The Hall Effect and Properties of Semiconductors

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Abstract

We investigated the Hall effect dependance on temperature in a germanium crystal. We confirmed that the conductance in the sample follows a power law in the intrinsic regime with $\chi_2^2 = 0.82$, at low temperatures, and an exponential in the extrinsic regime, at high temperatures. From the exponential fit we found band gap energy of germanium to be $E_g = 1.2(5) \times 10^{-19}\,\mathrm{J}$. The Hall coefficient follows the same trends, and this shows that the negative charge carrier density of germanium decreases as temperature increases in the intrinsic regime, and increases with temperature in the extrinsic regime. We also verify that the magnetoresistance of germanium follows a quadratic law with $\chi_3^2 = 0.902$.

Contents

1 Introduction

Some very interesting characteristics of crystals can be analyzed experimentally. Such characteristics are the resistivity of a crystal as a function of temperature, the hall voltage (this potential difference created by the separation of charges in the crystal when a magnetic field is applied on the crystal) as a function of temperature and how the intensity of the magnetic field affects the resistance of the crystal. In this experiment we will choose the germanium semiconductor as our crystal to analyze.

In order to analyze this characteristics we need to understand the theory behind them. The electrons in the semiconductor have access to different levels of energy. In semiconductors, these levels are grouped in bands: The Conduction Band and the Valence Band. The seperation between them is the energy gap. For insulators, electrons are in the Valence Band, the energy gap between Conduction and Valence Band is large and leaves no possibility for electrons to move around. In semiconductors, the energy band gap is much smaller and the majority of the electrons will be in the valence band but some will reach the conduction band and be charge carriers. For carriers that are negatives with density n and charge e for each charge, the current density $\mathbf{J}$ relates to the mean velocity $\mathbf{v}$ of the charge carriers by : $\mathbf{J} = en\mathbf{v}$. It also relates to the conductivity $\sigma = {}^{1}\!/\!_{\rho}$ by the equation $\mathbf{J} = \sigma\mathbf{E}$. The mean velocity $\mathbf{v}$ and the mean free path λ can be defined and we observe that λ goes like ${}^{1}\!/\!_{kT}$[**?**]. Combining those results, for semiconductors, it can be shown that the mean velocity $\mathbf{v} \approx \frac{e\lambda\mathbf{E}}{2\sqrt{3kTm}}$, where λ is the mean free path of the carriers, k is he Boltzmann constant, T is the temperature of the sample and m is the effective carrier mass. Using this relation and the fact that λ goes like ${}^{1}\!/\!_{kT}$ we find that the resistance R is related to the temperature T with:

$$R \propto T^{3/2} \rightarrow G \propto T^{-3/2} \tag{1}$$

This is the extrinsic conduction. However this equation is valid only if the number of carriers remains constant. At higher temperatures, we reach the limit where the carrier density varies with temperature. In this case the conductivity follows Maxwell distribution:

$$R \propto e^{a/T} \rightarrow G \propto e^{Eg/2kT}, \tag{2}$$

where E_g is the band-gap energy.

This is the intrinsic conduction. The conductance increases as electrons are able to reach the conduction band.

Before introducing next characteristic we are interested in the experiment, a common physical quantity, the mobility μ of the charge carriers, is defined as $\mu = \frac{\mathbf{v}}{\mathbf{E}} = \frac{\sigma}{ne}$. When a magnetic field is applied while current is driven through the sample, a force is induced separating the carriers in such a way that a potential difference is created across the sides of the sample. This potential is called the Hall Voltage V_H. This effect can also be quantified by the Hall coefficient defined to be

$$R_H = \frac{E_y}{J_x B} = \frac{V_H d}{IB} = \frac{1}{ne}, \tag{3}$$

whereE_y is the magnitude of the electric field created by the charge pile-up , $J_x$is the current density in the sample, B is the magnitude of the magnetic field and d is the thickness of the sample (for a rectangle model perpendicular to the field). Another way to quantify the effect is the Hall angle

$$\phi_H = \frac{V_y}{V_x} = \frac{E_y w}{E_x l} = \frac{w}{l}\mu_H B, \tag{4}$$

where V_x is the Hall voltage drop across a width w on the sample, and V_y is the voltage drop across a length l on the sample. E_x and E_y are the corresponding electric fields and μ_H corresponds to the mobility.

