On the Mechanics of New-Keynesian Models: Smoothing the Capital Controversy Out

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Abstract

Rupert and Šustek (2019) showed that, once we introduce endogenous capital to the canonical New-Keynesian model (i.e. Woodford (2003*a*); Galí (2015)), real interest rates may move in any direction after a positive monetary shock. According to them, this would prove that the real interest rate transmission channel of monetary policy is only observational, not structural, in that class of models. In this paper, we expose that such an identification problem for VARs can be circumvented by the inclusion of interest-rate smoothing in the Taylor rule – a feature as prevalent in middle-scale New-Keynesian models as capital itself. Moreover, we find that for realistic calibrations of the capital-to-output ratio, the canonical model augmented with endogenous capital may have trouble in fulfilling its local determinacy condition if interest-rate smoothing is low, whereas significant capital adjustment cost is still required for preventing output from overreacting after a monetary shock – a restriction we also point out under Rupert and Šustek (2019)'s original calibration.

1 Introduction

In a recent paper, Rupert and Šustek (2019) challenged the existence of a real interest rate channel of monetary policy transmission in textbook New-Keynesian models (Woodford

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(2003*a*); Galí (2015)). They showed that, after we introduce endogenous capital to such models, a monetary shock becomes consistent with the real interest rate moving in any direction.[1] According to the authors, this would prove that the aforementioned channel is only observational, not structural, in that class of models, raising serious concerns on the use of the latter for policy recommendations.[2] Moreover, that channel is a usual assumption employed in the identification of VARs with sign restrictions. In this paper, we show that this identification problem can be largely circumvented in the relevant parameter range by adding interest-rate smoothing to the Taylor rule – a feature as prevalent as capital in middle-scale New-Keynesian DSGE models (i.e. Smets and Wouters (2003)) – especially when the calibration adopts a realistic capital-to-output ratio. In this last case, the canonical model augmented with endogenous capital may have trouble in fulfilling its local determinacy condition if interest-rate smoothing is low, whereas a significant level of capital adjustment cost is required for preventing output from overreacting after a monetary shock – a restriction we also point out in the reference paper's model.

The importance of Rupert and Šustek (2019)'s result is straightforward. Although in general equilibrium all variables are determined simultaneously, the common view on the transmission of monetary policy in textbook New-Keynesian models relies on the real interest rate channel, which Ireland (2010) describes as follows:

> "A monetary tightening in the form of a shock to the Taylor rule that increases the short-term nominal interest rate translates into an increase in the real interest rate as well when nominal prices move sluggishly due to costly or staggered price setting. This rise in the real interest rate then causes households to cut back on their spending, as summarized by the IS curve. Finally, through the Phillips curve, the decline in output puts downward pressure on inflation, which adjusts only gradually after the shock."

Galí (2015, p. 5) also emphasizes the real interest rate channel when describing the short-

[1] Woodford (2003*a*, sec. 5.3.3) calls the lack of any effect of variations in private spending upon the economy's productive capacity one of the more obvious omissions in the baseline New-Keynesian model. He observes that although there are calibrations for which introducing endogenous capital results in similar dynamics for output and inflation after a monetary shock, the mechanisms within each model which generate these results are not too closely parallel.

[2] By their view, the apparent consistency of the canonical model with the real interest rate channel would be due to its equivalence to a set-up with infinite capital adjustment costs.

run non-neutrality of monetary policy in this class of models:

> "As a consequence of the presence of nominal rigidities, changes in short term nominal interest rates (whether chosen directly by the central bank or induced by changes in the money supply) are not matched by one-for-one changes in expected inflation, thus leading to variations in real interest rates. The latter bring about changes in consumption and investment and, as a result, on output and employment, because firms find it optimal to adjust the quantity of goods supplied to the new level of demand. In the long run, however, all prices and wages adjust, and the economy reverts back to its natural equilibrium."

Schematically, we should have that a positive monetary shock (ε_t^m) increases the real interest rate (r_t), *because of sticky prices*, leading to the reduction of consumption (C_t), output (Y_t), and, finally, inflation (π_t).

$$\uparrow \varepsilon_t^m \quad \Rightarrow \quad \underbrace{\uparrow r_t}_{\text{if prices are sticky}} \quad \Rightarrow \quad \downarrow C_t \quad \Rightarrow \quad \downarrow Y_t \quad \Rightarrow \quad \downarrow \pi_t$$

However, Rupert and Šustek (2019) propose a different story, that they argue would be more consistent with the actual mechanics of the model. The transmission does not operate through a real interest rate channel. First, equilibrium inflation is approximately determined as in a flexible-price model.[3] Second, output is pinned down by the New-Keynesian Phillips curve, interpreted here as, given the expected inflation trajectory, firms which cannot adjust prices will change output. Finally, the real rate only reflects the feasibility to keep consumption smooth when income changes. Once the authors introduce endogenous capital in the model, consistency with the real interest rate channel becomes *observational, not structural*, since a contractionary monetary shock reduces inflation and output, but the real interest rate can move in any direction depending on the parameterization of the shock persistence. After adding capital adjustment costs, the authors conclude that the canonical

[3]In Galí (2015)'s chapter 2, a canonical RBC model is augmented with a fixed-intercept interest rate rule to pin down inflation and, thus, a trajectory for the price level. Current inflation, as a deviation of its steady-state value, is determined by the expected path of real interest rate deviations from the steady state, as long as the Taylor principle is obeyed. Important to note that the steady-state value of the real interest rate and the intercept of the monetary policy rule coincide, assuming a zero inflation target. Woodford (2003*a*)'s chapter 1 shows the same idea in a partial-equilibrium monetary model where the sequence of real interest rates is exogenous.

model, without capital, is simply a limiting case where these costs are infinite. According to this view, we are supposed to have monetary transmission as follows:

$$\uparrow \varepsilon_t^m \quad \Rightarrow \quad \downarrow \pi_t \quad \Rightarrow \quad \underbrace{\downarrow Y_t}_{\text{if prices are sticky}} \quad \Rightarrow \quad \downarrow C_t \quad \Rightarrow \quad \underbrace{?r_t}_{\substack{\text{depends on} \\ \text{the presence of capital}}}$$

We adopt the following modeling strategy to challenge the practical relevance of the aforementioned finding. First, we solve the textbook New-Keynesian model of Galí (2015) with capital and interest-rate-smoothing in the Taylor rule to show that the latter can circumvent the identification problem revealed by Rupert and Šustek (2019), especially with realistic calibrations of the capital-to-output ratio. Our finding that the real interest rate channel of monetary policy is reestablished with a common ingredient of middle-scale New-Keynesian models weakens the concerns for its correct identification in policy-oriented DSGE or VAR models. Then, we better qualify our result by exploring different combinations of interest-rate smoothing, steady-state capital-to-output ratio, and capital adjustment costs. We manage to show that the latter is needed to prevent output from overreacting after a monetary shock.

The next sections of this paper are structured as follows. Section 2 presents the related literature. Section 3 describes, solves, and analyzes the closed-economy New-Keynesian model with endogenous capital. Finally, section 4 concludes.

2 Related literature

This paper builds on Rupert and Šustek (2019), who scrutinize the mechanics of canonical New-Keynesian models (i.e. Woodford (2003*a*); Galí (2015)). They argue that the transmission mechanism of monetary policy in this class of models does not operate through the real interest rate channel, against the conventional view. Actually, equilibrium inflation would be determined similarly to when prices are flexible; output is, then, pinned down by the Phillips curve; and the real rate is just the one that smooths out consumption given the income change. As a consequence, a contractionary monetary shock (a positive shock to the intercept of the policy rule), while it reduces inflation and output, it can coexist with the real rate moving in any direction, depending fundamentally on the persistence of the shock. They make explicit this dynamics by comparing the 3-equation canonical model with an extension with endogenous capital. The observational similitude of the first model with the real interest rate channel would come from a supposed implicit assumption of infinite capital adjustment costs. Their finding could represent an additional challenge on the identification of VAR models with sign restrictions.

Brault and Khan (2019) modify Rupert and Šustek (2019)'s work to include frictions on changes in the flow of investments instead of on capital adjustment and obtain that the real interest rate always moves in the same direction as the monetary policy shock. That result does not depend on the size of the adjustment cost nor on the persistence of the monetary shock. The authors argue, then, that at least in contemporary (middle-scale) New-Keynesian models, the real interest rate channel is present, a point similar to ours but made with a different ingredient.

