DTU42136, Large Scale Optimization using Decomposition

Assignment 2, Dantzig Wolfe Reformulation
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1 Part A

1.1 Task 1

Col Name	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	y_1	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	y_2	Vector B
Vector C	292	453	359	219	268	736	291	443	403	498	400	967	-
cons 2.1	1	0	0	0	0	0	1	0	0	0	0	0	1
cons 2.2	0	1	0	0	0	0	0	1	0	0	0	0	1
cons 2.3	0	0	1	0	0	0	0	0	1	0	0	0	1
cons 2.4	0	0	0	1	0	0	0	0	0	1	0	0	1
cons 2.5	0	0	0	0	1	0	0	0	0	0	1	0	1
cons 3.1	14	20	6	16	10	-43	0	0	0	0	0	0	0
cons 3.2	0	0	0	0	0	0	14	20	6	16	10	-43	0

Table 1. Matrix A, Vector B and Vector C for SSCFLP

2 Task A.2

One way of applying Dantzig-Wolfe reformulation to this problem is by keeping constraints (2) in the master and convexifying constraints (3)-(5).

Col Name	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	y_1	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	y_2	Vector B
Vector C	292	453	359	219	268	736	291	443	403	498	400	967	-
cons 2.1	1	0	0	0	0	0	1	0	0	0	0	0	1
cons 2.2	0	1	0	0	0	0	0	1	0	0	0	0	1
cons 2.3	0	0	1	0	0	0	0	0	1	0	0	0	1
cons 2.4	0	0	0	1	0	0	0	0	0	1	0	0	1
cons 2.5	0	0	0	0	1	0	0	0	0	0	1	0	1

Table 2. Constraint Matrix for the Master Problem

Col Name	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	y_1	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	y_2	Vector B
Vector C	292	453	359	219	268	736	291	443	403	498	400	967	-
cons 3.1	14	20	6	16	10	-43	0	0	0	0	0	0	0
cons 3.2	0	0	0	0	0	0	14	20	6	16	10	-43	0

Table 3. Constraint Matrix for the Sub Problem

3 Task A.3

Branch 12-th variable to 1 and 0. Find branch_1 is better. Then branch 6-th variable to 1 and 0. Find branch_1.1 is the final result.

Col Name	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	y_1	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	y_2	Vector B
Vector C	292	453	359	219	268	736	291	443	403	498	400	967	-
cons 5.1	0	0	0	0	0	0	0	0	0	0	0	-1	-1
cons 5.2	0	0	0	0	0	-1	0	0	0	0	0	0	-1

Table 4. Extra Constraint Matrix for the Sub Problem in Branch

4 Task A.4

Col Name	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	y_1	Vector B
Vector C	292	453	359	219	268	736	-
cons 3.1	14	20	6	16	10	-43	0

Table 5. Constraint Matrix for the Sub Problem 1

Col Name	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	y_2	Vector B
Vector C	291	443	403	498	400	967	-
cons 3.1	14	20	6	16	10	-43	0

Table 6. Constraint Matrix for the Sub Problem 2

4.1 Test using Generated Data Set and Conclusion

Data Set	Method	numSub	Obj	Time	numIteration
A	Gurobi	-	2834.02	0.0017	-
A	Gurobi-Relax	-	3283	0.0024	-
A	DW-Relax	1	3038	0.316	17
A	DW-Relax	2	3038	0.328	13
B	Gurobi	-	7956.88	0.100	-
B	Gurobi-Relax	-	8612.0	0.0023	-
B	DW-Relax	1	8583.33	6.138	409
B	DW-Relax	10	8583.33	1.425	44
C	Gurobi	-	16228	2.42	-
C	Gurobi-Relax	-	15301.81	0.0042	-
C	DW-Relax	1	15836.01	50.349	1383
C	DW-Relax	20	15836.01	1.371	71

Table 7. Table of Results from Optimizations using Different Data Sets and Methods

We can get three conclusions from the table:

- The objective value from Dantzig-Wolfe Decomposition is better than that solved by Gurobi relaxation.
- The advantage of splitting sub-problems is more obvious, when there are more sub-problems. There will be less iterations and elapsed time.
- It's possible for Dantzig-Wolfe Decomposition to beat Gurobi in MILP, if there are many sub-problems and good branch-and-cut algorithm.

