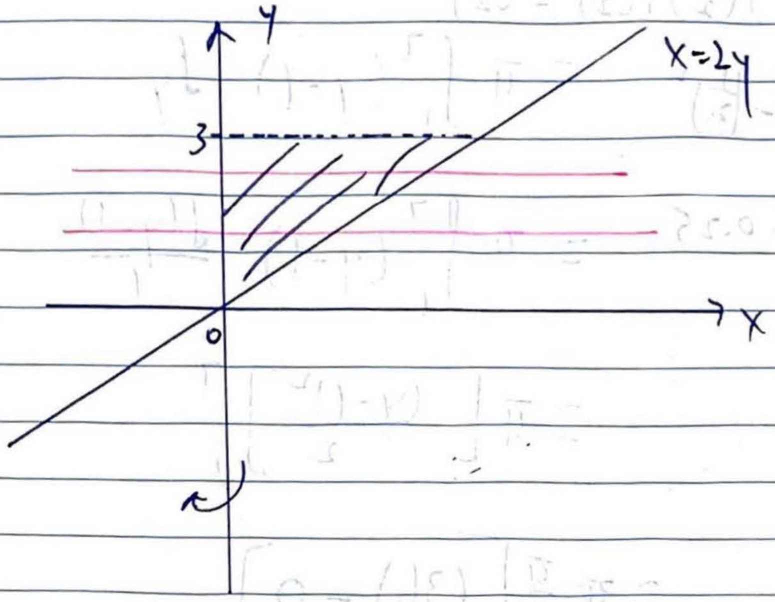
2.



Volume : $\int_0^3 \pi(2y)^2\, dy$

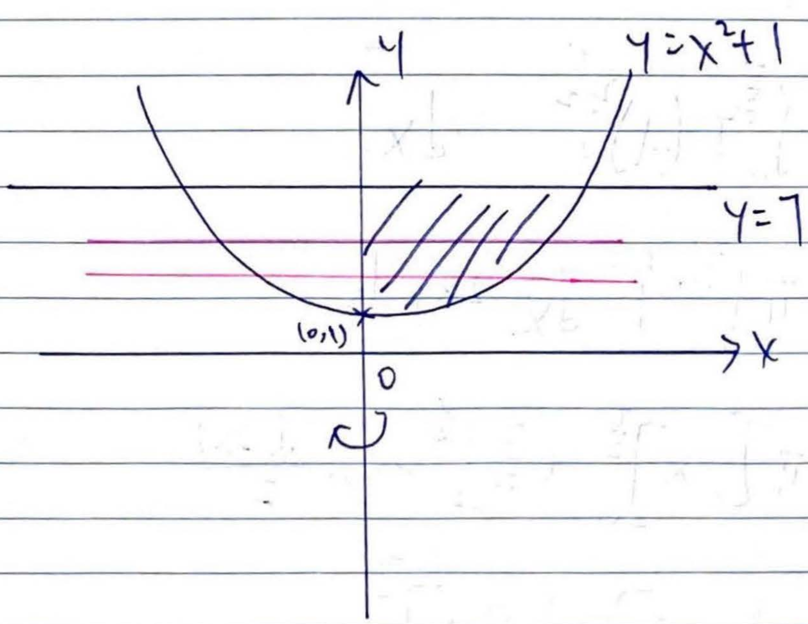
$= \pi \int_0^3 4y^2\, dy$

$= 4\pi \left[\frac{y^3}{3} \right]_0^3$

$= 4\pi \left[\frac{27}{3} - 0 \right]$

$= 36\pi$

4.



