})
- ***Actions*** **represent the decisions you can make which will cause you to transition from state to state**
  - Usually denoted by symbols a, A_t (sometimes u in control theory); set of all actions: $\mathcal{A} = \{a_1, a_2, \ldots\}$
  - Previous die example: a_1=roll, a_2=do nothing

State transitions and rewards

- **Only some states lead to other states; sometimes taking one action can lead to multiple potential states**
- **Model this using the transition matrix *T(s';s,a)***
  - *T(s';s,a)* = the probability to transition to state ***s'*** given that you take action ***a*** from state ***s***
  - 6-sided die example: $T(s_1;s_0,a_1)=2/6$, $T(s_2;s_0,a_1)=4/6$, $T(s_0;*,a_2)=1$, otherwise $T=0$
- **Sometimes rewards can be stochastic too; model this using reward matrix R(r;s,a)**
  - *R(r;s,a)* = probability of getting reward ***r*** when taking action ***a*** from state ***s***
  - If reward is deterministic, reward matrix ***R(s,a)*** = reward received from taking action ***a*** from state ***s***



The policy

- **How do you decide what to do in a given state?**
- **The *policy* function π determines which action to take**
  - Could be deterministic: $\pi(s) : \mathcal{S} \rightarrow \mathcal{A}$
  - Could be stochastic: $\pi(a|s) : \mathcal{S} \times \mathcal{A} \rightarrow [0, 1]$
  - (deterministic is special case of stochastic where all probability mass is located at one action)
- **The optimal policy is denoted by π*, which is the policy that maximizes the sum of future expected rewards (value function)**

Value functions

- **Two kinds of value functions:**
  - The state-value function ***V_π(s)***: the future reward you expect when starting from state ***s*** and following policy π
  - The action-value function ***Q_π(s,a)***: the future reward you expect when starting from state ***s*** and taking action ***a***, and following policy π thereafter

$$V_\pi(s) = \sum_{a \in \mathcal{A}} \pi(a|s) Q_\pi(s, a)$$

$$Q_\pi(s, a) = \mathbb{E}_{A \sim \pi} \left[\sum_{t'=t}^{T} \gamma^{t'-t} R_{t'} \middle| S_t = s_t, A_t = a \right]$$

How do you actually solve these equations?

- **Many different algorithms exist depending on size of state/action spaces, length of episodes, structure of reward/transition matrices…**
- **All of them use some form of the Bellman equation:**

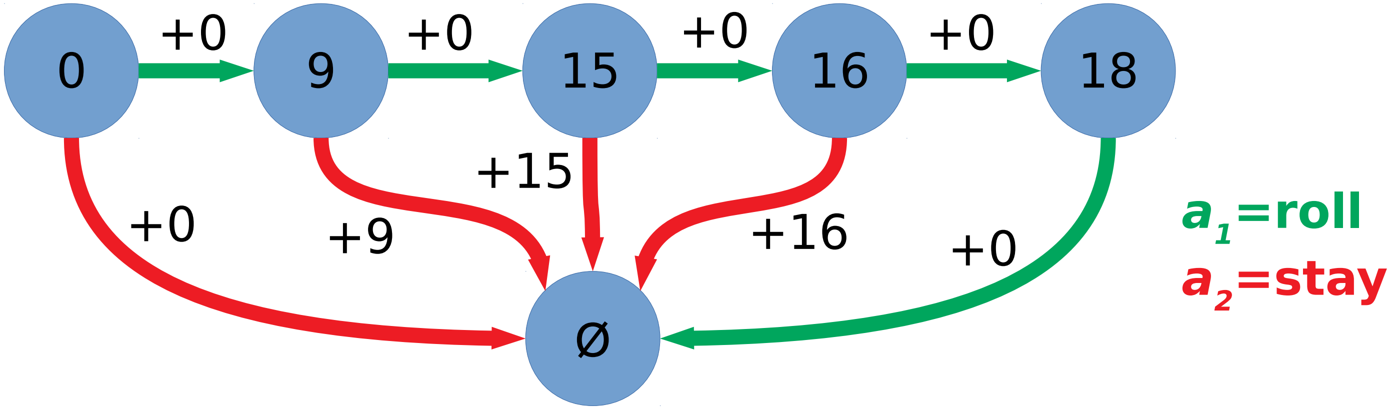
$$Q_\pi(s,a) \leftarrow \mathbb{E}_\pi\left[R(s,a) + \gamma Q(s',a')\right]$$

