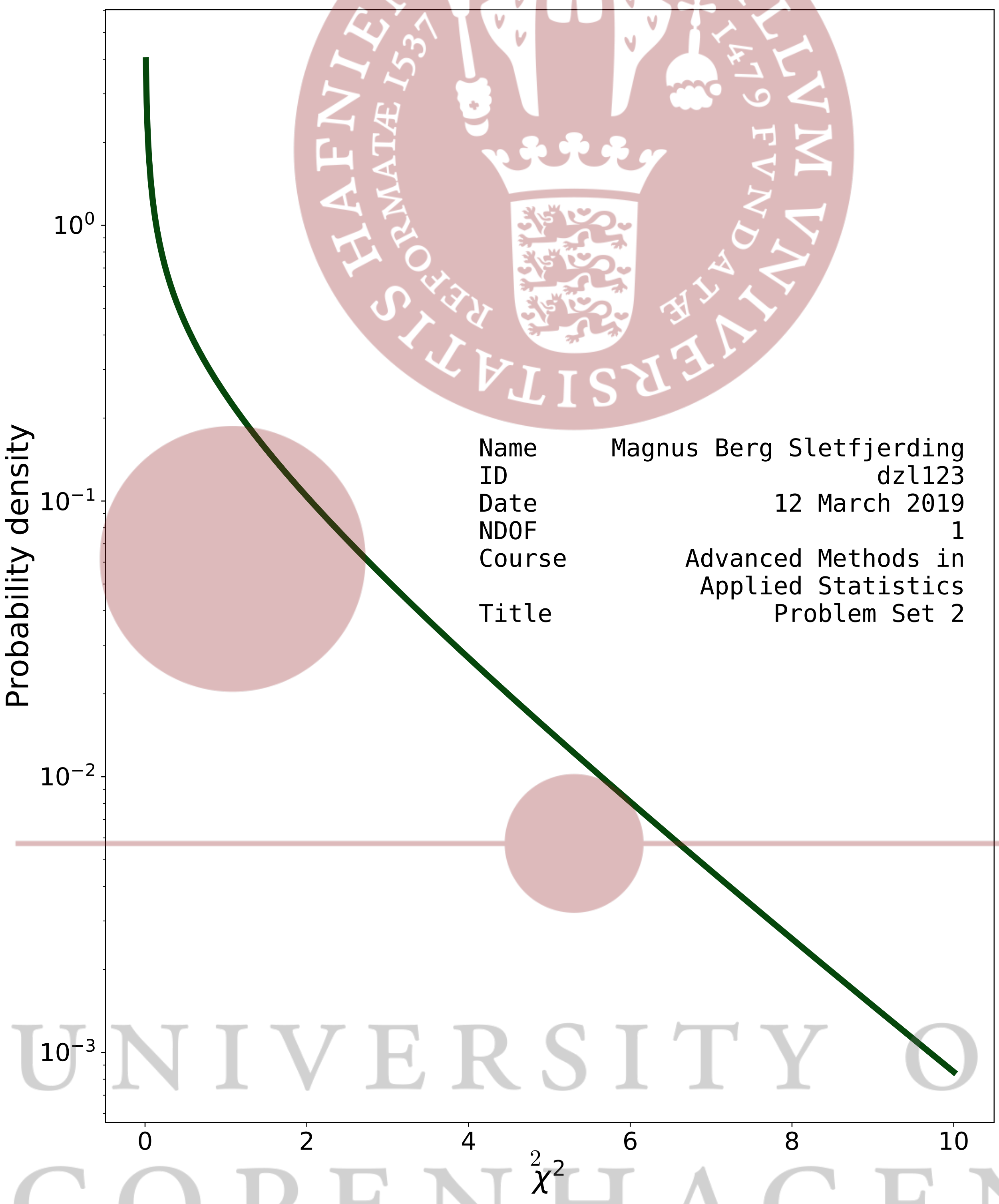

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Course	Advanced Methods in Applied Statistics
Title	Problem Set 2