$\cancel{y = x^2+1}$ Volume : $\int_1^7 \pi (\sqrt{y-1})^2 \, dy$

$= \pi \int_1^7 (y-1) \, dy$

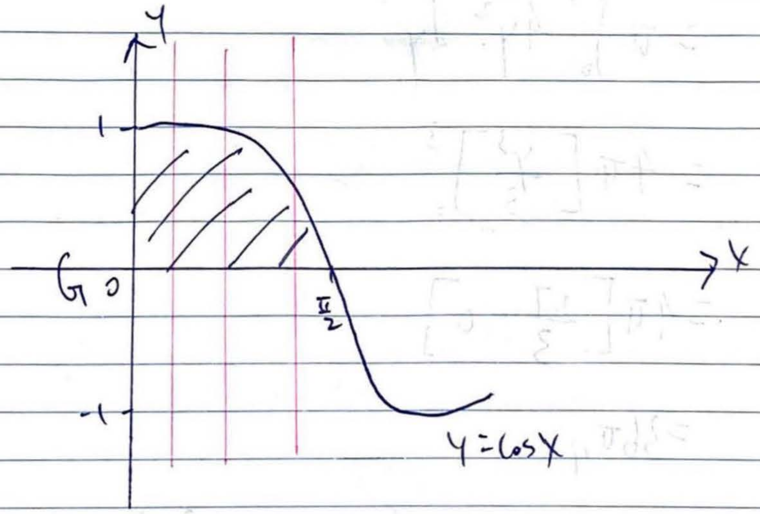
$= \pi \int_1^7 (y-1) \, \frac{d(y-1)}{1}$

$= \pi \left[\frac{(y-1)^2}{2} \right]_1^7$

$= \cancel{\pi} \frac{\pi}{2} \left[(36) - 0 \right]$

$= 18\pi$,,

6.



Volume : $\int_0^{\frac{\pi}{2}} \pi (\cos x)^2 \, dx$

$= \pi \int_0^{\frac{\pi}{2}} \frac{1}{2} (1 + \cos 2x) \, dx$

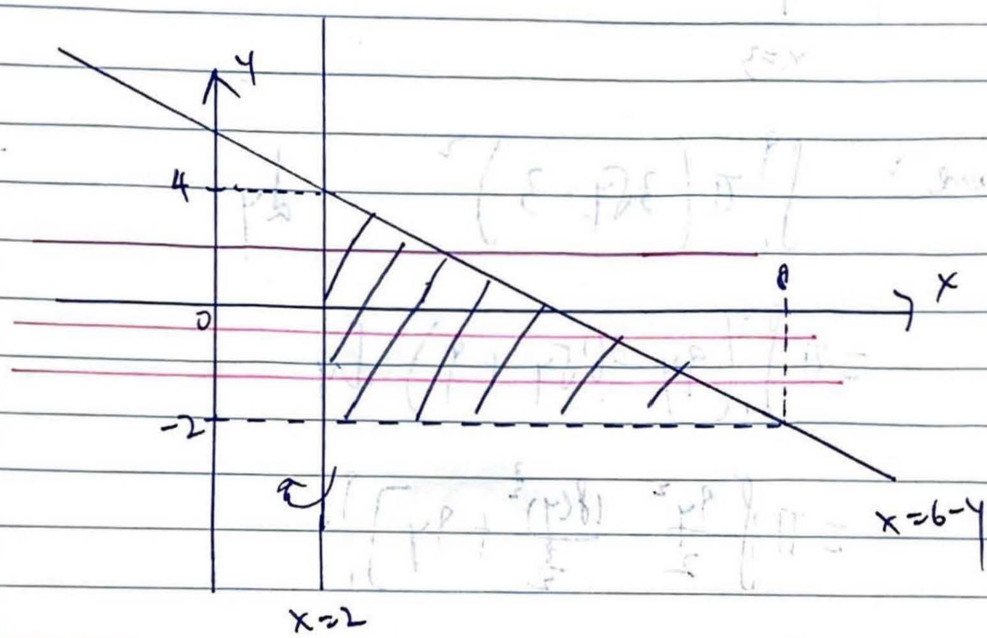
$= \frac{\pi}{2} \int_0^{\frac{\pi}{2}} 1 \, dx + \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \cos 2x \, \frac{d(2x)}{2}$

$= \frac{\pi}{2} \left[x \right]_0^{\frac{\pi}{2}} + \frac{\pi}{4} \left[\sin 2x \right]_0^{\frac{\pi}{2}}$

$$= \frac{\pi}{2}\left(\frac{\pi}{2}\right) + \frac{\pi}{4}(0)$$

$$= \frac{\pi^2}{4}$$ $_{//}$

8.