4.2 Task 5.B

$$H = \sum_{i \in I} h_i \tag{1}$$

$$\begin{aligned}
\min \quad & z && && (2)\\
\text{s.t.} \quad & \alpha_{ij} + \alpha_{ji} + \beta_{ij} + \beta_{ji} \geq 1 && \forall i, j \in I, i < j && (3)\\
& y_i + h_i \leq z && \forall i \in I && (4)\\
& x_i + w_i \leq W && \forall i \in I && (5)\\
& x_i + w_i \leq x_j + W(1 - \alpha_{ij}) && \forall i, j \in I && (6)\\
& y_i + h_i \leq y_j + H(1 - \beta_{ij}) && \forall i, j \in I && (7)\\
& x_i, y_i \geq 0 && \forall i \in I && (8)\\
& \alpha_{ij}, \beta_{ij} \in \{0, 1\} && \forall i, j \in I && (9)
\end{aligned}$$

$$\begin{aligned}
\max \quad & -z && && (10)\\
\text{s.t.} \quad & -\alpha_{ij} - \alpha_{ji} - \beta_{ij} - \beta_{ji} \leq -1 && \forall i, j \in I, i < j && (11)\\
& x_i - x_j + W\alpha_{ij} \leq W - w_i && \forall i, j \in I && (12)\\
& y_i - y_j + H\beta_{ij} \leq H - h_i && \forall i, j \in I && (13)\\
& y_i - z \leq -h_i && \forall i \in I && (14)\\
& x_i \leq W - w_i && \forall i \in I && (15)\\
& x_i, y_i \geq 0 && \forall i \in I && (16)\\
& \alpha_{ij}, \beta_{ij} \in \{0, 1\} && \forall i, j \in I && (17)
\end{aligned}$$

$$\begin{array}{cccccccccccccccccccc}
12 & 13 & 21 & 23 & 31 & 32 & 12 & 13 & 21 & 23 & 31 & 32 & 1 & 2 & 3 & 1 & 2 & 3 & z & b \\
-1 & . & -1 & . & . & . & -1 & . & -1 & . & . & . & . & . & . & . & . & . & . & -1 \\
. & -1 & . & . & -1 & . & . & -1 & . & . & -1 & . & . & . & . & . & . & . & . & -1 \\
. & . & . & -1 & . & -1 & . & . & . & -1 & . & -1 & . & . & . & . & . & . & . & -1 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & 1 & . & . & 1 & -h_1 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & 1 & . & 1 & -h_2 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & 1 & 1 & -h_3 \\
. & . & . & . & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & W - w_1 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . & W - w_2 \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & 1 & . & . & . & . & W - w_3 \\
W & . & . & . & . & . & . & . & . & . & . & . & 1 & -1 & . & . & . & . & . & W - w_1 \\
. & W & . & . & . & . & . & . & . & . & . & . & 1 & . & -1 & . & . & . & . & W - w_1 \\
. & . & W & . & . & . & . & . & . & . & . & . & -1 & 1 & . & . & . & . & . & W - w_2 \\
. & . & . & W & . & . & . & . & . & . & . & . & . & 1 & -1 & . & . & . & . & W - w_2 \\
. & . & . & . & W & . & . & . & . & . & . & . & -1 & . & 1 & . & . & . & . & W - w_3 \\
. & . & . & . & . & W & . & . & . & . & . & . & . & -1 & 1 & . & . & . & . & W - w_3 \\
. & . & . & . & . & . & H & . & . & . & . & . & . & . & . & 1 & -1 & . & . & H - h_1 \\
. & . & . & . & . & . & . & H & . & . & . & . & . & . & . & 1 & . & -1 & . & H - h_1 \\
. & . & . & . & . & . & . & . & H & . & . & . & . & . & . & -1 & 1 & . & . & H - h_2 \\
. & . & . & . & . & . & . & . & . & H & . & . & . & . & . & . & 1 & -1 & . & H - h_2 \\
. & . & . & . & . & . & . & . & . & . & H & . & . & . & . & -1 & . & 1 & . & H - h_3 \\
. & . & . & . & . & . & . & . & . & . & . & H & . & . & . & . & -1 & 1 & . & H - h_3
\end{array} \tag{18}$$