In a semiconductor the charge carriers can be negative carriers (carrier density n) or positive carriers (carrier density p). The force induced by the magnetic field, thus, push them in opposite direction along a $y$direction according to the sign of the carriers. Realizing that the net current must be zero and using the previous definition we can obtain an equation for the Hall coefficient in terms of the mobilities and the carriers densities:

$$R_H = \frac{\mu_+^2 p - \mu_-^2 n}{e(\mu_+ p - \mu_- n)} \tag{5}$$

These mobilities very with temperature. When $p > n$ we expect R_H to change sign at some temperature (as $p$increases and n decreases with increasing temperature). Such semiconductors are said to be p-type. Semiconductors which has $n > p$ are said to be n-type.

The resistance of the germanium sample is expected to change in the presence of a magnetic field because the magnetic force shifts electrical charges inside the sample. This affects the charge density, which is not uniform anymore, and this, in turn, affects the resistance. The difference between the resistivity in the presence of a magnetic field, ρ, and without a field, ρ_0, is defined as $\Delta\rho \equiv \rho - \rho_0$. For large values of B, $\Delta\rho$ is expected to have a quadratic dependence on B[**?**]

$$\Delta\rho \propto B^2. \tag{6}$$

Then, $K(R - R_0) = CB^2$, $R - R_0 = CB^2$ (K is absorbed in C), and $R = CB^2 + R_0$. Thus, the resistance R is expected to follow the form

$$R = C\cdot B^2 + D. \tag{7}$$

2 Function of conductance against temperature

We can show that the resistance R of a germanium crystal varies with temperature T as described in Eq. 1 at low temperatures and 2 at high temperatures. The setup used is shown in Fig. 1. We use a germanium sample connected to a thermocouple that measures the temperature of the crystal. The sample is cooled down with liquid nitrogen to approximately 150 K. The temperature increases to room temperature and then a DC power supply is used to power a heater connected to the sample to increase the temperature up to 400 K. During the experiment a current source supplies a constant current of 1mA to the crystal. A digital multimeter (DMM) reads the potentials from the sample holder, as well as the sample's temperature. These values are displayed on the DMM's screen and are recorded on a computer.

The sample's conductance was plotted against temperature. At low temperature when the conduction is mainly due to extrinsic process the number of carriers is constant and we expect it to follow a power law $G \propto T^{-3/2}$ (see Eq. 1). At high temperature, the carrier