$$\text{Volume} = \int_{-2}^{4} \pi (6-y-2)^2 \, dy$$

$$= \pi \int_{-2}^{4} (-y+4)^2 \, dy$$

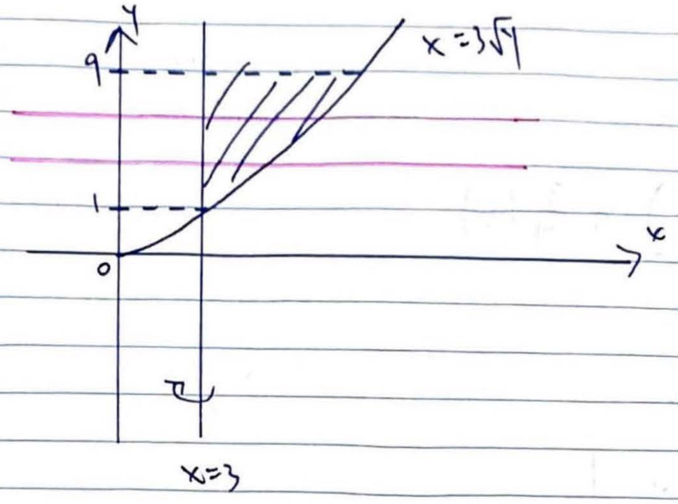
$$= -\pi \int_{-2}^{4} (-y+4)^2 \, d(-y+4)$$

$$= -\pi \left[\frac{(-y+4)^3}{3} \right]_{-2}^{4}$$

$$= -\frac{\pi}{3}[0] + \frac{\pi}{3}\left[(-(-2)+4)^3\right]$$

$$= 72\pi$$ $_{//}$

10.



Volume : $\int_1^9 \pi\left(3\sqrt{y}-3\right)^2 \, dy$

$= \pi\int_1^9 (9y - 18\sqrt{y} + 9)\,dy$

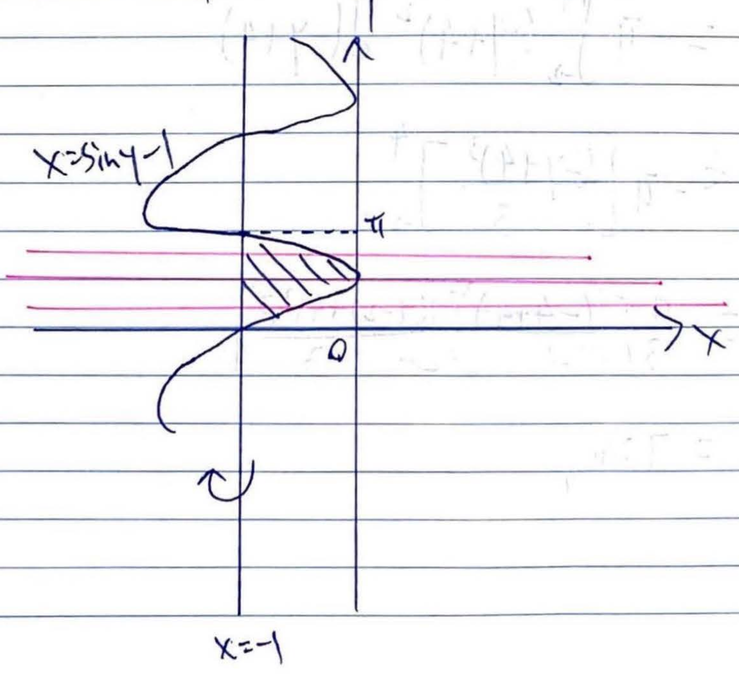
$= \pi\left[\frac{9y^2}{2} - \frac{18(y)^{\frac{3}{2}}}{\frac{3}{2}} + 9y\right]_1^9$

$= \pi\left[\frac{9(9)^2}{2} - 12(9)^{\frac{3}{2}} + 9(9)\right] - \pi\left[\frac{9(1)^2}{2} - 12(1)^{\frac{3}{2}} + 9(1)\right]$

$= \pi(121.5) - \pi(1.5)$

$= 120\pi$,,

12.