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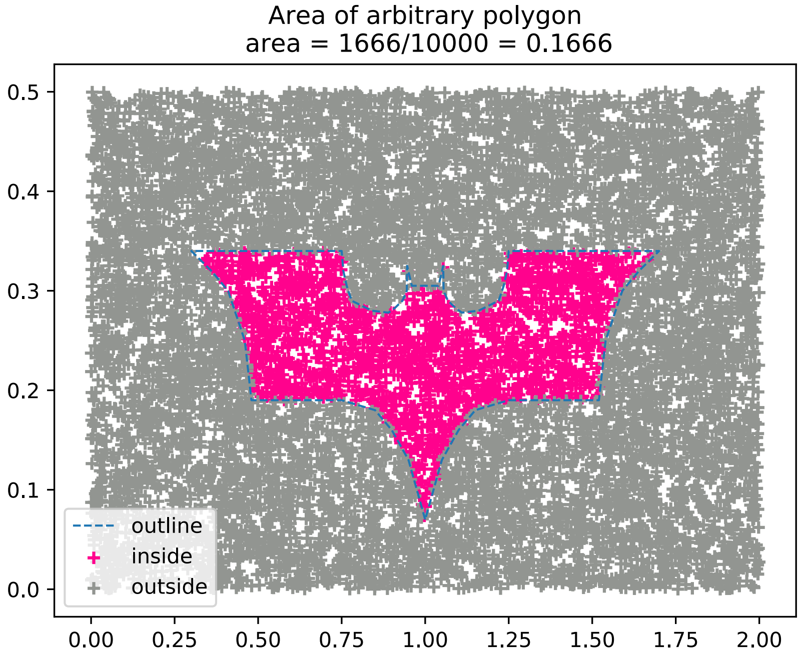

Figure 1: Problem 2

1 Problem 2 - Monte Carlo calculation of an arbitrarily drawn polygon

I was amused by the shape of this polygon. I generated points in the interval $x = (0, 2)$, $y = (0, 0.5)$ for the simple reason that its area is unity. Using the `shapely` package, it was simple to check whether a point were inside the polygon or not. I plotted the results, with the corresponding area in figure 1. The area (0.166) also seems appropriate upon visual inspection.

2 Problem 3 - Conditional probabilities for Slightly Evil Inc.

This section was by far the most tricky to get right.

2.1 Answers

My answers are contained in the output from my code:

```
  Problem 3a:
Current aggregate defective rate: 0.03275
```

```
If defective, a PM is 18.32% likely to come from the facility A2
If defective, a PM is 23.66% likely to come from the facility A5
  Problem 3b:
  facility  defective  new_defective
0       A1      0.020       0.022143
1       A2      0.040       0.051667
2       A3      0.100       0.155000
3       A4      0.035       0.038750
4       A5      0.031       0.031000
  Problem 3c:
   facility  defective  new_defective
0        A1      0.020       0.022037
1        A2      0.040       0.059500
2        A3      0.100       0.119000
3        A4      0.035       0.074375
4        A5      0.022       0.023800
5        A6      0.092       0.180303
6        A7      0.120       0.313158
7        A8      0.070       0.070000
8        A9      0.110       0.180303
9       A10      0.020       0.297500
10      A11      0.070       0.396667
11      A12      0.060       0.270455
12      A13      0.099       0.396667
13      A14      0.082       0.743750
```

2.2 Explanation

2.2.1 Problem 3a

Finding the chance a defective pacemaker is from A2 facility calls for the use of Bayes' Theorem:

$$P(A2|D) = \frac{P(D|A2) * P(A2)}{P(D)}$$

Applying the already found values:

$$P(A2|D) = \frac{0.04 * 0.15}{0.03275} = 18.32\%$$

Using the same formula, for all facilities, we find that:

$$P(A5|D) = \frac{0.031 * 0.25}{0.03275} = 23.66\%$$

Which is higher than all the others. It makes intuitive sense, too, as the only other candidate, facility A4, still produces fewer defective devices than

A5:

$$P(A4|D) = \frac{0.035 * 0.2}{0.03275} = 22.93\%$$

2.2.2 Problem 3b and 3c

Again, Bayes' Theorem shows itself handy.

In order to make all relative defective rates the same (20%) we need to adjust all but one of the defective rates up. (Analytically, it might make sense to increase the error rate for A5, as we know that it it the maximum.)

Doing so, I used the following formula to calculate the new defective rates from each factory:

$$P(D|A_i)_{new,j} = \frac{P(A_j)}{P(A_i)} P(D|A_i)_{old}$$

for all factories A_i, where j indicates which factory's defective rate remains constant.

At this point, I only accepted the set of new defective rates where

$$P(D|A_i)_{new} \geq P(D|A_i)_{old}, \forall A_i$$

Practically, this solution was scalable to be implemented on problem 3c as well.

A realization that would have made the solution easier I realized after finishing this, that an alternative solution would be to start with the relative defecive rate ($1.0/N$ where N is the number of factories) and from there calculate the defective rates from there. Nonwithstanding, I didn't implement this solution, as I believe that the original solution was sufficient (and scalable) enough to work for the whole problem.

3 Problem 4 - Seawater

3.0.1 Problem 4a

Numerical and graphical results are contained in figure 2.

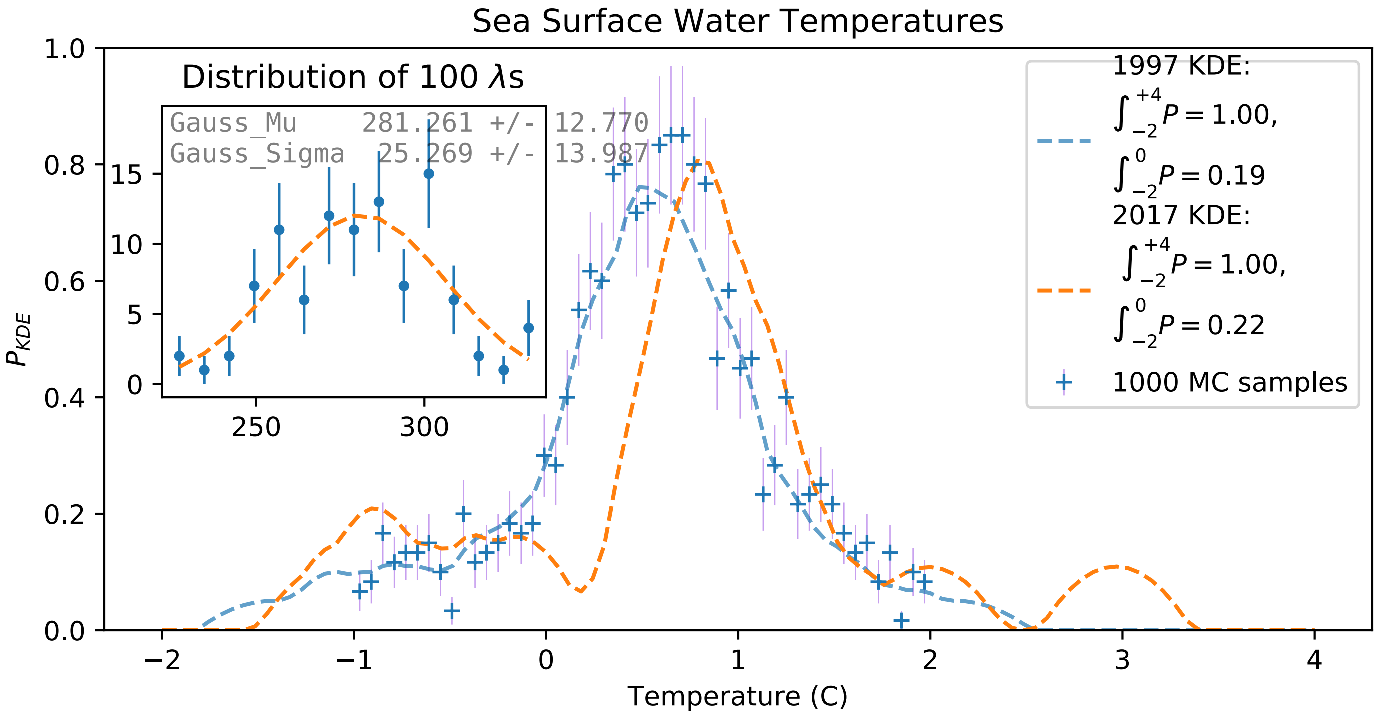

Figure 2: Problem 4. After generating one lambda, I was surprised when the value was several orders of magnitude larger than 1. I therefore generated 100 lambdas and plotted the distribution, which seems to fit nicely with a gaussian distribution. The distribution of the values as a gaussian with mean 285 and sigma 22 demonstrates that the values of lambda can be estimated to fall in the interval $(241, 329)$ with 95% confidence.

