p_{max} Derivation for Number of Timesteps

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We are more careful in our definitions now. Suppose we truncate our impact parameters to some p_{max} for perturbers with equal mass and velocities. Then, the characteristic time between encounters is given by

$$\delta t = \frac{m}{\rho \times \pi p_{max}^2 \times v}.$$

In this case, the truncation radius is defined from the threshold energy injection:

$$\frac{N_{enc} \Delta E(p_{max}, \theta = \pi/2, \phi = 0, v_0; a, M)}{E(a, M)} = \Delta \varepsilon,$$

where $N_{enc} = \text{int}(T/\delta t)$. Suppose we now allow the velocities to be picked out by some PDF. Then, forcing the truncation radius p_{max} to be independent of v. We define the maximum impact parameter as the threshold energy injection for the average cumulative encounter energy injection:

$$\frac{\langle N_{enc} \Delta E \rangle_V}{E(a, M)} = \Delta \varepsilon.$$

With this maximum impact parameter set, we can then calculate the average characteristic time between encounters for a system of perturbers truncated to this maximum impact parameter

$$\langle \delta t \rangle_V = \frac{m}{\rho \times \pi p_{max}^2} \times \left\langle \frac{1}{v} \right\rangle_V.$$

From this, the number of encounters is estimated to be

$$N_{enc} = \text{int}\left(\frac{T}{\langle \delta t \rangle_V} \right)$$

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p_{max} Note:

$$\frac{\langle N_{enc} \Delta E \rangle_V}{E} = \pi \left(\frac{\rho T}{m} \right) \times \frac{\bar{\mathcal{C}}}{E} \times \frac{4a^2 p_{max}^2}{(p_{max^2} - a^2)^2},$$

where

$$\bar{\mathcal{C}} = \frac{1}{2}(2GM)^2 \times \left\langle \frac{1}{v} \right\rangle_V$$

Solving for p_{max} follows the same procedure as v0.2 with appropriate replacements:

$$\begin{aligned}
N_{enc} \times \frac{\Delta E(p_{max}, \theta = \pi/2, \phi = 0, v_0; a, M)}{E(a, M)} &= \Delta \varepsilon \\
\implies \pi \left(\frac{\rho T}{m} \right) \times \frac{\bar{\mathcal{C}}}{E} \times \frac{4a^2 p_{max}^2}{(p_{max^2} - a^2)^2} &= \Delta \varepsilon \\
\implies \frac{4a^2 p_{max}^2}{(p_{max^2} - a^2)^2} &= \Delta \bar{\varepsilon} \\
\implies p_{max} &= a\sqrt{1 + 2\Delta\bar{\varepsilon}^{-1}(1 + \sqrt{1 + \Delta\bar{\varepsilon}})},
\end{aligned}$$

where $\bar{\mathcal{C}} = (2GM_p)^2 \times \langle v^{-1} \rangle / 2$ and all variables are set so we get the maximum value of p_{max}. That is, orientation of system ensures highest impact parameter corresponding to head-on encounter, density is set to the local halo density ($f = 1$), and the evolution time is set to $T = 10$ Gyr. Notice how now we are averaging beforehand, we are not setting values for v.

STRESS: THIS WAS DERIVED FOR THE CIRCULAR BINARIES.