

DTU42136, Large Scale Optimization using Decomposition

Assignment 2, Dantzig Wolfe Reformulation

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1 Part A

1.1 Task 1

Col Name	x ₁₁	x ₁₂	x ₁₃	x ₁₄	x ₁₅	y ₁	x ₂₁	x ₂₂	x ₂₃	x ₂₄	x ₂₅	y ₂	Vector B
Vector C	292	453	359	219	268	736	291	443	403	498	400	967	-
cons 2.1	1	0	0	0	0	0	1	0	0	0	0	0	1
cons 2.2	0	1	0	0	0	0	0	1	0	0	0	0	1
cons 2.3	0	0	1	0	0	0	0	0	1	0	0	0	1
cons 2.4	0	0	0	1	0	0	0	0	0	1	0	0	1
cons 2.5	0	0	0	0	1	0	0	0	0	0	1	0	1
cons 3.1	14	20	6	16	10	-43	0	0	0	0	0	0	0
cons 3.2	0	0	0	0	0	0	14	20	6	16	10	-43	0

Table 1. Matrix A, Vector B and Vector C for SSCFLP

2 Task A.2

One way of applying Dantzig-Wolfe reformulation to this problem is by keeping constraints (2) in the master and convexifying constraints (3)-(5).

Col Name	x ₁₁	x ₁₂	x ₁₃	x ₁₄	x ₁₅	y ₁	x ₂₁	x ₂₂	x ₂₃	x ₂₄	x ₂₅	y ₂	Vector B
Vector C	292	453	359	219	268	736	291	443	403	498	400	967	-
cons 2.1	1	0	0	0	0	0	1	0	0	0	0	0	1
cons 2.2	0	1	0	0	0	0	0	1	0	0	0	0	1
cons 2.3	0	0	1	0	0	0	0	0	1	0	0	0	1
cons 2.4	0	0	0	1	0	0	0	0	0	1	0	0	1
cons 2.5	0	0	0	0	1	0	0	0	0	0	1	0	1

Table 2. Constraint Matrix for the Master Problem

Col Name	x ₁₁	x ₁₂	x ₁₃	x ₁₄	x ₁₅	y ₁	x ₂₁	x ₂₂	x ₂₃	x ₂₄	x ₂₅	y ₂	Vector B
Vector C	292	453	359	219	268	736	291	443	403	498	400	967	-
cons 3.1	14	20	6	16	10	-43	0	0	0	0	0	0	0
cons 3.2	0	0	0	0	0	0	14	20	6	16	10	-43	0

Table 3. Constraint Matrix for the Sub Problem

3 Task A.3

Branch 12-th variable to 1 and 0. Find branch_1 is better. Then branch 6-th variable to 1 and 0. Find branch_1.1 is the final result.

Col Name	x ₁₁	x ₁₂	x ₁₃	x ₁₄	x ₁₅	y ₁	x ₂₁	x ₂₂	x ₂₃	x ₂₄	x ₂₅	y ₂	Vector B
Vector C	292	453	359	219	268	736	291	443	403	498	400	967	-
cons 5.1	0	0	0	0	0	0	0	0	0	0	0	-1	-1
cons 5.2	0	0	0	0	0	-1	0	0	0	0	0	0	-1

Table 4. Extra Constraint Matrix for the Sub Problem in Branch

4 Task A.4

Col Name	x ₁₁	x ₁₂	x ₁₃	x ₁₄	x ₁₅	y ₁	Vector B
Vector C	292	453	359	219	268	736	-
cons 3.1	14	20	6	16	10	-43	0

Table 5. Constraint Matrix for the Sub Problem 1

Col Name	x ₂₁	x ₂₂	x ₂₃	x ₂₄	x ₂₅	y ₂	Vector B
Vector C	291	443	403	498	400	967	-
cons 3.1	14	20	6	16	10	-43	0

