

$$1. \begin{cases} 5x - 4y = 6 \\ 2x - y = 3 \end{cases}$$

$$\begin{pmatrix} 5 & -4 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

$$\begin{vmatrix} 5 & -4 \\ 2 & -1 \end{vmatrix} = 3,$$

$$\therefore \begin{pmatrix} 5 & -4 \\ 2 & -1 \end{pmatrix}^{-1} \text{ exists}$$

$$\begin{pmatrix} 5 & -4 \\ 2 & -1 \end{pmatrix}^{-1} = \frac{1}{3} \begin{pmatrix} -1 & -(2) \\ -(-4) & 5 \end{pmatrix}^T$$

$$= \frac{1}{3} \begin{pmatrix} -1 & 4 \\ -2 & 5 \end{pmatrix},$$

$$\begin{pmatrix} 5 & -4 \\ 2 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 5 & -4 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & -4 \\ 2 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

$$I \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3} \begin{pmatrix} -1 & 4 \\ -2 & 5 \end{pmatrix} \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\therefore \begin{cases} x = 2 \\ y = 1 \end{cases}$$

$$2. \begin{cases} 7x + 2y = -1 \\ -6x - y = 3 \end{cases}$$

$$\begin{pmatrix} 7 & 2 \\ -6 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$\begin{vmatrix} 7 & 2 \\ -6 & -1 \end{vmatrix} = 5$$

$$\therefore \begin{pmatrix} 7 & 2 \\ -6 & -1 \end{pmatrix}^{-1} \text{ exists}$$

$$\begin{pmatrix} 7 & 2 \\ -6 & -1 \end{pmatrix}^{-1} = \frac{1}{5} \begin{pmatrix} -1 & -(-6) \\ -2 & 7 \end{pmatrix}^T$$

$$= \frac{1}{5} \begin{pmatrix} -1 & -2 \\ 6 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 7 & 2 \\ -6 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 7 & 2 \\ -6 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 & 2 \\ -6 & -1 \end{pmatrix}^{-1} \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$\begin{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 & 2 \\ -6 & -1 \end{pmatrix}^{-1} \begin{pmatrix} -1 \\ 3 \end{pmatrix} \end{bmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{5} \begin{pmatrix} -1 & -2 \\ 6 & 7 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{5} \begin{pmatrix} -5 \\ 15 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$\therefore \begin{cases} x = -1 \\ y = 3 \end{cases}$$

$$3. \begin{cases} x + 2y = 8 \\ y + z = 6 \\ -x + y + 2z = 7 \end{cases}$$

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \\ 7 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ -1 & 1 & 2 \end{vmatrix} = -1$$

$$\therefore \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}^{-1} \text{ exists}$$

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}^{-1} = - \begin{pmatrix} 1 & -(-1) & -1 \\ -(-4) & 2 & -(-1) \\ 2 & -(1) & 1 \end{pmatrix}^T$$

$$= - \begin{pmatrix} 1 & -4 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 4 & -2 \\ -1 & -2 & 1 \\ 1 & -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 8 \\ 6 \\ 7 \end{pmatrix}$$

$$I \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 & 4 & -2 \\ -1 & -2 & 1 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} 8 \\ 6 \\ 7 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix}$$

$$\therefore \begin{cases} x = 2, \\ y = 3, \\ z = 3 \end{cases}$$

4.
$$\begin{cases} x + y = 2 \\ 2y + 3z = 10 \\ x + 2z = 2 \end{cases}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 3 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 10 \\ 2 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 1 & 0 \\ 0 & 2 & 3 \\ 1 & 0 & 2 \end{vmatrix} = 7$$

$$\therefore \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 3 \\ 1 & 0 & 2 \end{pmatrix}^{-1} \text{ exists}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 3 \\ 1 & 0 & 2 \end{pmatrix}^{-1} = \frac{1}{7} \begin{pmatrix} 4 & -3 & -2 \\ -12 & 2 & -4 \\ 3 & -3 & 2 \end{pmatrix}$$

$$= \frac{1}{7} \begin{pmatrix} 4 & -2 & 3 \\ +3 & 2 & -3 \\ +2 & +1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 3 \\ 1 & 0 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 3 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 3 \\ 1 & 0 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 2 \\ 10 \\ 2 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 4 & -2 & 3 \\ +3 & 2 & -3 \\ +2 & +1 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 10 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{7} \begin{pmatrix} -6 \\ 20 \\ 10 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{6}{7} \\ \frac{20}{7} \\ \frac{10}{7} \end{pmatrix}$$

$$\therefore \begin{cases} x = -\frac{6}{7}, & z = \frac{10}{7}, \\ y = \frac{20}{7}, \end{cases}$$

5.
$$\begin{cases} x + 2y - 2z = 1 \\ -x - y + 3z = 4 \\ x + y - z = 0 \end{cases}$$

$$\begin{pmatrix} 1 & 2 & -2 \\ -1 & -1 & 3 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 2 & -2 \\ -1 & -1 & 3 \\ 1 & 1 & -1 \end{vmatrix} = 2, \quad \therefore \begin{pmatrix} 1 & 2 & -2 \\ -1 & -1 & 3 \\ 1 & 1 & -1 \end{pmatrix}^{-1} \text{ exists}$$

$$\therefore \begin{pmatrix} 1 & 2 & -2 \\ -1 & -1 & 3 \\ 1 & 1 & -1 \end{pmatrix}^{-1} = \frac{1}{2} \begin{pmatrix} -2 & -2 & 0 \\ -(-1) & 1 & -(-1) \\ 4 & -1 & 1 \end{pmatrix}^T$$