4 Problem 5 - Particles

4.1 Problem 5a and 5b

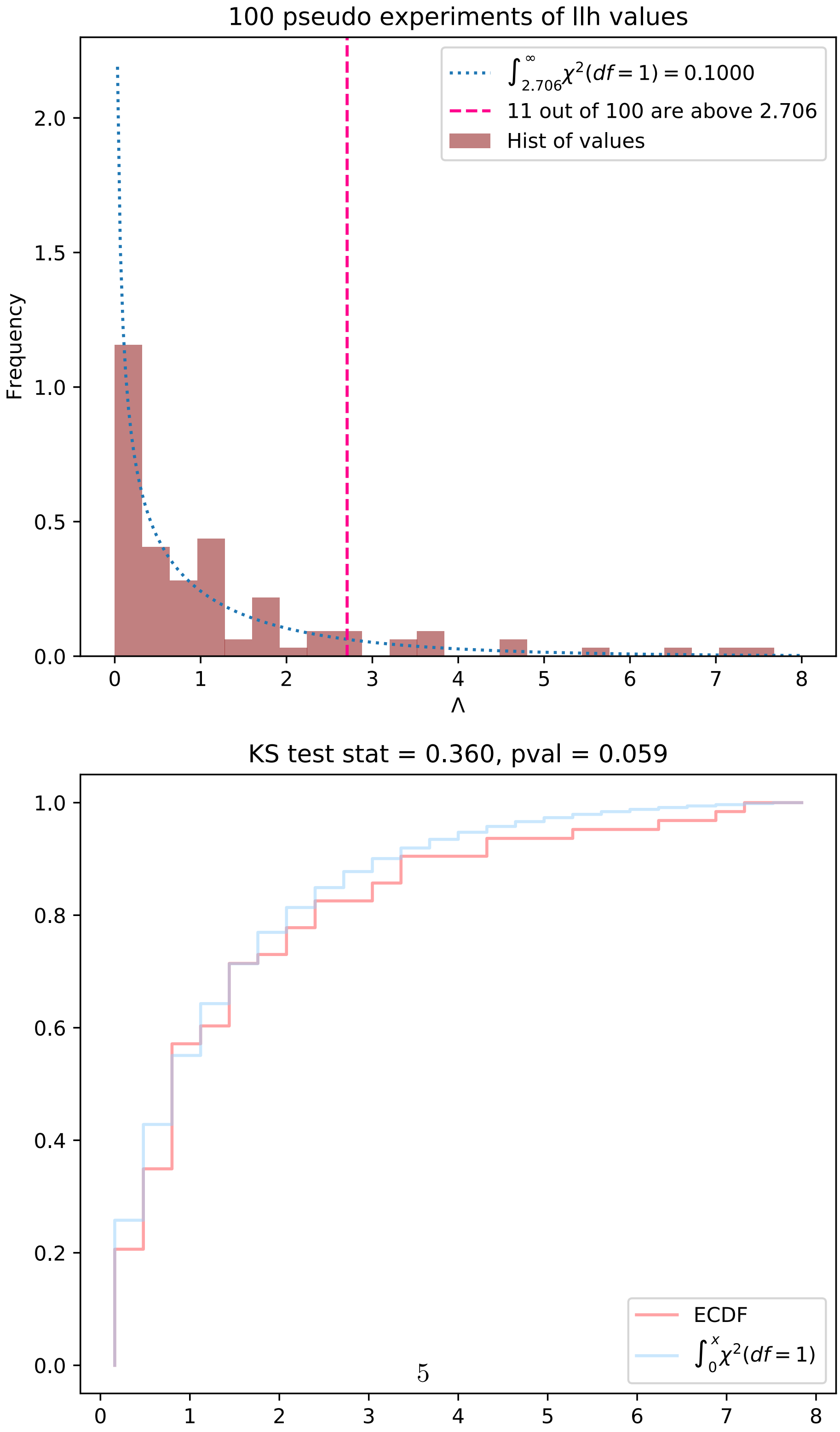

Figure 3: Problem 5

4.1.1 Problem 5b

Too see whether the distribution is actually chi squared distributed, I did a KS test as demonstrated in figure 2. Immidiately , I could see that the distributions lined up rather well. The p value close to 0.05 is also a good indicator that the values are chi square distributed, as one would expect if the samples are from the same distribution. As can also be seen in the figure, the number of pseudo-experiments with a test statistic over 2.706 is also comparable (11 observed vs 10 expected) to what one would expect from a chi square distributed test statistic.

