

# Supplementary Materials

January 28th, 2022

This document is for supporting the paper [Lee T. et al. (2022), "A Gaussian Process State-space Model for Sea Surface Temperature Reconstruction from the Alkenone Paleotemperature Proxy", *submitted*] that has been submitted to the 11th International Conference on Climate Informatics (CI2022) which will be held at Asheville, North Carolina, on May 9-13, 2022.

## S1 MULLER

This section contains figures that visualize the results from the calibration curve MULLER.

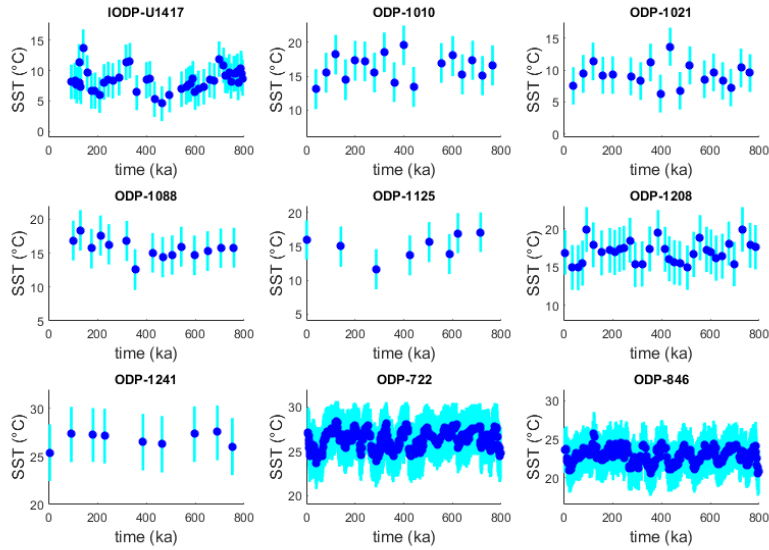


Figure S1: An overview of point-wise SST inference for nine sediment cores. In each panel, blue bars and dots are the 95% confidence intervals and medians of the posterior samples, respectively.

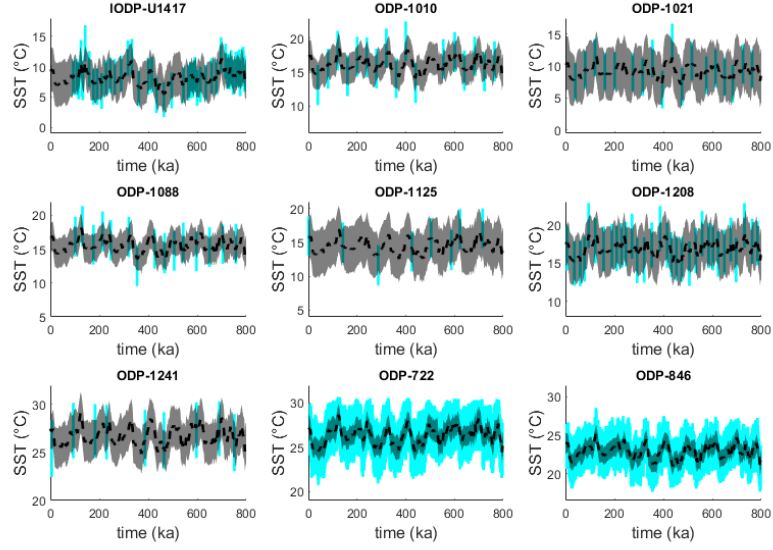


Figure S2: An overview of GPST SST inference for nine sediment cores. In each panel, blue bars are the 95% confidence intervals of the point-wise posterior samples. Black regions and dashed curves are the 95% confidence bands and medians of the GPST posterior samples, respectively.

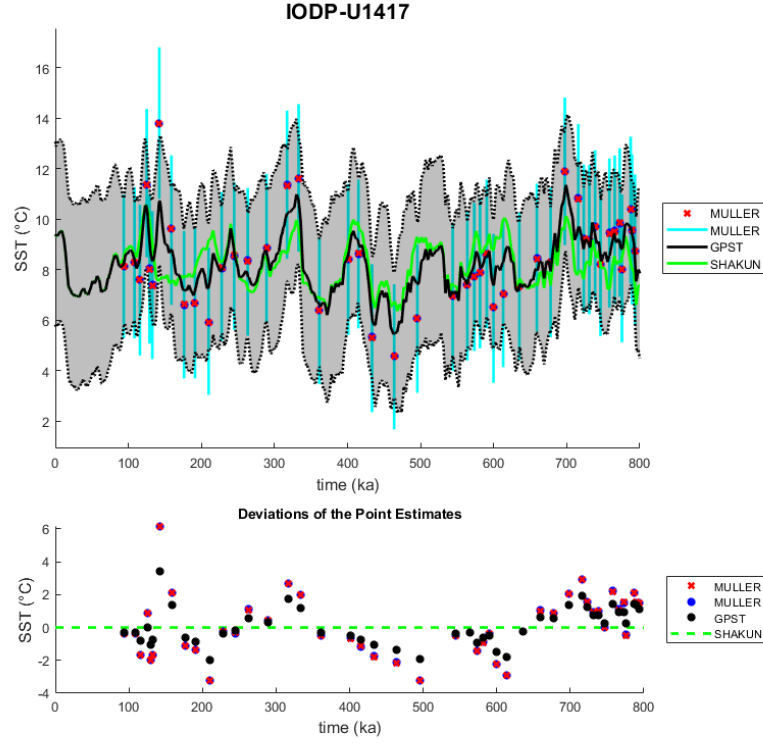


Figure S3: SST inference of the sediment core IODP-U1417. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^K$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

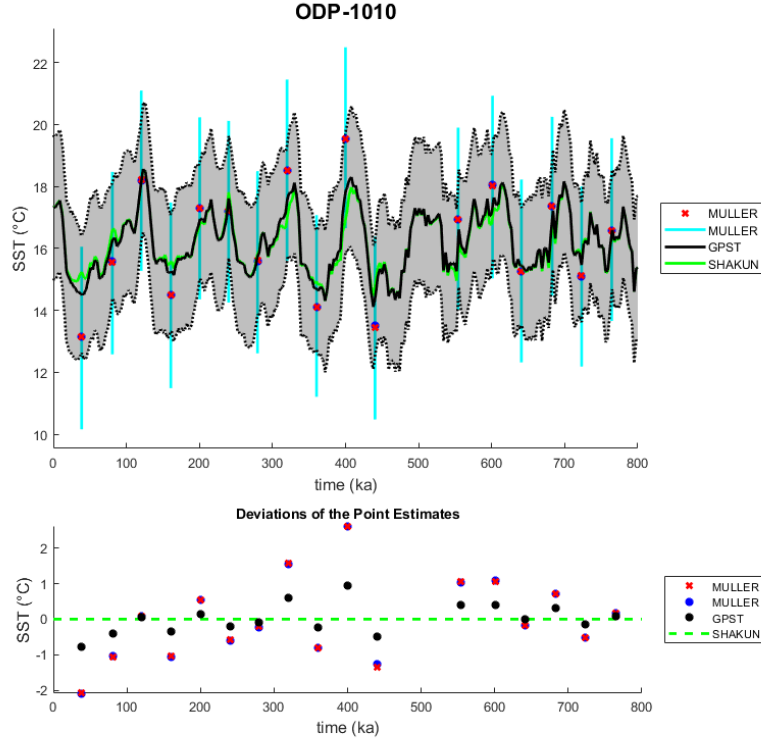


Figure S4: SST inference of the sediment core ODP-1010. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K'}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

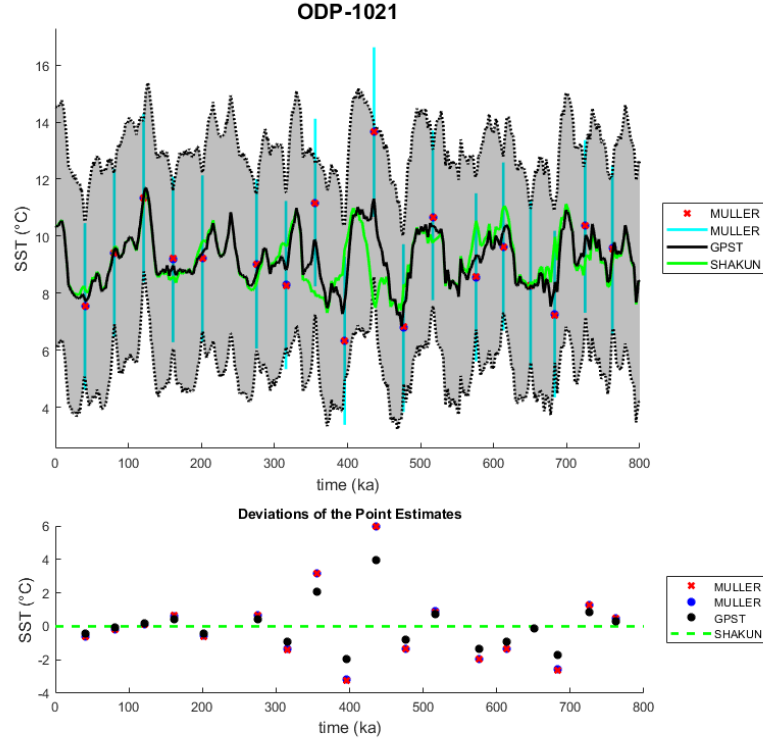


Figure S5: SST inference of the sediment core ODP-1021. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K/}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

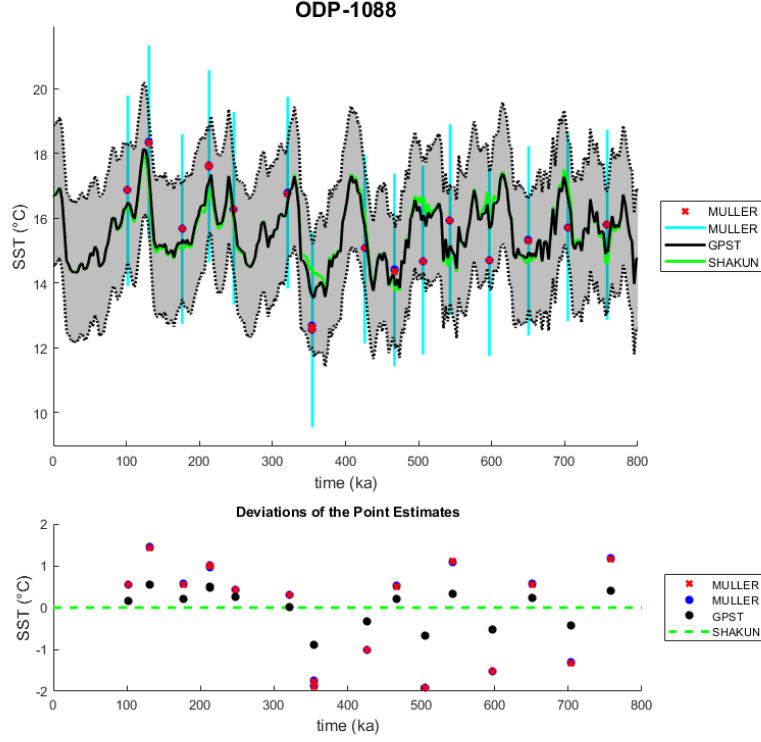


Figure S6: SST inference of the sediment core ODP-1088. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K'}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

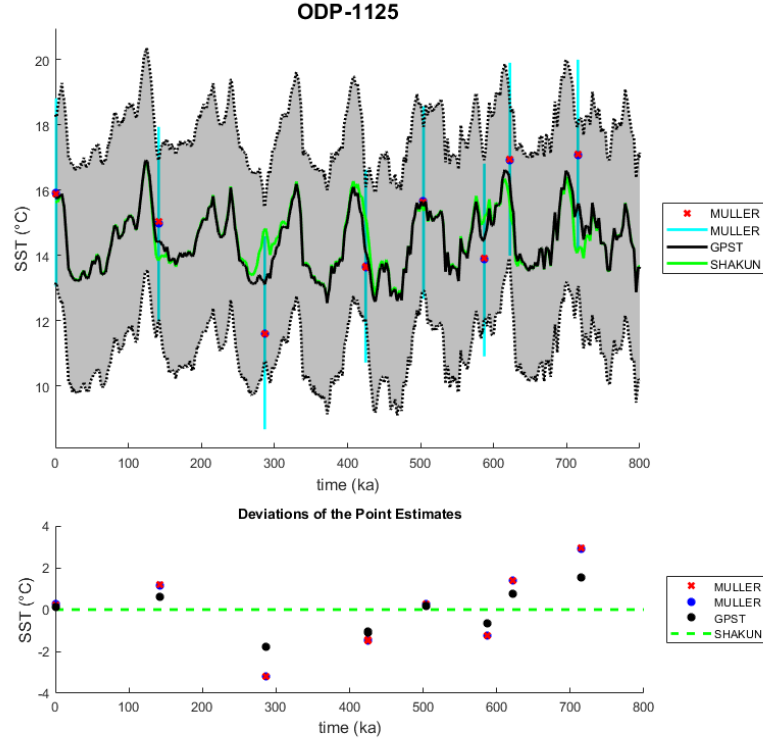


Figure S7: SST inference of the sediment core ODP-1125. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K/}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

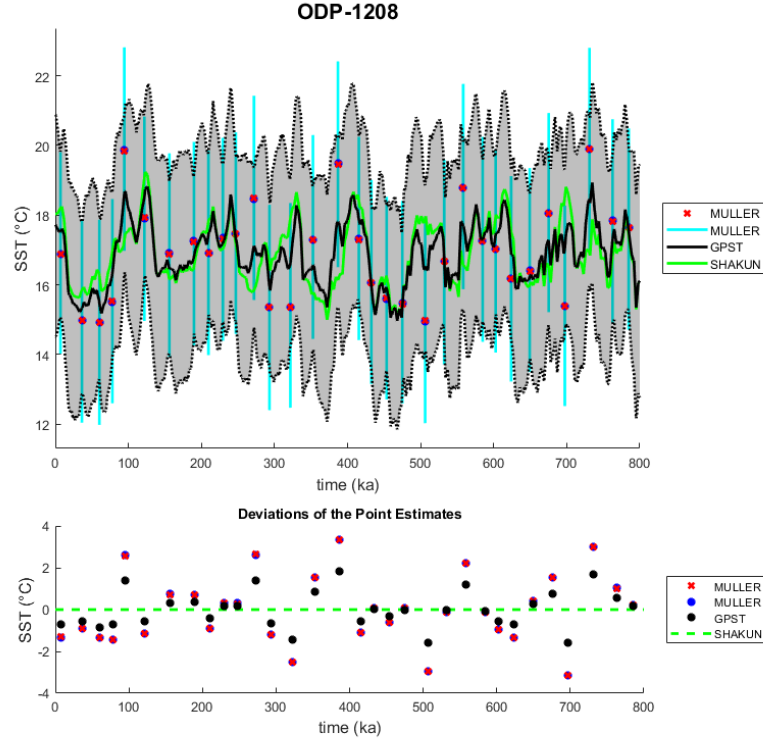


Figure S8: SST inference of the sediment core ODP-1208. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K/}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).



