Name Answers

EE3032 – Dr. Durant – Quiz 6
Winter 2019-2020, Week 6

Recall that the convolution integral is $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$.

Recall that the transfer function can be found by $H(\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt$.

1. (2 points) Given H(ω) = T sinc(ωT/2) exp(-jωT/2) and parameter T = 0.1 s, calculate H and present it in polar form for 8, 10, and 12 Hz sinusoidal inputs. Recall that, in general, H(ω) is a complex number.
2. (2 points) Let the system input x(t) = 50 sin(2π×8t + 30°). Calculate the (steady-state, sinusoidal) output using transfer function theory. Hint: having H in polar form will be useful.
3. (2 points) Now, consider a new system, where h(t) = δ(t+2) – δ(t-2). Describe in words how the output of this system relates to the input by taking advantage of the properties of the δ convolved with another function.
4. (2 points) Consider h(t) and explain why the system is BIBO stable.
5. (2 points) Calculate H(ω) for this system.

① $\omega = 2\pi F = 2\pi[8 \;\; 10 \;\; 12]$

$H(\omega) = T\,\mathrm{sinc}\left(\frac{\omega T}{2}\right) e^{-j\omega T/2} = 0.1\,\mathrm{sinc}(2\pi[0.4 \;\; 0.5 \;\; 0.6])\, e^{-j2\pi[0.4 \;\; 0.5 \;\; 0.6]}$

$= [0.0234\angle -0.8\pi \quad 0 \quad -0.0156\angle -1.2\pi]$

② f = 8 Hz, so use $H = 0.0234\angle -0.8\pi$ from #1.

$y(t) = 50 \cdot 0.0234 \sin(2\pi 8t + 30° - 0.8\pi)$

$= 1.17 \sin\left(2\pi 8t - \frac{19}{30}\pi\right) = 1.17 \sin(2\pi 8t - 114°)$

③ The system outputs a time-advanced copy of the signal 2s before the input arrives. It also adds this to an inverted + time-delayed copy of the input 2s after it arrives.

④ $\int |h(t)|dt = 2 < \infty$ ∴ BIBO stable

⑤ $H(\omega) = \int[\delta(t+2) - \delta(t-2)]e^{-j\omega t}dt = e^{-j\omega\cdot -2} - e^{-j\omega \cdot 2} = e^{j2\omega} - e^{-j2\omega}$

$= 2j\sin(2\omega)$