Table 6. Constraint Matrix for the Sub Problem 2

4.1 Test using Generated Data Set and Conclusion

Data Set	Method	numSub	Obj	Time	numIteration
A	Gurobi	-	2834.02	0.0017	-
A	Gurobi-Relax	-	3283	0.0024	-
A	DW-Relax	1	3038	0.316	17
A	DW-Relax	2	3038	0.328	13
B	Gurobi	-	7956.88	0.100	-
B	Gurobi-Relax	-	8612.0	0.0023	-
B	DW-Relax	1	8583.33	6.138	409
B	DW-Relax	10	8583.33	1.425	44
C	Gurobi	-	16228	2.42	-
C	Gurobi-Relax	-	15301.81	0.0042	-
C	DW-Relax	1	15836.01	50.349	1383
C	DW-Relax	20	15836.01	1.371	71

Table 7. Table of Results from Optimizations using Different Data Sets and Methods

We can get three conclusions from the table:

- The objective value from Dantzig-Wolfe Decomposition is better than that solved by Gurobi relaxation.
- The advantage of splitting sub-problems is more obvious, when there are more sub-problems. There will be less iterations and elapsed time.
- It's possible for Dantzig-Wolfe Decomposition to beat Gurobi in MILP, if there are many sub-problems and good branch-and-cut algorithm.

4.2 Task 5.B

$$H = \sum_{i \in I} h_i \quad (1)$$

$$\min \quad z \quad (2)$$

$$\text{s.t.} \quad \alpha_{ij} + \alpha_{ji} + \beta_{ij} + \beta_{ji} \geq 1 \quad \forall i, j \in I, i < j \quad (3)$$

$$y_i + h_i \leq z \quad \forall i \in I \quad (4)$$

$$x_i + w_i \leq W \quad \forall i \in I \quad (5)$$

$$x_i + w_i \leq x_j + W(1 - \alpha_{ij}) \quad \forall i, j \in I \quad (6)$$

$$y_i + h_i \leq y_j + H(1 - \beta_{ij}) \quad \forall i, j \in I \quad (7)$$

$$x_i, y_i \geq 0 \quad \forall i \in I \quad (8)$$

$$\alpha_{ij}, \beta_{ij} \in \{0, 1\} \quad \forall i, j \in I \quad (9)$$

$$\max \quad -z \quad (10)$$

$$\text{s.t.} \quad -\alpha_{ij} - \alpha_{ji} - \beta_{ij} - \beta_{ji} \leq -1 \quad \forall i, j \in I, i < j \quad (11)$$

$$x_i - x_j + W\alpha_{ij} \leq W - w_i \quad \forall i, j \in I \quad (12)$$

$$y_i - y_j + H\beta_{ij} \leq H - h_i \quad \forall i, j \in I \quad (13)$$

$$y_i - z \leq -h_i \quad \forall i \in I \quad (14)$$

$$x_i \leq W - w_i \quad \forall i \in I \quad (15)$$

$$x_i, y_i \geq 0 \quad \forall i \in I \quad (16)$$

$$\alpha_{ij}, \beta_{ij} \in \{0, 1\} \quad \forall i, j \in I \quad (17)$$

12	13	21	23	31	32	12	13	21	23	31	32	1	2	3	1	2	3	z	b
-1	.	-1	.	.	.	-1	.	-1	-1
.	-1	.	.	-1	.	.	-1	.	.	-1	-1
.	.	.	-1	.	-1	.	.	.	-1	.	-1	-1
.	1	.	.	1	$-h_1$
.	1	.	1	$-h_2$
.	1	1	$-h_3$
.	1	$W - w_1$
.	1	$W - w_2$
.	1	$W - w_3$
W	1	-1	$W - w_1$
.	W	1	.	-1	$W - w_1$
.	.	W	-1	1	$W - w_2$
.	.	.	W	1	-1	$W - w_2$
.	.	.	.	W	-1	.	1	$W - w_3$
.	W	-1	1	$W - w_3$
.	H	1	-1	.	.	$H - h_1$
.	H	1	.	-1	.	$H - h_1$
.	H	-1	1	.	.	$H - h_2$
.	H	1	-1	.	$H - h_2$
.	H	-1	.	1	.	$H - h_3$
.	H	-1	1	.	$H - h_3$

$$\max \quad -z \quad (19)$$

$$\text{s.t.} \quad \begin{cases} -\alpha_{ij} - \alpha_{ji} - \beta_{ij} - \beta_{ji} \leq -1 & \forall i, j \in I, i < j \\ x_i \leq W - w_i & \forall i \in I \\ y_i - z \leq -h_i & \forall i \in I \end{cases} \quad (20)$$

$$\begin{cases} x_i - x_j + W\alpha_{ij} \leq W - w_i & \forall i, j \in I \\ y_i - y_j + H\beta_{ij} \leq H - h_i & \forall i, j \in I \end{cases} \quad (21)$$

$$x_i, y_i \geq 0 \quad \forall i \in I \quad (22)$$

$$\alpha_{ij}, \beta_{ij} \in \{0, 1\} \quad \forall i, j \in I \quad (23)$$

5 Appendix

5.1 Task A.2 Detailed Result

```
1 ##### Begin Iteration #####
2 vec_pi = [-1.0e6 0.0 0.0 0.0 0.0]
3 vec_kappa = [0.0]
4
5 1-th iteration. obj_master = -1.0e6.
6
7 Reduced cost of 1-th sub model is 1.997714e6.
8 objCoef = -2286.0.
9 best reduced cost is 1.997714e6
10 vec_pi = [-1143.0 -998857.0 0.0 0.0 0.0]
11 vec_kappa = [0.0]
12
13 2-th iteration. obj_master = -1.0e6.
14
15 Reduced cost of 1-th sub model is 1.996818e6.
16 objCoef = -3182.0.
17 best reduced cost is 1.996818e6
18 vec_pi = [-1143.0 -448.0 -998409.0 0.0 0.0]
19 vec_kappa = [0.0]
20
21 3-th iteration. obj_master = -1.0e6.
22
23 Reduced cost of 1-th sub model is 1.996061e6.
24 objCoef = -3491.0.
25 best reduced cost is 1.996061e6
26 vec_pi = [-1143.0 -448.0 -378.5 -9.98031e5 0.0]
27 vec_kappa = [0.0]
28
29 4-th iteration. obj_master = -1.0e6.
30
31 Reduced cost of 1-th sub model is 1.9953635e6.
32 objCoef = -3362.0.
33 best reduced cost is 1.9953635e6
34 vec_pi = [-1143.0 -448.0 -378.5 -348.75 -9.97682e5]
35 vec_kappa = [0.0]
36
37 5-th iteration. obj_master = -1.0e6.
38
39 Reduced cost of 1-th sub model is 1.99482525e6.
40 objCoef = -3173.0.
41 best reduced cost is 1.99482525e6
42 vec_pi = [5.3081e5 -5.32401e5 -2.66355e5 -3.99314e5 -332740.0]
43 vec_kappa = [0.0]
44
45 6-th iteration. obj_master = -1.0e6.
46
47 Reduced cost of 1-th sub model is 2.3920628000000003e6.
48 objCoef = -4078.0.
49 best reduced cost is 2.3920628000000003e6
50 vec_pi = [1.32816e6 -1.32976e6 -6.65032e5 1.99275e6 -2.32613e6]
51 vec_kappa = [0.0]
52
```

```

53 7-th iteration. obj_master = -1.0e6.
54
55 Reduced cost of 1-th sub model is 8.637798333333332e6.
56 objCoef = -4029.0.
57 best reduced cost is 8.637798333333332e6
58 vec_pi = [-242.5 -448.0 -378.5 -312.0 -287.5]
59 vec_kappa = [-1801.0]
60
61 8-th iteration. obj_master = -3469.5.
62
63 Reduced cost of 1-th sub model is 1801.0.
64 objCoef = 0.0.
65 best reduced cost is 1801.0
66 vec_pi = [-727.385 -863.615 -586.308 -589.077 -564.577]
67 vec_kappa = [0.0]
68
69 9-th iteration. obj_master = -3330.9615384615386.
70
71 Reduced cost of 1-th sub model is 439.34615384615427.
72 objCoef = -3619.0.
73 best reduced cost is 439.34615384615427
74 vec_pi = [-397.875 -1193.13 -37.125 -808.75 -784.25]
75 vec_kappa = [0.0]
76
77 10-th iteration. obj_master = -3221.125.
78
79 Reduced cost of 1-th sub model is 761.25.
80 objCoef = -3218.0.
81 best reduced cost is 761.25
82 vec_pi = [-397.875 -812.5 -417.75 -808.75 -784.25]
83 vec_kappa = [0.0]
84
85 11-th iteration. obj_master = -3221.125.
86
87 Reduced cost of 1-th sub model is 475.875.
88 objCoef = -1515.0.
89 best reduced cost is 475.875
90 vec_pi = [-556.5 -1034.5 -513.0 -491.5 -467.0]
91 vec_kappa = [0.0]
92
93 12-th iteration. obj_master = -3062.5.
94
95 Reduced cost of 1-th sub model is 272.0.
96 objCoef = -1767.0.
97 best reduced cost is 272.0
98 vec_pi = [-556.5 -1129.75 -417.75 -763.5 -739.0]
99 vec_kappa = [544.0]
100
101 13-th iteration. obj_master = -3062.5.
102
103 Reduced cost of 1-th sub model is 73.5.
104 objCoef = -3980.0.
105 best reduced cost is 73.5
106 vec_pi = [-581.0 -1142.0 -381.0 -641.0 -690.0]
107 vec_kappa = [397.0]