Volume : $\int_0^{\pi} \pi\left(\sin y - 1 - (-1)\right)^2 dy$

$= \frac{\pi}{2}\int_0^{\pi} \left(\frac{1}{2}(1-\cos 2y)\right) dy$

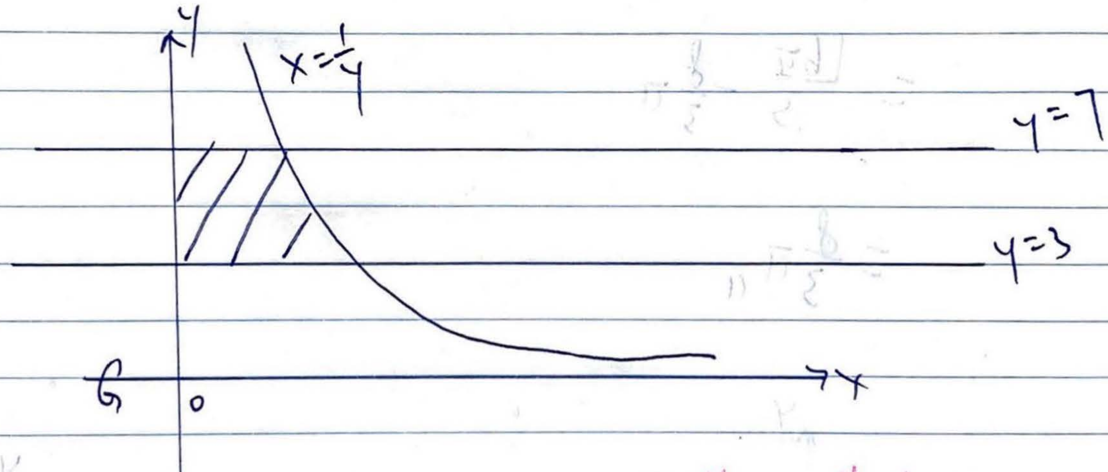
$= \frac{\pi}{2}\int_0^{\pi} (1-\cos 2y)\, dy$

$= \frac{\pi}{2}(\pi) - \pi\int_0^{\pi} \cos 2y \,\frac{d(2y)}{2}$

$= \frac{\pi^2}{2} - \frac{\pi}{2}\left[\sin 2y\right]_0^{\pi}$

$= \frac{\pi^2}{2}$,,

14.



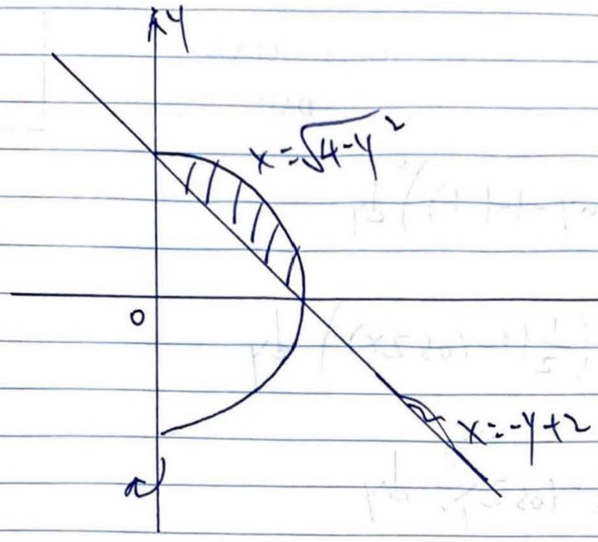
shell method

Volume : $\int_3^7 2\pi (y)\left(\frac{1}{y}\right) dy$

$= 2\pi\int_3^7 1\,dy$

$= 2\pi\left[(7)-(3)\right]$ $\quad = 8\pi$,,

16.



Volume : $\left(\int_0^2 \pi\left(\sqrt{4-y^2}\right)^2 dy\right) - \left(\frac{1}{3}\pi(2)^2(2)\right)$

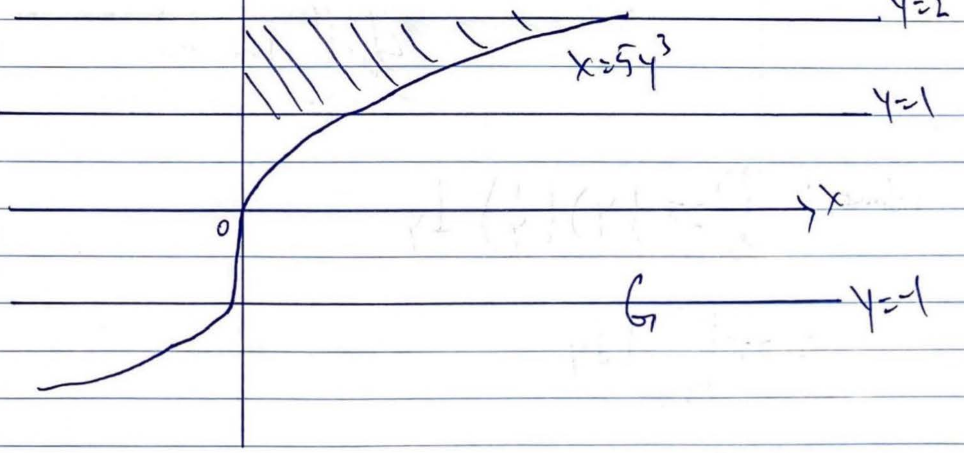
$= \pi\int_0^2 (4-y^2)\,dy - \left(\frac{8}{3}\pi\right)$

$= \pi\left[-\frac{y^3}{3}+4y\right]_0^2 - \left(\frac{8}{3}\pi\right)$

$= \frac{16\pi}{3} - \frac{8}{3}\pi$

$= \frac{8}{3}\pi$ //

18.



