EE3032, Winter 2019-'20, Homework 4 Solutions, Dr. Durant

Problem 2.14: Functions x(t) and h(t) are given by

$x(t) = sin(\pi t), 0 \leq t \leq 1; 0 \text{ otherwise}$

$h(t) = u(t)$

Determine $y(t) = x(t) * h(t)$.

The result of convolving with u(t) is to integrate the function from -∞ to t, so the result will be 0 until t=0, vary until t=1, and then be a constant value forever.

$$y(t) = \int_0^t sin(\pi t)dt = -\frac{1}{\pi}cos(\pi t)|_0^t = \frac{1 - cos(\pi t)}{\pi}$$

$$y(1) = \frac{1 - cos(\pi)}{\pi} = \frac{2}{\pi}$$

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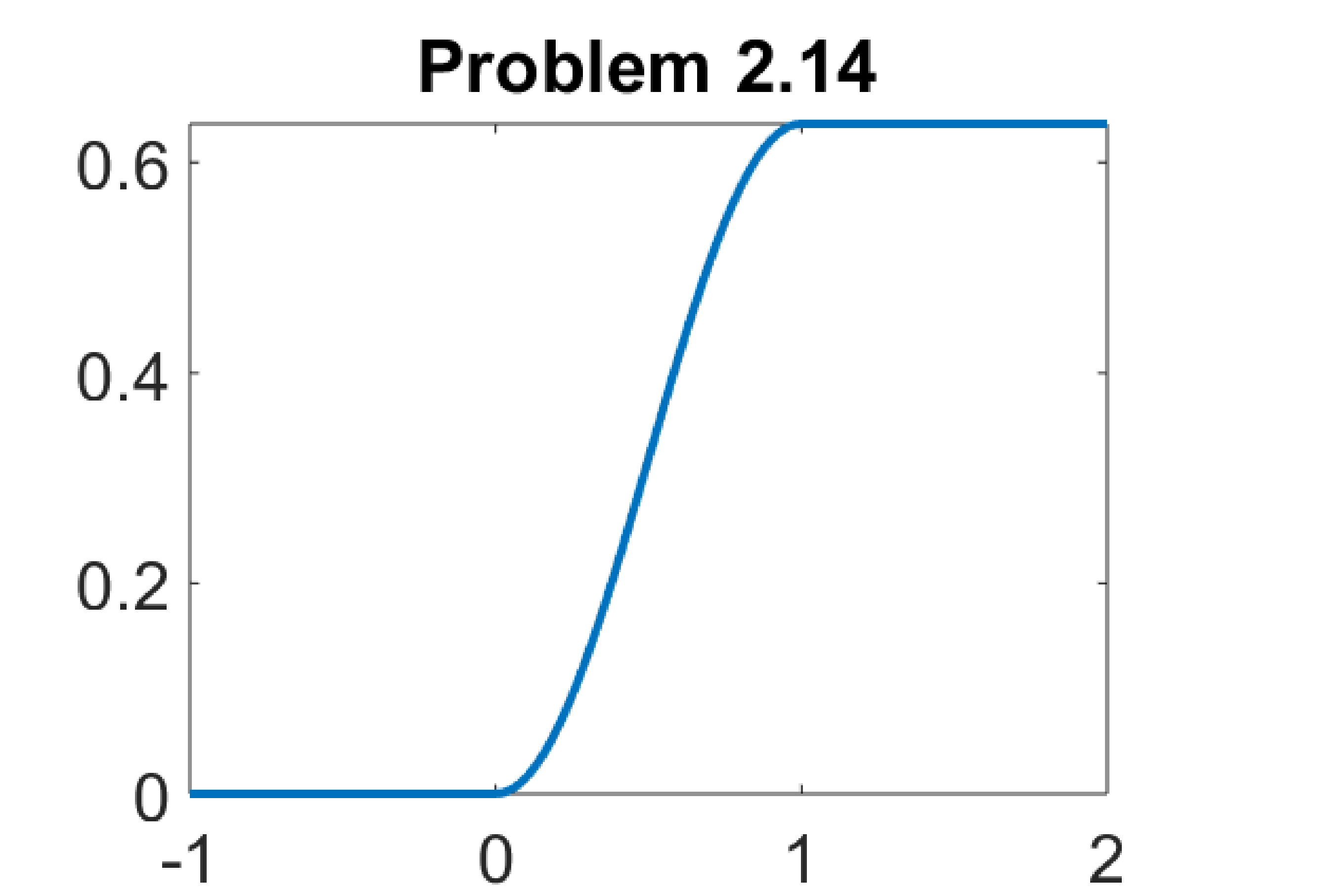
t = -1:0.02:2;
y = zeros(size(t));
r01 = 0 <= t & t <= 1;
y(r01) = (1-cos(pi*t(r01)))/pi;
y(t>=1) = 2/pi;
figure, plot(t,y), title('Problem 2.14')
```



Problem 2.15(b): Compute the following convolutions *without computing any integrals:* $u(t) * [2u(t) - 2u(t-3)]$

Again, convolving with u(t) gives the integral from -∞ to t of the other function. The other function is a height 2 pulse while t is between 0 and 3 s. So, the integral is a slope 2 ramp starting at 0, and flattinging off at t=3. This is accomplished by simply replacing "u" with "r" since ramp is the integral of step (is the integral of impulse).

$y(t) = 2r(t) - 2r(t-3)$

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t = -1:0.02:4;
y = zeros(size(t));
r0 = t>=0; y(r0) = 2*t(r0); % 2r(t)
r3 = t>=3; y(r3) = y(r3) - 2*(t(r3)-3); % y -= 2r(t-3)
figure, plot(t,y), title('Problem 2.15(b)')
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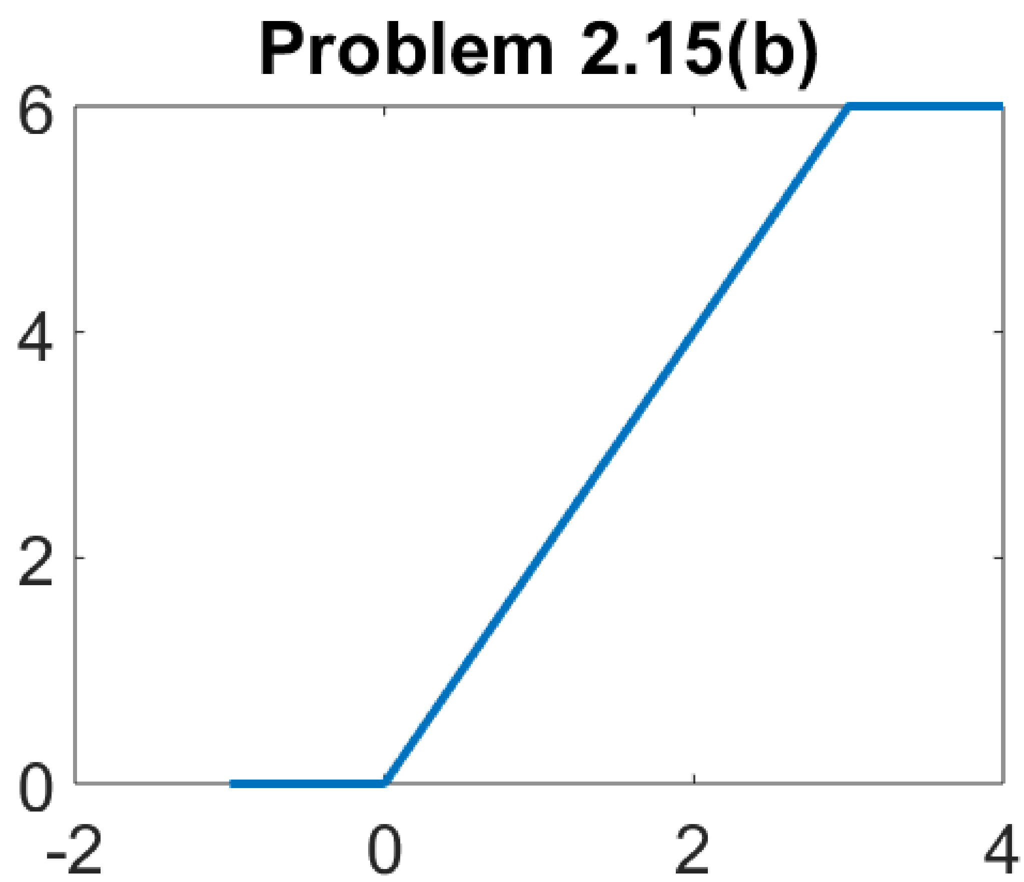