$$\max \quad -z \tag{19}$$

$$\text{s.t.} \quad \begin{cases} -\alpha_{ij} - \alpha_{ji} - \beta_{ij} - \beta_{ji} \leq -1 & \forall i, j \in I, i < j \\ x_i \leq W - w_i & \forall i \in I \\ y_i - z \leq -h_i & \forall i \in I \end{cases} \tag{20}$$

$$\begin{cases} x_i - x_j + W\alpha_{ij} \leq W - w_i & \forall i, j \in I \\ y_i - y_j + H\beta_{ij} \leq H - h_i & \forall i, j \in I \end{cases} \tag{21}$$

$$x_i, y_i \geq 0 \qquad \forall i \in I \tag{22}$$

$$\alpha_{ij}, \beta_{ij} \in \{0, 1\} \qquad \forall i, j \in I \tag{23}$$

5 Appendix

5.1 Task A.2 Detailed Result

```
#### Begin Iteration ###############################################################
vec_pi = [-1.0e6 0.0 0.0 0.0 0.0]
vec_kappa = [0.0]
-------------------------------------------------------------------------------------
1-th iteration. obj_master = -1.0e6.
-------------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 1.997714e6.
objCoef = -2286.0.
best reduced cost is 1.997714e6
vec_pi = [-1143.0 -998857.0 0.0 0.0 0.0]
vec_kappa = [0.0]
-------------------------------------------------------------------------------------
2-th iteration. obj_master = -1.0e6.
-------------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 1.996818e6.
objCoef = -3182.0.
best reduced cost is 1.996818e6
vec_pi = [-1143.0 -448.0 -998409.0 0.0 0.0]
vec_kappa = [0.0]
-------------------------------------------------------------------------------------
3-th iteration. obj_master = -1.0e6.
-------------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 1.996061e6.
objCoef = -3491.0.
best reduced cost is 1.996061e6
vec_pi = [-1143.0 -448.0 -378.5 -9.98031e5 0.0]
vec_kappa = [0.0]
-------------------------------------------------------------------------------------
4-th iteration. obj_master = -1.0e6.
-------------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 1.9953635e6.
objCoef = -3362.0.
best reduced cost is 1.9953635e6
vec_pi = [-1143.0 -448.0 -378.5 -348.75 -9.97682e5]
vec_kappa = [0.0]
-------------------------------------------------------------------------------------
5-th iteration. obj_master = -1.0e6.
-------------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 1.99482525e6.
objCoef = -3173.0.
best reduced cost is 1.99482525e6
vec_pi = [5.3081e5 -5.32401e5 -2.66355e5 -3.99314e5 -332740.0]
vec_kappa = [0.0]
-------------------------------------------------------------------------------------
6-th iteration. obj_master = -1.0e6.
-------------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 2.39206280000000003e6.
objCoef = -4078.0.
best reduced cost is 2.3920628000000003e6
vec_pi = [1.32816e6 -1.32976e6 -6.65032e5 1.99275e6 -2.32613e6]
vec_kappa = [0.0]
-------------------------------------------------------------------------------------
```

```
7-th iteration. obj_master = -1.0e6.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 8.6377983333333332e6.
objCoef = -4029.0.
best reduced cost is 8.6377983333333332e6
vec_pi = [-242.5 -448.0 -378.5 -312.0 -287.5]
vec_kappa = [-1801.0]
--------------------------------------------------------------------------------
8-th iteration. obj_master = -3469.5.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 1801.0.
objCoef = 0.0.
best reduced cost is 1801.0
vec_pi = [-727.385 -863.615 -586.308 -589.077 -564.577]
vec_kappa = [0.0]
--------------------------------------------------------------------------------
9-th iteration. obj_master = -3330.9615384615386.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 439.34615384615427.