The suspicion on the real interest rate channel of New-Keynesian models is not actually new. The seminal work of Kimball (1995) on the derivation of a real business cycle model with sticky prices – he called it Neo-Monetarist – already dedicated a whole section to discuss the unlikelihood of that channel. He concludes that, even when investment adjustment costs are introduced, parameter values perceived by him as "plausible" would imply that the real interest rate increases in response to a monetary expansion.[4] The "implausible" scenario would show up if either adjustment costs were "too high" or convergence back to the long-run equilibrium after a monetary shock were "too fast" – not unlike what Rupert and Šustek (2019) find. Nonetheless, here lies two distinctions from the Neo-Monetarist model and most New-Keynesian models that followed. First, while Kimball (1995) insisted on portraying monetary policy through a quantity equation with exogenous shocks to the money supply and no endogenous response of it, the New-Keynesian literature has followed the real-world trend of adopting nominal interest rate rules with an endogenous response to inflation. Especially when augmented with smoothing – as we propose in this paper – these last rules put in sharp relief the speed of the convergence back to a long-run equilibrium. Second, the parameterization deemed by him as "plausible" - an investment adjustment cost elasticity of 0.2 and a (labor-constant) elasticity of intertemporal substitution (EIS) for consumption of 0.2 - does not match modern estimations of these models, which find higher values for these parameters.[5]

[4]The model is linearized and, therefore, we assume a symmetrical response in case of a monetary contraction.

[5]With Bayesian methods, Smets and Wouters (2003) estimate the EIS to be 0.74 for the Euro Area, and Smets and Wouters (2007) estimate 0.68-0.72 for the U.S.. All values are posterior modes.

Our modification of the canonical model is empirically motivated. The presence of significant interest-rate smoothing in the response function of the FED is found by Coibion and Gorodnichenko (2012), whose results favor that source of policy inertia over serially correlated monetary shocks. Smets and Wouters (2007) estimate a middle-scale New-Keynesian DSGE model with Bayesian methods for the United States and find large interest-rate smoothing (above 0.7) as well as small monetary shock persistence (below 0.3) coefficients. Sack and Wieland (2000) and Woodford (2003*b*), by their turn, show that smoothing policy interest rates may be optimal from a welfare perspective. Despite the fact that such smoothing is a policy choice, we believe that the optimality of high levels of it and the empirical infrequency of low levels of it warrant the case of this paper.

3 New-Keynesian model before and after capital

In this section, we propose and solve a New-Keynesian toy model. First, the canonical version and, then, the model augmented with endogenous capital.

3.1 Canonical closed economy

Let us take the example of a closed economy without fiscal policy in which there is a one-period risk-free nominal bond available in zero net supply, and that its central bank adopts a fixed intercept Taylor rule. We expand here on the simplified presentation made by Rupert and Šustek (2019) of the canonical New-Keynesian model of Galí (2015), with minor changes in notation.

The simple model starts with 7 variables: real consumption, c_t; labor l_t; real output, y_t; real wage, w_t; real marginal cost, mc_t; nominal interest rate, i_t; and inflation, π_t. Overlined variables represent their steady-state values. There are 6 parameters: the subjective discount factor, β; the inverse of the elasticity of labor supply, η; the elasticity of substitution between intermediate goods, ε; the fraction of producers not adjusting prices at any given period, θ; the intercept of the Taylor rule, i; and the Taylor-rule coefficient that gauges the central bank's reaction to current inflation, ν. There is also an exogenous monetary shock variable, ξ_t^m.

Assuming a per-period utility function of the form (1) and an intermediate goods aggregator like (2), the equilibrium conditions of that economy are given by the Euler equation

(3) in conjunction with equations (4) to (9), namely the CPO of labor, the production function, the real marginal cost, the New-Keynesian Phillips Curve under Calvo pricing already linearized around a zero steady-state inflation, a Taylor rule, and the market-clearing condition.

$$u_t = \log(c_t) - \frac{l_t^{1+\eta}}{1+\eta} \tag{1}$$

$$y_t = \left[\int y(j)^{\varepsilon}\, \mathrm{d}j\right]^{\frac{1}{\varepsilon}} \tag{2}$$

$$\left.\begin{aligned}
\frac{1}{c_t} &= \beta \mathbb{E}_t\left(\frac{1}{c_{t+1}} \frac{1+i_t}{1+\pi_{t+1}}\right) && (3)\\
\frac{w_t}{c_t} &= l_t^{\eta} && (4)\\
y_t &= l_t && (5)\\
\mathrm{mc}_t &= w_t && (6)\\
\pi_t &= \frac{(1-\theta)(1-\theta\beta)}{\theta}\left(\mathrm{mc}_t - \overline{\mathrm{mc}}\right) + \beta \mathbb{E}_t \pi_{t+1} && (7)\\
i_t &= i + \nu\pi_t + \xi_t^m && (8)\\
y_t &= c_t && (9)
\end{aligned}\right\}$$

As usual, the previous equilibrium conditions can be simplified to a three-equation system linearized around a steady state ($\overline{\pi} = 0$, $\overline{y} = 1$). This is possible by first linearizing (3) and substituting the market-clearing condition (9) into it. Then, eliminating (9), (5), and (6) through the substitution of their respective expressions for c_t, l_t, and w_t into (4), which is later linearized such that $\hat{y}_t \equiv \frac{y_t - \overline{y}}{\overline{y}}$. Finally, the Taylor Rule (8) is rewritten as deviations of the interest rate from its steady-state value such that $\hat{i}_t = i_t - i$.

$$\left.\begin{aligned}
-\hat{y} &= -\mathbb{E}_t \hat{y}_{t+1} + \hat{i}_t - \mathbb{E}_t \pi_{t+1} && (10)\\
\pi_t &= \Omega \hat{y}_t + \beta \mathbb{E}_t \pi_{t+1} && (11)\\
\hat{i}_t &= \nu\pi_t + \xi_t^m && (12)
\end{aligned}\right\}$$

where

$$\Omega \equiv \frac{(1+\eta)(1-\theta)(1-\theta\beta)}{\theta} > 0 \tag{13}$$

Notice that when prices are fully flexible, $\theta \to 0$, then $\Omega \to \infty$, whereas when prices are fixed, $\theta \to 1$, then $\Omega \to 0$. As Rupert and Šustek (2019) comment, it is useful to think of Ω as a weight that gauges the solution coefficients of the system between the fully flexible and the fixed price regime.

We can proceed further by substituting the policy rule (12) into (10) so that we reduce the system to only two equations:

$$\left.\begin{aligned} -\hat{y} &= -\mathbb{E}_t\,\hat{y}_{t+1} + \nu\pi_t + \xi_t^m - \mathbb{E}_t\pi_{t+1} \qquad (14)\\ \pi_t &= \Omega\hat{y}_t + \beta\mathbb{E}_t\pi_{t+1} \qquad (15)\end{aligned}\right\}$$

We assume that the monetary shock follows an AR(1) process given by $\xi_t^m = \rho^m\xi_{t-1}^m + \epsilon_t^m$, where $\rho^m \in [0,1)$ and ϵ_t^m is i.i.d. N(0,1). Solving the model with the method of undetermined coefficients, also known as guess-and-verify, by conjecturing $\hat{y}_t = a\xi_t^m$ and $\pi_t = b\xi_t^m$, where a and b are the coefficients we want to obtain, and discarding explosive paths for output and inflation leads to

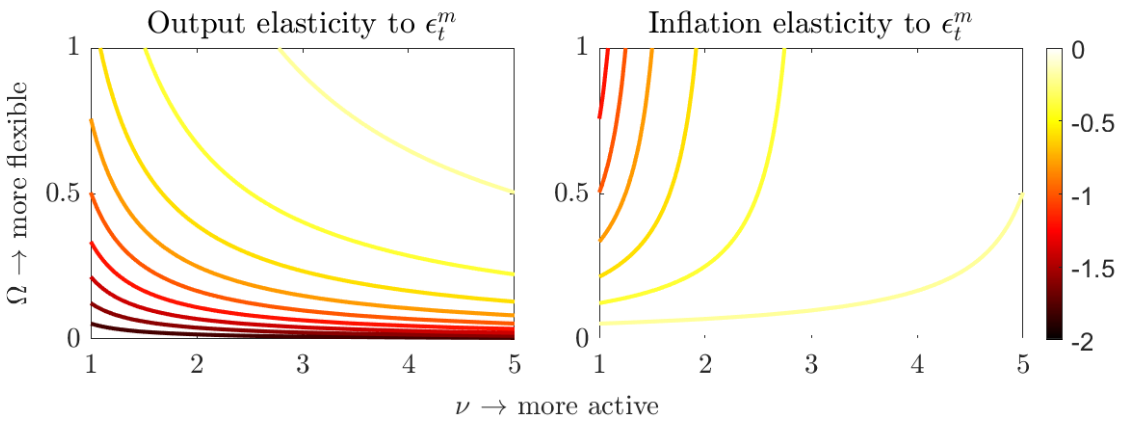
$$a = -\frac{1-\beta\rho^m}{(1-\rho^m)(1-\beta\rho^m)+\Omega(\nu-\rho^m)} < 0 \qquad (16)$$

$$b = -\frac{1}{(1-\rho^m)(1-\beta\rho^m)\Omega^{-1}+(\nu-\rho^m)} < 0 \qquad (17)$$

where both coefficients imply that a positive monetary shock always reduces inflation and output in the canonical New-Keynesian model.