Problem 2.16(a): Compute the following convolutions *without computing any integrals:* $\delta(t-2) * [u(t) - 3u(t-1) + 2u(t-2)]$

Consider the impulse at t=2 to be the impulse response. This shows that the system just delays the input (2nd signal) by 2 s. So, we have

$y(t) = u(t-2) - 3u(t-3) + 2u(t-4)$

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t = -1:0.05:5;
y = (t>=2) - 3*(t>=3) + 2*(t>=4)
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y = 1×121
     0     0     0     0     0     0     0     0     0     0     0     0     0     0     0
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figure, plot(t,y), title('Problem 2.16(a)')
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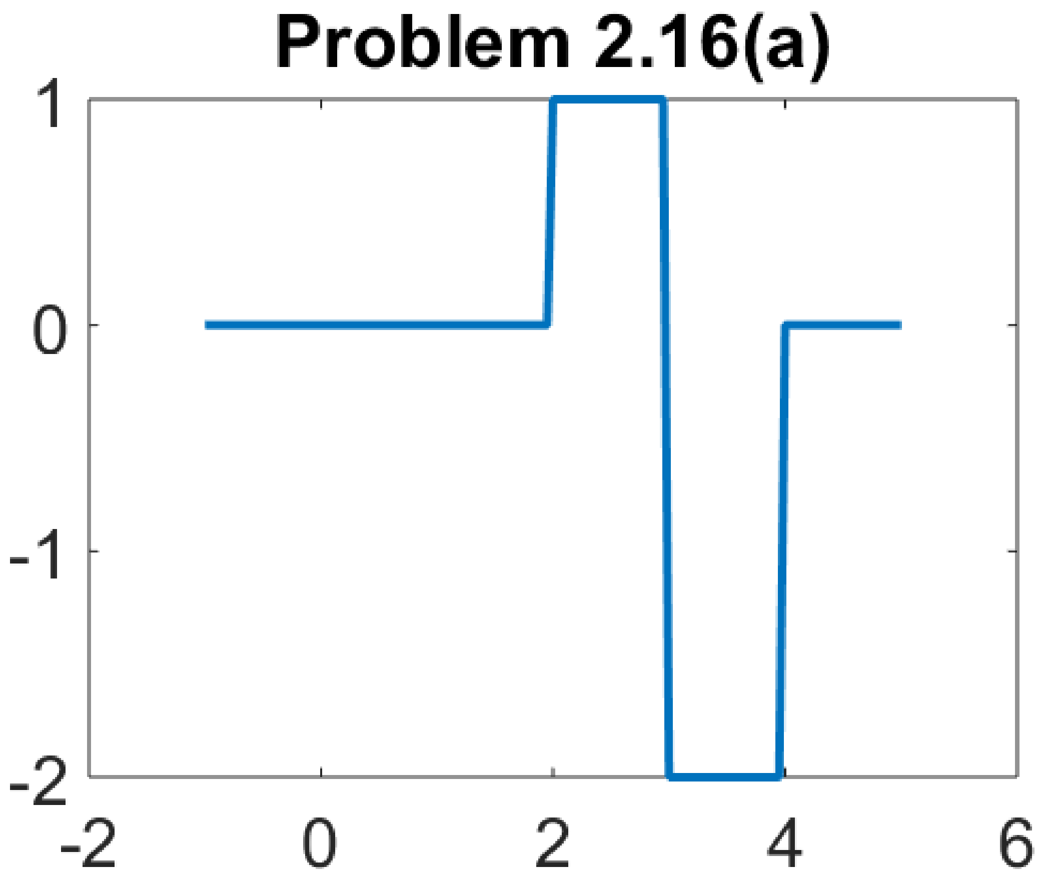

Problem 2.22(b,f) Determine whether or not each of the LTI systems whose impulse responses are specified below are (i) causal and/or (ii) BIBO stable.

(b) $h(t) = (1 - |t|)[u(t+1) - u(t-1)]$

This is ***not*** causal since it is non-zero for negative t due to u(t+1) starting at t=-1. This indicates outputs occur up to 1 second before inputs occur.

The system ***is*** stable since h(t) is absolutely integrable. h(t) is a triangle with base on $t \in [-1, 1]$ and height 1. Therefore its (absolute) area is 1, which is finite as required for stability.

(f) $h(t) = \dfrac{1}{t+1} u(t)$

This **is** causal due to multiplication by u(t) forcing a 0 value for t<0.

The system ***is not*** stable. The indefinite integral is ln(t+1), which diverges as t approaches infinity.

Problem 2.27: Prove the following statements.