objCoef = -3619.0.
best reduced cost is 439.34615384615427
vec_pi = [-397.875 -1193.13 -37.125 -808.75 -784.25]
vec_kappa = [0.0]
--------------------------------------------------------------------------------
10-th iteration. obj_master = -3221.125.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 761.25.
objCoef = -3218.0.
best reduced cost is 761.25
vec_pi = [-397.875 -812.5 -417.75 -808.75 -784.25]
vec_kappa = [0.0]
--------------------------------------------------------------------------------
11-th iteration. obj_master = -3221.125.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 475.875.
objCoef = -1515.0.
best reduced cost is 475.875
vec_pi = [-556.5 -1034.5 -513.0 -491.5 -467.0]
vec_kappa = [0.0]
--------------------------------------------------------------------------------
12-th iteration. obj_master = -3062.5.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 272.0.
objCoef = -1767.0.
best reduced cost is 272.0
vec_pi = [-556.5 -1129.75 -417.75 -763.5 -739.0]
vec_kappa = [544.0]
--------------------------------------------------------------------------------
13-th iteration. obj_master = -3062.5.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 73.5.
objCoef = -3980.0.
best reduced cost is 73.5
vec_pi = [-581.0 -1142.0 -381.0 -641.0 -690.0]
vec_kappa = [397.0]
```

```
--------------------------------------------------------------------------------
14-th iteration. obj_master = -3038.0.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 22.0.
objCoef = -3468.0.
best reduced cost is 22.0
vec_pi = [-581.0 -1142.0 -381.0 -1028.0 -1077.0]
vec_kappa = [1171.0]
--------------------------------------------------------------------------------
15-th iteration. obj_master = -3038.0.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 530.0.
objCoef = -3671.0.
best reduced cost is 530.0
vec_pi = [-581.0 -1142.0 -381.0 -663.0 -712.0]
vec_kappa = [441.0]
--------------------------------------------------------------------------------
16-th iteration. obj_master = -3038.0.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 44.0.
objCoef = -3325.0.
best reduced cost is 44.0
vec_pi = [-581.0 -1120.0 -403.0 -641.0 -690.0]
vec_kappa = [397.0]
--------------------------------------------------------------------------------
17-th iteration. obj_master = -3038.0.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 0.0.
best reduced cost is 0.0
################################################################################
Optimization Done After 17 Iterations.
obj_master = -3038.0
################################################################################
#### 4/4,  Print Result ######################################################
lambda_10 = 0.33333333333333337, sub = 1,
extreme point = Any[1.0, 0.0, 0.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, -0.0, 0.0, 1.0].
lambda_12 = 0.3333333333333333, sub = 1,
extreme point = Any[1.0, -0.0, -0.0, 1.0, 1.0, 1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0].
lambda_14 = 0.3333333333333333, sub = 1,
extreme point = Any[0.0, 1.0, 1.0, 1.0, 0.0, 1.0, -0.0, 1.0, 1.0, -0.0, 1.0, 1.0].
x_{1,1} = 0.6666666666666667
x_{1,2} = 0.3333333333333333
x_{1,3} = 0.3333333333333333
x_{1,4} = 1.0
x_{1,5} = 0.6666666666666667
x_{1,6} = 1.0
x_{1,7} = 0.33333333333333337
x_{1,8} = 0.6666666666666667
x_{1,9} = 0.6666666666666667
x_{1,11} = 0.3333333333333333
x_{1,12} = 0.6666666666666667
Elapsed time is 0.7346489429473877 seconds.
#### End #####################################################################
```

