# One-way to quantum computation

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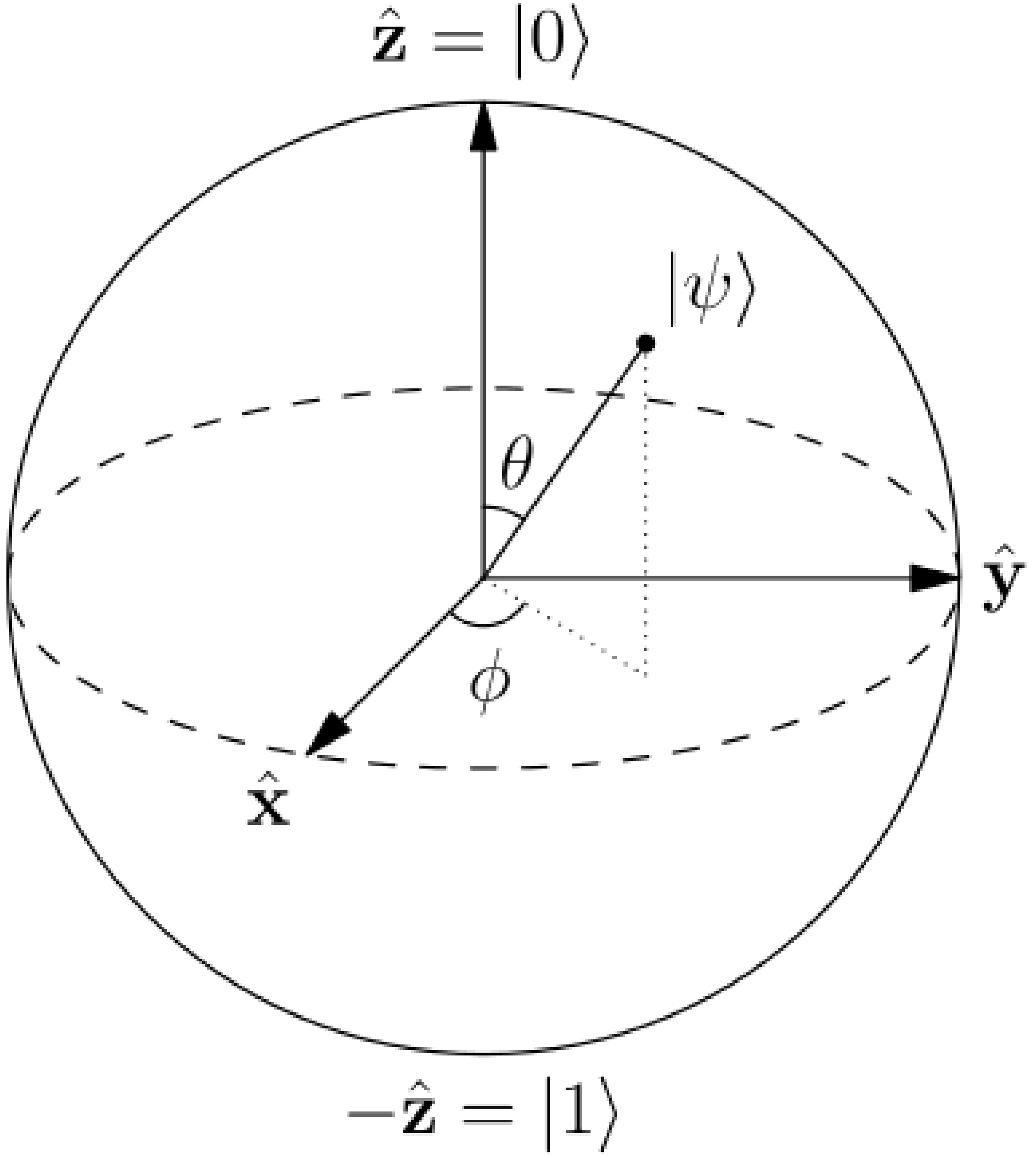
# Outline

- Qubits
- Quantum circuit model
- Cluster states
- One-way quantum computer
- Single-qubit measurements
- Applications
- Implementation and obstacles

http://www.slate.com/blogs/future_tense/2013/05/16/google_nasa_buy_d_wave_2_quantum_computer_what_will_they_do_with_it.html