Figure 1 plots coefficients a and b for different values of ν and Ω, under the calibration of Rupert and Šustek (2019)[6]. As expected, flexible prices make output elasticity goes to zero at the same time that inflation elasticity is maximum. Moreover, a more active monetary policy reduces both elasticities.

[6] The following calibration includes parameters that will be incorporated later to the model in this paper: $\beta = 0.99$, $\eta = 1$, $\varepsilon = 0.83$, $\theta = 0.7$, $\nu = 1.5$, $\rho^m = 0.5$, $\alpha = 0.3$, $\delta = 0.025$.



Note: Darker colors imply higher (absolute) elasticity values. Calibration: $\beta = 0.99$, $\nu = 1.5$, and $\rho^m = 0.5$.

Figure 1: Output and inflation elasticity to a monetary shock

The real interest rate as deviation from its steady-state value can be obtained from the Fisher identity, $\hat{R}_t = \hat{i}_t - \mathbb{E}_t \pi_{t+1}$. Substituting our solution, we have

$$\hat{R}_t = \underbrace{\left(1 - \underbrace{\frac{1}{1 + \frac{1-\rho^m}{\nu-\rho^m} \frac{1-\beta\rho^m}{\Omega}}}_{\in[0,1]} \right)}_{\geq 0} \xi_t^m \tag{18}$$

which implies that the real interest rate always increases/decreases right after a positive/negative monetary shock, consistent with the presence of a real interest rate transmission channel of monetary policy.

3.2 Endogenous capital and interest-rate smoothing

Rupert and Šustek (2019) argue that the real interest rate channel of monetary transmission in canonical New-Keynesian models is only observational, not structural, for, after introducing endogenous capital, a positive monetary shock can shift the real interest rate in any direction depending on the persistence of the shock, while the output gap becomes negative and inflation falls regardless of that persistence. This should raise concerns on the correct identification of VARs with sign restrictions, for example. We show, now, that adopting interest-rate-smoothing in the Taylor Rule – a prevalent feature of actual monetary policy practice as well as of middle-scale New-Keynesian models, such as Smets and Wouters

(2003) and Smets and Wouters (2007) – can deliver impulse-response functions with the sign consistent with the real interest rate channel in the empirically-relevant parameter range. This finding largely minimizes the identification problem from an empirical perspective.

We build on their model. Rupert and Šustek (2019) assume that there is an economy-wide rental market of capital, so that at every period capital can be rented by the firms. In that sense, capital is not firm-specific.[7] Moreover, they assume that whenever households change their stock of capital there is a quadratic adjustment cost, $-\frac{\kappa}{2}\left(k_{t+1}-k_t\right)^2$, where k_t is the stock of capital inherited from the previous period, and $\kappa \geq 0$ is a parameter that governs the size of the adjustment cost in terms of foregone real income.

Resuming from the canonical model of section 3.1, there is a new Euler equation for the capital asset (19), where $\delta \in (0,1)$ is a depreciation rate, and q_t is the price of capital in terms of current consumption, Tobin's q, such that $q_t \equiv 1+\kappa\left(k_{t+1}-k_t\right)$. The production function (5) is replaced by (20), which incorporates capital and labor in a proportion consistent with constant returns to scale, where α is the Cobb-Douglas coefficient of capital. Equation (21) is the condition for the optimal mix of capital and labor in production, which comes from the CPO of the firm. The marginal cost (6) is updated to include the rent on capital (22). The resources constraint (9) must account, now, for the investment flow so markets can clear (23). Finally, we substitute the previous Taylor rule (8), also adopted by Rupert and Šustek (2019), for one with interest-rate-smoothing (24), whose persistence is governed by $\rho^i \in [0,1)$.

$$\left.\begin{aligned}
\frac{1}{c_t} &= \beta \mathbb{E}_t\left(\frac{1}{c_{t+1}}\left(\frac{r_{t+1}-\delta}{q_t}+\frac{q_{t+1}}{q_t}\right)\right) && (19)\\
y_t &= k_t^{\alpha} l_t^{1-\alpha} && (20)\\
\frac{w_t}{r_t} &= \frac{1-\alpha}{\alpha}\left(\frac{k_t}{l_t}\right) && (21)\\
\mathrm{mc}_t &= \left(\frac{r_t}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} && (22)\\
y_t &= c_t + k_{t+1} - (1-\delta)\,k_t + \frac{\kappa}{2}\left(k_{t+1}-k_t\right)^2 && (23)\\
i_t &= \rho^i i_{t-1} + \left(1-\rho^i\right)(i+\nu\pi_t) + \xi_t^m && (24)
\end{aligned}\right\}$$

[7] Altig et al. (2011) estimate a New-Keynesian DSGE model for the U.S. and find that this modeling choice for introducing endogenous capital results in firms enduring long spells before readjusting prices on average, up to 9 quarters. They show that firm-specific capital can turn that spell more aligned with empirical evidence from micro data, say, once a year.

After substituting equation (21) into (22) by eliminating r_t, and substituting equation (4) into (20) so as to eliminate l_t, the model is log-linearized around the non-stochastic steady state ($\overline{\pi} = 0$, $\overline{y} = 1$). For any variable X, $\hat{X} \equiv \frac{X_t - \overline{X}}{\overline{X}}$, with exception of $\hat{i}_t \equiv i_t - \overline{i}$ and $\hat{r}_t \equiv r_t - \overline{r}$. After that, it is possible to eliminate $\hat{r}_t$, $\hat{\text{mc}}_t$, $\hat{w}_t$, i_t and $\hat{l}_t$, in order to obtain the following reduced system with only 4 equations.

$$\left.\begin{aligned} -\hat{c} &= -\mathbb{E}_t\,\hat{c}_{t+1} + \rho^i\,\hat{i}_{t-1} + \left(1-\rho^i\right)\nu\pi_t - \mathbb{E}_t\,\pi_{t+1} + \xi_t^m \quad &(25)\\ -\hat{c} &= -\mathbb{E}_t\,\hat{c}_{t+1} + \mathbb{E}_t\,\hat{g}_{t+1} + r\,\mathbb{E}_t\left(\hat{c}_{t+1} + \frac{1+\eta}{1-\alpha}\hat{y}_{t+1} - \frac{1+\alpha\eta}{1-\alpha}\hat{k}_{t+1}\right) \quad &(26)\\ \pi_t &= \Psi\left(\frac{\eta+\alpha}{1-\alpha}\hat{y}_t - \alpha\frac{1+\eta}{1-\alpha}\hat{k}_t + \hat{c}_t\right) + \beta\mathbb{E}_t\,\pi_{t+1} \quad &(27)\\ \hat{y}_t &= \frac{\overline{c}}{\overline{y}}\hat{c}_t + \frac{\overline{k}}{\overline{y}}\hat{k}_{t+1} - (1-\delta)\frac{\overline{k}}{\overline{y}}\hat{k}_t \quad &(28)\end{aligned}\right\}$$

where $\Psi \equiv \chi\frac{(1-\theta)(1-\theta\beta)}{\theta}$, such that when prices are flexible, $\Psi \to \infty$. Moreover, $G_{t+1} \equiv \frac{q_{t+1}}{q_t}$ is the capital gain, so $\hat{g}_t = \hat{q}_t - \hat{q}_{t-1} = \overline{\kappa}\left(\hat{k}_{t+1} - \hat{k}_t\right) - \overline{\kappa}\left(\hat{k}_t - \hat{k}_{t-1}\right)$, where $\overline{\kappa} = \kappa\overline{k}$.