Volume : $\int_1^2 2\pi(y+1)(5y^3)\ dy$

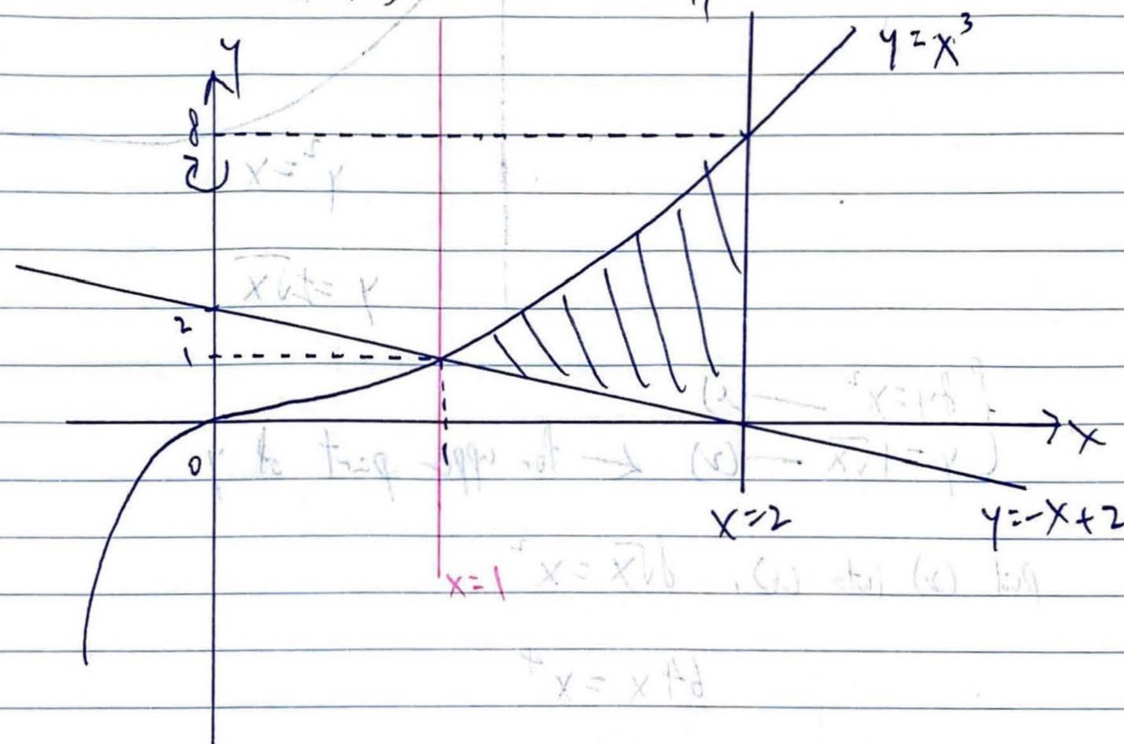
$= 2\pi \int_1^2 (5y^4+5y^3)\, dy$

$= 2\pi \left[y^5 + \frac{5y^4}{4} \right]_1^2$

$= 2\pi \left[(52) - \left(\frac{9}{4}\right) \right]$

$= 2\pi \left(\frac{199}{4} \right) = \frac{199}{2}\pi$ //

20.