# Qubits

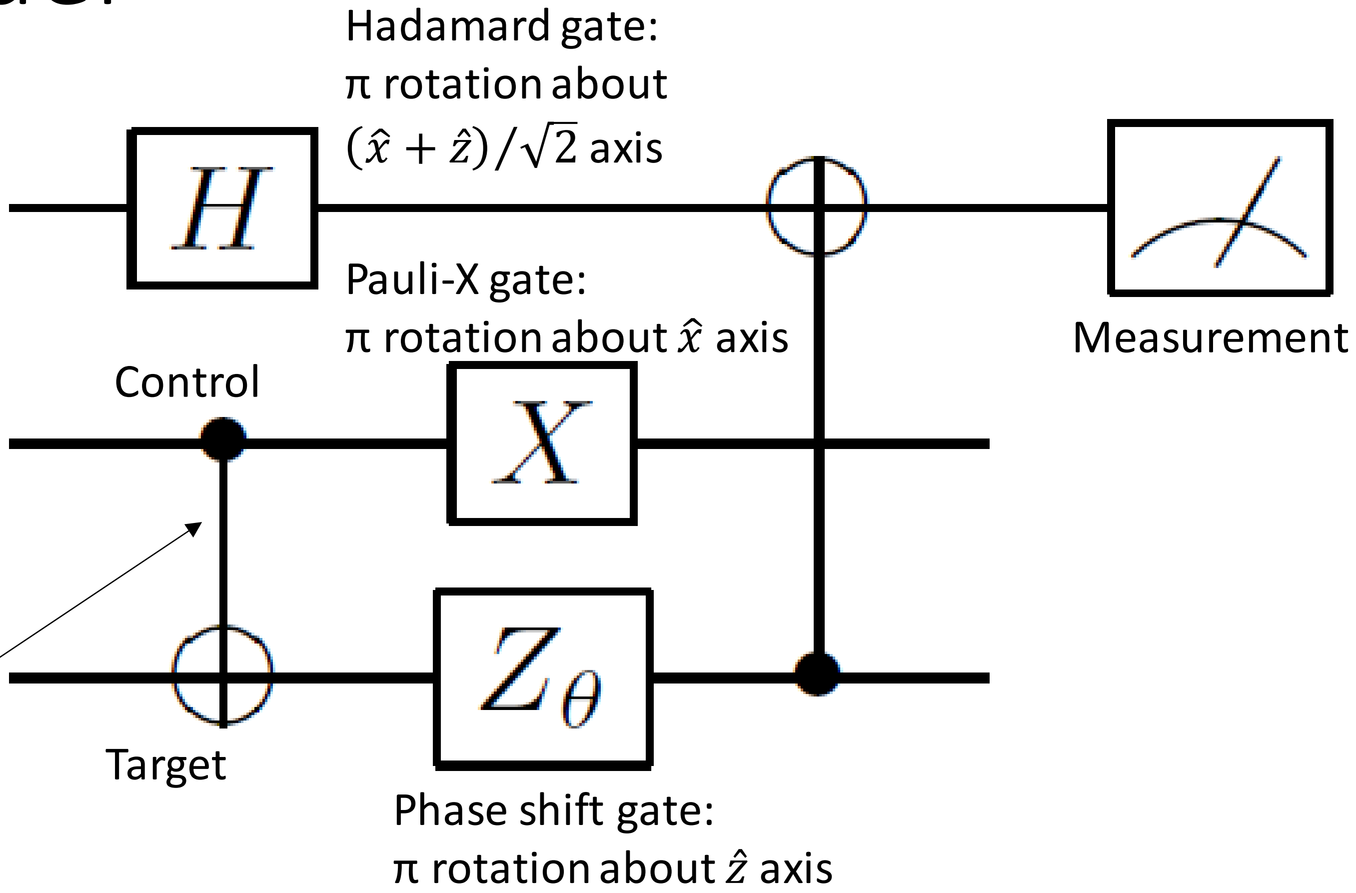
- Quantum bits
- Computational basis:
  - $|0\rangle$ and $|1\rangle$
- Additional superposition state
- Two-state quantum system
  - Spin of an electron
  - Polarisation of a photon
- Bloch sphere
  - Probabilities: θ
  - Phase: ϕ




http://upload.wikimedia.org/wikipedia/commons/thumb/f/f4/Bloch_Sphere.svg/163px-Bloch_Sphere.svg.png

