Are Charge Carriers Using Performance Enhancing Dopants?

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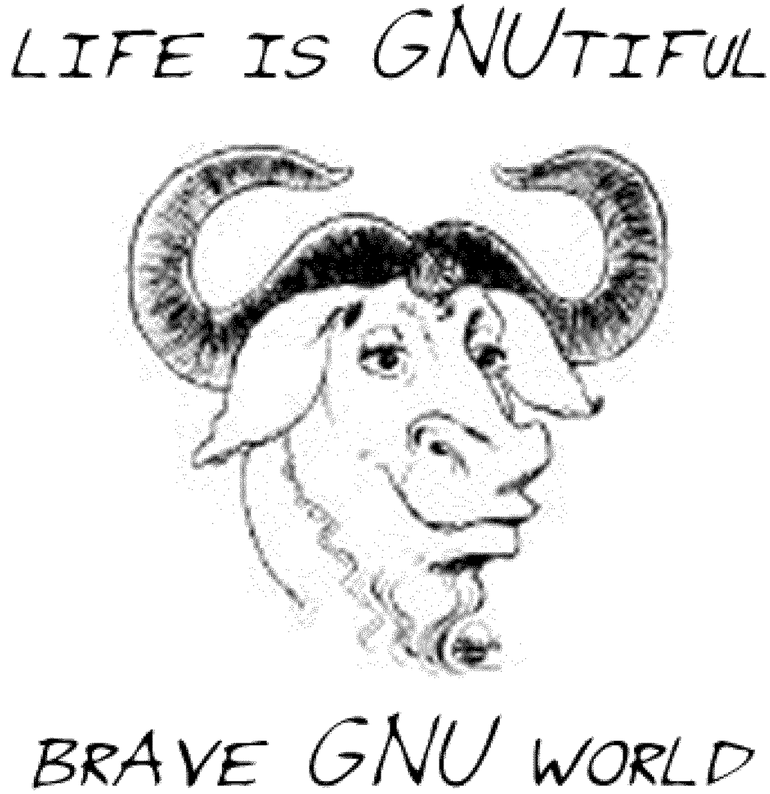

Figure 1: Brave Gnu World

Abstract

This document superceeds any previous document, and where contradictions arise is to be taken as the authoritative version. This document obseletes the scripts `hall_dimensions`, `hall_view_raw`, `hall_resistivity`, and `hall_voltage`. Current version is now in β release, suggestions for improvement are welcomed.

1 Rambling Introduction

Edwin Hall discovered the hall effect in the same year, 1879, as Einstein was born. He was a graduate student at the time. He then went to work as a professor at Harvard in the one of the first real physics laboratories in America. The Jefferson Laboratory as it was called was, like the Macdonald Laboratory here at McGill, built without any ferromagnetic material. Unfortuanately the Jefferson Laboratory was built using bricks which contained trace amounts of magnetic iron oxide, so all was for naught, Macdonald on the other hand was built from wood. With historical hind sight we can see that neither the failure nor the success of these ventures were critical due to the discoveries made by Rutherford, incidentally many of them in the Macdonald Laboratory. While what is currently refered to as modern physics was emerging into the light, Bell Labs was formed in New Jersey as AT&T and Western Electric's R&D merged. By 1937, Bell Labs' George Stibitz had created the world's first digital computer. Soon, at the Prinston Institute for Advanced Study, John Von Neumann a brilliant mathematician and theoretical physicist saw how this technology could be applied, and quickly designed the computer architecture which now bears his name, and is still the basis of most computer designs. In 1947 Haynes and Shockley, back at Bell Labs, were experimenting with semiconductors and created the first transistor. So, by 1969 transistorized computers were becoming relatively common place, once again at Bell Labs, Dennis Ritchie and Ken Thompson were busy creating both UNIX and the language C. Finally, in 1998, Horst Stormer of Bell Labs, and two former Bell Labs researchers, Robert Laughlin and Daniel Tsui, win the Nobel Prize for their discovery of the fractional quantum Hall effect.

In this experiment you will be studying the Hall Effect in a doped germanium semiconductor, you will be analysing your data on a digital computer with a Von Neumann based architecture with a Unix derived operating system using software written in C.

2 Data Collection

I think this part is pretty elementary. Here are a few guidelines:

2.1 Simple Filenames

Remember the data aquisition environment is braindead. It follows the 8.3 rule for filenames, there is a implicit dot in the name, before the dot there may be as many as eight non-dot characters, followed by a dot and then upto three non-dot characters. Additional characters beyond these limits will be ignored. So, use short, simple filenames.

2.2 Crashes

Due to the fact the DMM is being treated by DOS as a device, DOS will sometime crash the program if the DMM takes a long time to respond. The DMM filters the input signal, and makes multiple measurements, if the data seems noisy, it tries again. This is a feature, the next revision of the data collection software will likely be Linux based, and I'm sure life will be better. If the program crashes, just restart it with a new filename, to analysis program will concatenate the data fragments.

2.3 Data Transfer to Unix

This is covered in Hall Effect lab manual somewhere.

3 Sizing up the Enemy

First we must add the data processing tools to the exectutable path, maybe one day this will be automated.

```
[bofh@odin Hall_Effect]$ addpath labs
[bofh@odin Hall_Effect]$
```

I have stuffed my data into a directory, oddly enough called *data*.

```
[bofh@odin Hall_Effect]$ ls -l data
total 272
-rw-r--r--   1 bofh     daemon      84923 Oct 13 22:51 forward
-rw-r--r--   1 bofh     daemon      96683 Oct  6 21:26 nofield
-rw-r--r--   1 bofh     daemon      92297 Oct 13 22:51 reverse
```

```
[bofh@odin Hall_Effect]$
```