```

```

108
109 14-th iteration. obj_master = -3038.0.
110
111 Reduced cost of 1-th sub model is 22.0.
112 objCoef = -3468.0.
113 best reduced cost is 22.0
114 vec_pi = [-581.0 -1142.0 -381.0 -1028.0 -1077.0]
115 vec_kappa = [1171.0]
116
117 15-th iteration. obj_master = -3038.0.
118
119 Reduced cost of 1-th sub model is 530.0.
120 objCoef = -3671.0.
121 best reduced cost is 530.0
122 vec_pi = [-581.0 -1142.0 -381.0 -663.0 -712.0]
123 vec_kappa = [441.0]
124
125 16-th iteration. obj_master = -3038.0.
126
127 Reduced cost of 1-th sub model is 44.0.
128 objCoef = -3325.0.
129 best reduced cost is 44.0
130 vec_pi = [-581.0 -1120.0 -403.0 -641.0 -690.0]
131 vec_kappa = [397.0]
132
133 17-th iteration. obj_master = -3038.0.
134
135 Reduced cost of 1-th sub model is 0.0.
136 best reduced cost is 0.0
137 #####
138 Optimization Done After 17 Iterations.
139 obj_master = -3038.0
140 #####
141 ##### 4/4, Print Result #####
142 lambda_10 = 0.3333333333333337, sub = 1,
143 extreme point = Any[1.0, 0.0, 0.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, -0.0, 0.0, 1.0].
144 lambda_12 = 0.3333333333333333, sub = 1,
145 extreme point = Any[1.0, -0.0, -0.0, 1.0, 1.0, 1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0].
146 lambda_14 = 0.3333333333333333, sub = 1,
147 extreme point = Any[0.0, 1.0, 1.0, 1.0, 0.0, 1.0, -0.0, 1.0, 1.0, -0.0, 1.0, 1.0].
148 x_{1,1} = 0.6666666666666667
149 x_{1,2} = 0.3333333333333333
150 x_{1,3} = 0.3333333333333333
151 x_{1,4} = 1.0
152 x_{1,5} = 0.6666666666666667
153 x_{1,6} = 1.0
154 x_{1,7} = 0.3333333333333337
155 x_{1,8} = 0.6666666666666667
156 x_{1,9} = 0.6666666666666667
157 x_{1,11} = 0.3333333333333333
158 x_{1,12} = 0.6666666666666667
159 Elapsed time is 0.7346489429473877 seconds.
160 ##### End #####