# Quantum circuit model

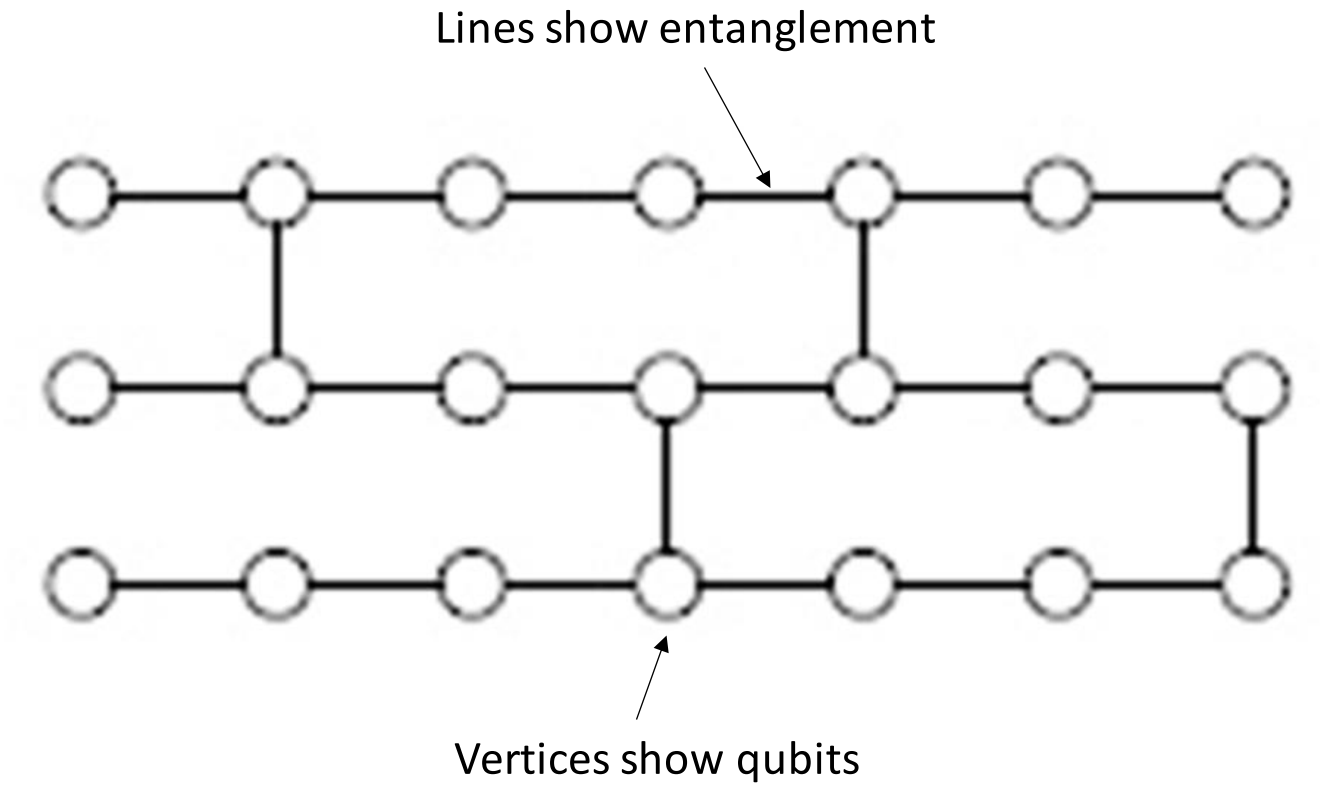
- Probabilistic
- Quantum wires: time
- One-qubit logic gates
  - Reversible transformations
  - Example: Pauli-X gate
    - Equivalent to NOT gate
    - $X\,\psi = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} b \\ a \end{pmatrix}$
- Two-qubit logic gates
  - Example: Controlled NOT gate
- Universal model