I guess the first useful task would be to make sure the data is what we think it is. The data is timestamped in a Y2K compliant manner, you can cross reference this with the entries in your log-book.

```
[bofh@odin Hall_Effect]$ hall_process -i data/*
data/forward: 918 points
Acquired over the interval: Fri Oct  4 12:46:48 1996 -Fri Oct  4 18:05:02 1996
Temperature range: 144.567 - 403.414 K
V[1] =   -2.605 -    0.343 mV V[2] =   -0.685 -    3.426 mV
V[3] =   27.443 -  122.508 mV V[4] =    9.998 -   10.045 mV
V[5] =   10.534 -   49.690 mV V[6] =   12.480 -   56.154 mV
V[7] =    9.872 -   39.903 mV V[8] =   -4.141 -    5.725 mV

data/nofield: 1048 points
Acquired over the interval: Thu Oct  3 17:40:59 1996 -Thu Oct  3 23:41:21 1996
Temperature range: 143.347 - 397.253 K
V[1] =   -0.087 -    0.119 mV V[2] =    1.375 -    5.190 mV
V[3] =   23.988 -  119.317 mV V[4] =    9.984 -   10.044 mV
V[5] =    9.378 -   48.606 mV V[6] =   10.816 -   54.521 mV
V[7] =    7.389 -   37.356 mV V[8] =   -4.172 -    5.426 mV

data/reverse: 1010 points
Acquired over the interval: Sat Oct  5 08:29:40 1996 -Sat Oct  5 14:19:29 1996
Temperature range: 128.373 - 394.284 K
V[1] =    2.514 -    3.258 mV V[2] =    3.498 -    8.389 mV
V[3] =   20.936 -  123.452 mV V[4] =    9.964 -   10.042 mV
V[5] =    8.039 -   50.383 mV V[6] =    9.081 -   56.083 mV
V[7] =    5.394 -   37.331 mV V[8] =   -4.566 -    5.282 mV

[bofh@odin Hall_Effect]$
```

These look to me like the data files of Lucky Tiger. The `-i` option means just show the info, do not perform the default action, which would not make a lot of sense in this case. Let us first examine the datafile `nofield`.

```
[bofh@odin Hall_Effect]$ hall_process data/nofield
data/nofield: 1048 points
Acquired over the interval: Thu Oct  3 17:40:59 1996 -Thu Oct  3 23:41:21 1996
Temperature range: 143.347 - 397.253 K
V[1] =   -0.087 -    0.119 mV V[2] =    1.375 -    5.190 mV
V[3] =   23.988 -  119.317 mV V[4] =    9.984 -   10.044 mV
V[5] =    9.378 -   48.606 mV V[6] =   10.816 -   54.521 mV
V[7] =    7.389 -   37.356 mV V[8] =   -4.172 -    5.426 mV

[bofh@odin Hall_Effect]$
```

```
[bofh@odin Hall_Effect]$ ls -l data/*
-rw-r--r--   1 bofh     daemon      84923 Oct 13 22:51 data/forward
-rw-r--r--   1 bofh     daemon      96683 Oct  6 21:26 data/nofield
-rw-r--r--   1 bofh     daemon       5026 Oct 20 11:42 data/nofield.1
-rw-r--r--   1 bofh     daemon       4903 Oct 20 11:42 data/nofield.2
-rw-r--r--   1 bofh     daemon       4799 Oct 20 11:42 data/nofield.3
-rw-r--r--   1 bofh     daemon       4902 Oct 20 11:42 data/nofield.4
-rw-r--r--   1 bofh     daemon       4908 Oct 20 11:42 data/nofield.5
-rw-r--r--   1 bofh     daemon       4879 Oct 20 11:42 data/nofield.6
-rw-r--r--   1 bofh     daemon       4885 Oct 20 11:42 data/nofield.7
-rw-r--r--   1 bofh     daemon      14036 Oct 20 11:42 data/nofield.8
-rw-r--r--   1 bofh     daemon      92297 Oct 13 22:51 data/reverse
[bofh@odin Hall_Effect]$
```

The data have been binned by temperature, using a default bin width of 1 K. Lets see how the temperature varied with time.

I created the following script which I called `T-vs-t.gnu`:

```
set nologscale
set xlabel "Time (minutes)"
set ylabel "Temperature (K)"
plot "<awk '{print $1/60,$2}' data/nofield.8" title ' ' with dots
```

I can look at the graph using the command

```
[bofh@odin Hall_Effect]$ gnuview T-vs-t.gnu
[bofh@odin Hall_Effect]$
```

Ok, perhaps even the more hardened veterans of my 339 lab are wondering "What on earth is that plot statement in the script!??" What I am doing here is using awk as a preprocessor for the graph data. The time data in `nofield.8` is in seconds, I am using awk to convert this into minutes. If the first character of a filename is a "<" then `gnuplot` executes the rest of the filename string as a command[1]

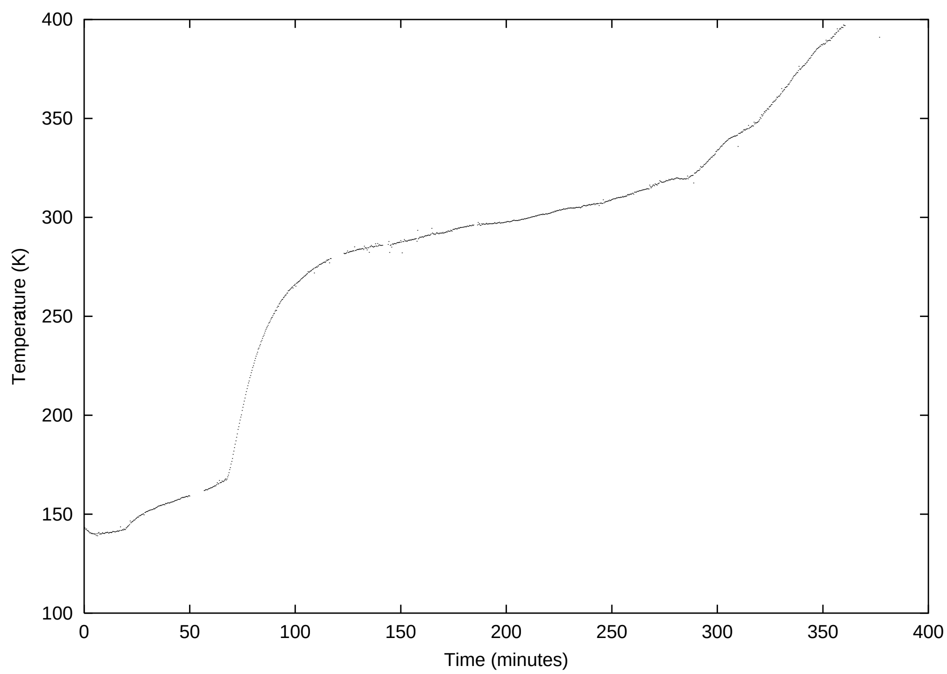

Figure 2: The Temperature of the Crystal During Data Collection

We can see that the temperature was changing rapidly between 175 K and 250 K. This may mean we need to use a larger bin width in this region to get statistical data.

