Summary of Monte Carlo Algorithm

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Each binary is simulated using an iterative algorithm. Suppose the simulation begins at t_0 and the encounter timesteps are given by $t_1 \leq t_2 \leq \ldots \leq t_N$, where $t_{n+1} = t_n + \delta t$ for $n = 0, \ldots, N-1$. At time t_n, the orbital parameters of a binary are given by (a_n, e_n, ψ_n). The orbital phase ψ_n specifies the effective orbital time τ_n necessary for evolving the binary forward in time. We first generate the observed separations of binaries after n encounters by calculating the projected physical separation of the binary at some random time between t_n and t_{n+1}. The effective orbital time corresponding to observation is given by $\tau_n + U[0,1]\ \frac{\delta t}{P(a_n)}$. We use this in Kepler's equation to calculate the orbital phase and, then, the physical separation at the observation time. The projected physical separation is calculated using $s = r\sin(i)$ (or, equivalently, $s = r\cos(i)$ due to random inclination angles), where i is the random inclination assigned to the binary. These separations are saved as a histogram of binary projected separations that are converted to the scattering matrix $S(s|a', t_i)$. Following this observation step, we evolve the binary to the next encounter time t_{n+1} as in the observation step to obtain an orbital phase and physical separation during the encounter. We calculate the effect of the encounter on binary orbital parameters using the equation for the impulse $\Delta\vec{v}$ and translate the impulse to evolved orbital parameters (a_{n+1}, e_n, ψ_n) using the line of equations connecting them in Sec. III of the paper. This procedure is repeated for N times. At the final timestep, a final set of observed separations is stored as a histogram for the scattering matrix . The entire binary evolution algorithm is described in Box .

Binary Evolution Algorithm

Initialization

1. Load binary population uniformly from bin centered at a'_j
2. Load perturber population
3. Load encounter timesteps
4. Generate encounter parameters corresponding to all binaries and timesteps
5. Load effective orbital time matrix used for unperturbed binary evolution

Evolve binaries from t_k to t_{k+1} for $k = 0, 1, \ldots, N-1$. For each binary,

1. Convert $\psi_k \to \tau_k$
2. Calculate and store binary projected physical separation histogram corresponding to timestep $t_k = k\ \delta t$, where the observation time is sampled randomly as: $\tau_{obs} = \tau_k + U[0,1]\ \frac{\delta t}{P(a_k)}$
3. Load encounter parameters $\vec{x}_k = (p_k, \theta_k, \phi_k, \gamma_k, v_{p},_k\ )$
4. Evolve the pre-encounter binary orbital parameters to post-encounter orbital parameters using the impulse approximation:

$$(a_k, e_k, \psi_k) \longrightarrow (a_{k+1}, e_{k+1}, \psi_{k+1})$$

5. Repeat the loop up to the final timestep N

Store final projected separation histogram observed after the Nth encounter.

These steps give the scattering matrix corresponding to the initial semimajor axis bin a_j: $\{S(s_i|a_j, t_k)\}_{i,k}$

Repeat this for all a_j-bins to form the scattering matrix: $\{S(s_i|a_j, t_k)\}_{i,j,k}$