$$= \frac{1}{2} \begin{pmatrix} -2 & 0 & 4 \\ 2 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & -2 \\ -1 & -1 & 3 \\ 1 & 1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 2 & -2 \\ -1 & -1 & 3 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 2 & -2 \\ -1 & -1 & 3 \\ 1 & 1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2} \begin{pmatrix} -2 & 0 & 4 \\ 2 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -2 \\ 6 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$$

$$\therefore \begin{cases} x = -1 \\ y = 3 \\ z = 2 \end{cases}$$

6.
$$\begin{cases} x+2y+2z=8 \\ x+y+z=5 \\ 2x+y+2z=9 \end{cases}$$

$$\begin{pmatrix} 1 & 2 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \\ 9 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 2 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = -1$$

$$\therefore \begin{pmatrix} 1 & 2 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix}^{-1} \text{ exists}$$

$$\begin{pmatrix} 1 & 2 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix}^{-1} = \frac{1}{-1} \begin{pmatrix} 1 & -0 & -1 \\ -1 & 2 & -3 \\ 0 & -1 & -1 \end{pmatrix}^T$$

$$= - \begin{pmatrix} 1 & -2 & 0 \\ 0 & -2 & 1 \\ -1 & 3 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 2 & 0 \\ 0 & 2 & -1 \\ 1 & -3 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 2 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 2 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 8 \\ 5 \\ 9 \end{pmatrix}$$

$$\begin{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 & 2 & 0 \\ 0 & 2 & -1 \\ 1 & -3 & 1 \end{pmatrix} \begin{pmatrix} 8 \\ 5 \\ 9 \end{pmatrix} \end{bmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

$$\therefore \begin{cases} x = 2, \\ y = 1, \\ z = 2, \end{cases}$$

13.
$$\begin{cases} x - y + z = 6 \\ 3x - 2y - 2z = 4 \\ x - y - z = 0 \end{cases}$$

$$\begin{pmatrix} 1 & -1 & 1 \\ 3 & -2 & -2 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ 0 \end{pmatrix}$$

$$\begin{vmatrix} 1 & -1 & 1 \\ 3 & -2 & -2 \\ 1 & -1 & -1 \end{vmatrix} = -2$$

$$\therefore \begin{pmatrix} 1 & -1 & 1 \\ 3 & -2 & -2 \\ 1 & -1 & -1 \end{pmatrix}^{-1} \text{ exists,}$$

$$\begin{aligned} \therefore \begin{pmatrix} 1 & -1 & 1 \\ 3 & -2 & -2 \\ 1 & -1 & -1 \end{pmatrix}^{-1} &= \frac{1}{-2} \begin{pmatrix} 0 & -1 & -1 \\ -1 & -2 & -1 \\ 4 & -5 & 1 \end{pmatrix}^T \\ &= \frac{1}{-2} \begin{pmatrix} 0 & -2 & 4 \\ 1 & -2 & 5 \\ -1 & 0 & 1 \end{pmatrix} \end{aligned}$$

$$\begin{pmatrix} 1 & -1 & 1 \\ 3 & -2 & -2 \\ 1 & -1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & -1 & 1 \\ 3 & -2 & -2 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 \\ 3 & -2 & -2 \\ 1 & -1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 6 \\ 4 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-2} \begin{pmatrix} 0 & -2 & 4 \\ 1 & -2 & 5 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 6 \\ 4 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{-2} \begin{pmatrix} -8 \\ -2 \\ -6 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$$

$$\therefore \begin{cases} x = 4, \\ y = 1, \\ z = 3. \end{cases}$$

14.
$$\begin{cases} 3x + 3y + z = -8 \\ 5x + 4y + z = -12 \\ x + 4y + 2z = -7 \end{cases}$$

$$\begin{pmatrix} 3 & 3 & 1 \\ 5 & 4 & 1 \\ 1 & 4 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -8 \\ -12 \\ -7 \end{pmatrix}$$

$$\begin{vmatrix} 3 & 3 & 1 \\ 5 & 4 & 1 \\ 1 & 4 & 2 \end{vmatrix} = 1$$

$$\therefore \begin{pmatrix} 3 & 3 & 1 \\ 5 & 4 & 1 \\ 1 & 4 & 2 \end{pmatrix}^{-1} \text{ exists}$$

$$\begin{pmatrix} 3 & 3 & 1 \\ 5 & 4 & 1 \\ 1 & 4 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 4 & -9 & 16 \\ -12 & 5 & -9 \\ -1 & -(-2) & 3 \end{pmatrix}^T$$

$$\begin{pmatrix} 3 & 3 & 1 \\ 5 & 4 & 1 \\ 1 & 4 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 4 & -2 & -1 \\ -9 & 5 & 2 \\ 16 & -9 & -3 \end{pmatrix},$$

$$\begin{pmatrix} 3 & 3 & 1 \\ 5 & 4 & 1 \\ 1 & 4 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 3 & 3 & 1 \\ 5 & 4 & 1 \\ 1 & 4 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 & 3 & 1 \\ 5 & 4 & 1 \\ 1 & 4 & 2 \end{pmatrix}^{-1} \begin{pmatrix} -8 \\ -12 \\ -7 \end{pmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{pmatrix} 4 & -2 & -1 \\ -9 & 5 & 2 \\ 16 & -9 & -3 \end{pmatrix} \begin{pmatrix} -8 \\ -12 \\ -7 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$$

$$\therefore \begin{cases} x = -1 \\ y = -2 \\ z = 1 \end{cases}$$