```

5.2 Task A.3 Detailed Result

```

1 ##### 1/4, Set vecModelSub #####

```

```

2 12-th variable in 1-th sub-problem is branched to 1.
3 6-th variable in 1-th sub-problem is branched to 1.
4 ##### 2/4, Set modelMas #####
5 ##### 3/4, Begin Optim #####
6 ##### Begin Iteration #####
7 vec_pi = [-1.0e6 0.0 0.0 0.0 0.0]
8 vec_kappa = [0.0]
9
10 1-th iteration. obj_master = -1.0e6.
11
12 Reduced cost of 1-th sub model is 1.997714e6.
13 objCoef = -2286.0.
14 best reduced cost is 1.997714e6
15 vec_pi = [-1143.0 -998857.0 0.0 0.0 0.0]
16 vec_kappa = [0.0]
17
18 2-th iteration. obj_master = -1.0e6.
19
20 Reduced cost of 1-th sub model is 1.996818e6.
21 objCoef = -3182.0.
22 best reduced cost is 1.996818e6
23 vec_pi = [-1143.0 -448.0 -998409.0 0.0 0.0]
24 vec_kappa = [0.0]
25
26 3-th iteration. obj_master = -1.0e6.
27
28 Reduced cost of 1-th sub model is 1.996061e6.
29 objCoef = -3491.0.
30 best reduced cost is 1.996061e6
31 vec_pi = [-1143.0 -448.0 -378.5 -9.98031e5 0.0]
32 vec_kappa = [0.0]
33
34 4-th iteration. obj_master = -1.0e6.
35
36 Reduced cost of 1-th sub model is 1.9953635e6.
37 objCoef = -3362.0.
38 best reduced cost is 1.9953635e6
39 vec_pi = [-1143.0 -448.0 -378.5 -348.75 -9.97682e5]
40 vec_kappa = [0.0]
41
42 5-th iteration. obj_master = -1.0e6.
43
44 Reduced cost of 1-th sub model is 1.99482525e6.
45 objCoef = -3173.0.
46 best reduced cost is 1.99482525e6
47 vec_pi = [5.3081e5 -5.32401e5 -2.66355e5 -3.99314e5 -332740.0]
48 vec_kappa = [0.0]
49
50 6-th iteration. obj_master = -1.0e6.
51
52 Reduced cost of 1-th sub model is 2.3920628000000003e6.
53 objCoef = -4078.0.
54 best reduced cost is 2.3920628000000003e6
55 vec_pi = [1.32816e6 -1.32976e6 -6.65032e5 1.99275e6 -2.32613e6]
56 vec_kappa = [0.0]

```



```

57
58 7-th iteration. obj_master = -1.0e6.
59
60 Reduced cost of 1-th sub model is 8.63779833333332e6.
61 objCoef = -4029.0.
62 best reduced cost is 8.63779833333332e6
63 vec_pi = [-242.5 -448.0 -378.5 -312.0 -287.5]
64 vec_kappa = [-1801.0]
65
66 8-th iteration. obj_master = -3469.5.
67
68 Reduced cost of 1-th sub model is 235.0.
69 objCoef = -2992.0.
70 best reduced cost is 235.0
71 vec_pi = [-425.278 -604.667 -456.833 -416.444 -391.944]
72 vec_kappa = [-1122.11]
73
74 9-th iteration. obj_master = -3417.277777777774.
75
76 Reduced cost of 1-th sub model is 223.5555555555543.
77 objCoef = -3619.0.
78 best reduced cost is 223.5555555555543
79 vec_pi = [-313.5 -604.667 -456.833 -304.667 -280.167]
80 vec_kappa = [-1345.67]
81
82 10-th iteration. obj_master = -3305.5.
83
84 Reduced cost of 1-th sub model is 215.8333333333326.
85 objCoef = -3871.0.
86 best reduced cost is 215.8333333333326
87 vec_pi = [-313.5 -496.75 -402.875 -196.75 -334.125]
88 vec_kappa = [-1561.5]
89
90 11-th iteration. obj_master = -3305.5.
91
92 Reduced cost of 1-th sub model is 88.5.
93 objCoef = -3517.0.
94 best reduced cost is 88.5
95 vec_pi = [-313.5 -452.5 -469.25 -241.0 -356.25]
96 vec_kappa = [-1473.0]
97
98 12-th iteration. obj_master = -3305.5.
99
100 Reduced cost of 1-th sub model is 88.75.
101 objCoef = -3686.0.
102 best reduced cost is 88.75
103 vec_pi = [-313.5 -453.0 -380.5 -241.5 -268.0]
104 vec_kappa = [-1649.0]
105
106 13-th iteration. obj_master = -3305.5.
107
108 Reduced cost of 1-th sub model is 44.0.
109 objCoef = -3307.0.
110 best reduced cost is 44.0
111 vec_pi = [-313.5 -497.0 -380.5 -285.5 -312.0]