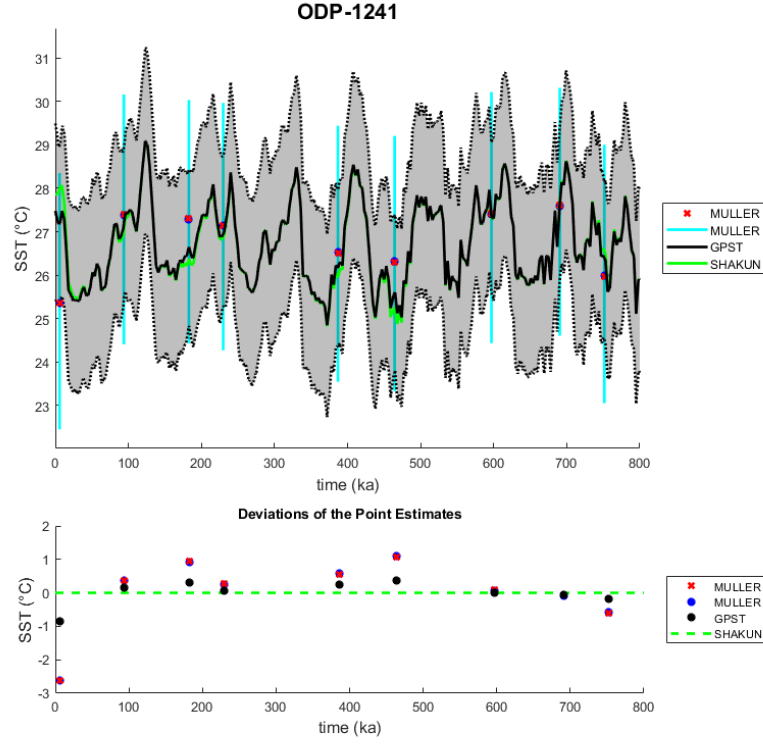


Figure S9: SST inference of the sediment core ODP-1241. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K/}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

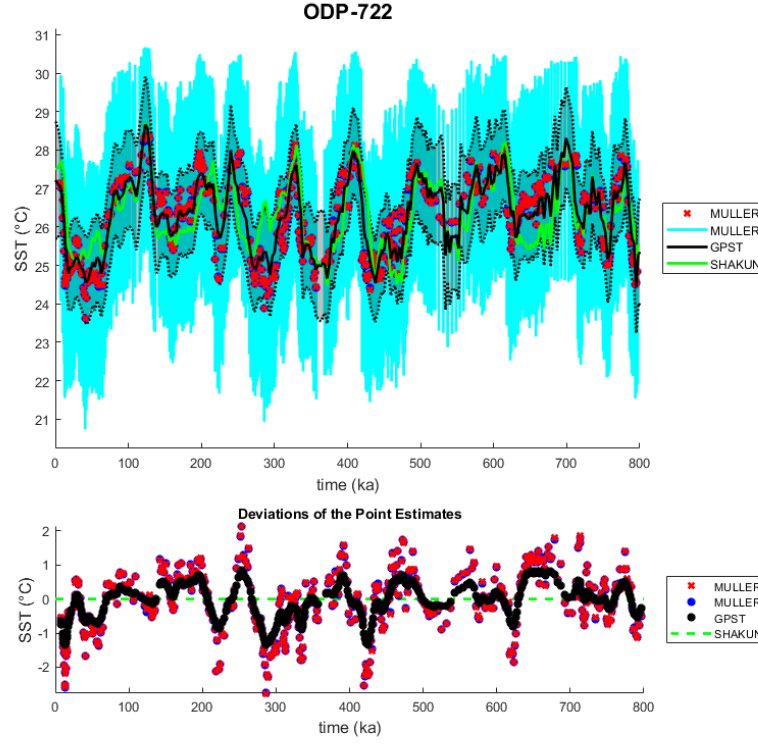


Figure S10: SST inference of the sediment core ODP-722. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K'}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

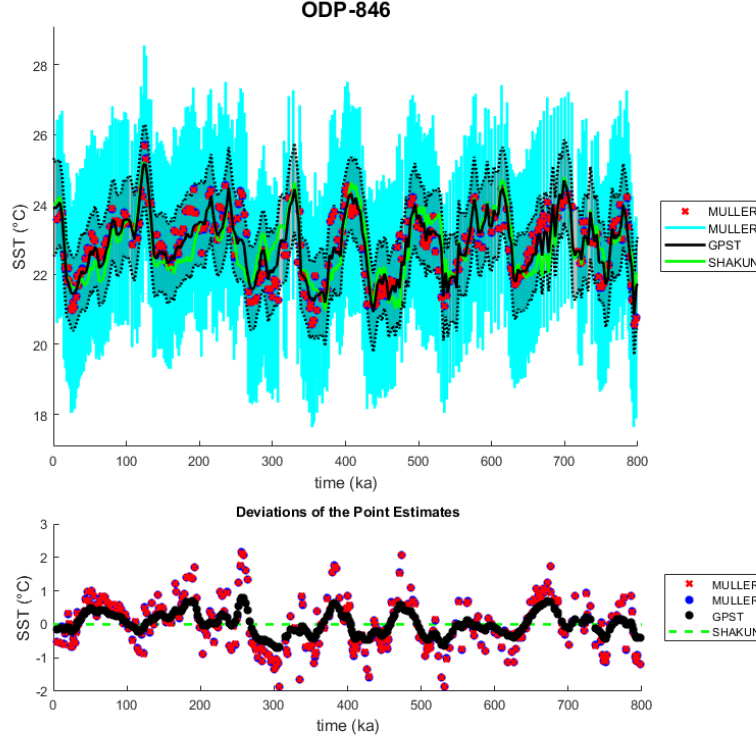


Figure S11: SST inference of the sediment core ODP-846. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K'}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

Core	Variance (point-wise)	Variance (GPST)
IODP-U1417	3.2752	1.5209
ODP-1010	3.4217	1.5006
ODP-1021	3.2281	1.4927
ODP-1088	2.5257	1.4121
ODP-1125	3.1616	1.1209
ODP-1208	1.9682	0.8070
ODP-1241	0.5852	0.4012
ODP-722	1.0772	0.7920
ODP-846	0.9397	0.7188

Figure S12: Variances of the SST estimates (medians of the posterior samples).

## 1 S2 BAYSPLINE

2 This section contains figures that visualize the results from the calibration curve  
3 BAYSPLINE.

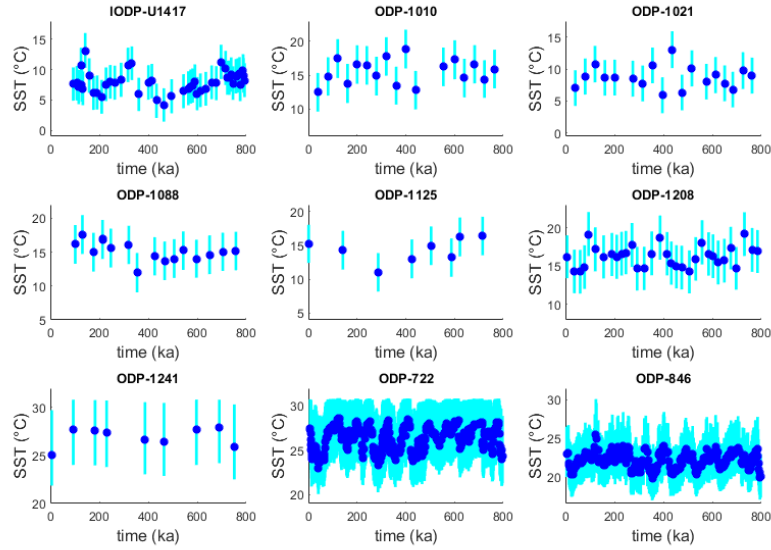


Figure S13: An overview of point-wise SST inference for nine sediment cores. In each panel, blue bars and dots are the 95% confidence intervals and medians of the posterior samples, respectively.

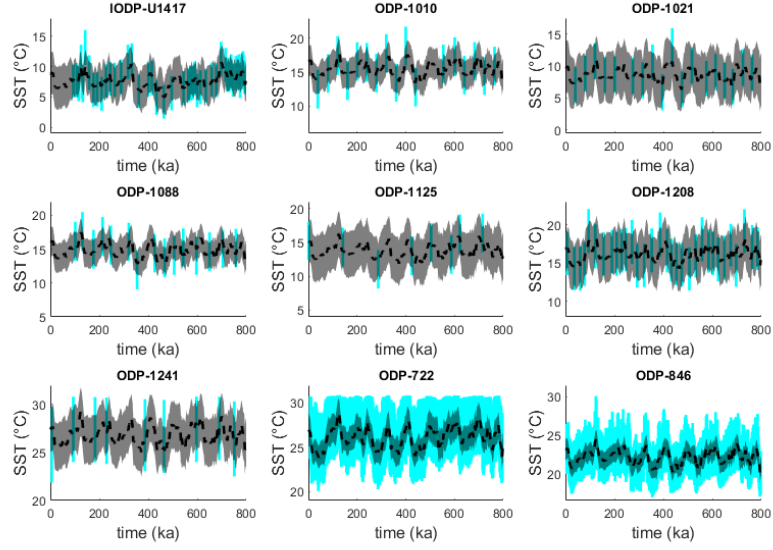


Figure S14: An overview of GPST SST inference for nine sediment cores. In each panel, blue bars are the 95% confidence intervals of the point-wise posterior samples. Black regions and dashed curves are the 95% confidence bands and medians of the GPST posterior samples, respectively.