(a) Parallel combinations of BIBO-stable systems are BIBO stable.

Parallel systems means that each system receives the same output and we ***add*** their outputs.

Consider N stable systems. Let c0 be the finite bound on the input. Then, c1..cN are the finite bounds on the outputs. These outputs are added, so we can add their bounds, yielding the bound for the composite system as c* = c1+c2+...+c(N-1)+cN. We must further assume that N is finite, which is reasonable in this problem. Then c* must be finite since a finite sum of finite numbers is finite. Since c* exists and is finite, we have a finite bound on the system output, which is the definition of BIBO stability.

(b) Series connections of causal systems are causal.

Systems in series mean each system receives as input the output of the previous system. Consider N causal systems. Let tN be the first time for which hN(t) is not 0; since the systems are causal, tN >= 0 for all N. We proved in class that we can get the impulse response of a composite of systems in series by convolving all the impulse respones. We also showed, via the width property, that the the first time for which the result of convolution is non-zero is the sum of when each of the inputs is non-zero. By induction, we can then say that if we convolve N functions the first time that the result is non-zero is the sum of all the tN. All tN are >= 0. Therefore the sum of all tN is >= 0 and the result is causal. (Note: in this case, we did ***not*** need to require that there be a finite number of systems.)

Problem 2.29(c,d,f) An LTI system has the frequency response function $H(\omega) = \dfrac{1}{j\omega + 3}$. Compute the output if the inputs is

(c) x(t) = 5 cos(4t)

Calculate the gain and phase shift: $H(4) = \dfrac{1}{j4+3} = 0.2\angle -0.9273$

$y(t) = 5 \times 0.2 \cos(4t - 0.9273) = \cos(4t - 0.9273)$

(d) $x(t) = \delta(t)$

Recognize that $h(t) = e^{-3t}u(t)$ yields the given transfer function. Then, h(t) = y(t) since it is given that the input is the unit impulse.

(f) x(t) = 1

$H(0) = \frac{1}{3}$

y(t) = 1/3

Problem 2.31: An LTI system has the impulse response $h(t) = e^{-12t}u(t)$. The input to the system is $x(t) = 12 + 26\cos(5t) + 45\cos(9t) + 80\cos(16t)$. Compute the output y(t).

Recognize that the corresponding $H(\omega) = \dfrac{1}{j\omega + 12}$. And we have 4 components (including DC) with...

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omega = [0 5 9 16];
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A = [12 26 45 80];
H = 1./ (1j*omega+12);
disp([abs(H); angle(H)]) % show in polar form
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    0.0833    0.0769    0.0667    0.0500
         0   -0.3948   -0.6435   -0.9273
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Y = A.*abs(H); % amplitude = original amplitude times gain
disp(Y)
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    1.0000    2.0000    3.0000    4.0000
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All the cosines were input with no phase shift, so the phase shifts due to H are the resulting phases. Putting it all together...

$$y(t) = 1 + 2\cos(5t - 0.3948) + 3\cos(9t - 0.6435) + 4\cos(16t - 0.9273)$$