To check whether the negative response of real interest rates to a positive monetary shock remains as an identification problem, we sweep the combinations of parameter values for $\rho^m \in [0 : 0.1 : 0.9]$ and $\rho^i \in [0 : 0.1 : 0.9]$. As we could not find in Rupert and Šustek (2019) the $\overline{k}$ used by the authors, we tested different values until find one that approximately matches their impulse response functions: $\overline{k} = 5.5$, or 1.375 times the annual output.[8] Table 1 displays the sign of the reaction of the real interest rate right after the shock for all combinations under $\overline{K} = 5.5\overline{Y}$, $\delta = 0.025$, and $\kappa = 0.0$. Tables 2 and 3 increase κ to 0.1 and 0.5, respectively. As one can see, under the hypothesis of no adjustment costs, ρ^i must be at least 0.9 to guarantee a positive response under all values of ρ^m. However, a small adjustment cost is already enough to largely increase the parameter range consistent with a real interest rate channel of monetary policy.

[8] In Rupert and Šustek (2016), the authors set $\overline{k} = 7.0938$. Brault and Khan (2019) use $\overline{k} = 8.4568$.

	$\rho^i = 0$	ρ^i =0.1	ρ^i =0.2	ρ^i =0.3	ρ^i =0.4	ρ^i =0.5	ρ^i =0.6	ρ^i =0.7	ρ^i =0.8	ρ^i =0.9
$\rho^m = 0$	+	+	+	+	+	+	+	+	+	+
ρ^m =0.1	-	-	-	-	+	+	+	+	+	+
ρ^m =0.2	-	-	-	-	-	-	-	+	+	+
ρ^m =0.3	-	-	-	-	-	-	-	-	+	+
ρ^m =0.4	-	-	-	-	-	-	-	-	-	+
ρ^m =0.5	-	-	-	-	-	-	-	-	-	+
ρ^m =0.6	-	-	-	-	-	-	-	-	-	+
ρ^m =0.7	-	-	-	-	-	-	-	-	-	+
ρ^m =0.8	-	-	-	-	-	-	-	-	-	+
ρ^m =0.9	-	-	-	-	-	-	-	-	+	+

Note: + indicates that the real interest rate increases right after a positive monetary shock; - indicates that it decreases; empty cells indicate that the Blanchard and Kahn conditions are not satisfied.

Table 1: Parameter sweep with $\overline{K} = 5.5\overline{Y}$, $\delta = 0.025$, and $\kappa = 0.0$

	$\rho^i = 0$	ρ^i =0.1	ρ^i =0.2	ρ^i =0.3	ρ^i =0.4	ρ^i =0.5	ρ^i =0.6	ρ^i =0.7	ρ^i =0.8	ρ^i =0.9
$\rho^m = 0$	+	+	+	+	+	+	+	+	+	+
ρ^m =0.1	+	+	+	+	+	+	+	+	+	+
ρ^m =0.2	+	+	+	+	+	+	+	+	+	+
ρ^m =0.3	+	+	+	+	+	+	+	+	+	+
ρ^m =0.4	+	+	+	+	+	+	+	+	+	+
ρ^m =0.5	+	+	+	+	+	+	+	+	+	+
ρ^m =0.6	-	-	+	+	+	+	+	+	+	+
ρ^m =0.7	-	-	-	+	+	+	+	+	+	+
ρ^m =0.8	-	-	-	+	+	+	+	+	+	+
ρ^m =0.9	-	-	-	+	+	+	+	+	+	+

Note: + indicates that the real interest rate increases right after a positive monetary shock; - indicates that it decreases; empty cells indicate that the Blanchard and Kahn conditions are not satisfied.

Table 2: Parameter sweep with $\overline{K} = 5.5\overline{Y}$, $\delta = 0.025$, and $\kappa = 0.1$

	$\rho^i = 0$	ρ^i =0.1	ρ^i =0.2	ρ^i =0.3	ρ^i =0.4	ρ^i =0.5	ρ^i =0.6	ρ^i =0.7	ρ^i =0.8	ρ^i =0.9
$\rho^m = 0$	+	+	+	+	+	+	+	+	+	+
ρ^m =0.1	+	+	+	+	+	+	+	+	+	+
ρ^m =0.2	+	+	+	+	+	+	+	+	+	+
ρ^m =0.3	+	+	+	+	+	+	+	+	+	+
ρ^m =0.4	+	+	+	+	+	+	+	+	+	+
ρ^m =0.5	+	+	+	+	+	+	+	+	+	+
ρ^m =0.6	+	+	+	+	+	+	+	+	+	+
ρ^m =0.7	+	+	+	+	+	+	+	+	+	+
ρ^m =0.8	+	+	+	+	+	+	+	+	+	+
ρ^m =0.9	-	+	+	+	+	+	+	+	+	+

Note: + indicates that the real interest rate increases right after a positive monetary shock; - indicates that it decreases; empty cells indicate that the Blanchard and Kahn conditions are not satisfied.

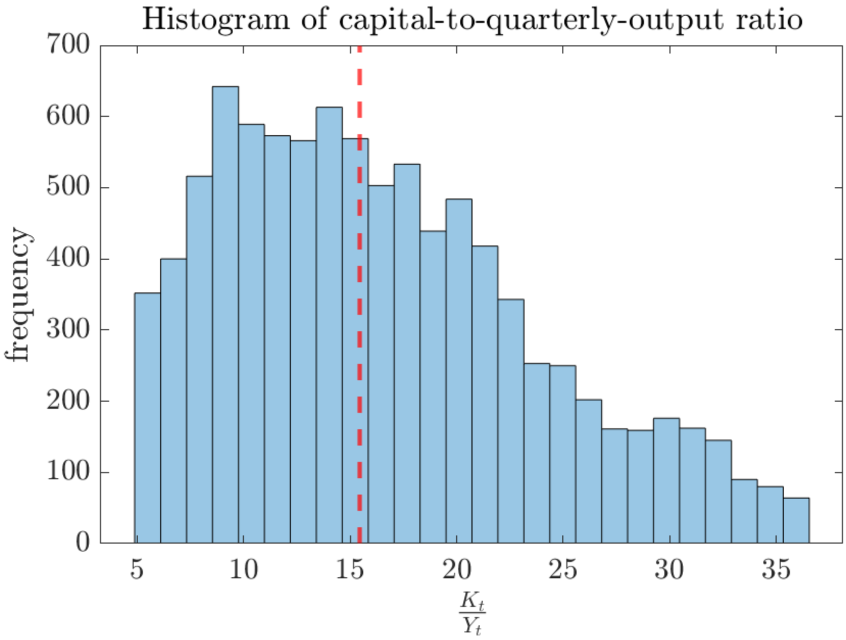
Table 3: Parameter sweep with $\overline{K} = 5.5\overline{Y}$, $\delta = 0.025$, and $\kappa = 0.5$

Thus, how restricting is the switching behavior of the real interest rate to a positive monetary shock for the estimation of VARs and DSGEs? We have seen that in the presence of interest-rate-smoothing, an empirically validated (Coibion and Gorodnichenko (2012)), theoretically desirable (i.e. Woodford (2003*b*), Sack and Wieland (2000)), and prevalent feature of middle-scale DSGE models (i.e. Smets and Wouters (2003) estimates $\rho^i = 0.956$ for the Euro Area; Smets and Wouters (2007) estimates $\rho^i = 0.75 - 0.84$ for the United States), a plausibly small adjustment cost is enough to reestablish the sign consistency with the real interest rate channel. Under no adjustment cost at all, $\rho^i > 0.9$ would be enough.

What we argue next is that the determinacy of the canonical model augmented with endogenous capital depends on the steady-state stock of capital, and $\overline{K} = 5.5\overline{Y}$ is much smaller than what is empirically estimated. In Figure 2, we plot the distribution of the capital-to-output ratio calculated with annual data from the Penn World Table 10.0 (Feenstra, Inklaar and Timmer (2015)) for 183 countries from 1950 to 2019.[9] Annual output is divided by 4 as the time frequency of our model is quarterly. For the sake of clarity, we omit from the graph values below percentile 5 and above percentile 95 of the distribution. The red-dashed line marks the median of the full distribution, at $\overline{K} = 15.4\overline{Y}$, much larger than the calibration of

[9] The annual output series is Real GDP at constant 2017 national prices (in mil. 2017US$); the capital series is Capital stock at constant 2017 national prices (in mil. 2017US$).