Figure 3. is shown to clarify the various reference points. Notice that my labeling of the dimensions is different from that used by Melissinos!

[1] Actually to create `T-vs-t.gnu` I executed

```
qp -x "Time (minutes)" -y "Temperature (K)" "<awk 'print \$1/60,\$2' data/nofield.8" > T-vs-t.gnu
```

But I suspect most people didn't want to know that.

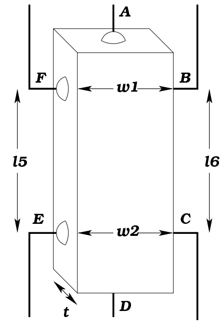

Figure 3: The Crystal

Let us now examine the voltages dropped between the connections E and F, and connections C and B. These are V_5, and V_6 respectively.

Another hastily concocted `gnuplot` script, `Vlat.gnu`

```
set nologscale
set xlabel "Temperature (K)"
set ylabel "Voltage Drop (mV)"
plot "<awk '{print $1,1000*$2,1000*$3}' data/nofield.5" using 1:2:3 title 'V5' with error, \
     "<awk '{print $1,1000*$2,1000*$3}' data/nofield.6" using 1:2:3 title 'V6' with error
```

As you can see the awk trickery has been used to convert volts into millivolts for easy of viewing.

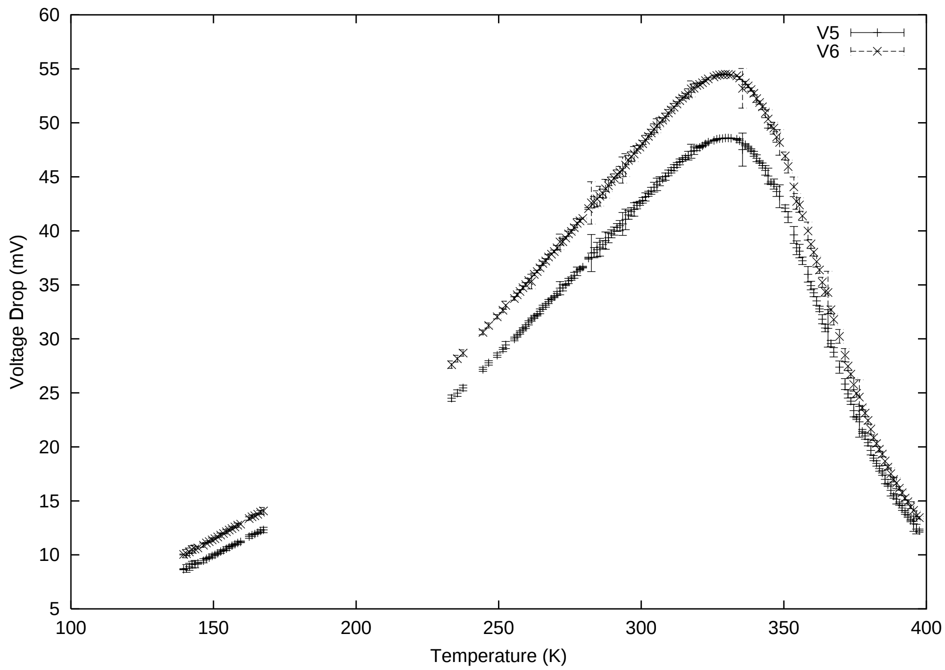

Figure 4: Voltages V_5 and V_6

As predicted there is a data deficit in the region where the temperature of the crystal was increasing rapidly. To compensate for this, we can increase the binning width.

```
[bofh@odin Hall_Effect]$ hall_process -b 5 -o data/nofield-b5 data/nofield
Setting binning width to 5 K
Using data/nofield-b5 as the output file
data/nofield: 1048 points
Acquired over the interval: Thu Oct  3 17:40:59 1996 -Thu Oct  3 23:41:21 1996
Temperature range: 143.347 - 397.253 K
V[1] =   -0.087 -    0.119 mV V[2] =    1.375 -    5.190 mV
V[3] =   23.988 -  119.317 mV V[4] =    9.984 -   10.044 mV
V[5] =    9.378 -   48.606 mV V[6] =   10.816 -   54.521 mV
V[7] =    7.389 -   37.356 mV V[8] =   -4.172 -    5.426 mV

[bofh@odin Hall_Effect]$
```

Modifying `Vlat.gnu` to `Vlat-b5.gnu`

```
set nologscale
set xlabel "Temperature (K)"
set ylabel "Voltage Drop (mV)"
plot "<awk '{print $1,1000*$2,1000*$3}' data/nofield-b5.5" using 1:2:3 title 'V5' with er-
ror, \
     "<awk '{print $1,1000*$2,1000*$3}' data/nofield-b5.6" using 1:2:3 title 'V6' with error
```