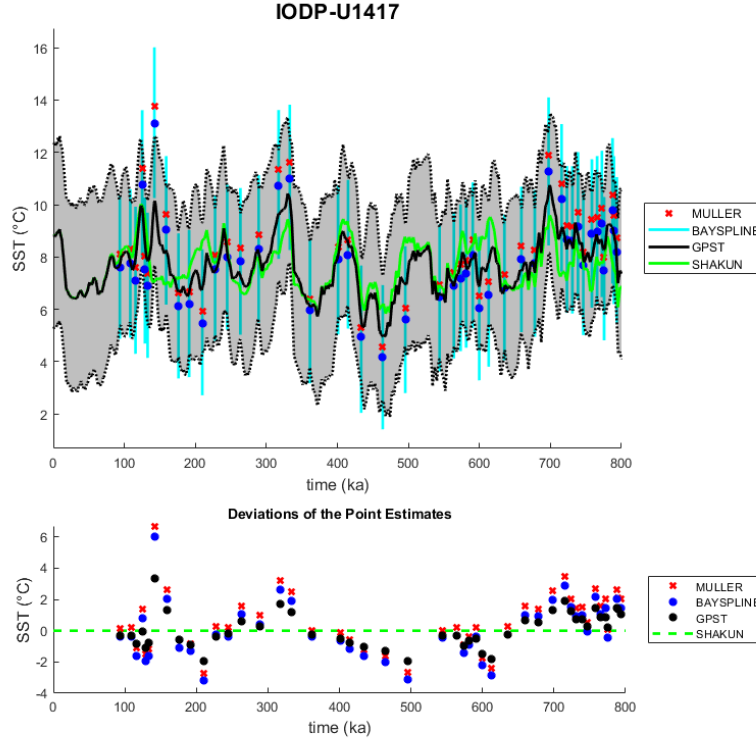


Figure S15: SST inference of the sediment core IODP-U1417. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K/}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

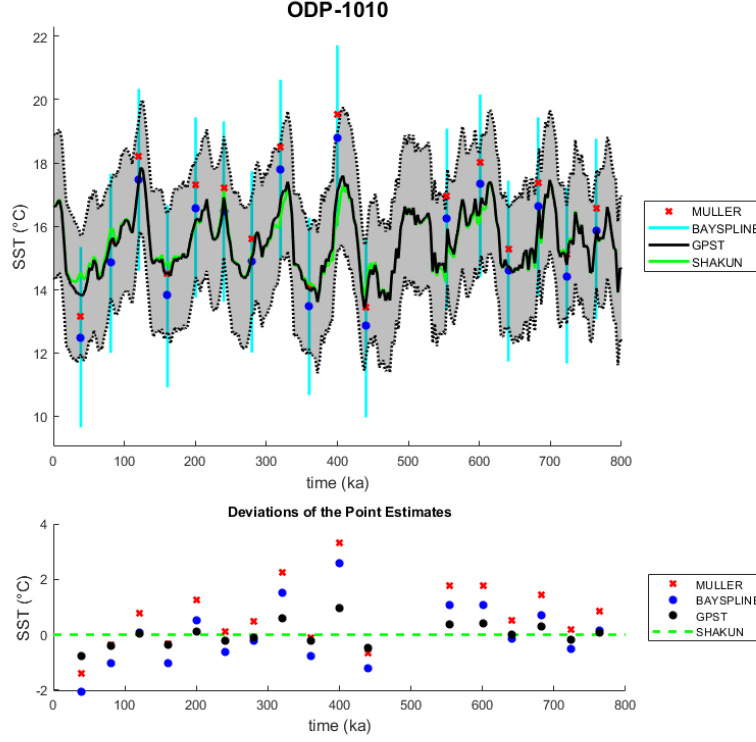


Figure S16: SST inference of the sediment core ODP-1010. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K/}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

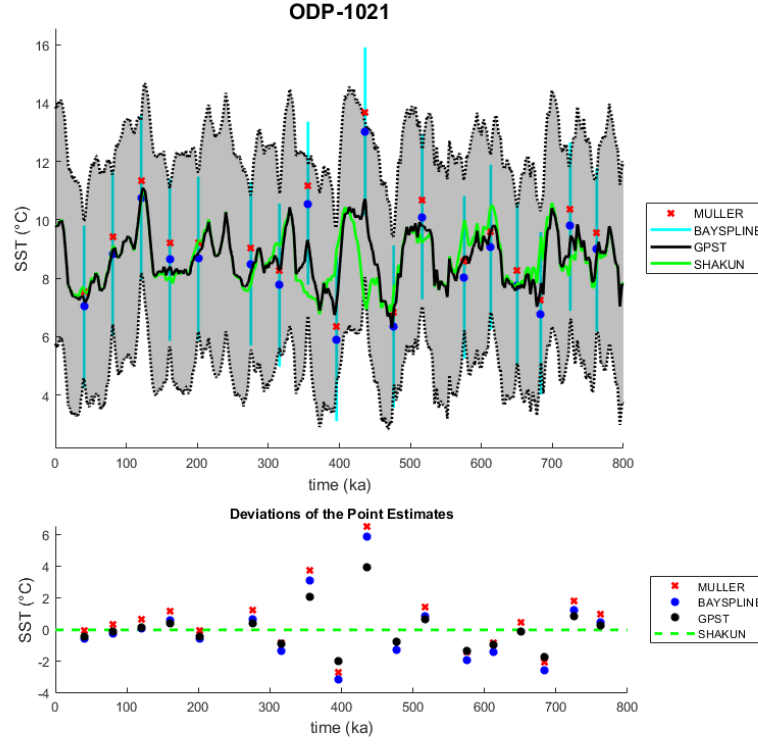


Figure S17: SST inference of the sediment core ODP-1021. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K'}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).



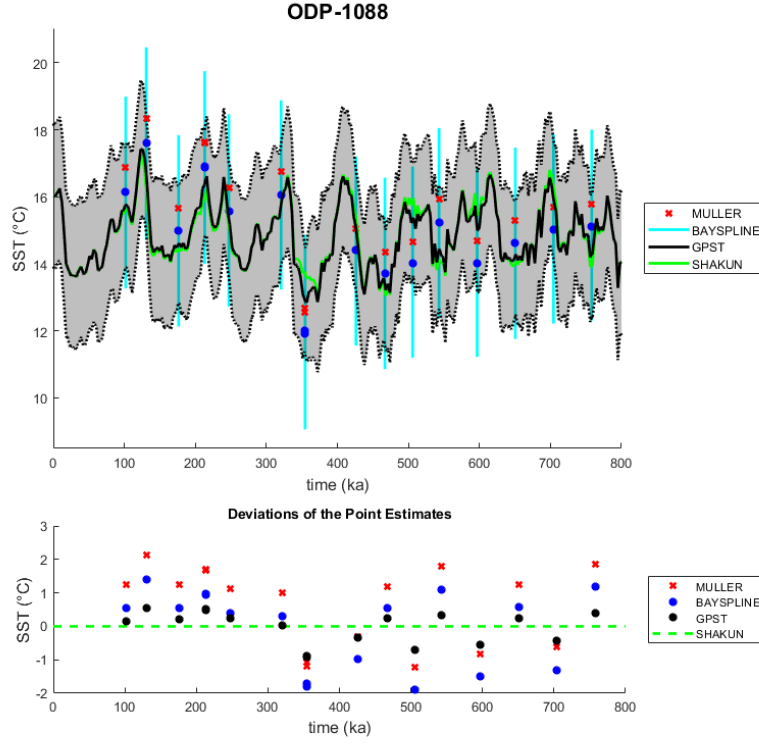


Figure S18: SST inference of the sediment core ODP-1088. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K/}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

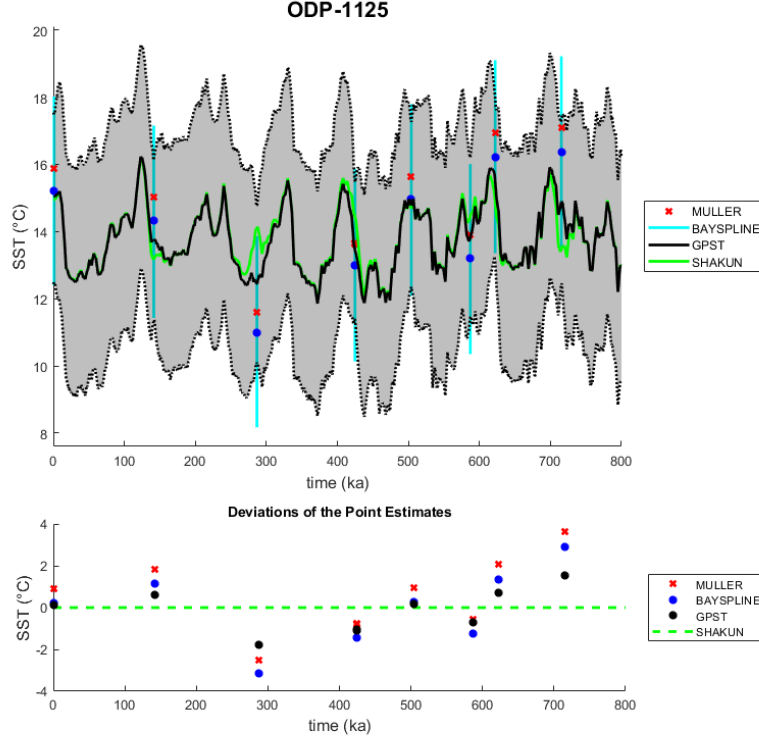


Figure S19: SST inference of the sediment core ODP-1125. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K/}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

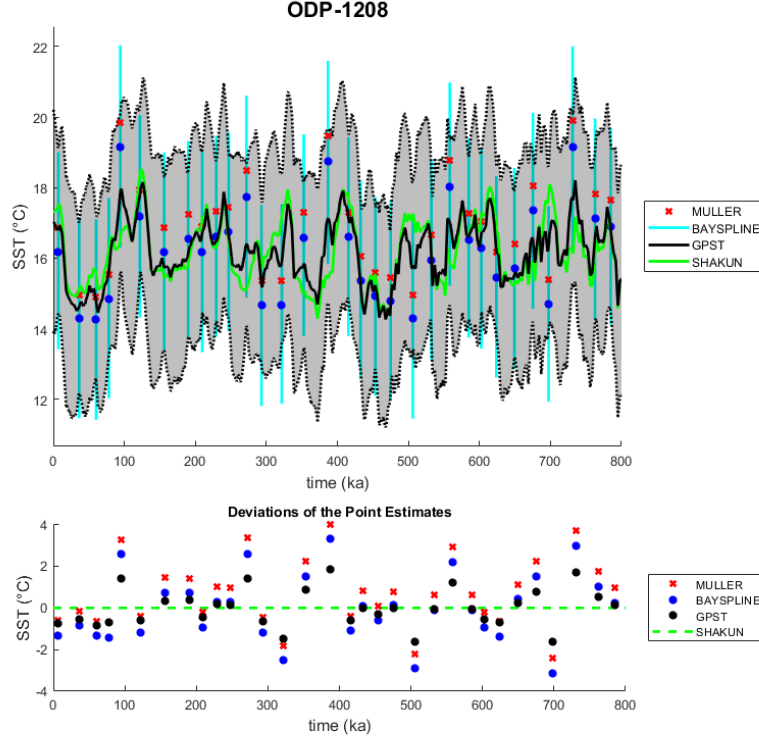


Figure S20: SST inference of the sediment core ODP-1208. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^K$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

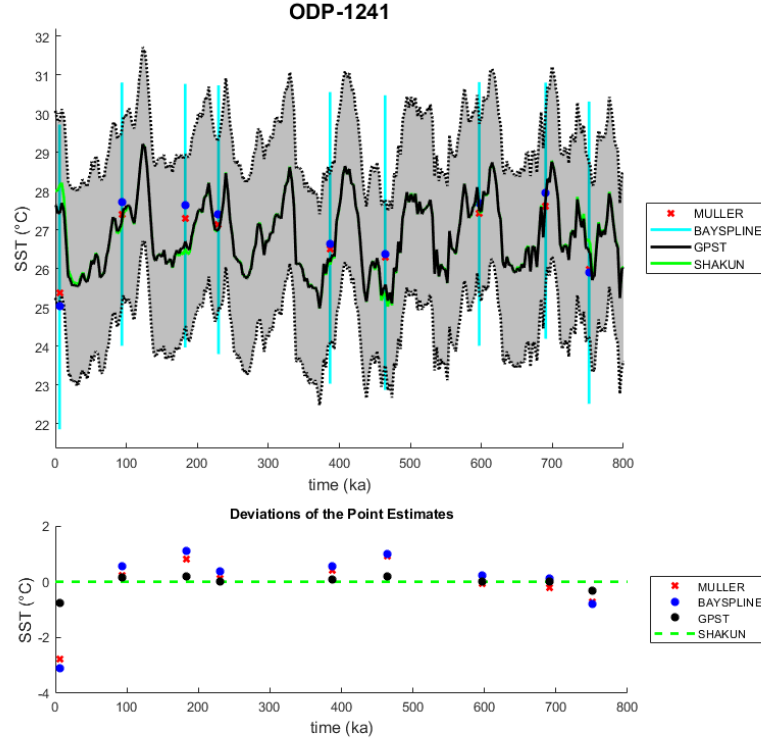


Figure S21: SST inference of the sediment core ODP-1241. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K'}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

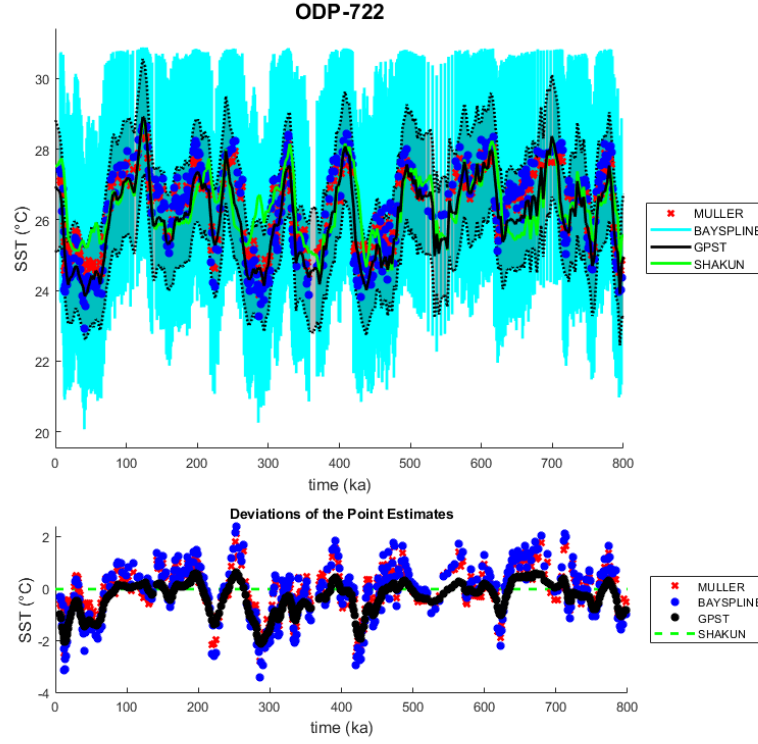


Figure S22: SST inference of the sediment core ODP-722. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K'}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

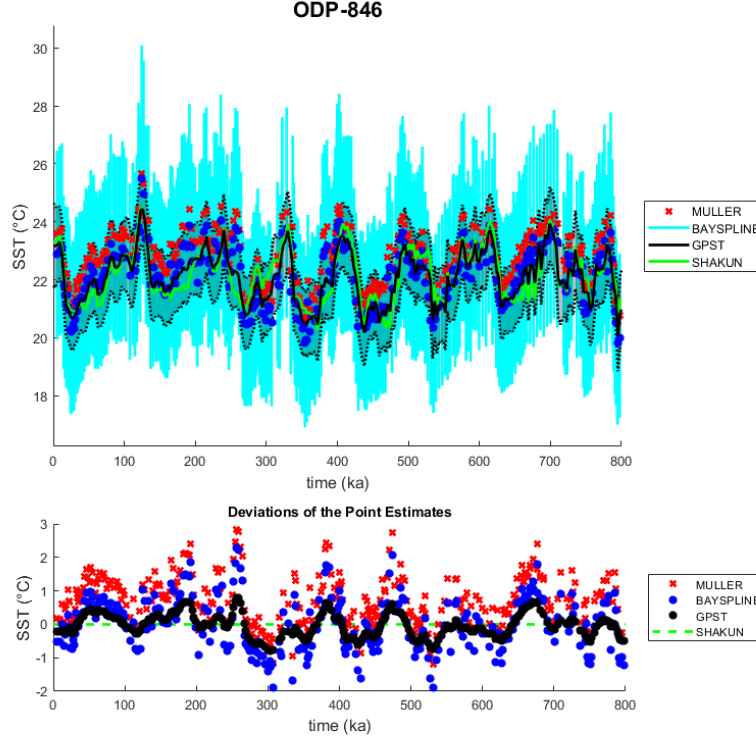


Figure S23: SST inference of the sediment core ODP-846. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^K$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

Core	Variance (point-wise)	Variance (GPST)
IODP-U1417	3.0908	1.4755
ODP-1010	3.3182	1.4989
ODP-1021	3.0555	1.4509
ODP-1088	2.4512	1.4504
ODP-1125	3.0613	1.1127
ODP-1208	1.9145	0.8076
ODP-1241	1.0000	0.5221
ODP-722	1.8043	1.2084
ODP-846	1.0435	0.7268

Figure S24: Variances of the SST estimates (medians of the posterior samples).

### S3 HGPR

This section contains figures that visualize the results from the calibration curve HGPR.

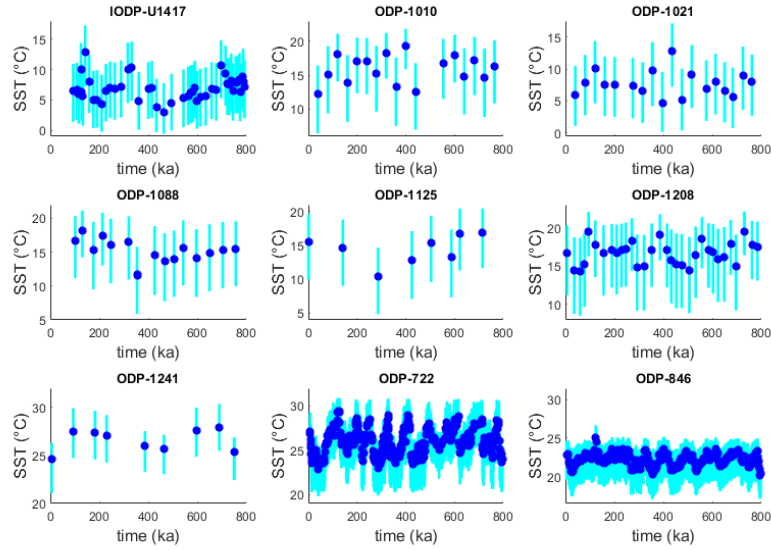


Figure S25: An overview of point-wise SST inference for nine sediment cores. In each panel, blue bars and dots are the 95% confidence intervals and medians of the posterior samples, respectively.

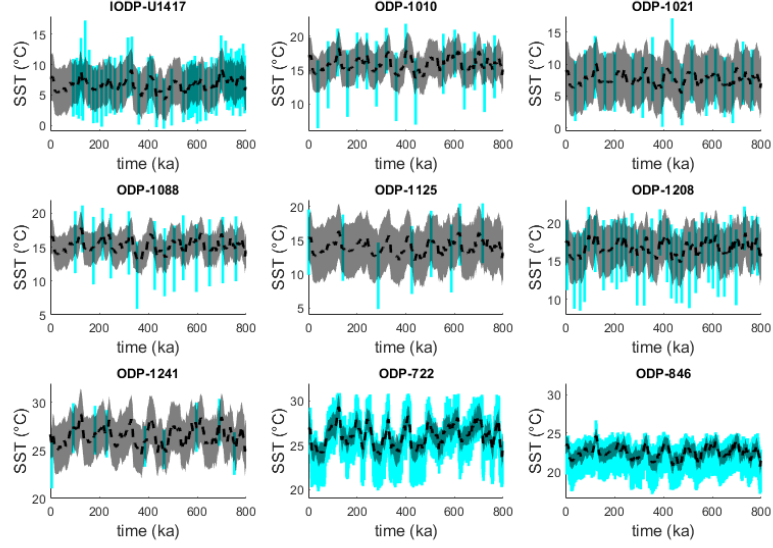


Figure S26: An overview of GPST SST inference for nine sediment cores. In each panel, blue bars are the 95% confidence intervals of the point-wise posterior samples. Black regions and dashed curves are the 95% confidence bands and medians of the GPST posterior samples, respectively.



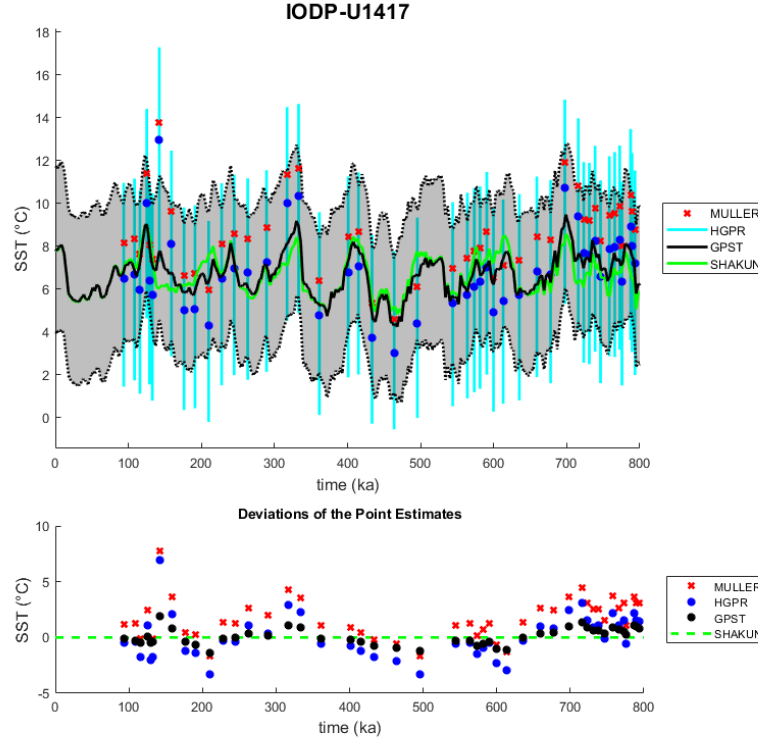


Figure S27: SST inference of the sediment core IODP-U1417. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K/}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

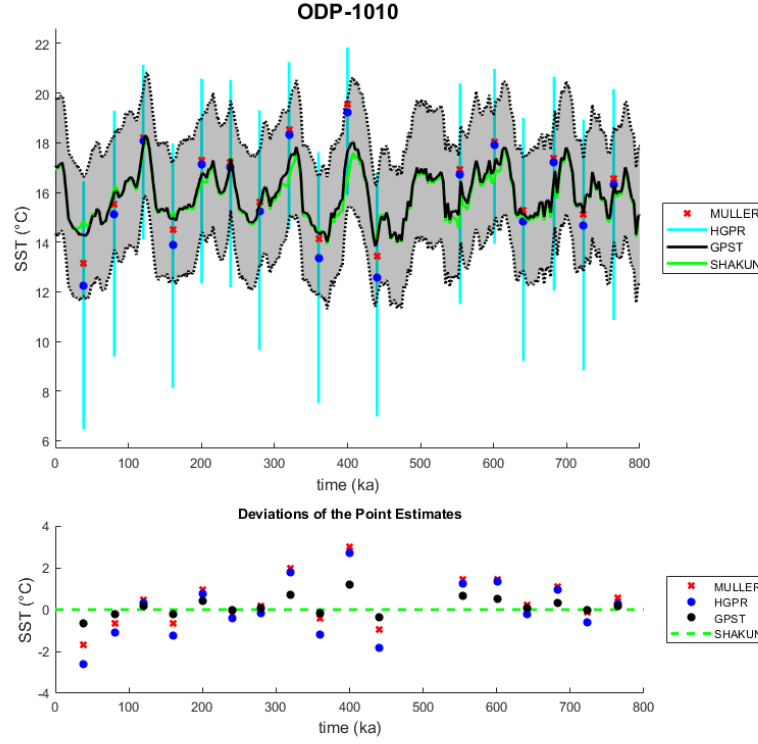


Figure S28: SST inference of the sediment core ODP-1010. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K/}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

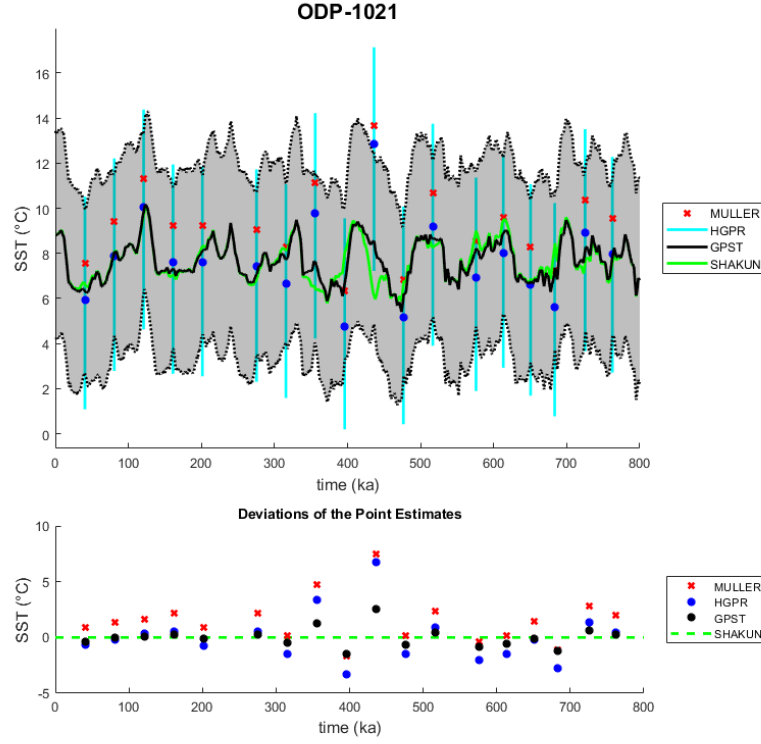


Figure S29: SST inference of the sediment core ODP-1021. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K'}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

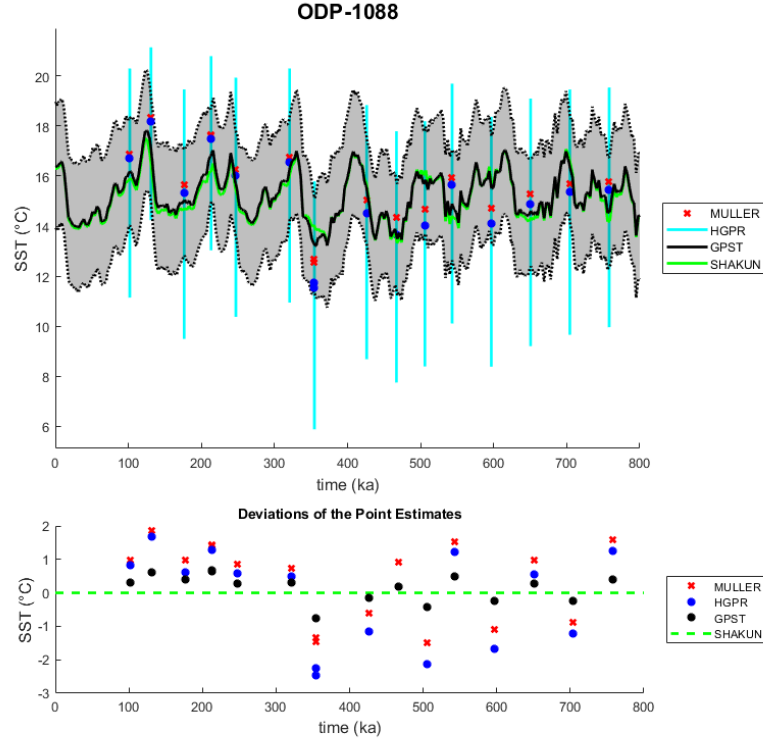


Figure S30: SST inference of the sediment core ODP-1088. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K/}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

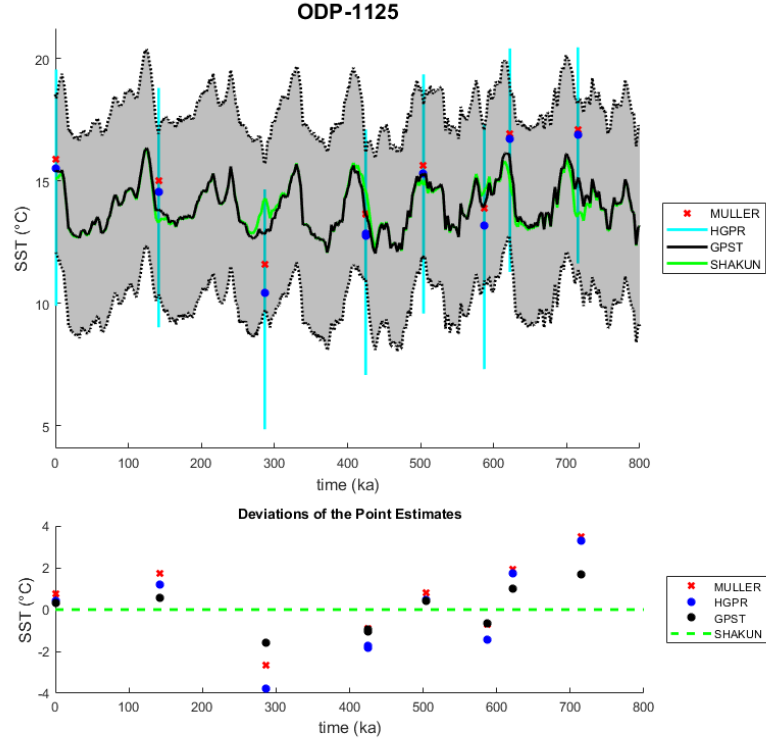


Figure S31: SST inference of the sediment core ODP-1125. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K/}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

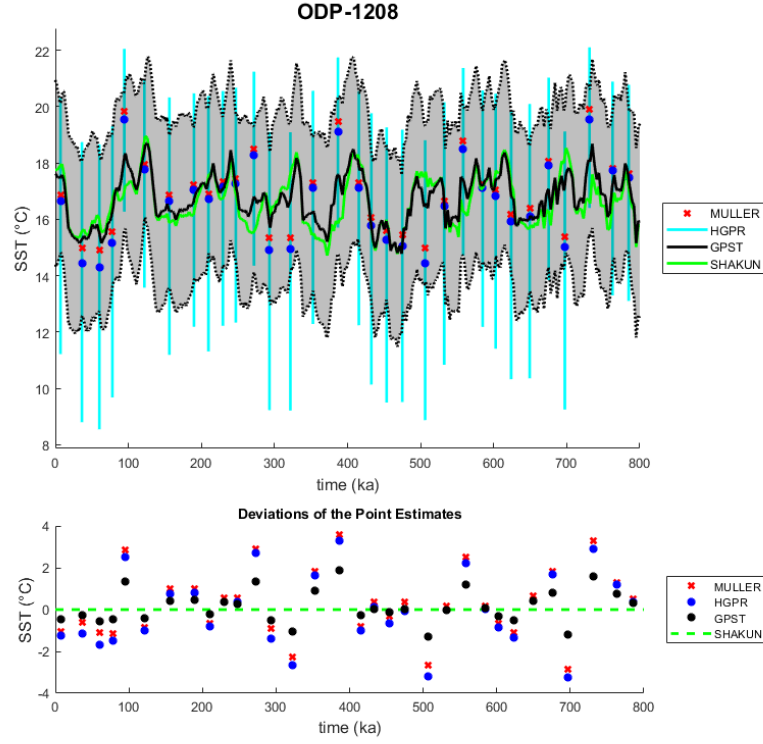


Figure S32: SST inference of the sediment core ODP-1208. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K/}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

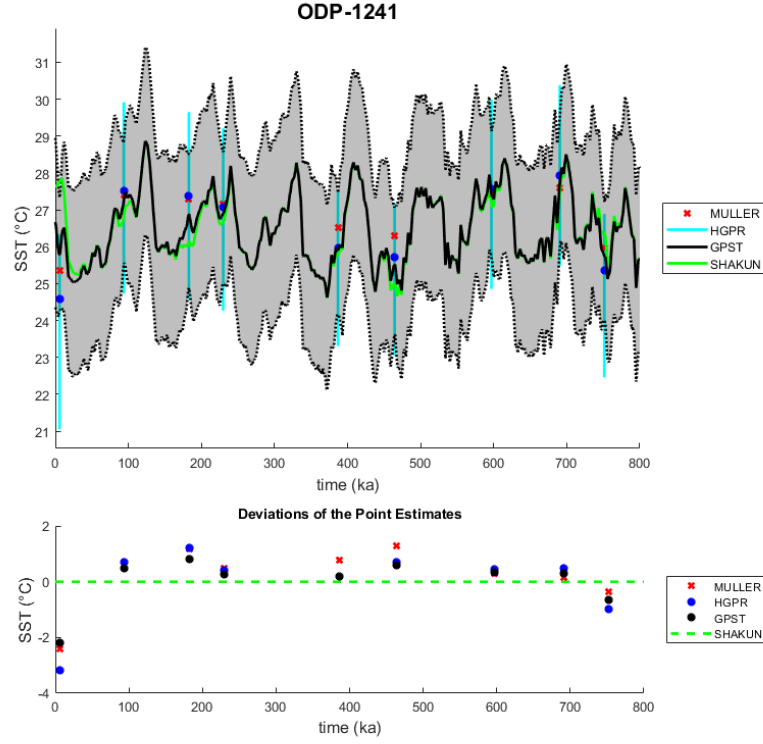


Figure S33: SST inference of the sediment core ODP-1241. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{K/}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

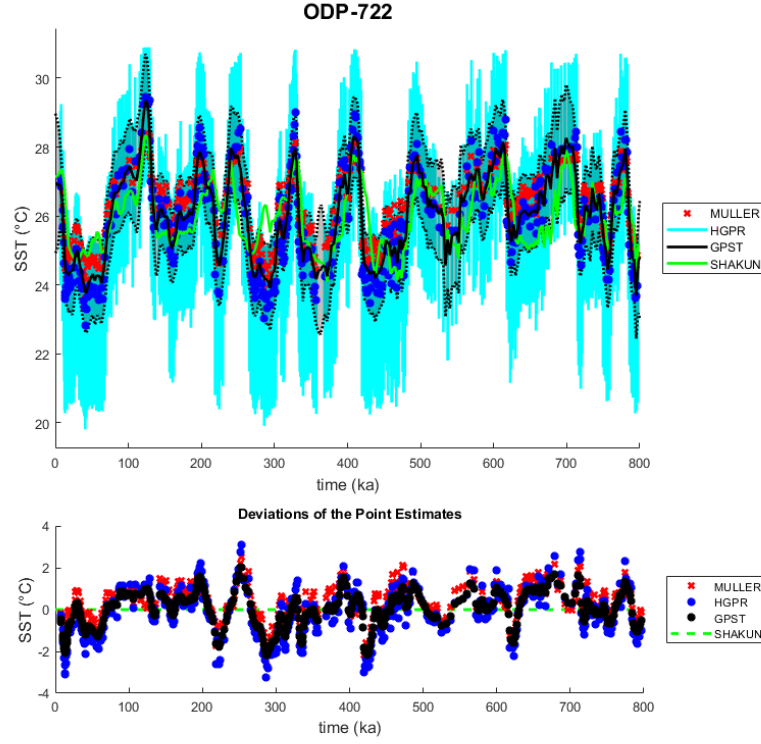


Figure S34: SST inference of the sediment core ODP-722. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^{Kx}$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).



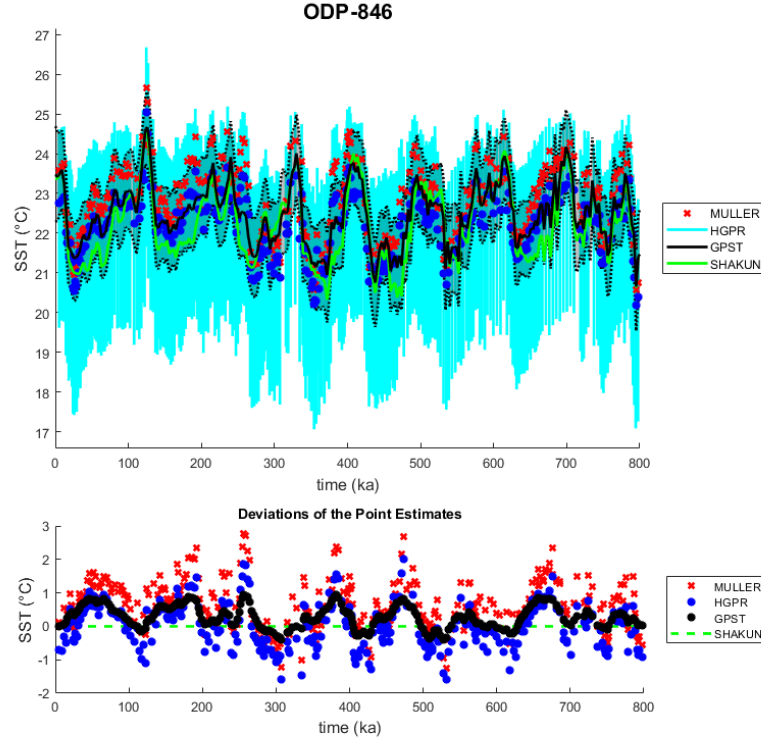


Figure S35: SST inference of the sediment core ODP-846. In the upper panel, the black shaded region and curve are the 95% confidence band and median of GPST SST inference, respectively. Blue bars and dots are those of point-wise SST inference, respectively. Red crosses are simple linear conversion of  $U_{37}^K$  proxies into SSTs. The green curve is the translated SST prior. In the lower panel, departures from the translated SST prior are shown for each type of SST estimations (medians).

Core	Variance (point-wise)	Variance (GPST)
IODP-U1417	3.7307	1.0430
ODP-1010	4.3621	1.5478
ODP-1021	3.8778	0.9835
ODP-1088	3.4479	1.4860
ODP-1125	4.4441	1.2378
ODP-1208	2.1641	0.7451
ODP-1241	1.3830	0.7534
ODP-722	2.1466	1.4622
ODP-846	0.6317	0.5230

Figure S36: Variances of the SST estimates (medians of the posterior samples).