5.2 Task A.3 Detailed Result

```
#### 1/4,  Set vecModelSub ###################################################
```

```
12-th variable in 1-th sub-problem is branched to 1.
6-th variable in 1-th sub-problem is branched to 1.
#### 2/4,  Set modelMas ############################################################
#### 3/4,  Begin Optim ############################################################
#### Begin Iteration ##############################################################
vec_pi = [-1.0e6 0.0 0.0 0.0 0.0]
vec_kappa = [0.0]
--------------------------------------------------------------------------------
1-th iteration. obj_master = -1.0e6.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 1.997714e6.
objCoef = -2286.0.
best reduced cost is 1.997714e6
vec_pi = [-1143.0 -998857.0 0.0 0.0 0.0]
vec_kappa = [0.0]
--------------------------------------------------------------------------------
2-th iteration. obj_master = -1.0e6.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 1.996818e6.
objCoef = -3182.0.
best reduced cost is 1.996818e6
vec_pi = [-1143.0 -448.0 -998409.0 0.0 0.0]
vec_kappa = [0.0]
--------------------------------------------------------------------------------
3-th iteration. obj_master = -1.0e6.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 1.996061e6.
objCoef = -3491.0.
best reduced cost is 1.996061e6
vec_pi = [-1143.0 -448.0 -378.5 -9.98031e5 0.0]
vec_kappa = [0.0]
--------------------------------------------------------------------------------
4-th iteration. obj_master = -1.0e6.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 1.9953635e6.
objCoef = -3362.0.
best reduced cost is 1.9953635e6
vec_pi = [-1143.0 -448.0 -378.5 -348.75 -9.97682e5]
vec_kappa = [0.0]
--------------------------------------------------------------------------------
5-th iteration. obj_master = -1.0e6.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 1.99482525e6.
objCoef = -3173.0.
best reduced cost is 1.99482525e6
vec_pi = [5.3081e5 -5.32401e5 -2.66355e5 -3.99314e5 -332740.0]
vec_kappa = [0.0]
--------------------------------------------------------------------------------
6-th iteration. obj_master = -1.0e6.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 2.3920628000000003e6.
objCoef = -4078.0.
best reduced cost is 2.3920628000000003e6
vec_pi = [1.32816e6 -1.32976e6 -6.65032e5 1.99275e6 -2.32613e6]
vec_kappa = [0.0]
```

```
--------------------------------------------------------------------------------
7-th iteration. obj_master = -1.0e6.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 8.637798333333332e6.
objCoef = -4029.0.
best reduced cost is 8.637798333333332e6
vec_pi = [-242.5 -448.0 -378.5 -312.0 -287.5]
vec_kappa = [-1801.0]
--------------------------------------------------------------------------------
8-th iteration. obj_master = -3469.5.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 235.0.
objCoef = -2992.0.
best reduced cost is 235.0
vec_pi = [-425.278 -604.667 -456.833 -416.444 -391.944]
vec_kappa = [-1122.11]
--------------------------------------------------------------------------------
9-th iteration. obj_master = -3417.2777777777774.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 223.55555555555543.
objCoef = -3619.0.
best reduced cost is 223.55555555555543
vec_pi = [-313.5 -604.667 -456.833 -304.667 -280.167]
vec_kappa = [-1345.67]
--------------------------------------------------------------------------------
10-th iteration. obj_master = -3305.5.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 215.83333333333326.
objCoef = -3871.0.
best reduced cost is 215.83333333333326
vec_pi = [-313.5 -496.75 -402.875 -196.75 -334.125]
vec_kappa = [-1561.5]
--------------------------------------------------------------------------------
11-th iteration. obj_master = -3305.5.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 88.5.
objCoef = -3517.0.
best reduced cost is 88.5
vec_pi = [-313.5 -452.5 -469.25 -241.0 -356.25]
vec_kappa = [-1473.0]
--------------------------------------------------------------------------------
12-th iteration. obj_master = -3305.5.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 88.75.
objCoef = -3686.0.
best reduced cost is 88.75
vec_pi = [-313.5 -453.0 -380.5 -241.5 -268.0]
vec_kappa = [-1649.0]
--------------------------------------------------------------------------------
13-th iteration. obj_master = -3305.5.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 44.0.
objCoef = -3307.0.
best reduced cost is 44.0
vec_pi = [-313.5 -497.0 -380.5 -285.5 -312.0]
```

```
vec_kappa = [-1517.0]
--------------------------------------------------------------------------------
14-th iteration. obj_master = -3305.5.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 22.5.
objCoef = -3283.0.
best reduced cost is 22.5
vec_pi = [-336.0 -497.0 -403.0 -263.0 -312.0]
vec_kappa = [-1472.0]
--------------------------------------------------------------------------------
15-th iteration. obj_master = -3283.0.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 0.0.
best reduced cost is 0.0
################################################################################
Optimization Done After 15 Iterations.
obj_master = -3283.0
################################################################################
#### 4/4,  Print Result ######################################################
lambda_15 = 1.0, sub = 1,
extreme point = Any[-0.0, 0.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 0.0, 0.0, 0.0, 1.0].
x_{1,3} = 1.0
x_{1,4} = 1.0
x_{1,5} = 1.0
x_{1,6} = 1.0
x_{1,7} = 1.0
x_{1,8} = 1.0
x_{1,12} = 1.0
################################################################################Elapsed time
#### End #####################################################################
```