4.2 problem 5c

Using all 20000 events for a LLH test, I get the following values for the likelihoods.

For the null hypothesis $H_0 : b = 1.0$, I get an LLH of $LLH_0 = -29653.08$, for the best fit parameters $\sigma_t = 0.618, b = 1.0$.

For the test hypothesis $H_1 : b \neq 1.0$, I get an LLH of $LLH_1 = -29652.74$, for the best fit parameters $\sigma_t = 0.619, b = 0.993$.

Hence,

$$\Lambda = -2 * (LLH_0 - LLH_1) = 1.358$$

Which gives us a chi square integrated value

$$\int_{1.358}^{\infty} \chi^2(df = 1) = 0.244$$

Where we have 1 degree of freedom between the two models (one free parameter).

In order to reject a hypothesis at a 3 sigma threshold, the result needs to have a test statistic (chi square) value higher than 9. ($p = 0.0027$) As this model does not have this, we can NOT reject the null hypothesis.

I was surprised that I found such a large difference in the two approaches to finding a lambda, and tried to part 5a and 5b with RANDOM chunks of data, instead of splitting periodically. However, I found no significant changes between the randomly sampled chunks and the periodically split chunks. Thus, I can conclude that the different λ values are not due to a designed way of making the data chunks, and that the ordering of the points are uninmportant. The file in question is attached as `p5_llhratios_random.pdf`.

In order to get a bit wiser on these results, I plotted the two results in figure 4, from which we can clearly see that the two functions are substantially equivalent.

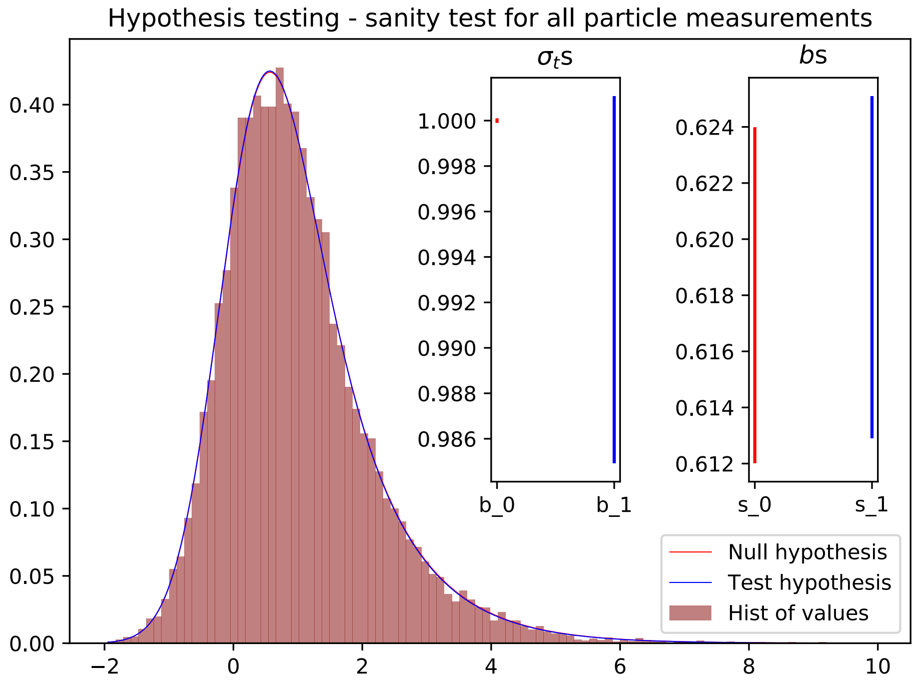

Figure 4: Visualization of the null hypothesis vs test hypothesis for problem 5c. Subplots show the 1 sigma symmetric error returned from minuit, except for b_0, which was fixed. Clearly, the two are not distinguishable using conventional chi-by-eye techniques.