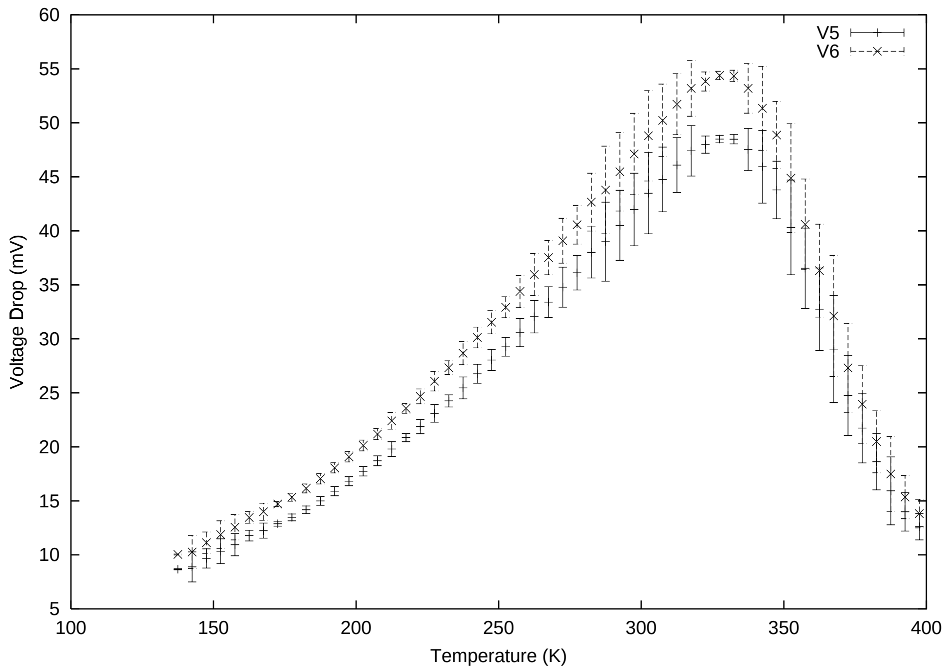

Figure 5: Voltages V_5 and V_6, (5 K bin size)

Good, the data in the sparse region has appeared, and looks reasonable. Observe however, that the error bars in the densely populated regions are now proportional to the slope of the graph. This is because our statistical analysis assumes that the contents of a particular bin are normally distributed about some average value. However in these regions, assuming the slope to be constant, the distribution will effectively be rectangular. What to do?! Your best option is to collect better data, if this is not possible then you have to choose some bin width which gives a reasonable trade-off, or perhaps analyse the data in a bitwise fashion.

4 But What About The Resistivity?

$$\rho = \frac{RA}{l} \tag{1}$$

Ohm's law states

$$R = \frac{V}{i} \tag{2}$$

where V is the voltage drop across the resistance due to the current i.
A can be determined from your measurements of the crystal. Off the top of my head I would say,

$$A = \frac{1}{2}\left[w_1 + w_2\right] t \tag{3}$$

$$A = \frac{1}{2}\left[2.73 \pm 0.01\text{mm} + 2.72 \pm 0.01\text{mm}\right] 0.85 \pm 0.01\text{mm} = 2.32 \pm 0.03\text{mm}^2 \tag{4}$$

So,

$$\rho = \frac{A}{li} V \tag{5}$$

So for V_5,

$$\rho_5 = \frac{2.32 \pm 0.03\text{mm}^2}{(3.27 \pm 0.01\text{mm})(1.00 \pm 0.01\text{mA})} V_5 = (0.71 \pm 0.01 \text{ mm mA}^{-1}) V_5 = (0.71 \pm 0.01 \text{ m A}^{-1}) V_5 \tag{6}$$

This sounds like a classic awk task.

```
[bofh@odin Hall_Effect]$ awk '{ print $1,0.71*$2,sqrt($2*$2*1e-4+0.5041*$3*$3)}'
 data/nofield-b2.5 > data/nofield-resistivity.5
[bofh@odin Hall_Effect]$
```

The following script displays this manipulated data,

```
set nologscale
set xlabel "Temperature (K)"
set ylabel "Resistivity (Ohm cm)"
plot "<awk '{print $1,100*$2,100*$3}' data/nofield-resistivity.5" using 1:2:3 title '' with error
```

The result is shown on figure 6

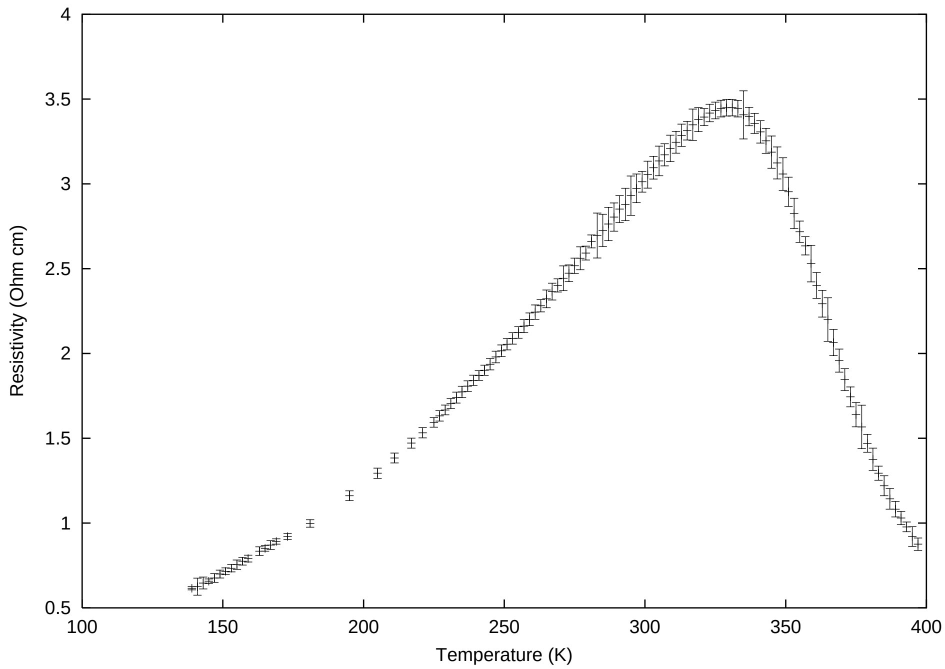

Figure 6: Resistivity of sample

4.1 Extrinsic Conduction

From figure 6. we are probably safe in assuming the majority of carriers are extrinsic below 280 K.