```

```

112 vec_kappa = [-1517.0]
113
114 14-th iteration. obj_master = -3305.5.
115
116 Reduced cost of 1-th sub model is 22.5.
117 objCoef = -3283.0.
118 best reduced cost is 22.5
119 vec_pi = [-336.0 -497.0 -403.0 -263.0 -312.0]
120 vec_kappa = [-1472.0]
121
122 15-th iteration. obj_master = -3283.0.
123
124 Reduced cost of 1-th sub model is 0.0.
125 best reduced cost is 0.0
126 #####
127 Optimization Done After 15 Iterations.
128 obj_master = -3283.0
129 #####
130 #### 4/4, Print Result #####
131 lambda_15 = 1.0, sub = 1,
132 extreme point = Any[-0.0, 0.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 0.0, 0.0, 0.0, 1.0].
133 x_{1,3} = 1.0
134 x_{1,4} = 1.0
135 x_{1,5} = 1.0
136 x_{1,6} = 1.0
137 x_{1,7} = 1.0
138 x_{1,8} = 1.0
139 x_{1,12} = 1.0
140 ##### Elapsed time
141 ##### End #####

```

5.3 Task A.4 Detailed Result

```

1 ##### Begin Iteration #####
2 vec_pi = [-1.0e6 0.0 0.0 0.0 0.0]
3 vec_kappa = [0.0 0.0]
4
5 1-th iteration. obj_master = -1.0e6.
6
7 Reduced cost of 1-th sub model is 998972.0.
8 objCoef = -1028.0.
9 Reduced cost of 2-th sub model is 998742.0.
10 objCoef = -1258.0.
11 best reduced cost is 998972.0
12 vec_pi = [-1028.0 -998972.0 0.0 0.0 0.0]
13 vec_kappa = [0.0 0.0]
14
15 2-th iteration. obj_master = -1.0e6.
16
17 Reduced cost of 1-th sub model is 998519.0.
18 objCoef = -1481.0.
19 Reduced cost of 2-th sub model is 998299.0.
20 objCoef = -1701.0.
21 best reduced cost is 998519.0
22 vec_pi = [-1028.0 -453.0 -998519.0 0.0 0.0]
23 vec_kappa = [0.0 0.0]
24

```

```

25 3-th iteration. obj_master = -1.0e6.
26
27 Reduced cost of 1-th sub model is 998160.0.
28 objCoef = -1387.0.
29 Reduced cost of 2-th sub model is 997896.0.
30 objCoef = -2104.0.
31 best reduced cost is 998160.0
32 vec_pi = [-764.0 -717.0 -623.0 -997896.0 0.0]
33 vec_kappa = [0.0 0.0]
34
35 4-th iteration. obj_master = -1.0e6.
36
37 Reduced cost of 1-th sub model is 997677.0.
38 objCoef = -1606.0.
39 Reduced cost of 2-th sub model is 997124.0.
40 objCoef = -2159.0.
41 best reduced cost is 997677.0
42 vec_pi = [1.99459e6 -998394.0 -998300.0 -219.0 0.0]
43 vec_kappa = [-997677.0 0.0]
44
45 5-th iteration. obj_master = -1.0e6.
46
47 Reduced cost of 1-th sub model is 2.992823e6.
48 objCoef = -1548.0.
49 Reduced cost of 2-th sub model is 1.994881e6.
50 objCoef = -1813.0.
51 best reduced cost is 2.992823e6
52 vec_pi = [-1204.0 -497.0 -403.0 -219.0 -997897.0]
53 vec_kappa = [220.0 0.0]
54
55 6-th iteration. obj_master = -1.0e6.
56
57 Reduced cost of 1-th sub model is 997629.0.
58 objCoef = -1655.0.
59 Reduced cost of 2-th sub model is 997443.0.
60 objCoef = -1658.0.
61 best reduced cost is 997629.0
62 vec_pi = [4.97518e5 -4.99219e5 4.97451e5 -996794.0 -4.99176e5]
63 vec_kappa = [220.0 0.0]
64
65 7-th iteration. obj_master = -1.0e6.
66
67 Reduced cost of 1-th sub model is 1.4945265e6.
68 objCoef = -1223.0.
69 Reduced cost of 2-th sub model is 1.4941045e6.
70 objCoef = -1908.0.
71 best reduced cost is 1.4945265e6
72 vec_pi = [-444.8 -1147.4 -511.8 -760.6 -573.6]
73 vec_kappa = [111.2 0.0]
74
75 8-th iteration. obj_master = -3327.0.
76
77 Reduced cost of 1-th sub model is 541.5999999999999.
78 objCoef = -1767.0.
79 Reduced cost of 2-th sub model is 108.79999999999995.

