In[22]:=
$$\mathbf{r}[\theta\_, \phi\_] := \begin{pmatrix} \mathbf{Sin}[\theta]\,\mathbf{Cos}[\phi] \\ \mathbf{Sin}[\theta]\,\mathbf{Sin}[\phi] \\ \mathbf{Cos}[\theta] \end{pmatrix}$$

In[23]:=
```
s = r[θ, ϕ]
xc = r[θc, ϕc]
```

Out[23]= {{Cos[ϕ] Sin[θ]}, {Sin[θ] Sin[ϕ]}, {Cos[θ]}}

Out[24]= {{Cos[ϕc] Sin[θc]}, {Sin[θc] Sin[ϕc]}, {Cos[θc]}}

In[25]:=
```
"Change of Basis Matrix"
cob[θ_, ϕ_] := Transpose[{{Sin[θ] Cos[ϕ], Cos[θ] Cos[ϕ], -Sin[ϕ]}, {Sin[θ] Sin[ϕ], Cos[θ] Sin[ϕ], Cos[ϕ]}, {Cos[θ], -Sin[θ], 0}}]
```

Out[25]= Change of Basis Matrix

In[27]:=
```
"Projection of s wrt tangent plane normal"
sproj = (Dot[Transpose[s], xc])[[1, 1]]
```

Out[27]= Projection of s wrt tangent plane normal

Out[28]= Cos[θ] Cos[θc] + Cos[ϕ] Cos[ϕc] Sin[θ] Sin[θc] + Sin[θ] Sin[θc] Sin[ϕ] Sin[ϕc]

In[29]:=
```
"Project s to tangent plane"
Simplify[s + (1 - sproj) xc]
```

Out[29]= Project s to tangent plane

Out[30]=
$$\Big\{\{\mathrm{Cos}[\phi]\,\mathrm{Sin}[\theta] - \mathrm{Cos}[\phi c]\,\mathrm{Sin}[\theta c]\,(-1 + \mathrm{Cos}[\theta]\,\mathrm{Cos}[\theta c] + \mathrm{Cos}[\phi]\,\mathrm{Cos}[\phi c]\,\mathrm{Sin}[\theta]\,\mathrm{Sin}[\theta c] + \mathrm{Sin}[\theta]\,\mathrm{Sin}[\theta c]\,\mathrm{Sin}[\phi]\,\mathrm{Sin}[\phi c])\},$$
$$\{\mathrm{Sin}[\theta]\,\mathrm{Sin}[\phi] - \mathrm{Sin}[\theta c]\,\mathrm{Sin}[\phi c]\,(-1 + \mathrm{Cos}[\theta]\,\mathrm{Cos}[\theta c] + \mathrm{Cos}[\phi]\,\mathrm{Cos}[\phi c]\,\mathrm{Sin}[\theta]\,\mathrm{Sin}[\theta c] + \mathrm{Sin}[\theta]\,\mathrm{Sin}[\theta c]\,\mathrm{Sin}[\phi]\,\mathrm{Sin}[\phi c])\},$$
$$\Big\{\mathrm{Cos}[\theta]\,\mathrm{Sin}[\theta c]^2 - \mathrm{Cos}[\theta c]\,(-1 + \mathrm{Cos}[\phi]\,\mathrm{Cos}[\phi c]\,\mathrm{Sin}[\theta]\,\mathrm{Sin}[\theta c] + \mathrm{Sin}[\theta]\,\mathrm{Sin}[\theta c]\,\mathrm{Sin}[\phi]\,\mathrm{Sin}[\phi c])\Big\}\Big\}$$

In[31]:=
```
"Change basis to θ, ϕ unit vectors"
Simplify[Dot[cob[θc, ϕc], s + (1 - sproj) xc]]
```

Out[31]= Change basis to θ, ϕ unit vectors

Out[32]= {{1}, {Cos[θc] Cos[ϕ - ϕc] Sin[θ] - Cos[θ] Sin[θc]}, {Sin[θ] Sin[ϕ - ϕc]}}

In[33]:=
```
"Shift coordinates so xc is the origin in new coord sys"
snew = Simplify[Dot[cob[θc, φc], s + (- sproj) c]]
```

Out[33]= Shift coordinates so xc is the origin in new coord sys

Out[34]=
$$\left\{\left\{-\left(\left(\text{Cos}[\theta]\,\text{Cos}[\theta c]+\text{Cos}[\phi-\phi c]\,\text{Sin}[\theta]\,\text{Sin}[\theta c]\right)\right.\right.$$
$$\left.\left(-1+c\,\text{Cos}[\theta c]+c\,\text{Cos}[\phi c]\,\text{Sin}[\theta c]+c\,\text{Sin}[\theta c]\,\text{Sin}[\phi c]\right)\right)\Big\},$$
$$\left\{-\text{Cos}[\theta]\left(\text{Sin}[\theta c]-c\,\text{Cos}[\theta c]\,\text{Sin}[\theta c]+c\,\text{Cos}[\theta c]^2\left(\text{Cos}[\phi c]+\text{Sin}[\phi c]\right)\right)-\right.$$
$$\frac{1}{4}\,\text{Cos}[\phi-\phi c]\,\text{Sin}[\theta]\left(2\,c\,\text{Cos}[\theta c]^2+c\left(-3+\text{Cos}[2\,\theta c]\right)+\right.$$
$$\left.\left.4\,\text{Cos}[\theta c]\left(-1+c\,\text{Cos}[\phi c]\,\text{Sin}[\theta c]+c\,\text{Sin}[\theta c]\,\text{Sin}[\phi c]\right)\right)\right\},$$
$$\left\{-c\,\text{Cos}[\theta]\,\text{Cos}[\theta c]\left(\text{Cos}[\phi c]-\text{Sin}[\phi c]\right)+\text{Sin}[\theta]\right.$$
$$\left(-\text{Cos}[\phi]\left(c\,\text{Cos}[\phi c]^2\,\text{Sin}[\theta c]+\text{Sin}[\phi c]-c\,\text{Cos}[\phi c]\,\text{Sin}[\theta c]\,\text{Sin}[\phi c]\right)+\right.$$
$$\left.\left.\left.\text{Sin}[\phi]\left(\text{Cos}[\phi c]-c\,\text{Cos}[\phi c]\,\text{Sin}[\theta c]\,\text{Sin}[\phi c]+c\,\text{Sin}[\theta c]\,\text{Sin}[\phi c]^2\right)\right)\right\}\right\}$$

In[35]:=
```
"Write coordinates where x ~ φ and y ~ θ"
snewflip2d[θ_, φ_, θc_, φc_] := {{Sin[θ] Sin[φ - φc]}, {-Cos[θc] Cos[φ - φc] Sin[θ] + Cos[θ] Sin[θc]}}
```

Out[35]= Write coordinates where x ~ ϕ and y ~ θ

In[37]:=
```
"Test Case"
snewflip2d[π / 2 - π / 6, 0, π / 2 , 0]
```

Out[37]= Test Case

Out[38]=
$$\left\{\{0\},\left\{\frac{1}{2}\right\}\right\}$$