Wild Binary Segmentation

A Monte Carlo-like approach to localizing changepoints in data

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The Problem

Let's say we have a noisy time series data.

- Financial data
- Student satisfaction over time
- Traffic Data
- Biomolecular activity

We would prefer to separate this data into separate chunks that are easier to work with. But how?!

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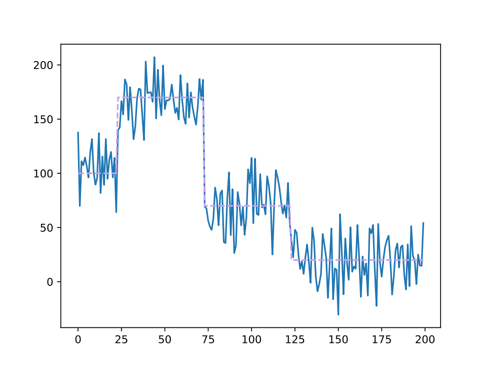

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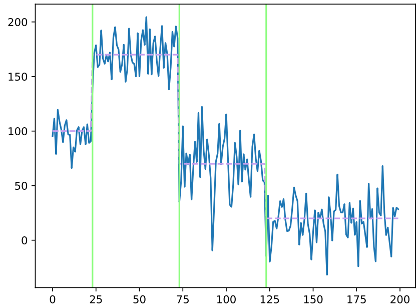
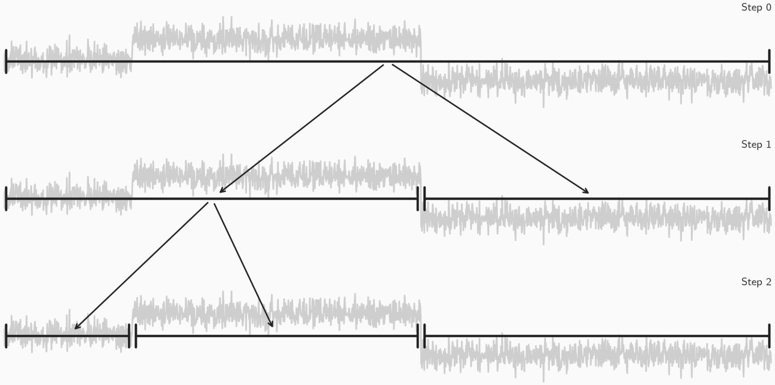


Binary Segmentation

This is the classical way of finding changepoints in data.



It's quick, easy to understand, and easy to conceptualize.

Binary Segmentation - math

Assume that our data can be like so:

$$X_t = f_t + \varepsilon_t, t = 1, \ldots, T$$

where f_t is a one-dimensional signal which has an unknown number of changepoints N, with unknown locations $\eta_1, \ldots, \eta_N$, and ε_t is a normal random variable centered at 0.

Binary Segmentation maximizes this statistic:

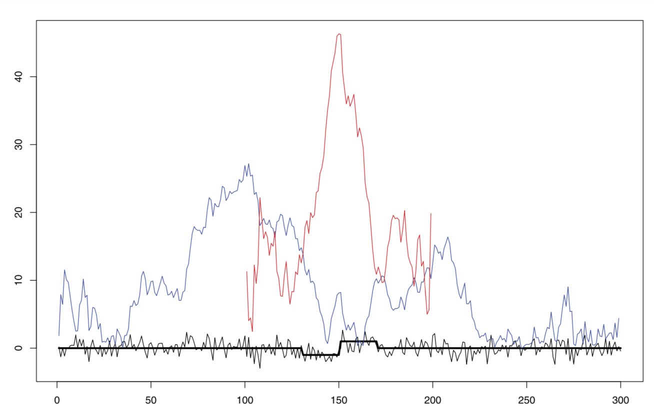
$$\tilde{X}^b_{s,e} = \sqrt{\frac{e-b}{n(b-s+1)}} \sum_{t=s}^{b} X_t - \sqrt{\frac{b-s+1}{n(e-b)}} \sum_{t=b+1}^{e} X_t$$

where $s \leq b < e$ and $n = e - s + 1$.

What are we left with? The **most likely changepoint,** b_0

So Binary Segmentation is perfect?

Not quite. Sometimes, the test statistic used can end up being maximized at places where there is no changepoint!!



Hopefully there will be a better method in the next slide!

Wild Binary Segmentation

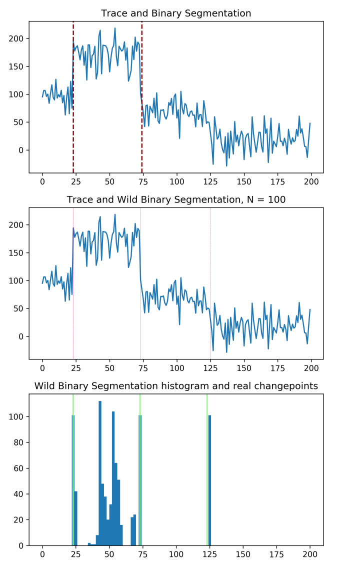
Before doing any Binary Segmentation, make a set F_T^M of M randomly sampled **intervals** $[s_m, e_m], m = 1, \ldots, M$ where $[s_m, e_m]$ have been drawn from $1, \ldots, M$.

Once that's done, do binary segmentation on ALL the intervals, but only choose the b_0^m that completely maximizes $\tilde{X}_{s,e}^b$

Higher chance that the fit will find the "right" values.

An example

- I generated some data to work on, and implemented the two algorithms
- This looks more or less like what I work with on the daily
- Normal Binary Segmentation does not find the third(last) changepoint
- Wild Binary Segmentation (1000 iterations) found the true changepoints for the most part



Conclusion

- $WBS > BS$
- Implementing algorithms yourself is hard
- BS is implemented in the `ruptures` package
- No WBS in Python 3.7