

Homework #0

28 Tháng Sáu 2019 9:37 CH

1. Continuous-Time Sinusoidal Generation.

a.

$$c(t) = \sin(2\pi fct) \text{rect}(t - 1/2)$$

$$C(f) = F\{c(t)\} = F\{\sin(2\pi fct) \text{rect}(t - 1/2)\}$$

$$C(f) = F\{\sin(2\pi fct)\} * F\{\text{rect}(t - 1/2)\} \quad F\{w_1(t)w_2(t)\} = W_1(f) * W_2(f)$$

$$F\{A \sin(2\pi f_0 t + \phi)\} = j \frac{A}{2} [-e^{-j\phi} \delta(f - f_0) + e^{-j\phi} \delta(f + f_0)]$$

$$*/ F\{\sin(2\pi fct)\} = j \frac{1}{2} [\delta(f + f_0) - \delta(f - f_0)]$$

$$F\{\text{rect}(t)\} = T \text{sinc}(fT) \quad \& \quad F\{w(t - t_0)\} = W(f) e^{-j2\pi f t_0}$$

$$*/ F\{\text{rect}(t - 1/2)\} = e^{-j\pi f} \text{sinc}(f)$$

$$\Rightarrow C(f) = j \frac{1}{2} [\delta(f + f_0) - \delta(f - f_0)] e^{-j\pi f} \text{sinc}(f)$$

$$w(t) * \delta(t + a) = w(t + a)$$

$$C(f) = j \frac{1}{2} [e^{-j\pi(f+f_0)} \text{sinc}(f + f_0) - e^{-j\pi(f-f_0)} \text{sinc}(f - f_0)]$$

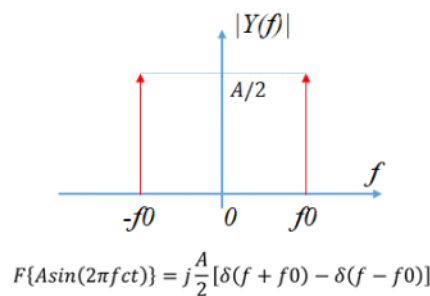
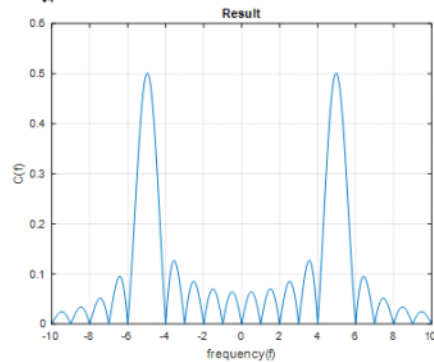
c.

Bandwith of $c(t) = 2\text{Hz}$ (4 – 6Hz)

Bandwith of two-side sine = 0

$$\text{Total Energy} = \int_0^\infty |C(f)|^2 df = 0.25J \text{ (Matlab)}$$

$$\int_{-6}^6 |C(f)|^2 df = 0.2265J = 90.6\% \text{Total Energy (Matlab)}$$



2. Downconversion.

a.

$$y(t) = x(t)\cos(\omega_0 t)$$

$$\Rightarrow Y(f) = F\{y(t)\} = X(f) * F\{\cos(\omega_0 t)\}$$

$$F\{\cos(\omega_0 t)\} = \frac{1}{2}[\delta(f + f_0) + \delta(f - f_0)]$$

$$Y(f) = X(f) * \frac{1}{2}[\delta(f + f_0) + \delta(f - f_0)]$$

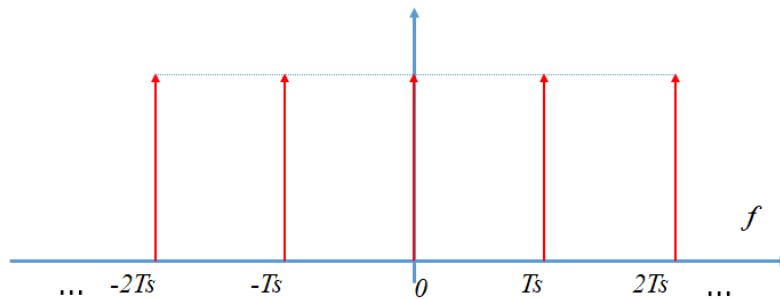
$$Y(f) = \frac{1}{2}[X(f) * \delta(f + f_0) + X(f) * \delta(f - f_0)]$$

$$Y(f) = \frac{1}{2}[X(f + f_0) + X(f - f_0)]$$

3. Sampling in Continuous Time.

a,b.

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$



Period = T_s (The sampling duration)

$$x(t) = x(t + nT_s)$$

c.

$$p(t) = \frac{1}{T_S} (1 + 2 \cos(\omega_s t) + 2 \cos(2\omega_s t))$$

$$F\{\cos(\omega_s t)\} = \pi[\delta(\omega + \omega_s) + \delta(\omega - \omega_s)]$$

$$F\{1\} = 2\pi\delta(\omega)$$

$$= \frac{2\pi}{T_S} (\delta(\omega) + \delta(\omega + \omega_s) + \delta(\omega - \omega_s) + \delta(\omega + 2\omega_s)$$

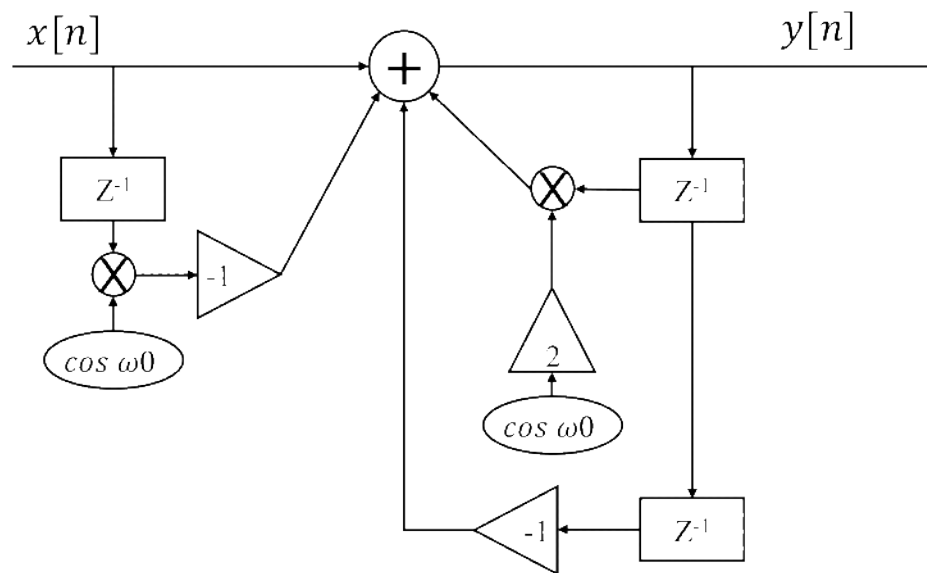
$$+ \delta(\omega - 2\omega_s) \dots)$$

$$p(t) = \omega_s \sum_{n=-\infty}^{\infty} \delta(\omega + n\omega_s)$$

4. Discrete-Time Sinusoidal Generation.

a.

$$y[n] = (2 \cos \omega_0) y[n-1] - y[n-2] + x[n] - (\cos \omega_0) x[n-1]$$



b.

The initial condition is zero

c.

$$y[n] = (2 \cos \omega_0) y[n-1] - y[n-2] + x[n] - (\cos \omega_0) x[n-1]$$

$$Y(z) = (2 \cos \omega_0) z^{-1} Y(z) - z^{-2} Y(z) + X(z) - (\cos \omega_0) z^{-1} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - (\cos \omega_0) z^{-1}}{1 - 2(\cos \omega_0) z^{-1} + z^{-2}}$$