1 Problem 2 - Monte Carlo calculation of an arbitrarily drawn polygon

I was amused by the shape of this polygon. I generated points in the interval $x = (0, 2) and y = (0, 0.5)$ for the simple reason that its area is unity. Using the `shapely` package, it was simple to check whether a point were inside the polygon or not. I plotted the results, with the corresponding area in figure 1. The area (0.166) also seems appropriate upon visual inspection.

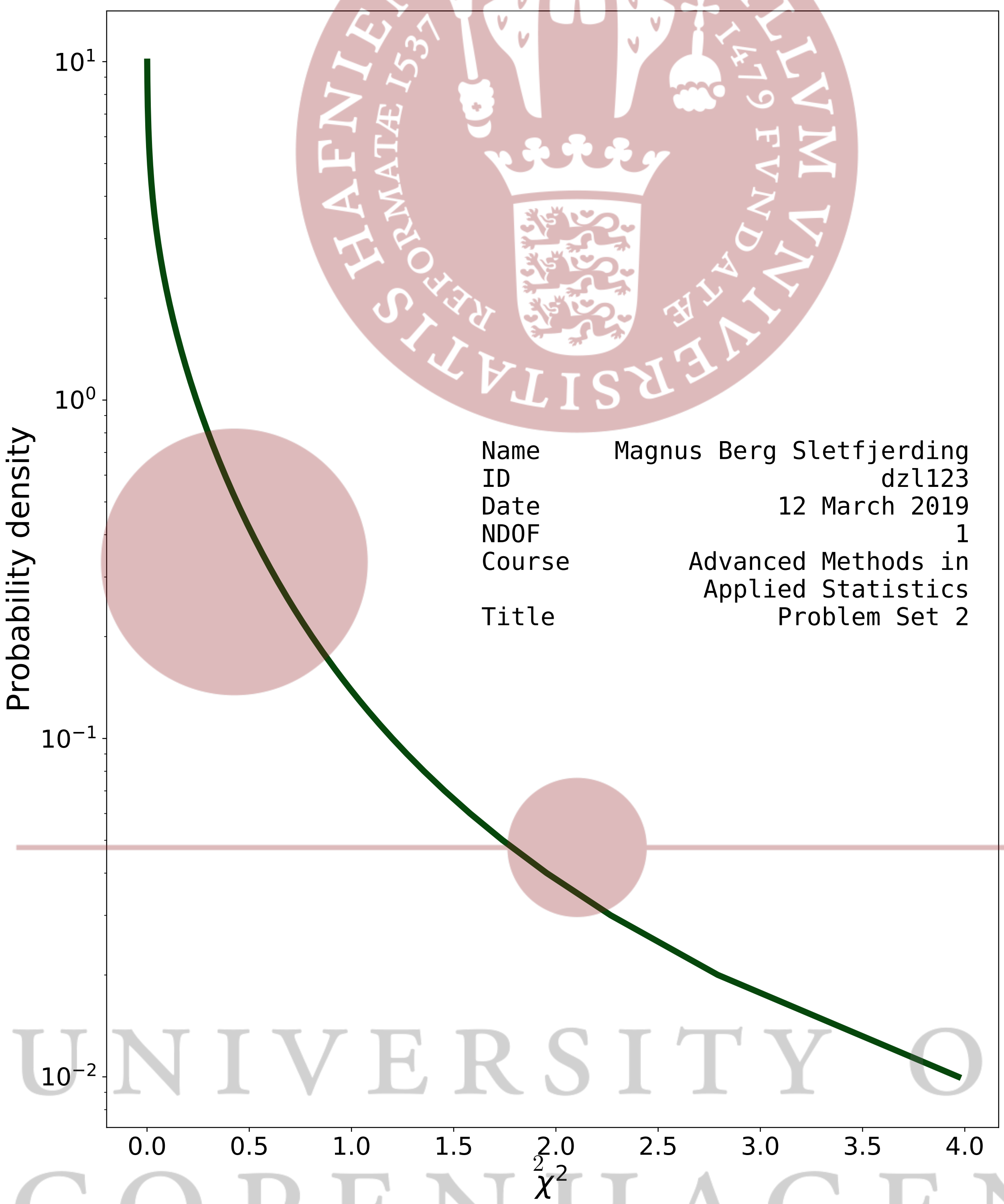
2 Problem 3 - Conditional probabilities for Slightly Evil Inc.