Rupert and Šustek (2019). Table 4 displays the capital-to-quarterly-output ratio of some selected countries in the sample for the years 1960 and 2019. We repeat the parameter sweeps in Tables 5 and 6 with that capital level as the steady state, and we show that Blanchard-Kahn conditions for determinacy of the system are not even met under more realistic capital-to-output ratios for most of the parameter combinations. With an adjustment cost as small as $\kappa = 0.1$, the identification problem disappears.



Note: Histogram of capital-to-quarterly-output ratio calculated with annual data from the Penn World Table 10.0 (Feenstra, Inklaar and Timmer (2015)) for 183 countries from 1950 to 2019. The red-dashed line marks the median of the full distribution. Values below percentile 5 and above percentile 95 are censored in the graph.

Figure 2: Histogram of capital-to-quarterly-output ratio

Year	Country	Annual GDP	Capital Stock	$\frac{\overline{K}}{\overline{Y}}$
1960	Brazil	US$ 285,613	US$ 1,095,861	15.3
2019	Brazil	US$ 3,042,119	US$ 13,716,488	18.0
1960	Canada	US$ 310,435	US$ 1,198,070	15.4
2019	Canada	US$ 1,874,187	US$ 8,757,840	18.7
1960	China	US$ 689,787	US$ 1,739,456	10.1
2019	China	US$ 20,572,606	US$ 81,726,344	15.9
1960	Germany	US$ 1,092,642	US$ 4,206,044	15.4
2019	Germany	US$ 4,314,068	US$ 20,957,202	19.4
1960	France	US$ 592,031	US$ 3,638,362	24.6
2019	France	US$ 2,965,339	US$ 18,013,436	24.3
1960	United Kingdom	US$ 782,240	US$ 3,447,930	17.6
2019	United Kingdom	US$ 3,016,695	US$ 15,374,464	20.4
1960	Japan	US$ 609,690	US$ 1,707,238	11.2
2019	Japan	US$ 5,099,254	US$ 26,138,818	20.5
1960	Mexico	US$ 274,372	US$ 1,012,672	14.8
2019	Mexico	US$ 2,406,410	US$ 10,934,025	18.2
1960	United States	US$ 3,510,945	US$ 14,711,919	16.8
2019	United States	US$ 20,563,592	US$ 69,059,072	13.4
1960	South Africa	US$ 134,600	US$ 417,390	12.4
2019	South Africa	US$ 732,852	US$ 2,892,806	15.8

Table 4: Capital-to-quarterly-output ratio of some selected years and countries

	$\rho^i = 0$	ρ^i =0.1	ρ^i =0.2	ρ^i =0.3	ρ^i =0.4	ρ^i =0.5	ρ^i =0.6	ρ^i =0.7	ρ^i =0.8	ρ^i =0.9
$\rho^m = 0$									+	+
ρ^m =0.1									+	+
ρ^m =0.2									+	+
ρ^m =0.3									-	+
ρ^m =0.4									-	+
ρ^m =0.5									-	-
ρ^m =0.6									-	-
ρ^m =0.7									-	-
ρ^m =0.8									-	-
ρ^m =0.9									-	-

Note: + indicates that the real interest rate increases right after a positive monetary shock; - indicates that it decreases; empty cells indicate that the Blanchard and Kahn conditions are not satisfied.

Table 5: Parameter sweep with $\overline{K} = 15.4\overline{Y}$, $\delta = 0.025$, and $\kappa = 0$

	$\rho^i = 0$	ρ^i =0.1	ρ^i =0.2	ρ^i =0.3	ρ^i =0.4	ρ^i =0.5	ρ^i =0.6	ρ^i =0.7	ρ^i =0.8	ρ^i =0.9
$\rho^m = 0$								+	+	+
ρ^m =0.1								+	+	+
ρ^m =0.2								+	+	+
ρ^m =0.3								+	+	+
ρ^m =0.4								+	+	+
ρ^m =0.5								+	+	+
ρ^m =0.6								+	+	+
ρ^m =0.7								+	+	+
ρ^m =0.8								+	+	+
ρ^m =0.9								+	+	+

Note: + indicates that the real interest rate increases right after a positive monetary shock; - indicates that it decreases; empty cells indicate that the Blanchard and Kahn conditions are not satisfied.

Table 6: Parameter sweep with $\overline{K} = 15.4\overline{Y}$, $\delta = 0.025$, and $\kappa = 0.1$

3.3 The mechanics

Now, we make explicit the solution for the real interest rate by the method of undetermined coefficients, and compare it to the exposition of Rupert and Šustek (2019). We assume $\hat{c}_t = a_0\hat{k}_t + a_1\xi_t^m + a_2\hat{\imath}_{t-1}$; $\hat{k}_{t+1} = f_0\hat{k}_t + f_1\xi_t^m + f_2\hat{\imath}_{t-1}$; $\pi_t = b_0\hat{k}_t + b_1\xi_t^m + b_2\hat{\imath}_{t-1}$; and $\hat{y}_t = d_0\hat{k}_t + d_1\xi_t^m + d_2\hat{\imath}_{t-1}$. The set of coefficients to be determined is $\{a_0, a_1, a_2, b_0, b_1, b_2, d_0, d_1, d_2, f_0, f_1, f_2\}$.

With the log-linearized Fisher relation, $\hat{R}_t = \hat{\imath}_t - \mathbb{E}_t\pi_{t+1}$, and the Euler equation (25) we can write:

$$
\begin{aligned}
\hat{R}_t &= \mathbb{E}_t\,\hat{c}_{t+1} - \hat{c}_t \\
&= \underbrace{\left(a_0 f_0 - a_0 + a_2\left(1-\rho^i\right)\nu b_0\right)\hat{k}_t + \left(a_0 f_2 - a_2 + a_2\rho^i + a_2\left(1-\rho^i\right)\nu b_2\right)\hat{\imath}_{t-1}}_{=0 \text{ at the shock}} \\
&\quad + \left(\underbrace{\rho^m a_1 - a_1 + a_2\left(1-\rho^i\right)\nu b_1 + a_2}_{\text{indirect effect of capital}} \qquad \underbrace{+a_0 f_1}_{\text{direct effect of capital}}\right)\xi_t^m
\end{aligned}
$$

where $\hat{R}$ is the log-deviation of the gross real interest rate. Unfortunately, the system does not seem to have a reduction that allows the direct interpretation of the sign of the coefficients. Instead, analysis must be done heuristically and numerically.[10] When we remove interest-rate-smoothing, that is, when $\rho^i = 0$, $a_2 = 0$, $f_2 = 0$, $b_2 = 0$, and $d_2 = 0$, the model is the same as the one portrayed in Rupert and Šustek (2019).

Under the benchmark calibration, with no adjustment costs and no interest-rate smoothing, the direct effect of capital to the real interest rate from a monetary shock is negative for all possible values of ρ^m, while the indirect effect is mostly positive. The indirect effect is larger than the direct one only at the lowest range of ρ^m, what can be seen in Figure 3. In Figure 4, we show that raising κ to 0.1 increases both components of the total effect, amplifying the range consistent with the real interest rate channel. In Figure 5, we introduce interest-rate smoothing, by setting $\rho^i = 0.5$, with no capital adjustment costs. In that case, the total

[10] The system is non-linear in the coefficients, what means that multiple solutions may exist. Therefore, we impose the following restrictions to single out a solution: $a_0 > 0$, $d_0 > 0$, $a_2 \leq 0$, $b_2 \leq 0$, and $d_2 \leq 0$. By that, we expect consumption to increase with the inherited stock of capital, as well as to decrease with the lagged nominal interest rate. Additionally, we expect output and inflation to also decrease with the lagged nominal interest rate. These signs are validated by the policy functions obtained from first-order perturbation in Dynare (Adjemian et al., 2011).

effect curve becomes flatter near the zero axis. Raising the adjustment cost to $\kappa = 0.1$, as in Figure 6, is enough to bring the total effect curve above zero for all possible values of ρ^m.