- **Consider one of the simplest: value iteration:**

1. Randomly initialize *Q(s,a)*
2. Select policy by maximizing value function: $\pi(s) = \underset{a}{\operatorname{argmax}}\, Q(s,a)$
3. Use Bellman equation to update estimate of *Q*
4. Repeat 2-3 until convergence

An example MDP

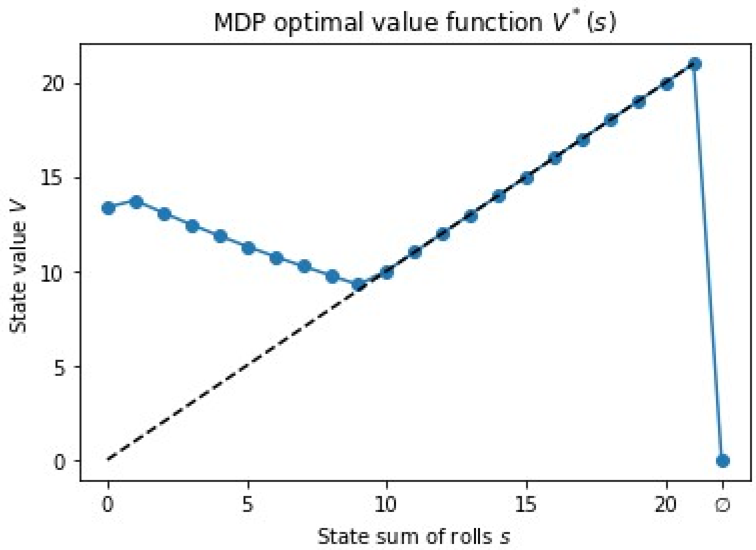
- **Let's look at a relatively simple example to solve – roll or stay?**
- **You have a 20-sided die, and you get to roll repeatedly until the sum of your rolls either gets as close as possible to 21 or you bust**
- **Your score is the numerical value of the sum of your rolls; if you bust, you get zero.**
- **What is the optimal strategy?**