Volume : $\int_1^2 2\pi\, x(x^3+x-2)\ dx$

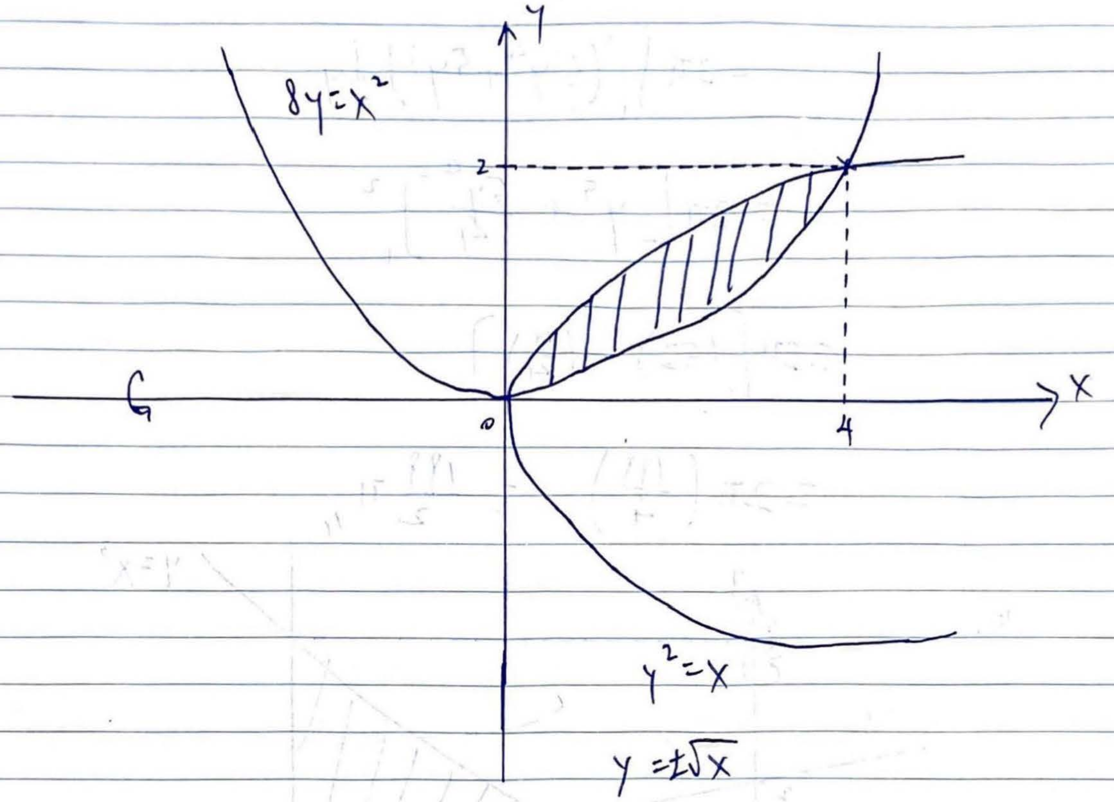
$= 2\pi \int_1^2 (x^4+x^2-2x)\, dx$

$= 2\pi \left[\frac{x^5}{5} + \frac{x^3}{3} - x^2 \right]_1^2$

$= 2\pi\left[\left(\frac{76}{15}\right) - \left(-\frac{7}{15}\right)\right]$

$= 2\pi\left[\frac{83}{15}\right] = \frac{166}{15}\pi$

22.



$\begin{cases} 8y = x^2 & \text{—(1)} \\ y = +\sqrt{x} & \text{—(2)} \end{cases}$ ← for upper part of y

put (2) into (1), $8\sqrt{x} = x^2$

$64x = x^4$

$0 = x^4 - 64x$

$0 = x(x^3 - 64)$

$\therefore\ x = 0$ or $(x^3 - 64) = 0$

$x^3 = 64$

$x = 4$, $y = +\sqrt{4} = 2$

$$\text{Volume: } \left(\int_0^4 \pi (r)^2 h \, dx\right) - \frac{488\pi}{5}$$

$$= \left(\int_0^4 \pi \left(\frac{x^2}{8}\right)^2 (4) \, dx\right) - \frac{488\pi}{5}$$