M. Nielson, *Cluster State Quantum Computation,* arXiv:quant-ph/0504097v2 (2005)

# Cluster states

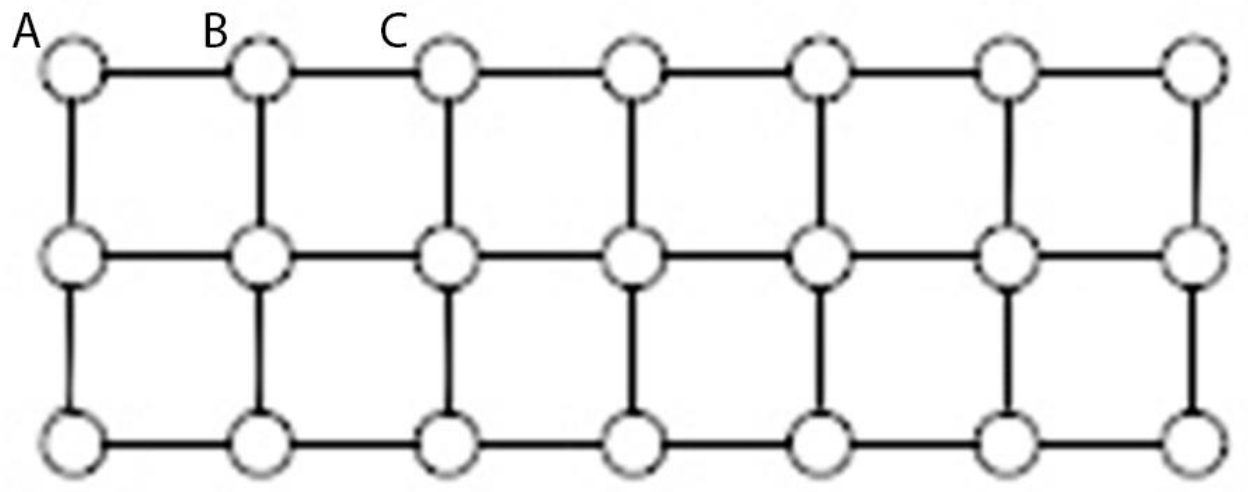
- Resource state for one-way quantum computer
- Particular graph state:
  - Multi-qubit system
  - Highly entangled
- Two-dimensional lattice sufficient for quantum computation




http://www.pieter-kok.staff.shef.ac.uk/index.php?nav=research&sub=cluster

# Entanglement

- Inseparable wave function
- Qubit A is entangled with C if B is entangled with each of A and C
- Completely entangled state for each qubit entangled to its neighbours
- Why do we want this arrangement?

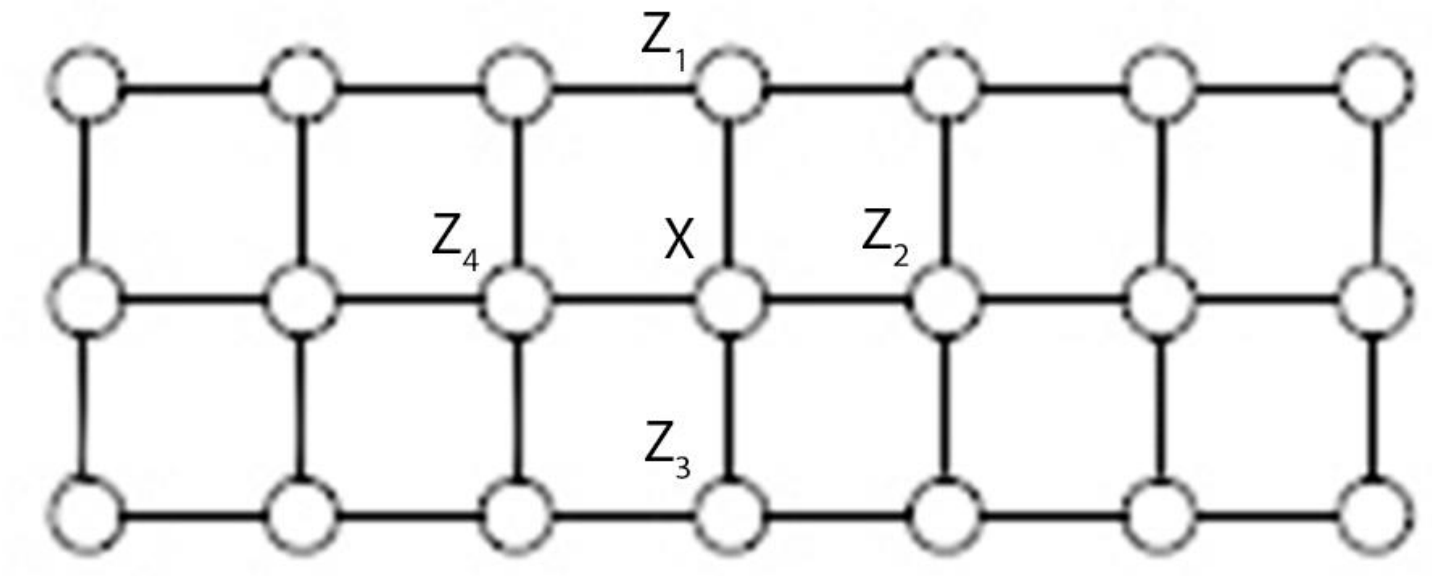



Original image from:
http://www.pieter-kok.staff.shef.ac.uk/index.php?nav=research&sub=cluster

# Universal cluster states

- Defined by stabilizer operators:

$$S_v \equiv X_v \bigotimes_{v'} Z_{v'}$$




- Such that

$$S_v|\psi\rangle = |\psi\rangle$$

- Pauli matrices

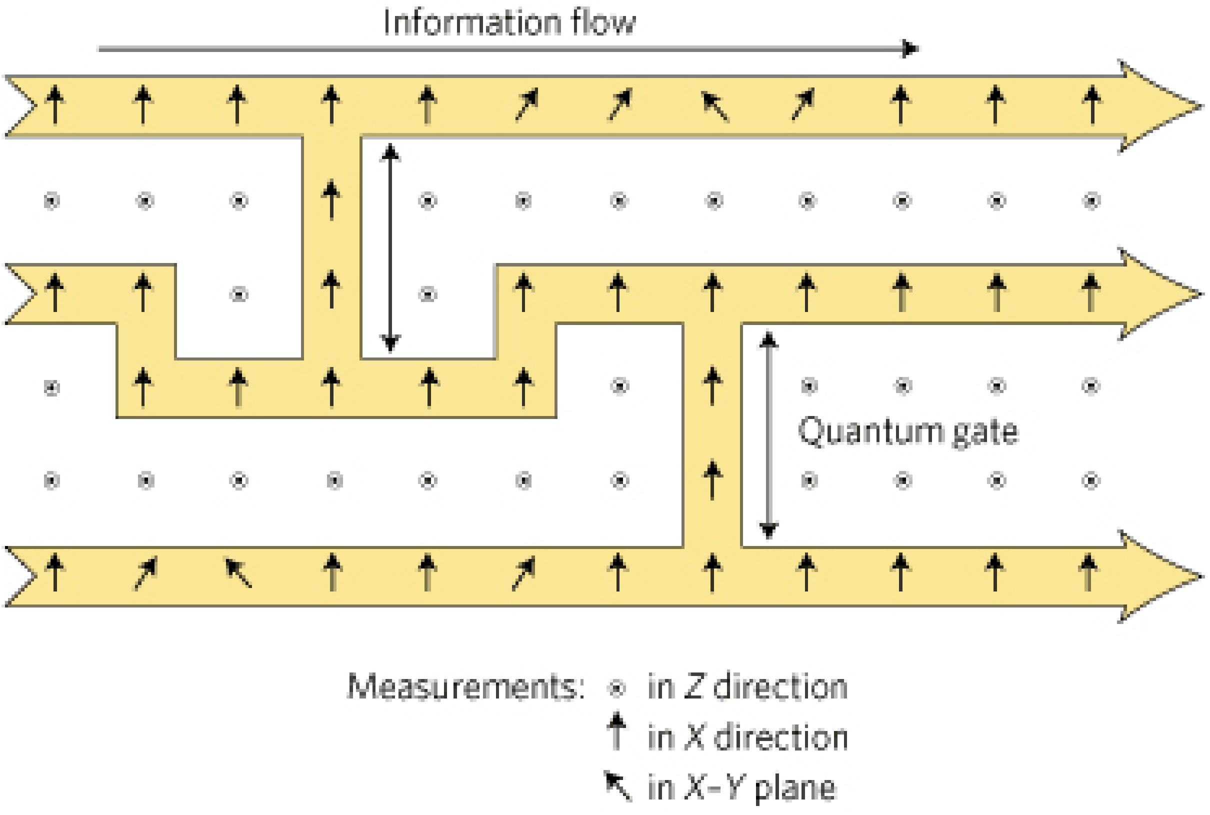
$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Universal resource state
  - Any computation can be performed from this substrate

Original image:
http://www.pieter-kok.staff.shef.ac.uk/index.php?nav=research&sub=cluster

# One-way quantum computer

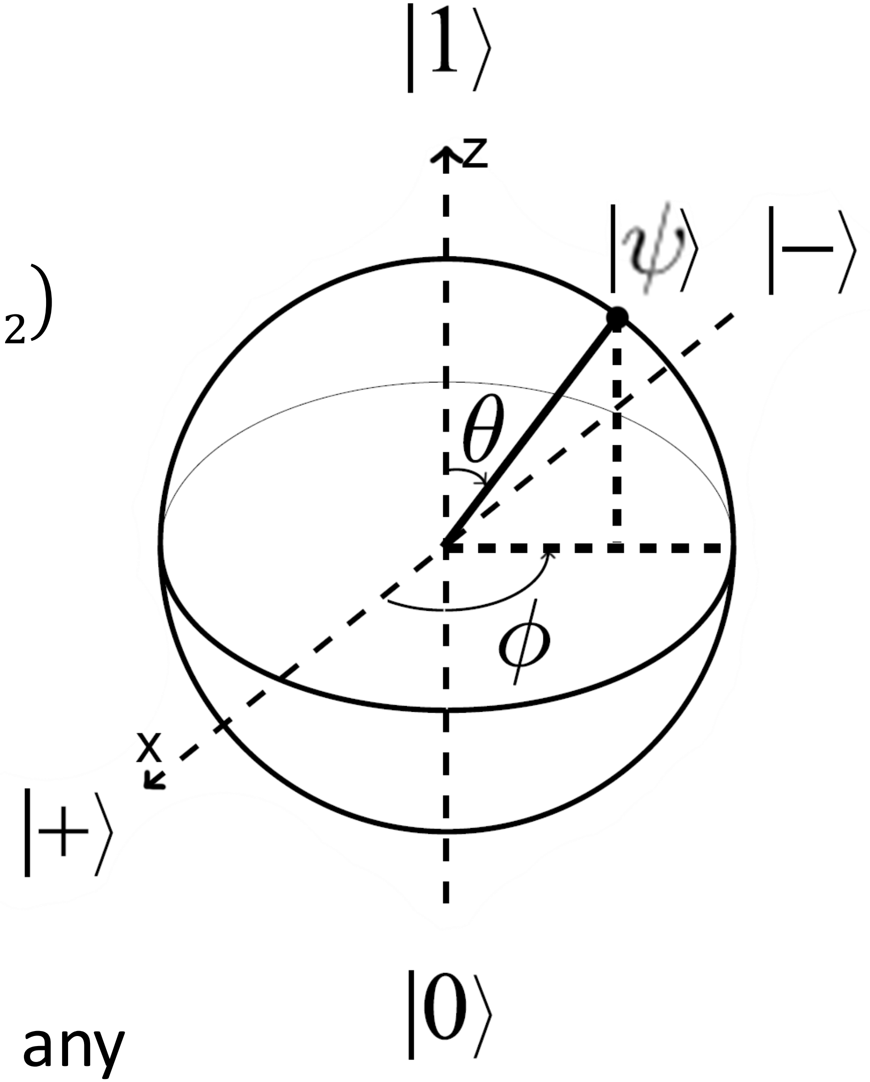
- Sequence of single-qubit measurements
  - Irreversible: one-way
  - Individual measurements random
  - Each influences next measurement:
    - Classical feed-forward
- Order and basis choices of measurement pattern form algorithm
- *Physical* and *encoded* qubits




http://www.nature.com/nphys/journal/v5/n1/box/nphys1157_BX2.html

# Single-qubit measurements

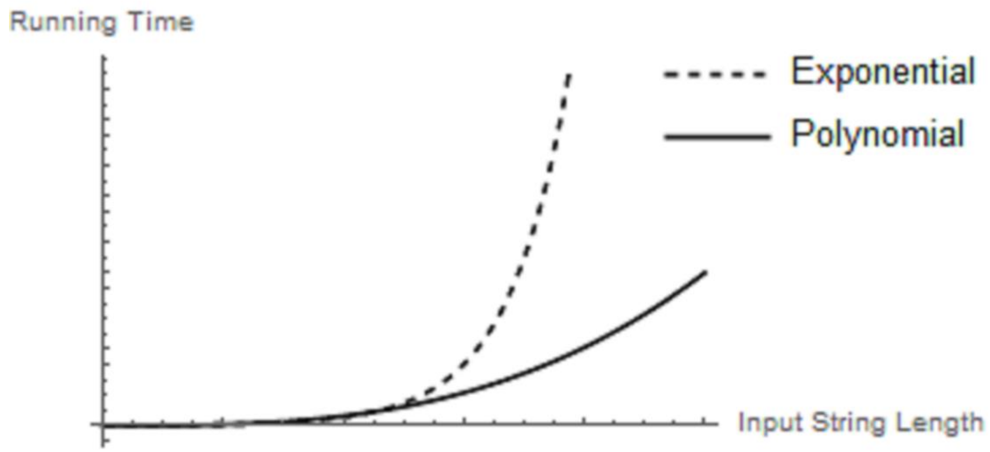
- Two-qubit example
  - $|\psi\rangle_1 = \alpha|0\rangle_1 + \beta|1\rangle_1$ and $|+\rangle_2 = \frac{1}{\sqrt{2}}\left(|0\rangle_2 + |1\rangle_2\right)$
- Perform CZ operation on qubits:
  - $\frac{1}{\sqrt{2}}\left(\alpha|0\rangle_1|+\rangle_2 + \beta|1\rangle_1|-\rangle_2\right)$
- Measure projection of qubit 1 in x,y-plane.
- Qubit 2 projected into state of either:
  - $\alpha|+\rangle_2 + e^{i\phi}\beta|-\rangle_2$
  - $\alpha|+\rangle_2 - e^{i\phi}\beta|-\rangle_2$
- Repeat protocol a further two times to perform any rotation by choosing subsequent $\phi$ based on previous measurement outcomes




Original image: Brown, Briegel, *One-way Quantum Computation,* arXiv:quant-ph/0504097v2 (2006)

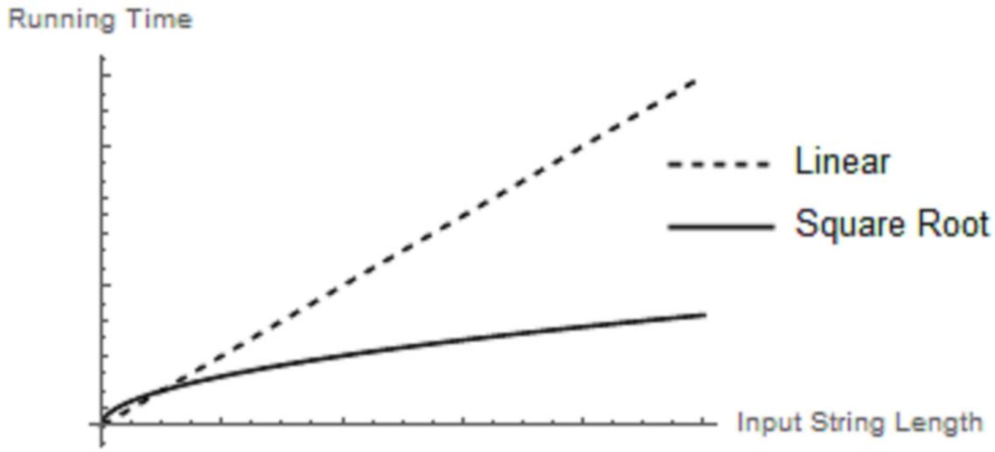
# Applications

- Time complexity:
  - running time as a function of input size




- Shor’s Algorithm
  - Prime factorisation of integers
  - Public-key cryptosystems, eg. RSA

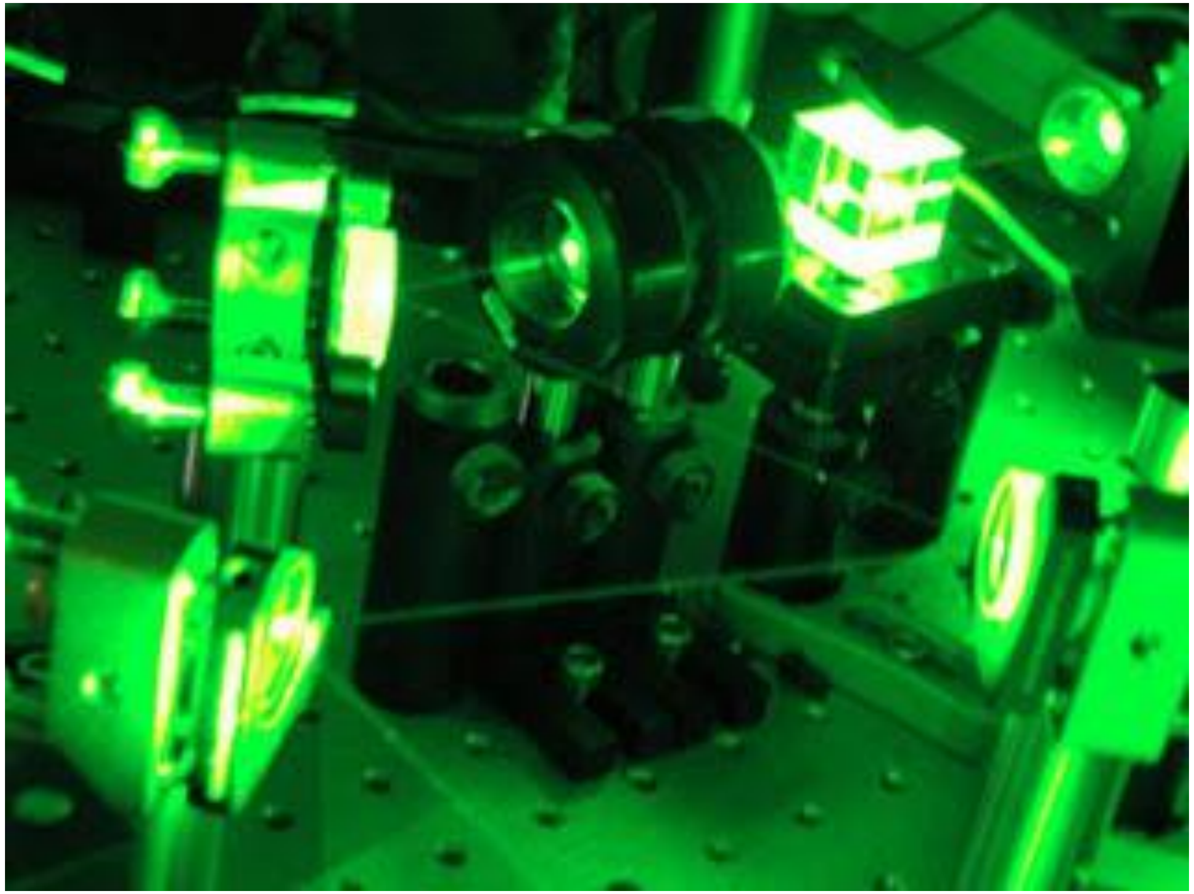
- Determination of small pieces of information from large numbers of intermediate results




- Grover’s Algorithm
  - Database searching

# Implementation and Obstacles

- Basic principles demonstrated
  - Four-qubit cluster state generated using photon polarisation states, and Grover's Algorithm implemented on it in 2005
- Decoherence and scalability
  - Optical quantum computers
  - Solid-state quantum computers
- Problem reduced to:
  - Preparing a specific state
  - Performing single qubit measurements



Next Big Future, *A new scheme for photonic quantum computing,* http://nextbigfuture.com/2011/10/new-scheme-for-photonic-quantum.html (accessed 23 Nov 2014)

# Quantum computation in one-way

- Built on a universal resource:
  - The cluster state
- Performs calculations by single-qubit measurements only
- Applications:
  - Grover’s and Shor’s algorithms demonstrate the potential
- Obstacles:
  - Decoherence and scalability
- One-way computers offer a single universal device which can efficiently perform calculations intractable on classical computers