5.3 Task A.4 Detailed Result

```
#### Begin Iteration ##########################################################
vec_pi = [-1.0e6 0.0 0.0 0.0 0.0]
vec_kappa = [0.0 0.0]
--------------------------------------------------------------------------------
1-th iteration. obj_master = -1.0e6.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 998972.0.
objCoef = -1028.0.
Reduced cost of 2-th sub model is 998742.0.
objCoef = -1258.0.
best reduced cost is 998972.0
vec_pi = [-1028.0 -998972.0 0.0 0.0 0.0]
vec_kappa = [0.0 0.0]
--------------------------------------------------------------------------------
2-th iteration. obj_master = -1.0e6.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 998519.0.
objCoef = -1481.0.
Reduced cost of 2-th sub model is 998299.0.
objCoef = -1701.0.
best reduced cost is 998519.0
vec_pi = [-1028.0 -453.0 -998519.0 0.0 0.0]
vec_kappa = [0.0 0.0]
--------------------------------------------------------------------------------
```

```
3-th iteration. obj_master = -1.0e6.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 998160.0.
objCoef = -1387.0.
Reduced cost of 2-th sub model is 997896.0.
objCoef = -2104.0.
best reduced cost is 998160.0
vec_pi = [-764.0 -717.0 -623.0 -997896.0 0.0]
vec_kappa = [0.0 0.0]
--------------------------------------------------------------------------------
4-th iteration. obj_master = -1.0e6.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 997677.0.
objCoef = -1606.0.
Reduced cost of 2-th sub model is 997124.0.
objCoef = -2159.0.
best reduced cost is 997677.0
vec_pi = [1.99459e6 -998394.0 -998300.0 -219.0 0.0]
vec_kappa = [-997677.0 0.0]
--------------------------------------------------------------------------------
5-th iteration. obj_master = -1.0e6.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 2.992823e6.
objCoef = -1548.0.
Reduced cost of 2-th sub model is 1.994881e6.
objCoef = -1813.0.
best reduced cost is 2.992823e6
vec_pi = [-1204.0 -497.0 -403.0 -219.0 -997897.0]
vec_kappa = [220.0 0.0]
--------------------------------------------------------------------------------
6-th iteration. obj_master = -1.0e6.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 997629.0.
objCoef = -1655.0.
Reduced cost of 2-th sub model is 997443.0.
objCoef = -1658.0.
best reduced cost is 997629.0
vec_pi = [4.97518e5 -4.99219e5 4.97451e5 -996794.0 -4.99176e5]
vec_kappa = [220.0 0.0]
--------------------------------------------------------------------------------
7-th iteration. obj_master = -1.0e6.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 1.4945265e6.
objCoef = -1223.0.
Reduced cost of 2-th sub model is 1.4941045e6.
objCoef = -1908.0.
best reduced cost is 1.4945265e6
vec_pi = [-444.8 -1147.4 -511.8 -760.6 -573.6]
vec_kappa = [111.2 0.0]
--------------------------------------------------------------------------------
8-th iteration. obj_master = -3327.0.
--------------------------------------------------------------------------------
Reduced cost of 1-th sub model is 541.5999999999999.
objCoef = -1767.0.
Reduced cost of 2-th sub model is 108.79999999999995.
```

```
objCoef = -2311.0.
best reduced cost is 541.5999999999999