Once again we call for awk,

```
[bofh@odin Hall_Effect]$ awk '($1 < 280) {print $0}' data/nofield-resistivity.5
> data/extrinsic.5
[bofh@odin Hall_Effect]$
```

Since we want to have a log-log plot, but of a somewhat restricted range we need to do some tricks to make `gnuplot` make an acceptable plot.

```
set logscale xy
set xtics (150,175,200,250,300)
set ytics (.6,.8,1,2,3,4)
set xlabel "Temperature (K)"
set ylabel "Resistivity (Ohm cm)"
plot [140:300][.6:4] "<awk '{print $1,100*$2,100*$3}' data/extrinsic.5" using 1:2:3 title '' with e
```

Which produces

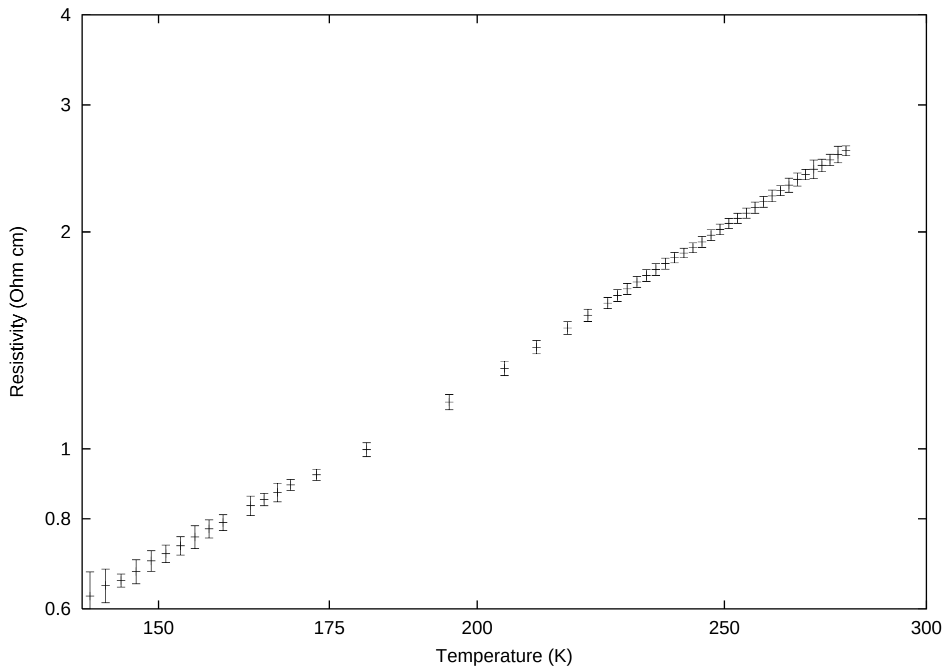

Figure 7: The Extrinsic Region

We can use `moosefit` to fit the data to a function of the form

$$\rho = \alpha T^{\beta} \tag{7}$$

I would use the following script, `extrinsic.moo`

```
function rho = alpha * T ^ beta
independent T
alpha = 3e-6
beta = 1.5
data "data/extrinsic.5"
```

Which can be loaded from the commandline

```
[bofh@odin Hall_Effect]$ moosefit extrinsic.moo
[bofh@odin Hall_Effect]$
```

Don't forget to set the wieighting to `instrumental`

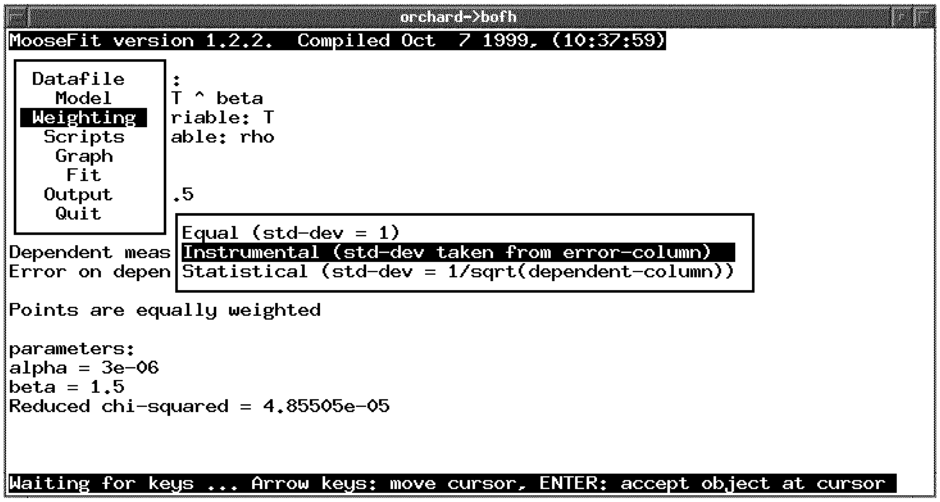

Figure 8: Set the weighting to instrumental

Actually do the fit, then you can save the result

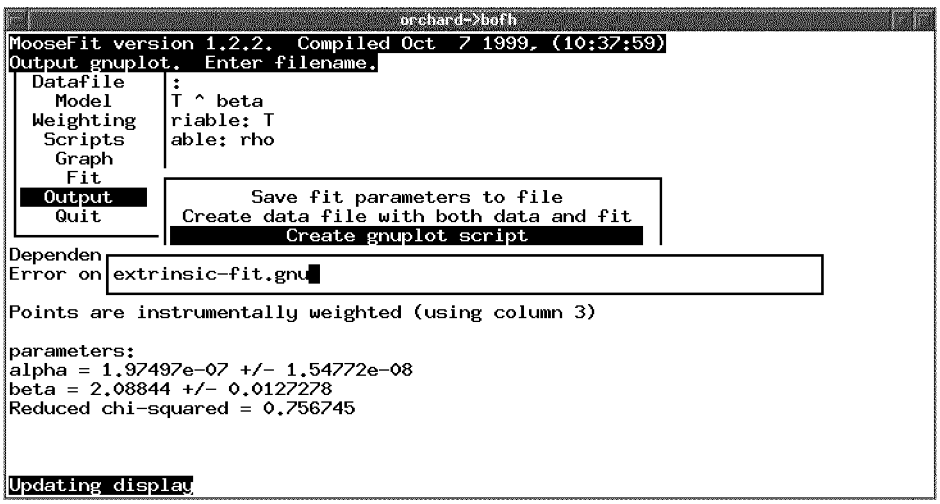

Figure 9: Save the graph

We can then edit this script to make it very similar to `extrinsic.gnu`, as follows

```
rho(T) = alpha * T ** beta

alpha = 1.97497e-07
beta = 2.08844

set logscale xy
set xtics (150,175,200,250,300)
set ytics (.6,.8,1,2,3,4)
set xlabel "Temperature (K)"
set ylabel "Resistivity (Ohm cm)"
plot [140:300][.6:4] \
  100 * rho(x) title 'fit', \
  "<awk '{print $1,100*$2,100*$3}' data/extrinsic.5" using 1:2:3 title 'data' with error
```