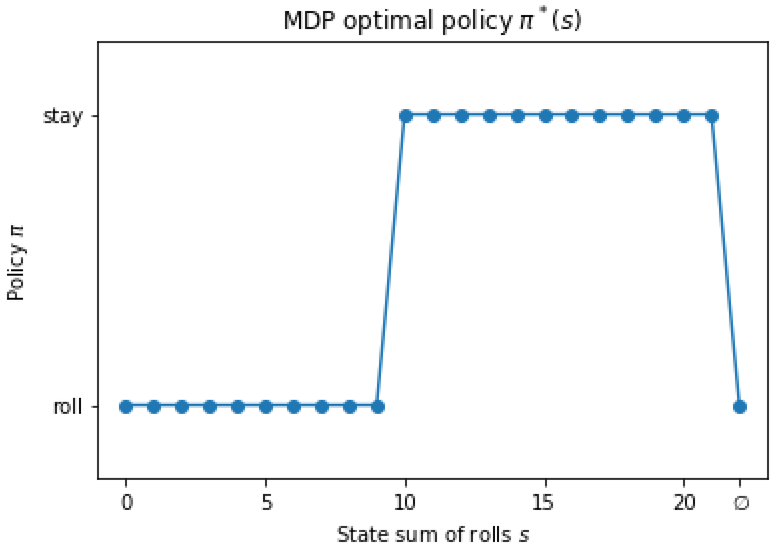


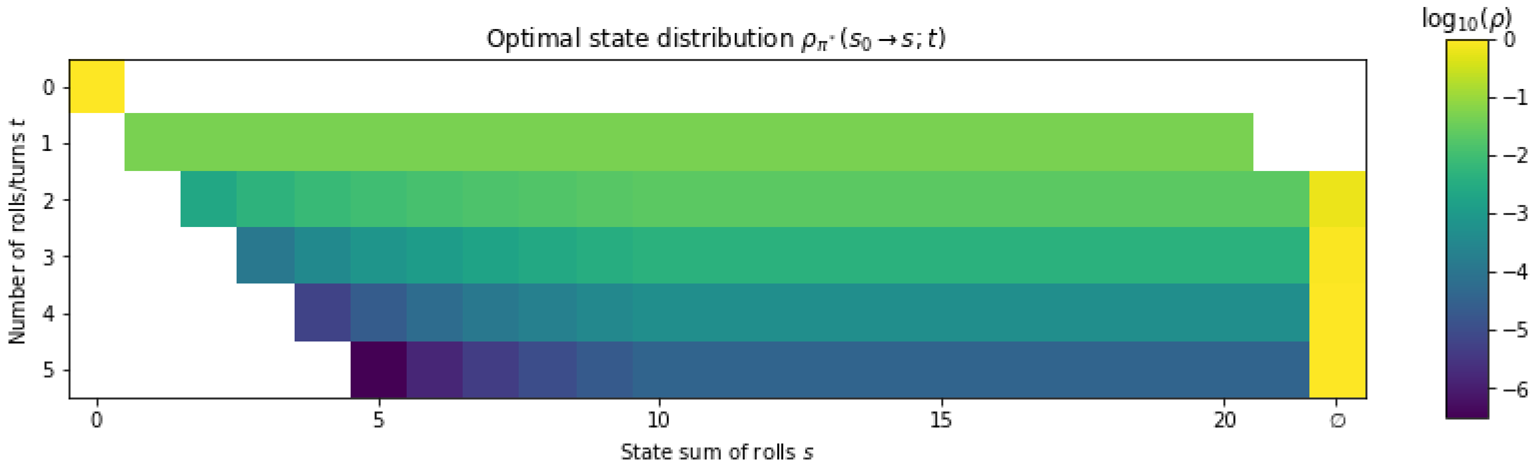
Using `pymdptoolbox`

- **Let's solve the example from the previous slide in python.**
- **We will use *pymdptoolbox*, a toolkit for solving discrete-time MDP's**
- **For more details, see the github repo or the API documentation**
- **Install with `` `pip install pymdptoolbox` ``**
- **Check out my demo jupyter notebook**

Example MDP solution





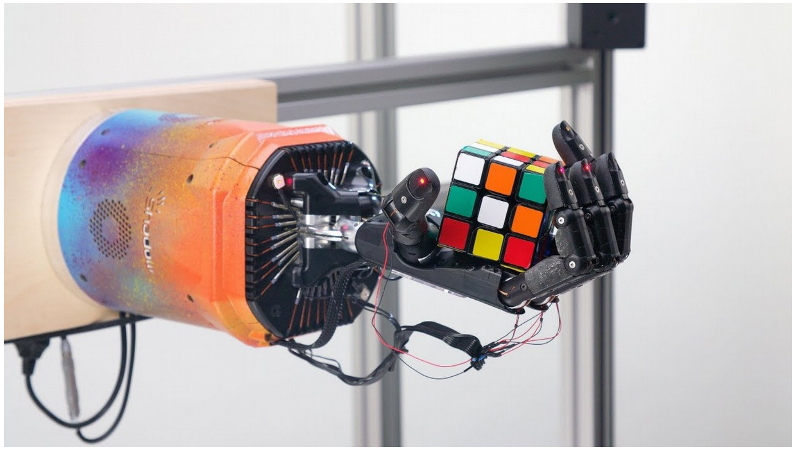


Working with more complex problems

- **Previous example was very simple compared to most realistic problems**
- **For example, maybe you're trying to plan purchase orders for your supply chain**
  - If you have 10 warehouses, each with 1,000 different products which can have stocking levels between 0-100, and you want to plan your orders for the next couple months… that's a 1M-dimensional state space and 1M-dimensional action space – would need a transition matrix with 10^18 elements…
- **For very large systems, you may have to resort to approximate solutions**
  - Bucket similar products together
  - Find better state/action representations – use dimensional reduction
  - Use function approximation to represent value/policy functions
- **Some problems have continuous state/action spaces and can't be formulated with matrices at all**
  - For example: self-driving car - state is position+orientation+velocity vectors, actions are amount of gas/brake/wheel angle; all probably continuous variables
  - Would need to use a completely different approach from the *value iteration* method we used

Reinforcement learning

- **Reinforcement learning: type of machine learning that learns how to act optimally in an environment to maximize rewards**
- **MDP’s are the underlying mathematical framework for RL – if you know MDP’s you can easily learn RL**
- **Very advanced algorithms that try to solve MDP’s in environments with enormous state spaces by only sampling states and actions encountered in simulation**
- **Can use neural networks to approximate complex value and policy functions**



Reference material

- ***`Pymdptoolbox`*; Steven Cordwell**
  - *https://pymdptoolbox.readthedocs.io/en/latest/api/mdptoolbox.html*
- ***Introduction to Reinforcement Learning*; Sutton, Barto:**
  - http://incompleteideas.net/sutton/book/the-book-2nd.html
- ***Spinning up in Deep RL*; OpenAI**
  - https://spinningup.openai.com/en/latest/
- **r/reinforcementlearning; Reddit**
  - https://www.reddit.com/r/reinforcementlearning/

Thank you!