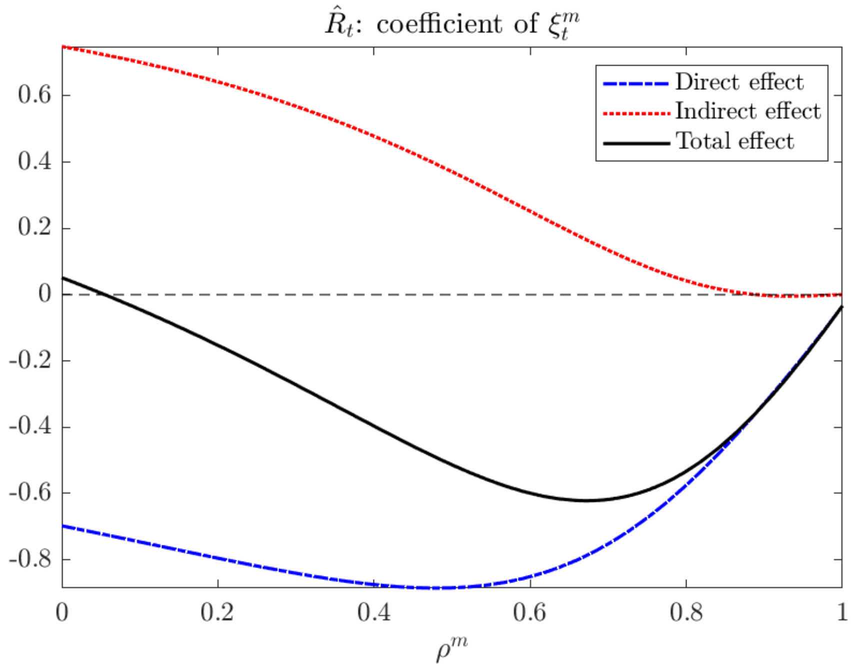


Figure 3: Decomposition of the effect of capital to $\hat{R}_t$ from a monetary shock when $\overline{K} = 5.5$, $\rho^i = 0$ and $\kappa = 0$

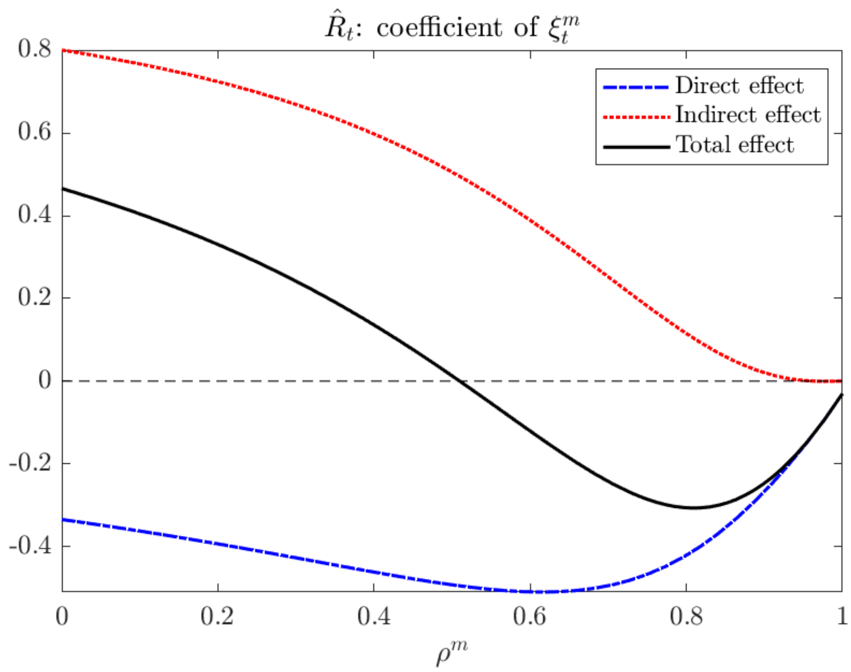


Figure 4: Decomposition of the effect of capital to $\hat{R}_t$ from a monetary shock when $\overline{K} = 5.5$, $\rho^i = 0$ and $\kappa = 0.1$

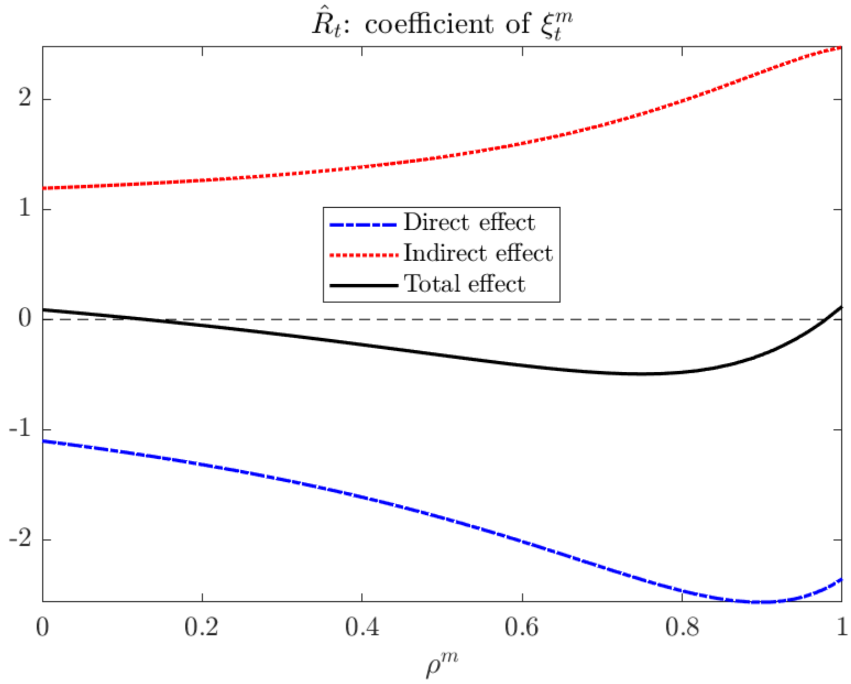


Figure 5: Decomposition of the effect of capital to $\hat{R}_t$ from a monetary shock when $\overline{K} = 5.5$, $\rho^i = 0.5$ and $\kappa = 0$

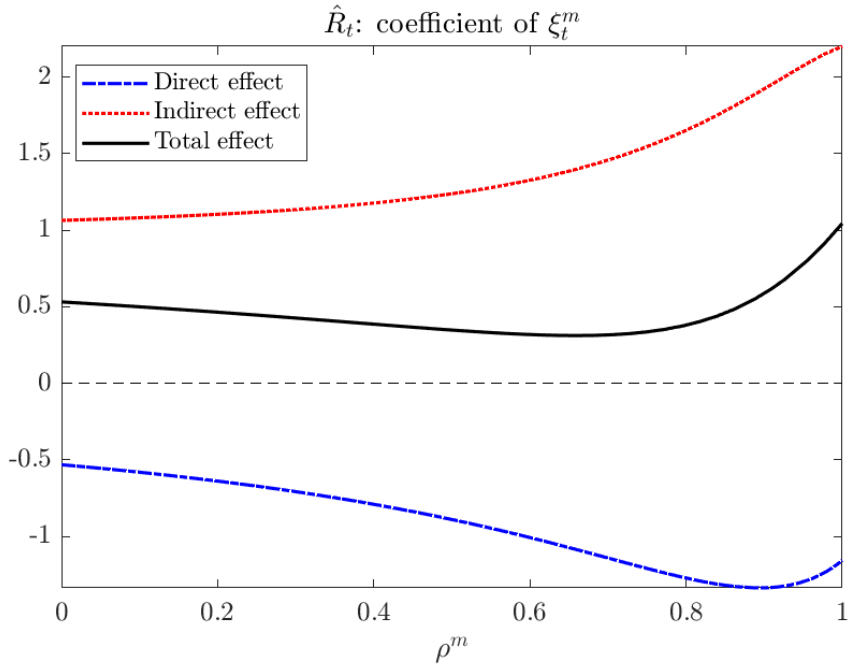


Figure 6: Decomposition of the effect of capital to $\hat{R}_t$ from a monetary shock when $\overline{K} = 5.5$, $\rho^i = 0.5$ and $\kappa = 0.1$

3.4 Impulse response functions

Next, we plot the impulse response functions of the New-Keynesian model augmented with endogenous capital, adjustment costs, and interest rate smoothing. We calibrate the standard-deviation of the monetary shock to 1 p.p.. Graphs display % deviations from steady-state values, except interest rates which are measured in p.p. deviations from steady-state values. As expected, in all figures, output, consumption, and inflation respond negatively at the event of a contractionary shock. The capital stock also reduces, but with a lag, due to our timing convention. The nominal interest rate, by its turn, may react positively or negatively, as the last response sign is a consequence of inflation expectations, and inflation itself, reducing by a large amount due to the persistence of the monetary shock, an already documented pattern (Galí (2015) and Woodford (2003*a*)).