```

```

80 objCoef = -2311.0.
81 best reduced cost is 541.5999999999999
82 vec_pi = [-770.0 -931.0 -403.0 -741.0 -790.0]
83 vec_kappa = [308.0 0.0]
84
85 9-th iteration. obj_master = -3327.0.
86
87 Reduced cost of 1-th sub model is 478.0.
88 objCoef = -1515.0.
89 Reduced cost of 2-th sub model is 145.0.
90 objCoef = -2156.0.
91 best reduced cost is 478.0
92 vec_pi = [-292.0 -453.0 -639.0 -499.0 -548.0]
93 vec_kappa = [-176.0 -720.0]
94
95 10-th iteration. obj_master = -3327.0.
96
97 Reduced cost of 1-th sub model is 280.0.
98 objCoef = -1582.0.
99 Reduced cost of 2-th sub model is 720.0.
100 objCoef = -0.0.
101 best reduced cost is 720.0
102 vec_pi = [-336.0 -1365.0 -403.0 -263.0 -1180.0]
103 vec_kappa = [264.0 0.0]
104
105 11-th iteration. obj_master = -3283.0.
106
107 Reduced cost of 1-th sub model is 868.0.
108 objCoef = -1816.0.
109 Reduced cost of 2-th sub model is 735.0.
110 objCoef = -1810.0.
111 best reduced cost is 868.0
112 vec_pi = [-532.0 -973.0 -599.0 -739.0 -788.0]
113 vec_kappa = [544.0 0.0]
114
115 12-th iteration. obj_master = -3087.0.
116
117 Reduced cost of 1-th sub model is 0.0.
118 Reduced cost of 2-th sub model is 147.0.
119 objCoef = -2213.0.
120 best reduced cost is 147.0
121 vec_pi = [-581.0 -875.0 -648.0 -641.0 -690.0]
122 vec_kappa = [397.0 0.0]
123
124 13-th iteration. obj_master = -3038.0.
125
126 Reduced cost of 1-th sub model is 0.0.
127 Reduced cost of 2-th sub model is 0.0.
128 best reduced cost is 0.0
129 #####
130 Optimization Done After 13 Iterations.
131 obj_master = -3038.0
132 #####
133 ##### 4/4, Print Result #####
134 lambda_7 = 0.33333333333333337, sub = 2,

```

```

135 extreme point = Any[1.0, 1.0, 1.0, 0.0, 0.0, 1.0].
136 lambda_16 = 0.3333333333333333, sub = 1,
137 extreme point = Any[0.0, 1.0, 1.0, 1.0, 0.0, 1.0].
138 lambda_18 = 0.6666666666666666, sub = 1,
139 extreme point = Any[1.0, -0.0, -0.0, 1.0, 1.0, 1.0].
140 lambda_21 = 0.3333333333333326, sub = 2,
141 extreme point = Any[0.0, 0.0, 0.0, 0.0, 0.0, 0.0].
142 lambda_24 = 0.3333333333333337, sub = 2,
143 extreme point = Any[-0.0, 1.0, 1.0, -0.0, 1.0, 1.0].
144 x_{1,1} = 0.6666666666666666
145 x_{1,2} = 0.3333333333333333
146 x_{1,3} = 0.3333333333333333
147 x_{1,4} = 1.0
148 x_{1,5} = 0.6666666666666666
149 x_{1,6} = 1.0
150 x_{2,1} = 0.3333333333333337
151 x_{2,2} = 0.6666666666666667
152 x_{2,3} = 0.6666666666666667
153 x_{2,5} = 0.3333333333333337
154 x_{2,6} = 0.6666666666666667
155 Elapsed time is 0.3501169681549072 seconds.
156 ##### End #####

```