$$= \frac{\pi}{2} \int_0^4 x^4 \, dx - \frac{488\pi}{5}$$

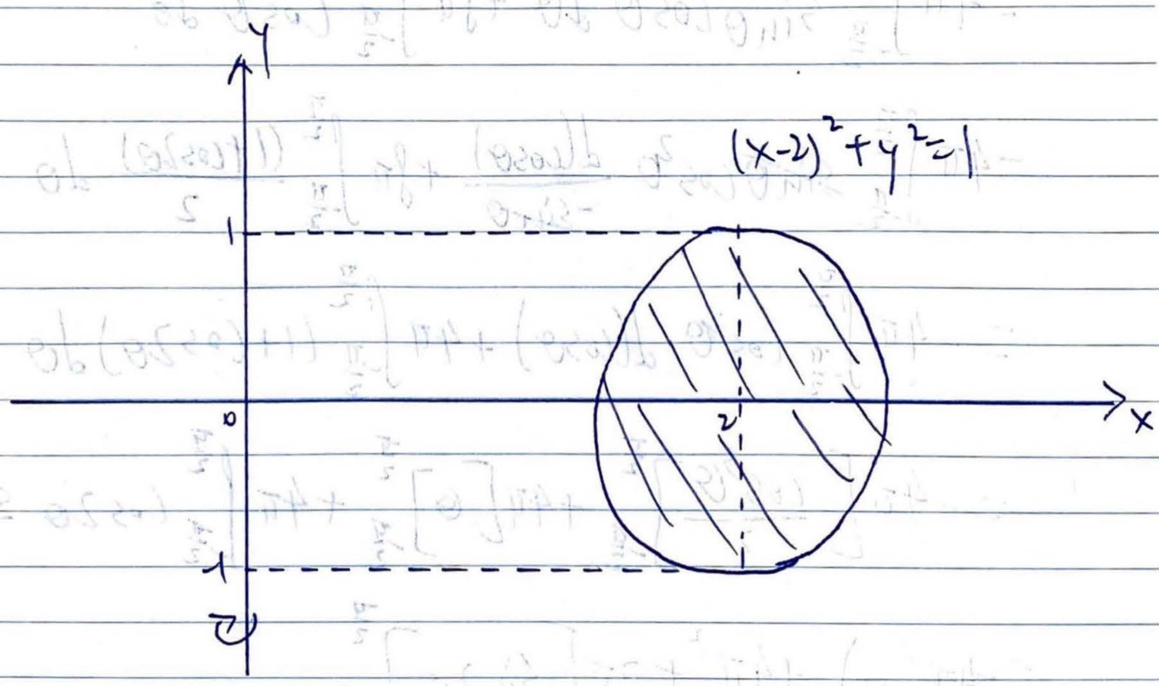
$$= \frac{\pi}{2} \left[\frac{x^5}{5}\right]_0^4 - \frac{488\pi}{5}$$

$$= \frac{\pi}{2}\left(\frac{1024}{5}\right) - \frac{488\pi}{5}$$

$$= \frac{512\pi}{5} - \frac{488\pi}{5}$$

$$= \frac{24\pi}{5}$$

23.



$(x-2)^2 + 0^2 = 1$

$x - 2 = 1$ or $x - 2 = -1$

$x = 3$ $\quad$ $x = 1$

$y^2 = 1 - (x-2)^2$

$$y = \pm\sqrt{1-(x-2)^2}$$

Volume : $2\int_1^3 2\pi\, x\sqrt{1-(x-2)^2}\, dx$

$$= 4\pi\int_1^3 x\sqrt{1-(x-2)^2}\, dx$$

$$= 4\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin\theta+2)\sqrt{1-\sin^2\theta}\,\cos\theta\, d\theta$$ let $(x-2) = \sin\theta$

$dx = \cos\theta\, d\theta$

$$= 4\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin\theta+2)\cos^2\theta\, d\theta$$

when $x=3$, $\theta = \frac{\pi}{2}$

$$= 4\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin\theta\cos^2\theta + 2\cos^2\theta)\, d\theta$$ when $x=1$, $\theta = -\frac{\pi}{2}$

$$= 4\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin\theta\cos^2\theta\, d\theta + 8\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta$$

$$= 4\pi\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \cancel{\sin\theta}\cos^2\theta\, \frac{d(\cos\theta)}{-\cancel{\sin\theta}} + 8\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{(1+\cos 2\theta)}{2}\, d\theta$$

$$= -4\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta\, d(\cos\theta) + 4\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (1+\cos 2\theta)\, d\theta$$

$$= -4\pi\left[\frac{\cos^3\theta}{3}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + 4\pi\left[\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + 4\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos 2\theta\, \frac{d(2\theta)}{2}$$

$$= -4\pi(0) + 4\pi^2 + 2\pi\left[\sin 2\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= 4\pi^2{}_{4}$$