vec_pi = [-770.0 -931.0 -403.0 -741.0 -790.0]
vec_kappa = [308.0 0.0]
-----------------------------------------------------------------------------
9-th iteration. obj_master = -3327.0.
-----------------------------------------------------------------------------
Reduced cost of 1-th sub model is 478.0.
objCoef = -1515.0.
Reduced cost of 2-th sub model is 145.0.
objCoef = -2156.0.
best reduced cost is 478.0
vec_pi = [-292.0 -453.0 -639.0 -499.0 -548.0]
vec_kappa = [-176.0 -720.0]
-----------------------------------------------------------------------------
10-th iteration. obj_master = -3327.0.
-----------------------------------------------------------------------------
Reduced cost of 1-th sub model is 280.0.
objCoef = -1582.0.
Reduced cost of 2-th sub model is 720.0.
objCoef = -0.0.
best reduced cost is 720.0
vec_pi = [-336.0 -1365.0 -403.0 -263.0 -1180.0]
vec_kappa = [264.0 0.0]
-----------------------------------------------------------------------------
11-th iteration. obj_master = -3283.0.
-----------------------------------------------------------------------------
Reduced cost of 1-th sub model is 868.0.
objCoef = -1816.0.
Reduced cost of 2-th sub model is 735.0.
objCoef = -1810.0.
best reduced cost is 868.0
vec_pi = [-532.0 -973.0 -599.0 -739.0 -788.0]
vec_kappa = [544.0 0.0]
-----------------------------------------------------------------------------
12-th iteration. obj_master = -3087.0.
-----------------------------------------------------------------------------
Reduced cost of 1-th sub model is 0.0.
Reduced cost of 2-th sub model is 147.0.
objCoef = -2213.0.
best reduced cost is 147.0
vec_pi = [-581.0 -875.0 -648.0 -641.0 -690.0]
vec_kappa = [397.0 0.0]
-----------------------------------------------------------------------------
13-th iteration. obj_master = -3038.0.
-----------------------------------------------------------------------------
Reduced cost of 1-th sub model is 0.0.
Reduced cost of 2-th sub model is 0.0.
best reduced cost is 0.0
#############################################################################
Optimization Done After 13 Iterations.
obj_master = -3038.0
#############################################################################
#### 4/4,  Print Result ###################################################
lambda_7 = 0.33333333333333337, sub = 2,
```

```
extreme point = Any[1.0, 1.0, 1.0, 0.0, 0.0, 1.0].
lambda_16 = 0.3333333333333333, sub = 1,
extreme point = Any[0.0, 1.0, 1.0, 1.0, 0.0, 1.0].
lambda_18 = 0.6666666666666666, sub = 1,
extreme point = Any[1.0, −0.0, −0.0, 1.0, 1.0, 1.0].
lambda_21 = 0.33333333333333326, sub = 2,
extreme point = Any[0.0, 0.0, 0.0, 0.0, 0.0, 0.0].
lambda_24 = 0.33333333333333337, sub = 2,
extreme point = Any[−0.0, 1.0, 1.0, −0.0, 1.0, 1.0].
x_{1,1} = 0.6666666666666666
x_{1,2} = 0.3333333333333333
x_{1,3} = 0.3333333333333333
x_{1,4} = 1.0
x_{1,5} = 0.6666666666666666
x_{1,6} = 1.0
x_{2,1} = 0.33333333333333337
x_{2,2} = 0.6666666666666667
x_{2,3} = 0.6666666666666667
x_{2,5} = 0.33333333333333337
x_{2,6} = 0.6666666666666667
Elapsed time is 0.3501169681549072 seconds.
#### End ##########################################################################
```