3.4.1 The identification problem

Figures 7 and 8 display the effect of a positive one-standard-deviation monetary shock to the model variables under different specifications for ρ^m without adjustment costs and without smoothing. This is, basically, the identification problem revealed by Rupert and Šustek (2019), as the real interest rate goes up right after the shock in the first figure, but it goes down in the second one.

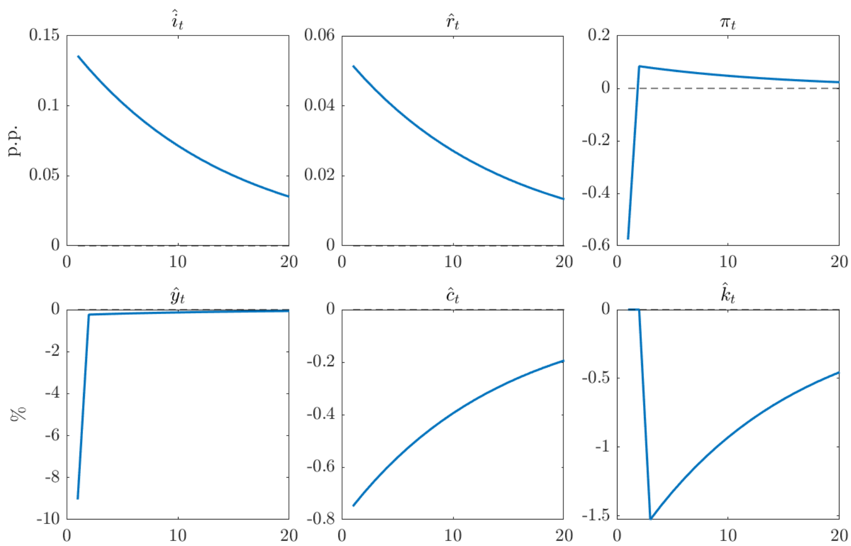


Figure 7: Impulse response function to a 1-standard-deviation monetary shock under $\rho^m = 0$, $\rho^i = 0$, $\frac{\overline{K}}{\overline{Y}} = 5.5$, and $\kappa = 0$

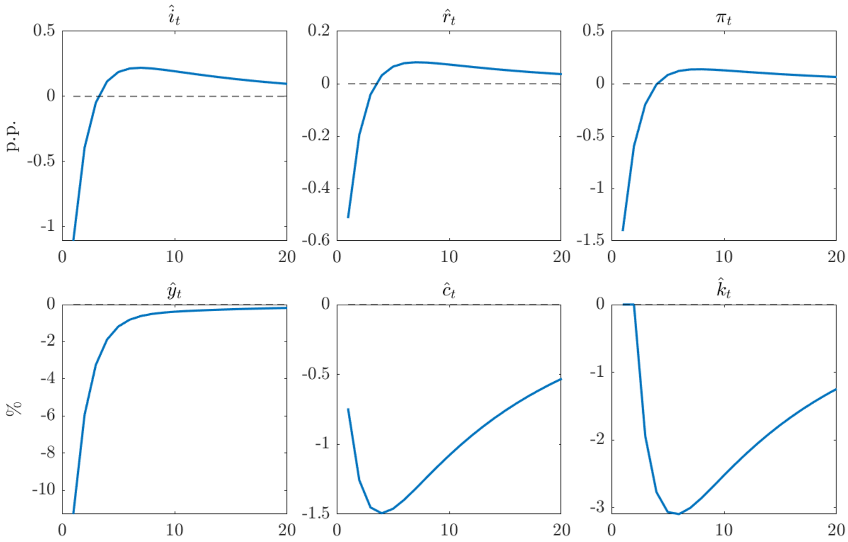


Figure 8: Impulse response function to a 1-standard-deviation monetary shock under $\rho^m = .50$, $\rho^i = 0$, $\frac{\overline{K}}{\overline{Y}} = 5.5$, and $\kappa = 0$

3.4.2 Fixing with high interest-rate smoothing

Figures 9 and 10 show that a high level of interest-rate smoothing ($\rho^i = 0.9$) is able to reestablish the observational consistency with the real interest rate channel of monetary policy transmission. Despite that, in this case without capital adjustment costs, output overreacts becoming unrealistic.

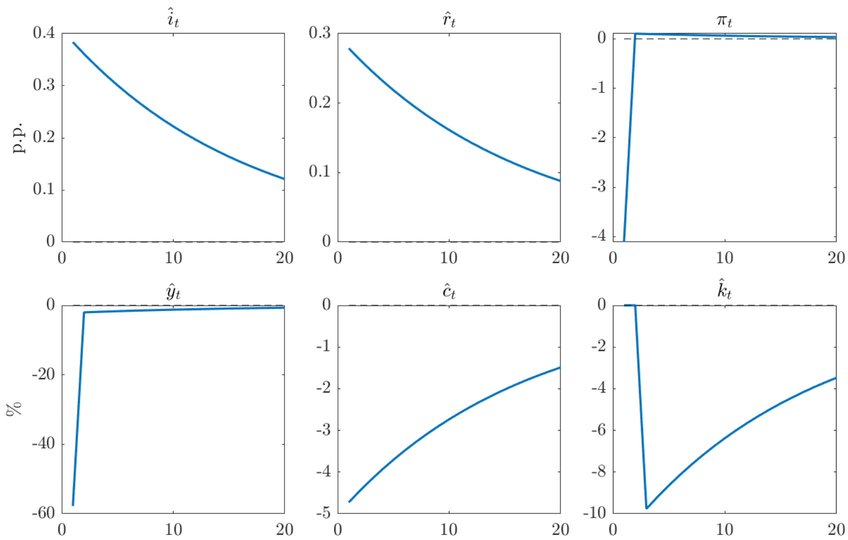


Figure 9: Impulse response function to a 1-standard-deviation monetary shock under $\rho^m = 0$, $\rho^i = 0.9$, $\frac{\overline{K}}{\overline{Y}} = 5.5$, and $\kappa = 0$

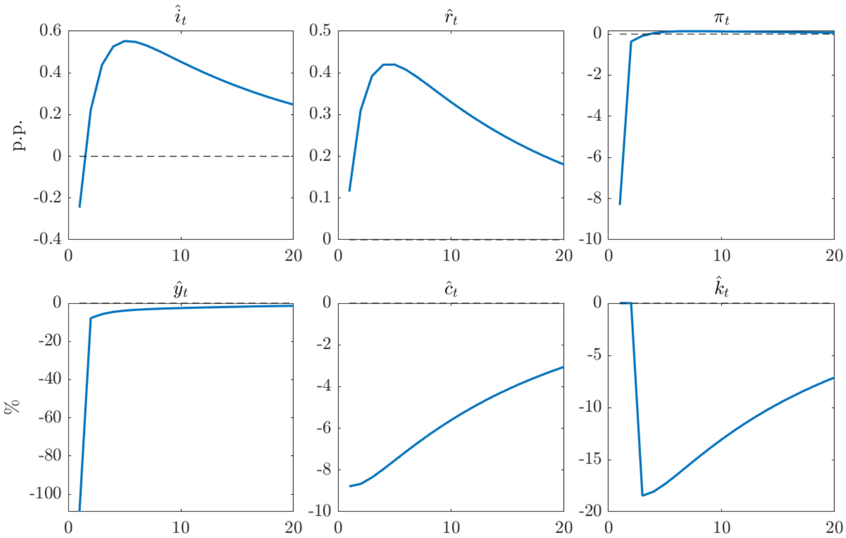


Figure 10: Impulse response function to a 1-standard-deviation monetary shock under $\rho^m = .50$, $\rho^i = 0.9$, $\frac{\overline{K}}{\overline{Y}} = 5.5$, and $\kappa = 0$

3.4.3 Fixing with low interest-rate smoothing and small adjustment cost

Figures 11 and 12 show that the solution for the identification problem can also be obtained by combining a low level of smoothing with a small adjustment cost. The latter prevents output from overreacting right after the shock.