23 另一做法！(consider the function)

let 步驟之前做過，
可以參考之前

$$\text{volume} = 2\int_1^3 2\pi x\sqrt{1-(x-2)^2}\,dx$$

$$= 4\pi\int_1^3 x\sqrt{1-(x-2)^2}\,dx$$

$$= 4\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin\theta+2)\sqrt{1-\sin^2\theta}\cos\theta\,d\theta$$

$$= 4\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin\theta+2)\cos^2\theta\,d\theta$$

$$= 4\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin\theta\cos^2\theta+2\cos^2\theta)\,d\theta$$

consider the function is odd or even

$$= 4\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cancel{\sin\theta}\cos^2\theta\,\frac{d(\cos\theta)}{-\cancel{\sin\theta}} + 8\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^2\theta\,d\theta$$

$$= -4\pi\left[\frac{\cos^3\theta}{3}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + 8\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^2\theta\,d\theta$$

$$= -4\pi(0) + 8\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^2\theta\,d\theta$$

consider $f(\theta) = \cos^2\theta$

$f(-\theta) = \cos^2(-\theta)$
$= \cos^2\theta$

$\therefore f(-\theta) = f(\theta)$

$\therefore f(\theta) = \cos^2\theta$ is an even function.

if $f(x)$ is even function.

$$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$$

$$= -4\pi(0) + 16\pi \int_0^{\frac{\pi}{2}} \cos^2\theta \, d\theta$$

$$= 16\pi \int_0^{\frac{\pi}{2}} \frac{1+\cos 2\theta}{2} \, d\theta$$

$$= 8\pi \int_0^{\frac{\pi}{2}} (1+\cos 2\theta) \, d\theta$$

$$= 8\pi\left(\frac{\pi}{2}\right) + 8\pi \int_0^{\frac{\pi}{2}} \cos 2\theta \, \frac{d(2\theta)}{2}$$

$$= 4\pi^2 + 4\pi \int_0^{\frac{\pi}{2}} \cos 2\theta \, d(2\theta)$$

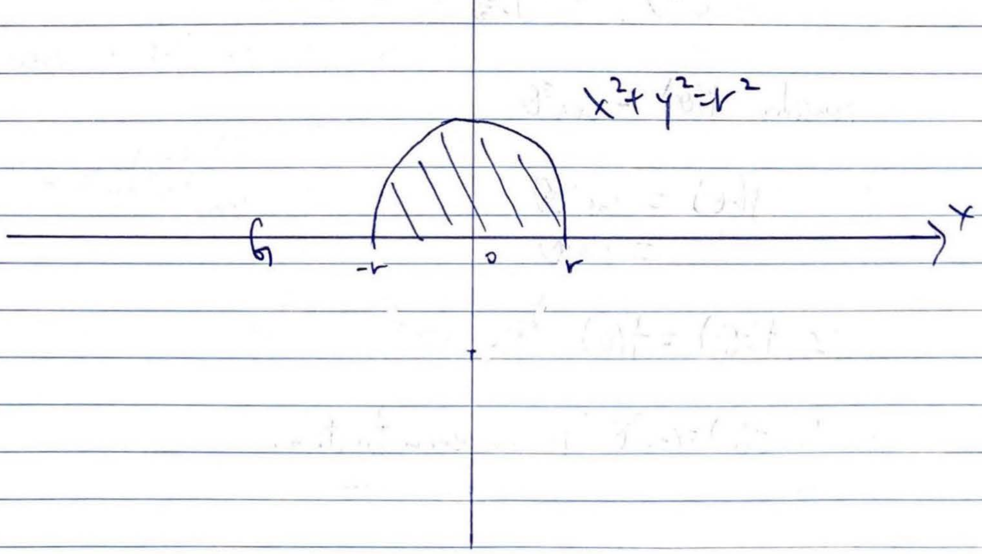
$$= 4\pi^2 + 4\pi \left[\sin 2\theta\right]_0^{\frac{\pi}{2}}$$

$$= 4\pi^2 + 4\pi\left[(\sin\pi) - \sin(-\pi)\right]$$

$$= 4\pi^2 + 4\pi(0)$$

$$= 4\pi^2 \; //$$

24. consider the graph $x^2+y^2=r^2$



Volume $: \int_a^b \pi y^2 \, dx$

$$= \int_{-r}^{r} \pi \left(r^2 - x^2\right) dx$$

$$= \int_{-r}^{r} \left(\pi r^2 - \pi x^2\right) dx$$

$$= \pi r^2 \int_{-r}^{r} 1 \, dx - \pi \int_{-r}^{r} x^2 \, dx$$

$$= \pi r^2 \left[x \right]_{-r}^{r} - \pi \left[\frac{x^3}{3} \right]_{-r}^{r}$$

$$= 2\pi r^3 - \frac{2}{3}\pi r^3$$

$$= \frac{4}{3}\pi r^3 \;//$$

—End—