This section was by far the most tricky to get right.

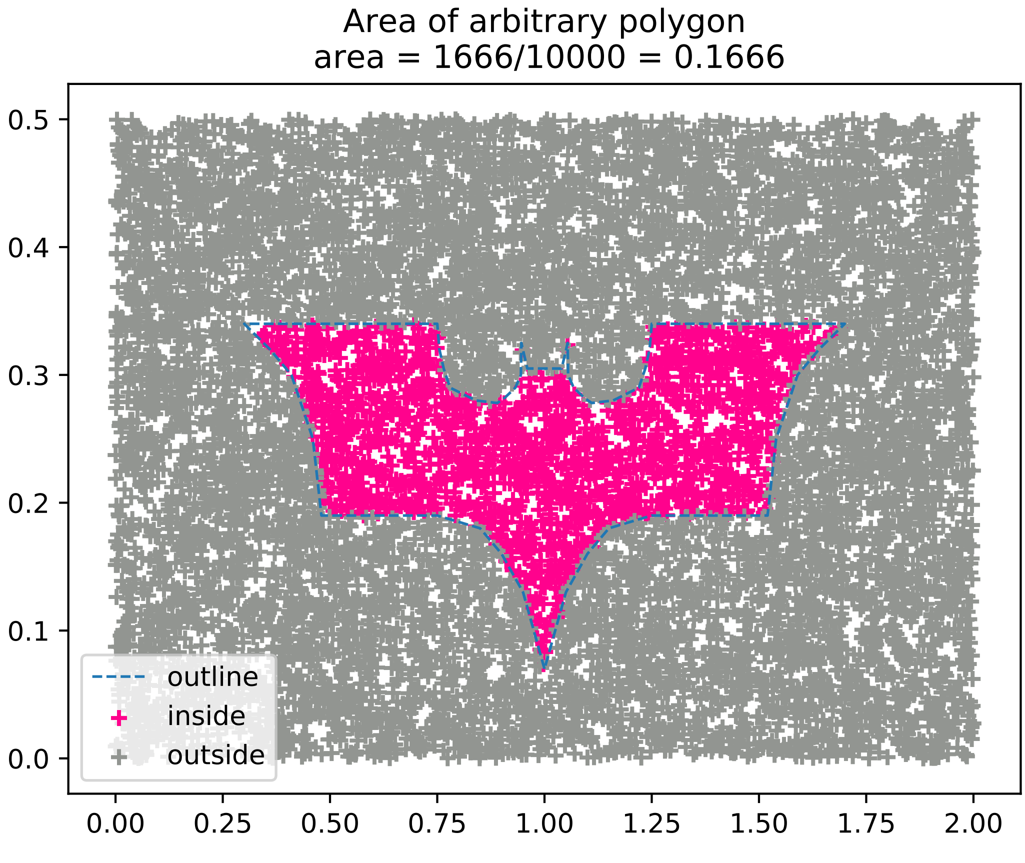
2.1 Answers

My answers are contained in the output from my code:

```
  Problem 3a:
Current aggregate defective rate: 0.03275
If defective, a PM is 18.32% likely to come from the facility A2
If defective, a PM is 23.66% likely to come from the facility A5
  Problem 3b:
  facility  defective  new_defective
0       A1      0.020       0.022143
1       A2      0.040       0.051667
2       A3      0.100       0.155000
3       A4      0.035       0.038750
4       A5      0.031       0.031000
  Problem 3c:
   facility  defective  new_defective
0        A1      0.020       0.022037
1        A2      0.040       0.059500
2        A3      0.100       0.119000
3        A4      0.035       0.074375
4        A5      0.022       0.023800
5        A6      0.092       0.180303
6        A7      0.120       0.313158
7        A8      0.070       0.070000
8        A9      0.110       0.180303
9       A10      0.020       0.297500
10      A11      0.070       0.396667
11      A12      0.060       0.270455
12      A13      0.099       0.396667
```