Which will be rendered as

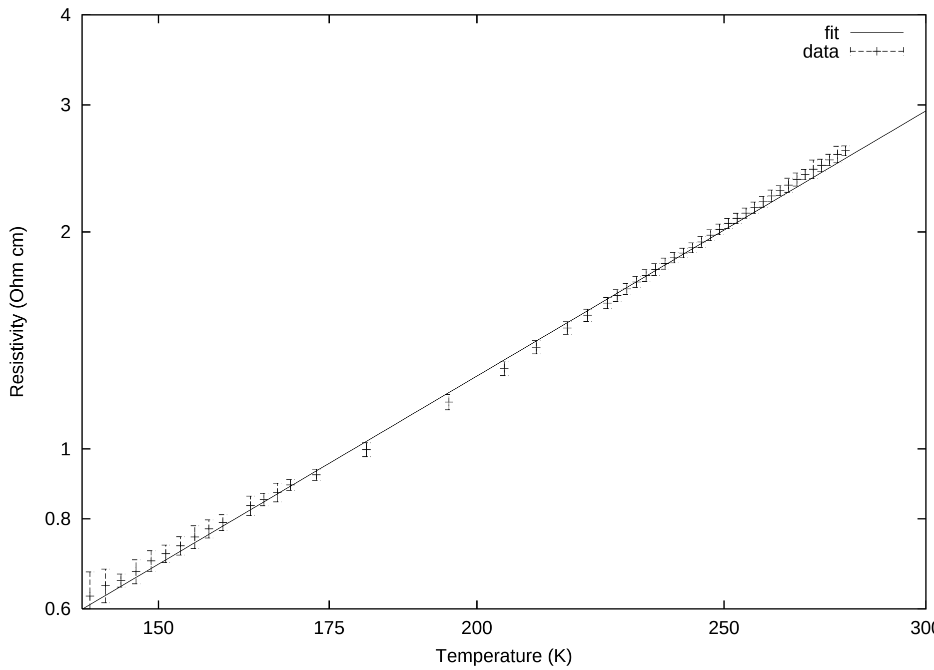

Figure 10: The fit to the data

Well that is a bit on the ugly side, in my humble opinion that is. As mentioned a few sections back, it wouldn't hurt to take good data, then reasonable arguments can be made. Initial celebrations based on the small χ^2 are ill founded, since the error bars are obviously too large, and there is a definite systematic difference between the data and the fit.[2]

4.2 Intrinsic Conduction

If you were thinking "let's use `awk`" you are catching on.

```
[bofh@odin Hall_Effect]$ awk '($1 > 350) {print $0}' data/nofield-resistivity.5
> data/intrinsic.5
[bofh@odin Hall_Effect]$
```

In the intrinsic conduction model we are looking for an expression of the form

$$\rho = Ce^{\frac{E_g}{2kT}} \tag{8}$$

Given $k = 1.3805 \times 10^{-23}$ J K^{-}1, and 1 eV = 1.602×10^{-19} J we can write the following `moosefit` script.

```
function rho = C * exp (5802 * Eg / T)
independent T
C = 1e-7
Eg = 0.700000
data "data/intrinsic.5"
```

Once again, the resulting `gnuplot` script can be hacked to make as follows

```
rho(T) = C * exp (5802 * Eg / T)

C = 7.67991e-07
Eg = 0.642238

set logscale y
```

[2] This is an example of a motivational technique — get the listener psyched up.

```
set ytics (.6,.8,1,2,3,4)
set xlabel "Temperature (K)"
set ylabel "Resistivity (Ohm cm)"
plot [][.6:4] \
  100 * rho(x) title ’fit’, \
  "<awk ’{print $1,100*$2,100*$3}’ data/intrinsic.5" using 1:2:3 title ’data’ with error
```

