HKALE 2006 AL Pure Mathematics Paper 2 Q5

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(a) Find $\int \ln y \, dy$.

(b) Find the volume of the solid of revolution generated by revolving the region bounded by the curve $y = 2^{x^2}$ and the straight line $y = 2$ about the y-axis. (6 marks)

(a) $\int \ln y \, dy$ integration by parts

$= [y \ln y] - \int y \, d(\ln y)$

$= [y \ln y] - \int y \frac{dy}{y}$

$= [y \ln y] - \int dy$

$= [y \ln y] - y + C$ //

(b) The volume : $\int_0^1 2\pi (x)(2-y) \, dx$

$= 2\pi \int_0^1 x(2 - 2^{x^2}) \, dx$

$= 2\pi \int_0^1 (2x - 2^{x^2} x) \, dx$

$= 2\pi \int_0^1 2x \, dx - 2\pi \int_0^1 2^{x^2} x \, dx$

$$= 2\pi\left[x^2\right]_0^1 - 2\pi\int_0^1 x\, 2^{x^2}\frac{d(x^2)}{2x}$$

$$= 2\pi\left[(1)-(0)\right] - \pi\int_0^1 2^{x^2}\, d(x^2)$$

$$\int a^x\, dx = \frac{a^x}{\ln(a)}$$

$$= 2\pi - \pi\left[\frac{2^{x^2}}{\ln(2)}\right]_0^1$$

$$= 2\pi - \pi\left[\frac{2^{(1)^2}}{\ln(2)} - \frac{2^{(0)^2}}{\ln(2)}\right]$$

$$= 2\pi - \pi\left[\frac{2}{\ln(2)} - \frac{1}{\ln(2)}\right]$$

$$= 2\pi - \pi\left[\frac{1}{\ln(2)}\right]$$

$$= 2\pi - \frac{\pi}{\ln(2)}$$