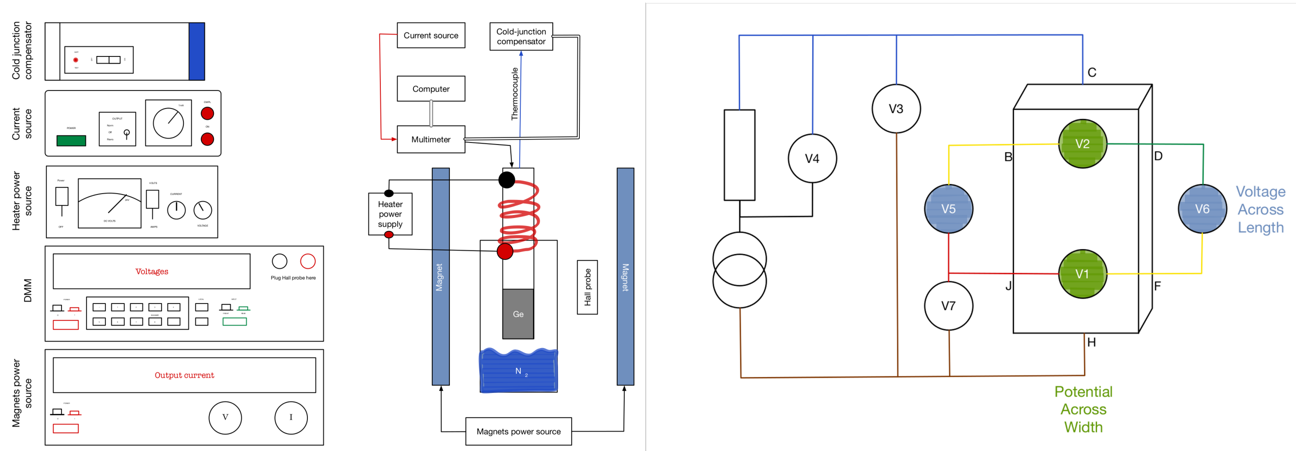

Figure 1: Schematic of the apparatus for the experiment. On the left, the dials of each of the relevant parts, in the center, the connections between parts and position of the sample, and on the right the circuit embedded on the sample holder. The sample is connected to the voltmeter, which feeds the potential measurements to the computer, and supplies the current to the sample. The thermocouple labelled is embedded into the sample holder, and connected to the voltmeter via a cold-junction compensator.

density varies with temperature and the conductance follows $G \propto e^{b/T}$ (see Eq. 2). We fitted the graph of conductance against temperature in the low temperature region with a function of the form $a \times (x - x_0)^{3/2}$, with parameters a and x_0. We also fitted the graph in a high temperature region with a function of the form $a \times e^{bx}$, with parameters are a and b.

The χ_2^2 for the power fit was found to be $\chi^2_{V_5} = 0.82$ for V_5 and $\chi^2_{V_6} = 0.85$ for V_6, indicating that G does follow a power law at low temperatures. The parameters for the exponential fit were, for V_5 : $a = 2.5(3) \times 10^{-4}\,\Omega^{-1}$, $b = 4.54(5) \times 10^3\,\mathrm{K}$ and the χ_2^2 is $\chi^2_{V_5} = 1.07$ and $\chi^2_{V_6} = 1.03$ for V_6, and are plotted in Fig. 2.

At high temperature the conductance follows maxwell equation $G \propto e^{\frac{E_g}{2kT}}$, where E_g is the band gap energy. In order to find a value for the band gap energy, we use the fit of the Conductance versus temperature graph in the high temperature region where the fit is exponential. Comparing the two equation we can see that $E_g = b \times 2k = 1.2(5) \times 10^{-19}\,\mathrm{J}$, where $b = 4.54(5) \times 10^3\,\mathrm{K}$.

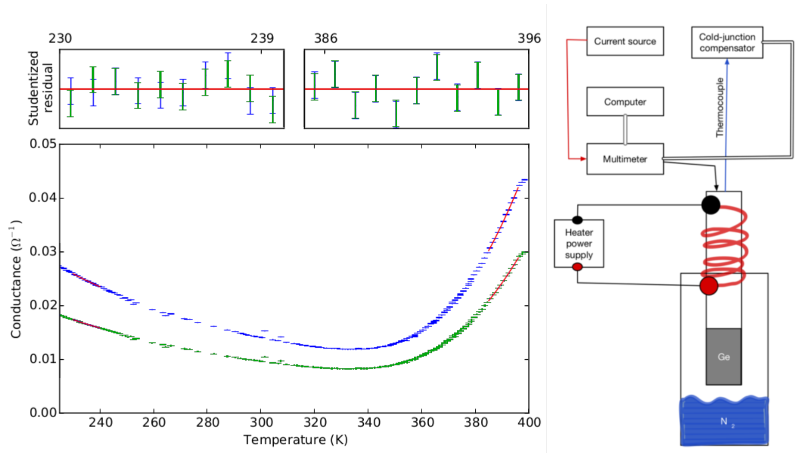

Figure 2: Fits at low and high temperature region for graphs of conductance versus temperature when V_5 (blue) is applied and when V_6 (green) is applied with the parameters of the exponential fit $a = 2.5(3) \times 10^{-4}$ and $b = 4.54(5) \times 10^3$ for V_5 and $a = 4.54(5) \times 10^3$ and $b = 4.48(5) \times 10^3$ for V_6. On the right is the way the apparatus is connected for this section of the experiment.

