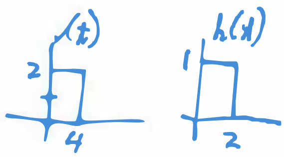
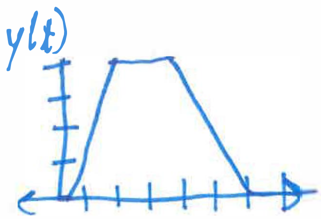
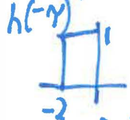
EE3032-4- Homework 3 - Dr. Durant - October, 2019

2.10 b)





Graphical convolution



$h(t-\tau)$ is shifted t to right

Regions:

- $t \le 0$, no overlap
- $0 \le t \le 2$, partial
- $2 \le t \le 4$, full
- $4 \le t \le 6$, partial
- $t \ge 6$, no

$y(t)$

0

$2t \leftarrow 2 \cdot 1 \cdot t$

4

$2(6-t)$

0

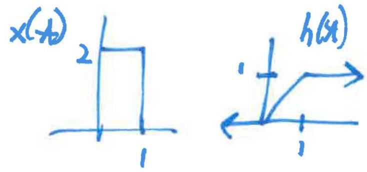
height product

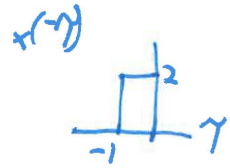
× h

$2 \cdot 1 \cdot t$ width

width from t to 6, $\int_t^6$

2.13 a,b) Analytic convolution. Flip $x(t)$ since it is simpler/finite support.





$y(t) = \int h(\tau)\, x(t-\tau)\, d\tau$

Region	$y(t)$
$t \le 0$	0
$0 \le t \le 1$	$\int_0^t t \cdot 2\, dt = \int_0^t \tau \cdot 2\, d\tau = \tau^2 \big|_0^t = t^2$
$1 \le t \le 2$	$\int_{t-1}^{1} \tau \cdot 2\, d\tau + \int_{1}^{t} 1 \cdot 2\, d\tau = \tau^2 \big|_{t-1}^{1} + 2\tau \big|_1^t = 1-(t-1)^2+2t-2$
$t > 2$	$\int_{t-1}^{t} 1 \cdot 2\, d\tau = 2\tau \big|_{t-1}^{t} = 2t-(2t-2) = 2$

↑ h ↑ x

Overlap is $[0,t]$, Use to simplify h & x expressions

More Formally

$= 1 - t^2 + 2t + -1 + 2t - 2$

$= -(t^2 - 4t + 2)$

Interval $[t-1, t]$ of $x(-\tau)$ is broken by h @ $t=1$

Check: $y(1^-) \stackrel{?}{=} y(1^+)$

$t^2 \big|_1 = -(t^2 - 4t + 2)\big|_1$

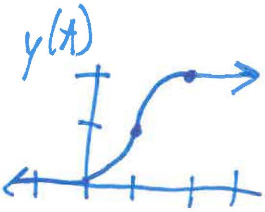
$1 = 1 \checkmark$

$y(2^-) \stackrel{?}{=} y(2^+)$

$-(t^2 - 4t + 2)\big|_2 = 2$

$-(4 - 8 + 2) = 2$

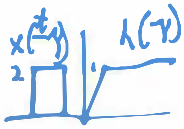
$2 = 2 \checkmark$



2.13 b) Graphical

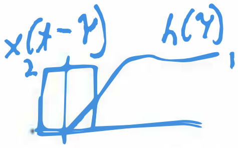
Region

$t \leq 0$



No overlap

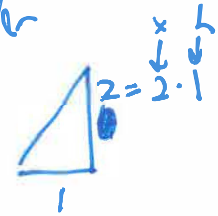
$0 \leq t \leq 1$



Overlap, 0th · 1st order = 1st order

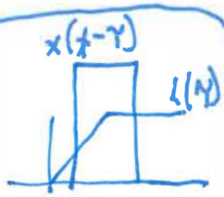
$\int \rightarrow$ 2nd order, parabola

$y(1) =$ Area of rect · triangle $=$



$= 1$

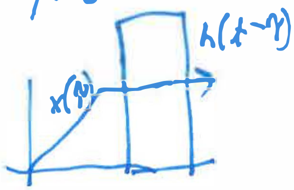
$1 \leq t \leq 2$



Area increasing, but at slowing rate, losing parabola area on left, so convex down on this region.

$y(2) =$ Area of rect · rect $= \underbrace{2 \cdot 1 \cdot 1}_{x\ h\ t} = 2$

$t \geq 2$



$y(t) =$ Area of rect · rect $= \ldots 2$

2.15 a) $y(t) = \overbrace{u(t)}^{\text{Gives } \int_0^t \cdot d\tau \text{ of other function. Use sampling property}} * [\delta(t) - 3\delta(t-1) + 2\delta(t-2)]$

$= u(t) - 3u(t-1) + 2u(t-2)$

Or!: Use sampling directly: $h(t) * (A\delta(t-B)) = Au(t-B)$
Samples value A @ $t = B$.

c) $y(t) = u(t) * [(t-1)u(t-1)] = u(t) * r(t-1) = \int_0^t r(\tau-1)\,d\tau = \frac{(\tau-1)^2}{2}$

Integral of $r(t) = t^2/2, t > 0$, area is parabola. Delay by 1s (time-invariant)

2.17 b) $y(t) = e^{-2t}u(t) * e^{-3t}u(t) = x(t) * h(t)$

$$= \int_0^t e^{-2\tau} e^{-3(t-\tau)} d\tau$$

$$= e^{-3t}\int_0^t e^{-2\tau} e^{3\tau} d\tau = e^{-3t}\int_0^t e^{\tau} d\tau = e^{-3t} \cdot e^{\tau}\Big|_0^t = e^{-3t}(e^{t} - e^{0})$$

$$= e^{-2t} - e^{-3t}, \quad t > 0$$

2.22 a, c, e

BIBO stable $\Leftrightarrow$ absolutely integrable $\Leftrightarrow$ $\int h(t)dt$ is finite

(a) $h(t) = e^{-|t|}$ $\boxed{\text{not causal}}$ since $h(-3) = e^{-3} \neq 0$
e.g.

$$\int |h(t)| dt = \underbrace{2\int_0^t}_{\text{symmetry}} e^{-\tau} d\tau = 2 \cdot -e^{-\tau}\Big|_0^t = -2(e^{-t} - 1) = 2(1 - e^{-t})$$

$$\lim_{t \to \infty} = 2 < \infty \therefore \boxed{\text{stable}}$$

(c) $h(t) = e^{2t}u(-t)$ $\boxed{\text{not causal}}$ since $h(t) \neq 0$ when $t < 0$

$$\int |h(t)| = \int_{-\infty}^0 e^{2\tau} d\tau = \frac{1}{2}e^{2\tau}\Big|_{-\infty}^0 = \frac{1}{2}(1 - 0) = \frac{1}{2} \underbrace{< \infty}_{\text{informal notation for "is finite"}} \therefore \boxed{\text{stable}}$$

(e) $h(t) = \cos(2t)u(t)$ $\boxed{\text{causal}}$ since $u(t)$ enforces $h(t) = 0 \underbrace{\forall}_{\text{"for all"}} t < 0$

$$\int |h(t)| dt = \int_0^\infty |\cos(2t)| dt \to \infty$$

area grows w/o bound