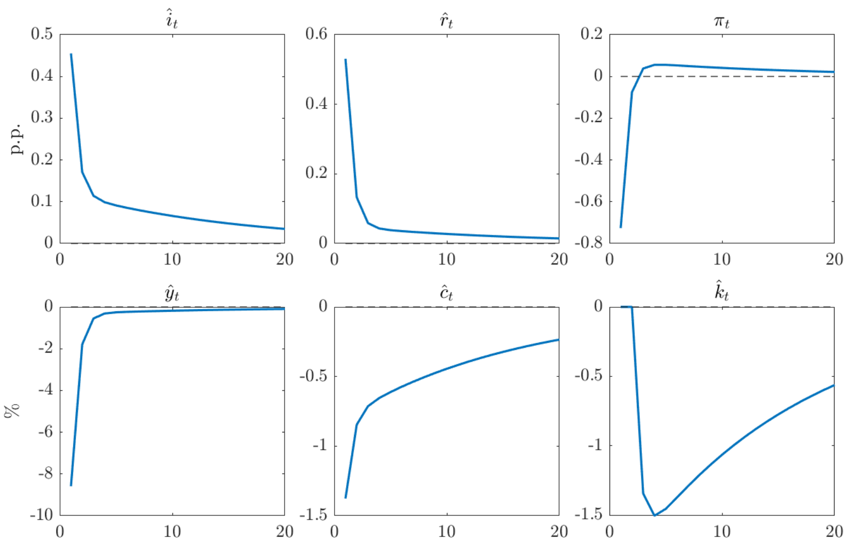


Figure 11: Impulse response function to a 1-standard-deviation monetary shock under $\rho^m = 0$, $\rho^i = 0.5$, $\frac{\overline{K}}{\overline{Y}} = 5.5$, and $\kappa = 0.1$

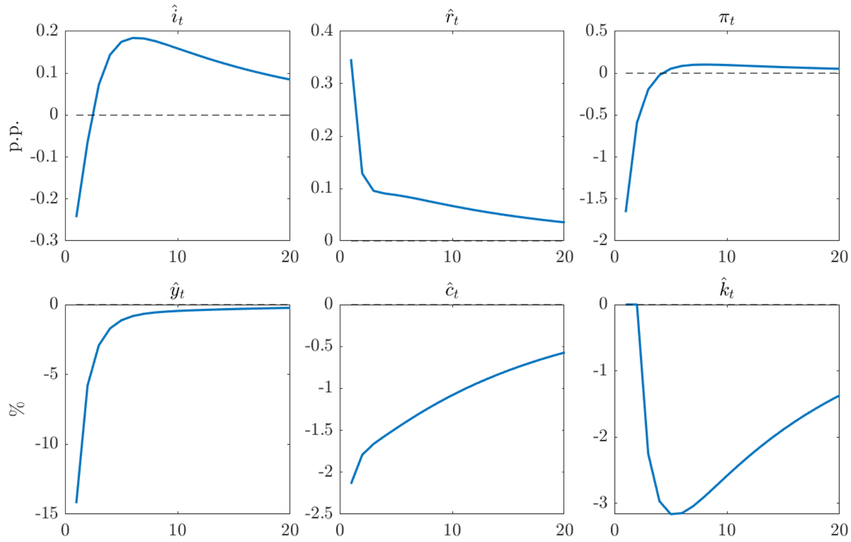


Figure 12: Impulse response function to a 1-standard-deviation monetary shock under $\rho^m = .50$, $\rho^i = 0.5$, $\frac{\overline{K}}{\overline{Y}} = 5.5$, and $\kappa = 0.1$

3.4.4 Fixing with high interest-rate smoothing, high adjustment cost, and realistic capital-to-output ratio

Finally, Figures 13 and 14 show that, for a realistic calibration of the capital-to-output ratio, a high level of interest-rate smoothing with high adjustment cost is also successful in reverting the sign of the real interest rate. The more persistent are both the smoothing and the exogenous shock, the higher the capital adjustment cost necessary to prevent output from overreacting after a monetary shock. This fact suggests that, although the identification problem raised by Rupert and Šustek (2019) can be circumvented with interest-rate smoothing in the Taylor rule, more realistic calibrations of the capital-to-output ratio still demand a considerable level of adjustment cost for the model to deliver adequate numerical responses.

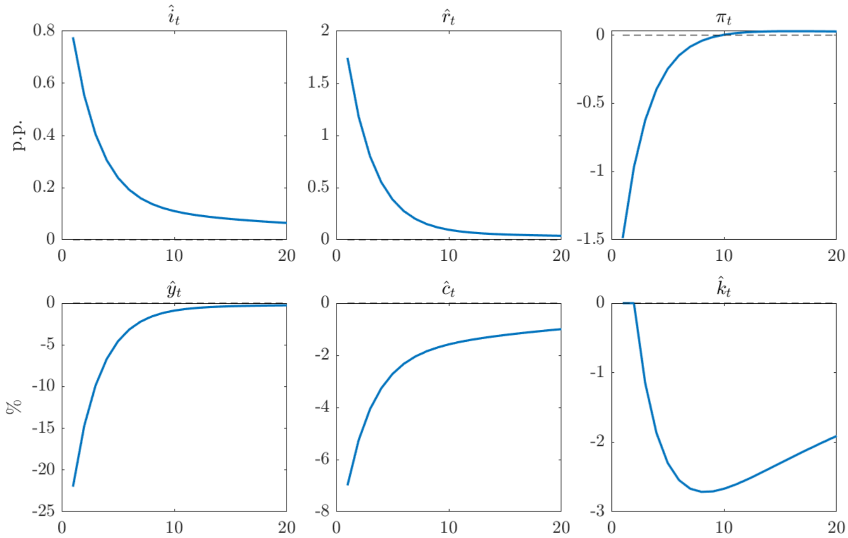


Figure 13: Impulse response function to a 1-standard-deviation monetary shock under $\rho^m = 0$, $\rho^i = 0.9$, $\frac{\overline{K}}{\overline{Y}} = 15.4$, and $\kappa = 0.5$

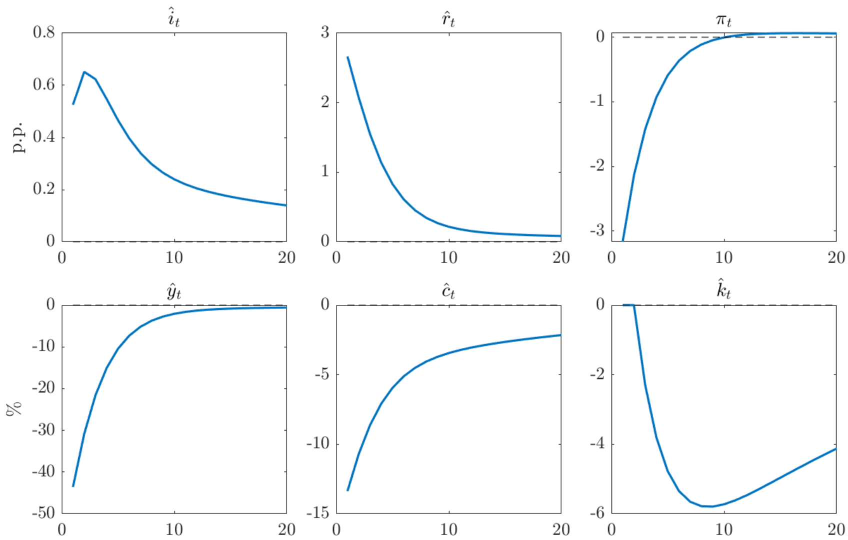


Figure 14: Impulse response function to a 1-standard-deviation monetary shock under $\rho^m = .50$, $\rho^i = 0.9$, $\frac{\overline{K}}{\overline{Y}} = 15.4$, and $\kappa = 0.5$

4 Conclusion

This paper showed that the identification problem of canonical New-Keynesian models augmented with endogenous capital, revealed by Rupert and Šustek (2019), can be circumvented by the inclusion of empirically-validated interest-rate smoothing in the Taylor rule, a feature as prevalent in middle-scale New-Keynesian models as capital itself. The sign of the real interest rate right after a positive monetary shock is positive under realistic parameters, reestablishing the observational consistency of the real interest rate channel of monetary policy transmission.

Moreover, we found that for realistic calibrations of the capital-to-output ratio, the canonical model augmented with capital may have trouble in fulfilling its local determinacy condition if interest-rate smoothing is low, while a considerable level of capital adjustment cost is required for preventing output from overreacting after a monetary shock. This last problem is aggravated the higher the persistence of the shock and the smoothing coefficient in the Taylor rule.

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