3 Hall coefficient as a function of Temperature

The next step in the experiment is to determine the Hall coefficient of the germanium sample as a function of temperature. This is done by repeating the previous measurements, with the magnets powered to supply uniform magnetic field of 500 mT, as shown on the right in Fig. 3. The Hall coefficient is computed from V_5 using Eq. 3, and the Hall mobility from the manufacturer given sample characteristics (Insert reference here) using Eq. 4. As shown on the plot, the Hall coefficient is decreasing in the extrinsic regime, meaning the number of negative charge carriers is increasing with temperature, as expected in Eq. 5. In the intrinsic regime, the number of positive charge carriers is increasing.

4 Magneto-resistance against magnetic field intensity

In this section, we seek to determine how the resistance of the germanium sample is affected by the magnetic field. We put the sample inside the magnet. There is a constant current of 1 mA passing through the sample. We record the potential of the Hall probe V_{probe}, and the potential V_5 (see Fig. 1). We varied the magnetic field from 50 to 500 mT. The resistance R, we divide V_5 by the current of 1 mA. The magnetic field B depends on V_{probe} according

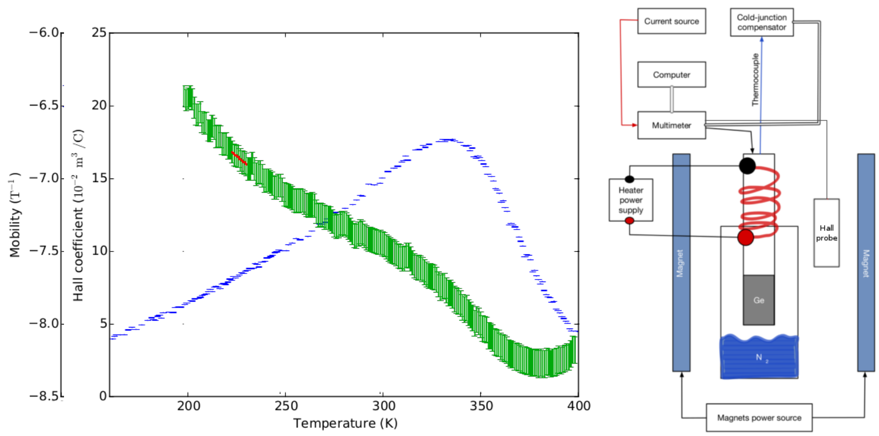

Figure 3: Hall coefficient (blue) and mobility (green) against temperature, and the way the apparatus is connected during this section of the experiment. The magnets are on, and the Hall probe is used to measure the magnetic field.

$B = \gamma + \delta \cdot V_{probe}$. Our probe was calibrated prior to the experiment, and has parameters $\gamma = 9.62$ mT, and $\delta = 1.8949$ T/V. The plot of the R against B is displayed in Figure 4.

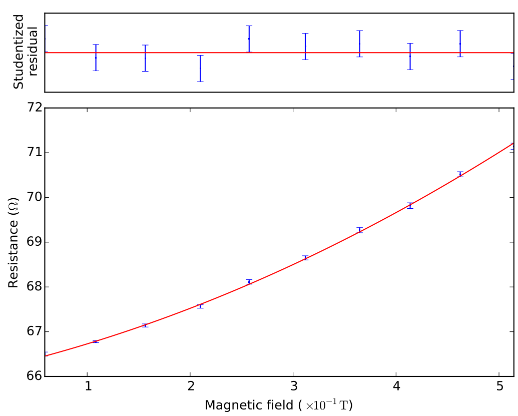

Figure 4: The resistance R (Ω) against the magnetic field B (10^{-1}T) of the germanium sample. R increases quadratically with B. The results of the fit are shown in Tab. ??.

We used a fit of the form $R = a(B - B_0)^2 + b$, where B_0 is a constant. The parameter values and χ^2_3 are written in Table ??. The χ^2_3 is 0.902, thus we conclude that the resistance of germanium does indeed depend on the magnetic field quadratically.