to generate the following graph

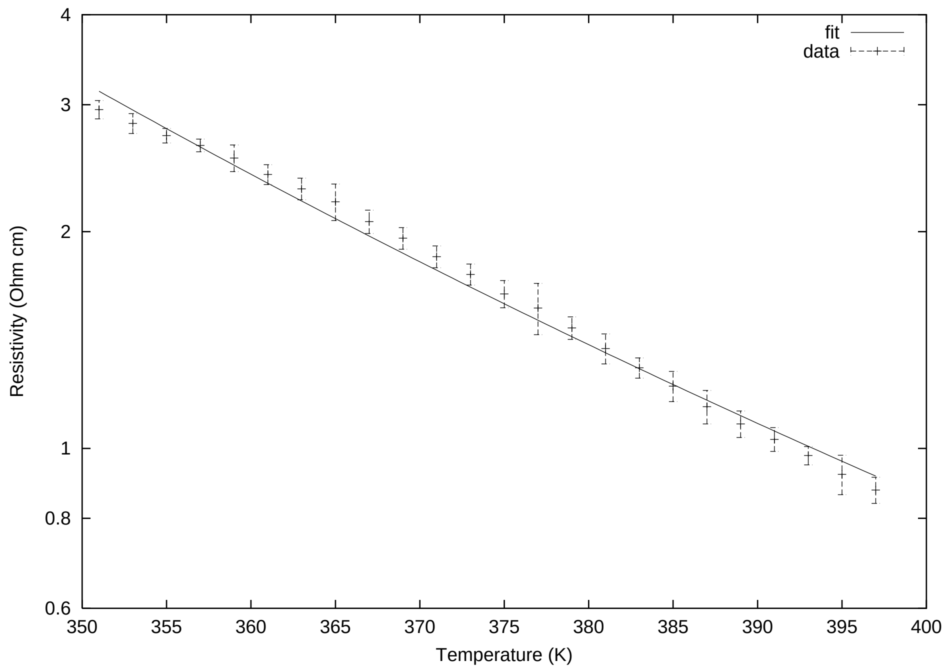

Figure 11: Resistivity in the Intrinsic Region

5 I Thought We Were Going To Look At The Hall Effect

Given two datafiles taken under the same conditions, excepting the orientation of the crystal within the magnetic field, we can do a binwise subtraction of the two datasets. For this to work, the two datasets should be identically binned by `hall_process`.

```
[bofh@odin Hall_Effect]$ hall_process -b 2 data/forward
Setting binning width to 2 K
data/forward: 918 points
Acquired over the interval: Fri Oct  4 12:46:48 1996 -Fri Oct  4 18:05:02 1996
Temperature range: 144.567 - 403.414 K
V[1] =   -2.605 -    0.343 mV V[2] =   -0.685 -    3.426 mV
V[3] =   27.443 -  122.508 mV V[4] =    9.998 -   10.045 mV
V[5] =   10.534 -   49.690 mV V[6] =   12.480 -   56.154 mV
V[7] =    9.872 -   39.903 mV V[8] =   -4.141 -    5.725 mV

[bofh@odin Hall_Effect]$ hall_process -b 2 data/reverse
Setting binning width to 2 K
data/reverse: 1010 points
Acquired over the interval: Sat Oct  5 08:29:40 1996 -Sat Oct  5 14:19:29 1996
Temperature range: 128.373 - 394.284 K
V[1] =    2.514 -    3.258 mV V[2] =    3.498 -    8.389 mV
```

```
V[3] =   20.936 -  123.452 mV V[4] =    9.964 -   10.042 mV
V[5] =    8.039 -   50.383 mV V[6] =    9.081 -   56.083 mV
V[7] =    5.394 -   37.331 mV V[8] =   -4.566 -    5.282 mV

[bofh@odin Hall_Effect]$
```

Then reverse field datafile can be subtracted from the forward field datafile,

```
[bofh@odin Hall_Effect]$ hall_subtract data/forward.1 data/reverse.1 | awk '{pri
nt $1,$2/2,$3/2}' > data/hall_voltage.1
data/forward.1: 145 - 403 K in 2 K bins
data/reverse.1: 135 - 393 K in 2 K bins
[bofh@odin Hall_Effect]$
```

Notice that awk is being used to divide the result by two.

```
set nologscale
set xlabel "Temperature (K)"
set ylabel "Hall Voltage (mV)"
plot "<awk '{print $1,1000*$2,1000*$3}' data/hall_voltage.1" using 1:2:3 title '' with error
```

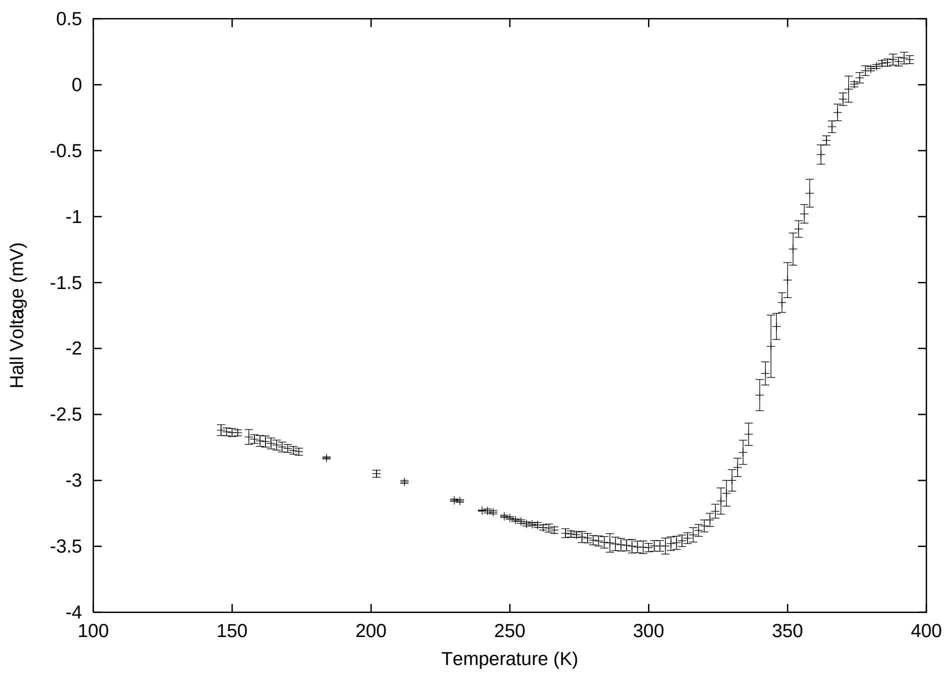

Figure 12: Hall Voltage

But wait, there is more, we can use a similar operation to divide this dataset in a binwise manner by the voltage due to resistivity, lets say for example V_5. A potentially sticky situation now arises, which V_5, from zero field, an average of forward and reverse fields? I'm goint to arbitrarily choose zero field.

```
[bofh@odin Hall_Effect]$ hall_process -b 2 -o data/zerofield data/nofield
Setting binning width to 2 K
Using data/zerofield as the output file
data/nofield: 1048 points
Acquired over the interval: Thu Oct  3 17:40:59 1996 -Thu Oct  3 23:41:21 1996
Temperature range: 143.347 - 397.253 K
V[1] =   -0.087 -    0.119 mV V[2] =    1.375 -    5.190 mV
V[3] =   23.988 -  119.317 mV V[4] =    9.984 -   10.044 mV
V[5] =    9.378 -   48.606 mV V[6] =   10.816 -   54.521 mV
V[7] =    7.389 -   37.356 mV V[8] =   -4.172 -    5.426 mV
```

```
[bofh@odin Hall_Effect]$ hall_quotient data/hall_voltage.1 data/zerofield.5 > data/hall_angle.1.5
data/hall_voltage.1: 146 - 394 K in 2 K bins
data/zerofield.5: 139 - 397 K in 2 K bins
[bofh@odin Hall_Effect]$
```

```
set nologscale
set xlabel "Temperature (K)"
set ylabel "Hall Angle"
plot "data/hall_angle.1.5" using 1:2:3 title '' with error
```

