Enzyme Kinetics

$$(1) \quad E + S \xrightarrow{k_1 \atop k_2} ES \xrightarrow{k_3} E + P$$

The rate of change of free enzyme E is given by:

$$\frac{d(E)}{dt} = -k_1[E][S] + k_2[ES] + k_3[ES]$$

The rate of change of substrate S is given by:

$$\frac{d[S]}{dt} = -k_1[E][S] + k_2[ES]$$

The rate of change of enzyme-substrate complex ES is given by:

$$\frac{d[ES]}{dt} = k_1[E][S] - (k_2 + k_3)[ES]$$

The rate of change of product P is given by:

$$\frac{d[P]}{dt} = k_3[ES]$$

(2) The code is as follows:

(16) dt = 0.001 # min

```
(1) import numpy as np
(2)import matplotlib.pyplot as plt
(3)
(4)# Define the rate constants
(5)k1 = 100 \# 1/min/\mu M
(6)k2 = 600 \# 1/min
(7)k3 = 150 # 1/min
(8)
(9)# Define the initial concentrations
(10) E0 = 1
                # µM
(11) S0 = 10 \# \mu M
     ES0 = 0
(12)
(13)
     P0 = 0
(14)
(15) # Define the time step and simulation duration
```

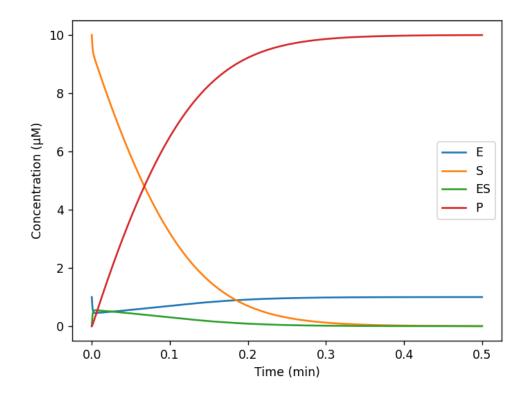
```
(17) t_max = 0.5 # min
(18)
(19) # Define the initial conditions
     y0 = np.array([E0, S0, ES0, P0])
(20)
(21)
(22)
      # Define the function
(23)
(24)
(25) def f(t, y):
          E, S, ES, P = y
(26)
          dEdt = -k1*E*S + k2*ES + k3*ES
(27)
(28)
          dSdt = -k1*E*S + k2*ES
(29)
          dESdt = k1*E*S - k2*ES - k3*ES
          dPdt = k3*ES
(30)
          return np.array([dEdt, dSdt, dESdt, dPdt])
(31)
(32)
(33) # Define the function for the fourth-order Runge-Kutta method
(34)
      # traditional classical runge-kutta
(35)
(36)
(37) def rk4_step(f, t, y, dt):
(38)
          k1 = f(t, y)
          k2 = f(t + dt/2, y + k1*dt/2)
(39)
(40)
          k3 = f(t + dt/2, y + k2*dt/2)
(41)
          k4 = f(t + dt, y + k3*dt)
(42)
          return y + (k1 + 2*k2 + 2*k3 + k4)*dt/6
(43)
(44)
(45) # Perform the simulation
(46)
     t vals = np.arange(0, t max+dt, dt)
(47) y_{vals} = np.zeros((len(t_vals), len(y0)))
(48)
     y_vals[0, :] = y0
(49) for i in range(len(t_vals)-1):
(50)
          y_vals[i+1, :] = rk4_step(f, t_vals[i], y_vals[i, :], dt)
(51)
(52)
     # Plot the results
(53) plt.figure('Runge Kutta numerical results')
      plt.plot(t_vals, y_vals[:, 0], label='E')
(54)
(55) plt.plot(t_vals, y_vals[:, 1], label='S')
     plt.plot(t_vals, y_vals[:, 2], label='ES')
(56)
(57) plt.plot(t_vals, y_vals[:, 3], label='P')
(58)
      plt.xlabel('Time (min)')
(59) plt.ylabel('Concentration (μM)')
     plt.legend()
(60)
```

(61) plt.show()

The calculation result as follows:

	T=0	T=0.05	T=0.1	T=0.15	T=0.2	T=0.25	T=0.3	T=0.35	T=0.4
	(µM)	(µM)	(µM)	(µM)	(µM)	(µM)	(µM)	(µM)	(µM)
Е	1	0.56	0.69	0.83	0.91	0.96	0.98	0.99	0.99
S	10	5.84	3.19	1.56	0.69	0.29	0.12	0.04	0.02
ES	0	0.43	0.30	0.17	0.08	0.04	0.01	0.006	0.002
P	0	3.72	6.50	8.27	9.22	9.67	9.86	9.94	9.98

The Runge Kutta numerical results figure as follows:



(3). According to the Michaelies-Menten equation, the relationship between the velocity V and the concentration of substrate S can be described as follows:

$$V = \frac{v_m[s]}{k_m + [s]} \tag{1}$$

where v_m is the maximum velocity of the reaction, [S] is the concentration of the substrate, and k_m is the Michaelis-Menten constant which represents the concentration of substrate required to achieve half of the maximum velocity.

From the formula [1], when the concentration of S is low, the formula can be simplified to:

$$V \approx \frac{v_m}{k_m} [s]$$
 [2]

This relationship is approximately linear, meaning that the velocity increases proportionally with the concentration of the substrate.

However, as the concentration of S increases, the term [S] becomes the denominator of the formula [1] and the velocity V saturates to the maximum value v_m , the formula can be simplified to:

$$V \approx v_m$$
 [3]

The figure is as follows:

