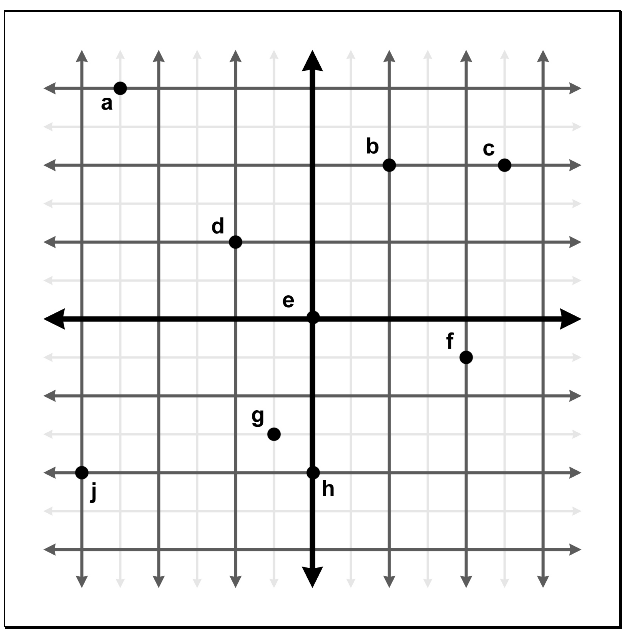
Answers

Chapters 1-7

Chapter 2

1) *Give the coordinates of the following points:*



a	(-2.5, 3)
b	(1, 2)
c	(2.5, 2)
d	(-1, 1)
e	(0, 0)
f	(2, -0.5)
g	(-0.5, -1.5)
h	(-2, 0)
j	(-3, -2)

2) *List the 48 different possible ways that the 3D axes may be assigned to the directions "north," "east" and "up." Identify which of these combinations are left-handed, and which are right-handed.*

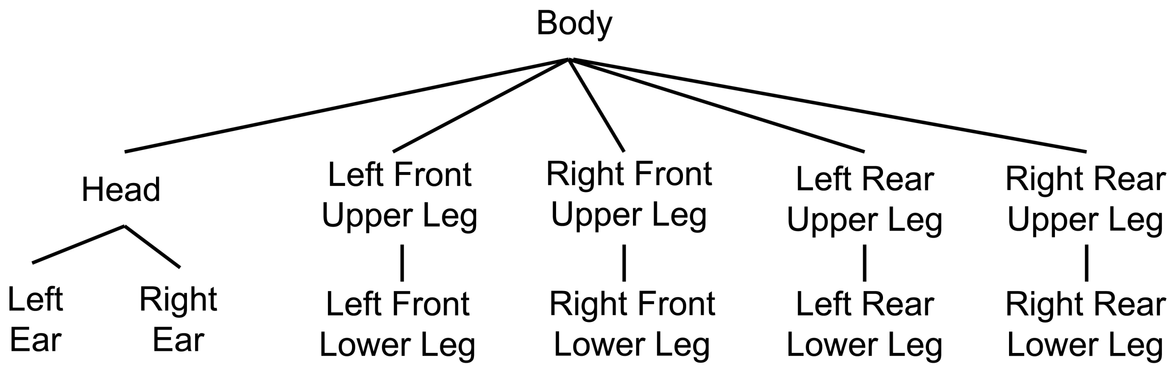
North	East	Up	Hand	North	East	Up	Hand
+x	+y	+z	Left	+x	+z	+y	Right
+x	+y	–z	Right	+x	+z	–y	Left
+x	–y	+z	Right	+x	–z	+y	Left
+x	–y	–z	Left	+x	–z	–y	Right
–x	+y	+z	Right	–x	+z	+y	Left
–x	+y	–z	Left	–x	+z	–y	Right
–x	–y	+z	Left	–x	–z	+y	Right
–x	–y	–z	Right	–x	–z	–y	Left
+y	+x	+z	Right	+y	+z	+x	Left
+y	+x	–z	Left	+y	+z	–x	Right
+y	–x	+z	Left	+y	–z	+x	Right
+y	–x	–z	Right	+y	–z	–x	Left
–y	+x	+z	Left	–y	+z	+x	Right
–y	+x	–z	Right	–y	+z	–x	Left
–y	–x	+z	Right	–y	–z	+x	Left
–y	–x	–z	Left	–y	–z	–x	Right
+z	+x	+y	Left	+z	+y	+x	Right
+z	+x	–y	Right	+z	+y	–x	Left
+z	–x	+y	Right	+z	–y	+x	Left
+z	–x	–y	Left	+z	–y	–x	Right
–z	+x	+y	Right	–z	+y	+x	Left
–z	+x	–y	Left	–z	+y	–x	Right
–z	–x	+y	Left	–z	–y	+x	Right
–z	–x	–y	Right	–z	–y	–x	Left

3) *In a popular modeling program 3D Studio Max, the default orientation of the axes is for +x to point right, +y to point forward, and +z to point up. Is this a left- or right-handed coordinate space?*

Right-handed.

Chapter 3

1) *Draw a nested space hierarchy tree for the sheep described in Section 3.3, assuming that its head, ears, upper legs, lower legs, and body move indemendently.*



2) *Suppose our object axes are transformed to world axes by rotating them counterclockwise around the* y*-axis by 42°and then translating six units along the* z*-axis and 12 units along the* x*-axis. Describe this transformation from the perspective of the object.*

Imagine a point on the object, in object space. As the axes are rotating counterclockwise, the point is actually rotating *counterclockwise* relative to the axes. Then, as the axes translate by [12, 0, 6], the point translates [-12, 0, -6] relative to the axes.

3) *Which coordinate space is the most appropriate in which to ask the following questions?*

a) *Is my computer in front of or behind me?* Object space. If we know the position of the computer within our object space, this question is a trivial matter of checking for a positive z value. (Assuming the conventions from 2.3.4)

b) *Is the book east or west of me?* Inertial space is the easiest space to make this test. Again, assuming the conventions from 2.3.4, the book is east of us if the x-coordinate of the book's position in our inertial space is positive, and west if this value is negative. Alternatively, we could answer the question in world space, by comparing the x-coordinate of the book in world space, with our own world space x-coordinate.

c) *How do I get from one room to the other?* Pathfinding-type querries are usually made in world space.

d) *Can I see my computer?* The "camera space" for our viewpoint is the most natural coordinate space to use for this question.

Chapter 4

1) *Let:*

$$\mathbf{a} = \begin{bmatrix} -3 & 8 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 4 \\ 0 \\ 5 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} 16 \\ -1 \\ 4 \\ 6 \end{bmatrix}$$

a) *Identify* **a**, **b**, *and* **c**, *as row or column vectors, and give the dimension of each vector.*

a is a 2D row vector. **b** is a 3D column vector. **c** is a 4D column vector.

b) *Compute* $\mathbf{b}_y+\mathbf{c}_w+\mathbf{a}_x+\mathbf{b}_z$.

$$\begin{aligned} \mathbf{b}_y &= 0 \\ \mathbf{c}_w &= 6 \\ \mathbf{a}_x &= -3 \\ \mathbf{b}_z &= 5 \\ \mathbf{b}_y + \mathbf{c}_w + \mathbf{a}_x + \mathbf{b}_z &= 0 + 6 + (-3) + 5 \\ &= 8 \end{aligned}$$

2) *Identity the quantities in each of the following sentences as scalar or vector. For vector quantities, give the magnitude and direction. (Note: some directions may be implicit.)*

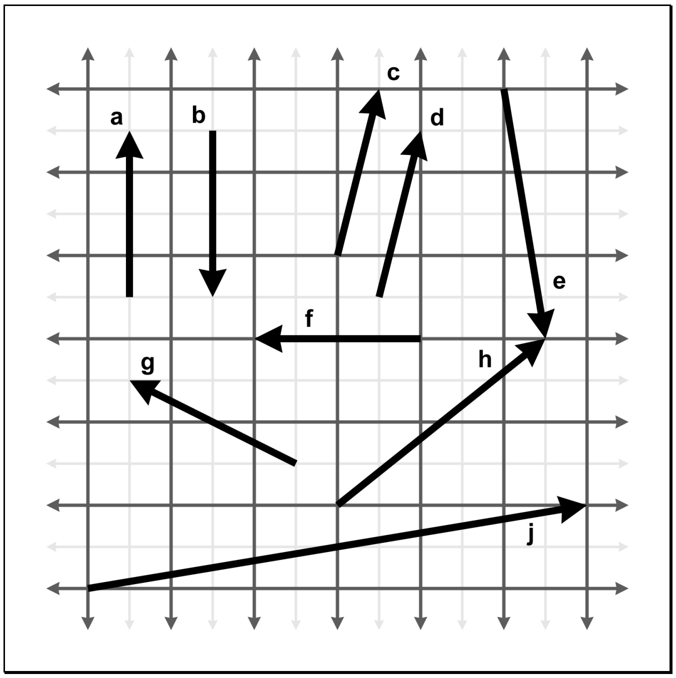
a) *How much do you weigh?* Weight is a calar quantity.

b) *Do you have any idea how fast you were going?* Speed is a scalar quantity.

c) *It's two blocks north of here.* "Two blocks north" is a vector quantity, since it specified a magnitude ("two blocks") and a direction ("north").

d) *We're cruising from Los Angeles to New York at 600mph, at an altitude of 33,000ft.* Speed (600mph) is a scalar quantity. However, since we know we are traveling from Los Angeles to New York, we could assume an eastward direction, which would provide a direction, making it a velocity, which is a vector quantity. Altitude (33,000ft) is a scalar quantity.

3) *Give the values of the following vectors:*



a	[0, 2]
b	[0, -2]
c	[0.5, 2]
d	[0.5, 2]
e	[0.5, -3]
f	[-2, 0]
g	[-2, 1]
h	[2.5, 2]
j	[6, 1]

4) *Identify the following statements as true or false. If the statement is false, explain why.*

a) *The size of a vector in a diagram doesn't matter; we just need to draw it in the right place.* **False**. Size matters; so does direction. A vector does not express a "position," and so we can draw in on a diagram anywhere that is convenient. See 4.2.2.

b) *The displacement expressed by a vector can be visualized as a sequence of axially aligned displacements*. **True**. See Figure 4.5 on page 40.

c) *These axially aligned displacements from the previous question must occur in order.* **False**. They can occur in any order, due to commutative nature of vector addition. See page 40.

d) *The vector [x, y] gives the displacement from the point (x, y) to the origin.* **False**. It gives the opposite displacement – from the origin to the point.

Chapter 5

1) *Evaluate the following vector expressions:*

a) $-\begin{bmatrix} 3 & 7 \end{bmatrix} = \begin{bmatrix} -3 & -7 \end{bmatrix}$

b)
$$\begin{aligned}\left\| \begin{bmatrix} -12 & 5 \end{bmatrix} \right\| &= \sqrt{(-12)^2 + 5^2} \\ &= \sqrt{144 + 25} \\ &= \sqrt{169} \\ &= 13\end{aligned}$$

c)
$$\begin{aligned}\left\| \begin{bmatrix} 8 & -3 & 1/2 \end{bmatrix} \right\| &= \sqrt{8^2 + (-3)^2 + (\frac{1}{2})^2} \\ &= \sqrt{64 + 9 + \frac{1}{4}} \\ &= \sqrt{64 + 9 + \frac{1}{4}} \\ &= \sqrt{73\frac{1}{4}} \\ &\approx 8.5586\end{aligned}$$

d) $3\begin{bmatrix} 4 & -7 & 0 \end{bmatrix} = \begin{bmatrix} 3 \cdot 4 & 3 \cdot -7 & 3 \cdot 0 \end{bmatrix} = \begin{bmatrix} 12 & -21 & 0 \end{bmatrix}$

e) $\begin{bmatrix} 4 & 5 \end{bmatrix}/2 = \begin{bmatrix} 4/2 & 5/2 \end{bmatrix} = \begin{bmatrix} 2 & 5/2 \end{bmatrix}$

2) *Normalize the following vectors:*

a)

$$\begin{aligned}
\begin{bmatrix} 12 & 5 \end{bmatrix}_{\text{norm}} &= \frac{\begin{bmatrix} 12 & 5 \end{bmatrix}}{\left\|\begin{bmatrix} 12 & 5 \end{bmatrix}\right\|} \\
&= \frac{\begin{bmatrix} 12 & 5 \end{bmatrix}}{\sqrt{12^2 + 5^2}} \\
&= \frac{\begin{bmatrix} 12 & 5 \end{bmatrix}}{13} \\
&= \begin{bmatrix} \frac{12}{13} & \frac{5}{13} \end{bmatrix}
\end{aligned}$$

b)

$$\begin{aligned}
\begin{bmatrix} 8 & -3 & 1/2 \end{bmatrix}_{\text{norm}} &= \frac{\begin{bmatrix} 8 & -3 & 1/2 \end{bmatrix}}{\left\|\begin{bmatrix} 8 & -3 & 1/2 \end{bmatrix}\right\|} \\
&= \frac{\begin{bmatrix} 8 & -3 & 1/2 \end{bmatrix}}{\sqrt{8^2 + (-3)^2 + (1/2)^2}} \\
&= \frac{\begin{bmatrix} 8 & -3 & 1/2 \end{bmatrix}}{\sqrt{64 + 9 + (1/4)}} \\
&= \frac{\begin{bmatrix} 8 & -3 & 1/2 \end{bmatrix}}{\sqrt{293/4}} \\
&\approx \frac{\begin{bmatrix} 8 & -3 & .5 \end{bmatrix}}{8.5586} \\
&\approx \begin{bmatrix} .9347 & -.3505 & .05842 \end{bmatrix}
\end{aligned}$$

3) *Evaluate the following vector expressions:*

a)

$$\begin{bmatrix} 3 \\ 10 \\ 7 \end{bmatrix} - \begin{bmatrix} 8 \\ -7 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 - 8 \\ 10 - (-7) \\ 7 - 4 \end{bmatrix} = \begin{bmatrix} -5 \\ 17 \\ -3 \end{bmatrix}$$

b)

$$3\begin{bmatrix} a \\ b \\ c \end{bmatrix} - 4\begin{bmatrix} 2 \\ 10 \\ -6 \end{bmatrix} = \begin{bmatrix} 3a \\ 3b \\ 3c \end{bmatrix} - \begin{bmatrix} 8 \\ 40 \\ -24 \end{bmatrix} = \begin{bmatrix} 3a - 8 \\ 3b - 40 \\ 3c - 24 \end{bmatrix}$$

4) *Compute the distance between the following pairs of points:*

a)

$$\begin{aligned}
\text{distance}\left(\begin{bmatrix} 3 \\ 10 \\ 7 \end{bmatrix}, \begin{bmatrix} 8 \\ -7 \\ 4 \end{bmatrix}\right) &= \sqrt{(3-8)^2 + (10-(-7))^2(7-4)^2} \\
&= \sqrt{(-5)^2 + 17^2(-3)^2} \\
&= \sqrt{25 + 289 + 9} \\
&= \sqrt{323} \\
&\approx 17.9722
\end{aligned}$$

b)

$$\begin{aligned}
\text{distance}\left(\begin{bmatrix} 10 \\ 6 \end{bmatrix}, \begin{bmatrix} -14 \\ 30 \end{bmatrix}\right) &= \sqrt{(10-(-14))^2 + (6-30)^2} \\
&= \sqrt{24^2 + (-24)^2} \\
&= \sqrt{576 + 576} \\
&= \sqrt{1152} \\
&= 24\sqrt{2} \\
&\approx 33.9411
\end{aligned}$$

5) *Evaluate the following vector expressions:*

a)

$$\begin{bmatrix} 2 \\ 6 \end{bmatrix} \cdot -38 = \begin{bmatrix} (2)(-38) \\ (6)(-38) \end{bmatrix} = \begin{bmatrix} -76 \\ -228 \end{bmatrix}$$

Note: Although the above problem is valid, the notation isn't the same as the notation used in the book. That's because it contained a typo. The problem should have read:

$$\begin{bmatrix} 2 \\ 6 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 8 \end{bmatrix} = (2)(-3) + (6)(8) = -6 + 48 = 42$$

b)

$$
\begin{aligned}
3\begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix} \cdot \left(\begin{bmatrix} 8 \\ -2 \\ 3/2 \end{bmatrix} + \begin{bmatrix} 0 \\ 9 \\ 7 \end{bmatrix} \right) &= \begin{bmatrix} (3)(-2) \\ (3)(0) \\ (3)(4) \end{bmatrix} \cdot \begin{bmatrix} 8+0 \\ -2+9 \\ 3/2+7 \end{bmatrix} \\
&= \begin{bmatrix} -6 \\ 0 \\ 12 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 7 \\ 17/2 \end{bmatrix} \\
&= (-6)(8) + (0)(7) + (12)(17/2) \\
&= -48 + 0 + 102 \\
&= 54
\end{aligned}
$$

6) *Compute the angle between the vectors [1, 2] and [-6, 3].*

From 5.10.2, we solve for the angle using the dot product:

$$
\begin{aligned}
\theta &= \operatorname{acos}\left(\frac{\begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} -6 & 3 \end{bmatrix}}{\left\| \begin{bmatrix} 1 & 2 \end{bmatrix} \right\| \left\| \begin{bmatrix} -6 & 3 \end{bmatrix} \right\|} \right) \\
&= \operatorname{acos}\left(\frac{(1)(-6) + (2)(3)}{\sqrt{1^2 + 2^2}\sqrt{(-6)^2 + 3^2}} \right) \\
&= \operatorname{acos}\left(\frac{-6+6}{\sqrt{5}\sqrt{45}} \right) \\
&= \operatorname{acos} 0 \\
&= 90^\circ
\end{aligned}
$$

7) *Given the two vectors*

$$\mathbf{v} = \begin{bmatrix} 4 \\ 3 \\ -1 \end{bmatrix}, \mathbf{n} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix}$$

Separate ***v*** *into components that are perpendicular and parallel to* ***n****. (****n*** *is a unit vector.)*

See 5.10.3.

$$\begin{aligned}
\mathbf{v}_{\parallel} &= \mathbf{n}\frac{\mathbf{v}\cdot\mathbf{n}}{\|\mathbf{n}\|^2} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix} \frac{\begin{bmatrix} 4 \\ 3 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix}}{1^2} \\
&= \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix} \left(\begin{bmatrix} 4 \\ 3 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix} \right) \\
&= \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix} \left(\frac{4\sqrt{2}}{2} + \frac{3\sqrt{2}}{2} + (-1)(0) \right) \\
&= \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix} \frac{7\sqrt{2}}{2} = \begin{bmatrix} (\frac{\sqrt{2}}{2})(\frac{7\sqrt{2}}{2}) \\ (\frac{\sqrt{2}}{2})(\frac{7\sqrt{2}}{2}) \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} 7/2 \\ 7/2 \\ 0 \end{bmatrix} \\
\mathbf{v}_{\perp} &= \mathbf{v} - \mathbf{v}_{\parallel} = \begin{bmatrix} 4 \\ 3 \\ -1 \end{bmatrix} - \begin{bmatrix} 7/2 \\ 7/2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1/2 \\ -1 \end{bmatrix}
\end{aligned}$$

8) *Compute the value of*

$$\begin{bmatrix} 3 \\ 10 \\ 7 \end{bmatrix} \times \begin{bmatrix} 8 \\ -7 \\ 4 \end{bmatrix} = \begin{bmatrix} (10)(4) - (7)(-7) \\ (7)(8) - (3)(4) \\ (3)(-7) - (10)(8) \end{bmatrix} = \begin{bmatrix} 40 - (-49) \\ (56) - (12) \\ (-21) - (80) \end{bmatrix} = \begin{bmatrix} 89 \\ 44 \\ -101 \end{bmatrix}$$

9) *A man is boarding a plane. The airline has a rule where no carry-on item may be more than 2ft long, 2ft wide, or 2ft tall. He has a very valuable sword that is three feet long. He is able to carry the sword on board with him. How is he able to do this? What is the longest possible item that he could carry on?*

The man is able to board the plane by placing his sword diagonally in a cube-shaped box that is 2ft long, 2ft tall, and 2ft wide. The length of the longest item he could carry is:

$$\left\| \begin{bmatrix} 2 & 2 & 2 \end{bmatrix} \right\| = \sqrt{2^2 + 2^2 + 2^2} = \sqrt{12} = 2\sqrt{3} \approx 3.4641$$

which is about 41.5 inches. (Of course, nowadays, he would be arrested and would not be allowed to board the plane at all! This question was written before the recent increase airport security.)

10) *Verify Figure 5.7 on page 56 mathematically.*

$$\begin{aligned} &\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} + \mathbf{e} + \mathbf{f} \\ &= \begin{bmatrix} -1 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 \\ -2 \end{bmatrix} + \begin{bmatrix} -6 \\ 4 \end{bmatrix} + \begin{bmatrix} -1 \\ -3 \end{bmatrix} \\ &= \begin{bmatrix} -1 + 1 + 3 + (-1) + (-6) + (-1) \\ 3 + 3 + (-2) + (-2) + 4 + (-3) \end{bmatrix} \\ &= \begin{bmatrix} -5 \\ 3 \end{bmatrix} \end{aligned}$$

11) *Is the coordinate system used in Figure 5.13 on page 63 a left-handed or right-handed coordinate system?*

Left-handed.

12) *Assume that Texas is flat. A minute of latitude is approximately 1.15 miles in length. At the authors' latitude (see section 3.2.1), a minute of longitude is approximately 0.97 miles in length. There are 60 minutes in one degree of latitude or longitude. How far apart are the authors?*

First, we need to convert degrees and minutes to miles. The latitudinal and longitudinal distances are:

$$\begin{array}{llcl} \text{Latitude} & 33^\circ 11' - 33^\circ 01' & = & 10' & \approx & 11.5\text{miles} \\ \text{Longitude} & 97^\circ 07' - 96^\circ 59' & = & 8' & \approx & 7.76\text{miles} \end{array}$$

FIXME

Chapter 7

1) *Given the following matrices:*

$$\mathbf{A} = \begin{bmatrix} 13 & 4 & -8 \\ 12 & 0 & 6 \\ -3 & -1 & 5 \\ 10 & -2 & 5 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 15 & 8 \\ -7 & 3 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0 & 1 & 3 \end{bmatrix} \quad \mathbf{E} = \begin{bmatrix} a & g \\ b & h \\ c & i \\ d & j \\ f & k \end{bmatrix} \quad \mathbf{F} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

a) *For each matrix **A** through **F** above, give the dimensions of the matrix and identify the matrix as square and/or diagonal.*

Matrix	Dimensions	Square	Diagonal
A	4×3	No	No
B	2×2	Yes	Yes
C	2×2	Yes	No
D	1×3	No	No
E	5×2	No	No
F	4×1	No	No

b) *Determine if the following matrix multiplications are allowed, and if so, give the dimensions of the resulting matrix.*

Product	**Dimensions**
DA	Undefined
AD	Undefined
BC	2×2
AF	Undefined
E^TB	Undefined
DFA	Undefined

c) *Compute the following transpositions:*

$$\mathbf{A}^T = \begin{bmatrix} 13 & 4 & -8 \\ 12 & 0 & 6 \\ -3 & -1 & 5 \\ 10 & -2 & 5 \end{bmatrix}^T = \begin{bmatrix} 13 & 12 & -3 & 10 \\ 4 & 0 & -1 & -2 \\ -8 & 6 & 5 & 5 \end{bmatrix}$$

$$\mathbf{E}^T = \begin{bmatrix} a & g \\ b & h \\ c & i \\ d & j \\ f & k \end{bmatrix}^T = \begin{bmatrix} a & b & c & d & f \\ g & h & i & j & k \end{bmatrix}$$

$$\mathbf{B}^T = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix}^T = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix}$$

2) *Compute the following products:*

a)

$$\begin{aligned} \begin{bmatrix} 1 & -2 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} -3 & 7 \\ 4 & 1/3 \end{bmatrix} &= \begin{bmatrix} (1)(-3)+(-2)(4) & (1)(7)+(-2)(1/3) \\ (5)(-3)+(0)(4) & (5)(7)+(0)(1/3) \end{bmatrix} \\ &= \begin{bmatrix} -3+(-8) & 7+(-2/3) \\ -15+0 & 35+0 \end{bmatrix} \\ &= \begin{bmatrix} -11 & 19/3 \\ -15 & 35 \end{bmatrix} \end{aligned}$$

b)

$$
\begin{aligned}
& \begin{bmatrix} 3 & -1 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 3 \\ 5 & 7 & -6 \\ 1 & -4 & 2 \end{bmatrix} \\
&= \begin{bmatrix} (3)(-2)+(-1)(5)+(4)(1) & (3)(0)+(-1)(7)+(4)(-4) & (3)(3)+(-1)(-6)+(4)(2) \end{bmatrix} \\
&= \begin{bmatrix} (-6)+(-5)+4 & 0+(-7)+(-16) & 9+6+8 \end{bmatrix} \\
&= \begin{bmatrix} -7 & -23 & 23 \end{bmatrix}
\end{aligned}
$$

3) *Manipulate the following matrix product to remove the parenthesis:*

$$
\begin{aligned}
& \left((\mathbf{AB})^T (\mathbf{CDE})^T\right)^T \\
&= \left((\mathbf{CDE})^T\right)^T \left((\mathbf{AB})^T\right)^T \\
&= (\mathbf{CDE})(\mathbf{AB}) \\
&= \mathbf{CDEAB}
\end{aligned}
$$

4) *What type of transformation is represented by the following 2D matrix:*

$$
\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}
$$

Extracting the basis vectors [0,-1] and [1,0] and drawing them on a coordinate grid, we see that the transformation matrix performs a clockwise rotation about the origin by 90 degrees.