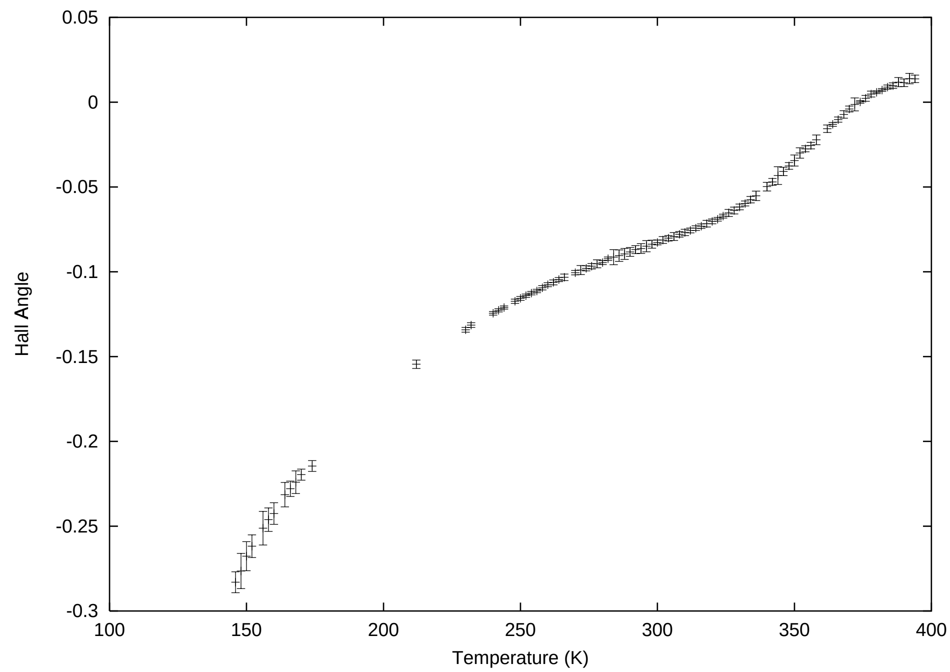

Figure 13: Hall Angle

This data might be more useful if it was converted to something physically meaningful, such as mobility. At this point I pause to rant about the content of the course hand-out. Thus far I have contained my disbelief at the log-log / log-lin plots of ρ vs $1000/T$, but let us examine the graph on page 96 of Appendix 1, and contrast it with the equation above it.

$$\mu_H = \phi \left(\frac{l}{w} \right) H^{-1} \tag{9}$$

The units of μ_H are given as cm^2 V^{-1} s^{-1}. The units of l and w cancel, and the ϕ is unitless which implies that the units of H would be V s cm^{-2} which is odd because these are actually units of magnetic flux density, normally denoted by a B, whereas H is usually denotes magnetic field intensity (A m^{-1}), and

$$\mathbf{B} = \mu \mathbf{H} \tag{10}$$

where μ is the permeability of the medium in which the magnetic field is present [3], and has absolutely no relation to the μ_H refered to in equation 9.

So, what we should have is

$$\mu_H = \phi \frac{l}{w} B^{-1} \tag{11}$$

[3] μ_0 the permiability free space is $4\pi \times 10^{-7}$ H m^{-1} so this is not an insignificant difference!

The magnetic flux density was measured as 3.3 ± 0.1 kilogauss, or 0.33 ± 0.01 T

$$\mu = \phi \frac{3.27 \pm 0.01 \text{ mm}}{2.73 \pm 0.01 \text{ mm}} \left(0.33 \pm 0.01 \text{ T}\right)^{-1} = 3.6 \pm 0.1 \text{ V s m}^{-2} \tag{12}$$

What? You want to use `awk`? I was going to suggest we do something in MicroSoft QuickBASIC, but if you insist ...

```
[bofh@odin Hall_Effect]$ awk '{print $1,3.6*$2,sqrt(13.175*$3*$3+$2*$2*.0124)}'
data/hall_angle.1.5 > data/mobility.1.5
[bofh@odin Hall_Effect]$
```

Added a couple of enhanced postscript commands to a `gnuplot` script just to keep things interesting

```
set logscale xy
set xtics (150,175,200,250,300,350,400)
set ytics (.01,.02,.05,.1,.2,.5,1,2)
set xlabel "Temperature (K)"
set ylabel "Hall Mobility (m{^2} V^{-1} s^{-1})"
plot [140:400][.02:1] \
  "<awk '{print $1,$2,$3}' data/mobility.1.5" using 1:2:3 title "+ve mobility" \
  with error pt 9 ps 0.75, \
  "<awk '{print $1,-$2,$3}' data/mobility.1.5" using 1:2:3 title "-ve mobility" \
  with error pt 11 ps 0.75
```

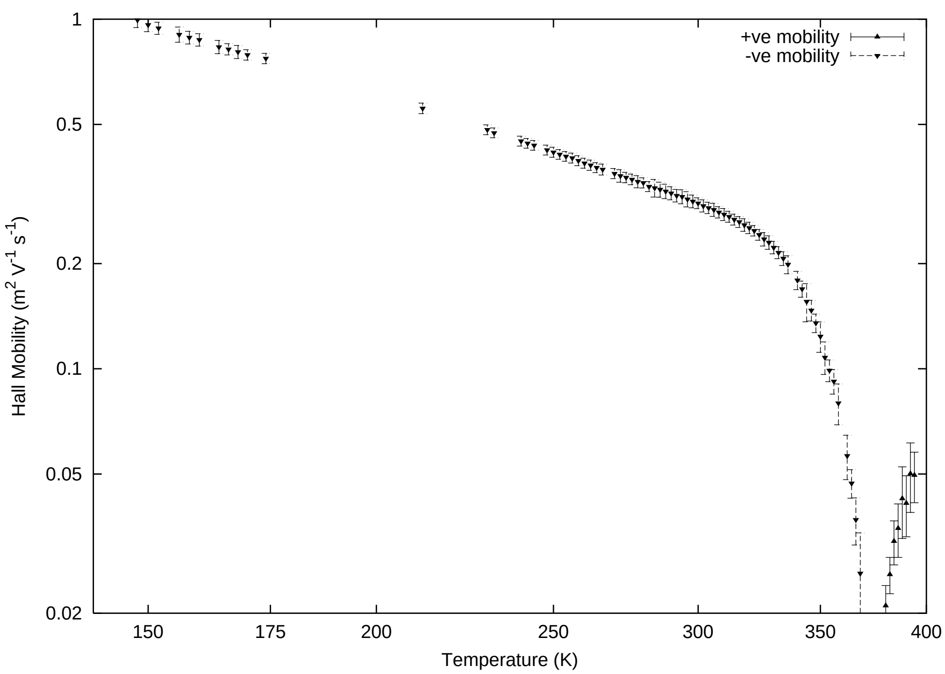

Figure 14: The Hall Mobility

Well, I think that is probably enough information to get you past the mine fields, and on to the road to ruin.