UNIVERSITY OF COPENHAGEN



Area of arbitrary polygon
area = 1666/10000 = 0.1666
0.5
0.4
0.3
0.2
0.1
0.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
outline
inside
outside

```
13        A14       0.082         0.743750
```

2.2 Explanation

2.2.1 Problem 3a

Finding the chance a defective pacemaker is from A2 facility calls for the use of Bayes' Theorem:

$$P(A2|D) = \frac{P(D|A2) * P(A2)}{P(D)}$$

Applying the already found values:

$$P(A2|D) = \frac{0.04 * 0.15}{0.03275} = 18.32\%$$

Using the same formula, for all facilities, we find that:

$$P(A5|D) = \frac{0.031 * 0.25}{0.03275} = 23.66\%$$

Which is higher than all the others. It makes intuitive sense, too, as the only other candidate, facility A4, still produces fewer defective devices than A5:

$$P(A4|D) = \frac{0.035 * 0.2}{0.03275} = 22.93\%$$

2.2.2 Problem 3b and 3c

Again, Bayes' Theorem shows itself handy.

In order to make all relative defective rates the same (20%) we need to adjust all but one of the defective rates up. (Analytically, it might make sense to increase the error rate for A5, as we know that it it the maximum.)

Doing so, I used the following formula to calculate the new defective rates from each factory:

$$P(D|A_i)_{new,j} = \frac{P(A_j)}{P(A_i)} P(D|A_i)_{old}$$

for all factories A_i, where j indicates which factory's defective rate remains constant.

At this point, I only accepted the set of new defective rates where

$$P(D|A_i)_{new} \geq P(D|A_i)_{old}, \forall A_i$$

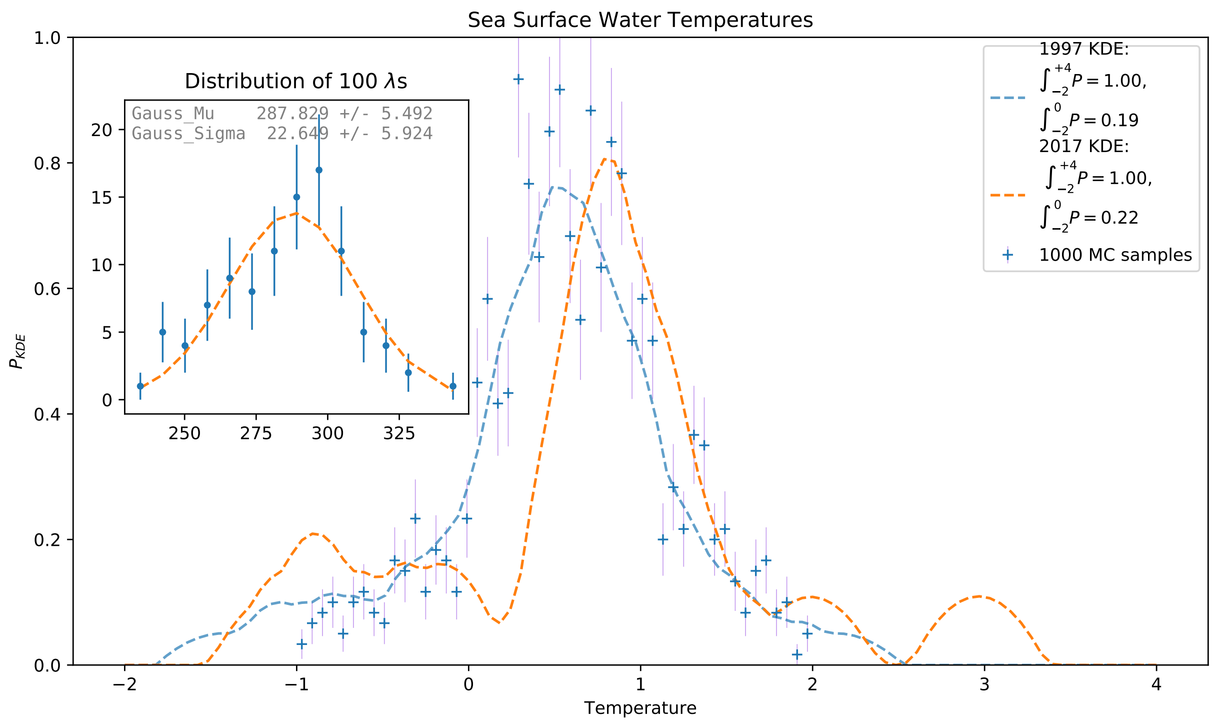
Practically, this solution was scalable to be implemented on problem 3c as well.

A realization that would have made the solution easier I realized after finishing this, that an alternative solution would be to start with the relative defecive rate ($1.0/N$ where N is the number of factories) and from there calculate the defective rates from there. Nonwithstanding, I didn't implement this solution, as I believe that the original solution was sufficient (and scalable).

3 Problem 4 - Seawater

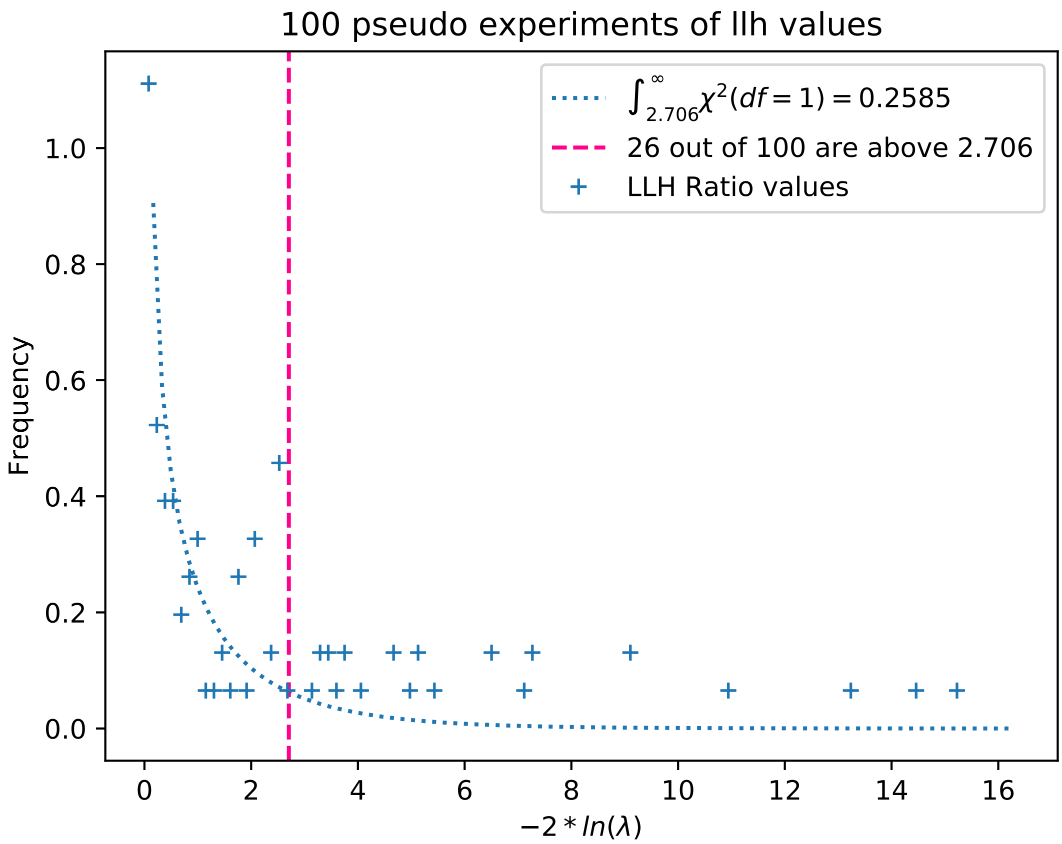
3.0.1 Problem 4a

See the figure 3.0.1.



4 Problem 5 - Particles

4.1 Problem 5a and 5b



4.2 problem 5c